

Risk-Taking, Rent-Seeking, and Investment when Financial Markets are Noisy*

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Abstract

We analyze investment and risk-taking by firms and identify a new distortion due to market imperfections and shareholder incentives. First we show that noisy information aggregation introduces a rent-shifting motive and leads to inefficient investment. These inefficiencies are particularly severe if upside risks are coupled with near constant returns to scale. Second we consider four applications of our model that give rise to excess leverage, negative welfare effects of transparency, excess sensitivity of investment to stock prices, and dynamically inconsistent firm behavior. Our third contribution is to evaluate various welfare improving policy interventions.

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1 Introduction

Asset markets play an important role in aggregating information about the value of firms. By pooling together the dispersed knowledge of individual actors, information contained in share prices reflects investor expectations about a firm's future earnings. If financial markets are efficient (in the sense that the firm's share price equals expected future dividends conditional on all publicly available information) and the firm's decision imposes no direct externalities, the share price also aligns shareholder incentives with social welfare, even if shareholders sell their shares before these returns are realized. This eliminates the need to actively monitor or regulate firms or intervene in financial markets in order to induce socially efficient investment decisions and channel financial resources to their most productive uses. Any attempt to intervene can only reduce social welfare.

Critics of the *laissez-faire* view argue that it enables inefficient rent-seeking behavior by shareholders and managers, exposing governments and society to excessive risks and costs of corporate failures. This critique is inconsistent with an efficient markets view, in which these risks are just part of an efficient reallocation process. But what departures from market efficiency would induce firms to act inefficiently and through what channels? How do such inefficiencies depend on firm or market characteristics? How should society intervene to limit its exposure to excessive financial or technological risks? And how robust are the lessons of the *laissez-faire* approach to small departures from market efficiency?

To answer these questions and assess the validity of the *laissez-faire* view beyond the efficient markets benchmark, we propose a model of corporate investment and risk-taking in which financial markets aggregate information with noise. Consider a firm whose incumbent shareholders take an investment decision before selling a fraction of their shares in a financial market populated by informed and noise traders. The share price then emerges as a noisy signal pooling dispersed investor information about the firm's value. We explore how the information aggregation friction in the financial market affects the initial shareholder's ability to capture the returns from their investment, and how the firm's decisions depart from socially efficient levels. Finally, we return to the regulatory questions and argue that even small frictions in financial markets provide an important rationale for regulating firm behavior.

The central point of our analysis is that financial market imperfections result in a rent-shifting motive for incumbent shareholders. With noisy information aggregation, the share price is not just a noisy but also a biased estimate of the firm's dividends. *Ex post*, the market-clearing share price must partially absorb shocks to demand and supply of securities, since informed traders are

not willing or able to perfectly arbitrage perceived gaps between prices and expected fundamental values. This amplifies price fluctuations relative to the information about expected dividend values that is aggregated through the market.

The amplification of price fluctuations ex post results in a wedge between the expected market value of the firm's equity and the expected value of its dividends ex ante. This wedge is a transfer from final to initial shareholders, in other terms, a rent. We show that the sign and magnitude of the wedge depend on the firm's risk and return characteristics and its investment decisions, and therefore distorts shareholders' investment incentives. If the firm's investment is characterized by upside risk, the expected market return exceeds expected dividend return, and the firm invests more than the efficient level. If instead the investment is characterized by downside risk, the expected market return falls short of dividend values and the firm invests too little. These inefficiencies become particularly pronounced if the firm's technology has nearly constant returns to scale, in which case (i) the surplus from the investment is small, and (ii) the firm's investment choice is highly sensitive to expected market returns, so that the scope for rent-shifting becomes very large. If near constant returns are coupled with upside risks, even small frictions in financial markets can have very large efficiency consequences – so large in fact that the firm generates negative expected surplus, while incumbent shareholders take excessive risks purely to capture rents from selling their shares.

As our second contribution, we consider four applications to demonstrate the relevance of rent-shifting incentives for firm behavior. First, we introduce leverage by augmenting our model of investment with a choice of external borrowing with *costly state verification* (Townsend, 1979; Gale and Hellwig, 1985). We show that incumbent shareholders seek to take on excessive leverage, as this tilts the distribution of equity returns towards the upside, inflating the market value of equity shares, and at the same time allows them to scale up the investment to increase shareholder rents. If informational frictions are sufficiently important, shareholders may even borrow to invest in projects with negative expected returns, just to gain from the upside distortion in equity prices. In contrast, with efficient equity markets, private and social returns from borrowing and investment are aligned, and costly state verification can only reduce borrowing relative to the first-best.

Second, we show that public information disclosures about the firms' investment returns may be socially harmful, when rent-seeking motives are sufficiently severe. While better public information is always welfare improving when investment is socially efficient, here it enables the incumbent shareholders to fine-tune their rent-seeking activities. This result mirrors Morris and Shin's (2002) finding on the social value of public information in beauty contest games, but is based on a different

model and mechanism in which inefficiency results from rent-seeking incentives that are based on financial market frictions.

Third, we consider an application of our model with informational feedbacks from stock prices to investment decisions, and show how rent-seeking incentives can account for excess sensitivity of investment to share prices, and a low co-movement of investment with future returns.

Fourth, with sequential decision-making by successive shareholder generations, market imperfections and rent-shifting lead to dynamically inconsistent, and Pareto-inferior firm decisions.

As our third contribution, we evaluate various policies that may improve upon the market allocations. The incumbent shareholders' rent-shifting incentives offer a rationale for regulatory or policy interventions. We show how regulatory oversight, transaction taxes or direct market interventions can restore efficiency either by directly controlling firm decisions or indirectly by altering the incumbent shareholder's perception of upside risk. The optimal intervention policy has the property of leaning against return asymmetries as a counterweight to the market imperfections.

Finally, we show that our analysis is robust to the inclusion of agency conflicts between shareholders and managers. Inefficiency stems solely from a misalignment between incumbent shareholder preferences and social surplus that is caused by market frictions, which is unrelated to agency conflicts inside the firm. Shareholder discretion to design CEO incentives instead facilitates rent-seeking, contracts such as stock options that serve to provide optimal incentives when markets are efficient now serve to extract shareholder rents, and restrictions on executive pay offer an additional margin for welfare-improving policy interventions. In fact, in some cases, a regulator can restore efficiency by limiting executive compensation to untraded restricted equity contracts, without requiring any further knowledge of the firm's characteristics or the financial market structure. The separation of ownership and control may even be socially desirable, if the associated agency costs limit incumbent shareholders' ability to shift rents. This is in complete opposition to the *laissez-faire* tradition, which views agency frictions as an impediment to optimal firm decisions, but trusts shareholders and managers to design contracts to deal with these costs in a socially efficient manner without regulatory oversight. As another extension we also show that our main conclusions are robust to considering a more general structure of the financial markets.

Related Literature. Our model of the financial market builds on models of noisy information aggregation (Grossman and Stiglitz, 1980, Hellwig, 1980, Diamond and Verrecchia, 1981), or more specifically the formulation in Albagli, Hellwig and Tsyvinski (2011, henceforth AHT) which offers return implications for arbitrary securities in a non-linear noisy REE model. We extend AHT by endogenizing security cash flows as the outcome of firm decisions.

Our discussion of market value vs. dividends mirrors the comparison between efficiency and revenue maximization in auction theory. However, auction theory takes securities as given to focus on market design, while we take the market as given and endogenize investment and security returns.¹ Our discussion of firm behavior and optimal regulation draws connections to the large literature on time consistency, the value of commitment, and optimal regulation. We identify market imperfections as a source of time-inconsistent firm behavior.

The corporate finance literature instead views agency frictions between firms and managers as a source of inefficient firm decisions, and emphasizes market-based incentives to discipline firms and managers.² The literature on informational feedbacks in asset markets emphasizes the social value of information contained in share prices for firm decisions.³ In our analysis instead, market signals contain valuable information, but they are not unbiased, and these biases in turn distort firm incentives. The core distinction of our analysis is that shareholders (or their agents, i.e. managers) can no longer be trusted to take decisions that are in the firms' or society's long-term interests, once market imperfections distort incentives.

Some papers emphasize the limitations of market-based incentives. Closest to our work, Bolton, Scheinkman, and Xiong (2006) show that it may be in the interest of incumbent shareholders to induce inefficient investments aimed at boosting short-term share prices. But they consider a market in which traders "agree to disagree" about the informativeness of profit signals, which creates a speculative component in prices that shareholders manipulate through their investment. Our model instead features a common prior, which endogenously limits shareholder disagreement and enables us to go much further in discussing welfare and policy interventions. Moreover, mis-pricing doesn't arise from specific speculation motives, but depends on cash-flow risks allowing the possibility of both systematic over- and under-valuation, and over- and under- investment at equilibrium.

Other papers instead emphasize that market-based incentives may exacerbate agency conflicts when managers have superior information and manipulate the information contained in stock prices. Stein (1988) argues that managers may manipulate earnings to limit the risk that asymmetric information causes temporary price dips that invite disadvantageous takeovers. Stein (1989) argues

¹Moreover, the noisy information aggregation model (and the AHT model in particular) shares close similarities with common value auctions following Milgrom (1981).

²See Tirole (200?) and references therein for a broad overview. The classic paper by Holmstrom and Tirole (1993) emphasizes the social value of tying managerial incentives to market prices, under the background hypothesis that financial markets are efficient.

³See Dow and Gorton (1997), Dow and Rahi (2003), Goldstein and Guembel (2008), and Goldstein, Ozdenoren, and Yuan (2013).

that earnings manipulation is pervasive if price-based compensation induces short-termism by managers. Benmelech, Kandel, and Veronesi (2010) take a longer-term view of the lifecycle of firms, and argue that while early on share price compensation leads to high effort, once firms mature, managers have an incentive to manipulate earnings to hide the decline in growth opportunities from shareholders. In these models, managers can manipulate information contained in stock prices but not systematically fool shareholders. In our model instead, information cannot be manipulated, and distortions are in incumbent shareholders' interest.

Finally, several recent empirical studies provide evidence consistent with some of our central predictions. In our model, upside cash-flow risk will systematically lead to excessive investment, the more so the larger the information frictions and the bias towards short-term results by incumbent shareholder. Gilchrist, Himmelberg and Huberman (2005) find that an increase in belief heterogeneity (proxied by analyst forecast dispersion) leads to an increase in new equity issuance, Tobin's Q, and real investment. Polk and Sapienza (2009) find a positive relation between abnormal investment and stock overpricing (proxied by discretionary accruals), a pattern that is more prevalent in firms they identify as being more opaque (higher R&D intensity), as well as in firms with shorter shareholders' horizons (proxied by share turnover). Moreover, excessive investment is typically followed by abnormally low returns, especially in firms with the aforementioned characteristics.

Section 2 introduces our model and derives the equilibrium characterization in the financial market. Section 3 analyzes rent-seeking behavior by incumbent shareholders, investment distortions and the associated welfare losses. Section 4 discusses applications to risk-taking with leverage, welfare effects of public disclosures, stock price sensitivity of investment, and dynamic inconsistency. Section 5 discusses regulatory and policy interventions. Section 6 considers managerial incentives and alternative information structures. Section 7 concludes.

2 Baseline model

Our model has three stages. In the first stage, incumbent shareholders in a firm decide on an observable investment decision $k \geq 0$. In the second stage, they sell a fraction $\alpha \in (0, 1]$ of the shares to outside investors. At the final stage, the firm's cash flow $\Pi(\theta, k)$ is realized and paid to the final shareholders. The cash flow $\Pi(\theta, k)$ is as a strictly increasing function of a stochastic fundamental $\theta \in \mathbb{R}$, strictly concave in k , and has increasing differences. The fundamental θ is distributed according to $\theta \sim \mathcal{N}(0, \lambda^{-1})$.

The *ex post efficient investment* $k^{FB}(\theta)$ maximizes $\Pi(\theta, k)$. The *ex ante efficient investment*

k^* maximizes $\mathbb{E}(\Pi(\theta, k))$, given the information available at stage 1.

2.1 Stage 2: Description of the Market Environment.

There are two types of outside investors: a unit measure of risk-neutral informed traders, who are indexed by i , and noise traders.

Informed traders observe a private signal $x_i \sim \mathcal{N}(\theta, \beta^{-1})$, which is i.i.d. across traders (conditional on θ). After observing x_i , an informed trader submits a price-contingent demand schedule $d_i(\cdot) : \mathbb{R} \rightarrow [0, \alpha]$, to maximize expected wealth $w_i = d_i \cdot (\Pi(\theta, k) - P)$. That is, informed traders cannot short-sell, and can buy at most α units of the shares. An informed trader's strategy is then a function $d(x_i, P) \in [0, \alpha]$ of the private signal and the price.

Noise traders place an order to purchase a random quantity $\alpha\Phi(u)$ of shares, where $u \sim \mathcal{N}(0, \delta^{-1})$ is independent of θ , and δ^{-1} is a measure of demand noise.

