Breaking the Spell with *Credit-Easing*: Self-Confirming Credit Crises in Competitive Search Economies*

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Abstract

We analyze an economy where banks are uncertain about firms' investment opportunities and, as a result, credit tightness – due to banks' pessimistic beliefs – can result in excessive risk-taking. In the competitive credit market, banks announce credit contracts and firms apply to them, as in a directed search model. The Central Bank can affect banks' liquidity costs by changing its lending rate. We show that high-risk Self-Confirming Equilibria coexist with a low-risk Rational Expectations Equilibrium in this competitive search economy. Misperceptions never disappear in a Self-Confirming Equilibrium. For most economies (parameters), lowering the CB policy rate is ineffective. However, a *credit-easing* policy can be an effective experiment, breaking the high-risk (low-credit) Self-Confirming Equilibrium. Since the latter does not arise from a *coordination failure*, the implications of the model differ from models of Self-Fulfilling credit freezes. For example, 'horizontal banking integration' can solve *coordination failures*, but not the *misperceptions* between lenders and borrowers which may require their 'vertical integration'. We emphasize the social value of experimentation, often neglected in the recent literature that vindicates *robust decision making* as a form of good governance for central banks. (*JEL*: D53, D83, D84, D92, E44, E61, G01, G20, J64.)

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1 Introduction

The financial crisis of 2008 and the subsequent 2010 euro crisis have been characterized by two elements common to most financial crisis: high uncertainty regarding the state of the economy – with the asset and sovereign debt markets respectively playing the main role – and, as a consequence, generalized high lending rates to the private sector resulting, in some cases, in credit freezes. Even if by the end of the 20th Century central bank policies in advanced economies seemed to have converged (e.g. price stability as the main objective and the interest rate as the main policy instrument) the reaction of the main western banks to the crises has not been uniform. The most generalized reaction has been to apply conventional central bank policies of lowering the cost of money or, more generally, of facilitating liquidity and credit to private banks. Unfortunately, these policies have seldom translated into a substantial improvement of the credit conditions for private firms, deepening the recession is some countries. Nevertheless, already at the beginning of the financial crisis the Federal Reserve Bank implemented a more daring unconventional policy of *credit easing* (the TALF, which we discuss below) successfully *breaking the spell* of a financial market freeze (the ABS market) and, as a side effect, increasing the Fed revenues; not an uncommon experience of central banks in times of financial crisis. Despite of this 'success' other central banks – for example, the ECB – have not implemented similar unconventional policies, as if the Fed had just been lucky or the circumstances – say, in the euro crisis – had been different.

Was the Fed right, or better informed, or just lucky? More broadly, how should a central bank react when conventional policies are not effective in improving credit market conditions and knows no more than the private sector? Or, is the *credit freeze* problem a *coordination failure* that could be effectively solved with a better horizontal integration within the banking system?

Unfortunately, our macro-money-finance theoretical toolkit is very limited to reply to these and related questions. In particular, as we discuss below in more detail, existing models tend to give and advantage to central banks with respect to private banks (better information, less incentive problems, etc.) and, if not, answer 'just luck' to the first question. Or, alternatively, postulate private *coordination failures* and, therefore, answer affirmatively to the last question. With this paper, we would like to contribute to our toolkit by developing a theory (of Self-Confirming Equilibrium) which we think helps to answer the broader second question and, in doing so, provides a new *rationale* for unconventional monetary policies in times of economic high uncertainty.

We explore the macroeconomic consequences of individual uncertainty about others' agents opportunities and corresponding payoffs. In particular, we study a competitive economy where, in equilibrium, individual financial intermediaries (banks) – possibly, due to a collective 'bad experience" – can have pessimistic beliefs about firms' capacity or incentive to make low-risk investments. With such beliefs, banks charge a risk-premium to cover their expected losses. Facing high interest loans, firms' optimal investment is a risky project. As a result, the behavior of firms *self-confirms* banks' pessimistic beliefs and these beliefs persist in a high-risk equilibrium. It is in this context *credit-easing* policy, implemented through the banking system (e.g. subsidizing credit) can result in an immediate 'learning-by-doing' experience for the banks, breaking the high-risk (low-credit) Self-Confirming Equilibrium.

1.1 The model components

In our model, the competitive credit market consists of a continuum of firms and banks. The latter post credit contracts and the former apply for these contracts in order to finance their investment projects. The interest rate on the loan defines the credit contract and, consequently, the type of project and size of the investment that maximizes firm's profits. In the simplest version of our model, we assume that, at any point in time, firms can only invest in one project, which can be either a risk-free project, which involves a per-unit cost, or a risky project with zero per-unit cost. Firms' expected profits depend on the probability that their loan application is accepted and, if so, on the expected net return of their project. When a risky project fails firms only repay the principal of the loan (the *pledgeable* part) and, therefore, the lender-bank also bears part of the risk, which is compensated by the loan's interest risk-premium. Firms' have *rational expectations* in making their choice conditional on the existing menu of debt contracts.

Firms in our model can be non-banking financial intermediaries. In particular, intermediaries of Asset Backed Securities (ABS) that by incurring a per-unit monitoring cost can guarantee a 'safe' ABS package, while the latter becomes risky if they do not incur the cost. In contrast with models of *Self-fulfilling credit freezes* (e.g. Bebchuk and Goldstein 2011) a firm's project returns does not depend on other firms' financial conditions. As it will see, in our model *credit freezes* can arise even if there is no *coordination failure*.

Banks are financial intermediaries that borrow money from the Central Bank in order to provide loans to individual firms. There is free entry in this industry and banks cannot default on their Central Bank obligations, including CB lending-rate payments (i.e. default is too costly for banks). Banks know their costs with certainty but they are uncertain about their revenues, since they have to anticipate firm's reaction to their credit offers not knowing in which project firms will invest. We weaken the rational expectations hypothesis, with respect to banks, by assuming that their beliefs only need to be 'locally-rational' in equilibrium, meaning that they satisfy two conditions: first, as in directed search-models, marginal variations of loans' interest rates are expected to be compensated by marginal variations on the number of applications; second, banks expect to loose if they offer non-equilibrium debt contracts and these beliefs are locally correct (i.e. for small, non-marginal, deviations); however, they may not be correct for large deviations that would result on a different choice of project. For example, banks may wrongly believe that offering a lower interest rate will not cover their expected losses since they miss-perceive the investments firms will undertake when borrowing at low interest rates. This equilibrium is a *Self-Confirming Equilibrium* and, only when banks' correctly perceive firms' reactions to all their offers, it is a *Rational Expectations Equilibrium*; i.e. only when the second rationality requirement is global. In our model, the *Rational Expectations Equilibrium* is unique.

In our model the Central Bank does not have superior information than private banks; nor has a better commitment technology than them, as it happens in the work of Karadi and Gertler (2011) on credit easing. However, the CB maximize social welfare – not a specific objective, such as price stability, – and, furthermore, has access to society's resources (tax revenues). These two classical components of public policy provide a *rationale* for a *credit-easing* as a *social experimentation* policy. Furthermore, in our model, the central bank cannot substitute (sidestep) private banks, nor can the macro social experiment be effectively substituted by a small 'natural experiment'. This role for social-monetary policy contrast with the work of Chari *et al.* (2010) on credit easing, where the central bank policy is ineffective.

It should be noticed that the mechanism that we explore is pervasive. If a market collapses, or does not exist, agents' *subjective beliefs* regarding their expected profits if the market opens are unlikely to coincide with the *objective beliefs* corresponding to a Rational Expectations Equilibrium. In fact, there may even be generalized 'biased *subjective pessimistic beliefs*' if the collapse of the market has been the result of a collective bad experience. Furthermore, if firms are aware that, in the event the market successfully opens (i.e. there are unexploited trading possibilities), perfect competition will prevail, the prospect of making zero-profits may deter them from incurring the costs (risks) of exploring the market. Such exploration becomes a public good and social policy may be needed.

For example, we could simplify our model assuming Bertrand competition among banks supplying unlimited credit lines at posted interest rates to all firms. The main results of the paper (existence of a high-risk SCE, which is not REE, and effects of policy interventions) would prevail. However, credit lines always involve advertising, processing and monitoring activities resulting in banks' costs and delays, which we identify with search-matching frictions that we would like our theory to account for. As we show, reducing these frictions increases investment and welfare but does not change the structure of equilibria or the equilibrium prices; in other words, our results are robust to changes in market structure (of degrees of frictions and competition) and other modelling details. We now discuss our results on monetary policies.

1.2 The Central Bank policy interventions

The Central Bank affects the banks' cost of liquidity by changing its lending rate. A conventional policy of lowering the CB policy rate reduces the set of economies for which an economy has a high-risk Self-Confirming Equilibrium (SCE). For economies within this set, a lower cost of money is ineffective. For an economy outside (close to the boundary of) this set, the same policy can have a radical effect if the economy is in a SCE and only the Rational Expectations low-interest rate equilibrium exists after the policy has been introduced.

In our economies neither firms or banks have a problem of shortage of capital or liquidity: firms apply for loans without constraints at the interest rates specified by banks and, similarly, banks can borrow unlimited amounts from the Central Bank at its lending rate. Therefore, there is no role for an unconventional policy of capital injections from the CB to the banking system. If the latter is trapped in a high-risk Self-Confirming Equilibrium such a policy will be completely ineffective: it would only rebalance the respective balance sheets and we abstract from capital-requirements regulation issues.

Should the Central Bank pursue an unconventional policy of directly lending to firms? And, if so, how? Given that, as we have said, there are no capital shortages, the only reason could be if by doing so the CB could break a high-risk Self-Confirming equilibrium shifting the economy to the more efficient no-risk Rational Expectations Equilibrium. However, in our economies, the Central Bank does not have any informational advantage, wouldn't the CB have the same mis-perceptions than private banks? In our model the answer is yes; in fact, it can even be more pessimistic than the private sector. However, as we have already emphasized the CB and private banks have different – social vs. private – objectives. The difference translates into a different assessment of the value of experimentation in the dynamic formulation of the model.

The dynamic model is basically a repeated version of the static model just described. In particular, in the simple version with two types of projects, one safe and one risky, the state of the economy has two components: the per-unit cost of the safe project and the probability of success of the risky project. Miss-perceptions by banks does not mean that they have degenerate beliefs assessing, for example, zero probability to the safe technology being available, but simply that their subjective beliefs are distorted with respect to the objective distribution generating the state of the economy. In our model once self-confirming mis-perceptions have been falsified, the economy remains in the unique rational expectations equilibrium and, therefore, the policy intervention only needs to be implemented temporarily.

As explained, free entry in the banking industry implies that if banks (mistakenly) assign a low probability to firms investing is safe projects may have no incentive to 'experiment' with low interest rates, even if posting a credit line with very low interest rate will eventually dissipate the misperception. In contrast, a welfare maximizing central bank may have a positive value for such experimentation – even if it attaches the same, or even lower, probability to the safe technology – since it values the move to a more efficient equilibrium against the cost of the temporary implementation of the *credit-easing* policy. In fact, in our parameterization this difference turns out to be large enough as to offset all but the most pessimistic beliefs.

However, as we have also emphasized, the Central Bank does not have the technology to intermediate with firms and, therefore, cannot directly lend to them at a low interest rate. Banks must act as the 'transmission' for CB policies. The *credit-easing* policy in our economies is a policy of subsidizing banks' risky loans, creating a *wedge* between the relatively high interest rate charged by banks and the relatively low interest rate faced by firms. This wedge modifies the behavior of the banks as if they had revised their beliefs giving higher probability to firms investing in safe projects. Therefore, in our model, the *credit-easing* policy produces a direct 'learning-by-doing' effect in the banking system which cannot be achieved with other (possibly, less costly) forms of experimentation, such as a 'natural experiment' randomizing the policy across banks (which, in addition, may not be politically feasible).

1.3 The contribution of the paper

In sum, our theory provides a different perspective for central bank policy in times of high economic uncertainty. This perspective relies on the central bank maximizing social welfare and being able to subsidize bank lending; not just having a narrow objective of 'price stability' and a single, interest rate, instrument. It sets the central bank in the same footing than the private banks. It also helps to explains why more conventional monetary policies may not be effective in these contexts and why different banks may react differently. However, the theory predicts that the reluctance to apply daring *credit-easing* policies may only be justified by very pessimistic CB beliefs[1].

Our work is also novel in bringing the concept of Self-Confirming Equilibrium to a competitive environment. This concept, was pioneered in game theory by Fudenberg and Levine (1993) and in macroeconomics by Sargent (1999). In both contexts, the beliefs of a non-negligible agent are misspecified out-of-equilibrium. This has two consequences. First, the agent's deviation from the equilibrium path can disrupt the equilibrium and detect the miss-perception; something which is not possible in a competitive environment where individual agents cannot affect prices. Second, the individual and social value of experimentation is basically the same. We develop a model of *competitive economies with search frictions* where Self-Confirming Equilibria may exists, resulting in interesting policy implications.

1.4 The roadmap

In Section 2 we describe, more formally, the components of the model. In section 3 we define and characterize Self-confirming equilibria. In Section 4 we state and compare the equilibria of the model. Section 5 illustrates the disconnect between public and social value of experimenting credit-easing. Section 6 illustrates policy implementation. In Section 7 we describe the TALF experiment as an example (which turned out to be successful) of the *credit-easing* postulated in our model; then we compare our model with a model of multiple REE (Self-fulfilling credit freezes).

