Inspecting the Mechanism:

Leverage and the Great Recession in the Eurozone*

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PRELIMINARY

Abstract

We provide a first comprehensive account of the dynamics of Eurozone countries from the creation of the Euro to the Great recession. We model each country as an open economy within a monetary union and we analyze the dynamics of private leverage, fiscal policy and capital flows. We show that our parsimonious model can replicate the time-series for output, employment, and net exports of Eurozone countries between 2000 and 2012. We then use the model to perform counterfactual experiments. To do so we propose a new identification strategy using the dynamics of U.S. states as a control group that did not suffer from sudden stops of capital flows. We then ask how periphery countries would have fared if they had followed more conservative fiscal policies during the boom. We find that periphery countries could have stabilized successfully their employment. For Greece, even an a-cyclical policy during the boom would have made a large difference during the bust. For Ireland, Spain and Portugal, the policy would have had to be very conservative during the boom. For Ireland, given the size of the private leverage boom, such a policy would have required buying back almost all of the public debt. This suggests that fiscal policy is unlikely to be sufficient as a stabilization tool in case of a large private credit boom and sheds light on the value of macro-prudential regulations.

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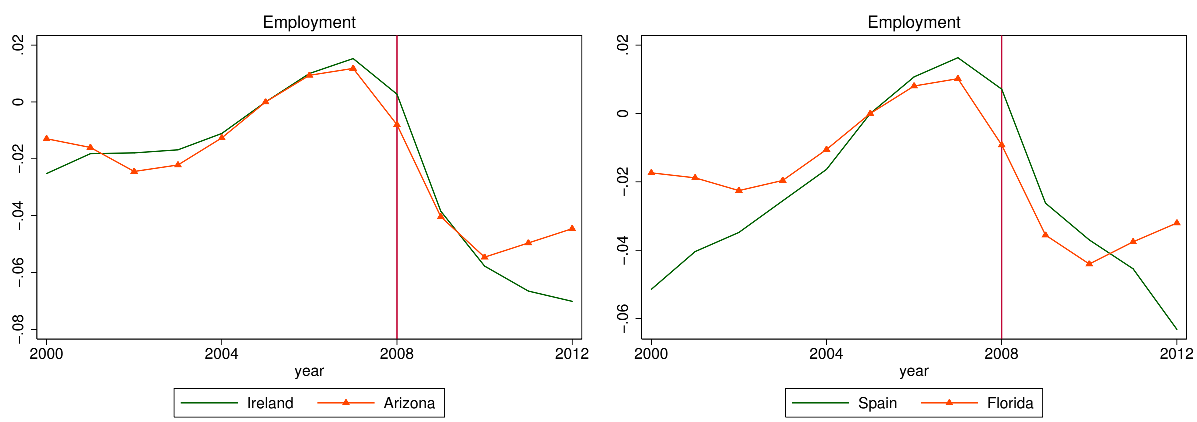
There is wide disagreement about the nature of the eurozone crisis. Some see the crisis as driven by fiscal indiscipline or fiscal austerity, others by external imbalances and sudden stops, others by excessive private leverage. Most observers understand that these "usual suspects" have played a role but without a quantification of their respective importance and without some understanding of how they relate to each other, it is difficult to frame policy prescriptions on macroeconomic policies and on reforms of the eurozone. Moreover, given the scale of the crisis, understanding the dynamics of the Eurozone is a major challenge for macroeconomics. A key reason for this disagreement is that we lack a theoretical and empirical framework that would enable us to identify and quantify the importance of these mechanisms and ultimately to run counterfactual experiments. The objective of this paper is to make progress on these fronts. To do this we propose a simple model that allows to inspect the mechanisms at work by focusing on three types of shocks: household leverage, fiscal policy and interest rate spreads. A key challenge is then to empirically identify private leverage shocks that are orthogonal to shocks on fiscal policy and shocks on spreads. To help us identify the eurozone shocks, we use the US as a benchmark. The US experience is of great use for us because of both its similarities and its differences with the eurozone experience.

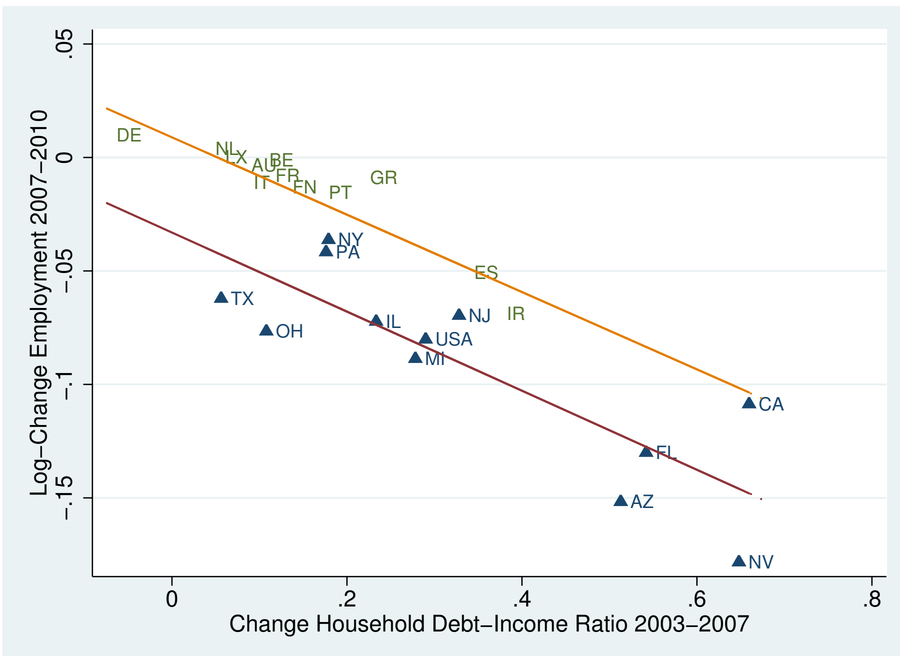
Figure 1: Employment Rates in Ireland, Arizona, Spain and Florida.



To illustrate this, we take the example of Nevada and Ireland because their increase in household debt to income ratio during the boom years were very large and similar. Figure 1 which shows deviations of the employment rates (normalized to zero in 2005) is striking. The employment boom and bust are almost identical up to 2010 but diverges after that. This suggests that the fundamental mechanisms at work in both regions were also similar up to 2010. But Nevada stands apart from Ireland in that it did not experience a sudden stop after 2010 because there was no concern on its public debt let alone on its remaining in the dollar zone. A similar, although less striking, pattern emerges when we compare Spain and Florida. Again, divergence is clear after 2010.

A salient feature of the great recession in both the US and the eurozone is that regions that have experienced the largest swings in household borrowing have also experienced the largest declines in employment and output. Figure 2 illustrates this feature of the data, by plotting the change in employment during the credit crunch (2007-2010) against the change in household debt-to-income ratios during the preceding boom (2003-2007) for the largest US states[1] and Eurozone countries.

Figure 2: First Stage of the Great Recession: Household Borrowing predicts Employment Bust in the US and the EZ

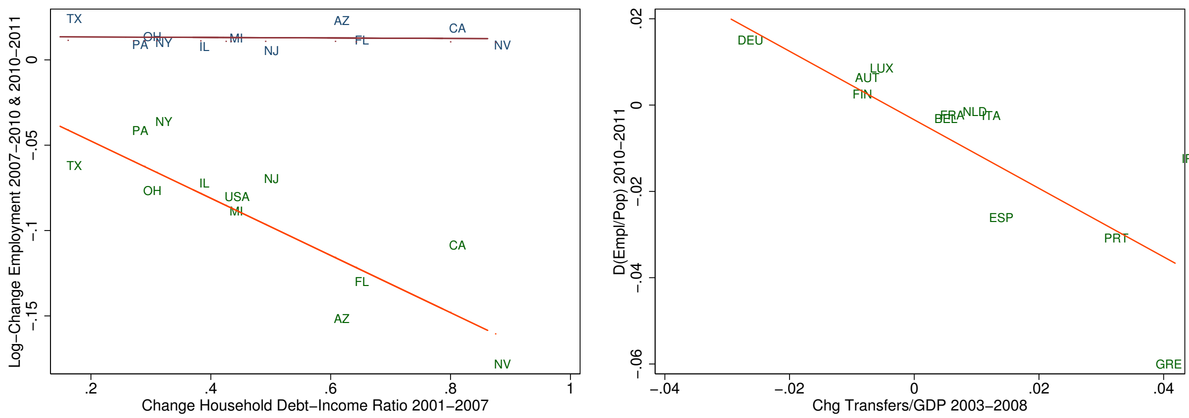


The American and European cross-sectional experiences look strikingly similar in this respect on the period 2007-2010. This suggests that the shock faced by these two economies were similar in nature on that period. Moreover this suggests that the structural parameters that govern the way the economy reacts to a deleveraging shock may also be similar in the two monetary zones.

The key difference between the US and the eurozone experience is the sudden stop in capital flows starting in 2010 in the later. The eurozone stands apart from the US but also historically as we do not know of any other historical example of a sudden stop among countries or states inside a monetary union although sudden stops have been frequent in the 19th and 20th centuries (see Accominotti and Eichengreen (2013)). Contrary

[1]State level household debt for the US comes from the Federal Reserve Bank of New York, see Midrigan and Philippon (2010).

Figure 3: Second Stage of the Great Recession: Fiscal Imbalances Predict Double Dip in Europe



to the eurozone, the US states did not experience any shock on spreads in borrowing costs and no fear an a potential exit of the dollar zone. This allows us, for the eurozone, to identify the part of the private deleverage dynamics that is not due to the spreads shocks by the private deleveraging predicted in the US on the period 2008-2012. We call this the "structural" private leverage shock.

Figure 3 illustrates the differences between the American and European experiences during the later stage of the recession. Starting in the Spring of 2010, sovereign spreads widen and several European countries find it difficult to borrow on financial markets. The US and EZ experiences then start to diverge. While US states grow (slowly) together, eurozone countries experience drastically different growth rates and employment. A state variable that correlates well with labor markets performance in 2010-2011 in the Eurozone is the change in social transfers during the boom. Eurozone countries where spending on transfers (and also government expenditures) increased the most from 2003 to 2008 are those that are now experiencing severe recessions in the later stage. This suggests that in the second stage past fiscal policy, because of its effect on accumulated debt, had an impact on the economy through spreads and the constraint on fiscal policy it generated after 2010. This is an hypothesis we will analyze.

As noted in Midrigan and Philippon (2010), the pattern of figure 2is at odds with the predictions of standard models of financing frictions. Such models predict that a tightening of borrowing constraints at the household level leads to a decline in consumption but, due to wealth effects, to an increase in the supply of labor. In this paper, we analyze a model where borrowing limits on "impatient" agents drive consumption, income, the saving decisions of "patient" agents and employment in small open economies belonging to a monetary union. We introduce nominal wage rigidities which translate the change of nominal expenditures into employment. We first consider the predictions of the model taking as given the observed series for private debt, fiscal policy and interest rate spreads between 2000 and 2012. This reduced form simulation reproduces very well what was observed across Eurozone countries both during the boom during the recession starting in 2008 for employment, nominal GDP, consumption and wages. We then identify structural shocks for household debt, interest rate spreads and fiscal policy using the US experience to predict the household debt shock. Finally we feed our model with these structural shocks and run counterfactual experiments on fiscal policy and macro prudential policies. This allows us to analyze whether a more countercyclical fiscal policy or policy that would have restricted private lending would have weakened the boom and bust cycle.

We find that private leverage is the key driver of both the boom and the bust dynamics. This is especially true in Ireland. An exception is Greece for which fiscal policy is the main driver of macroeconomic dynamics. Spain is an intermediate case. We ask how periphery countries would have fared if they had followed more conservative fiscal policies during the boom than they actually pursued. Such policies would have reduced the spreads and fiscal austerity during the bust. We find that periphery countries would then have stabilized their employment. This is especially true for Greece and Ireland, less so for Spain and Portugal. For Ireland however such policy would have entailed entering the bust with no public debt. This suggests that fiscal policy alone cannot act as a stabilization tool in case of a massive private credit boom. This sheds light on the importance of macro-prudential policy.

Relation to the literature

Our paper is related to three lines of research: (i) macroeconomic models with credit frictions, (ii) monetary economics, (iii) sudden stops and sovereign defaults. We discuss the connections of our paper to each topic. Following Bernanke and Gertler (1989), many macroeconomic papers introduce credit constraints at the entrepreneur level (Kiyotaki and Moore (1997), Bernanke et al. (1999), or Cooley et al. (2004)). In all these models, the availability of credit limits *corporate* investment. As a result, credit constraints affect the economy by affecting the size of the capital stock. Curdia and Woodford (2009) analyze the implication for monetary policy of imperfect intermediation between borrowers and lenders. Gertler and Kiyotaki (2010) study a model where shocks that hit the financial intermediation sector lead to tighter borrowing constraints for entrepreneurs. We model shocks in a similar way. The difference is that our borrowers are households, not entrepreneurs, and, we argue, this makes a difference for the model's cross-sectional implications. Models that emphasize firm-level frictions cannot reproduce the strong correlation between household-leverage and employment at the micro-level, unless the banking sector is island-specific, as in the small open economy "Sudden Stop" literature (Chari et al. (2005), Mendoza (2010)). This "local lending channel" does not appear to be operative across U.S. states, however, presumably because business lending is not very localized[2]. Our framework is also related to heterogeneous-agent macroeconomic models such as Krusell and Smith (1998), and models in the tradition of Campbell and Mankiw (1989), that feature impatient and patient consumers. This type of models has been used by Gali et al. (2007) to analyze the impact of fiscal policy on consumption and by Eggertsson and Krugman (2012) to analyze macroeconomic dynamics during the Great Recession.

Papers in the sudden stop literature have aimed at reproducing the stylized facts of these crises in emerging markets. According to Korinek and Mendoza (2013) the key characteristics of a sudden stop are 1) a sharp, sudden reversal in international capital flows, which is typically measured as a sudden increase in the current account 2) a deep recessions and 3) sharp changes in relative prices, including exchange rate depreciations. The eurozone crisis shares the two first characteristics even if the pace of current account adjustment in the euro area is slower than for non euro area countries (such as Bulgaria, Latvia and Lithuania) and past experiences of emerging markets crises (see Merler and Pisani-Ferry (2012) for a discussion). Substitution of private-capital inflows by public inflows, especially Eurosystem financing, partly explains this difference. The third characteristic of an emerging market sudden stop has been absent in the eurozone crisis: there has been (so far) no currency depreciation and no sudden and large change in goods relative prices between countries hit at different degrees by a sudden stop (Greece, Spain, Ireland, Portugal and Italy) and the rest of the eurozone. That these countries belong to a monetary union means the eurozone sudden stop stands apart. These differences are important for the choice of modeling approach. The sudden stop literature on emerging markets (see Mendoza (2010) and Korinek and Mendoza (2013) for example) has focused on a Fisherian amplification mechanism where debts are denominated in different units than incomes and collateral. This is not the case in our model as we study countries that belong to a monetary union. Another difference is that the sudden stop literature in emerging markets has focused on the sudden imposition of an external credit constraint (see Mendoza and Smith (2006) and Christiano and Roldos (2004) for example) or on transaction costs on international financial markets with multiple equilibria, as in Martin and Rey (2006). Our model integrates, for the first time to our knowledge, both a domestic credit crunch and a sudden stop produced by a spike in interest rate so that we can compare the impact of both on macroeconomic aggregates. The role of interest rates in our model relates our work to the paper of Neumeyer and Perri (2005). In their paper, as in ours, the economy is subject to interest rate shocks that generate a sudden stop in the form of a current account reversal. However, the mechanism is very different. In Neumeyer and Perri (2005), real

[2]For instance, Mian and Sufi (2010) find that the predictive power of household borrowing remains the same in counties dominated by national banks. It is also well known that businesses entered the recession with historically strong balanced sheets and were able to draw on existing credit lines Ivashina and Scharfstein (2008).

interest rates movements either exogenous or induced by productivity shocks amplify the effect of the latter on production because they induce a working capital shortage. In our model, the increase in interest rate generates a demand shock through a fall in consumption.

Even if the bulk of the literature on sudden stops has put credit constraints at the center of the story, Gopinath (2004) and Aguiar and Gopinath (2007) have focused on an alternative explanation with TFP shocks taking center stage. Gopinath (2004) proposes a model with a search friction to generate asymmetric responses to symmetric shocks. A search friction in foreign investors entry decision into emerging markets creates an asymmetry in the adjustment process of the economy: An increase in traded sector productivity raises GDP on impact and it continues to grow to a higher long-run level. On the other hand, a decline in traded sector productivity causes GDP to contract in the short run by more than it does in the long-run. A related approach is the possibility of growth shocks as explored in Aguiar and Gopinath (2007). Because of the income effect, a negative shock leads to a fall in consumption and an increase in the trade balance. Aguiar and Gopinath (2007) do not study the response of the labor market but it is well known that income effects tend move consumption and hours in opposite directions.