The aggregate demand for shares is $D(\theta, P) = \int d(x, P)d\Phi(\sqrt{\beta}(x-\theta)) + \alpha\Phi(u)$, where $\Phi(\sqrt{\beta}(x-\theta))$ represents the cross-sectional distribution of private signals x_i conditional on θ , and $\Phi(\cdot)$ denotes the cdf of a standard normal distribution. The orders submitted by informed and noise traders are executed at a market-clearing price P such that $D(\theta, P) = \alpha$.

Let $H(\cdot|x, P)$ denote the traders' posterior cdf of θ , conditional on observing a private signal x , and a market-clearing price P . A *noisy Rational Expectations Equilibrium* at stage 2 consists of a demand function $d(x, P)$, a price function $P(\theta, u; k)$, and posterior beliefs $H(\cdot|x, P)$, such that $d(x, P)$ is optimal given the shareholder's beliefs $H(\cdot|x, P)$; $P(\theta, u; k)$ clears the market for all (θ, u) and k ; and $H(\cdot|x, P)$ satisfies Bayes' Rule whenever applicable.

2.2 Stage 2: Equilibrium Characterization.

For a given k , our first result characterizes the equilibrium share price in the unique noisy Rational Expectations Equilibrium, in which demand schedules are non-increasing in P . Monotonicity restrictions arise naturally if trading takes place through limit orders.

Proposition 1 *Equilibrium Characterization and Uniqueness*

Define $z \equiv \theta + 1/\sqrt{\beta} \cdot u$. In the unique equilibrium in which the informed traders' demand $d(x, P)$ is non-increasing in P , the market-clearing price function is

$$P(z, k) = \mathbb{E}(\Pi(\theta, k) | x = z, z). \tag{1}$$

The variable z is normally distributed with mean θ and precision $\beta\delta$, and is a sufficient statistic for the information conveyed through the share price. This characterization of $P(z, k)$ gains its significance from the comparison with the share's expected dividend value $V(z, k)$:

$$V(z, k) = \mathbb{E}(\Pi(\theta, k) | z). \quad (2)$$

The equilibrium share price differs systematically from the expected dividend value: Both are characterized as expected dividends conditional on the information contained in z . However, the share price treats the signal z as if it had precision $\beta + \beta\delta$ (equal to the sum of the private and the price signal precision), when in reality its precision is only equal to $\beta\delta$. Hence, the market price is based on an expectation of the marginal return to the investment level k that conditions places a higher weight on the market signal z , relative to its objective information content.

The price $P(z, k)$ reacts more strongly to the realization of z than $V(z, k)$ as a result of market clearing with heterogenous information. Both the price and the expected dividend reflect the public information z that is conveyed through the price. However in addition, the price must shift to equate demand and supply: if θ increases, all trader become more optimistic through the observation of private signals, and hence demand more. Of course, demand also increases when u rises, due to the order by noise traders. To compensate for this increase in demand that arises with a shift in z , the market price must respond to these shocks by more than what is consistent merely with the information provided through the price.

We refer to the difference $\Omega(z, k) = P(z, k) - V(z, k)$ between price and expected dividend value as the *information aggregation wedge*.⁴

2.3 Stage 1: Investment Decision

At the first stage, the incumbent shareholders decide on an investment level k to maximize the expected value of their equity:

$$\max_{k \geq 0} \mathbb{E} \{ \alpha P(z; k) + (1 - \alpha) \Pi(\theta, k) \}, \quad (3)$$

where $P(z; k)$ is characterized by (1). Therefore, the incumbent shareholder's objective differs from expected dividends by the expected information aggregation wedge $\alpha \mathbb{E}(\Omega(z, k))$.

⁴We discussed properties of this wedge at length in Albagli, Hellwig, and Tsyvinski (2011). There, we also show that its core properties are robust to general, non-normal distributional assumptions, risk aversion, and arbitrary position limits. The functional form assumptions and the restriction of the demand of the indormed traders to lie in $[0, 1]$ are convenient for comparative statics, but not important for the economic forces in play.

Our efficiency criterion $\mathbb{E}(\Pi(\theta, k))$ focuses on the value of the firm to final shareholders.⁵ The expected wedge $\alpha\mathbb{E}(\Omega(z, k))$ enters the initial shareholders' preferences but not the social surplus, because the share price is a pure transfer from final to initial shareholders. Noisy information aggregation resulting in a non-zero wedge thus introduces a pure rent-seeking motive into incumbent shareholder preferences.

Departures from efficiency arise because initial shareholders' sell a fraction of shares and informational frictions in the equity market drive a wedge between share prices and expected dividends. Initial shareholders have no incentive to deviate from efficiency, if they were to hold on to all their shares, i.e. in the limit where $\alpha \rightarrow 0$.⁶ In an *efficient market*, i.e. if $P(z; k) = V(z; k)$, the share price aligns initial and final shareholders incentives and motivates the former to invest efficiently, even if they are planning to sell some of their shares.⁷

With noisy information aggregation, initial and final shareholder incentives are no longer aligned. Two properties of the information aggregation wedge are key for generating this conflict of interests: First, the wedge does not just add noise to stock prices, which would average out from an ex ante perspective, but it responds systematically to the price realization (in other words, P and $P - V$ are not orthogonal to each other). Second, the investment decision taken at the first stage influence the magnitude of the wedge, and therefore the pricing of shares in the market.⁸

3 Investment distortions from market frictions

In this section, we explore investment distortions due to noisy information aggregation in a simple example.

Suppose that the firm's dividend is $\Pi(\theta, k) = R(\theta)k - C(k)$, where $R(\cdot)$ is a positive, increasing function of the firm's fundamental (interpreted as the return to the investment k), $C(k) = k^{1+\chi}/(1+\chi)$ denotes the cost of investment, and χ the firm's degree of return to scale.

⁵Any discussion of welfare and efficiency remains incomplete if the noise traders' motives for buying the shares is not specified. We can justify $\mathbb{E}(\Pi(\theta, k))$ as our welfare measure on utilitarian grounds, if noise traders are risk-neutral over dividends and their demand stems from a preference shock as in Duffie, Garleanu and Pedersen (2005).

⁶The only (minor) modification relative to Albagli, Hellwig, and Tsyvinski (2011) is the normalization of demand by α instead of 1. The parameter α thus isolates the impact of market prices on shareholder incentives without affecting market conditions themselves.

⁷ $P(z, k) = V(z, k)$ could result for example with free entry of uninformed arbitrageurs as in Kyle (1985), or when there is a public signal z , but no private information, and no heterogeneity among informed traders, so that they must be indifferent about buying at equilibrium. This also corresponds to the limiting case of our model with $\beta \rightarrow 0$.

⁸While we base our analysis on noisy information aggregation, other models with market frictions and limits to arbitrage may lead to similar results if mis-pricing has similar incentive effects.

The efficient investment k^* sets $C'(k^*) = \mathbb{E}(R(\theta))$. The initial shareholders instead choose \hat{k} to equate the marginal cost of investment to a weighted average of expected market return $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\}$ and expected dividend return $\mathbb{E}(R(\theta))$:

$$C'(\hat{k}) = \alpha \mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} + (1 - \alpha) \mathbb{E}(R(\theta)). \quad (4)$$

Proposition 2 *Market frictions cause investment distortions.*

$\hat{k} \underset{\geq}{\underset{\leq}} k^*$ whenever $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} \underset{\geq}{\underset{\leq}} \mathbb{E}(R(\theta))$. $|\hat{k}/k^* - 1|$ is increasing in α .

The difference between $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\}$ and $\mathbb{E}(R(\theta))$ corresponds to the marginal impact of k on the expected information aggregation wedge. Whenever the expected wedge is positive, the expected market return to investment exceeds the expected fundamental return, so the initial shareholders find it optimal to over-invest to enhance the over-valuation of their shares. When instead the expected wedge is negative, the initial shareholders want to under-invest in order to limit the under-valuation of their shares. The initial shareholders thus influence the expected over- or under-valuation of their shares through the choice of k .

The extent of over- or under-investment depends on characteristics of the return distribution $R(\cdot)$ and on the parameters determining how much noisy information aggregation distorts share prices. We compute $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\}$ as a market-implied prior expectation of $R(\theta)$, i.e. as the expectation of $R(\theta)$ with respect to a distribution over θ that is normal with mean zero, and a market-implied variance λ_p^{-1} that turns out to be strictly larger than the true fundamental variance λ^{-1} . From an ex ante perspective the market thus attributes too much weight to the tail realizations of the θ , relative to the objective prior distribution. The ratio $\lambda_p^{-1}/\lambda^{-1}$ governs by how much the market overweighs the tail realizations. The over-weighting of tails ex ante is just a reflection of the fact that the market assigns a larger weight to the market signal z than the objective posterior over fundamentals does.

The direction of the distortion then depends on which of the two tail risks is more important. Following Albagli, Hellwig, and Tsyvinski (2011), a return $R(\cdot)$ is *symmetric* if $R'(\theta) = R'(-\theta)$ for all $\theta > 0$. A return $R(\cdot)$ is *dominated by upside risks*, if $R'(\theta) \geq R'(-\theta)$ for all $\theta > 0$, and *dominated by downside risks* if $R'(\theta) \leq R'(-\theta)$ for all $\theta > 0$. Thus we classify returns by comparing marginal gains and losses at fixed distances from the prior mean to determine whether returns are steeper on the upside or on the downside. The following result is a direct corollary of Theorem 2 in AHT.

Proposition 3 *Upside (downside) risk induces over- (under-)investment*

(i) **Sign:** If R has symmetric risk, then $\hat{k} = k^*$. If R is dominated by upside risk, then $\hat{k} > k^*$. If R is dominated by downside risk, then $\hat{k} < k^*$.

(ii) **Comparative Statics w.r.t. λ_P^{-1} :** If R is dominated by upside or downside risk, then $|\hat{k}/k^* - 1|$ is increasing in λ_P^{-1} .

If returns are dominated by upside risk (for example, if $R(\theta)$ is convex), the comparison of expected market and fundamental returns is dominated by the upper tail, so the firm over-invests. When returns are dominated by downside risk (for example, if $R(\theta)$ incorporates mainly the risk of a failure if the fundamentals are low), the comparison is dominated by the lower tail, and the firm under-invests. These distortions are larger if information frictions are more severe.

Our next result provides comparative statics of investment distortions and efficiency losses with respect to the returns to scale χ and the ratio $\mathbb{E}\{\mathbb{E}(R(\theta)|x=z, z)\}/\mathbb{E}(R(\theta))$ of expected market to dividend returns. Investment distortions are defined as $|\hat{k}/k^* - 1|$. The efficiency loss $\Delta = 1 - \hat{V}/V^*$ is defined as the percentage loss in expected dividends $\hat{V} = \mathbb{E}(R(\theta)) \cdot \hat{k} - C(\hat{k})$, relative to the ex ante efficient benchmark $V^* = \mathbb{E}(R(\theta)) \cdot k^* - C(k^*)$. If $\Delta > 1$, the distortion is so severe that expected dividends are negative.

Proposition 4 : Rent manipulation and efficiency losses increase with market frictions and returns to scale.

(i) **Comparative Statics:** $|\hat{k}/k^* - 1| = \Delta = 0$ only if $\mathbb{E}\{\mathbb{E}(R(\theta)|x=z, z)\} = \mathbb{E}(R(\theta))$ or $\chi \rightarrow \infty$. $|\hat{k}/k^* - 1|$ and Δ are decreasing in χ and increasing in $|\mathbb{E}\{\mathbb{E}(R(\theta)|x=z, z)\}/\mathbb{E}(R(\theta)) - 1|$.

(ii) **Bounded Distortions on the Downside:** If $\mathbb{E}\{\mathbb{E}(R(\theta)|x=z, z)\} < \mathbb{E}(R(\theta))$, then $\lim_{\chi \rightarrow 0} \hat{k}/k^* = 0$ and $\lim_{\chi \rightarrow 0} \Delta = 1$.

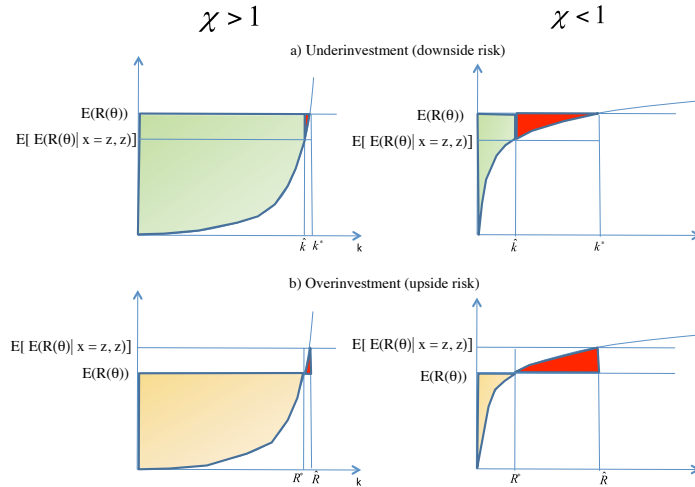
(iii) **Unbounded Distortions on the Upside:** $|\hat{k}/k^* - 1|$ and Δ become infinitely large, if either $\mathbb{E}\{\mathbb{E}(R(\theta)|x=z, z)\}/\mathbb{E}(R(\theta)) \rightarrow \infty$, or $\chi \rightarrow 0$ and $\mathbb{E}\{\mathbb{E}(R(\theta)|x=z, z)\} > \mathbb{E}(R(\theta))$.