[1] This statement is, obviously, conditional on the theory. That is, on abstracting from elements that can play a role in times of financial crises (e.g. the financial position of firms) and on the CB being aware of the possibilities that a *credit-easing* can offer.

2 A model of competitive search for credit

2.1 Investment choice

A firm $i \in [0,1]$ has access to two kinds of projects $\varsigma \in \{s, r\}$, respectively a safe and a risky one, which differ for the likelihood of success and per-unit adoption cost. Each project yields the same conditional per-unit gross return after one period irrespective of the type: $1 + y$ in case of success, whereas 1 in case of failure. Safe projects do not fail, but their adoption requires a fix per unit cost of $k \in \Re^{+}$ which sums up to the repayment of the bank's loan. Risky project do not have any fix per-unit additional cost, but they are successful only with a probability $\alpha_i \in (0,1)$.

A firm can obtain liquidity to invest in a project from a bank. A contract in the credit market is a credit line which provides for an unlimited amount of credit at a fixed interest rate $1 + R$ which has to be paid back at the end of the life of the project (one period). Only in case of success the repayment is fully enforceable, otherwise a firm can default on the interest payment as only capital is pledgeable. Finally, a firm choosing to invest $I_i \in \Re_+$ incurs in a quadratic cost $I_i^2/2$ irrespective of the type ς of the project. Therefore, a *risky* project gives a net expected return of

$$\Pi\left(R, \omega_i, r, I_i\right) \equiv \alpha_i \left(y - R\right) I_i - \frac{1}{2} I_i^2,$$

whereas a *safe* project gives

$$\Pi\left(R, \omega_i, s, I_i\right) \equiv \left(y - k_i - R\right) I_i - \frac{1}{2} I_i^2,$$

where $\omega_i \equiv \{\alpha_i, k_i\}$ denote the realization of a state of nature which is distributed according to a density function[2] $\phi\left(\omega, i\right)$ which assigns the probability that $\omega_i = \omega$ where $\omega \in \Omega \equiv (0,1) \times \Re_+$. In other words $\phi\left(\omega, i\right)$ controls for the heterogeneity of firms. Nevertheless, the assumption that firms are homogeneous, that is $\omega_i = \omega$ for each $i \in [0,1]$, is without loss of generality to the main results of this paper. Therefore to keep the presentation simple and concise we will consider the case of homogeneity of firms and drop the i index.

The investment policy of a firm is a mapping $\mathrm{f}\left(R, \omega\right) : \Re_+ \times \Omega \to \{s, r\} \times \Re_+$ giving for each couple $\left(R, \omega\right)$ a couple $\mathrm{f}\left(R, \omega\right) = \left(\varsigma^*\left(R\right), I^*\left(R\right)\right)$ such that

$$\mathrm{f}\left(R, \omega\right) = \underset{\{\varsigma \in \{s,r\}, I \in \Re_+\}}{\arg\max} \; \Pi\left(R, \omega, \varsigma, I\right), \tag{1}$$

is an optimal reply to an offer R. In particular, for a given R, the firm will adopt a technology that delivers the highest return. Hence, the value to a firm of a credit line at an interest rate R is denoted by $\Pi\left(R, \omega, \mathrm{f}\left(R, \omega\right)\right)$, or simply $\Pi\left(\mathrm{f}\left(R, \omega\right)\right)$ in short. A participation constraint imposes $\Pi\left(\mathrm{f}\left(R, \omega\right)\right) \geq 0$. For a given ω, the safe project is the most profitable when the return of the safe

[2] From here onward we will use tildes to denote random variables as opposed to realizations.

technology is sufficiently high or equivalently the offered contract is sufficiently low, that is

$$R \leq y - \frac{k}{1-\alpha},$$

to which we will refer as the adoption frontier of firms. Therefore, any credit contract characterized by R is tightened to a type of investment conditional on a certain realization of the state of nature ω. Moreover, for any $R \neq y - k/(1-\alpha)$, there always exists a open neighborhood of interest rates around R such that the optimal choice of the type of project $\varsigma^*(R)$ remains constant. On the other hand, $I^*(R)$ continuously varies for marginal changes of R on the whole support of positive reals.

2.2 Banks

A banks borrows money in the interbank market, or equivalently at a rate determined by the central bank (CB), R_{CB} and lends to firms. As said, banks do not observe the technological choice of firms. Nonetheless we can assume without loss of generality that banks know firms are homogeneous. The investment choice affects the expected profit of a bank in a contractual relationship with a firm at a given interest rate R. The type of technology determines the degree of pleadgeability of the investment, and so the expected repayment rate of the loan. In case of a risky type of project the firm will be able to repay back $I(1+R)$ only with probability α, whereas with probability $(1-\alpha)$ only the pledgeable part, the principal, will be paid back. Therefore the expected per-unit return on the loan is given by

$$\pi(r, R, R_{CB}) = \alpha R - R_{CB},$$

in case of risky adoption, and

$$\pi(s, R, R_{CB}) = R - R_{CB}$$

in case of safe adoption. Notice that a bank cannot in any case default on its own interbank market loan which is fully collateralized; hence the per-unit interest payment of a bank R_{CB} does not depend on α. Participation of a bank requires $R \geq R_{CB}$ in case of safe adoption and $R \geq R_{CB}/\alpha$ in case of risky adoption. The value to a bank of an open credit line at a rate R is therefore

$$Y(R, \mathrm{f}(R,\omega)) \equiv I^*(R)\,\pi(\varsigma^*(R), R, R_{CB}).$$

Notice that whereas for a firm is enough knowing the offered rate R to evaluate the value of an open credit line, a bank has instead to know the investment policy of a firm which crucially depends on the realization of the state of nature ω which is only dirercty observable to firms, but not by banks. In other words, banks cannot screen firms for project quality.

2.3 Matching in the credit market

To decribe how banks and firm match to sign a credit contract we introduce a competitive direct search framework as introduced by Moen (1997) along a simplified variant described by Shi (2006). The probability that a match is formed is described by a matching function which is a map $\Re_+^2 \to \Re_+$ from a couple (a, o) - being respectively the measure of applicants and the measure of offers - to $x(a, o)$ being a flow of new firm-bank matches. The matching function $x(a, o)$ encapsulates a search friction assumed in the competitive credit market. Following standard assumptions , let x be concave and homogeneous of degree one in (a, o) with continuous derivatives. Let $p = x(a, o)/a = x(1, \theta) = p(\theta)$ denote the probability for applicants to sign a credit contract, and $q = x(a, o)/o = q(\theta)$ the probability that an offer will generate a contract, where θ is the credit market tightness a/o. Let $\lim_{\theta \to 0} p(\theta) = \lim_{\theta \to \infty} q(\theta) = 1$ and $\lim_{\theta \to \infty} p(\theta) = \lim_{\theta \to 0} q(\theta) = 0$. For the sake of simplicity and without loss of generality, we will assume that the matching function has a Cobb-Douglas form

$$x(a, o) = Aa^\gamma o^{1-\gamma}$$

so that $p(\theta) = A\theta^{\gamma-1}$ and $q(\theta) = A\theta^\gamma$. This assumption, which is standard in the literature, ensures a constant elasticity to the fraction of vacancies and illiquid firms.

The search is *directed*, meaning that at a certain interest rate R there is a subset of illiquid firms and banks with open credit lines looking for a match at that specific R. The number of matches in the submarket R is $x(a(R), o(R))$, where $a(R)$ is the measure of firms and $v(R)$ the measure of vacant credit lines searching for a counterpart in a credit contract at an interest rate R. The arrival rates of trading partners for firms and banks in this market are thus $p(\theta(R))$ and $q(\theta(R))$, respectively, where $\theta(R) = a(R)/o(R)$ is the specific tightness[3] associated to the submarket R. Both firms and banks are free to move between submarkets. Once the match is formed any amount of credit is provided at a rate R. We will say that a submarket is active if there is at least a vacancy posted.

The timing in the market within one period is the following:

1. a bank can borrow at a rate R_{CB}, controlled by the central bank, in the interbank market;

2. a bank chooses at which R opens a full-allotment credit line, filled with probability $q(R)$;

[3] The tightness is a ratio representing the number of firms looking for a credit line *per-unit of vacant open lines.* This means that the tightness is independent of the absolute number of vacancies open in a certain market. The matching function is just a one-to-one map between p and q through a ratio θ. In other words, suppose an equilibrium is associated with a particular probability to obtain credit $\bar{p}$, then the matching function gives a $\theta = p^{-1}(\bar{p})$ and so a $\bar{q} = q\left(p^{-1}(\bar{p})\right)$ that is a probability of filling a vacant line in that submarket. The latter argument is independent on how many vacancies are open in that particular submarket. With a single vacancy open (resp. a measure ε of vacancies open), θ (resp. $\varepsilon\theta$) will be the expected number of firms searching in that submarket.

3. a firm chooses to which posted R to apply for credit, with success probability $p(R)$;
4. a firm also chooses: the technology ς and the size I of the investment;
5. if the project is successful a firm pays back $(1+R)I$ to the bank, and only I otherwise;
6. a bank pays back their loan $(1+R)I$ *irrespective* of the success of the project success.

Banks bear two kind of risks: one is associated with the probability that a vacancy is not filled, one other originates from the partial enforceability of the posted contracts. An entrepreneur (firm) instead does not incur any cost if she does not match or if her project fail. Nevertheless, the exposure to risk of banks depends on firms' choices. when a project fails, which occurs with an exogenous probability, a bank looses interest repayments whereas it is enforced to repay the interests on its own loan. The return on a project can be secured adopting a possibly more costly type of investment. A firm adopts the type of investment in its own interest according to its payoff structure which depends on the state of the world ω. In particular, to optimally solve their problem banks need to anticipate the probability that a posted contract finds a match and firms' reaction once matched, that is they need to identify in which state of the nature they act.

2.4 Offers and Applications in the credit market

Banks are first movers in the search: they choose whether to enter in the market and eventually post a credit line in a submarket. A credit line is a contract by which banks commit to lend any amount of liquidity to successful applicants at a fix rate R. The ex-ante value of a credit line for a bank is given by

$$V(R,\omega) \equiv q(R)\, Y(R, \mathrm{f}(R,\omega)), \tag{2}$$

and the cost of posting an offer is c. Therefore a bank posts an offer R to solve

$$\max_R \mathrm{E}^{\beta}\left[V(R,\omega) - c\right], \tag{3}$$

where β is the system of subjective beliefs held by banks on the realization of the state of the word ω. In particular, let define the E^{β} operator as follows

$$\mathrm{E}^{\beta}\left[(\cdot)\right] \equiv \int (\cdot)\ \beta(\omega, i)\ did\omega,$$

where $\beta(\tilde{\omega}_i)$ is the banks' subjective probability density function of the distribution of types across the population. Notice that for a bank to solve (3) it has to anticipate the reaction of firms $\mathrm{f}(R,\omega)$ to the posted R.

Let us denote by H the set of offers publicy posted by banks. Firms choose to which posted contract $R \in H$ to send its application for funds. Once matched at the targeted R a firm implement its investment policy $\mathrm{f}(R,\omega)$, namely, a vector of optimal choices in response to a couple (R,ω). The objective of a firm is to maximize ex-ante profit

$$J(R) \equiv p(R)\,\Pi(\mathrm{f}(R,\omega))\,.$$

Therefore a firm sends an application to the posted $R \in H$ to solve

$$\max_{R\in H} J(R),$$

where notice any expectation from the side of firms is involved as they have all the elements to act optimally. Nevertheless the choice of a firm is constrained by the set H of available offers which is predetermined to the choice of firms by banks.

3 Equilibria

3.1 Free entry and competitive search

Given the competitive nature of the banking sector, banks enter in the market untill profits are strictly positive. Therefore at the equilibrium free entry requires $\mathrm{E}^{\beta}[V(R,\omega)-c]=0$.

On the side of firms instead, competitive search determines the tightness of each submarket for which an offer has been announced. In particular, consider the case where two different offers R' and R'' are publicly announced. In case $J(R') > J(R'')$ then firms will be more willing to send applications to get the R'' contract rather than the R'. As a consequence of a larger number of applications in the submarket R', the probability of matching $p(R')$ must decrease lowering $J(R')$. Symmetrically, as a consequence of a smaller number of applications in the submarket R', the probability of matching $p(R'')$ must increase enhancing $J(R'')$. Therefore at the equilibrium, for a given set of posted contracts $H = \{R_1, R_2, R_3, ...\}$, the tightness is determined by

$$\bar{J} = p\left(\boldsymbol{\theta}\left(R,\bar{J}\right)\right)\Pi(\mathrm{f}(R,\omega)) \tag{4}$$

where $J(R) = \bar{J}$ is constant for each $R \in H$. In particular, $\bar{J}$ is the ex-ante utility that firms expect from participating to the market. Therefore there exists a unique tightness associated to each interest rate R which is conditional to a given level of ex-ante utility guaranteed by the participation of firms to the market.

3.2 Competitive SSCE and REE

In this section we present the definition of strong self-fulfilling equilibrium (SSCE) and we contrast it to the notion of self-fulfilling equilibrium (SCE) and the classical one of rational expectation equilibria (REE).