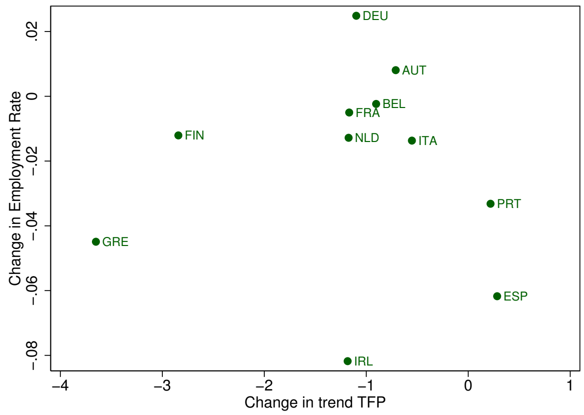
Shocks to trend TFP growth might be important in emerging markets, but they do not seem to explain the dynamics of euro area countries over the past 5 years. With the exception of Greece, countries that were hit by a sudden stop (Greece, Ireland, Italy, Spain, Portugal) are not those for which the reversal in TFP growth is the largest[3] Moreover, as illustrated by figure 4, there is no correlation between the differential in TFP growth (between the periods 2008-2012 and 2000-2007) and employment growth during the bust (2008-2012)[4]. Spain is a prime example. It is the only eurozone country that experienced an acceleration of its TFP growth during the crisis. Sanguinetti and Fuentes (2012) show that *"The Spanish economy experienced significantly weaker labour productivity growth than other OECD economies and failed to catch up with the most advanced economies in the period 1996-2007... The relatively weak performance largely reflects the low growth of total factor productivity within a wide range of sectors, with very limited impact of composition effects, while the capital stock and educational attainment of the workforce have grown relatively strongly."* We conclude that we need to look somewhere else for an explanation to the business cycles and to sudden stops in the eurozone. We focus on leverage, interest rates and fiscal policy dynamics.

In the eurozone, financial integration has also - until the most recent crisis - led to large cross-border lending. Most closely connected to our paper is the work of Midrigan and Philippon (2010), Guerrieri and

[3]If the differential of TFP is computed on the period 2010-2012, this conclusion remains unchanged.

[4]A simple panel regression among Eurozone members on the period 2002-2012 of GDP growth on TFP growth, with or without year fixed effects and country fixed effects reveals no significant correlation between GDP and TFP growth.

Figure 4: Changes in Trend TFP Do Not Explain the European Crisis



Notes: Horizontal Axis is average TFP growth during 2008-2012 minus average TFP growth during 2000-2007 (source: Conference Board). Vertical Axis is change in employment rate between 2008 and 2012.

Lorenzoni (2010) and Eggertsson and Krugman (2012) who also study the responses of an economy to a household-level credit crunch. Consistent with our results, Mian and Sufi (2012) show that differences in the debt overhang of households across U.S. counties partly explain why unemployment is higher in some regions than others. Schmitt-Grohe and Uribe (2012) emphasize the role of downward wage rigidity in the Eurozone recession. Our paper is also related to the literature on sovereign default (see Eaton and Gersovitz (1982), Arellano (2008) and Mendoza and Yue (2012)) that models default as a strategic decision with a tradeoff between gains from forgone repayment and the costs of exclusion from international credit markets. The objective of our paper however is to analyze how the sovereign default risk can affect the real economy through the impact it can have on liquidity available to households. The paper by Corsetti et al. (2013) considers a "sovereign risk channel," through which sovereign default risk spills over to the rest of the economy, raising funding costs in the private sector. Finally the paper is related to the recent research on fiscal multipliers at the regional level (Nakamura and Steinsson (2014), Farhi and Werning (2013)).

In Section 1 we present the model. In Section 2, we analyze how nominal dynamics and employment respond to different shocks and in Section 3, we compare the model predictions with leverage dynamics, fiscal policy and sudden stops with the observed path of the Eurozone countries. We do this both for the reduced form shocks and for the structural shocks. These serve to conduct the final exercise on counterfactual policies. Section 4 concludes.

1 Model

Our model can be interpreted as a large country with a collection of regions (e.g., USA), or a monetary union with a collection of states (e.g., EZ). The key assumption are that these regions share a common currency, and that agents live and work in only one region.

We study a small open economy that trades with other regions of the currency area as in Gali and Monacelli (2008). Each region j produces a tradable domestic good and is populated by households who consume the domestic good and an aggregate of foreign goods. Following Mankiw (2000) and more recently Eggertsson and Krugman (2012), we assume that households are heterogenous in their degree of time preferences. More precisely, in region j, there is a fraction χ_j of impatient households, and $1-\chi_j$ of patient ones. Patient households (indexed by $i = s$ for savers) have a higher discount factor than borrowers (indexed by $i = b$ for borrowers): $\beta \equiv \beta_s > \beta_b$. Saving and borrowing are in units of the currency of the monetary union.

1.1 Within period trade and production.

Consider household i in region j at time t. Within period, all households have the same log preferences over the consumption of home (h), foreign goods (f), and labor supply n:

$$u_{i,j,t} = \alpha_j \log\left(\frac{C^h_{i,j,t}}{\alpha_j}\right) + (1-\alpha_j)\log\left(\frac{C^f_{i,j,t}}{1-\alpha_j}\right) - \nu\left(N_{i,j,t}\right)$$

With these preferences, households of region$j$ spend a fraction α_j of their income on home goods, and $1-\alpha_j$ on foreign goods. The parameter α_j measures how closed the economy is, because of home bias in preferences or specialization in trade. The demand functions are then:

$$\begin{aligned} p^h_{j,t} C^h_{i,j,t} &= \alpha_j X_{i,j,t}, \\ p^f_t C^f_{i,j,t} &= (1-\alpha_j) X_{i,j,t}. \end{aligned}$$

where

$$X_{i,j,t} \equiv p^h_{j,t} C^h_{i,j,t} + p^f_t C^f_{i,j,t}$$

measures the spending of household i in region j in period t, $p^h_{j,t}$ is the price of home goods in country j and p^f_t is the price index of foreign goods. This gives the indirect utility

$$U\left(X_{i,j,t}, p_{ijt}\right) = \log\left(X_{i,j,t}\right) - \log p_{j,t} - \nu\left(n_{i,j,t}\right),$$

where the CPI of country j is $\log p_{j,t} = \alpha_j \log p^h_{j,t} + (1-\alpha_j)\log p^f_{j,t}$, the PPI is $p^h_{j,t}$, and the terms of trade are $\frac{p^f_t}{p^h_{j,t}}$. From the perspective of country j, foreign demand for home good $F_{j,t}$ is exogenous, and we assume a unit elasticity with respect to export price $p^h_{j,t}$. Production is linear in labor $n_{j,t}$, and competitive, so $p^h_{j,t} = w_{j,t}$. Market clearing in real terms requires:

$$N_{j,t} = \chi_j C^h_{b,j,t} + (1-\chi_j)\, C^h_{s,j,t} + \frac{F_{j,t}}{p^h_{j,t}} + \frac{G_{j,t}}{p^h_{j,t}},$$

where $G_{j,t}$ are nominal government expenditures. Note that we assume that the government spends only on domestic goods. Define nominal domestic product as

$$Y_{j,t} \equiv p^h_{j,t} N_{j,t}$$

and total private expenditures as

$$\bar{X}_{j,t} \equiv \chi_j X_{b,j,t} + (1-\chi_j) X_{s,j,t}.$$

It is useful to write the market clearing condition in nominal terms (in euros)

$$Y_{jt} = \alpha_j \bar{X}_{j,t} + F_{j,t} + G_{j,t}. \tag{1}$$

Each household supplies labor at the prevailing wage and receives wage income net of taxes $(1-\tau_{j,t})\, w_{j,t} N_{j,t}$. They also receive transfers from the government $T_{j,t}$. We assume that wages are sticky and we ration the labor market uniformly across households. This assumption simplifies the analysis because we do not need to keep track separately of the labor income of patient and impatient households within a country. Not much changes if we relax this assumption, except that we loose some tractability.[5]

[5]In response to a negative shock, impatient households would try to work more. The prediction that hours increase more for credit constrained households appears to be counter-factual however. One can fix this by assuming a low elasticity of labor supply, which essentially boils down to assuming that hours worked are rationed uniformly in response to slack in the labor market. Assuming that the elasticity of labor supply is small (near zero) also means that the natural rate does not depend on fiscal policy. In an extension we study the case where the natural rate is defined by the labor supply condition in the pseudo-steady state $\nu'\left(n^\star_i\right) = (1-\tau_j)\,\frac{w_j}{x_{i,j}}$. We can then ration the labor market relative to their natural rate: $n_{i,j,t} = \frac{n^\star_i(\tau)}{\sum_i n^\star_i(\tau)} n_{j,t}$ where $n^\star_i(\tau)$ is the natural rate for household i in country. This ensures consistency and convergence to the correct long run

1.2 Inter-temporal budget constraints

Let $B_{j,t}$ be the amount borrowed by impatient households. It is the face value of the debt issued in $t-1$ and due in period t. It will be convenient to define disposable income (after tax and transfers but before interest payments) as

$$\tilde{Y}_{j,t} \equiv (1-\tau_{j,t})\, Y_{j,t} + T_{j,t}.$$

The budget constraint of impatient households in country j is then

$$\frac{B_{j,t+1}}{1+r_{b,j,t}} + \tilde{Y}_{j,t} = X_{b,j,t} + B_{j,t}, \tag{2}$$

where $r_{b,j,t}$ is the nominal borrowing rate between t and $t+1$. Borrowing is subject to the exogenous limit $B^h_{j,t}$:

$$B_{j,t} \leq B^h_{j,t}. \tag{3}$$

Savers save at the nominal rate $r_{s,j,t}$ and their budget constraint is:

$$S_{j,t} + \tilde{Y}_{j,t} = X_{s,j,t} + \frac{S_{j,t+1}}{1+r_{s,j,t}}, \tag{4}$$

so their Euler equation is

$$\frac{1}{X_{s,j,t}} = \mathbb{E}_t\left[\frac{\beta\,(1+r_{s,j,t})}{X_{s,j,t+1}}\right]. \tag{5}$$

Note that financial markets clear in two ways in our model. For the impatient agents, given that they are quantity constrained, interest rates do not affect their borrowing. For the patient agents, their saving is determined by the interest rates through the Euler equation.

For now we keep the notations general by using a country and agent specific rate, $r_{i,j,t}$. We will analyze cases where rates differ across countries in the monetary union, and cases where rates differ across agents within a country. The government budget constraint is

$$\frac{B^g_{j,t+1}}{1+r_{g,j,t}} + \tau_{j,t} Y_{jt} = G_{j,t} + T_{j,t} + B^g_{j,t}, \tag{6}$$

where $B^g_{j,t}$ is public debt issued by government j at time t.

equilibrium. Steady state changes in the natural rate are quantitatively small, however, so the dynamics that we study are virtually unchanged. See Midrigan and Philippon (2010) for a discussion.

1.3 Exports and foreign assets

Nominal exports are $F_{j,t}$ and nominal imports are $(1-\alpha_j)\,X_{j,t}$ since the government does not buy imported goods while private agents spend a fraction $1-\alpha_j$ on foreign goods. So net exports are:

$$E_{j,t} = F_{j,t} - (1-\alpha_j)\,X_{j,t}. \tag{7}$$

The net foreign asset position of the country is at the *end of period* t, measured in market value, is

$$A_{j,t} \equiv (1-\chi_j)\,\frac{S_{j,t+1}}{1+r_{s,j,t}} - \chi_j \frac{B^h_{j,t+1}}{1+r_{b,j,t}} - \frac{B^g_{j,t+1}}{1+r_{g,j,t}}. \tag{8}$$

Adding up the budget constraints, we have the spending equation

$$X_{j,t} + p^h_{j,t} G_{j,t} = Y_{j,t} + \chi_j \left(\frac{B^h_{j,t+1}}{1+r_{b,j,t}} - B^h_{j,t}\right) - (1-\chi_j)\left(\frac{S_{j,t+1}}{1+r_{s,j,t}} - S_{j,t}\right) + \frac{B^g_{j,t+1}}{1+r_{g,j,t}} - B^g_{j,t} \tag{9}$$

Total spending (public and private) equals total income (GDP) plus total net borrowing. If we combine with the market clearing condition (1), we get the current account condition

$$CA_{j,t} \equiv A_{j,t} - A_{j,t-1} = E_{j,t} + \bar{r}_{j,t-1} A_{j,t-1},$$

where $\bar{r}_{j,t}$ is defined as the weighted average interest rate. The closed economy corresponds to $A=0$ and endogenous interest rate r. The small open region corresponds to exogenous r and endogenous A. It will often be convenient to rewrite (9) with disposable income as

$$(1-\alpha_j)\,\tilde{Y}_{j,t} = \alpha_j \chi_j \left(\frac{B^h_{j,t+1}}{1+r_{b,j,t}} - B^h_{j,t}\right) - \alpha_j\,(1-\chi_j)\left(\frac{S_{j,t+1}}{1+r_{s,j,t}} - S_{j,t}\right) + F_{j,t} + \frac{B^g_{j,t+1}}{1+r_{g,j,t}} - B^g_{j,t}. \tag{10}$$

1.4 Employment and Inflation

The system above completely pins down the dynamics of nominal variables: $Y_{j,t}, X_{i,j,t}, \ldots$. Employment (real output) is given by

$$N_{j,t} = \frac{Y_{j,t}}{p^h_{j,t}}.$$

We need to specify the dynamics of inflation. Letting $n^\star$ denote the natural rate of unemployment, we assume the following Phillips curve:

$$\frac{p^h_{j,t} - p^h_{j,t-1}}{p^h_{j,t-1}} = \kappa \left(N_{j,t} - N^\star\right) \tag{11}$$

2 Nominal Dynamics

We now study the nominal dynamics of the small open economy. The main challenge is to pin down the behavior of savers. Our first task is to understand why and how savers do, or do not, react to certain shocks.

2.1 Savers' Inter-temporal Budget

The complete objective function of the savers is

$$\sum_{t\geq 0} \beta^t \left(\log\left(X_{s,j,t}\right) - \log\left(\bar{p}_{j,t}\right) - \nu\left(N_{j,t}\right)\right).$$

Prices are additively separable thanks to log preferences. In addition, our rationing rule for labor combined with vanishingly small labor supply elasticity implies that $n_{j,t}$ depends only on aggregate variables. The problem of the savers can therefore be written as:

$$\max \textstyle\sum_{t\geq 0} \beta^t \log\left(X_{s,j,t}\right)$$
$$X_{s,j,t} + \tfrac{S_{j,t+1}}{1+r_{s,j,t}} = S_{j,t} + \tilde{Y}_{j,t}$$

where $\tilde{y}_{j,t}$ and $r_{s,j,t}$ are both random variables.[6] Let us focus first on the budget constraint, and its Ricardian properties. Let us first define the k-period discount rate from the savers' perspective as

$$R_{j,t,k} \equiv \left(1 + r_{s,j,t}\right) .. \left(1 + r_{s,j,t+k-1}\right).$$

[6]This is a well-studied problem but we will not do justice to all of its interesting aspects. In particular, we will neglect (for now) the role of precautionary savings and use a certainty equivalent approach by linearizing the Euler equation. Because of precautionary savings, we know that the interest rate consistent with finite savings must be such that $\beta\left(1+r\right) < 1$. We consider the limit where aggregate shocks are small and $\beta\left(1+r\right)$ is close to one.

with the convention $R_{j,t,0} = 1$. We can then write the inter-temporal budget constraint of savers as

$$\mathbb{E}_t \sum_{k=0}^{\infty} \frac{X_{s,j,t+k}}{R_{j,t,k}} = S_{j,t} + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\tilde{Y}_{j,t+k}}{R_{j,t,k}}. \tag{12}$$

The key idea is to relate this to the inter-temporal current account for the country. To do so, we define the financing spreads ϕ for private borrowers and for the government as

$$1 - \phi_{b,j,t} \equiv \frac{1 + r_{s,j,t}}{1 + r_{b,j,t}}, \text{ and } 1 - \phi_{g,j,t} \equiv \frac{1 + r_{s,j,t}}{1 + r_{g,j,t}}.$$

Then we have the following Lemma

Lemma 1. *The inter-temporal current account condition is*

$$(1 - \alpha_j) \left((1 - \chi_j) S_{j,t} - \chi_j B^h_{j,t} + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\tilde{Y}_{j,t+k}}{R_{j,t,k-1}} \right) = (1 - \chi_j) S_{j,t} - \chi_j B^h_{j,t} - B^g_{j,t} + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{F_{j,t+k}}{R_{j,t,k}} - \Phi_{j,t},$$

where Φ_t is the net present value of financing costs

$$\Phi_{j,t} \equiv \alpha_j \chi_j \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\phi_{b,j,t+k-1}}{R_{j,t,k}} B_{j,t+k} + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\phi_{g,j,t+k-1}}{R_{j,t,k}} B^g_{j,t+k}.$$

Proof. See Appendix. □

On the left we have private wealth (discounted at the savers's rate) and $1 - \alpha_j$ is the share of wealth spent on imports. On the right we have net foreign assets plus the value of exports.