(iv) **Negative Expected Dividends:** Expected dividends are negative ($\Delta > 1$), whenever

$$\alpha \left(\frac{\mathbb{E}\{\mathbb{E}(R(\theta)|x=z, z)\}}{\mathbb{E}(R(\theta))} - 1 \right) > \chi. \quad (5)$$

The ratio of expected market to dividend returns governs the initial shareholders' incentive to distort their marginal costs. The returns to scale parameter χ in turn translates these marginal cost distortions into investment distortions. When the firm operates close to constant returns (low χ), and the optimal investment is very sensitive to return expectations, the incentive to manipulate the rent, as well as the associated efficiency losses can become very large. At the other

Figure 1: Investment distortions and efficiency losses



extreme, investment distortions and welfare losses are small if marginal costs are very sensitive to the investment level (high χ).

Figure 1 illustrates the comparative statics described by proposition 4. Here, we plot marginal costs and expected market and fundamental returns, for high and low values of χ , and for the case with over- and under-investment respectively. In all cases, the red triangular area corresponds to the welfare loss, relative to the efficient investment level k^* . In the top two graphs, there is under-investment. The green area corresponds to the realized social surplus \hat{V} , while the maximal surplus V^* corresponds to the combined green and red areas. In the lower two panels, we plot the case with over-investment. Here, the yellow area corresponds to V^* , the welfare loss corresponds to the red area, and the realized surplus \hat{V} to the difference between the yellow and the red areas. In both cases, a lower value of χ leads to a larger impact of frictions on investment and welfare.

In extreme cases, welfare losses exceed 100% of the first-best welfare level, i.e. the firm's investment generates negative expected cash flows, and thus destroys value. This occurs, whenever the elasticity of marginal costs χ is less than the return distortion, which is given by the distance of the return ratio from 1, multiplied by the fraction of shares sold, and corresponds to the lower right panel. Even a small departure from the efficient markets benchmark (in terms of $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} / \mathbb{E}(R(\theta))$) can thus have very large efficiency consequences for firms that operate near constant returns, and with investments that are characterized by upside risk. On the other hand, with under-investment the firm's expected dividends always remain positive.⁹

⁹Note that further distortions emerge if the firm can choose between different return profiles. While a social planner wants the firm to maximize the fundamental return $\mathbb{E}\{R(\theta)\}$, the initial shareholders have a preference for

To summarize, this section shows two sets of results. First, we characterize how the return characteristics of the investment (upside or downside risks) determine the sign of the information wedge (and, consequentially, the degree of the rent seeking) and lead to inefficient under- or over-investment. Second, we show how magnitude of the inefficiency depends on the firm's technology and the return characteristics of the investment. If near constant returns are coupled with upside risks, even small frictions in financial markets can have very large efficiency consequences – so large in fact that the firm generates negative expected surplus.

4 Applications

In this section, we develop applications of our model to excess risk-taking and leverage, provision of public information, sensitivity of investment to share prices, and managerial incentive contracts. To simplify we set $\alpha = 1$, i.e. initial shareholders only care about the market value of equity.

4.1 Leverage and Risk-Taking

As our first application, we introduce borrowing and leverage in the baseline model. Suppose that at the first stage the firm has cash reserves w , and chooses to invest $k \geq 0$ in a project to obtain a stochastic dividend $R(\theta)k$ at the final stage. If $k > w$, the difference $k - w$ must be funded through borrowing from external lenders. The lending relation is subject to *costly state verification*: lenders incur a cost $\varepsilon R(\theta)k$ to verify the firm's realized cash flows ex post.

A standard result is that the optimal contract takes the form of a standard debt contract (b, B) , where $b \geq k - w$ is the initial loan size, and B is the promised repayment. There exists a default threshold $\hat{\theta}$, such that $B = R(\hat{\theta})k$. Whenever $\theta \geq \hat{\theta}$, the lender is repaid in full. Whenever $\theta < \hat{\theta}$, the borrower defaults, and the lender optimally decides to monitor the borrower's cash flows and recovers the cash flows, net of monitoring costs, $(1 - \varepsilon)R(\theta)k$. Assuming that the lender and borrower do not discount between the initial and final stages, the lender breaks even whenever

$$\frac{b}{k} \leq (1 - \varepsilon) \int_{-\infty}^{\hat{\theta}} R(\theta) d\Phi(\sqrt{\lambda}\theta) + R(\hat{\theta}) \left(1 - \Phi(\sqrt{\lambda}\hat{\theta})\right).$$

Combined with the constraint on the initial loan size, this yields the following restriction on the firm's *leverage ratio* w/k :

$$\frac{w}{k} \geq 1 - \mathbb{E}(R(\theta)) + \varepsilon \int_{-\infty}^{\hat{\theta}} R(\theta) d\Phi(\sqrt{\lambda}\theta) + \int_{\hat{\theta}}^{\infty} \left(R(\theta) - R(\hat{\theta})\right) d\Phi(\sqrt{\lambda}\theta) \quad (6)$$

maximizing expected market returns $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\}$. This skews decisions in favor of upside risks.

We assume that the RHS of this inequality reaches a unique minimum for some value $\tilde{\theta} < 0$.¹⁰ We further assume that this minimum value of w/k is strictly positive so that there exists no loan contract at which the project is completely self-funding. The final shareholder dividend is $k \cdot \max \left\{ R(\theta) - R(\hat{\theta}), 0 \right\}$, so the shareholder's expected dividend is $k \int_{\hat{\theta}}^{\infty} \left(R(\theta) - R(\hat{\theta}) \right) d\Phi(\sqrt{\lambda}\theta)$. Let θ^* and k^* denote the threshold fundamental and investment level for the debt contract that maximizes the the expected fundamental return on cash

$$\rho(w/k, \hat{\theta}) = (k/w) \cdot \int_{\hat{\theta}}^{\infty} \left(R(\theta) - R(\hat{\theta}) \right) d\Phi(\sqrt{\lambda}\theta). \quad (7)$$

Therefore, the firm's efficient repayment threshold is θ^* , and it borrows and invests only if the expected return on cash $\rho(w/k, \theta^*)$ exceeds 1. We recall the following standard results with regards to borrowing with costly state verification frictions:

(i) *Credit restrictions*: If $\mathbb{E}(R(\theta)) > 1$, the loan size is endogenously restricted (relative to an infinite investment level at first-best), due to the leverage ratio constraint.

(ii) *Investment in projects with positive excess returns*: If $\mathbb{E}(R(\theta)) \leq 1$, the incumbent shareholders do not seek any credit to invest in the project and prefer to hold cash.

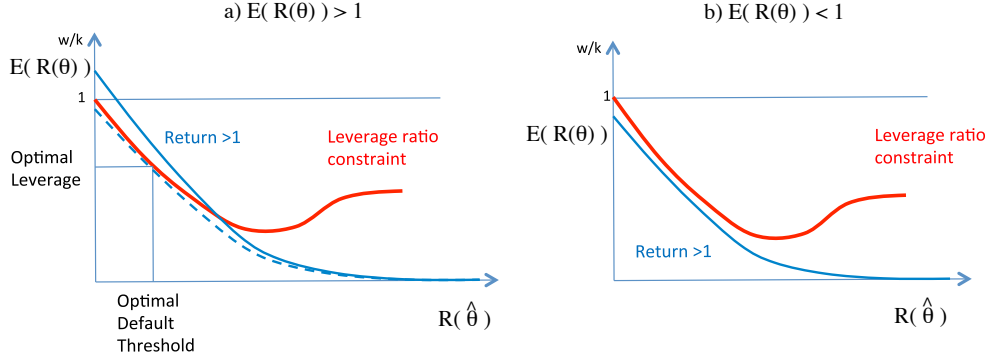
(iii) *Disciplining effect of credit spreads*: The optimal loan contract achieves the second-best, i.e. θ^* and k^* maximize social surplus by optimally trading off loan size against bankruptcy costs.

If equity and debt are traded in an efficient financial market at prices equal to conditional expectations of their corresponding cash flows, such markets do not alter the original lenders' break-even constraint, the borrowers' incentives, or the optimal loan contract. Efficient financial markets thus impose *market discipline* on borrowers and lenders to agree on a loan contract that maximizes the social surplus from the firm's investment, given the agency friction.

Figure 2 illustrates the optimal lending contract with costly state verification and efficient financial markets. We plot the borrower's and lender's indifference curves over pairs of leverage and repayment promise $(w/k, R(\hat{\theta}))$. A lender breaks even only with contracts that lie on or above the leverage ratio constraint (6), which is represented by the red curve. The borrowers only accepts contracts that lie on or below the solid blue line, at which the shareholder's expected returns on internal funds equal 1. In the left panel, when $E(R(\theta)) > 1$, the red and blue curves intersect, so there exist loan contracts that allow the borrower to earn excess returns and the lender to break even. From (7), the borrowers' indifference curves over different contracts simply scale the solid

¹⁰There exists a unique minimum if $R(\theta)/R'(\theta) \cdot \frac{\sqrt{\lambda}\phi(\sqrt{\lambda}\theta)}{1-\Phi(\sqrt{\lambda}\theta)}$ is strictly increasing, which is guaranteed if the distribution of returns $R(\cdot)$ satisfies the monotone hazard rate property (or equivalently, $R'(\cdot)/R(\cdot)$ not increasing too fast, relative to $\frac{\sqrt{\lambda}\phi(\sqrt{\lambda}\theta)}{1-\Phi(\sqrt{\lambda}\theta)}$), and this minimum is reached at $\tilde{\theta} < 0$ if $R'(0)/R(0) \leq 2\varepsilon\sqrt{\lambda}/\sqrt{2\pi}$.

Figure 2: Optimal Lending with Costly State Verification



blue line by ρ^{-1} , for different return levels ρ . The borrowers' return on internal funds is maximized at the tangency point between the red and the dashed blue line. In the right panel, $E(R(\theta)) \leq 1$, and there exists no contract to which the two parties would ever agree.

Borrowing with noisy information aggregation. Suppose now that the initial borrowers sell their equity share in a market with information aggregation frictions. The lenders on the other hand hold their claim to maturity, so their break-even constraint is unchanged. Following proposition 1, the equity claim is priced at $P(k, z) = k \cdot \mathbb{E} \left(\max \left\{ R(\theta) - R(\hat{\theta}), 0 \right\} | x = z, z \right)$, and the expected market value of equity is $k \int_{\hat{\theta}}^{\infty} \left(R(\theta) - R(\hat{\theta}) \right) d\Phi(\sqrt{\lambda_P} \theta)$, where λ_P^{-1} measures the information frictions in the equity market. Our next proposition characterizes the optimal debt contract when the financial market is noisy:

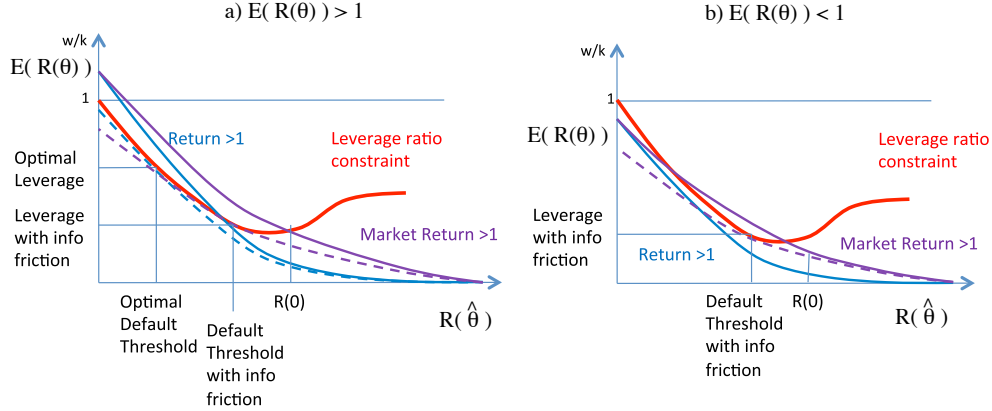
Proposition 5 Market frictions cause excessive leverage.

(i) **Excessive leverage** If $\mathbb{E}(R(\theta)) > 1$ and borrowers' indifference curves flatten with λ_P^{-1} , then $\hat{k} > k^*$ and $\hat{\theta} > \theta^*$ and \hat{k} and $\hat{\theta}$ are increasing in λ_P^{-1} .

(ii) **Inefficient investment:** There exists $\bar{R} < 2$, such that if $\lim_{\theta \rightarrow \infty} R(\theta) > \bar{R}$ and λ_P^{-1} is sufficiently large, incumbent shareholders borrow and invest $\hat{k} > w$.