Definition 1 *A strong self-confirming equilibrium (SSCE) is a set of posted contracts H^* such that, for each $R^* \in H^*$:*

(i) firms maximize expected profits

$$R^* = \arg\sup_{R \in H^*} p\left(\boldsymbol{\theta}\left(R, J^*\right)\right) \Pi(\mathrm{f}\left(R, \omega\right)) \tag{5}$$

and $J^* = p\left(\boldsymbol{\theta}\left(R', J^*\right)\right) \Pi(\mathrm{f}\left(R', \omega\right))$ *for any* $R' \in H^*$;

(ii) banks maximize expected profits

$$R^* = \arg\sup_{R \in \Re} \mathrm{E}^{\beta}\left[V\left(R\right)\right] \tag{6a}$$

$$s.t. \quad \bar{J} = p\left(\boldsymbol{\theta}\left(R, \bar{J}\right)\right) \mathrm{E}^{\beta}\left[\Pi\left(\mathrm{f}\left(R, \omega\right)\right)\right] \tag{6b}$$

where $J\left(R\right) = \bar{J}$ *is constant for each* $R \in \Re$ *and* $\mathrm{E}^{\beta}\left[V\left(R^*\right)\right] = c$;

(iii) banks correctly anticipates firms reaction locally: there is an open neighbourhood of R^*, $\Im\left(R^*\right)$, *such that for any* $R \in \Im\left(R^*\right)$

$$\mathrm{E}^{\beta}\left[V\left(R\right)\right] = \mathrm{E}^{\phi}\left[V\left(R\right)\right],$$

that is, the subjective valuation of banks is equal to the objective one.

The first requirement implies optimality from the side of the firms so that (19) defines the tightness of the submarkets. Notice that $\boldsymbol{\theta}\left(R, J^*\right)$ depends on the ex-ante utility granted to firms at equilibrium conditions, and does not depend on individual choices.

The second condition requires that a bank posts a R^* that globally maximizes its expected value of a credit line. The relevant expectation is the one conditional to the bank subjective beliefs summarized by β.

The third condition restricts banks' beliefs about firms' actions to be correct in a neighborhood of an equilibrium R^*. This is also a stronger beliefs' restriction of the one usually assumed in the notion of Self-Confirming Equilibrium which does not contemplate any belief restriction out of equilibrium. This is also a stronger beliefs' restriction of the one usually assumed in the directed search literature to get rid of trembling-hand imperfect equilibria, which involves only correct beliefs on first-order marginal perturbation of the equilibrium tighteness function.

What is crucial in the definition of SSCE is that it does not require banks to have correct beliefs about out-of-equilibrium behavior that is far away from the equilibrium considered. This leaves open the possibility that at a self-confirming equilibrium banks are not actually maximizing in the *whole* domain of their actions, *although they believe they do*. In fact banks' unbiased beliefs about firms' payoffs are only required limited to the neighborhood of the contract that is implemented, and whose effects are therefore observed. Dominant contracts out of a neighborhood of the equilibrium could be wrongly believed by banks to be strictly dominated. Since such contracts will be never posted, then in equilibrium there do not exist counterfactual observations that could confute wrong beliefs.

A REE is a stronger notion than a SSCE requiring that no agent holds wrong out-of-equilibrium beliefs. In the present model this equals to impose that banks' unbiased beliefs about firms' payoffs. In such a case the equilibrium contract is the one which objectively yields the highest reward with respect to every possible feasible contract.

Definition 2 *A rational expectation equilibrium (REE) is a self-confirming equilibrium $H^\star$ for which:*

iii-bis) banks correctly anticipates liquidity demand, and so the corresponding type of investment, at any feasible R, that is,

$$\mathrm{E}^{\beta}\left[V\left(R\right)\right]=\mathrm{E}^{\phi}\left[V\left(R\right)\right],$$

for any $R\in\Re$.

A REE obtains from a tightening of condition (iii) in the definition of a SSCE. This implies that every $R^\star\in H^\star$ is such that banks can exactly forecast their payoffs out of the equilibrium, as they can correctly anticipate firms' responses. Therefore condition ii) of the definition of a SSCE becomes

$$R^\star=\arg\sup_{R\in\Re}\mathrm{E}^{\phi}\left[V\left(R\right)\right] \tag{7}$$

subject to $\bar{J}=p\left(\boldsymbol{\theta}\left(R,\bar{J}\right)\right)\mathrm{E}^{\phi}\left[\Pi\left(\mathrm{f}\left(R,\omega\right)\right)\right]$, in the case of a REE. That is, posting in the submarket $R^\star$ is a globally dominant strategy both from an objective and a subjective point of view.

3.3 Characterization of the Equilibria

In this section we provide a characterization of the equilibrium developping a non-marginal technique that can be generally used to identify the equilibrium of serch and matching economies with non convex payoff structures and potentially multiple local maxima. As we will show the popular Hosios condition can be recovered from this more general analysis.

Proposition 1 *Consider two credit lines posted respectively at R_1 and R_2. From the point of view of a single atomistic bank*

$$\mathrm{E}^{\beta}\left[V\left(R_1,\omega\right)\right]\geq\mathrm{E}^{\beta}\left[V\left(R_2,\omega\right)\right]$$

if and only if

$$\mathrm{E}^{\beta}\left[\mu\left(R_1,\omega\right)\right]\geq\mathrm{E}^{\beta}\left[\mu\left(R_2,\omega\right)\right] \tag{8}$$

with

$$\mu\left(R,\omega\right)\equiv\Pi\left(\mathrm{f}\left(R,\omega\right)\right)^{\frac{\gamma}{1-\gamma}}Y\left(R,\mathrm{f}\left(R,\omega\right)\right),$$

for any profile of contracts offered by other banks.

Proof. Postponed to the appendix.

Corollary *A set of contracts H^* is a SSCE if for a given system of subjective beliefs β, any $R^*\in H^*$ is such that*

$$\mathrm{E}^{\beta}\left[\mu\left(R^*,\omega\right)\right]=\mathrm{E}^{\phi}\left[\mu\left(R^*,\omega\right)\right]\geq\mathrm{E}^{\beta}\left[\mu\left(R,\omega\right)\right], \tag{9}$$

for any $R \in \Re$, and there exists a neighbourhood $\Im(R^*)$ of the equilibrium such that

$$\mathrm{E}^{\beta}\left[\mu\left(R^*,\omega\right)\right]=\mathrm{E}^{\phi}\left[\mu\left(R^*,\omega\right)\right]\geq\mathrm{E}^{\phi}\left[\mu\left(R,\omega\right)\right], \tag{10}$$

for any $R \in \Im(R^*)$. A SSCE is a REE if and only if (9) and (10), both hold for any $R \in \Re$.

Notice one important feature of the condition above. With $\gamma = 0$ when all the surplus is extracted by banks (10) becomes $Y(R^*, \mathrm{f}(R^*,\omega)) \geq Y(R, \mathrm{f}(R,\omega))$, that is at the equilibrium only the interim payoff of banks is maximized as firms will always earn zero. With $\gamma = 1$ instead when the whole surplus is extracted by firms (10) becomes $\Pi(R^*,\omega) \geq \Pi(R,\omega)$, that is only the interim payoff of firms is maximized as banks will always earn nothing. Of course, (10) is satisfied locally by any interior SSCE. In other words, (10) is a condition that implies the local holding of Hosios (1990) condition in the case of homogeneous firms.

Proposition 2 *An interior (i.e. when firms strictly prefer a type of project) SSCE is characterized by a single contract $H^* = \{R^*\}$ satisfying*

$$\mathrm{E}^{\beta}\left[\mu\left(R^*,\omega\right)\left(\gamma\frac{\Pi'(\mathrm{f}\left(R^*,\omega\right))}{\Pi(\mathrm{f}\left(R^*,\omega\right))}+(1-\gamma)\frac{Y'\left(R^*,\mathrm{f}\left(R^*,\omega\right)\right)}{Y\left(R^*,\mathrm{f}\left(R^*,\omega\right)\right)}\right)\right]=0, \tag{11}$$

for a given parameterization of the functions $\Pi(\mathrm{f}(R,\omega))$ and $Y(R, \mathrm{f}(R,\omega))$.

Proof. Postponed to the appendix.

The proposition above states that a self-confirming equilibrium is such that all the matches occur at a unique R^*. In the case of homogeneous firms SSCEs are *locally* optimal in the sense of the Hosios (1990) condition. In fact, at a SSCE with homogeneous firms we have that

$$1-\gamma=\frac{Y\left(R^*,\mathrm{f}\left(R^*,\omega\right)\right)}{Y\left(R^*,\mathrm{f}\left(R^*,\omega\right)\right)-\frac{Y'\left(R^*,\mathrm{f}\left(R^*,\omega\right)\right)}{\Pi'\left(\mathrm{f}\left(R^*,\omega\right)\right)}\Pi(\mathrm{f}\left(R^*,\omega\right))},$$

that is, the fraction of the surplus (properly evaluated) going to banks - the term on the right-hand side - reflects the elasticity of the matching function with respect to the fraction of illiquid firms in the market. This is exactly the condition for which banks internalize the social cost of opening a new vacancy.

4 SSCE in the credit market

In this section we state the equilibrium of the model presented above. Since at a SSCE banks believe that, in expectation, they are playing a REE, it is convenient to firslty describe the set of possible REEs, and then the set of SSCE. It is convenient to define $\bar{R} \equiv \max\{y - k/(1-\alpha), R_{CB}\}$, then the following holds.

Proposition 3 *For given R_{CB}, there exists a unique threshold value $\hat{\alpha}(k) \in (\underline{\alpha}, \overline{\alpha})$, with*

$$\underline{\alpha}=\frac{y-\hat{R}_s-k}{y-\hat{R}_s}\quad \textit{and}\quad \overline{\alpha}=\frac{y-R_{CB}-k}{y-R_{CB}},$$

which is strictly decreasing in k such that:

(i) if $\alpha < \hat{\alpha}$ *then there exists a unique "safe" REE characterized by* $R_s^\star \equiv \min\left(\bar{R}, \hat{R}_s\right)$ *where*

$$\hat{R}_s \equiv \frac{1-\gamma}{2}(y-k) + \frac{1+\gamma}{2} R_{CB}; \tag{12}$$

(ii) if $\alpha \geq \hat{\alpha}$ *and* $\hat{R}_r > \bar{R}$, *then there exists a "risky" REE characterized by* $R_r^\star \equiv \min\left(r, \hat{R}_r\right)$ *where*

$$\hat{R}_r \equiv \frac{1-\gamma}{2} y + \frac{1+\gamma}{2\alpha} R_{CB}; \tag{13}$$

(iii) a "safe" REE and a "risky" REE both exist when $\alpha = \hat{\alpha}$ *or*

$$\frac{R_{CB}}{y} \geq \alpha \geq \frac{y-k-R_{CB}}{y-R_{CB}},$$

which requires $y \geq (y - R_{CB})^2/r$. *In the latter both equilibria are degenerate, that is,* $(R_s^\star, R_r^\star) = (R_{CB}, y)$.

Proof. Postponed to the appendix.

$\hat{R}^s$ and $\hat{R}^r$ represent interior solutions, namely they are the contracts which locally maximize banks' profits when no participation constraints are binding; $R_s^\star$ and $R_r^\star$ instead account for the possibility that participation constraints bind. In particular, $R_r^\star > R_s^\star$, that is, ceteris paribus, risky projects imply higher interest rates. Nevertheless, the profit of a firm and of a bank can be higher when a risky project is implemented depending on parameters. Notice that at the risky equilibrium banks' participation constraint $\alpha R_r^\star - R_{CB} \geq 0$ is always satisfied whenever firms' participation constraint $r - R_r^\star \geq 0$ is too: in fact $r \geq R_r^\star$ implies $r \geq R_{CB}/\alpha$ which in turn yields $R_r^\star \geq R_{CB}/\alpha$.

The proposition below states the possibility of a SCE

Proposition 4 *For a given* R_{CB} *there is a sufficiently high* $\mathrm{E}^\beta[k]$ *such that for* $\alpha < \hat{\alpha}$ *a unique SSCE that is not REE exists characterized by* $R_r^* = \min\left(y, \hat{R}_r\right)$ *with* $\hat{R}_r > \bar{R}$. *Otherwise only REE exist.*

Proof (sketch). By construction it is $\mathrm{E}^\phi[\mu(R_r^*, \omega)] = \mathrm{E}^\beta[\mu(R_r^*, \omega)] \geq \mathrm{E}^\phi[\mu(R, \omega)]$ for any $R \in \Im(R_r^*)$ where $\Im(R_r^*)$ is an open neighborhood of R_r^*. Since $\alpha < \hat{\alpha}$ then there exists at least a $R' \leq y - k/(1-\alpha)$ such that $\mathrm{E}^\phi[\mu(R', \omega)] > \mathrm{E}^\phi[\mu(R_r^*, \omega)]$. Still this does not prevent $\mathrm{E}^\beta[\mu(R_r^*, \omega)] \geq \mathrm{E}^\beta[\mu(R', \omega)]$ since for any $R \in \Im(R_r^*)$ with $R_r^* > 0$, $\mathrm{f}(R, \omega)$ does not reveal the realization k. In particular, $\mu(R', \omega)$ is weakly decreasing in k from which the thesis.

On the other hand, there could not exist a safe SSCE that is not REE. Suppose such an equilibrium exists, then it would arise as a corner solution posted at the frontier $\bar{R}$ because interior safe SSCE are always REE. Nevertheless, by definition of a SSCE, agents would have correct beliefs for marginal deviations from the equilibrium that in this case would provide information about the actual α. Therefore at a SSCE posted along the frontier $\bar{R}$ agents would know the actual α. Hence banks can correctly forecast f(R, ω) at any R, and so they cannot sustain a safe SSCE that is not a REE. A contradiction arises. A safe SCE that is not REE can instead exists. ■

Figure 1 illustrates a baseline configuration of the economy in a case with homogeneous projects[4]. Let us firstly focus on panel A. The feasible range of equilibrium interest rates compatible with the adoption of a safe (risky) technology is the region below (above) the dotted curve representing the adoption frontier of firms. For any α value R_r and R_s are denoted by respectively the upper and lower solid/dashed lines. In particular, the solid line denotes the unique REE. For $\alpha < \hat{\alpha}$ a risky SSCE coexists with a safe REE (the threshold $\hat{\alpha}$ is denoted by a vertical dotted line).