If we combine the Lemma with the inter-temporal budget constraint (12), we have the following proposition

Proposition 1. *When interest rates are the same within a country ($\Phi_{j,t} = 0$) the nominal spending of patient agents $X_{s,j,t}$ does not react to private credit shocks ($B_{j,t}$) or to fiscal policy (neither $G_{j,t}$ nor $T_{j,t}$).*

Proposition 1 clarifies the behavior of savers. In particular, their spending reacts neither to $G_{j,t}$ nor to $T_{j,t}$. This result is related to – but also different from – Ricardian equivalence. To understand it, one needs to focus on the budget constraint of the patient households. Clearly, shocks to interest rates will affect this budget constraint and also the Euler equation, so we know that they will affect spending $X_{s,j,t}$. What is surprising is that, even though changes in the borrowing constraints of impatient agents or changes in fiscal policy have a direct impact on disposable income $\tilde{Y}_{j,t}$, savers do not react. The reason is that patient agents know that higher spending today – which increases output – means higher interest payments in the future – which decreases spending and output. The key result is that the *net present value* of disposable income does not change as long as rates are the same within the country. Changes in $\bar{B}, G, T, \tau$ have no effect on the permanent income of patient agents. Shocks to foreign demand, on the other hand, affect consumption expenditures of patient households because they affect their permanent income. Of course, even when expenditures remain constant, this does not mean that real consumption remains constant. In fact real consumption always changes because prices (wages) always react to changes in aggregate spending. These results depend on our using the preferences of Cole and Obstfeld (1991). Farhi and Werning (2013) discuss the implications of these preferences for government multipliers in currency unions.

2.2 Scaling and Spreads

Proposition 1 explains why savers do not react to demand shocks. But savers react to other shocks. We consider an economy subject to four series of shocks: the borrowing limit of the impatient households $B^h_{j,t}$, foreign demand $F_{j,t}$, interest rates, and fiscal policy. We assume that interest rates are time varying and country-specific, but they are the same for all agents within a country:

$$r_{b,j,t} = r_{s,j,t} = r_{g,j,t}.$$

Savers react to interest rates and to foreign demand shocks. We are going to assume that the variance of these shocks is small and linearize the Euler equation (5) as

$$\mathbb{E}_t \left[X_{s,j,t+1} \right] \approx \beta \left(1 + r_{j,t} \right) X_{s,j,t}.$$

The equivalent equation for the monetary union as a whole is:

$$\mathbb{E}_t \left[X^*_{s,t+1} \right] \approx \beta \left(1 + r^*_t \right) X^*_{s,t}.$$

with r^*_t the interest for the monetary union as a whole. We define the spread shock as:

$$1 + \rho_{j,t} \equiv \frac{1 + r_{j,t}}{1 + r^*_t}$$

We show in the Appendix that if we scale all our variables by $X_{s,t}^*$:

$$x_{s,j,t} \equiv \frac{X_{s,j,t}}{X_{s,t}^*} \tag{13}$$

Then we have

$$\mathbb{E}_t\left[x_{s,j,t+1}\right] \approx \left(1+\rho_{j,t}\right) x_{s,j,t}$$

From now on we work with scaled variables with the patient budget constraint:

$$x_{s,j,t} + \frac{\beta}{1+\rho_{j,t}} s_{j,t+1} = s_{j,t} + \tilde{y}_{j,t}$$

2.3 Shocks and Policies

We make the following assumptions about the shocks:

Assumption A1.

- *Exogenous shocks are such that* $\mathbb{E}_t\left[b_{j,t+2}^h\right] = b_{j,t+1}^h$, $\mathbb{E}_t\left[f_{j,t+1}\right] = f_{j,t}$, *and* $\mathbb{E}_t\left[\rho_{j,t+1}\right] = 0$*;*
- *Fiscal policy is such that* $\mathbb{E}_t\left[b_{j,t+2}^g\right] = b_{j,t+1}^g$*;*
- *The variance of interest rates and foreign demand is small, and* β_b *is small enough that* $b_{j,t} = b_{j,t}^h$ *at all times.*

The first point says that the shocks are permanent and that spreads are iid. The second point defines a class of fiscal policies. These assumptions are important for the expectations of the agents in the model. The last point is purely technical. It allows us to linearize Euler equations. Notice that we do not need to assume that shocks to $B_{j,t}^h$ are small or that fiscal policy shocks are small. We assume that the shocks are small enough that impatient households find it optimal to borrow up to the constraint (this is a joint restriction on the discount factor and the size of the shocks).

We now look for decision rules for the savers $\{s_{j,t}\}_{t=1,2...}$ and the other variables of the model, $y_{j,t}$, $p_{j,t}^h$, etc. The following Lemma is used repeatedly

Lemma 2. *Under A1, savings dynamics satisfy* $\mathbb{E}_t\left[s_{j,t+2}\right] = s_{j,t+1}$.

In fact, we show in the Appendix that

$$\frac{s_{j,t+1}}{1+\rho_{j,t}} - s_{j,t} = \frac{\alpha_j \chi_j}{1-\alpha_j \chi_j}\left(\frac{b^h_{j,t+1}}{1+\rho_{j,t}} - b^h_{j,t}\right) + \frac{1}{1-\alpha_j \chi_j}\left(\frac{b^g_{j,t+1}}{1+\rho_{j,t}} - b^g_{j,t} + \frac{\rho_{j,t}}{1+\rho_{j,t}}\frac{f_{j,t}}{1-\alpha_j}\right) \tag{14}$$

Savings inherit the dynamic properties of $b^h_{j,t}$ and $b^g_{j,t}$. Since $\mathbb{E}_t\left[\rho_{j,t+1}\right] = 0$, this validates our conjecture that $\mathbb{E}_t\left[s_{j,t+2}\right] = s_{j,t+1}$.

Proposition 1 becomes

Corollary 1. *Under A1, nominal spending of patient agents follows*

$$\Delta x_{s,j,t} = \frac{\Delta\left[f_{j,t}\right]}{1-\alpha_j} + \Omega_{j,t}\left(\rho_{j,t}, \rho_{j,t-1}\right)$$

where $\Omega_{j,t}\left(0,0\right) = 0$.

This result is important because it says that in response to shocks to $B^h_{j,t}$ or shocks to fiscal policy – including shocks to $p^h_{j,t} g_{j,t}$ – we not only have $x_{s,j,t} = \mathbb{E}_t\left[x_{s,j,t+1}\right]$ but in fact $x_{s,j,t} = x_{s,j,t-1}$. As explained earlier, the reason is that patient agents know that higher spending today – which increases output – means higher interest payments in the future – which decreases spending and output. The key result is that the *net present value* of disposable income does not change. Since on average $\beta\left(1+r\right) = 1$, savers optimally choose to keep $x_{s,j,t}$ exactly constant.

2.4 Impulse Responses

For simplicity, we assume that the tax rate is τ_j is constant. The five equilibrium conditions of the model are

1. $s_{j,t+1} = \left(1+\rho_{j,t}\right)\left(s_{j,t} + \tilde{y}_{j,t}\right) - \mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$
2. $\left(1-\alpha_j\right)\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right] = \frac{r}{1+r}\left(\alpha_j\left(1-\chi_j\right)s_{j,t+1} - \alpha_j\chi_j\bar{b}_{j,t+1} - b^g_{j,t+1}\right) + f_{j,t}$.
3. $\left(1-\alpha_j\right)\tilde{y}_{j,t} = \alpha_j\chi_j\left(\frac{b^h_{j,t+1}}{1+r_{j,t}} - b^h_{j,t}\right) + \alpha_j\left(1-\chi_j\right)\left(s_{j,t} - \frac{s_{j,t+1}}{1+r_{j,t}}\right) + \frac{b^g_{j,t+1}}{1+r_{j,t}} - b^g_{j,t} + f_{j,t}$.
4. $g_{j,t} + T_{j,t} - \tau_j y_{j,t} = \frac{b^g_{j,t+1}}{1+r_{j,t}} - b^g_{j,t}$
5. $\tilde{y}_{j,t} = \left(1-\tau_j\right)y_{j,t} + T_{j,t}$

The five unknown endogenous variables are $\tilde{y}_{j,t}$, $y_{j,t}$, $b^g_{j,t}$, $s_{j,t+1}$, and $\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$. The exogenous shocks are $B^h_{j,t}$, $F_{j,t}$ and $r_{j,t}$. The policy shocks variables are $T_{j,t}$ and $g_{j,t}$. The state space of predetermined endogenous variables is $s_{j,t}$ and $b^g_{j,t}$.

Policy Rules We need to specify the policy function of the government in order to compute the impulse responses. [7] We are interested in simple rules that deliver the property that the public debt follows a random walk: $\mathbb{E}_t\left[b^g_{j,t+2}\right] = b^g_{j,t+1}$. Automatic stabilizers have become a characteristic of modern fiscal systems in all OECD countries and the income tax is a strong component of automatic stabilizers (see Fatas and Mihov (2012)). We assume that spending and transfers are predetermined. From the government budget constraint, this means that a recession at time t automatically increases government debt at time t. To maintain fiscal stability, we specify that transfers adjust from t to $t+1$ to keep public debt constant thereafter: $\mathbb{E}_t\left[b^g_{j,t+2}\right] = b^g_{j,t+1}$. More precisely, we specify the general policy rule as follows

1. Fiscal variables $p^h_{j,t}g_{j,t}$ and $T_{j,t}$ are pre-determined. Government debt $b^g_{j,t}$ is determined in equilibrium at time t.

2. Set transfers $T_{j,t+1}$ for next period so that $\mathbb{E}_t\left[b^g_{j,t+2}\right] = b^g_{j,t+1}$ assuming a martingale for $p^h_{j,t}g_{j,t}$.

   (a) The expected budget constraint is

   $$\tau_j \mathbb{E}_t\left[y_{j,t+1}\right] = g_{j,t} + T_{j,t+1} + \frac{r}{1+r} b^g_{j,t+1}$$

   (b) By definition we have

   $$\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right] = \left(1-\tau_j\right)\mathbb{E}_t\left[y_{j,t+1}\right] + T_{j,t+1}$$

   (c) Therefore (recall that $\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$ is part of our solution at time t), $T_{j,t+1}$ is given by

   $$T_{j,t+1} = \tau_j \mathbb{E}_t\left[\tilde{y}_{j,t+1}\right] - \left(1-\tau_j\right) g_{j,t} - \left(1-\tau_j\right)\frac{r}{1+r} b^g_{j,t+1}$$

[7] Here is a simple example, that we do not use in the quantitative experiments, but that can help understand the model. If the only shocks are changes in the private debt constraint $B^h_{j,t}$, a government trying to stabilize its economy could react to these shocks with the following simple rule:

$$b^g_{j,t+1} - b^g_{j,t} = -\gamma\alpha_j\chi_j\left(b^h_{j,t+1} - b^h_{j,t}\right),$$

with $0 \le \gamma \le 1$. We then get

$$s_{j,t+1} - s_{j,t} = (1-\gamma)\frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\left(b^h_{j,t+1} - b^h_{j,t}\right).$$

Nominal GDP is a more volatile when the economy is more closed and when the share of impatient borrowers is larger. We see that perfect stabilization of $s_{j,t}$ is theoretically possible by choosing $\gamma = 1$. We can check that this rule also implies a constant disposable income.

We present in Figures (5), (6), (7) and (8) the simple impulse reaction functions[8] that illustrate the impact of shocks on household debt ($b_{j,t}$), public spending ($g_{j,t}$), interest rates ($r_{j,t}$) and foreign demand ($f_{j,t}$). An increase in household debt generates a boom in nominal GDP, employment, consumption of impatient households (but not of patient ones who increase their saving, as explained in Proposition 1) and imports. Public debt falls but the net foreign asset position deteriorates. A fiscal expansion has qualitatively similar effects except that in this case public debt increases, although it decreases in percentage of GDP. The GDP multipliers for household debt and government spending both increase with α_i and χ_i. The reason is that a higher share of spending on domestic goods reduces leakage through imports. A lower share of patient agents in the economy implies that there is less increase in aggregate saving following either an increase in private or public debt.

An increase in interest rates is very different because it induces patient households to save more so it reduces their expenditures and generates a recession (fall in nominal GDP and in employment) that obliges impatient households to reduce their spending. Imports fall and the net foreign asset position improves. Because of lower tax revenues, the recession increases public debt. Finally, an increase in foreign demand permanently increases nominal GDP which induces patient households to increase their saving. Consumption of both patient and impatient households increase. The net foreign asset position improves. Public debt falls because of higher tax revenues.

[8] For these impulse response functions, we use the following parameters: $\alpha = 0.75, \chi = 0.5, r = 0.05, \kappa = 0.3, \tau = 0.4$. Prices, wages and employment are normalized to unity at time $t = 0$. The debt to income ratio is set at 60% for impatient households at time $t = 0$, so that the household debt to income ratio is 30%. The government debt to GDP ratio is set at 50% and the net foreign asset position over GDP is set at zero at time $t = 0$.

Figure 5: Private Credit Expansion

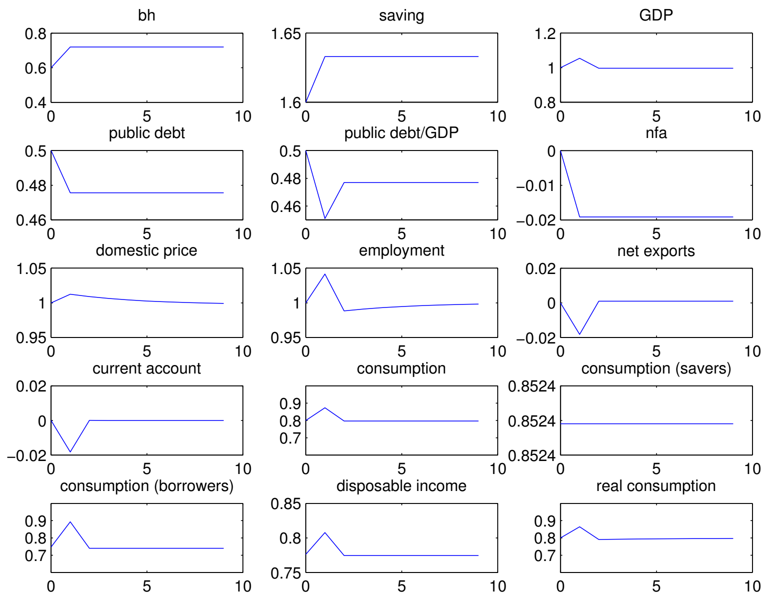


Figure 6: Fiscal Expansion

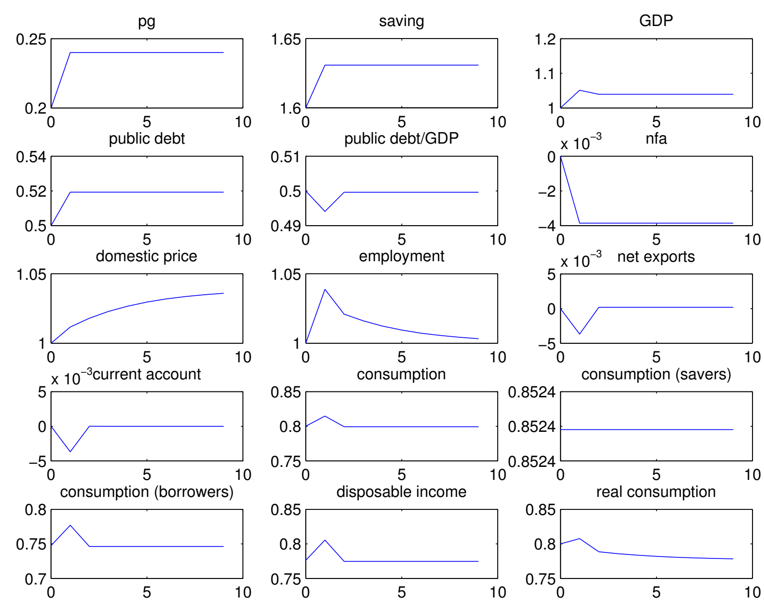


Figure 7: Interest rate shock

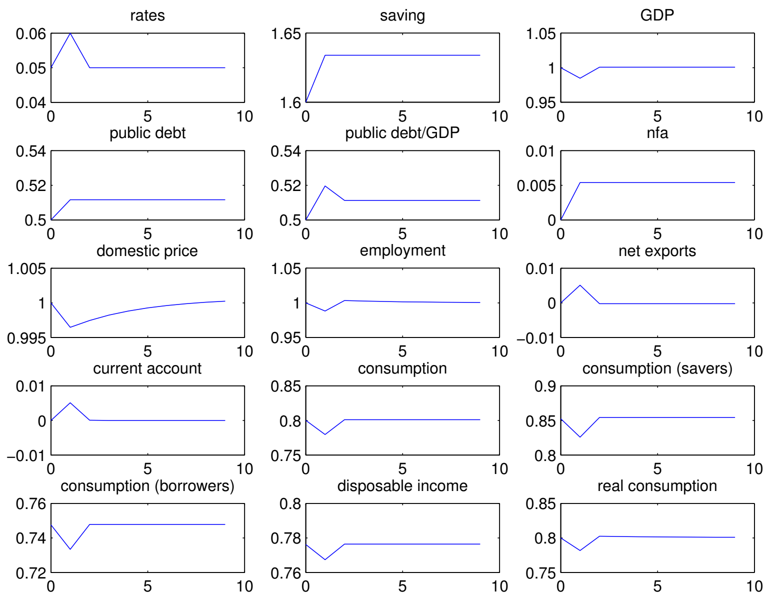
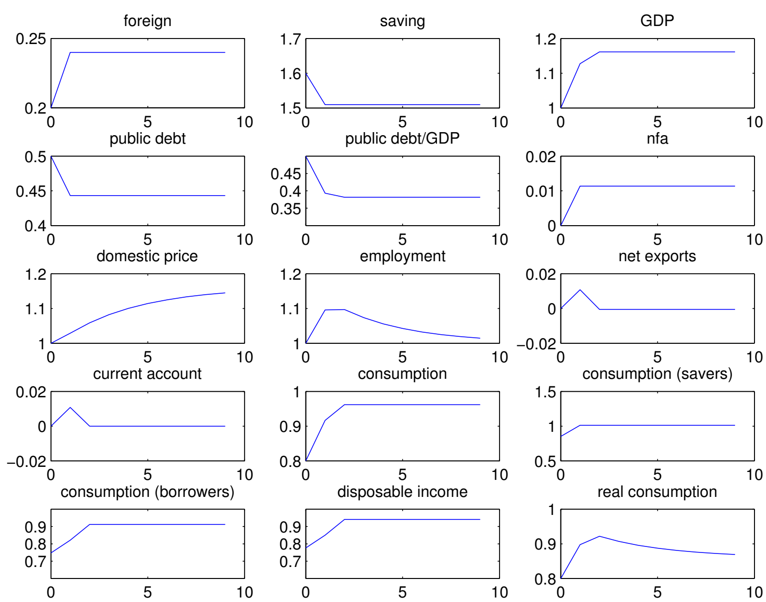


Figure 8: Foreign demand shock



3 Quantitative Experiments

We next analyze a cross-sectional experiment to compare the model predictions and the data. We describe the sources of the cross-sectional and aggregate data we use in the Appendix. We use data for 11 Eurozone countries from 2000 to 2012: Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, Netherlands and Portugal and calibrate the shocks on the observed data.