This proposition shows how noisy information aggregation in equity markets distorts borrowing incentives. First, shareholders seek excessive leverage whenever market frictions flatten the shareholders' perceived tradeoff between leverage and default risk, or equivalently whenever $\int_{\hat{\theta}}^{\infty} \left(R(\theta) - R(\hat{\theta}) \right) d\Phi(\sqrt{\lambda_P} \theta) / \left(1 - \Phi(\sqrt{\lambda_P} \hat{\theta}) \right)$, i.e. the expected market value of equity conditional on repayment, is increasing in λ_P^{-1} . This is the case whenever $R(\cdot)$ is symmetric or dominated by upside risk, and even when $R(\cdot)$ is dominated by downside risk, but $\hat{\theta}$ or λ_P^{-1} are sufficiently

Figure 3: Lending with information aggregation frictions in equity markets



large.¹¹ Second, if market frictions are sufficiently severe, shareholders choose to invest even in projects with negative excess returns in order to chase upside gains.

Figures 3 illustrates the effects of financial market frictions on shareholder incentives. The solid purple line plots the shareholders' indifference curve for a return of 1 on internal funds when $\lambda_P^{-1} > \lambda^{-1}$. The difference between the solid blue and purple lines corresponds to the shareholder rents. These rents are maximized when equity share only includes upside risk, i.e. $\hat{\theta} = 0$. Therefore, for any $R(\hat{\theta}) < R(0)$, information frictions flatten the shareholders' indifference curves, while for $R(\hat{\theta}) > R(0)$ the opposite is true.¹² The shareholders other indifference curves are again obtained by scaling the purple line by ρ^{-1} .

In the left panel, $\mathbb{E}(R(\theta)) > 1$, so it's efficient to invest. With market frictions, the shareholder's expected returns increase (i.e. the purple line shifts up), and they are more willing to accept higher default risk in return for leverage (the curve becomes flatter), because the marginal increase in borrowing costs is partly offset by the shift towards upside risk. Because shareholders do not fully internalize the marginal costs of default, they opt for excessive leverage.

In the right panel, the level effect of shareholder rents on market returns dominates. Here the expected excess returns are too low to make the investment profitable, but with market distortions, shareholders nevertheless have an incentive to invest, and even take out very high leverage to

¹¹In the case of downside risk and low values of $\hat{\theta}$, the result is ambiguous due to the fact that market expectations of returns on internal funds are below fundamental expectations of returns on internal funds.

¹²The intercept of the indifference curve as $R(\hat{\theta}) \rightarrow 0$ is equal to $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\}$; here we have drawn the case where $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} = \mathbb{E}(R(\theta))$. If $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} > \mathbb{E}(R(\theta))$, the purple line is always higher than the blue line, while if $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} < \mathbb{E}(R(\theta))$, the purple and blue lines intersect, and the market expectations of equity returns are below the fundamental ones for sufficiently low values of $R(\hat{\theta})$.

maximize the gains from upside distortions in the equity market.

To summarize, leverage is an ideal tool to increase incumbent shareholder rents through two channels: It allows the firm to choose a higher value of \hat{k} and thus scale up its risk-taking, and it shifts equity returns to the upside which increases expected over-pricing for a given investment level. By taking a levered bet, incumbent shareholders capture rents from final shareholders with the complicity of lenders. Market frictions thus undermine market discipline.¹³

4.2 Social Value of Public Information

Here, we enrich the model of section 3 to consider the impact of public information on market prices, expected market returns and investment. As our main result, we show that public information disclosures may have adverse welfare effects: while more accurate information should in principle enable firms to make better investment decisions, here this information can also be mis-used as an additional margin along which incumbent shareholders optimize their rent. If the rent-seeking motive is sufficiently important, the provision of noisy public information is welfare reducing. Enhanced transparency is welfare increasing only once the information provision is sufficiently precise to crowd the market-generated signal z out from investor expectations.

Formally, suppose that incumbent shareholders and outside investors can condition their decisions on a public signal $y \sim \mathcal{N}(\theta, \kappa^{-1})$. For simplicity, we suppose that investment returns are log-normal, i.e. $R(\theta) = e^\theta$. Once we adjust beliefs to include the information contained in the public signal, the equilibrium share prices and optimal investment decisions are characterized as before in proposition 1 and equation 4.¹⁴ The share price $P(y, z; k)$ and expected dividend value $V(y, z; k)$ are then conditioned on y as well as z and take the form $P(y, z; k) = \mathbb{E}\{R(\theta) | x = z, z, y\} \cdot k - C(k)$ and $V(y, z; k) = \mathbb{E}(R(\theta) | z, y) \cdot k - C(k)$. The market price thus overweighs the market information z , but reduces the weight attached to the public signal y .

The investment $k(y)$ is also a function of y . The efficient investment decision $k^*(y)$ maximizes $\mathbb{E}(V(y, z; k))$, solves the first-order condition $C'(k^*(y)) = \mathbb{E}(R(\theta) | y) = e^{\frac{\kappa}{\lambda + \kappa} y + \frac{1}{2}(\lambda + \kappa)^{-1}}$, and incorporates the information contained in y according to Bayes' Rule. As the signal becomes infinitely precise, $k^*(y)$ converges in probability to the first-best decision rule $k^{FB}(\theta)$. The incumbent shareholder's optimal investment maximizes the expected market price, condi-

¹³Information frictions in bond markets would have counter-vailing effects, as they would induce original lenders to over-estimate the tail risk of default. See Albagli, Hellwig and Tsyvinski (2014) for bond pricing with noisy information aggregation in a dynamic variant of this model.

¹⁴That is, $\theta | y \sim \mathcal{N}(\kappa / (\lambda + \kappa) y; (\lambda + \kappa)^{-1})$ and $y \sim \mathcal{N}(0; \lambda^{-1} + \kappa^{-1})$

tional on y : $C'(\hat{k}(y)) = \mathbb{E}\{\mathbb{E}(R(\theta)|x=z, z)|y\} = e^{\frac{\kappa}{\lambda+\kappa}y + \frac{1}{2}\lambda_P^{-1}}$, where the market-implied uncertainty λ_P^{-1} is now a function of κ . Therefore, the investment distortion is independent of y : $\hat{k}(y)/k^*(y) = e^{\frac{1}{2\chi}(\lambda_P^{-1} - (\lambda+\kappa)^{-1})} > 1$ is constant and depends only on the gap between market uncertainty λ_P^{-1} and objective uncertainty $(\lambda + \kappa)^{-1}$.

Now, using the same notation as earlier, let \hat{V} denote the expected dividend at the equilibrium contract, and V^* the expected welfare when investment is efficient. Our next proposition shows that if the frictions are sufficiently severe to cause expected dividends to be negative, then provisions of noisy public information may lead to further reduction in welfare.

Proposition 6 *Noisy public news may reduce welfare.*

If $1 + \chi < e^{\frac{1}{2\delta(\beta+\beta\delta)}}$, then there exists $\hat{\kappa} > 0$ such that $\hat{V} < 0$ and \hat{V} is decreasing in κ for $\lambda + \kappa \leq \hat{\kappa}$. Moreover, $\hat{\kappa} \rightarrow \infty$ as $\chi \rightarrow 0$.

This result shows that information disclosures can be welfare-reducing, if the information offers initial shareholders an additional flexibility to optimize rent-seeking behavior. An improvement in information unambiguously increases V^* , but this is not sufficient to guarantee an overall welfare increase if at the same time the loss from distortions $V^* - \hat{V}$ gets worse. Taking derivatives of \hat{V} w.r.t. κ , we have $\frac{\partial \hat{V}}{\partial \kappa} = \hat{V}/V^* \frac{\partial V^*}{\partial \kappa} + \frac{\partial \hat{V}/V^*}{\partial \kappa} V^*$. Although \hat{V}/V^* is increasing in $\lambda + \kappa$ (i.e. better information reduces the relative welfare loss), \hat{V}/V^* is negative if $\lambda + \kappa$ is sufficiently low. The second term of this expression is therefore strictly positive, but the first term is negative when $\lambda + \kappa$ is sufficiently low. Moreover, as $\chi \rightarrow 0$, $V^* \rightarrow 0$, so the scope for distortions always dominates the gains from better information. Thus, if rent-seeking is sufficiently severe, noisy public information reduces welfare. Public information is welfare-improving only once it is sufficiently precise to crowd out the price distortion, resulting in more efficient firm decisions and positive surplus.

Proposition 6 contributes to the literature on the social value of public information. Morris and Shin (2002) and Angeletos and Pavan (2007) show that public disclosures may be welfare-reducing, if strategic complementarities distort agents' responses to information. Although Morris and Shin (2002) cite financial markets as a leading motivation for the reduced-form zero-sum coordination motives embedded in the beauty contest game, coordination in financial markets was not modeled explicitly. Amador and Weill (2010) obtain a similar result but focus on informational externalities. Proposition 6 also features a distortion in the market response to public information, but the welfare losses arise because of the rent-seeking behavior by shareholders, not coordination motives or informational externalities.

4.3 Stock-Price Sensitivity of Investment

Here, we consider how market frictions distort the use of information aggregated through share prices resulting in excess sensitivity of investment to share prices.¹⁵ We modify our benchmark model by assuming that the initial shareholders publicly commit to a price-contingent investment rule $k(\cdot)$, allowing for informational feedback effects from the share price to investment. The market price perfectly anticipates the investment level that will realize at a given price, and the initial shareholders internalize the impact of their decision rule on the share price.¹⁶

Information Feedback. We depart from our baseline model by assuming that the investment rule is a price-contingent, or equivalently z -contingent function $k(z)$, which investors perfectly anticipate given the realization of the share price. We further assume that $\mathbb{E}(R(\theta) | x = z, z) / \mathbb{E}(R(\theta) | z)$ is increasing in z , and there exists a unique \hat{z} s.t. $\mathbb{E}(R(\theta) | x = z, z) \geq \mathbb{E}\{R(\theta) | z\}$ for $z \geq \hat{z}$.¹⁷ For a given $k(z)$, the equilibrium share price and expected dividend value take the form

$$P(z) = \mathbb{E}(R(\theta) | x = z, z) \cdot k(z) - C(k(z)) \quad \text{and} \quad V(z) = \mathbb{E}(R(\theta) | z) \cdot k(z) - C(k(z)).$$

The price function $P(z)$ must be strictly monotonic in z to constitute an equilibrium. This is no longer guaranteed once k depends on z , but due to an envelope condition, this holds automatically for $\alpha = 1$, i.e. if the chosen investment rule maximizes the share price conditional on z .¹⁸

The efficient investment rule sets $k^*(z)$ such that $C'(k^*(z)) = \mathbb{E}(R(\theta) | z)$ and incorporates the information contained in the price according to Bayes' Rule. Our first result shows that informational feedback from share prices to investment creates endogenous upside risk.¹⁹

Proposition 7 *Information feedback generates endogenous upside risk:*

- (i) **Increased Shareholder Rents:** $\mathbb{E}(\Omega(z, k^*(z))) > \mathbb{E}(\Omega(z, k^*(\hat{z})))$.

¹⁵This application subsumes results from the working paper Albagli, Hellwig, and Tsyvinski (2011).

¹⁶This commitment could result from internal reporting and decision procedures that make it difficult to reverse the initial decision ex post. Importantly we assume that final shareholders are not able to renegotiate the investment ex post, even though they would have incentives to do so.

¹⁷Many return distributions satisfy this condition, provided that they are unbounded above.

¹⁸For $\alpha < 1$, monotonicity may fail without additional restrictions on the shape of returns if the expected market return $\mathbb{E}(R(\theta) | x = z, z)$ is significantly below the objective return, $\mathbb{E}(R(\theta) | z)$, and the initial shareholders put sufficient weight on the latter (i.e. α is close to 0). It suffices to assume that $\lim_{\theta \rightarrow -\infty} R(\theta) \geq \mathbb{E}\{R(\theta) | \hat{z}\} / (1 + \chi)$, or to assume that the firms' dividend includes an additional component that is increasing in θ but not affected by investment. See the working paper Albagli, Hellwig, and Tsyvinski (2011) for further discussion.

¹⁹This proposition follows from aligning investment with the information contained in the share price and holds with any strictly increasing investment rule, not just the efficient one.

(ii) **Endogenous Upside Risk:** $\mathbb{E}(\Omega(z, k^*(z))) > 0$ if either $\mathbb{E}(\mathbb{E}(R(\theta) | x = z, z)) \geq \mathbb{E}(R(\theta))$, or $\mathbb{E}(\mathbb{E}(R(\theta) | x = z, z)) < \mathbb{E}(R(\theta))$ and χ is sufficiently close to 0.

(iii) **Unbounded Rents:** $\lim_{\chi \rightarrow 0} \mathbb{E}(\Omega(z, k^*(z))) = \infty$, for any $R(\cdot)$.