Panel B plots the corresponding levels of social welfare for the REE and SSCE equilibria as a function of the aggregate risk in the economy, measured in terms of cost-per-vacancy c. Notice that since banks run at zero profits, then the social welfare coincides with the profits of firms. Social welfare is increasing in α (and so decreasing in R_r) when the economy is on a risky equilibrium, whereas it is decreasing whenever $\alpha \in (\underline{\alpha}, \bar{\alpha})$ for which the safe equilibrium arises as a corner contract constrained by the firms' profitability constraint. In the other regions where corner contract arises, namely when $R_r = y$ and $R_s = R_{CB}$, the economy displays no aggregate investment because, respectively firms and banks, are indifferent to participating or not to the market. For values of $\alpha < \underline{\alpha}$ instead, when the equilibrium contract is $\hat{R}_s$ the aggregate investment is insensitive to α.

Panel C and D illustrate the individual maximization problem of a single bank for a specific value $\alpha = 0.72$ when all the others post equilibrium contracts. The REE is the safe equilibrium whereas the SSCE is the risky one. This is evident from the inspection of panel C. The dotted line denotes the actual payoff that a bank would obtain conditional on all other banks posting at the risky equilibrium. The risky SSCE equilibrium contract R_r corresponds to a local maximum of the dotted line where $V(R_r) - c$ takes value zero due to free entry. Posting R_r is a *locally*-optimal action since marginal deviations from that contract would produce ex-ante negative profits. Nevertheless, the risky equilibrium is not a REE because lowering the interest rate up to the point where firms will adopt safe projects would yield a strictly positive ex-ante profits. The solid line in panel C denotes the actual payoff that a bank would obtain conditional on all other banks posting at the safe equilibrium. The safe SSCE equilibrium contract R_s corresponds to the absolute maximum of the solid line where $V(R_s) - c$ takes value zero as a consequence of free entry. Posting R_s is a *globally*-optimal action since any deviation from such contract would produce ex-ante negative profits.[5]

[4] We will maintain this simplifying assumption in all following examples.

[5] Concluding on the description of figure 1, Panel D plots the probabilities $q(R)$ (decreasing in R) and $p(R)$ (increasing in R) at the SCE and the REE. Their evolution reflect the effects of directed search. For a given equilibrium, the higher (lower) the interest rate posted by a bank, the lower (higher) the probability of filling that vacancy, and the higher (lower) the probability of firms obtaining funds at that rate.

Nevertheless, the risky SSCE equilibrium, and not the safe REE, can be sustained for sufficiently pessimistic beliefs on the value of k. For the sake of clarity, let me provide an extreme example. Suppose banks believe that with probability one $k = 0.0073$ instead of $k = 0.0042$. Such a beliefs in fact is never confuted by observable produced at the risky SSCE where no firm will implement the safe technology. The case with $k = 0.0073$ is displayed, with the same convention of figure 1, in figure 2. Notice that the risky SSCE in figure 1 is exactly the unique REE in figure 2. In fact as long as safe project are not adopted in equilibrium, the two economies are observationally equivalent: not only at the risky equilibrium but also for any marginal deviation from that equilibrium. The two pictures only differ for *large* out-of-equilibrium deviations which could trigger, in the first case but not in the second, a change in the type of investment.

5 Private vs Social Value of Experimentation

This section discusses the disconnect between the public and the private value of experimentation. In the first subsection we build up a simple example to clarify that a SSCE does not exclude that banks put a positive probability on a true state of the world. It just requires that they expect deviations would generate losses. Given such beliefs, banks do not have any incentive to experiment neither individually nor in cooperation with other banks. Nevertheless, the social value of experimentation largely overcomes the private evaluation of banks, as it includes the current and future increase in social welfare, which is constituted by firms' profit. This is why a benevolent policy maker would use resources in order to provide evidence of firms' reaction at low interest rates.

5.1 The private value of experimentation

To structure our discussion we will rely on a simple example that captures the main mechanism at work without loss of generality. Suppose the economy is at $\alpha = 0.72$ and $k_l = 0.0043$ as the one in figure 1, but banks also assign some probability to the case $k_h = 0.007$ plotted in figure 2. Banks posting at the risky equilibrium cannot discriminate between the two cases. The lines in the left panel of figure 5 display the expected payoff of a single bank, when all other bank post contracts at the risky SSCE, for different beliefs of banks about k. This difference shows up for interest rates lower than $\bar{R} = 1 - k_l/(1-\alpha)$, the adoption frontier with a low k. The red dashed curve corresponds to the expected payoff curve of a bank putting zero probability on k_l in analogy to the case of figure 2. The blue dashed curve denotes the expected payoff curve of a bank putting probability one on k_l in analogy to the case of figure 1. Finally the solid blue denotes the expected payoff curve of a bank that is just indifferent between posting interest rates that would reveal k (at the inside limit of the frontier) and posting at the risky equilibrium. This curve obtains exactly when k_l is believed with probability 0.213.

Notice that from the point of view of an individual bank, the evaluation of the benefits of a deviation concerns the expected return of one period only. In fact, free entry will guarantee zero expected profits on the best contract one period after. More precisely, the survival of low-interest contracts signals viable opportunities - meaning non-negative profits - which encourages other banks to enter in the market. Hence, for a risky SSCE which is not REE all banks need to believe k_l with a probability strictly lower than 0.213. Otherwise, more optimistic banks would open the market for low-interest rates providing evidence of worthy business.

Private experimentation cannot be solved by banks cooperating among each other. Banks with pessimistic beliefs simply judge not worth exploring new markets. A pessimism trap is not the result of a coordination problem among banks - as usually assumed in multiple equilibria models - but rather it concerns a creditor's uncertainty about the counterpart's payoff, within a single credit relationship. This clarifies that, in contrast to models of multiple equilibria, banks' nationalization is not a solution, whereas policies aiming to incentive bottom-up integration would be effective. Notice that firms and banks do not have incentive to cooperate. On one hand, firms have always incentive to signal that they will implement a safe project to obtain a lower interest rate and so higher profits[6]. On the other hand, a bank that finances risky project at low interest rates incurs in losses that pushes it out of the market. Moreover, banks have no incentive to implement a collective punishment strategy. It is costly for a bank to refuse a matching with a firm that lied in the past: such a firm would be not worse than others on the market. Finally notice that banks run in zero-profit, so they do not have resources to incentive cooperation or finance "explorers".

To sum up, when the low-fix-cost economy is the actual state of the world but banks believe it with a probability $\zeta < 0.213$, then conforming to the SSCE prescriptions is a dominant action from the point of view of a subjective Bayesian agent. Nevertheless, the absence of observable counterfactuals in equilibrium prevents learning about the true state. In this sense a SSCE does not require banks ignoring the presence of safe investment opportunities; it instead requires banks wrongly believe that with high probability is too costly for firms investing in a safe portfolio. This uncertainty cannot be solved by horizontal integration across banks, but it could instead be internalized through vertical integration.

5.2 The social value of experimentation

The social value of experimentation can be largely superior to the private one. This is true even if the policy maker is more pessimistic than banks. The role of the policy maker is to invest a fraction of current available resources in the interest of the collectivity, banks *and* firms, in order to provide a public good: the experimentation of low-interest rate credit markets. In other words, the aim

[6]Keep in mind that once the banks post a contract they cannot renegotiate the contract once the demand for funds has been realized.

of the policy is to *produce* a piece of information which has an extremely high social value but cannot be internalized by current private transactions.

In an intertemporal perspective the objective of the CB is to maximize the social welfare

$$\mathrm{W}_t = \mathrm{E}^{\beta}\left[\sum_{\tau=0}^{\infty} \delta^{\tau} \mathrm{w}_{t+\tau}\right]$$

with $\mathrm{w}_t = J_t^* - T_t + V_t(R^*) - c$, where δ is a discount factor weighting the welfare of future generations, J_t^* and $V_t(R^*) - c$ denote the expected profit of respectively firms and banks at the equilibrium; T_t is a tax intended to cover the eventual cost of social experimentation. Following our previous example suppose that the CB believes, like banks[7], that the probability of the low-fix-cost economy (the one of figure 1) is $\zeta < 0.213$. Imagine the experiment is conducted at time t. Whatever the way the experiment is conducted (we will discuss this issue later) the experiment publicly reveals the state of the word at time $t+1$. Hence, since time $t+1$ onwards the economy will be on a REE. The ex-ante social evaluation of the experiment would yield

$$\mathrm{W}_t = \mathrm{E}^{\beta}[\Delta \mathrm{w}_t] + \zeta \frac{\delta}{1-\delta}(J^s - J^r),$$

which depends on: the one period difference between firms' profit (social welfare) at the two potential REE, $J^s - J^r$, the social discount factor δ, the expected probability of a successful experimentation ζ, and the expected gain or cost of the social experiment on the current generation $\mathrm{E}^{\beta}[\Delta \mathrm{w}_t]$. The last term varies with the details of the implementation of the social experiment which we discuss below. The first term, $J^s - J^r$, instead is a well defined positive value being equal to the increase in firms' REE profits (banks will earn zero due to free entry). This is the dividend of the social experiment which is evaluated according to the social discount factor. A more conservative (progressive) policy maker, that is one with a lower (higher) δ, will be less (more) incline to social experimentation. For $\delta \to 0$ the evaluation of the experiment will only concern the eventual benefit to the current generation. With a $\delta \to 1$ instead the policy maker has incentive to experiment every good state of the world which receives a strictly positive probability, no matter what is the cost in terms of the current generation.

6 Policy as Experimentation

The aim of this section is to provide a discussion of policy interventions. First we consider the impact of a conventional policy of reduction of the cost of money. Second, we explore the impact of an universal bank's subsidy as the one adopted in the TALF experience. The policy maker induces a social experiment providing subsidies to banks to cover their expected losses. Once low-interest

[7] By continuity, all the following arguments work equally with a policy maker that is more pessimist than the private sector.

rate market are opened firms' incentives unfold and a SSCE eventually breaks. Although such a market could be created subsidizing a zero-measure set of banks who are lately imitated by other banks, the authority maximizes the value of experimentation providing the subsidy to all banks. This is because social learning takes time, whereas a large-scale subsidy instantaneously realizes the social benefits of successful experimentation "learning by doing".

6.1 The effect of lowering the policy interest rate

Figure 3 illustrates the effect of lowering the CB interest rate from 0.01 to 0.005 in the economy described by figure 1. A lower cost of money provides for lower interest rates on equilibrium contracts. In particular, such a policy can reduce the set of economies where a risky SSCE exists. This happens when the optimal risky contract became bounded by the adoption frontier. In fact, in panel A and B, the dotted blue line exhibits a discontinuity in the range of α for which $\hat{R}^r \leq \bar{R}$. This is a case in which the best risky contract lies at the outside limit of the adoption frontier (i.e. posting a $R^{'} = \bar{R} + \varepsilon$ with $\varepsilon > 0$ infinitesimal), but posting a contract at the inside limit of the frontier (i.e. posting a $R' = \bar{R} - \varepsilon$ with $\varepsilon > 0$ infinitesimal) gives a higher payoff. This is plotted in panel C. Therefore the set of risky SSCE is reduced by a decrease of the CB interest rate. At the same time, the aggregate investment at the REE increases.

Nevertheless, the effect of a decrease in the cost of money can be offset by a decrease in project return y. This scenario is illustrated by figure 4. y is lowered from 0.03 to 0.025, all the rates shift downwards but now the frontier $\bar{R}$ also follows the move. The scenario is qualitatively the same as in figure 1. This demonstrates that lowering the interest rates can have only a limited impact. More importantly, the possibility to lower policy rates is constrained below by the existence of the zero-lower bound which are binds exactly when the likelihood of a self-confirming credit crisis is higher, that is, in presence of higher aggregate credit risk in the economy. Next section is devoted to the analysis of non-conventional policies that are able to break the spell more effectively.

6.2 *Credit-Easing* through banks' subsidies

The revelation of firms' true incentives through the opening of out-of-equilibrium credit markets breaks self-confirming failures, correcting wrong beliefs. The result can be achieved though a credit-easing policy, that is direct lending from the CB to the firms. Nevertheless, it is realistic to assume that the central bank does not have lending facilities, so that it has to "use the market", that is, it should provide the right incentive for banks to set low interest rates.