3.1 Calibration

In order to map the observed data into the model we rebase the data with aggregate Eurozone spending, as in Equation (13). We compute a benchmark in the same manner for consumption, government spending, transfers and unit labor costs. The normalized data is the ratio of the observed data to this benchmark level. This enables us therefore to interpret these as deviations from the benchmark levels for each data series we observe. For both the household debt and the government debt, the rebased levels are the ratios of debt to the benchmark levels of GDP. Aggregate variables (employment, GDP, consumption, transfers, government spending and taxes, exports and transfers...) are analyzed either in per capita terms or in ratios of GDP. For employment per capita, we take the deviation with respect to 2001 with an index of 1 for that year. 2001 is the base year for consumption and unit labor costs (index 1 in 2001). The normalized data are given in figures (19) and (20) in the Appendix. Note also that government spending is adjusted for expenditures on bank recapitalization.

The parameters that serve in the simulations are given in Table (1).

Table 1: Parameters

Parameter Name		*Value*
Annual Discount Factor	β	0.98
Domestic share of consumption	α_j	country specific
Share of credit constrained households	χ_j	country specific
Phillips curve parameter	κ	0.2

For the country specific domestic share of consumption, α_j, we rely on the paper by Bussiere et al. (2011) who compute the total import content of expenditure components, including the value of indirect imports. For consumption expenditures and for our sample our countries the average implied domestic share in 2005 (the latest date in their study) is 27.3%. The lowest is 66.4% for Belgium and the highest is 78.7% for Italy. For the country specific share of credit constrained borrowers, χ_j, we use the Eurosystem Household

Finance and Consumption Survey (HFCS)[9]. This survey has been used recently by Mendicino (2014) and Kaplan et al. (2014) to quantify the share of hand-to-month households. The later paper defines these as consumers who spend all of their available resources in every pay-period, and hence do not carry any wealth across periods. They argue that measuring this behavior using data on net worth (as consistent with heterogeneous-agent macroeconomic models) is misleading because this misses what they call the wealthy hand-to-mouth households. These are households who hold sizable amounts of wealth in illiquid assets (such as housing or retirement accounts), but very little or no liquid wealth, and therefore consume all of their disposable income every period. They define hand-to-mouth consumers as those households in the survey whose average balances of liquid wealth are positive but equal to or less than half their earnings. We use a related measure by Mendicino (2014), who for each country computes the fraction of household with liquid assets below two months of total households gross income to approximate the share of credit constrained households. The average for our set of countries is 48% with a maximum of 64.8% for Greece and a minimum of 34.7% for Austria. Ireland did not participate in the survey so for this country we use the average of the eurozone. These two parameters, the share of credit constrained households (χ_j) and the domestic share of consumption (α_j), for our panel of countries are shown on figure (9).

Figure 9: Share of credit constrained households (χ_j) and domestic share of consumption (α_j)

To transform observed aggregate household debt in the data into household debt $b_{j,t}$ in the model which is debt per impatient household (with share ψ_j), we take the household to benchmark income ratio for each country and then divide it by the share of impatient in the country χ_j. We use $t_0 = 2002$ as our base year.

Given the absence of an intermediate goods sector in our model, we cannot use the value of gross exports

[9]The survey took place in 2010. In Greece and Spain, the data were collected in 2009 and 2008-09 respectively.

as a measure of foreign demand, $F_{j,t}$. The trade linked to international production networks has been well documented (see for example Baldwin and Lopez-Gonzalez (2013)). In the context of our model, we need to measure the domestic value added that is associated with final consumption in the rest of the world, which corresponds to value added based exports. As detailed in the data appendix, we use the data from the OECD-WTO Trade in Value-Added (TiVA) initiative to measure domestic value added embodied in gross exports.

3.2 Reduced Form Simulations

We first consider the predictions of the model taking as given the observed series for private debt (b^h), fiscal policy (g, T, τ) and interest rate spreads (ρ_t). To run the simulations, we first need to set the initial conditions in a particular year (we use 2002).

1. Natural employment and prices are normalized to $n^\star = 1$ and $p^h_{j,t_0} = 1$ (so nominal GDP is normalized in the base year to: $y_{j,t_0} = 1$)

2. Variables set to their observed values are: $\bar{b}_{j,t_0}$, T_{j,t_0}, g_{j,t_0}, b^g_{j,t_0}, b^g_{j,t_0-1}, r_{j,t_0}. We then get $\bar{x}_{f,t_0}$, τ_j and $\tilde{y}_{j,t_0}$ from market clearing and budget constraints:

   (a) Foreign demand $\bar{x}_{f,t_0}$ is chosen to match net exports $e_{j,t_0} = \frac{1}{\alpha_j}\left(f_{j,t_0} - (1-\alpha_j)\left(y_{j,t_0} - g_{j,t_0}\right)\right)$

   (b) Get τ_j from the government budget constraint $g_{j,t_0} + T_{j,t_0} - \tau_j y_{j,t_0} = \frac{b^g_{j,t_0+1}}{1+r_{j,t_0}} - b^g_{j,t_0}$

   (c) Disposable income at time t_0 is $\tilde{y}_{j,t_0} = (1-\tau_j)\, y_{j,t_0} + T_{j,t_0}$

3. Savers' assets s_{j,t_0} and s_{j,t_0-1} are chosen to solve the equilibrium conditions

   (a) $s_{j,t} = (1+\rho_{j,t})\left(s_{j,t-1} + \tilde{y}_{j,t}\right) - \mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$

   (b) $(1-\alpha_j)\,\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right] = \frac{r}{1+r}\left(\alpha_j(1-\chi_j)\, s_{j,t+1} - \alpha_j\chi_j b^h_{j,t+1} - b^g_{j,t+1}\right) + f_{j,t}$.

   (c) $(1-\alpha_j)\,\tilde{y}_{j,t} = \alpha_j\chi_j\left(\frac{b^h_{j,t+1}}{1+r_{j,t}} - b^h_{j,t}\right) + \alpha_j(1-\chi_j)\left(s_{j,t} - \frac{s_{j,t+1}}{1+r_{j,t}}\right) + \frac{b^g_{j,t+1}}{1+r_{j,t}} - b^g_{j,t} + f_{j,t}$.

We then feed exogenous processes for the different shocks (using rebased values) for observed household debt $b^h_{j,t}$, fiscal policy $\tau_{j,t}$, $T_{j,t}$ and $g_{j,t}$, foreign demand $f_{j,t}$ and interest rate spreads $\rho_{j,t}$. For each country, we simulate the path between 2001 and 2012 of nominal GDP $y_{j,t}$, nominal consumption $x_{j,t}$, employment $n_{j,t}$, net exports $e_{j,t}$ and public debt $b^g_{j,t}$.

The normalized data on observed shocks that serve to feed the model for each country are shown in figures (19), (20), (21) and (22) in appendix.

Just to be clear, there is no degree of freedom in our simulations of nominal variables. There is *no* parameter which is set to match any moment in the data. The model is entirely constrained by observable micro estimates and by equilibrium conditions. The only parameter that we can adjust is the slope of the Phillips curve κ but it does not affect the GDP in euros, it only pins down the allocation of nominal GDP between prices (unit labor cost) and quantities (employment).

Figures (10), (11), (12), (13), (14), show the simulated and observed nominal GDP, net exports, employment, prices and public debt. The reduced form model reproduces very well the cross sectional dynamics in the euro zone for nominal GDP and net exports. In particular, it replicates well the boom and bust dynamics on GDP and the current account reversal for the crisis hit countries. For employment, the model does also well for the crisis countries and for countries that were hit less severely. This is also true for unit labor costs, apart from Italy in the later period and also to some extent Greece. Finally, the reduced form model also generates public debt dynamics that are very close to the observed data.

Figure 10: Simulated and observed nominal GDP; all shocks

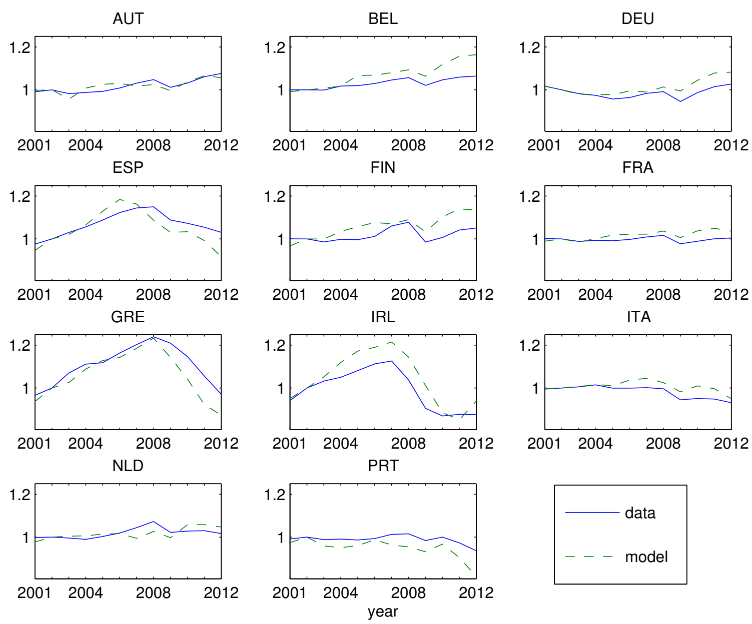


Figure 11: Simulated and observed net exports

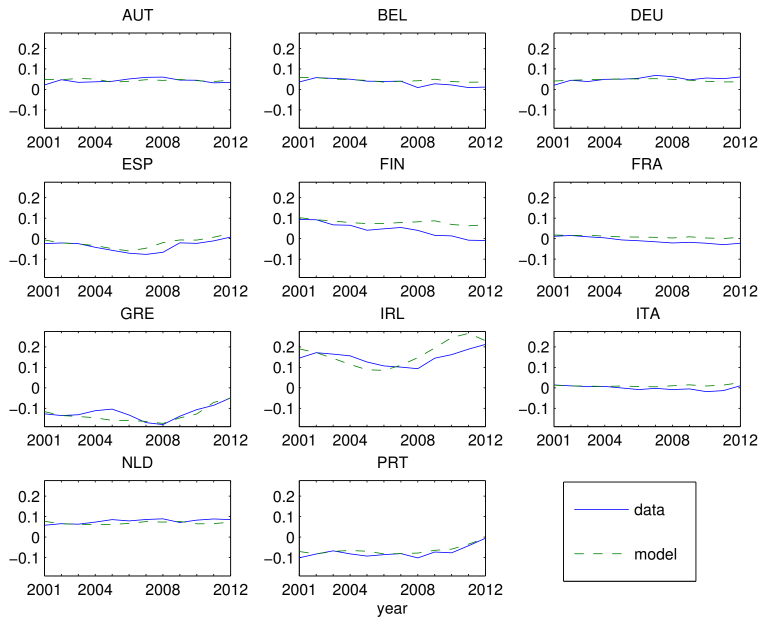


Figure 12: Simulated and observed employment

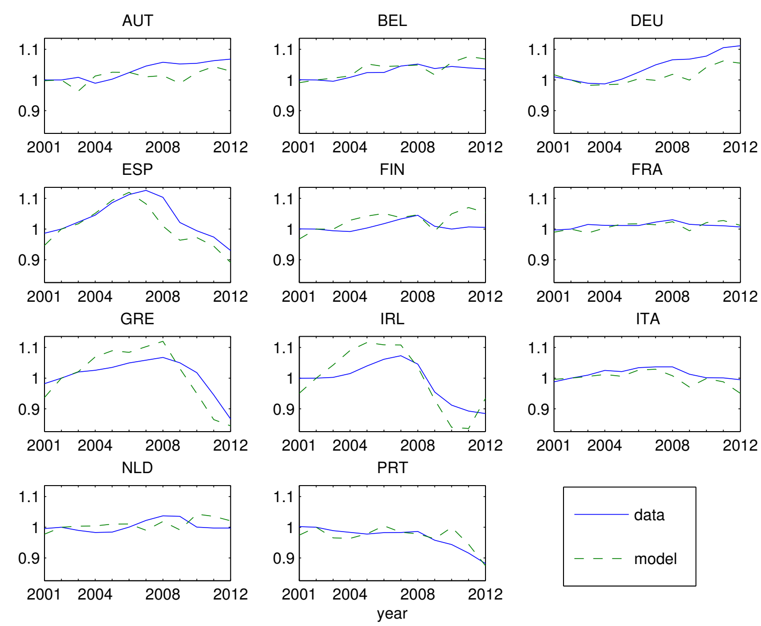


Figure 13: Simulated and observed unit labor costs

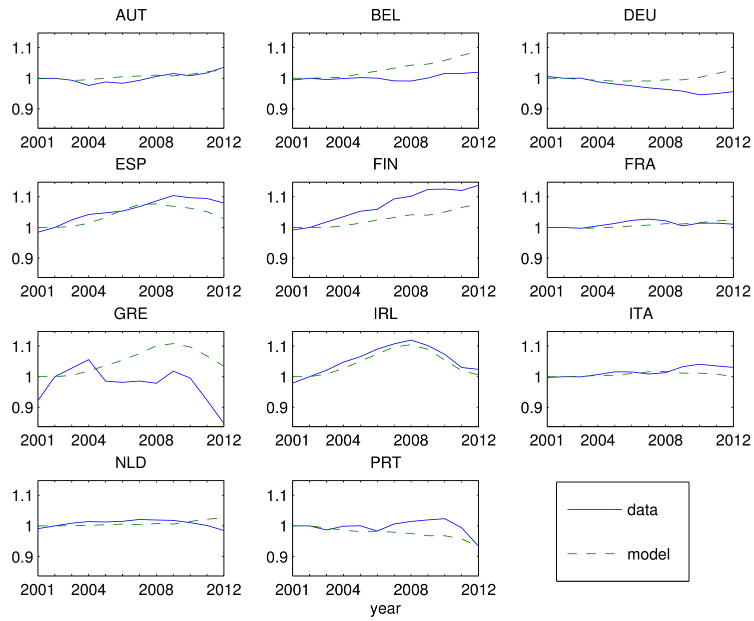
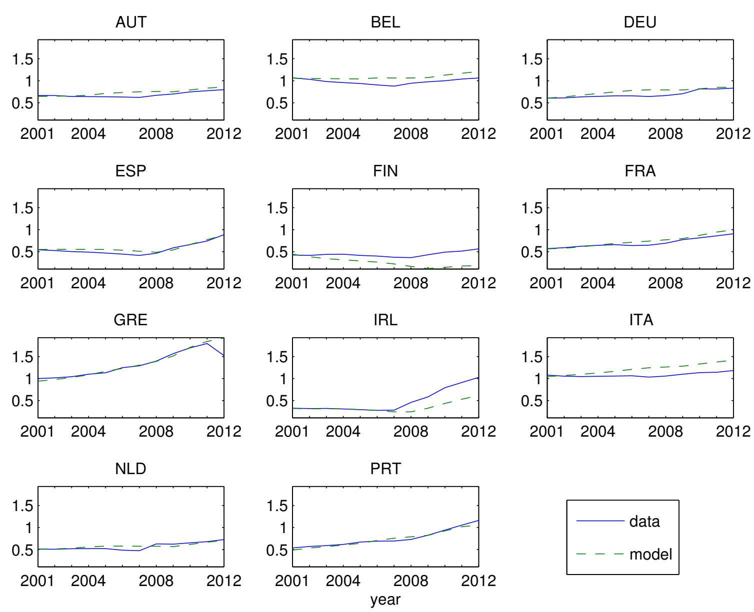


Figure 14: Simulated and observed government debt



3.3 Structural Experiments

We now present our main experiments. The goal is to provide counter-factual simulations of what would have happened to each country had it followed a different set of policies.