Therefore, by responding to the news in z , the efficient investments tilts the initial shareholder's expected rents to the upside. We can write the expected wedge as a sum of two terms:

$$\mathbb{E}(\Omega(z, k^*(z))) = \mathbb{E}(\Omega(z, \mathbb{E}(k^*(z)))) + \text{cov}(k^*(z); \mathbb{E}(R(\theta) | x = z, z) - \mathbb{E}\{R(\theta) | z\})$$

The term $\mathbb{E}(\Omega(z, \mathbb{E}(k^*(z))))$, corresponds to the expected wedge when investment is fixed at its unconditional expectation $\mathbb{E}(k^*(z))$. The term $\text{cov}(k^*(z); \mathbb{E}(R(\theta) | x = z, z) - \mathbb{E}\{R(\theta) | z\})$ captures the informational feedback from share prices to investment. This term is strictly positive. The informational feedback leads to more risk-taking on the upside and less risk-taking on the downside, and therefore creates an endogenous element of upside risk, which increases the initial shareholders' rents. If the underlying return is symmetrically distributed or dominated by upside risk, this feedback effect strengthens the original upside bias. If instead the underlying risk was dominated by the downside, then the endogenous investment response mitigates the downside exposure or overturns it if investment is sufficiently responsive to z . In the limiting case of constant returns to scale, investment is infinitely responsive to z , and incumbent shareholder rents are positive and arbitrarily large, regardless of the underlying $R(\cdot)$.

Investment Distortions with Informational Feedback. Next, we discuss how rent-seeking by incumbent shareholders leads to excess sensitivity of investment to stock prices. The initial shareholders choose $\hat{k}(z)$ to satisfy the first-order condition $C'(\hat{k}(z)) = \mathbb{E}(R(\theta) | x = z, z)$. Therefore, investment $\hat{k}(z)$ is dictated by market expectations: investment responds more to z than would be justified from an objective point of view. By shifting the weight given to z in the direction of the market's expectations, investment becomes more sensitive to the information, but also the noise, contained in the stock price. These results are summarized in the next proposition:

Proposition 8 Market noise causes excess investment volatility

(i) **Excess investment sensitivity:** $\hat{k}(z)/k^*(z)$ is increasing in z .

(ii) **Fundamentals vs. Market Noise:** For any $\beta > 0$, as $\delta \rightarrow 0$, $\text{Var}(\hat{k}(z)) \rightarrow \infty$ and $\text{Corr}(\hat{k}(z), R(\theta)) \rightarrow 0$.

(iii) **Unbounded rents and welfare losses:** For any $\beta > 0$, as $\delta \rightarrow 0$, $\mathbb{E}(\Omega(z, \hat{k}(z))) \rightarrow \infty$ and $\mathbb{E}(V(z, \hat{k}(z))) \rightarrow -\infty$.

We can interpret this result as another manifestation of the rent-seeking argument. Here, by setting investment conditional on z , incumbent shareholders are given an additional margin along which to optimize their rents. Since shareholder rents are increasing in the sensitivity of investment to z , they take advantage through an investment rule that is excessively responsive to market information. This causes excess volatility in investment: on the upside, shareholders prefer to over-invest in order to maximize the positive rents they extract from inflated share prices. On the downside, they under-invest to limit the losses they incur from the market price being below the fundamental value. Our model thus links investment volatility to stock market volatility by tying investment decisions to market expectations of returns.

This channel not only increases the volatility of investment, but it also disconnects investment from fundamentals. A high price translates into a higher investment level, but if the market is sufficiently noisy, prices may be almost exclusively driven by market noise, and carry little information about fundamental returns, yet investment responds aggressively in order to capture rents on the upside or limit losses on the downside. In the limiting case where investment is orthogonal to fundamentals, the feedback from prices to investment leads to arbitrarily large shareholder rents, and arbitrarily negative surplus.

4.4 Time-inconsistent firm decisions

So far we have assumed that incumbent shareholders control the decisions taken by the firm. In a dynamic context, however, the identity of incumbent shareholders changes over time as the firms' equity is traded in the market. In other words, today's future shareholders will be tomorrow's incumbent shareholders in charge of firm decisions. Here, we show that the conflict of interest between successive shareholder generations results in inefficient, and time-inconsistent firm decisions.

We extend our model to include an ex ante decision that is controlled by incumbent shareholders before the market opens, and an ex post decision that is controlled by final shareholders and taken after the firm has been traded. Suppose that the firm's final dividend is $\Pi(\theta; k, l) = e^\theta k^\sigma l^{1-\sigma} - l - C(k)$, where k is the "investment" decision controlled by incumbent shareholders and l the "labor" decision controlled by final shareholders. The surplus-maximizing choice of k and l takes the form

$$l^*(z) = k^* \cdot (1 - \sigma)^{\frac{1}{\sigma}} \mathbb{E} \left(e^\theta | z \right)^{\frac{1}{\sigma}} \quad \text{and} \quad C'(k^*) = \sigma (1 - \sigma)^{\frac{1-\sigma}{\sigma}} \mathbb{E} \left(\mathbb{E} \left(e^\theta | z \right)^{\frac{1}{\sigma}} \right).$$

Taking as given the price $P(k, z) = \mathbb{E}(\Pi(\theta; k, l(z)) | x = z, z)$ they have to pay, this is also the set of choices k and l that maximize the final shareholder payoffs.²⁰

²⁰As in the previous application, the price function must be monotone in z . Here, we abstract from the issue and

The incumbent shareholders instead would choose k and l to maximize the expected price, and they would like to commit to a decision rule

$$\hat{l}(z) = \hat{k} \cdot (1 - \sigma)^{\frac{1}{\sigma}} \mathbb{E} \left(e^\theta | x = z, z \right)^{\frac{1}{\sigma}} \quad \text{and} \quad C'(\hat{k}) = \sigma (1 - \sigma)^{\frac{1-\sigma}{\sigma}} \mathbb{E} \left(\mathbb{E} \left(e^\theta | x = z, z \right)^{\frac{1}{\sigma}} \right)$$

which maximizes the expected market value of the firm. Their preferred initial investment level \hat{k} exceeds k^* (since $\mathbb{E} \left(\mathbb{E} \left(e^\theta | x = z, z \right)^{\frac{1}{\sigma}} \right) > \mathbb{E} \left(\mathbb{E} \left(e^\theta | z \right)^{\frac{1}{\sigma}} \right)$), and $\hat{l}(z)$ is more sensitive to z than the final shareholders' preferred choice.

Consider now the dynamic game in which incumbent shareholders first choose k , and final shareholders then choose l after the realization of the market price (and correspondingly, z). The two choices \tilde{k} and $\tilde{l}(z)$ satisfy the first-order conditions

$$\begin{aligned} \tilde{l}(z) &= \tilde{k} \cdot (1 - \sigma)^{\frac{1}{\sigma}} \mathbb{E} \left(e^\theta | z \right)^{\frac{1}{\sigma}} \quad \text{and} \\ C'(\tilde{k}) &= \sigma (1 - \sigma)^{\frac{1-\sigma}{\sigma}} \mathbb{E} \left(\mathbb{E} \left(e^\theta | z \right)^{\frac{1}{\sigma}} \left[1 + \frac{1}{\sigma} \left(\frac{\mathbb{E} \left(e^\theta | x = z, z \right)}{\mathbb{E} \left(e^\theta | z \right)} - 1 \right) \right] \right). \end{aligned}$$

This equilibrium maximizes neither the incumbent nor the final shareholders' welfare. In fact, the following proposition shows that the resulting choices are strictly inside the frontier between incumbent and final shareholders' welfare. Therefore, the equilibrium of decisions made by successive shareholder generations no longer coincides with the maximization of a well-defined common welfare criterion, such as expected discounted values of firm cash flows.

Proposition 9 *Market Frictions cause dynamically inconsistent firm behavior*

Whenever $\tilde{k} \neq k^$, the equilibrium choices of \tilde{k} and $\tilde{l}(z)$ are strictly Pareto-inferior.*

The equilibrium is strictly inside the Pareto frontier between incumbent and final shareholders, because the input mix of \tilde{k} and $\tilde{l}(z)$ is not set at its cost-minimizing level: While the capital-labor ratio is set at the surplus-maximizing level, the initial investment choice is distorted by incumbent shareholders looking to maximize shareholder rents. Consider therefore a shift in k and $l(z)/k$ that keeps $k^\sigma l(z)^{1-\sigma}$ constant for each z . This shift keeps the firm's exposure to θ unchanged, and therefore doesn't alter the final shareholders expected payoffs. The shift in the capital-labor ratio only has second-order effects on total surplus, while the change k results in first-order gains, if it brings k closer to k^* , hence resulting in a welfare gain for incumbent shareholders. By shifting to a lower capital-labor ratio, a lower level of investment, and a higher level of labor, the firm can lower its input costs $\mathbb{E}(l(z)) + C(k)$ and incumbent shareholder payoffs without altering its exposure to θ and the final shareholders' payoffs.²¹

assume that z can be inferred from P .

²¹It's straight-forward to extend this argument to include disagreement about the response to the public signal z .

Time inconsistency results from the interaction of complementarity between the incumbent and final shareholders' decisions with the disagreement between them due to rent-seeking by incumbent shareholders. To see this, consider the following example, in which the two decisions are completely separable: there are two rounds of investment and trading, each with an iid draw of θ_t and u_t . Investment k_t is decided by period t shareholders, and dividend $\Pi(\theta_t, k_t)$ collected by period $t + 1$ shareholders, as in the model discussed in sections 2 and 3. In this case, date 0 and date 1 shareholders perfectly agree on the choice of k_1 to maximize the expected price extracted from final (date 2) shareholders, and the expected shareholder rent from the second investment is passed on to the incumbent shareholders through the share price.²²

To summarize, with information frictions in financial markets, rent-seeking incentives by incumbent shareholders leads to an inefficient equilibrium with dynamically inconsistent firm behavior, which is no longer consistent with the maximization of a well-defined criterion of expected discounted value of cash flows. As in the case of consumers with time-inconsistent preferences, endogenous preference reversals generate a positive value to commitment, or a motive to restrict the set of feasible options ex ante. Here, the current generation of shareholders would like to commit future shareholders to decisions that increase the current share prices. Such commitment is privately valuable but socially harmful, if it enforces rent-seeking strategies that are in the interest of incumbent shareholders, but do not maximize social surplus.

5 Regulation and Policy Interventions

In this section, we discuss policy interventions to correct the inefficiencies resulting from rent-shifting by incumbent shareholders. A policy maker could alter incumbent shareholder incentives either directly through regulatory oversight of firm decisions, or indirectly by altering the shareholders' returns in the market through tax policies or market interventions. At the efficient markets benchmark, any intervention of either kind only distort investment behavior and thus reduces welfare. Our model offers an efficiency rationale for these interventions.

Direct Regulatory Oversight: Size Caps or Floors. A regulation that limits the firm's investment level to $\hat{k} \in [\underline{k}, \bar{k}]$, i.e. a cap \bar{k} and a floor \underline{k} , directly rules out the possibility of over-

²²The potential for time-inconsistency may arise even if the identity of the controlling shareholder does not change. For example, suppose that all decisions are controlled by an incumbent owner who floats a minority share $\alpha < 1/2$ of the firm in the market. This incumbent owner will want to commit ex ante to a strategy that maximizes a weighted average of the market value and fundamental value of the firm, but once the share α is sold, he would want to re-optimize the subsequent decisions to maximize the firm's fundamental value.

or under-investment. The optimal cap sets $\bar{k} = k^*$ to limit over-investment, or $\underline{k} = k^*$ to limit under-investment. In our application to borrowing with costly state verification, such a regulation corresponds to a *capital requirement* on the firm, i.e. a cap on the amount of leverage the firm is allowed to take on, regardless of the interest it is prepared to pay. The optimal caps and floors depend on the firm's return characteristics and technologies.

Financial Transaction Tax. An uncontingent tax on share sales modifies incentives because it shifts a part of the shareholder rents to the tax authorities. With an uncontingent tax τ on the proceeds of share sales, the shareholders maximize $\mathbb{E}((1 - \tau)\alpha P(z; k) + (1 - \alpha)V(z, k))$. A tax on share sales thus reduces the relative weight on the share price from α to $\alpha(1 - \tau)/(1 - \alpha\tau)$. This reduces the externality, but unless the tax is completely confiscatory ($\tau = 1$), such a tax can never fully correct for the externality.

A contingent tax $\tau(z)$ that conditions on the market price has the potential to be more effective. Such a tax modifies the incumbent shareholder's objective function to $\alpha\mathbb{E}((1 - \tau(z))P(z, k)) + (1 - \alpha)\mathbb{E}(\Pi(\theta, k))$, and a tax function implements k^* if and only if $\mathbb{E}\{(1 - \tau(z))P_k(z, k^*)\} = 0$.