In accordance to what happened with the TALF program (see later discussion), we will explore the introduction of a bank subsidy which covers eventual banks' credit losses such that the per-unit-return on a risky project becomes

$$\pi\left(r, R, R_{CB}\right) = \left(\alpha + (1-\alpha)\,\mathrm{sub}\right) R - R_{CB}, \tag{14}$$

where sub $\in (0,1)$ is the fraction of losses covered in case of project failure (which occurs with probability $1-\alpha$). The payoff in case of safe projects adoption $\pi(r, R, R_{CB})$ remains unaffected. We will also investigate what is the optimal size of the intervention focusing on the two extreme scenarios: a *large-scale* subsidy provided to all active banks, or a controlled experiment, which only provide the subsidy to a *zero-measure* set of banks. There is an implicit trade-off in this choice. On one hand, the socially valuable information can be produced experimenting on a zero measure set of banks with a negligible expected cost. On the other hand, the impact of a controlled experiment relies on social learning that operates with a lag of one period. In other words, a controlled experiment would imply by construction $\mathrm{E}^{\beta}[\mathrm{w}_t] = 0$, whereas the expected value of a large scale intervention depends on the expected impact on the current generation. In particular, suppose a CB believe k_l with probability ζ, then a *large-scale* policy which incentives all banks to post a contract R_d yields

$$\mathrm{E}^{\beta}[\Delta \mathrm{w}_t] = (1-\zeta)(\underbrace{\frac{\alpha^2}{2}(y-R_d)^2 p_r(R_d)}_{\text{firms' profit in case } k_h} - \underbrace{\alpha(y-R_d)(1-\alpha)\mathrm{sub}R_d q_r(R_d)}_{\text{cost of the subsidy}}) + \\ + \zeta \underbrace{p_s(R_d)\frac{1}{2}(y-k-R_d)^2}_{\text{firms' profit in case } k_l} - \underbrace{p_r(R_r)\frac{\alpha^2}{2}(y-R_r)^2}_{\text{firms' profit at the SSCE}},$$

where the probabilities $p_s(R_d)$ and $p_r(R_d)$ denote the likelihood of a matching for a firm when all other firms adopt respectively the safe and the risky portfolio at R_d. In particular,

$$R_d = \arg\max_{R}\{\mathrm{E}^{\beta}[\mu(R,\omega)]\},$$

with $\mu(R,\omega)$ being calculated according to (14), represents the optimal contract induced by a level of subsidy. The policy maker maximizes (15) given that banks react optimally to the incentive created by sub. For the experiment to be implemented it is necessary that the subsidy is large enough for $R_d \leq \bar{R}$, that is, banks have interest in experimenting the safe adoption region.

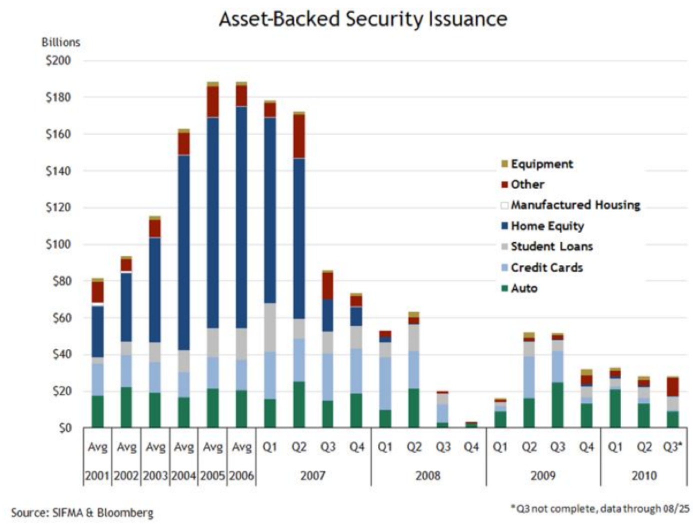
The social evaluation formula (15) rules out the possibility that the policy maker can smooth forward the social costs of experimentation. In such a case the large scale solution is always preferred to the controlled experiment as it generates either instantaneous benefits or negligible losses for all the generations to come. This requires that the policy maker has some reserves or that the credit sector is small with respect to the size of the whole economy. This leads to the result stated by the following proposition.

Proposition. *In case intergenerational transfers are possible, the social experiment will be always conducted on a large scale. Otherwise, the social experiment is conducted on a large scale if and only if* $\mathrm{E}^{\beta}[\Delta \mathrm{w}_t] > 0$ *provided* $R_d \leq \bar{R}$.

Figure 5 helps to get a better understanding of the impact of a bank subsidy in the two scenarios of a large-scale or zero-measure policy intervention. The upper-right panel plots the expected payoff of a single bank, when all other bank post contracts at the risky SSCE, in the case of an universal subsidy of sub = 0.36 (corresponding to the 36% of coverage) for the same three cases and with the same conventions of the upper-left panel. Notice that the subsidy has two contrasting effects. On one hand, a subsidy reduces the loss in the k_h scenario, so it increases the value of an individual deviation into safe territory (that is, the dashed red curve is higher). On the other hand, it reduces the distance between the equilibrium value of the risky interest rate - the risky option is less risky because of the subsidy - and the adoption frontier (the dashed blue curve decreases). This last effect implies a lower competitive advantage from experimenting lower interest rates, that is, less firms will apply to the out-of-equilibrium offer and so the value of an individual deviation lowers. The lower-left panel of figure 5 plots the expected payoff of a single bank, when all other bank post contracts at the risky SSCE, in the case of a subsidy of sub = 0.36 is provided to this bank only (or to a zero-measure set of banks). In this case the whole pay-off curve for risky adoption shifts left-up as subsidized banks can benefit of a rent due to higher liquidity demand. It has no effect instead on the payoff curve corresponding to safe adoption. A trade-off arises also in this case: the value of posting in the safe area does not decrease, but the value of posting in the risky area increases well above zero-profits because of the competitive advantage allowed by the subsidy.

The net result of the trade-off in the two cases is depicted in the lower-right panel of figure 5. The curve assigns for each value of sub the maximal probability that banks put on k_l that makes them to sustain a risky SSCE. The graph shows that the outcome of the subsidy is the same irrespective of the size of the intervention. This is not surprising because the individual evaluation criterion (20) is independent from the profile of others' actions. A subsidy has initially a positive effect since it lowers the minimal belief that sustain a SSCE. Nevertheless, as the subsidy approaches the totality of expected losses, the negative effect prevails and the subsidy makes more difficult to break a risky SSCE. Nevertheless, at the limit of sub = 1 (and higher values) the banks are induced to post at the (inside of the) adoption frontier for no matter which belief given that the central bank is absorbing the whole risk. This way the authority can induce an experiment using the market.

Figure 6 finally depicts the evaluation of $\Delta \mathrm{w}_t$ for the case of figure 5 with an fully-coverage subsidy (sub = 1) provided on a large scale, as a function of ζ the probability that the value of k is k_l. The subsidy is such that all banks are induced to post at the adoption frontier independently of their beliefs. Notice that for $\zeta = 0$ the expected social value is negative. In such a case, the risky equilibrium is a REE which already guarantees the social optimum. Nevertheless, as soon as ζ increases, the expected value becomes positive, that is, the expected increase in firms' profits largely overcomes the expected cost of the experimental policy. Unless the policy maker has very pessimistic beliefs about k_l, it will find optimal to run the subsidy on a large scale, even if the eventual costs of an unsuccessful policy cannot be transferred to future generations. This is true also in case the policy maker is more pessimistic than private banks.



7 Discussion

7.1 The collapse of the ABS market and the TALF policy

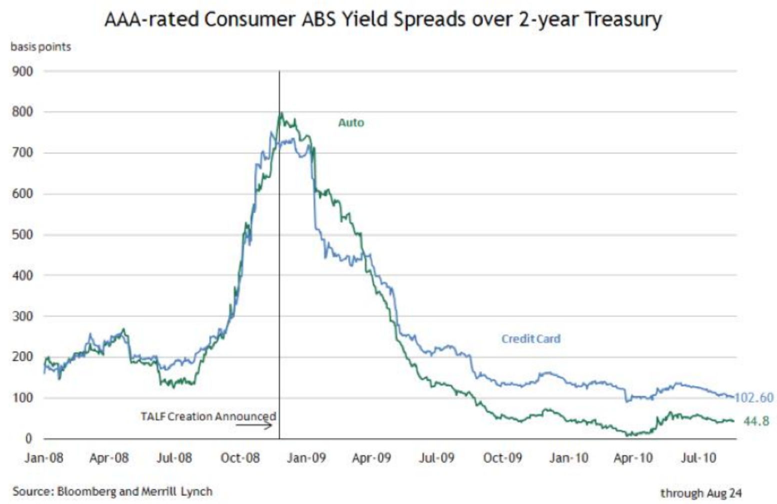
One of the most interesting example of successful credit-easing occurred during the 2008-2009 crisis of the Asset Backed Securities (ABS) market in US. In the second half of 2007 the ASB market has experienced a sudden contraction after a constant increase in volumes since early 2000.[8] The crash was mostly driven by lower-than-expected returns in the housing markets which depressed the value of subprime home equities. The dramatic increase in perceived risk and the lack of confidence in rating agencies did even not spare an abrupt freeze of the AAA-rated ABS segment whose interest-rate rose at exceptionally high levels reflecting unusually high risk premiums[9]. As a consequence private liquidity collapsed rapidly, and investors directed available resources to quality assets like treasury bills which almost doubled their daily volumes of trade from 40 to 80 USD billions during 2008-2009.

Within this context the Fed stepped in with the lunch of the Term Asset Backed Securities Lending Facility (TALF) which supplied about 71 billions of non-recourse loans[10] at lower interest rates, to any U.S. company provided of

[8] New issuances of consumer ABS plunged from $50 billion per quarter of new originations in 2007 to only $4 in the last quarter of 2008.

[9] "This can be seen in the fact that AAA-rated student loan tranches, with underlying loans 97% guaranteed by the federal government, climbed to yield levels as much as 400 basis points over LIBOR." Speech by William C. Dudley, President and Chief Executive Officer on the 4th. of June 2009.

[10] Meaning that if the economy performs very badly and the securities fall sharply in value, an investor can put the collateral that secures its TALF loan back to the Fed, only losing the collateral haircut.



highly rated (AAA and AAA-) collateral. This intervention was made primarily to sustain the credit market in a period of high perceived counterpart risk. More precisely, the Fed acted as a borrower of last resort taking the risk of experimenting contractual conditions which were perceived as too risky by the private sector.

Nevertheless, despite malign prophecies welcoming the birth of the programs, on the 30th of September 2010, the Fed announced that more than 60 percent of the TALF loans have been repaid in full, with interest, ahead of their legal maturity dates. In other words, the more favorable conditions offered by the Fed, instead of triggering an adverse selection process, have been the prelude of a remarkable business performance and expansion of consumption credit.

Should we conclude then that the market was mispricing highly rated ABS? Not necessarily in the sense that probably at such high interest rates failures to repay the loans could have been much more likely. According to our SSCE theory excessive pessimist could have prevented the private sector from experimenting and so revealing the existence of profit opportunities in the high-quality segment of the ABS market. In the absence of the Fed intervention such bias could have not been corrected, self-sustaining a suboptimal outcome with major consequences for the already tatty American economy.

7.2 The contrast with Self-fulfilling credit freezes

A different explanation of the TALF effectiveness could be instead given in terms of multiple REE with the action of the Fed helping coordination. Bebchuk and Goldstein's (2011) (B&G from now) model of credit freezes provides an excellent multiple REE benchmark. It is useful to have a first contrast of both models to clarify the different policy prescription that the two approaches generate.

In the B&G model, there are bad firms with useless projects and good firms whose projects can have positive returns if, and only if, the (parameterized) state of the economy is sufficiently good (high) and enough banks lend to firms. When the state of the economy is sufficiently good (bad) all banks lend (do not lend) to firms. For intermediate values, there is a coordination problem among banks and if they have precise information about the state of the economy there two rational expectations equilibria coexist: one with lending and one without. Following the *global-games* approach of Morris and Shin (2004) they consider the case where banks have imprecise information and, therefore, their decision to lend or not is given by a threshold value of the signal received. Correspondingly, there is a threshold value of the state of the economy determining a region characterized by inefficient credit freezes.

There are interesting difference between the models. First, in B&G the inefficient Self-fulfilling credit-freeze equilibrium is a manifestation of a *coordination failure*, while in our model the inefficient Self-Confirming low-credit equilibrium is a manifestation of a *misperceptions*. As a result there is a 'structural policy' fundamental difference: Self-fulfilling credit-freezes can be avoided by concentrating the banking system in the B&G model, while banking concentration does not resolve the *misperceptions*, as said, a vertical integration between borrowers and lenders would be needed. Second, in B&G a Self-fulfilling credit freeze is associated with low risk in financial markets (in fact, only riskless government bonds are traded), while in our low-credit Self-confirming equilibrium is characterized by high risk investments. Third, in B&G an economy falls into a Self-fulfilling credit freeze equilibrium as a result of an adverse shock to banks' capital, in ours into a Self-confirming low-credit equilibrium because of an increase in risk in the financial sector.

There are interesting similarities too. Our (two-variables) state of the economy plays a similar role that the B&G state of the economy. Similarly, the analogy can be extended to consider that our Self-confirming high-risk (low credit) equilibrium correspond – even if they are qualitatively different – to their rational expectations Self-fulfilling credit freezes. Keeping this analogy, both models have a common property: policies that are effective in 'destroying' the low-credit equilibrium only need to be implemented temporarily: until they produce the desired 'cleansing effect'.

Regarding the three mentioned policy interventions of the Central Bank a brief comparison of their effects is as follows:

Lowering the lending rate: it has a similar effect in both models: for certain states of the economy the policy destroys the low credit equilibrium, while for other (not at the margin) states the policy is ineffective.

Capital injections into the banking system: in B&G Central Bank can be effective since they raise the amount of lending and, therefore, they also reduce the set of economies for which a Self-fulfilling credit freeze equilibrium exists. In contrast, in our model as we have already mentioned such a policy is ineffective.