We consider two policies:

- fiscal policy. We consider a different fiscal policy during the boom years. This leads to lower debt levels in some countries.
- macro-prudential policies. We ask what would have happened if the government had restricted private lending.

The identification is based on the following assumptions:

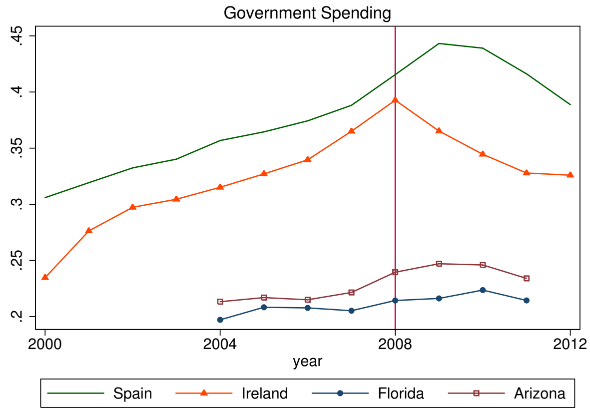
Private leverage. We use the US as a control group. More precisely, we estimate the following model for deleveraging in a panel of U.S. states

$$b_{j,t}^{h,US} = \sum_{k=1}^{K} \alpha_k^{US} b_{j,t-k}^{h,US} + \epsilon_{j,t}$$

for $t = 2008, .., 2012$ and $j = 1, ..52$. Note that $b_{j,t}^{h}$ is detrended household debt defined exactly as for the Eurozone. The idea is that these private leverage bubbles reflected various global and financial factors: low real rates, financial innovations, regulatory arbitrage of the Basel rules by banks, real estate bubbles, etc. To a large extent these forces were present both in Europe and in the US. The difference of course is that there was no sudden stops within the US. Hence, we interpret the US experience as representative of a deleveraging outcome in a monetary union without sudden stops.

Another issue is whether fiscal policy was not also active in the US. Perhaps private debt bubbles were associated with large fiscal revenues and large spending. This probably happened to some extent, but compared to the Eurozone, these effects are very small (of course we are only talking about cross-sectional variation in government spending). Figure 15 shows this for two states and two countries. A regression for all the states and all the countries shows that the link between private debt and government spending was 4 times smaller in the US than in Europe.

Figure 15: Government Spending

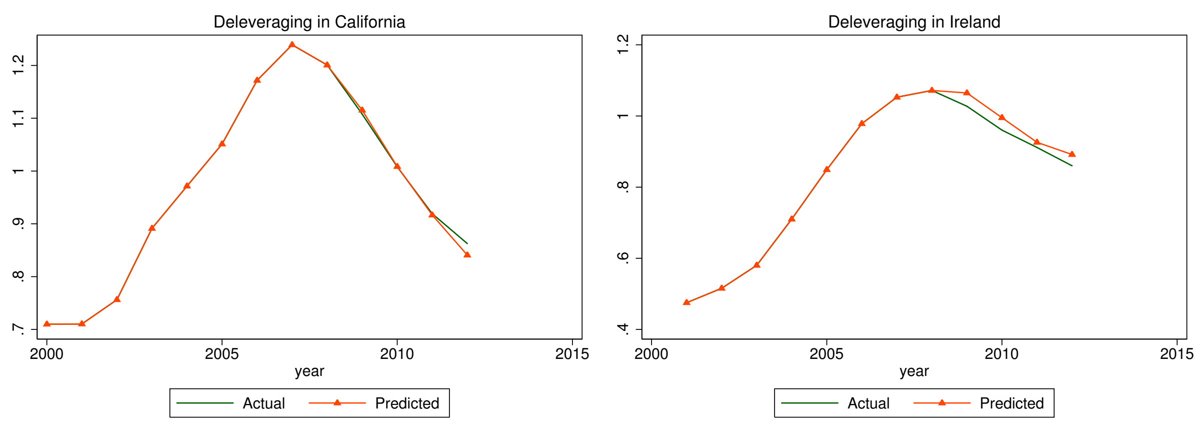


We therefore argue that the US provides a benchmark for private deleveraging without sudden stops, and with relatively neutral (cross-sectional) fiscal policy. We then take the estimated coefficients α_k^{US} and use them to construct predicted deleveraging in Eurozone countries:

$$\hat{b}_{j,t}^{h} = \sum_{k=1}^{K} \alpha_k^{US} b_{j,t-k}^{h,}$$

for t=2008, .., 2012 and j=1..11. Figure 16 illustrates the results for California and Ireland. The model predicts a somewhat slower deleveraging in Ireland than actually happened. This is also the case for other countries that experienced a sudden stop.

Figure 16: Private Deleveraging



Structural Equations of the Model. We now posit the following structural equations. Observed private leverage is given by:

$$b^h_{j,t} = \lambda^{bh}\hat{b}^h_{j,t} + \lambda^{\rho h}\rho_{j,t} \tag{15}$$

Private leverage is what it would have been based on the US experience minus the impact of the spread. This impact can capture funding costs for banks and also the fact that debt demand by impatient agents is not entirely interest inelastic as we have assumed in our model so far.

The second equation captures bond pricing:

$$\rho_{j,t} = \sigma_{j,t} \times \left(\lambda^{g\rho} b^g_t + \lambda^{h\rho} b^h_t\right) \tag{16}$$

where σ captures the possibility of a sudden stop. This equation says that funding cost diverge when there is a sudden stop and that the extent to which this happens depends on public and private debt. This captures financial frictions associated with high leverage (debt overhang, risk shifting, adverse selection, runs, etc.)

We need to estimate the coefficients λ's on the period 2008-2012. We do so by running instrumental variable regressions. Clearly,$\hat{b}^h_{j,t}$, the deleveraging in Eurozone countries predicted by the US experience, is a valid instrument for $b^h_{j,t}$. For government debt for $b^g_{j,t}$, we use debt lagged three years as an instrument.

The coefficients (with standard errors in parenthesis) are shown in Table (2) :

Table 2: Coefficients Estimated with Instrumental Variables.

λ^{bh}	$\lambda^{\rho h}$	$\lambda^{g\rho}$	$\lambda^{h\rho}$
0.967	-0.418	6.05	3.2
(0.007)	(0.071)	(0.96)	(0.015)

Policy Functions The last task is to specify the alternative policy functions. We assume that the government attempts to stabilize the economy but is constrained by its cost of funds. We also assume that it can potentially react asymmetrically to an increase and a decrease of household debt so that stabilization is not symmetric over the cycle.

$$\begin{aligned} b^g_{j,t+1} - b^g_{j,t} &= -\gamma^{hU}\alpha_j\chi_j\left(b^h_{j,t+1} - b^h_{j,t}\right) - \gamma^\rho \rho_{j,t} \; if \; b^h_{j,t+1} > b^h_{j,t} \\ b^g_{j,t+1} - b^g_{j,t} &= -\gamma^{hD}\alpha_j\chi_j\left(b^h_{j,t+1} - b^h_{j,t}\right) - \gamma^\rho \rho_{j,t} \; if \; b^h_{j,t+1} < b^h_{j,t}. \end{aligned} \tag{17}$$

Counterfactual experiments For the counterfactual experiments, we use the cross sectional time series for household debt and spreads that are constructed using the estimates from the structural regressions in equations (15) and (16). Given that the fiscal policy function in equation (17) maps changes in household debt and the level of spreads to changes in government debt, we can calculate the time series for government debt on the period 2001-12, using debt in 2000 as an initial point. To simulate the counterfactual fiscal policy, transfers and taxes are fixed at their 2002 levels and the level of government spending is calibrated so that the path of government debt implied by the model matches the time series for government debt in equation (17).

It is important to distinguish 3 possible histories.

First there is the actual history, which corresponds to governments behaving in a certain way during the boom, then trying to enact counter-cyclical policies in the bust, but also being constrained by sudden stops. All of these forces create the actual path of fiscal policy (i.e. government debt). Then there are two counter-factual policies. One is: what would have happened without sudden stops? We can obtain this by removing the feed-back loop between fiscal policy and spreads. In this case, we want the government to be able to stabilize the economy. This allows us to calibrate the parameters γ^{hD}, γ^{hU}. When we add the sudden stop, we then want to reproduce a path of fiscal policy that is close to the actual one. This pins down γ^{ρ}

In other words, we choose our policy parameters such that:

1. absent sudden stops the government would have stabilized employment;

2. the impact of sudden stops is such that it forces the predicted policy to be close to the actual ones.

This leads us to choose the parameters given in Table (3) :

Table 3: Policy coefficients

γ^{hU}	γ^{hD}	γ^{ρ}
0.5	2.5	2

We then ask how countries would have fared if they had followed different policies during the boom. A more conservative fiscal policy during the boom produces:

1. an alternative path for government debt during the boom

2. different amounts of fiscal slacks leading to different spreads

3. the spreads feeding back to the economy via the structural parameters and the Euler equation, and the actual, constrained, fiscal policy in the bust

We find in preliminary results that

- All countries would have stabilized successfully if they had had a more conservative policy during the boom;
- The spreads would have been smaller;
- The implied path for government debt is of course different from the actual one or the one simulated in the reduced form simulations. We know the later two are very similar. The counterfactual more conservative fiscal path is described in Table (4) and compared to the one simulated in the reduced form simulations which we interpret as the actual one.

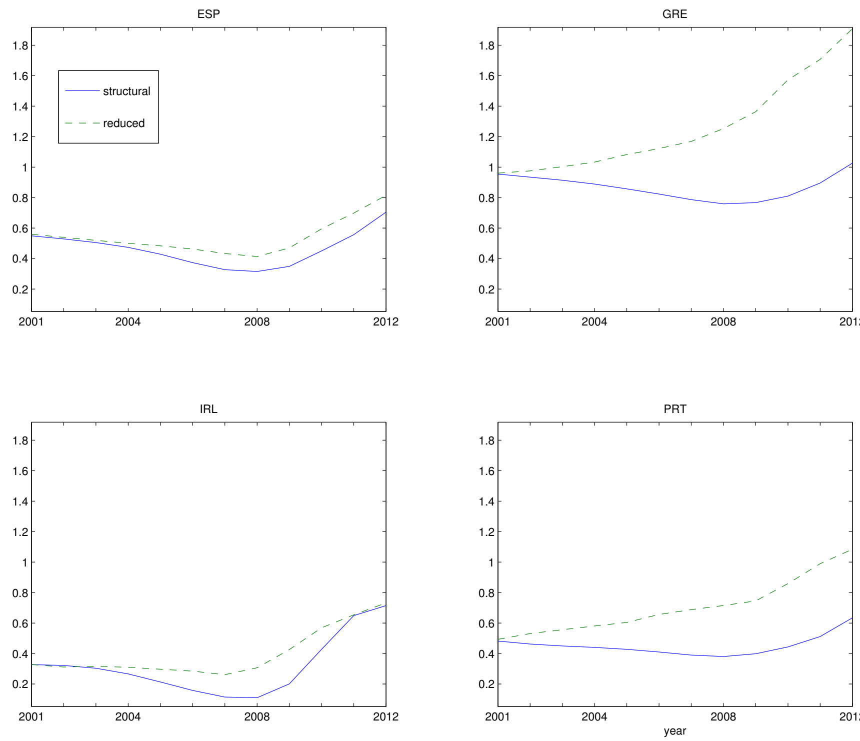
Table 4: Simulated (reduced form) and counterfactual debt to benchmark GDP ratio in 2008

Country	Simulated (reduced form)	Counter-factual	Difference
Spain	0.5	0.35	0.15
Ireland	0.35	0.05	0.3
Greece	1.2	0.8	0.4
Portugal	0.7	0.4	0.3

Greece is the one that deviates the most as such policy would entail a massive reduction in public debt in 2008 compared to the actual one. For Ireland, it implies extremely low levels of debt in 2008 (even negative in some calibrations). This suggests that fiscal policy is unlikely to be sufficient as a stabilization tool in case of a large private credit boom. For Spain, the difference in public debt is much smaller.

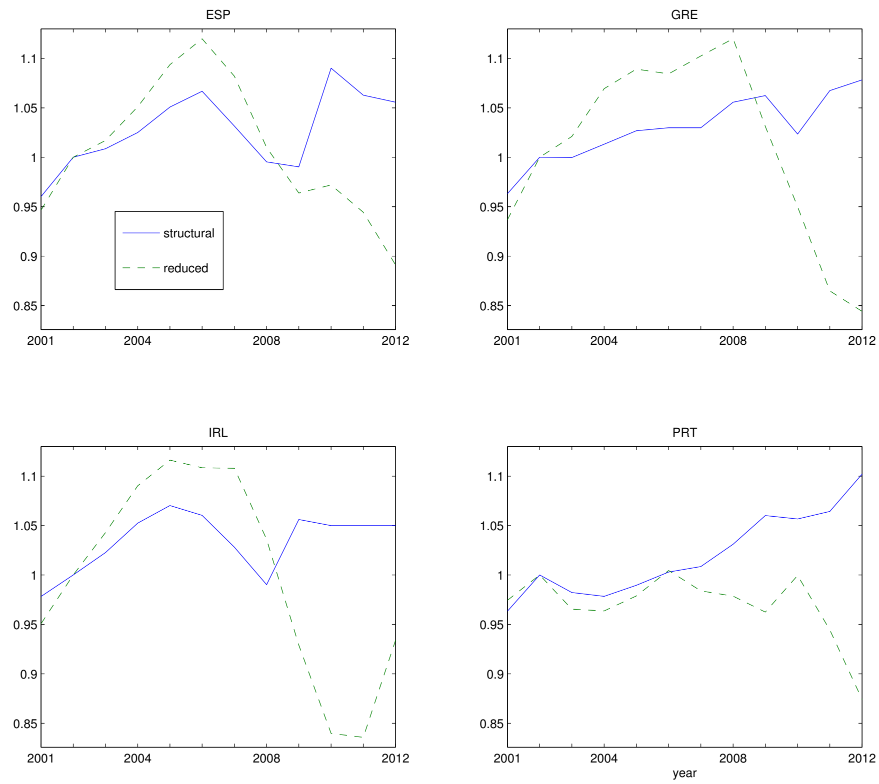
Figure (17) illustrates these differences in paths of actual and counterfactual public debt. Figure (18) shows the impact on employment. The difference is massive for employment in Greece and Ireland with, in 2011, an increase of around 25% in employment with respect to the observed path. In Portugal, and Spain the difference is around 14%.

Figure 17: Government Debt: Reduced Form Fiscal Policy and Simulated with Counterfactual Policy



"Reduced" represents the actual path of public debt. "Structural" uses the structural model with identified shocks and an alternative fiscal policy function.

Figure 18: Employment: Reduced form and Simulated with Counterfactual Policy



"Reduced" represents the actual path of public debt. "Structural" uses the structural model with identified shocks and an alternative fiscal policy function.

4 Conclusion

Understanding the dynamics of the Eurozone is the major challenge for macroeconomics and monetary economics. Eurozone countries have experienced extraordinary levels of real and financial volatility. Unemployment rates have diverged to an extent that nobody had anticipated. Financial flows have been extremely volatile and for the first time in history there have been major sudden stop in a common currency area.Yet very little progress has been made. While most observers recognize that private leverage, fiscal policy and sudden stops matter, no one has proposed a way to analyze them jointly, let alone propose a way to disentangle them.

Our paper makes three contributions. First, we present a model that accounts at the same time for domestic credit, fiscal policy, and current account dynamics. Second, we create a data set for 11 countries over 13 years that covers the variables of interest and deal with the various accounting issues. Third, and most importantly, we propose a new identification strategy that allows us to run counter-factual experiments on fiscal policy as well as macro-prudential policy.

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Appendix

A Model

A.1 Scaling

We have already defined the Euler equations

$$\mathbb{E}_t\left[X_{s,j,t+1}\right] \approx \beta\left(1 + r_{s,j,t}\right) X_{s,j,t}.$$

and

$$\mathbb{E}_t\left[X^*_{s,t+1}\right] \approx \beta\left(1 + r^*_{s,t}\right) X^*_{s,t}.$$

and the spread as

$$1 + \rho_{j,t} \equiv \frac{1 + r_{s,j,t}}{1 + r^*_{s,t}}$$

If we scale the budget constraint (assuming perfect foresight, or equivalently, neglecting the conditional variance of the aggregate shocks), we get

$$\frac{X_{s,j,t}}{X^*_{s,t}} + \frac{S_{j,t+1}}{X^*_{s,t+1}} \frac{\frac{X^*_{s,t+1}}{X^*_{s,t}}}{1 + r_{s,j,t}} = \frac{S_{j,t}}{X^*_{s,t}} + \frac{\tilde{Y}_{j,t}}{X^*_{s,t}}$$

and up to the usual approximation we have

$$x_{s,j,t} + \frac{\beta}{1 + \rho_{j,t}} s_{j,t+1} = s_{j,t} + \tilde{y}_{j,t}$$

A.2 Phillips Curve

$$\frac{\left(p^h_{j,t}\right)^2}{p^h_{j,t-1}} + \left(\kappa N^\star - 1\right) p^h_{j,t} - \kappa Y_{j,t} = 0$$

Defining $\Delta \equiv \left(\kappa n^\star - 1\right)^2 + 4\kappa \frac{Y_{j,t}}{p^h_{j,t-1}}$, we find that

$$\frac{p^h_{j,t}}{p^h_{j,t-1}} = \frac{1 - \kappa n^\star + \sqrt{\Delta}}{2}$$

Note that if $\frac{Y_{j,t}}{p^h_{j,t-1}} = n^\star$, then $\Delta = \left(\kappa n^\star + 1\right)^2$, and $\frac{p^h_{j,t}}{p^h_{j,t-1}} = 1$. We also experiment with asymmetric wage rigidity where wages are more flexible upward (when the output gap is positive) than downward (when the output gap is negative)[10]: $\kappa = \kappa_d 1_{n_{j,t} < n^\star} + \kappa_u 1_{n_{j,t} > n^\star}$ where $\kappa_d < \kappa_u$.