Proposition 10 *Contingent transaction taxes must lean against return asymmetries.*

For any $\tau(z)$, let $\hat{\tau}(z) = (\tau(z) - \mathbb{E}(\tau(z))) / (1 - \mathbb{E}(\tau(z)))$. A transaction tax policy $\tau(\cdot)$ implements k^* , if and only if

$$1 - \frac{\mathbb{E}(R(\theta))}{\mathbb{E}(\mathbb{E}(R(\theta)|x = z, z))} = \int_{-\infty}^{\infty} \left\{ \frac{\mathbb{E}(\mathbb{E}(R(\theta)|x = z', z')|z' \geq z)}{\mathbb{E}(\mathbb{E}(R(\theta)|x = z, z))} - 1 \right\} \left(1 - \Phi \left(\sqrt{\frac{\beta\delta}{\lambda + \beta\delta}} \lambda z \right) \right) d\hat{\tau}(z). \quad (8)$$

The tax policy affects incentives by shifting shareholder risk more towards the upside or the downside: if the tax function is increasing ($d\hat{\tau}(z) \geq 0$), then it taxes returns on the upside more heavily than returns on the downside. As a result, the shareholders' expected returns become less dominated by upside risk, which dampens investment incentives. If instead the tax function is decreasing ($d\hat{\tau}(z) \leq 0$), then it shifts the shareholder's expected returns to the upside, which strengthens investment incentives. Equation (8) spells out the condition on taxes and market returns that must be satisfied to implement k^* . If $R(\cdot)$ is dominated by upside (downside) risk, then k^* is implementable with a monotonic tax function only if the share price is taxed more heavily on the upside (downside). The LHS of equation (8) corresponds to the percentage change in expected shareholder returns from the investment that the tax policy must achieve to implement k^* . The term inside the integral on the RHS measures the contribution of a tax increment $d\hat{\tau}(z)$

to this shift in expected returns. The taxes or tax revenues do not have to be strictly positive on average, since incentives are shifted by the *relative* size of taxes on the upside vs. the downside.²³

Market Interventions. Market interventions are another channel through which a policy maker can influence investment incentives. The idea is again based on a strategy of “leaning against return asymmetries”: Suppose that a policy maker commits to buy shares at a guaranteed support price $\bar{P} > 0$. For a given k , and for any z such that $P(z, k) \equiv \mathbb{E}(R(\theta) | x = z, z) k - C(k) > \bar{P}$, this intervention has no effect on the market. However, if $P(z, k) < \bar{P}$, the policy maker buys a positive level of shares. This occurs whenever z falls below some threshold \hat{z} . The initial shareholders’ objective is then to maximize $\alpha \mathbb{E}(\max\{\bar{P}, P(z, k)\}) + (1 - \alpha) \mathbb{E}(\Pi(\theta, k))$. Clearly, this intervention strengthens the initial shareholders’ incentives to invest, so it is efficiency enhancing only in the case where the equilibrium resulted in under-investment.²⁴ The implemented investment \hat{k} satisfies the following first order condition:

$$C'(\hat{k}) = \frac{\alpha \int_{\hat{z}}^{\infty} \mathbb{E}(R(\theta) | x = z, z) d\Phi\left(\sqrt{\frac{\beta\delta}{\lambda+\beta\delta}} \lambda z\right) + (1 - \alpha) \mathbb{E}(R(\theta))}{1 - \alpha + \alpha \left(1 - \Phi\left(\sqrt{\frac{\beta\delta}{\lambda+\beta\delta}} \lambda \hat{z}\right)\right)} \quad (9)$$

This condition uniquely defines \hat{k} as a function of \hat{z} , and the associated value of \bar{P} then satisfies $\bar{P} = \mathbb{E}(R(\theta) | x = \hat{z}, \hat{z}) \hat{k} - C(\hat{k})$. We then arrive at the following result:

Proposition 11 *Implementing k^* through a market intervention.*

(i) Let z^* be defined by $\mathbb{E}(R(\theta)) = \mathbb{E}(\mathbb{E}(R(\theta) | x = z, z) | z \geq z^*)$. A market intervention with a support price \bar{P} implements k^* , if and only if $\bar{P} = \mathbb{E}(R(\theta) | x = z^*, z^*) k^* - C(k^*)$.

(ii) Direct market interventions generate negative expected revenues.

As in the case of the tax policy, the intervention must modify the expected market return from the investment to align it with the fundamental return $\mathbb{E}(R(\theta))$. Here, the shareholders’ return varies with k , only if the realized z exceeds a threshold \hat{z} , so the expected market return takes the form $\mathbb{E}(\mathbb{E}(R(\theta) | x = z, z) | z \geq \hat{z})$. The market return is aligned with fundamental returns, if \hat{z} is targeted to be equal to z^* .²⁵

²³Likewise, uncontingent dividend taxes have no effect on incentives, since the tax is fully passed through into the share price. Contingent dividend taxes on the other hand can be used to align initial and final shareholder incentives.

²⁴For over-investment situations, one would have to consider policies of short-selling equity at a pre-set price, so as to limit upside gains.

²⁵While the support price \bar{P} and the investment target \hat{k} are uniquely determined for a threshold signal \hat{z} , the reverse need not be true: The relation between \bar{P} and \hat{z} is not necessary monotonic, resulting in the possibility of multiple equilibria for the investment level and market outcome, for a given price support \bar{P} . In other words, implementation is not guaranteed to be unique.

However, such policy interventions do not come for free. Although the policy maker buys shares at prices below fundamental values, the policy maker is also exposed to a winner's curse problem when the informed traders offload their shares. The latter always dominates the former, so the policy maker's expected revenue is negative.

Winners and Losers from Policy Interventions: The focus on efficiency masks the fact that policy interventions have re-distributive effects. Because the rationale for policy emerges from the existence of shareholder rents, there is a natural conflict of interest over policy interventions between incumbent shareholders, and final shareholders and a social planner. Incumbent shareholders have a clear preference against policies that reduce the rents they can capture without a compensating subsidy (such as direct regulatory oversight), and a clear preference in favor of policies that provide incentives through explicit or implicit subsidies (such as equity purchases). Final shareholders and the social planner benefit as long as social surplus increases by more than rents and subsidies going to initial shareholders.

Therefore, policies must include a subsidy to incumbent shareholders in order to benefit both incumbent and final shareholders. Consider for example a transaction tax $\tau(z)$ that implements k^* . If this tax is revenue neutral (i.e. $\mathbb{E}(\tau(z)P(z, k^*)) = 0$), incumbent shareholders will be strictly worse off with the intervention, but the after-tax revenue $\mathbb{E}((1 - \tau(z))P(z, k^*))$ of incumbent shareholders can easily be re-scaled to a level at which they receive the same expected revenue as before the intervention, without altering incentives. Market interventions also subsidize incumbent shareholders, and the gains in efficiency may exceed the cost of this subsidy, at least if the intervention is not too large. In certain cases, this even applies to market interventions that implement k^* .

Informational Requirements: One characteristic of all these interventions is that they require that the regulator has access to a lot of firm-specific information about returns, costs or market-frictions. In these cases, the optimal policy offers a remedy that introduces a new distortion to exactly offset the rent-shifting incentive, and thus need to be finely adjusted to the firm's characteristics to achieve the optimum. The level of information required for these policies may thus seem unrealistic from a practical point of view, and this issue only gets compounded when firm decisions become more complex and possibly based on proprietary information that is not directly accessible to the regulator. We will return to this point below when comparing to policies that directly target managerial incentives.

6 Extensions

In this section we discuss two extensions of our model. First, we extend our model of firm decisions to the inclusion of agency frictions inside the firm, when decisions are delegated to a manager. Second, we consider different modeling assumptions about the information in financial markets. These extensions are interesting not only to illustrate the robustness of our main results along two important dimensions, but also because of the additional results that each of them provides.

6.1 Managerial Incentives

The corporate finance literature identifies agency frictions between shareholders and managers as the central source of imperfections in firm decision-making. Here, we argue that our results are robust to the inclusion of such agency frictions. In our model, agency costs do not alter the shareholders' rent-seeking incentives, since these derive from return perceptions in the market, which are independent of how decisions are taken inside the firm.

In addition, the presence of agency costs leads to additional interesting results that go against the conventional wisdom regarding the effects of agency frictions. First, those contracts that under an efficient markets view are viewed as quintessential for providing efficient managerial incentives (such as stock option compensation to incentivize risk-taking) now emerge as tools of choice to optimize shareholder rents. Second, when control is delegated to managers, the regulation of managerial contracts emerges as an additional margin for welfare-improving policy interventions. In fact, restrictions on CEO compensation may prove to be more effective at reigning in rent-seeking incentives than direct regulatory oversight or market interventions. For a broad class of agency cost models that we describe below, full efficiency can be restored by restricting executive compensation to be linear in final dividends, without any further knowledge of proprietary information about firm technologies, agency frictions and risks. Third, whereas agency costs are typically viewed as an impediment to efficient firm decision-making, we show that their presence may actually improve efficiency, if they reduce the incumbent shareholders' ability to incentivize rent-seeking.

We first consider incentive contracts with a risk-neutral manager, in which case incumbent shareholders can always implement their preferred action at no additional cost, before enriching the model to allow for risk aversion and managerial moral hazard.

Consider the baseline model from section 3 but assume that incumbent shareholders delegate the choice of the investment scale $k \geq 0$ to a risk-neutral manager. Incumbent shareholders pay the manager a wage $W(\cdot)$ that is contingent on the realization of final dividends Π .

A triplet $(W(\cdot), \hat{k}, P(z; k))$ is *implementable*, if (i) $P(z; k)$ is the market-clearing price function of a noisy rational expectations equilibrium in the financial market, and (ii) the manager's individual rationality and incentive compatibility constraints hold: $\mathbb{E}\{W(\Pi(\theta, \hat{k}))\} \geq \bar{w}$ and $\hat{k} \in \arg \max_k \mathbb{E}\{W(\Pi(\theta, k))\}$. The optimal contract $W(\cdot)$ maximizes expected shareholder returns $\mathbb{E}(P(z; k) - W(\Pi(\theta, k)))$, among the set of implementable wage contracts.

Let $\underline{k} = \lim_{\theta \rightarrow -\infty} k^{FB}(\theta)$ and $\bar{k} = \lim_{\theta \rightarrow \infty} k^{FB}(\theta)$. The interval (\underline{k}, \bar{k}) contains all investment levels that are efficient for some θ . We show that incumbent shareholders can incentivize the manager to choose their preferred investment level at no extra cost: any $\hat{k} \in (\underline{k}, \bar{k})$ is implementable through a wage contract with an expected wage payment $\mathbb{E}\{W(\Pi(\theta, k))\} = \bar{w}$.

Clearly, the efficient investment k^* is implemented if $W^*(\Pi) = \omega\Pi$, i.e. if the manager's compensation is linear in the final dividend. This is a "restricted equity" contract, since the manager cannot sell the equity share in the market. Compensation floors (i.e. stock options) and ceilings can be used to modify incentives, relative to this baseline. Any $k \in (\underline{k}, \bar{k})$ is implementable by a contract that adds either a floor or a ceiling to this baseline contract.

Proposition 12 (Almost) anything is implementable with equity, options, and caps.

In the baseline model of Section 3, any $k \in (k^, \bar{k})$ can be implemented with a contract of the form $W(\Pi) = \max\{\underline{W}, \omega\Pi\}$. Any $k \in (\underline{k}, k^*)$ can be implemented with a contract of the form $W(\Pi) = \min\{\bar{W}, \omega\Pi\}$.²⁶*

If $\hat{k} > k^*$, $W^*(\Pi)$ induces too little investment from the incumbent shareholder's perspective. An optimal contract adds a minimal compensation level into the benchmark contract $W^*(\Pi)$ to strengthen incentives and increase investment above k^* . If instead $\hat{k} < k^*$, the incentives provided by $W^*(\Pi)$ are too strong from the incumbent shareholders' perspective, and therefore the optimal contract includes a cap on total compensation. With market frictions, sophisticated incentive pay schemes for managers thus become an important tool to optimize shareholder rents. (ii).

Richer contract space: Since any investment level can be implemented without agency costs through a Π -contingent contract, investment distortions do not result from an artificial restriction on the contract space. Implementation possibilities only expand with a richer contract space.²⁷

²⁶Any $k \notin [\underline{k}, \bar{k}]$ cannot be implemented through any Π -contingent contract $W(\cdot)$

²⁷For example, \hat{k} could also be implemented through a P -contingent contract by compensating the manager through an unrestricted equity share, or participation in the share sale. This requires that k is perfectly observable - otherwise, the market would always attribute any fluctuation in the observation of k to noise, and the price would therefore not respond to variation in k . With Π -contingent wage contracts it suffices that contract terms are publicly known for the market to perfectly anticipate the implemented value of k and price equity accordingly.

Risk aversion and agency costs. Our core results also extend to the inclusion of non-trivial agency frictions. Suppose that the manager is risk-averse with preferences over wage payments characterized by an increasing concave utility function $U(\cdot)$, and that in addition to choosing an investment scale $k > 0$, the manager makes a hidden effort choice $e \in \{0, 1\}$, which affects the return distribution $R(\theta, e)$.²⁸ The choice of $e = 0$ entails a pecuniary private benefit $B \geq 0$. The cash flow takes the form $\Pi(\theta; k, e) = R(\theta, e)k - C(k)$.