***Credit-easing*:** while both models vindicate this policy as the most effective in reducing the probability of a low-credit equilibrium, there is an important policy difference while, in terms of efficiency, *Credit-easing* can be dominated by a policy of capital injections into the banking system in B&G, as we have seen this can never happen in our economies. Furthermore, one expects that it should be 'less costly' to resolve a *misperceptions* problem than a *coordination failure* with *credit-easing* this is an empirical question, but as we have seen, social experimentation with *credit-easing* doesn't need to be socially costly.

Furthermore, there are differences on what *credit-easing* is in both models. In B&G the policy is implemented by directly lending to firms and the CB is prevented from fully substituting the private bank system by assuming that it cannot discriminate between bad and good firms as private banks do. In our model, as we have seen, the CB cannot substitute the private banks and, therefore, implements the policy through them (which seems closer to the evidence; see below the TALF example). Finally, in our model *Credit-easing* is an explicit policy experiment in which cost and benefits are assessed and the different public-private valuations compared as part of the policy design, while in their model the policy design problem is not addressed (a common feature of Self-fulfilling equilibrium models).

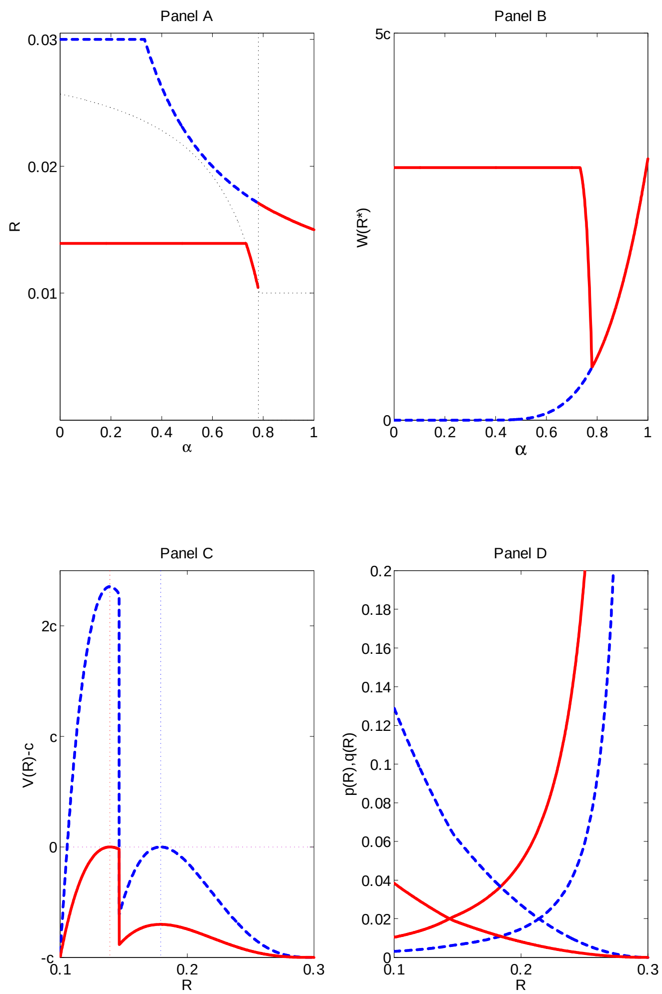


Figure 1: **Baseline parameterization.** Panel A: REE (solid) and SCE (dashed) in the (R, α)-space. Panel B: aggregate investment at the REE (solid) and the SCE (dashed). Panel C: payoff of a bank's individual deviation from the REE (solid) and the SCE (dashed) for $\alpha = 0.72$. Panel D: matching probabilities ($q(R)$ decreasing in R, $p(R)$ increasing in R) relative to a bank's individual deviation from the REE (solid) and SCE (dashed) for $\alpha = 0.72$. Other parameters taken fix across panels $y = 0.3$, $R_{CB} = 0.01$, $k = 0.0043$, $A = 0.02$, $\gamma = 0.5$, $c = 10^{-6}$.

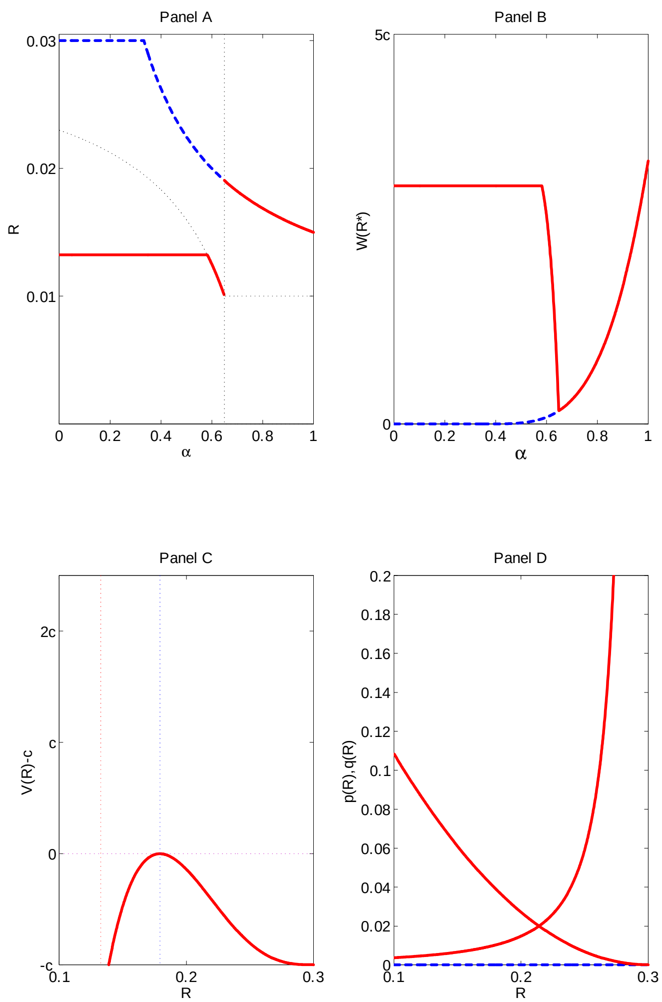


Figure 2: **Higher fix cost for safe projects ($k = 0.007$).** Panel A: REE (solid) and SCE (dashed) in the (R, α)-space. Panel B: aggregate investment at the REE (solid) and the SCE (dashed). Panel C: payoff of a bank's individual deviation from the REE (solid) and the SCE (dashed) for $\alpha = 0.73$. Panel D: matching probabilities ($q(R)$ increasing in R, $p(R)$ decreasing in R) relative to a bank's individual deviation from the REE (solid) and SCE (dashed) for $\alpha = 0.73$. Other parameters taken fix across panels $y = 0.3$, $R_{CB} = 0.01$, $A = 0.02$, $\gamma = 0.5$, $c = 10^{-6}$.

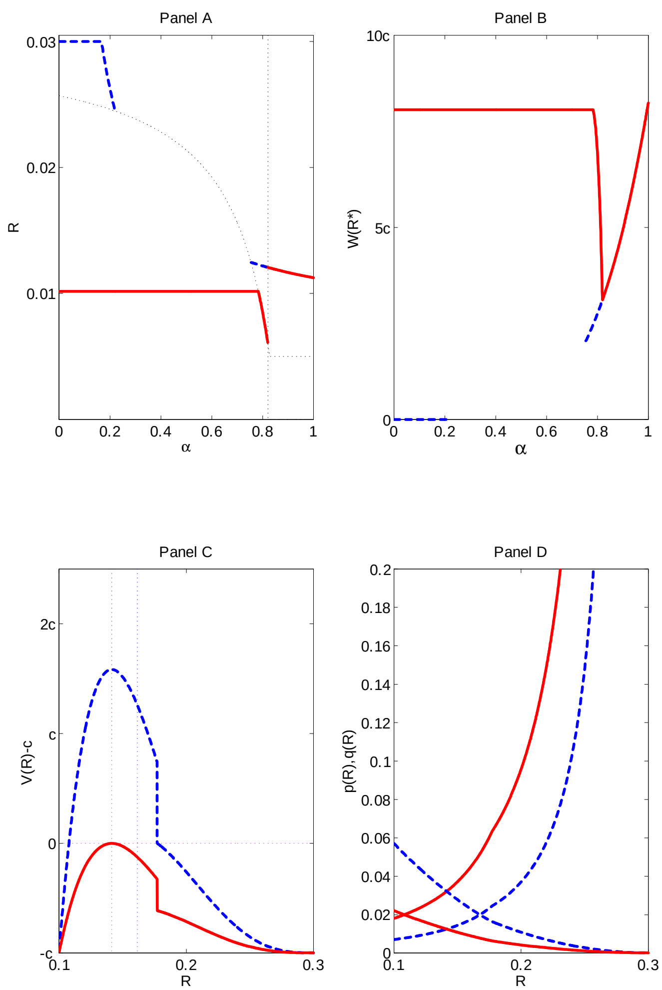


Figure 3: **Reduction in the cost of money** ($R_{CB} = 0.005$)**.** Panel A: REE (solid) and SCE (dashed) in the (R, α)-space. Panel B: aggregate investment at the REE (solid) and the SCE (dashed). Panel C: payoff of a bank's individual deviation from the REE (solid) and the SCE (dashed) for $\alpha = 0.72$. Panel D: matching probabilities ($q(R)$ decreasing in R, $p(R)$ increasing in R) relative to a bank's individual deviation from the REE (solid) and SCE (dashed) for $\alpha = 0.72$. Other parameters taken fix across panels $y = 0.3$, $k = 0.0043$, $A = 0.02$, $\gamma = 0.5$, $c = 10^{-6}$.

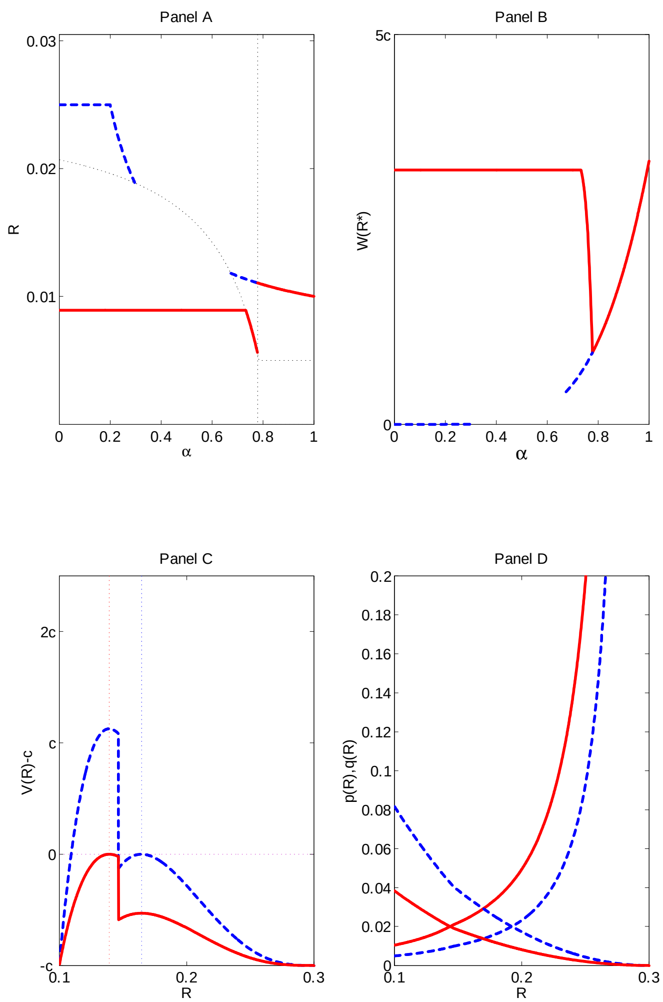


Figure 4: **Decrease in project returns** ($y = 0.025$). Panel A: REE (solid) and SCE (dashed) in the (R, α)-space. Panel B: aggregate investment at the REE (solid) and the SCE (dashed). Panel C: payoff of a bank's individual deviation from the REE (solid) and the SCE (dashed) for $\alpha = 0.72$. Panel D: matching probabilities ($q(R)$ decreasing in R, $p(R)$ increasing in R) relative to a bank's individual deviation from the REE (solid) and SCE (dashed) for $\alpha = 0.72$. Other parameters taken fix across panels $R_{CB} = 0.005$, $k = 0.0043$, $A = 0.02$, $\gamma = 0.5$, $c = 10^{-6}$.