[10] For empirical evidence on downward nominal wage rigidity, see for example, Schmitt-Grohe and Uribe (2012) on periphery countries in Europe.

A.3 Budget constraints.

Let us first rewrite the budget constraints and market clearing conditions. For the sake of generality, let us allow for three different interest rates for borrowers, savers, and the government: $r_{b,j,t}$, $r_{s,j,t}$, and $r_{g,j,t}$. Using the market clearing condition, and competition $p^h_{j,t} = w_{j,t}$, we get

$$y_{j,t} = \alpha_j \left(\chi_j x_{b,j,t} + (1-\chi_j)\, x_{s,j,t}\right) + f_{j,t} + g_{j,t}.$$

Nominal exports are $\bar{x}_f$, nominal imports are $(1-\chi_j)\, p^f_{j,t} c^f_{s,j,t} + \chi_j p^f_{j,t} c^f_{b,j,t}$ since the government does not buy imported goods. So net exports are

$$e_{j,t} = f_{j,t} - (1-\alpha_j)\left(\chi_j x_{b,j,t} + (1-\chi_j)\, x_{s,j,t}\right).$$

We define disposable (after-tax) income as

$$\tilde{y}_{j,t} \equiv (1-\tau_{j,t})\, y_{j,t} + T_{j,t}.$$

We can then write the system for *nominal variables*

- $x_{b,j,t} = \frac{b^h_{j,t+1}}{1+r_{b,j,t}} + \tilde{y}_{j,t} - b^h_{j,t}$, budget constraint of impatient agents
- $x_{s,j,t} = s_{j,t} + \tilde{y}_{j,t} - \frac{s_{j,t+1}}{1+r_{s,j,t}}$, budget constraint of patient agents
- $y_{j,t} = \alpha_j \left(\chi_j x_{b,j,t} + (1-\chi_j)\, x_{s,j,t}\right) + f_{j,t} + g_{j,t}$, market clearing
- $g_{j,t} + T_{j,t} - \tau_{j,t} y_{j,t} = \frac{b^g_{j,t+1}}{1+r_{g,j,t}} - b^g_{j,t}$, budget constraint of the government
- $e_{j,t} = \frac{1}{\alpha_j}\left(f_{j,t} - (1-\alpha_j)\,(y_{j,t} - g_{j,t})\right)$, definition of net exports

Combining the first four equations, we get market clearing at time t:

$$(1-\alpha_j)\,\tilde{y}_{j,t} = \alpha_j \chi_j \left(\frac{b^h_{j,t+1}}{1+r_{b,j,t}} - b^h_{j,t}\right) + \alpha_j (1-\chi_j)\left(s_{j,t} - \frac{s_{j,t+1}}{1+r_{s,j,t}}\right) + \frac{b^g_{j,t+1}}{1+r_{g,j,t}} - b^g_{j,t} + f_{j,t}. \tag{18}$$

It will often be useful to obtain a recursive equation for nominal GDP. Taking the first difference of (18) we get

$$\begin{aligned}(1-\alpha_j)\,\Delta\left[\tilde{y}_{j,t}\right] &= \chi_j \alpha_j \left(\beta \Delta\left[\frac{b^h_{j,t+1}}{1+\rho_{j,t}}\right] - \Delta\left[b^h_{j,t}\right]\right) - (1-\chi_j)\,\alpha_j \left(\beta\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}}\right] - \Delta\left[s_{j,t}\right]\right) \\ &\quad + \beta\Delta\left[\frac{b^g_{j,t+1}}{1+\rho_{j,t}}\right] - \Delta\left[b^g_{j,t}\right] + \Delta\left[f_{j,t}\right]\end{aligned}$$

A.4 Pseudo-Steady State

We consider a steady state with constant interest rates equal to the rate of time preference of savers, i.e. $\beta(1+r) = 1$ and the spread is zero: $\rho = 0$. The borrowing limit $\bar{b}_j$ is exogenous and we consider equilibria where the borrowing constraint 3 binds. Our notion of steady state is complicated by the fact that savings $s_{j,t}$ are history-dependent. We define a steady state as the long run equilibrium of an economy with initial savings s_j and government debt b_j^g, subject to no further shocks, constant government spending and constant government debt. All nominal quantities are constant and employment is at its natural rate $n^{\star}$.[11] The long-run equilibrium conditions are

$$\begin{aligned}
p_j^h n^{\star} &= \chi_j \alpha_j x_{b,j} + (1-\chi_j)\,\alpha_j x_{s,j} + f + g_j \\
x_{b,j} &= \tilde{y}_j - \frac{r}{1+r} b_j^h \\
x_{s,j} &= \tilde{y}_j + \frac{r}{1+r} s_j \\
\tau_j p_j^h n^{\star} &= g_j + T_j + \frac{r}{1+r} b_j^g
\end{aligned}$$

Nominal output (the price of home goods) is pinned down by

$$p_j^h n^{\star} = \alpha_j \left((1-\tau_j)\, p_j^h n^{\star} + T_j \right) + \alpha_j \frac{r}{1+r} \left((1-\chi_j)\, s_j - \chi_j \bar{b}_j \right) + f_j + g_j .$$

There are several ways to specify government policy. Here we assume that the policy is to keep government debt and nominal spending g_j constant. Long run nominal output is then given by

$$p_j^h n^{\star} = \frac{\alpha_j}{1-\alpha_j} r a_j + \frac{f_j}{1-\alpha_j} + g_j \tag{19}$$

where recall that we have defined a as net foreign assets. This equation shows the determinants of the long run price level. The long run price level depends on the exogenous components of spending: net asset income, foreign demand, and government spending. All these are inflationary. For a given tax rate τ_j, transfers are then chosen to satisfy the government's budget constraint:

$$T_j = \tau_j p_j^h n^{\star} - g_j - \frac{r}{1+r} b_j^g .$$

A.5 Euler Equation and Expected Income

We assume that interest rates are the same within a country: $r_{b,j,t} = r_{s,j,t} = r_{g,j,t}$. The crucial variable is now the response of savers' consumption. The Euler equation of savers is

$$\frac{1}{x_{s,j,t}} = (1+\rho_{j,t})\, \mathbb{E}_t \left[\frac{1}{x_{s,j,t+1}} \right] .$$

and we use the linear approximation

$$\mathbb{E}_t \left[x_{s,j,t+1} \right] = (1+\rho_{j,t})\, x_{s,j,t} .$$

Consider the following experiment. Savers enter the period with a given level of savings. Then there is a shock to interest rates. For instance, starting from a steady state where $\rho = 0$, if the new rate is such

[11] We consider here the case where labor supply is inelastic, so $n^{\star}$ is effectively exogenous. The case of elastic labor supply is presented in the Appendix. The differences are entirely predictable: $n^{\star}$ depends on τ and is not the same for borrowers and for savers. These effects are rather small and essentially irrelevant for the dynamics that we study, so we leave them out of the main body of the paper.

that $\rho > 0$, savings jumps up, and if the new rate is such that $\rho < 0$, consumption jumps up. The budget constraint at time t is

$$x_{s,j,t} = s_{j,t} + \tilde{y}_{j,t} - \frac{s_{j,t+1}}{1+\rho_{j,t}}$$

and the expected budget constraint at time $t+1$ is

$$\mathbb{E}_t \left[\frac{s_{j,t+2}}{1+\rho_{j,t+1}} \right] = s_{j,t+1} + \mathbb{E}_t \left[\tilde{y}_{j,t+1} - x_{s,j,t+1} \right].$$

Combining the budget constraints and the linearized Euler equation, we get

$$(1+\beta)s_{j,t+1} + \mathbb{E}_t \left[\tilde{y}_{j,t+1} \right] - \beta \mathbb{E}_t \left[\frac{s_{j,t+2}}{1+\rho_{j,t+1}} \right] = (1+\rho_{j,t}) \left(s_{j,t} + \tilde{y}_{j,t} \right). \tag{20}$$

since on average $\beta(1+r) = 1$.

A.6 Equilibrium Conditions of the Model

Detrending and assuming that rates are the same within a country, we have:

- $\frac{1}{x_{s,j,t}} = (1+\rho_{j,t}) \mathbb{E}_t \left[\frac{1}{x_{s,j,t+1}} \right]$, Euler equation
- $x_{b,j,t} = \frac{b^h_{j,t+1}}{1+\rho_{j,t}} + \tilde{y}_{j,t} - b^h_{j,t}$, budget constraint of impatient agents
- $x_{s,j,t} = s_{j,t} + \tilde{y}_{j,t} - \frac{s_{j,t+1}}{1+\rho_{j,t}}$, budget constraint of patient agents
- $y_{j,t} = \alpha_j \left(\chi_j x_{b,j,t} + (1-\chi_j) x_{s,j,t} \right) + f_{j,t} + g_{j,t}$, market clearing
- $g_{j,t} + T_{j,t} - \tau_{j,t} y_{j,t} = \frac{b^g_{j,t+1}}{1+\rho_{j,t}} - b^g_{j,t}$, budget constraint of the government
- $e_{j,t} = \frac{1}{\alpha_j} \left(f_{j,t} - (1-\alpha_j)(y_{j,t} - g_{j,t}) \right)$, net exports
- $\tilde{y}_{j,t} = (1-\tau_j) y_{j,t} + T_{j,t}$, disposable income
- $(1-\alpha_j) \tilde{y}_{j,t} = \alpha_j \chi_j \left(\frac{b^h_{j,t+1}}{1+\rho_{j,t}} - b^h_{j,t} \right) + \alpha_j (1-\chi_j) \left(s_{j,t} - \frac{s_{j,t+1}}{1+\rho_{j,t}} \right) + \frac{b^g_{j,t+1}}{1+\rho_{j,t}} - b^g_{j,t} + f_{j,t}$, market clearing (alternative)
- $n_{j,t} = \frac{y_{j,t}}{p^h_{j,t}}$, labor market
- $\frac{p^h_{j,t} - p^h_{j,t-1}}{p^h_{j,t-1}} = \kappa \left(n_{j,t} - n^\star \right)$, inflation

A.7 Martingales and *iid* Spreads

We simplify the analysis by assuming martingales for the exogenous shocks $b^h_{j,t}$ and $F_{j,t}$: $\mathbb{E}_t \left[b^h_{j,t+2} \right] = b^h_{j,t+1}$ and $\mathbb{E}_t \left[f_{j,t+1} \right] = f_{j,t}$. We also assume that the policy function of the government is such that $\mathbb{E}_t \left[b^g_{j,t+2} \right] = b^g_{j,t+1}$. Finally we assume that interest rates are iid:

$$\mathbb{E}_t \left[\rho_{j,t+1} \right] = 0$$

We then have the following Lemma.

Lemma 3. $\mathbb{E}_t\left[s_{j,t+2}\right]=s_{t+1}$

We guess and verify that this is correct. Suppose the Lemma is true, then we obtain two important equations. Equation (20) becomes

$$s_{j,t}=\left(1+\rho_{j,t}\right)\left(s_{j,t-1}+\tilde{y}_{j,t}\right)-\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right], \tag{21}$$

and expected market clearing at $t+1$ is

$$\left(1-\alpha_j\right)\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]=(1-\beta)\left(\alpha_j\left(1-\chi_j\right)s_{j,t+1}-\alpha_j\chi b^h_{j,t+1}-b^g_{j,t+1}\right)+f_{j,t}. \tag{22}$$

Therefore

$$\left(1-\alpha_j\right)s_{j,t+1}=\left(1-\alpha_j\right)\left(1+\rho_{j,t}\right)\left(s_{j,t}+\tilde{y}_{j,t}\right)-(1-\beta)\left(\alpha_j\left(1-\chi_j\right)s_{j,t+1}-\alpha_j\chi b^h_{j,t+1}-b^g_{j,t+1}\right)-\frac{f_{j,t}}{1-\alpha_j}.$$

Using market clearing at time t we get

$$\frac{s_{j,t+1}}{1+\rho_{j,t}}-s_{j,t}=\frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\left(\frac{b^h_{j,t+1}}{1+\rho_{j,t}}-b^h_{j,t}\right)+\frac{1}{1-\alpha_j\chi_j}\left(\frac{b^g_{j,t+1}}{1+\rho_{j,t}}-b^g_{j,t}+\frac{\rho_{j,t}}{1+\rho_{j,t}}\frac{f_{j,t}}{1-\alpha_j}\right) \tag{23}$$

Savings inherit the dynamic properties of $B^h_{j,t}$ and $b^g_{j,t}$. Since we assume small shocks and $\mathbb{E}_t\left[\rho_{j,t+1}\right]=0$, this validates our conjecture that $\mathbb{E}_t\left[s_{j,t+1}\right]=s_{j,t}$.

A.8 Equilibrium Conditions with Martingales and *iid* Spreads

Equilibrium conditions

1. $s_{j,t+1}=\left(1+\rho_{j,t}\right)\left(s_{j,t}+\tilde{y}_{j,t}\right)-\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$
2. $\left(1-\alpha_j\right)\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]=\frac{r}{1+r}\left(\alpha_j\left(1-\chi_j\right)s_{j,t+1}-\alpha_j\chi_j b^h_{j,t+1}-b^g_{j,t+1}\right)+f_{j,t}.$
3. $\left(1-\alpha_j\right)\tilde{y}_{j,t}=\alpha_j\chi_j\left(\frac{b^h_{j,t+1}}{1+\rho_{j,t}}-b^h_{j,t}\right)+\alpha_j\left(1-\chi_j\right)\left(s_{j,t}-\frac{s_{j,t+1}}{1+\rho_{j,t}}\right)+\frac{b^g_{j,t+1}}{1+\rho_{j,t}}-b^g_{j,t}+f_{j,t}.$
4. $g_{j,t}+T_{j,t}-\tau_j y_{j,t}=\frac{b^g_{j,t+1}}{1+\rho_{j,t}}-b^g_{j,t}$
5. $\tilde{y}_{j,t}=\left(1-\tau_j\right)y_{j,t}+T_{j,t}$

There are five equations and five unknowns: $\tilde{y}_{j,t}$, $y_{j,t}$, $b^g_{j,t+1}$, $s_{j,t+1}$, $\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$. The exogenous shocks are $b^h_{j,t}$, $f_{j,t}$ and $\rho_{j,t}$. The policy shocks are $T_{j,t}$ and $g_{j,t}$ and τ_j is constant. The state space of predetermined endogenous variable is $s_{j,t}$ and $b^g_{j,t}$.