The optimal contract design problem can then be analyzed in the usual two-step procedure. First, for each pair of choices (k, e) , let $W(k, e)$ denote the minimal expected wage cost (over all possible contracts) that implements the choice of (k, e) , subject to the manager's incentive compatibility and participation constraints:

$$\begin{aligned} W(k, e) &= \min_{W(\cdot)} \mathbb{E} \{W(\Pi(\theta; k, e))\} \\ \text{s.t. } (k, e) &\in \arg \max_{(k', e')} \mathbb{E} \{U(W(\Pi(\theta; k', e'))) + (1 - e')B\} \\ \bar{U} &\leq \mathbb{E} \{U(W(\Pi(\theta; k, e))) + (1 - e)B\} \end{aligned}$$

Second, we determine the pair (k, e) that maximizes the initial shareholder's expected payoffs:

$$(k, e) \in \arg \max_{(k', e')} \mathbb{E} \{ \alpha P(z; k', e') + (1 - \alpha) \Pi(\theta; k', e') - W(k', e') \},$$

where $P(z; k, e) = \mathbb{E}(\Pi(\theta; k, e) | x = z, z)$

is the equilibrium equity price function. The optimal investment \hat{k} satisfies the first order condition $\mathbb{E} \{ \mathbb{E}(R(\theta, e) | x = z, z) \} = C'(\hat{k}) + W_k(\hat{k}, e)$, and therefore modifies the marginal cost of investment by the marginal wage cost associated with higher investment. Likewise, the efficient investment k^* satisfies the first-order condition $\mathbb{E} \{ R(\theta, e) \} = C'(k^*) + W_k(k^*, e)$. Thus investment distortions are entirely due to distortions in the incumbent shareholders' objective. All our previous results are robust, once the marginal cost of investment is adjusted to include the agency cost component: Upside risk results in over-investment, while downside risk results in under-investment, and the easier it is to scale investment, the more pronounced the distortions are.

Wage payment by final shareholders: If the manager is paid by final shareholders, then dividends are $\Pi - W(\Pi)$, the share price is $P(z; k) = \mathbb{E}(\Pi - W(\Pi) | x = z, z)$, and incumbent shareholders

²⁸Without hidden effort, any k remains approximately implementable with a contract that offers the manager an arbitrarily small incentive component of the form characterized by Proposition 12, along with a fixed wage to meet the participation constraint, unless the manager is infinitely risk-averse. A non-trivial agency cost arises only if the manager is risk-averse and needs to be given a non-vanishing exposure to the firm's dividend risk for incentive reasons.

evaluate the cost of the managers' wage through the lens of market, instead of fundamental, expectations. The wage contract influences the expected market value of equity by shifting upside and downside risk between shareholders and managers. In this case, the first-order condition for efficient investment is $\mathbb{E}\{R(\theta, e)\} = C'(k^*) + \frac{\partial}{\partial k}\mathbb{E}\{W(\Pi(\theta; k^*, e))\}$, so incentives are distorted through market expectations of returns, as well as market expectations of marginal wage costs ($\frac{\partial}{\partial k}\mathbb{E}\{W(\Pi(\theta; k^*, e))\}$ vs. $\frac{\partial}{\partial k}\mathbb{E}\{\mathbb{E}(W(\Pi(\theta, k))|x=z, z)\}$).

Regulation of Executive Pay. When firm decisions are delegated to a manager, the regulation of executive compensation contracts offers an additional policy tool to influence firm decisions. If incumbent shareholders design CEO compensation to incentivize rent-shifting, then a regulator may limit investment distortions by limiting the incumbent shareholders' discretion in designing CEO compensation contracts.

We formalize this intuition when the manager is risk-neutral, as in proposition 12. Suppose that a regulator limits CEO compensation to a fixed set of $N + 1$ contracts, or linear combinations of these. Each contract $n = 0, \dots, N$ defines an expected compensation $T_n(k)$ to the manager as a function of the chosen k . Each $T_n(\cdot)$ is assumed to be bounded, concave and continuously differentiable. We let $T_0(k) = \mathbb{E}(R(\theta))k - C(k)$ denote the transfer associated with a restricted equity claim, and assume that this claim is included in the set of available contracts.

Proposition 13 *It's efficient to limit incentive pay to restricted equity.*

A set of contracts $\{T_n(\cdot)\}$ implements k^ if and only if $(\hat{k} - k^*)T'_n(k^*) \leq 0$ for all n .*

Proposition 13 suggests a simple regulation of executive pay as a remedy to rent-shifting by incumbent shareholders: just ban any form of performance pay other than restricted equity shares. This eliminates the incumbent shareholders' discretion and fully implements the efficient investment levels. On the other hand, as soon as initial shareholders have access to an instrument that allows them to distort the CEO's incentives in the desired direction they will find it optimal to include such an incentive component in the compensation contract.²⁹

Restricting CEO compensation to restricted equity has one important advantage over other policies: it is a simple one-size-fits-all policy that does not need to be adjusted to firm characteristics, return distributions or market frictions. This is an important distinction from the other

²⁹If negative positions are allowed (i.e. leveraging different securities against each other), the minimal qualification that $(\hat{k} - k^*)T'_n(k^*) \leq 0$ for all n must be tightened to $T'_n(k^*) = 0$ for all n . Therefore, a ban on the use of any compensation contract other than restricted equity is sufficient and quasi necessary to guarantee efficiency.

interventions we discussed, which all required far more firm-specific information about firm characteristics and market frictions. Therefore, limiting compensation to restricted equity also has no adverse effects when financial markets are efficient.

Regulation of executive compensation is the most effective remedy because it tackles the problem at its root by directly limiting the shareholder’s ability to incentivize rent-shifting. By directly promoting efficient CEO incentives, the regulator no longer has to worry about how market frictions affect incentives.

With non-trivial agency frictions, proposition 13 applies as long as the planner’s allocation is implementable through a restricted equity contract that is a linear function of final dividends. More generally, as long as market frictions introduce a wedge between initial shareholders’ incentives and social surplus, a social planner will gain from imposing some restrictions on executive contract design. In this context, the restricted equity contract emerges as a useful benchmark against which to measure the tradeoff between the fine-tuning of managerial incentives and the potential for rent-shifting that would emerge as one allows for a richer set of compensation contracts.³⁰

Social benefit of separating ownership from control. The separation of ownership from control and the associated agency conflicts that characterize large modern corporations may even be socially beneficial if this serves to temper incumbent shareholders’ ability to extract rents. This benefit is distinct from the “usual” comparative advantage arguments that the person best placed to own a firm may not be the person best placed to manage it, because it is obviously not internalized by incumbent shareholder preferences.

To illustrate this point, suppose that the firm chooses an investment level k to generate dividends $\Pi(\theta, k) = R(\theta)k - C(k)$, with $C(k) = 1/(1 + \chi)k^{1+\chi}$. Suppose that the expected wage cost of an investment level k is given by an increasing, convex function $\eta W(k)$, where the scaling parameter $\eta > 0$ that captures the severity of agency frictions in a reduced form. The incumbent shareholders implement an investment level $\hat{k}(\eta) \in \arg \max_{k'} \mathbb{E} \{P(z; k') - \eta W(k')\}$. The expected social surplus is $\mathbb{E} \left\{ \Pi(\theta; \hat{k}(\eta)) - \eta W(\hat{k}(\eta)) \right\}$.

Proposition 14 *Social benefit of separating ownership from control*

³⁰Prop. 13 also suggests that a partial approach to regulation that focuses on specific instruments (for example, a ban on stock option compensation, participation in share sales or limits on bonuses) will not have any impact as long as incumbent shareholders can create the same incentives through other means. A successful regulation must tightly specify the margins along which initial shareholders are given discretion to select the contracting terms, i.e. focus on what contracts are permitted, rather than which ones are ruled out, and how far the compensation is allowed to depart from the restricted equity benchmark.

Social surplus is increasing in η if the following inequality holds:

$$\frac{\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} - \mathbb{E}(R(\theta))}{\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\}} \frac{\frac{W'(\hat{k})\hat{k}}{W(\hat{k})}}{\chi + \frac{\eta W''(\hat{k})}{\eta W'(\hat{k}) + C'(\hat{k})} \left(\frac{W''(\hat{k})\hat{k}}{W'(\hat{k})} - \chi \right)} > 1. \quad (10)$$

Social surplus equals the difference between shareholder value, $\mathbb{E}\left\{P\left(z; \hat{k}(\eta)\right) - \eta W\left(\hat{k}(\eta)\right)\right\}$, and shareholder rents, $\{\mathbb{E}(\mathbb{E}(R(\theta) | x = z, z)) - \mathbb{E}(R(\theta))\} \hat{k}(\eta)$. An increase in η increases social surplus, if it reduces shareholder rents by more than it reduces shareholder value. The RHS of (10) represents the marginal impact of $\eta W(\hat{k})$ on shareholder value. The LHS of (10) represents the marginal impact of $\eta W(\hat{k})$ on shareholder rents, $(\mathbb{E}(\mathbb{E}(R(\theta) | x = z, z)) - \mathbb{E}(R(\theta))) \hat{k}_\eta(\eta) / W(\hat{k})$, which is the product of two terms: First, the gap between market and fundamental returns (relative to the market returns, or marginal costs) measures the shareholder's baseline *incentive* to distort investment. Second, the ratio of the elasticity of wages to investment in the numerator, relative to the elasticity of marginal costs to investment in the denominator governs how much a change in $\eta W(\hat{k})$ affects the shareholders' *ability* to distort investment. When η is close to zero, this denominator is close to the technological returns to scale parameter χ . Therefore, social surplus is increasing in η for sufficiently low η , if (i) the percentage gap between expected market and fundamental returns is sufficiently large, (ii) χ is small so that the potential for rent-shifting is large, and (iii) the elasticity of wages to \hat{k} is positive (negative) if shareholder rents are positive (negative), and sufficiently large in absolute terms, so that agency frictions effectively reign in the shareholder's ability to shift rents.

6.2 Financial Market Structure

So far, we have assumed that initial shareholders passively sell a fraction α of their shares to informed and noise traders, abstracting from proprietary information by incumbent shareholders, their possible exposure to liquidity shocks, or even heterogeneity and potential conflicts of interests, and focusing on the gap between $P(z, k)$ and $V(z, k)$ as the source of distortions. Here we argue that our results are robust to alternative information structures in the financial market: Assumptions about information and liquidity shocks affect the nature, sign and magnitude of shareholder rents and distortions, but they do not alter their impact on firm decisions.

We can decompose expected cash-flows of initial shareholders, informed traders and noise traders into three terms. First, the term $V(z, k)$ measures the expected final dividend from the firm. Second, the initial shareholders earn the price $P(z, k)$ for each share that they sell. The wedge

$\Omega(z, k) = P(z, k) - V(z, k)$ thus represents a pure transfer from the buyers to the sellers of equity shares. Third, we can compute the expected payoffs to informed traders in the financial market. Let $F(x|z)$ denote the distribution of the private signal x , conditional on observing a market signal z ; note that $x|z \sim \mathcal{N}\left(\beta\delta/(\lambda + \beta\delta) \cdot z; 1/\left((\lambda + \beta\delta)^{-1} + \beta^{-1}\right)\right)$. An informed buyer's expected payoff is

$$\begin{aligned} I(z, k) &= \int \max\{0, \mathbb{E}(\Pi(\theta, k) | x', z) - P(z)\} dF(x'|z) \\ &= k \cdot \int_z^\infty (\mathbb{E}(R(\theta) | x', z) - \mathbb{E}(R(\theta) | x = z, z)) dF(x'|z) \end{aligned}$$

$I(z, k)$ is strictly positive, and corresponds to an information rent, i.e. the option value of trading on private information. Since the aggregate payoff to buyers is $-\Omega(z, k)$ and informed buyers' expected payoff is $I(z, k) > 0$, the noise trader's expected payoff is $-\Omega(z, k) - I(z, k)$. By the Law of Iterated Expectations, $V(z, k) = \int \mathbb{E}(\Pi(\theta, k) | x', z) dF(x'|z)$, and

$$-\Omega(z, k) = k \cdot \int_{-\infty}^\infty (\mathbb{E}(R(\theta) | x', z) - \mathbb{E}(R(\theta) | x = z, z)) dF(x'|z) < I(z, k),$$

so that noise trader's expected payoffs is unambiguously negative. With this decomposition, we can extend our analysis to alternative assumptions about information and liquidity shocks:

1. *Private information by incumbent shareholders:* Suppose incumbent shareholders receive a private signal before deciding whether to sell their equity share. Shares are bought by a random measure of noise traders, given by $\Phi(u)$, as in the benchmark model. In this case, the supply of shares is endogenous and given by $\Pr(x < \hat{x}(P) | \theta) = \Phi(\sqrt{\beta}(\hat{x}(P) - \theta))$, while demand is $\Phi(u)$. The equilibrium characterization remains the same, with $\hat{x}(P) = z = \theta + 1/\sqrt{\beta} \cdot u$. But the initial shareholder objective changes, and is now given by $P(z, k) + I(z, k) = V(z, k) + \Omega(z, k) + I(z, k) > V(z, k)$, while the buyers' (noise traders') payoff is $-\Omega(z, k) - I(z, k) < 0$. In other words, the initial shareholder rents are no longer given by the gap between expected price and dividend value, but by the option value of trading on private information. Since this value is strictly positive and scales with k , we obtain that the initial shareholders now have a strict incentive to over-invest in order to capture a larger information rent regardless of the firm's risk characteristics.