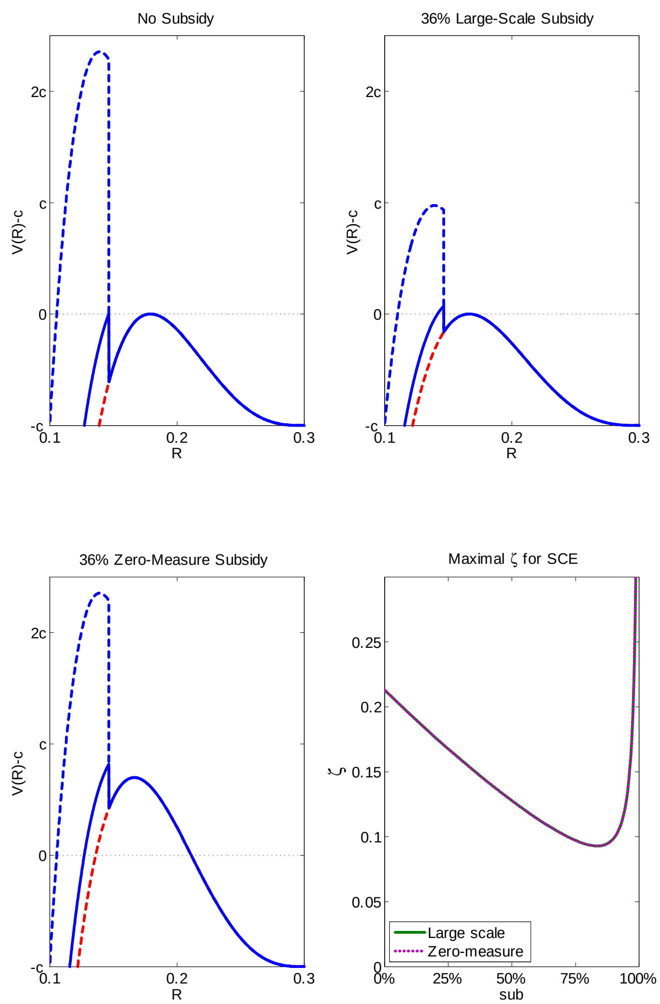


Figure 5: **Subsidy.** The blu line represents the expected payoff function when banks believe with probability 0.213 that $k = 0.0043$ and with probability 0.787 that $k = 0.007$. In the upper-left panel the subsidy is set to zero sub $= 0$ whereas in the upper-right and in the lower-left panels the central banks provides sub $= 0.1$ respectively to all banks and to a zero-measure set of thme. The curve in the lower-right panel assigns for each value of the subsidy sub the maximal probability that banks put on k_l that makes them to sustain a risky SCE. Other parameters taken fix across panels $\alpha = 0.72$, $y = 0.3$, $R_{CB} = 0.01$, $k = 0.0043$, $A = 0.02$, $\gamma = 0.5$, $c = 10^{-6}$.

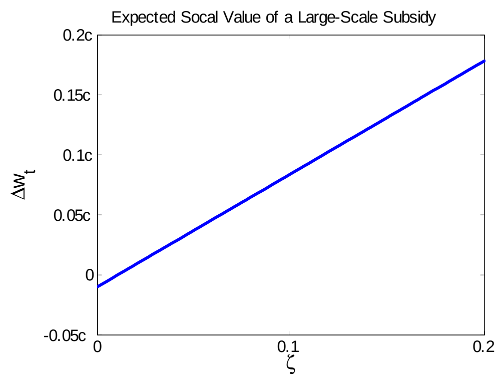


Figure 6: **Expected Social Value of a Large Scale Policy.** Figure 6 depicts the evaluation of $\Delta \mathrm{w}_t$ with sub $= 1-\alpha$ provided on a large scale, as a function of ζ the probability that the value of k is k_l. Other parameters taken fix across panels $\alpha = 0.72$, $y = 0.3$, $R_{CB} = 0.01$, $k = 0.0043$, $A = 0.02$, $\gamma = 0.5$, $c = 10^{-6}$.

Appendix

Proof of proposition 1. Consider a single bank that evaluates an individual deviation R from an equilibrium contract R^*. A bank knows that firms act with perfect information, that is, banks know that firms enter the submarket that yield the highest expected profit.

Therefore, a bank anticipates that for any possible state of the world the tightness linked to the submarket R is such that

$$\bar{J} = p\left(\boldsymbol{\theta}\left(R^*, \bar{J}\right)\right)\Pi\left(R^*\right) = p\left(\boldsymbol{\theta}\left(R, \bar{J}\right)\right)\Pi\left(R, \mathrm{f}\right) \tag{16}$$

where $\bar{J}$ is a constant ex-ante level of firm profits that a single bank cannot affect. For the sake of notational simplicity we denote $\mathrm{f}(R,\omega)$ as just f (and $\mathrm{f}(R^*,\omega)$ as f^*) when there is no ambiguity. Given the relation above the out-of-equilibrium function $p\left(\boldsymbol{\theta}\left(R, \bar{J}\right)\right)$ is obtained as

$$p\left(\boldsymbol{\theta}\left(R, \bar{J}\right)\right) = p\left(\boldsymbol{\theta}\left(R^*, \bar{J}\right)\right)\frac{\Pi\left(\mathrm{f}\right)}{\Pi\left(\mathrm{f}\right)}, \tag{17}$$

for any state of the world. Notice that if R decreases, $\Pi\left(\mathrm{f}\right)$ increases and so $p\left(\boldsymbol{\theta}\left(R, \bar{J}\right)\right)$ decreases,that is the probability for a firm to match decreases in R.

Nevertheless a bank does not know with certainty which state of the world has realized. According to (2), the expected value of a credit line is

$$V\left(R, \theta\left(R\right)\right) = \mathrm{E}^{\beta}\left[q\left(\theta\left(R\right)\right)Y\left(R, \mathrm{f}\right)\right]$$

for the individual bank posting the contract R whereas all the others posting R^*. Given banks' knowledge of the tightness function (17), we can write

$$V\left(R, \boldsymbol{\theta}\left(R, \bar{J}\right)\right) = \mathrm{E}^{\beta}\left[\boldsymbol{\theta}\left(R, \bar{J}\right)p\left(\boldsymbol{\theta}\left(R^*, \bar{J}\right)\right)\frac{\Pi\left(\mathrm{f}^*\right)}{\Pi\left(\mathrm{f}\right)}Y\left(R, \mathrm{f}\right)\right],$$

where we also used the fact $q(\theta) = \theta p(\theta)$. On the other hand, the bank who conforms to the equilibrium prescriptions expects

$$V\left(R^*, \boldsymbol{\theta}\left(R^*, \bar{J}\right)\right) = \boldsymbol{\theta}\left(R^*, \bar{J}\right) p\left(\boldsymbol{\theta}\left(R^*, \bar{J}\right)\right) Y\left(R^*, \mathrm{f}^*\right),$$

where notice we assume that there is no uncertainty at the equilibrium, that is $\mathrm{E}^{\beta}\left[V\left(R^*, \boldsymbol{\theta}\left(R^*, \bar{J}\right)\right)\right] = V\left(R^*, \boldsymbol{\theta}\left(R^*, \bar{J}\right)\right)$.

The latter is a weakly dominant strategy if and only if

$$\begin{aligned}\boldsymbol{\theta}\left(R^*, \bar{J}\right) p\left(\boldsymbol{\theta}\left(R^*, \bar{J}\right)\right) Y\left(R^*, \mathrm{f}^*\right) \geq \\ \geq \mathrm{E}^{\beta}\left[\boldsymbol{\theta}\left(R, \bar{J}\right) p\left(\boldsymbol{\theta}\left(R^*, \bar{J}\right)\right) \frac{\Pi\left(\mathrm{f}^*\right)}{\Pi\left(\mathrm{f}\right)} Y\left(R, \mathrm{f}^*\right)\right]\end{aligned}$$

or

$$\mathrm{E}^{\beta}\left[\frac{\boldsymbol{\theta}\left(R, \bar{J}\right)}{\boldsymbol{\theta}\left(R^*, \bar{J}\right)} \frac{\Pi\left(\mathrm{f}^*\right)}{\Pi\left(\mathrm{f}\right)} \frac{Y\left(R, \mathrm{f}\right)}{Y\left(R^*, \mathrm{f}^*\right)}\right] \leq 1 \tag{18}$$

To derive $\boldsymbol{\theta}\left(R, J^*\right)$ we can use the definition of $\bar{J}$ in (16), so that

$$\bar{J} = A\boldsymbol{\theta}\left(R, \bar{J}\right)^{\gamma-1} \Pi\left(\mathrm{f}\right)$$

implies

$$\boldsymbol{\theta}\left(R, \bar{J}\right) = C\Pi\left(\mathrm{f}\right)^{\frac{1}{1-\gamma}} \tag{19}$$

where $C \equiv \left(A/\bar{J}\right)^{\frac{1}{1-\gamma}}$ is a constant for any single deviation R. Hence, we obtain

$$\frac{\boldsymbol{\theta}\left(R, \bar{J}\right)}{\boldsymbol{\theta}\left(R^*, \bar{J}\right)} = \left(\frac{\Pi\left(\mathrm{f}^*\right)}{\Pi\left(\mathrm{f}\right)}\right)^{-\frac{1}{1-\gamma}},$$

which is true at any state of the world. Hence (18) becomes

$$\mathrm{E}^{\beta}\left[\left(\frac{\Pi\left(\mathrm{f}^*\right)}{\Pi\left(\mathrm{f}\right)}\right)^{-\frac{\gamma}{1-\gamma}} \frac{Y\left(R, \mathrm{f}\right)}{Y\left(R^*, \mathrm{f}^*\right)}\right] \leq 1$$

or

$$\mathrm{E}^{\beta}\left[\Pi\left(\mathrm{f}\right)^{\frac{\gamma}{1-\gamma}} Y\left(R, \mathrm{f}\right)\right] \leq \Pi\left(\mathrm{f}^*\right)^{\frac{\gamma}{1-\gamma}} Y\left(R^*, \mathrm{f}^*\right)$$

which is (20) in the main text. Notice that the result does not depend on which $\bar{J}$ is believed by banks.

If instead we consider two arbitrary single deviations R_1 and R_2 from R^* we get that the latter weakly dominates the former when

$$\begin{aligned}\mathrm{E}^{\beta}\left[\boldsymbol{\theta}\left(R_2, \bar{J}\right) p\left(\boldsymbol{\theta}\left(R^*, \bar{J}\right)\right) \frac{\Pi\left(\mathrm{f}\left(R^*, \omega\right)\right)}{\Pi\left(\mathrm{f}\left(R_2, \omega\right)\right)} Y\left(R_2, \mathrm{f}\left(R_2, \omega\right)\right)\right] \geq \\ \geq \mathrm{E}^{\beta}\left[\boldsymbol{\theta}\left(R_1, \bar{J}\right) p\left(\boldsymbol{\theta}\left(R^*, \bar{J}\right)\right) \frac{\Pi\left(\mathrm{f}\left(R^*, \omega\right)\right)}{\Pi\left(\mathrm{f}\left(R_1, \omega\right)\right)} Y\left(R_1, \mathrm{f}\left(R_1, \omega\right)\right)\right]\end{aligned}$$

or

$$\mathrm{E}^{\beta}\left[\boldsymbol{\theta}\left(R_2,\bar{J}\right)\frac{Y\left(R_2,\mathrm{f}\left(R_2,\omega\right)\right)}{\Pi\left(\mathrm{f}\left(R_2,\omega\right)\right)}\right]\geq\mathrm{E}^{\beta}\left[\boldsymbol{\theta}\left(R_1,\bar{J}\right)\frac{Y\left(R_1,\mathrm{f}\left(R_1,\omega\right)\right)}{\Pi\left(\mathrm{f}\left(R_1,\omega\right)\right)}\right]$$

which becomes

$$\mathrm{E}^{\beta}\left[C\Pi\left(R_2\right)^{\frac{1}{1-\gamma}}\frac{Y\left(R_2,\mathrm{f}\left(R_2,\omega\right)\right)}{\Pi\left(\mathrm{f}\left(R_2,\omega\right)\right)}\right]\geq\mathrm{E}^{\beta}\left[C\Pi\left(R_1\right)^{\frac{1}{1-\gamma}}\frac{Y\left(R_1,\mathrm{f}\left(R_1,\omega\right)\right)}{\Pi\left(\mathrm{f}\left(R_1,\omega\right)\right)}\right]$$

after using (19). Finally we get

$$\mathrm{E}^{\beta}\left[\mu\left(R_2\right)\right]\geq\mathrm{E}^{\beta}\left[\mu\left(R_1\right)\right], \tag{20}$$

where

$$\mu\left(R\right)\equiv\Pi\left(\mathrm{f}\left(R,\omega\right)\right)^{\frac{\gamma}{1-\gamma}}Y\left(R,\mathrm{f}\left(R,\omega\right)\right),$$

that is our operative criterion.

Therefore the best *interior* single deviation is the one that maximizes

$$\mathrm{E}^{\beta}\left[\mu\left(R\right)\right]$$

whose first order condition is

$$\mathrm{E}^{\beta}\left[\frac{\gamma}{1-\gamma}\Pi\left(\mathrm{f}\right)^{\frac{\gamma}{1-\gamma}}\frac{\Pi'(\mathrm{f})}{\Pi(\mathrm{f})}Y\left(R,\mathrm{f}\right)+\Pi\left(\mathrm{f}\right)^{\frac{\gamma}{1-\gamma}}Y'\left(R,\mathrm{f}\right)\right]=0$$

at any state of the world. Hence

$$\mathrm{E}^{\beta}\left[\left(\Pi\left(\mathrm{f}\right)^{\frac{\gamma}{1-\gamma}}Y\left(R,\mathrm{f}\right)\right)\left(\gamma\frac{\Pi'(\mathrm{f})}{\Pi(\mathrm{f})}+\left(1-\gamma\right)\frac{Y'\left(R,\mathrm{f}\right)}{Y\left(R,\mathrm{f}\right)}\right)\right]=0$$

whose deterministic solution is

$$\gamma\frac{\Pi'(\mathrm{f})}{\Pi(\mathrm{f})}+\left(1-\gamma\right)\frac{Y'\left(R,\mathrm{f}\right)}{Y\left(R,\mathrm{f}\right)}=0,$$

which reconciles with the solution under certainty we found using standard techniques.