Combining 1 and 2 we can get rid of $\mathbb{E}_t[\tilde{y}_{j,t+1}]$ to get a system with 4 unknowns

$$\begin{aligned}
(1-\alpha_j)\, s_{j,t+1} &= (1-\alpha_j)(1+\rho_{j,t})(s_{j,t}+\tilde{y}_{j,t}) - \frac{r}{1+r}\left(\alpha_j(1-\chi_j)\, s_{j,t+1} - \alpha_j\chi_j b^h_{j,t+1} - b^g_{j,t+1}\right) - f_{j,t} \\
(1-\alpha_j)\,\tilde{y}_{j,t} &= \alpha_j\chi_j\left(\frac{b^h_{j,t+1}}{1+\rho_{j,t}} - b^h_{j,t}\right) + \alpha_j(1-\chi_j)\left(s_{j,t} - \frac{s_{j,t+1}}{1+\rho_{j,t}}\right) + \frac{b^g_{j,t+1}}{1+\rho_{j,t}} - b^g_{j,t} + f_{j,t} \\
\frac{b^g_{j,t+1}}{1+\rho_{j,t}} - b^g_{j,t} &= g_{j,t} + T_{j,t} - \tau_j y_{j,t} \\
\tilde{y}_{j,t} &= (1-\tau_j)\, y_{j,t} + T_{j,t}
\end{aligned}$$

Policy Rule of Government We specify the general policy rule as follows

1. Fiscal variables $g_{j,t}$ and $T_{j,t}$ are pre-determined. Solve the 4*4 system above.
2. Get $b^g_{j,t+1}$ from
$$\frac{b^g_{j,t+1}}{1+\rho_{j,t}} = b^g_{j,t} + g_{j,t} + T_{j,t} - \tau_j y_{j,t}$$
3. Set transfers $T_{j,t+1}$ for next period so that $\mathbb{E}_t\left[b^g_{j,t+2}\right] = b^g_{j,t+1}$ assuming martingales for $p^h_{j,t} g_{j,t}$. Then the expected budget constraint is
$$\tau_j \mathbb{E}_t[y_{j,t+1}] = g_{j,t} + T_{j,t+1} + (1-\beta)\, b^g_{j,t+1}$$
and $\mathbb{E}_t[\tilde{y}_{j,t+1}]$ is known from the 4*4 system
$$\mathbb{E}_t[\tilde{y}_{j,t+1}] = (1-\tau_j)\,\mathbb{E}_t[y_{j,t+1}] + T_{j,t+1}$$
so we can solve for $T_{j,t+1}$:
$$T_{j,t+1} = \tau_j \mathbb{E}_t[\tilde{y}_{j,t+1}] - (1-\tau_j)\, g_{j,t} - (1-\tau_j)(1-\beta)\, b^g_{j,t+1}$$

A.9 The Case of Constant Spreads

Savers' Spending with Constant Spreads With constant interest rates, the Euler equation is simply $x_{s,j,t} = \mathbb{E}_t[x_{s,j,t+1}]$ and equation (21) becomes $s_{j,t+1} - s_{j,t} = \tilde{y}_{j,t} - \mathbb{E}_t[\tilde{y}_{j,t+1}]$. Combining market clearing at t and $t+1$, we get a simple equation

$$(1-\alpha_j)(\tilde{y}_{j,t} - \mathbb{E}_t[\tilde{y}_{j,t+1}]) = \alpha_j\chi_j\left(\bar{b}_{j,t+1} - B^h_{j,t}\right) - \alpha_j(1-\chi_j)(s_{j,t+1} - s_{j,t}) + b^g_{j,t+1} - b^g_{j,t}$$

Equation (23) becomes

$$s_{j,t+1} - s_{j,t} = \frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\left(\bar{b}_{j,t+1} - B^h_{j,t}\right) + \frac{1}{1-\alpha_j\chi_j}\left(b^g_{j,t+1} - b^g_{j,t}\right). \tag{24}$$

The first difference of the disposable income equation is

$$(1-\alpha_j)\,\Delta\left[\tilde{y}_{j,t}\right]=\chi_j\alpha_j\left(\frac{\Delta\left[\bar{b}_{j,t+1}\right]}{1+r}-\Delta\left[B^h_{j,t}\right]\right)-(1-\chi_j)\,\alpha_j\left(\frac{\Delta\left[s_{j,t+1}\right]}{1+r}-\Delta\left[s_{j,t}\right]\right)+\frac{\Delta\left[b^g_{j,t+1}\right]}{1+r}-\Delta\left[b^g_{j,t}\right]+\Delta\left[f_{j,t}\right]$$

Using equation (24), we then see that

$$\Delta\tilde{y}_{j,t}=\frac{\Delta s_{j,t+1}}{1+r}-\Delta s_{j,t}+\frac{\Delta\left[F_{j,t}\right]}{1-\alpha_j}. \tag{25}$$

Equation (25) has an important implication. Using the budget constraint of the patient agents, we get

$$\begin{aligned}\Delta x_{s,j,t} &= \Delta s_{j,t}+\Delta\tilde{y}_{j,t}-\frac{\Delta s_{j,t+1}}{1+r}\\ &= \frac{\Delta\left[f_{j,t}\right]}{1-\alpha_j}\end{aligned}$$

Therefore we have the following Lemma

Lemma 4. *The nominal spending of patient agents remains constant in response to any sequence of shocks to the debt of impatient agents and of the government.*

This is important because it means that our linear approximation for x is in fact exact for shocks to $\bar{b}_{j,t+1}$ and to $b^g_{j,t+1}$. Note that we can also express the evolution of disposable income as a function of exogenous shocks

$$\Delta\tilde{y}_{j,t}=\frac{\chi_j\alpha_j}{1-\alpha_j\chi_j}\left(\frac{\Delta b^h_{j,t+1}}{1+r}-\Delta b^h_{j,t}\right)+\frac{1}{1-\alpha_j\chi_j}\left(\frac{\Delta b^g_{j,t+1}}{1+r}-\Delta b^g_{j,t}\right)+\frac{\Delta\left[f_{j,t}\right]}{1-\alpha_j}$$

And for GDP we get

$$\Delta\left[(1-\tau_{j,t})\,y_{j,t}\right]=\frac{\chi_j\alpha_j\left(\frac{\Delta b^h_{j,t+1}}{1+r}-\Delta b^h_{j,t}\right)+\frac{\Delta b^g_{j,t+1}}{1+r}-\Delta b^g_{j,t}}{1-\alpha_j\chi_j}-\Delta T_{j,t}+\frac{\Delta\left[f_{j,t}\right]}{1-\alpha_j}$$

The fact that changes in transfers enter negatively reflect the fact that they have a smaller impact on output than direct spending. This is because transfers can be spent on foreign goods and can be saved by patient agents.

Simplest policy rule under constant interest rates In the model with constant interest rates, the simplest policy rule is

$$b^g_{j,t+1}-b^g_{j,t}=-\gamma\alpha_j\chi_j\left(b^h_{j,t+1}-b^h_{j,t}\right), \tag{26}$$

with $\gamma\leq 1$. We then get

$$s_{j,t+1}-s_{j,t}=(1-\gamma)\,\frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\left(b^h_{j,t+1}-b^h_{j,t}\right).$$

Perfect stabilization is possible with $\gamma = 1$. More generally we have

$$\tilde{y}_{j,t} - \tilde{y}_{j,t-1} = (1-\gamma)\frac{\chi_j \alpha_j}{1-\alpha_j \chi_j}\left(\frac{\Delta\left[b^h_{j,t+1}\right]}{1+r} - \Delta\left[b^h_{j,t}\right]\right)$$

so disposable income is partially stabilized or completely if $\gamma = 1$. From the budget constraint of the government,

$$p^h_{j,t} g_{j,t} + T_{j,t} - \tau_{j,t} y_{j,t} = \frac{b^g_{j,t+1}}{1+r} - b^g_{j,t},$$

this policy also implies a path for nominal primary deficits. Disposable income and primary deficits are uniquely pinned down by the policy rule (26) irrespective of the composition of government spending. The composition of government spending matters for GDP, however, since

$$y_{j,t} = \frac{\tilde{y}_{j,t} - T_{j,t}}{1-\tau_{j,t}}$$

We get

$$\Delta\left[\left(1-\tau_{j,t}\right) y_{j,t}\right] = (1-\gamma)\frac{\alpha_j \chi_j}{1-\alpha_j \chi_j}\left(\frac{\Delta\left[b^h_{j,t+1}\right]}{1+r} - \Delta\left[b^h_{j,t}\right]\right) - \Delta\left[T_{j,t}\right].$$

and if we assume a constant τ_j, we get

$$\mathbb{E}_t\left[y_{j,t+1}\right] - y_{j,t} = -\frac{s_{j,t+1} - s_{j,t}}{1-\tau_j} = -\frac{1-\gamma_j}{1-\tau_j}\frac{\alpha_j \chi_j}{1-\alpha_j \chi_j}\left(b^h_{j,t+1} - b^h_{j,t}\right)$$

These two equations show the tendency of mean reversion in this model. For net foreign assets, we get

$$\begin{aligned} a_{j,t} - a_{j,t-1} &= \left(1-\chi_j\right)\left(s_{j,t+1} - s_{j,t}\right) - \chi_j\left(b^h_{j,t+1} - b^h_{j,t}\right) - \left(b^g_{j,t+1} - b^g_{j,t}\right) \\ &= -\frac{\left(1-\alpha_j\right)\chi_j}{1-\alpha_j \chi_j}\left(b^h_{j,t+1} - b^h_{j,t} + b^g_{j,t+1} - b^g_{j,t}\right) = -\left(1-\alpha_j \chi_j \gamma_j\right)\frac{\chi_j\left(1-\alpha_j\right)}{1-\alpha_j \chi_j}\left(b^h_{j,t+1} - b^h_{j,t}\right) \end{aligned}$$

An increase in borrowing by impatient agents deteriorates the net foreign asset position of the country, but this deterioration is muted if the government conducts an active countercyclical fiscal policy with a high γ_j. Notice, however, that even when $\gamma_j = 1$, so that nominal GDP is perfectly stabilized, the net foreign position is not constant.

The evolution of net exports is given by:

$$e_{j,t} - e_{j,t-1} = \Delta\left[f_{j,t}\right] - \frac{\chi_j\left(1-\alpha_j\right)}{1-\alpha_j \chi_j}\left[\frac{\Delta b^h_{j,t+1}}{1+r} - \Delta b^h_{j,t} + \frac{\Delta b^g_{j,t+1}}{1+r} - \Delta b^g_{j,t}\right] \tag{27}$$

With the government rule of equation (26) we can get:

$$e_{j,t} - e_{j,t-1} = \Delta\left[f_{j,t}\right] - \left(1-\alpha_j \chi_j \gamma_j\right)\frac{\chi_j\left(1-\alpha_j\right)}{1-\alpha_j \chi_j}\left[\frac{b^h_{j,t+1} - b^h_{j,t}}{1+r} - \left(b^h_{j,t} - b^h_{j,t-1}\right)\right] \tag{28}$$

so that net exports decrease with borrowing of impatient agents even if the government conducts an active countercyclical fiscal policy with a high γ_j.

We can consider more general dynamics of the economy for arbitrary paths for transfers and spending T

and g that are consistent with a martingale for government debt. The dynamic equations are

- $\Delta\left[\tilde{y}_{j,t}\right]=\frac{\chi_j\alpha_j}{1-\alpha_j\chi_j}\left(\frac{\Delta b^h_{j,t+1}}{1+r}-\Delta b^h_{j,t}\right)+\frac{1}{1-\alpha_j\chi_j}\left(\frac{\Delta b^g_{j,t+1}}{1+r}-\Delta b^g_{j,t}\right)+\frac{\Delta[f_{j,t}]}{1-\alpha_j}$
- $(1-\tau_j)\,\Delta\left[y_{j,t}\right]=\frac{1}{1-\alpha_j\chi_j}\left(\alpha_j\chi_j\left(\Delta\left[\frac{b^h_{j,t+1}}{1+r}\right]-\Delta\left[b^h_{j,t}\right]\right)+\Delta\left[\frac{b^g_{j,t+1}}{1+r}\right]-\Delta\left[b^g_{j,t}\right]\right)-\Delta\left[T_{j,t}\right]+\frac{\Delta[f_{j,t}]}{1-\alpha_j}$
- $\Delta\left[\frac{b^g_{j,t+1}}{1+r}\right]-\Delta\left[b^g_{j,t}\right]=\Delta\left[g_{j,t}\right]+\Delta\left[T_{j,t}\right]-\tau_j\Delta\left[y_{j,t}\right]$

Combining we get

$$\left(1+\frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\tau_j\right)\Delta\left[y_{j,t}\right]=\frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\left(\Delta\left[\frac{b^h_{j,t+1}}{1+r}\right]-\Delta\left[b^h_{j,t}\right]\right)+\frac{1}{1-\alpha_j\chi_j}\Delta\left[p^h_{j,t}g_{j,t}\right]+\frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\Delta\left[T_{j,t}\right]+\frac{\Delta\left[f_{j,t}\right]}{1-\alpha_j} \tag{29}$$

A.10 Saver spending lemma with stochastic rates

A.10.1 Proof of Proposition 1

Savers solve the following problem:

$$\begin{gathered}\max\mathbb{E}_t\textstyle\sum_{t\geq 0}\beta^t\log\left(x_{s,j,t}\right)\\ s.t.\\ x_{s,j,t}+\frac{s_{j,t+1}}{1+r_{s,j,t}}=s_{j,t}+\tilde{y}_{j,t}\\ s_{j,t}>-B^h_{j,t}\end{gathered}$$

Let us integrate forward the budget constraint:

$$\sum_{k=0}^{K}\frac{x_{s,j,t+k}}{R_{j,t,k}}+\frac{s_{j,t+K+1}}{R_{j,t,K+1}}=s_{j,t}+\sum_{k=0}^{K}\frac{\tilde{y}_{j,t+k}}{R_{j,t,k}}$$

where the k-period ahead discount rate for $k\geq 1$ from the savers' perspective

$$R_{j,t,k}\equiv\left(1+r_{s,j,t}\right)..\left(1+r_{s,j,t+k}\right),$$

and the convention $R_{j,t,0}=1$. The next step is to use the resource constraint

$$\left(1-\alpha_j\right)\tilde{y}_{j,t}=\alpha_j\chi_j\left(\frac{b_{j,t+1}}{1+r_{b,j,t}}-b_{j,t}\right)-\alpha_j\left(1-\chi_j\right)\left(\frac{s_{j,t+1}}{1+r_{s,j,t}}-s_{j,t}\right)+F_{j,t}+\frac{b^g_{j,t+1}}{1+r_{g,j,t}}-b^g_{j,t}$$

Summing and rearranging the terms, we get

$$
\begin{aligned}
(1-\alpha_j)\left(\tilde{y}_{j,t}+\frac{\tilde{y}_{j,t+1}}{R_{j,t,1}}\right) &= \alpha_j\chi_j\left(\frac{b_{j,t+1}}{1+r_{b,j,t}}-\frac{b_{j,t+1}}{R_{j,t,1}}+\frac{1}{R_{j,t,1}}\frac{b_{j,t+2}}{1+r_{b,j,t+1}}-b_{j,t}\right)\\
&\quad -\alpha_j(1-\chi_j)\left(-s_{j,t}+\frac{1}{R_{j,t,1}}\frac{s_{j,t+2}}{1+r_{s,j,t+1}}\right)+F_{j,t}+\frac{\bar{x}_{f,t+1}}{R_{j,t,1}}\\
&\quad +\frac{b^g_{j,t+1}}{1+r_{g,j,t}}-\frac{b^g_{j,t+1}}{R_{j,t,1}}+\frac{1}{R_{j,t,1}}\frac{b^g_{j,t+2}}{1+r_{g,j,t+1}}-b^g_{j,t}
\end{aligned}
$$

Then define

$$
\begin{aligned}
1-\phi_{b,j,t} &\equiv \frac{1+r_{s,j,t}}{1+r_{b,j,t}}\\
1-\phi_{g,j,t} &\equiv \frac{1+r_{s,j,t}}{1+r_{g,j,t}}
\end{aligned}
$$

to write

$$
\begin{aligned}
(1-\alpha_j)\left(\tilde{y}_{j,t}+\frac{\tilde{y}_{j,t+1}}{R_{j,t,1}}+\frac{\tilde{y}_{j,t+2}}{R_{j,t,2}}\right) &= -\alpha_j\chi_j\left(b_{j,t}+\frac{\phi_{b,j,t}}{R_{j,t,1}}b_{j,t+1}+\frac{\phi_{b,j,t+1}}{R_{j,t,2}}b_{j,t+2}-\frac{1}{R_{j,t,2}}\frac{b_{j,t+3}}{1+r_{b,j,t+2}}\right)\\
&\quad +\alpha_j(1-\chi_j)\left(s_{j,t}-\frac{s_{j,t+3}}{R_{j,t,3}}\right)+F_{j,t}+\frac{\bar{x}_{f,t+1}}{R_{j,t,1}}+\frac{\bar{x}_{f,t+2}}{R_{j,t,2}}\\
&\quad -b^g_{j,t}-\frac{\phi_{g,j,t}}{R_{j,t,1}}b^g_{j,t+1}-\frac{\phi_{b,j,t+1}}{R_{j,t,2}}b^g_{j,t+2}+\frac{1}{R_{j,t,2}}\frac{b^g_{j,t+3}}{1+r_{g,j,t+2}}
\end{aligned}
$$

Therefore for a generic horizon K:

$$
\begin{aligned}
\sum_{k=0}^{K}\frac{(1-\alpha_j)\,\tilde{y}_{j,t+k}}{R_{j,t,k-1}} &= \alpha_j\left((1-\chi_j)\,s_{j,t}-\chi_j b_{j,t}\right)-b^g_{j,t}+\sum_{k=0}^{K}\frac{\bar{x}_{f,t+k}}{R_{j,t,k}}\\
&\quad -\alpha_j\chi_j\sum_{k=1}^{K}\frac{\phi_{b,j,t+k-1}}{R_{j,t,k}}b_{j,t+k}-\sum_{k=1}^{K}\frac{\phi_{g,j,t+k-1}}{R_{j,t,k}}b^g_{j,t+k}\\
&\quad -(1-\chi_j)\,\alpha_j\frac{s_{j,t+K+1}}{R_{j,t,K+1}}+\frac{1}{R_{j,t,K}}\left(\frac{\alpha_j\chi_j b_{j,t+K+1}}{1+r_{b,j,t+K}}+\frac{b^g_{j,t+K+1}}{1+r_{g,j,t+K}}\right)
\end{aligned}
$$

We take the limit and we impose a No-Ponzi condition

$$
\begin{aligned}
\lim_{K\to\infty}\mathbb{E}_t\left[\frac{s_{j,t+K+1}}{R_{j,t,K+1}}\right] &= 0\\
\lim_{K\to\infty}\mathbb{E}_t\left[\frac{1}{R_{j,t,K}}\frac{b_{j,t+K+1}}{1+r_{b,j,t+K}}\right] &= 0\\
\lim_{K\to\infty}\mathbb{E}_t\left[\frac{1}{R_{j,t,K}}\frac{b^g_{j,t+K+1}}{1+r_{g,j,t+K}}\right] &= 0
\end{aligned}
$$

The inter-temporal current account condition is

$$\begin{aligned}(1-\alpha_j)\,\mathbb{E}_t\sum_{k=0}^{\infty}\frac{\tilde{y}_{j,t+k}}{R_{j,t,k}} &= \alpha_j\,(1-\chi_j)\,s_{j,t}-\alpha_j\chi_j b_{j,t}-b^g_{j,t}+\mathbb{E}_t\sum_{k=0}^{\infty}\frac{\bar{x}_{f,t+k}}{R_{j,t,k}}\\ &\quad-\mathbb{E}_t\sum_{k=1}^{\infty}\frac{\alpha_j\chi_j\phi_{b,j,t+k-1}b_{j,t+k}+\phi_{g,j,t+k-1}b^g_{j,t+k}}{R_{j,t,k}}\end{aligned}$$

Define

$$\Phi_t\equiv\mathbb{E}_t\sum_{k=1}^{\infty}\frac{\alpha_j\chi_j\phi_{b,j,t+k-1}b_{j,t+k}+\phi_{g,j,t+k-1}b^g_{j,t+k}}{R_{j,t,k}}.$$

The inter-temporal budget constraint is then

$$(1-\alpha_j)\,\mathbb{E}_t\sum_{k=0}^{\infty}\frac{x_{s,j,t+k}}{R_{j,t,k}}=(1-\alpha_j\chi_j)\,s_{j,t}-\chi_j\alpha_j b_{j,t}-b^g_{j,t}+\mathbb{E}_t\sum_{k=0}^{\infty}\frac{\bar{x}_{f,t+k}}{R_{j,t,k}}-\Phi_t$$

The net present value of savers' spending depends on beginning of period net foreign assets $(1-\alpha_j\chi_j)\,s_{j,t}-\chi_j\alpha_j b_{j,t}-b^g_{j,t}$, the net present value of exports, and the cost of debt when there are spreads.

A.10.2 Proof of Lemma 2

We define $\rho_{j,t}$ as the deviation of the interest rate from its steady state value:

$$1+r_{j,t}\equiv(1+r)\,(1+\rho_{j,t})\,.$$

Note that we can rewrite equation (23) as

$$(1-\alpha_j\chi_j)\,\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}}-s_{j,t}\right]-\Delta\left[\frac{\rho_{j,t}}{1+\rho_{j,t}}\frac{F_{j,t}}{1-\alpha_j}\right]=\alpha_j\chi_j\Delta\left[\frac{\bar{b}_{j,t+1}}{1+\rho_{j,t}}-B^h_{j,t}\right]+\Delta\left[\frac{b^g_{j,t+1}}{1+\rho_{j,t}}-b^g_{j,t}\right]$$

or as

$$\Delta\left[(1-\alpha_j\chi_j)\,s_{j,t+1}-b^g_{j,t+1}-\alpha_j\chi_j\bar{b}_{j,t+1}\right]=\rho_{j,t}\left(\frac{F_{j,t}}{1-\alpha_j}+(1-\alpha_j\chi_j)\,s_{j,t}-\alpha_j\chi_j B^h_{j,t}-b^g_{j,t}\right)$$

We can write the change in disposable income as:

$$\begin{aligned}(1-\alpha_j)\,\Delta\left[\tilde{y}_{j,t}\right] &= \beta\left(\chi_j\alpha_j\Delta\left[\frac{\bar{b}_{j,t+1}}{1+\rho_{j,t}}-B^h_{j,t}\right]+\Delta\left[\frac{b^g_{j,t+1}}{1+\rho_{j,t}}-b^g_{j,t}\right]\right)-(1-\beta)\,\Delta\left[\chi_j\alpha_j B^h_{j,t}+b^g_{j,t}\right]\\ &\quad+\Delta\left[F_{j,t}\right]-(1-\chi_j)\,\alpha_j\left(\beta\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}}\right]-\Delta\left[s_{j,t}\right]\right)\end{aligned}$$

Using the first we get

$$(1-\alpha_j)\,\Delta\left[\tilde{y}_{j,t}\right]=\beta\,(1-\alpha_j)\,\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}}-s_{j,t}\right]-\beta\Delta\left[\frac{\rho_{j,t}}{1+\rho_{j,t}}\frac{F_{j,t}}{1-\alpha_j}\right]-(1-\beta)\,\Delta\left[\chi_j\alpha_jB^h_{j,t}+b^g_{j,t}-(1-\chi_j)\,\alpha_js_{j,t}\right]+\Delta\left[F_{j,t}\right]$$

or

$$\begin{aligned}(1-\alpha_j)\,\Delta\left[\tilde{y}_{j,t}\right] &= \beta\,(1-\alpha_j)\,\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}}\right]-(1-\alpha_j)\,\Delta\left[s_{j,t}\right]+\Delta\left[F_{j,t}\right]\\ &\quad -(1-\beta)\,\Delta\left[\chi_j\alpha_jB^h_{j,t}+b^g_{j,t}-(1-\alpha_j\chi_j)\,s_{j,t}\right]-\beta\Delta\left[\frac{\rho_{j,t}}{1+\rho_{j,t}}\frac{F_{j,t}}{1-\alpha_j}\right]\end{aligned}$$

or

$$\begin{aligned}(1-\alpha_j)\,\Delta\left[\tilde{y}_{j,t}\right] &= \beta\,(1-\alpha_j)\,\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}}\right]-(1-\alpha_j)\,\Delta\left[s_{j,t}\right]+\Delta\left[F_{j,t}\right]\\ &\quad +(1-\beta)\,\rho_{j,t}\left(\frac{\bar{x}_{f,t-1}}{1-\alpha_j}+(1-\alpha_j\chi_j)\,s_{j,t-1}-\alpha_j\chi_j\bar{b}_{j,t-1}-b^g_{j,t-1}\right)-\beta\Delta\left[\frac{\rho_{j,t}}{1+\rho_{j,t}}\frac{F_{j,t}}{1-\alpha_j}\right]\end{aligned}$$

Therefore we get

$$\Delta\left[\tilde{y}_{j,t}\right]=\beta\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}}\right]-\Delta\left[s_{j,t}\right]+\frac{\Delta\left[F_{j,t}\right]}{1-\alpha_j}+\Omega_{j,t}$$

where

$$\Omega_{j,t}\equiv\frac{(1-\beta)\,\rho_{j,t}\left(\frac{\bar{x}_{f,t-1}}{1-\alpha_j}+(1-\alpha_j\chi_j)\,s_{j,t-1}-\alpha_j\chi_j\bar{b}_{j,t-1}-b^g_{j,t-1}\right)-\beta\Delta\left[\frac{\rho_{j,t}}{1+\rho_{j,t}}\frac{F_{j,t}}{1-\alpha_j}\right]}{1-\alpha_j}$$

where the key point is that Ω is zero if ρ is zero.

Using the budget constraint of the patient agents, we get

$$\Delta x_{s,j,t}=\Delta s_{j,t}+\Delta\tilde{y}_{j,t}-\beta\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}}\right]=\frac{\Delta\left[F_{j,t}\right]}{1-\alpha_j}+\Omega_{j,t}$$

which proves the Lemma.

B Data & Calibration

B.1 Data

All economic data (employment, population, GDP, consumption, government debt, expenditures...) comes from Eurostat. The data on household debt comes from the BIS which itself compiled the data from national central banks. Credit covers all loans and debt securities and comes from both domestic and foreign lenders. The data on interest rates (10 year government bonds) come from the ECB. The spread is defined as the difference of the 10 year interest rate on government bonds with the median of the euro zone. We had to exclude Luxembourg for which data is available only starting in 2005. We also excluded eurozone countries that joined 2007 and later. We are left with 11 countries: Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, Netherlands and Portugal.

In order to map the observed data into the model we rebase the data in the following manner. We construct a benchmark level of GDP for each country and year: it is the GDP the country would have if it had the same per-capita growth rate as the whole eurozone and its actual population growth. Define

- $Y_{j,t}$: GDP in euros of country j, and $\bar{Y}_t$ for the Eurozone
- $N_{j,t}$: population of country j, and $\bar{N}_t$ for the Eurozone
- Benchmark GDP for country$j$ at time$t$:

$$\hat{Y}_{j,t} \equiv Y_{j,0} \frac{\bar{Y}_t}{\bar{Y}_0} \frac{N_{j,t}/\bar{N}_t}{N_{j,0}/\bar{N}_0}$$

We have experimented with two definitions of $\bar{Y}_t/\bar{Y}_0$: one is actual GDP, the other is the trend nominal growth before and after the crisis (starting in 2008 this benchmark growth rate is the average of the eurozone crisis, around 1%). Then we define the rebased GDP as

$$y_{j,t} \equiv \frac{Y_{j,t}}{\hat{Y}_{j,t}}$$

We compute rebased series for consumption, government spending by scaling by the benchmark GDP. For unit labor costs, we scale by the average unit labor cost in the eurozone. For employment per capita, we take the deviation with respect to 2001 with an index of 1 for that year. We define the rebased level as the ratio of debt to the benchmark levels of GDP:

$$b^g_{j,t} \equiv \frac{B^g_{j,t}}{\hat{Y}_{j,t}}.$$

Finally, when we map to our model, we must remember that we define $b_{j,t}$ as per capita debt, so in fact we have

$$\chi_j b_{j,t} \equiv \frac{B_{j,t}}{\hat{Y}_{j,t}}$$

The normalized data for the reduced form shocks for household and public debt, government expenditures and transfers, spreads and foreign demand are given in figures (19), (20), (21) and (22).

Figure 19: Household and public debt

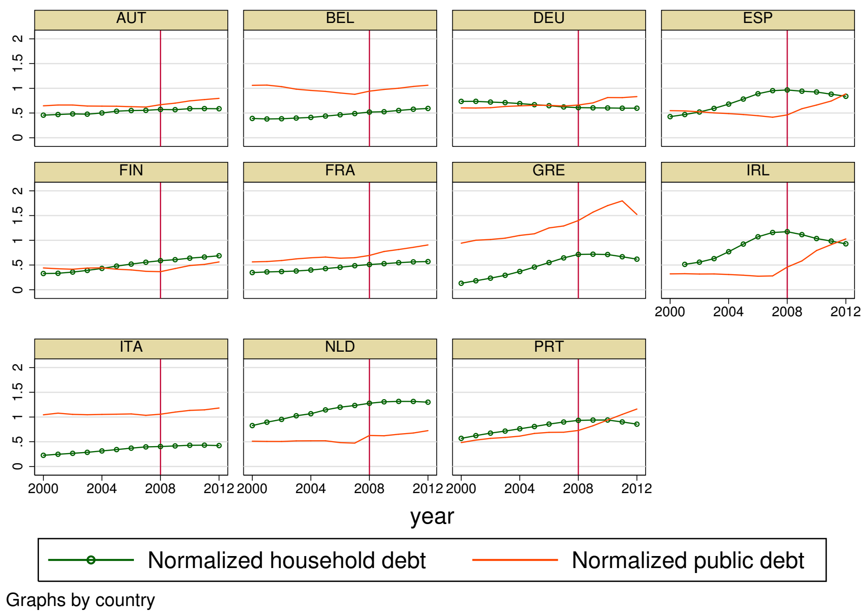


Figure 20: Government expenditures and transfers

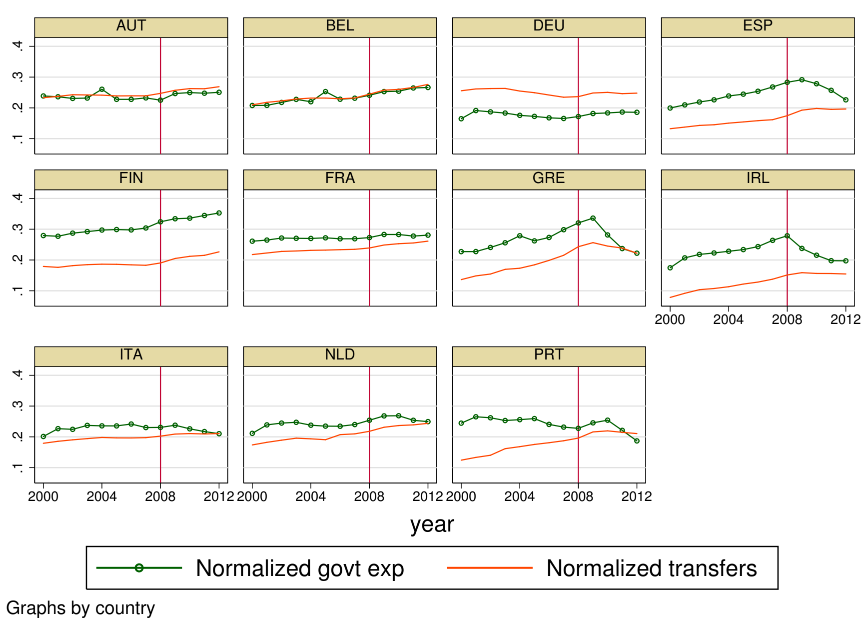


Figure 21: Spreads (10 year government bonds)

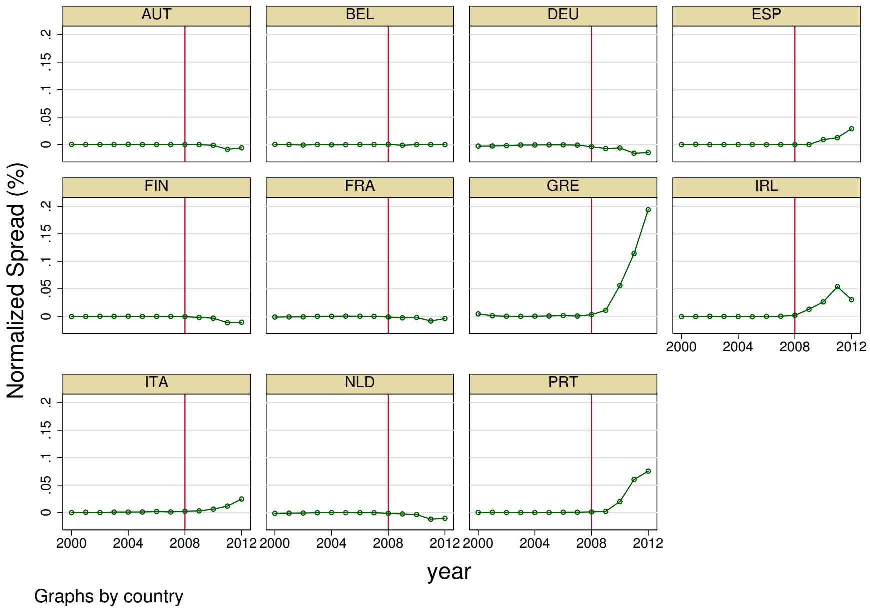
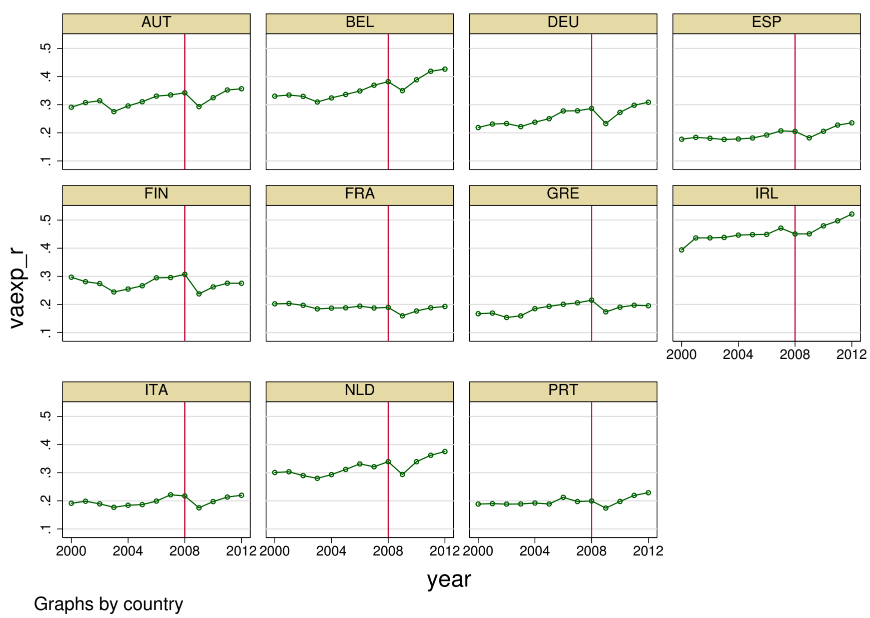


Figure 22: Value added exports



B.2 Simulation

To simulate the model and compare model outputs to realized data, we use as inputs to the model annual data on household debt, government spending and transfers, government debt[12] and interest rates for the simulation period 2001-2012. The level of output, the price level and the level of net foreign assets are set to their 2002 levels in the data. Foreign demand is set using data on exports in value added terms for 2001-2012, normalized so that the level of foreign demand in 2002 satisfies goods market clearing in the model. Finally, taxes are set so that the path for government debt implied by the model coincides exactly with the data. In the counterfactual experiment in which fiscal policy data are not used as inputs to the model, the fiscal rule described in the paper is used to set the level of transfers in each period, government spending is set constant at its 2002 level in the data and tax rates are set from data in each period.

[12]Instead of using government debt directly from data, we construct a simulated government debt series in order to avoid including factors that affect government debt in the data but are not in the model, such as bank recapitalizations, default, revenues from privatizations, etc. The simulated debt series is constructed by adding to $t-1$ period debt government expenditures including interest payments and subtracting tax revenues.