Proof of proposition 2 An interior SSCE R^* is a solution to

$$\begin{aligned}&\max_{R}q\left(R\right)Y\left(R,\mathrm{f}\right)\\&\text{s.t. }\bar{J}=p\left(R\right)\Pi(\mathrm{f})\end{aligned}$$

where $\mathrm{E}^{\beta}\left[\mathrm{f}\left(R,\omega\right)\right]=\mathrm{f}\left(R,\omega\right)$ for any $R\in\Im\left(R^*\right)$.[11] Therefore the following two FOCs

$$\begin{aligned}q'\left(R\right)Y\left(R,\mathrm{f}\right)+q\left(R\right)Y'\left(R,\mathrm{f}\right)&=0\\p'\left(R\right)\Pi\left(\mathrm{f}\right)+p\left(R\right)\Pi'\left(\mathrm{f}\right)&=0\end{aligned}$$

[11] To save on notation from here onwards we will omit dependence of functions from R.

have to be satisfied at the equilibrium R^*. Since $p(R) = A\theta(R)^{\gamma-1}$ and $q(R) = A\theta(R)^{\gamma}$ we can rewrite the latter as

$$(\gamma - 1)\Pi(\mathrm{f}) + \theta(R)\Pi(\mathrm{f})' = 0$$

and the former as

$$\gamma Y(R, \mathrm{f}) + \theta(R) Y'(R, \mathrm{f}) = 0$$

and finally use both to get rid of θ obtaining the relation

$$\gamma Y(R, \mathrm{f}) + (1-\gamma)\frac{\Pi(\mathrm{f})}{\Pi'(\mathrm{f})} Y'(R, \mathrm{f}) = 0,$$

or

$$\gamma \frac{\Pi'(\mathrm{f})}{\Pi(\mathrm{f})} + (1-\gamma)\frac{Y'(R, \mathrm{f})}{Y(R, \mathrm{f})} = 0$$

After simple manipulations we have that, at the equilibrium,

$$1 - \gamma = \frac{Y(R^*, \mathrm{f}^*)}{Y(R, \mathrm{f}^*) - \frac{Y'(R^*, \mathrm{f}^*)}{\Pi'(\mathrm{f}^*)}\Pi(\mathrm{f}^*)}$$

which demonstrates that the Hosios condition is met.

Proof of proposition 3 Here we deal with REE that is the case where banks' beliefs are correct also out of equilibrium.

FIRST STEP. The first step of the proof is to establish the set of best R contracts conditional to firms having incentives to adopt one specific type of technology. Call these locus best restricted contracts.

To obtain the best restricted contracts when *no participation constraints bind* it is enough to plug the explicit form of I and π into (11). In the case of a risky technology we have

$$\hat{R}_r = \frac{1-\gamma}{2} r + \frac{1+\gamma}{2\alpha} R_{CB}; \tag{21}$$

whereas in case of a safe technology

$$\hat{R}_s = \frac{1-\gamma}{2}(r - k) + \frac{1+\gamma}{2} R_{CB}. \tag{22}$$

(21) and (22) are the best interior contracts that a bank would provide if it was restricted to post a credit line respectively outside and inside the adoption frontier irrespective of any $\bar{J}$ value. This is intuitive since, as already noted, point ii) of definition of a SSCE does not link $\bar{J}$ to the actual level granted in equilibrium J^*.

Nevertheless, best interior contracts could be unfeasible due to participation constraint. To find the best restricted contracts when *at least one participation constraint is binding* for at least one type of agent, we make use of a simple convexity argument. In the abstract case where only one technology is available, the maximization problem is nicely convex and has a single absolute maximum. Therefore, ceteris paribus, the closest the offered contract to $\hat{R}_r$ (resp. $\hat{R}_s$), the higher the expected profits $V(R)$ of a bank. This implies that whenever for a level of α it is $\hat{R}_r > r$ then the best risky restricted-contract is r. Moreover, whenever for a level of α it is $\hat{R}_r < \bar{R}$ then the best risky restricted-contract is $\bar{R}$. Finally whenever for a level of α it is $\hat{R}_s > \bar{R}$ then the best safe-restricted contract is $\bar{R}$.

SECOND STEP. For best restriced-contracts to be REE, it is necessary (but still not sufficient) that in a neighborhood of the equilibrium other contracts does not yield a strictly larger profit. For (21) and (22) this condition is always satisfied as both emerge as solution of a well-defined maximization problem in R. So, both (21) and (22) are candidate for the next step.

Also in the case where the best risky-restricted contract is r, then a bank cannot do better locally as any local deviation from r will remains strictly outside the adoption frontier. Therefore also r is a candidate for the next step.

When instead best restricted contracts lie along the adoption frontier $\bar{R}$ the argument is more involved. First consider the case of a best risky-restricted contract along $\bar{R}$. We need to use the criterion (20) to assess whether or not a single bank has incentive to deviate posting a contract inside the adoption frontier (i.e. posting a $\bar{R} - \varepsilon$ with $\varepsilon > 0$ infinitesimal) when all others post a contract outside the adoption frontier (i.e. posting a $\bar{R} + \varepsilon$ with $\varepsilon > 0$ infinitesimal): in the cases where this is true there do not exist REE risky contracts along $\bar{R}$, otherwise we proceed to the third step.

A deviation from the risky equilibrium (all banks post $\bar{R} + \varepsilon$) into safe territory (the deviant posts $\bar{R} - \varepsilon$) is worth if and only if,

$$\frac{\pi(s, \bar{R}, R_{CB}) I(s, \bar{R})}{\pi(r, \bar{R}, R_{CB}) I(r, \bar{R})} > \left(\frac{\pi(s, \bar{R}, R_{CB}) I(r, \bar{R})}{\pi(r, \bar{R}, R_{CB}) I(s, \bar{R})} \right)^{\gamma} \tag{23}$$

which is a rearrangement of (20). Hence, we have

$$\frac{\left(\bar{R} - R_{CB}\right)(r - k - \bar{R})}{\left(\alpha\bar{R} - R_{CB}\right)\alpha(r - \bar{R})} > \left(\frac{\left(\bar{R} - R_{CB}\right)\alpha(r - \bar{R})}{\left(\alpha\bar{R} - R_{CB}\right)(r - k - \bar{R})} \right)^{\gamma} \tag{24}$$

and after substituting for $\bar{R} = r - \frac{k}{1-\alpha} > R_{CB}$,

$$\frac{\bar{R} - R_{CB}}{\alpha\bar{R} - R_{CB}} > \left(\frac{\bar{R} - R_{CB}}{\alpha\bar{R} - R_{CB}} \right)^{\gamma}.$$

For $\alpha\bar{R} - R_{CB} < 0$ banks do not have incentive to enter in the market for risky credit so that a risky REE does not exist in this case. When instead $\alpha\bar{R} - R_{CB} \geq 0$ then the inequality always holds for whatever $\gamma < 1$. Therefore we can conclude that a risky REE does not exist along the $\bar{R}$ frontier.

Let us turn attention to the case of a best safe-restricted contract along $\bar{R}$. We apply the criterion (23) along $\bar{R} = r - \frac{k}{1-\alpha} > R_{CB}$ to assess whether a single bank has incentive to post a contract outside the adoption frontier (i.e.

posting a $\bar{R}+\varepsilon$ with $\varepsilon > 0$ infinitesimal) when all others post a contract inside the adoption frontier (i.e. posting a $\bar{R}-\varepsilon$ with $\varepsilon > 0$ infinitesimal); in the cases where this is true there do not exist REE safe contracts along $\bar{R}$, otherwise we proceed to the third step. Applying (23) we have a relation which holds as a strict inequality whenever (24) is false. Therefore we can conclude that there could exist safe REE in the case $\hat{R}^s > \bar{R}$ along the $\bar{R}$ frontier.

THIRD STEP. This is the final step. Once selected the best restricted contracts (step 1) that are local maxima (step 2), we need to establish whether they are global maxima, that is, if they are REE. Now we apply (20) to the different cases, distinguishing between interior and corner contracts.

The relevant equation for an interior best restricted contract to be a REE when both type of interior best restricted contracts ($R^s = \hat{R}^s$ and $R^r = \hat{R}^r$) are feasible is

$$(1-\gamma)(1+\gamma)^{\frac{1+\gamma}{1-\gamma}} \left(\frac{r-k-R_{CB}}{2}\right)^{\frac{2}{1-\gamma}} > (1-\gamma)(1+\gamma)^{\frac{1+\gamma}{1-\gamma}} \left(\frac{r\alpha-R_{CB}}{2}\right)^{\frac{2}{1-\gamma}}$$

which holds is and only if $r-k/(1-\alpha) > 0$. Since a necessary condition for the existence of an interior best safe-restricted contract is that the adoption region is not empty, $\bar{R} > R_{CB}$, this condition always holds: whenever an interior best safe-restricted contract exists then it is a REE. When instead we confront an interior best safe-restricted contract with a corner best risky-restricted contract posted at r, then the right-hand side of the disequality takes value zero so that the disequality is trivially satisfied. *We conclude that whenever an interior best safe-restricted contract exists then it is a REE.*

When instead the best safe-restricted contract is posted at R_{CB}, that is when

$$\alpha > \bar{\alpha} \equiv \frac{r-k-R_{CB}}{r-R_{CB}},$$

whereas the best risky-restricted contract is an interior ($r\alpha - R_{CB} > 0$), we have

$$0 > (1-\gamma)(1+\gamma)^{\frac{1+\gamma}{1-\gamma}} \left(\frac{r\alpha-R_{CB}}{2}\right)^{\frac{2}{1-\gamma}}, \tag{25}$$

which is always true. This implies that *whenever an interior best risky-restricted contract co-exists with a corner best safe-restricted contract being posted at R_{CB}, the former is always the unique REE.*

When instead the best safe-restricted contract is posted at $\bar{R} \neq R_{CB}$, whereas the best safe-restricted contract is an interior, we have

$$\left(r-\frac{k}{1-\alpha}-R_{CB}\right)\left(\frac{\alpha k}{1-\alpha}\right)^{\frac{1+\gamma}{1-\gamma}} > (1-\gamma)(1+\gamma)^{\frac{1+\gamma}{1-\gamma}} \left(\frac{r\alpha-R_{CB}}{2}\right)^{\frac{2}{1-\gamma}}. \tag{26}$$

The right-hand side is always monotonically increasing in α. The left-hand side instead is always monotonically decreasing in α in the relevant case

$$\alpha > \underline{\alpha} \equiv \frac{r-\hat{R}^s-k}{r-\hat{R}^s} = \frac{(\gamma+1)(r-k-M)}{2k+(\gamma+1)(r-k-M)},$$

for which the interior best safe-restricted contract is on the adoption frontier ($I\left(\hat{R}_s\right) = I\left(\hat{R}_s\right)$), given that

$$\frac{\partial\left(\left(r-\frac{k}{1-\alpha}-R_{CB}\right)\left(\frac{\alpha k}{1-\alpha}\right)^{\frac{1+\gamma}{1-\gamma}}\right)}{\partial\alpha} = \frac{(1-\alpha)(1+\gamma)(r-k-R_{CB})-2k\alpha}{\alpha\left(1-\alpha\right)^2\left(1-\gamma\right)\left(\frac{\alpha k}{1-\alpha}\right)^{\frac{\gamma+1}{\gamma-1}}}.$$

Hence, we can conclude that

$$\left(r-\frac{k}{1-\alpha}-R_{CB}\right)\left(\frac{\alpha k}{1-\alpha}\right)^{\frac{1+\gamma}{1-\gamma}} = (1-\gamma)\left(1+\gamma\right)^{\frac{1+\gamma}{1-\gamma}}\left(\frac{r\alpha-R_{CB}}{2}\right)^{\frac{2}{1-\gamma}}$$

defines a threshold $\hat{\alpha}$, such that for $\alpha < \hat{\alpha}$ the corner best safe-restricted contract is the unique REE, whereas for $\alpha > \hat{\alpha}$ the interior best risky-restricted contract is the unique a REE. The zero measure case $\alpha = \hat{\alpha}$ is the only one where two non-degenerate REE exist. In particular, notice that since

$$\frac{\partial\left(\left(r-\frac{k}{1-\alpha}-R_{CB}\right)\left(\frac{\alpha k}{1-\alpha}\right)^{\frac{1+\gamma}{1-\gamma}}\right)}{\partial k} = \frac{(1-\alpha)(1+\gamma)(r-R_{CB})-2k}{k\left(1-\alpha\right)\left(1-\gamma\right)\left(\frac{\alpha k}{1-\alpha}\right)^{\frac{\gamma+1}{\gamma-1}}} < 0$$

is true whenever $(1-\alpha)(1+\gamma)(r-k-R_{CB})-2k\alpha < 0$. This implies that $\hat{\alpha}$ has to be decreasing in k.

When instead the best safe-restricted contract is posted at $\bar{R} \neq R_{CB}$, whereas the best safe-restricted contract is posted at r ($r\alpha - R_{CB} < 0$), we have

$$\left(r-\frac{k}{1-\alpha}-R_{CB}\right)\left(\frac{\alpha k}{1-\alpha}\right)^{\frac{1+\gamma}{1-\gamma}} > 0, \tag{27}$$

so that $\hat{\alpha} = \bar{\alpha}$. This implies that *whenever a corner best risky-restricted contract co-exists with a corner best safe-restricted contract being posted at $\bar{R} > R_{CB}$, the latter is always the unique REE.*

Finally *whenever a corner best risky-restricted contract being posted at r (which requires $R_{CB}/r \leq \alpha$) co-exists with a corner best safe-restricted contract being posted at R_{CB} (which requires $r - k/(1-\alpha) \leq R_{CB}$), the two arise as two degenerate REE.*

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