2. *Liquidity shocks to incumbent shareholders:* Suppose next that incumbent shareholders sell passively, but the fraction sold, given by $1 - \Phi(u)$, is random, while potential buyers of equity shares observe a private signals (i.e. noise traders are only on the seller's side). Now the demand for shares is given by $\Pr(x > \hat{x}(P) | \theta) = 1 - \Phi(\sqrt{\beta}(\hat{x}(P) - \theta))$, while supply is random. The equilibrium price characterization remains the same, with $\hat{x}(P) = z = \theta + 1/\sqrt{\beta} \cdot u$. But the initial shareholder objective changes to $V(z, k) - I(z, k)$, while the informed buyers earn $I(z, k)$.

In this case, the initial shareholders have an unambiguous interest to under-invest so as to reduce information rents conceded to informed buyers.

3. *Disagreement among shareholders:* If shareholders differ ex ante in their investment horizons, their access to private information, or their potential exposure to liquidity shocks, then this heterogeneity leads to disagreement among shareholders about the desired investment decisions. With heterogeneity, each incumbent shareholder will have their own preferred value of k . A voting perspective would suggest the implementation of the value of k that is preferred by the median shareholder. The departure from efficient markets is crucial for the emergence of shareholder disagreement: Under efficient markets, shareholder rents disappear, and hence shareholder incentives are all aligned with social surplus. Financial market frictions may therefore also play an important role in shaping disagreement between concurrent shareholders of a firm.

7 Conclusion

According to the efficient markets paradigm, the market provides *discipline* by offering the right valuation signals, thereby incentivizing firms and managers to take decisions that are in the social interest. Because of these disciplining effects of financial prices, firms and managers can be trusted to take decisions that are in the collective interest without further oversight and regulation.

Here, we have taken a different view of price formation in asset markets, one that is based on limits to information and limits to arbitrage. Once equity prices depart from the efficient markets benchmark, they induce a rent-seeking motive into shareholder objectives, since any deviation of share prices from fundamental values amounts to a zero-sum transfer from final to initial shareholders (or vice versa). Because of this rent-seeking element, initial shareholders can no longer be trusted to act in the interest of future shareholders or society. In other words, market discipline no longer works, and even has detrimental effects, if distorted valuations lead to distorted incentives. Rent-seeking becomes especially problematic if risks are tilted to the upside (limited liability) and easily scaleable, i.e. exactly for those types of financial institutions that specialize in the arbitrage activities required to align market and fundamental valuations. According to our theory, such corporations mainly exist to transfer rents to their incumbent shareholders, while the social surplus they provide may well be negative. In a dynamic context, shareholder rent-seeking results in time-inconsistent firm decisions, since at each point in time the current shareholders have an incentive to take decisions that are partly designed to extract rents from their successors, and thus not in the long-term interest of the corporation. This rent-seeking argument offers a rationale

for regulating corporate behavior or for policy interventions in the market, in the same vein as time-inconsistency of individual preferences offers a rationale for regulating consumer choices.

For expositional reasons, we have limited ourselves to a few particularly stark applications of our theory: distortions in risk-taking, excessive leverage, sensitivity of investment to stock prices, and potentially harmful side-effects of transparency. But our model suggests a theory of distortions and time inconsistent firm behavior which should be generally applicable to corporate decision making with frictions in financial markets. This suggests numerous other applications, such as capital structure choice, dividend payout policies, collective action problems between shareholders with different investment horizons, or real effects of credit distortions. These applications are all interesting in their own right, but their development would go far beyond the scope of this paper.

The empirical assessment of the mechanisms highlighted by our paper is another important direction for future work. In Albagli, Hellwig and Tsyvinski (2012, 2014), we argue that noisy information aggregation can account for some asset pricing anomalies that appear difficult to reconcile with no arbitrage principles, such as the large volatility in equity prices, or the very large spreads on corporate default risk. On the corporate side, there is ample suggestive evidence of excessive risk-taking by financial institutions, and more generally of patterns of corporate short-termism, which are more pronounced in companies without long-term shareholders, or with a stronger focus on shareholder value as measured by market returns. These observations are consistent with our model, but one would need to be far more systematic and comprehensive in order to measure and confirm the causal impact of market frictions on firm decisions, or to disentangle those effects from the impact of agency costs.

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8 Appendix: Proofs

Proof of Proposition 1:

Let $\hat{x}(P)$ denote the private signal of a trader who is just indifferent between buying and not buying the share at a given price P , so that $P = \int \Pi(\theta, k) dH(\theta|\hat{x}(P), P)$. Since $R(\cdot)$ is increasing in θ , $\int \Pi(\theta, k) dH(\theta|x, P)$ must be monotone in x , and any trader whose private signal exceeds $\hat{x}(P)$ must strictly prefer to purchase a share, while any trader whose signal is less than $\hat{x}(P)$ prefers not to buy. Thus, the total demand by the informed traders is $\alpha(1 - \Phi(\sqrt{\beta}(\hat{x}(P) - \theta)))$. Equating demand and supply, a price P clears the market in state (θ, u) if and only if $\hat{x}(P) = \theta + 1/\sqrt{\beta} \cdot u \equiv z$. Therefore, in any equilibrium, it must be the case that $\hat{x}(P(\theta, u; k)) = z$, and if P is a function of z only, then it must be invertible. But if $P(\cdot)$ is invertible, observing P is informationally equivalent to observing $\hat{x}(P) = z \sim \mathcal{N}(\theta, (\beta\delta)^{-1})$. Along the equilibrium path, the traders thus treat the signals $\hat{x}(P) \sim \mathcal{N}(\theta, (\beta\delta)^{-1})$ and $x \sim \mathcal{N}(\theta, \beta^{-1})$ as mutually independent normal signals and their posterior beliefs $H(\cdot|x, P)$ are given by

$$\theta|x, P \sim \mathcal{N}\left(\frac{\hat{\lambda}\mu + \beta x + \beta\delta\hat{x}(P)}{\hat{\lambda} + \beta + \beta\delta}, \left(\hat{\lambda} + \beta + \beta\delta\right)^{-1}\right).$$

Substitute $\hat{x}(P) = z$, we restate the informed traders' indifference condition in terms of z : $P(z, k) = \mathbb{E}(\Pi(\theta, k) | x = z, z)$. The expression for $V(z, k) = \mathbb{E}(\Pi(\theta, k) | z)$ is derived analogously using only the information from the market signal, $\theta|P \sim \mathcal{N}\left(\frac{\hat{\lambda}\mu + \beta\delta\hat{x}(P)}{\hat{\lambda} + \beta\delta}, \left(\hat{\lambda} + \beta\delta\right)^{-1}\right)$.

Uniqueness. If demand is restricted to be non-increasing in P , $\hat{x}(P)$ must be non-decreasing. If $\hat{x}(P)$ is strictly monotone everywhere, then it is invertible, P is informationally equivalent to $\hat{x}(P) = z$, and we arrive at the equilibrium characterized above. Suppose therefore that $\hat{x}(P)$ is flat over some range, i.e. $\hat{x}(P) = \hat{x}$ for $P \in (P', P'')$. Suppose further that for sufficiently low $\varepsilon > 0$, $\hat{x}(P)$ is strictly increasing over $(P' - \varepsilon, P')$ and $(P'', P'' + \varepsilon)$, and hence uniquely invertible.³¹ But then for $z \in (\hat{x}(P' - \varepsilon), \hat{x})$ and $z \in (\hat{x}, \hat{x}(P'' + \varepsilon))$, $P(z)$ is uniquely defined, and must be characterized as above, from the indifference condition for $\hat{x}(P) = z$. But since the function $P(z, k)$ defined above is continuous and strictly monotonic in z , it must be the case that $P' = P''$, contradicting the existence of an interval for which $\hat{x}(P)$ is flat.

Proof of Propositions 2 and 3:

Proposition 2 follows directly from arguments in the text. Proposition 3 is a direct corollary of THM 2 in Albagli, Hellwig and Tsyvinski (2011).

Proof of Proposition 4:

To simplify notation, let $\Upsilon = \alpha (\mathbb{E} \{ \mathbb{E} (R(\theta) | x = z, z) \} / \mathbb{E} (R(\theta)) - 1)$. We begin with the results concerning \hat{k}/k^* . Since $\hat{k}/k^* = (1 + \Upsilon)^{1/\chi}$, it is immediate that \hat{k}/k^* is increasing in Υ , equal to 1 if and only if $\Upsilon = 0$, and unbounded as $\Upsilon \rightarrow \infty$. Moreover, $\partial (\hat{k}/k^*) / \partial \chi^{-1} = \log(1 + \Upsilon) (1 + \Upsilon)^{1/\chi}$, which is positive if and only if $\Upsilon > 0$. Hence \hat{k}/k^* is decreasing in χ , if $\Upsilon < 0$ and increasing in χ , if $\Upsilon > 0$, which proves that investment distortions are worse, the lower is χ . Finally, if $\Upsilon > 0$, then clearly \hat{k}/k^* is unbounded as $\chi \rightarrow 0$, while if $\Upsilon < 0$, $\hat{k}/k^* \geq (1 - \alpha)^{1/\chi} > 0$. Next we consider comparative statics w.r.t. Δ . Since $\Delta = 1 + (1 + \Upsilon)^{1/\chi} (\Upsilon/\chi - 1)$, we have

$$\begin{aligned} \frac{\partial \Delta}{\partial \Upsilon} &= \frac{1}{\chi} \frac{1 + \chi}{\chi} (1 + \Upsilon)^{1/\chi} \frac{\Upsilon}{1 + \Upsilon} \text{ and} \\ \frac{\partial \Delta}{\partial \chi^{-1}} &= (1 + \Upsilon)^{1/\chi} (\Upsilon - \log(1 + \Upsilon) (1 - \Upsilon/\chi)), \end{aligned}$$

and one therefore obtains that $\Delta = 0$ iff $\Upsilon = 0$, Δ is increasing in Υ (and therefore positive) if $\Upsilon > 0$, and Δ is decreasing in Υ (and therefore again positive) if $\Upsilon < 0$. Furthermore, if $\Upsilon \geq \chi > 0$, it is clear that $\frac{\partial \Delta}{\partial \chi^{-1}} > 0$, while if $\Upsilon < \chi$, $\Upsilon - \log(1 + \Upsilon) (1 - \Upsilon/\chi) > \Upsilon - \Upsilon (1 - \Upsilon/\chi) > \Upsilon^2/\chi$ and so once again $\frac{\partial \Delta}{\partial \chi^{-1}} > 0$. The limiting behavior, the bounds, and the result that $\Delta > 1$ if $\Upsilon > \chi$ also follow immediately.

³¹It cannot be flat everywhere, because then informed demand would be completely inelastic, and there would be no way to absorb noise trader shocks.

Proof of Proposition 5:

Let $\omega(\hat{\theta})$ denote the lowest ratio $\frac{w}{k}$ that is consistent with the leverage ratio constraint. Then the incumbent shareholder's return takes the form

$$\rho(\omega(\hat{\theta}); \hat{\theta}) = \left(1/\omega(\hat{\theta})\right) \int_{\hat{\theta}}^{\infty} (R(\theta) - R(\hat{\theta})) d\Phi(\sqrt{\lambda_P}\theta) = \left(1/\omega(\hat{\theta})\right) \int_{\hat{\theta}}^{\infty} R'(\theta) (1 - \Phi(\sqrt{\lambda_P}\theta)) d\theta.$$

The optimal choice of $\hat{\theta}$ satisfies the first-order condition

$$0 = \frac{d \ln \rho(\omega(\hat{\theta}); \hat{\theta})}{d\hat{\theta}} = \frac{-R'(\hat{\theta}) (1 - \Phi(\sqrt{\lambda_P}\hat{\theta}))}{\int_{\hat{\theta}}^{\infty} R'(\theta) (1 - \Phi(\sqrt{\lambda_P}\theta)) d\theta} - \frac{\omega'(\hat{\theta})}{\omega(\hat{\theta})}.$$

(i) The result holds if $\hat{\theta}$, or equivalently $\frac{\partial \ln \rho(\omega(\hat{\theta}); \hat{\theta})}{\partial \hat{\theta}} = \frac{\rho_{\hat{\theta}}(\omega(\hat{\theta}); \hat{\theta})}{\rho(\omega(\hat{\theta}); \hat{\theta})}$ is decreasing in λ_P . But

$$\frac{\partial^2 \ln \rho(\omega(\hat{\theta}); \hat{\theta})}{\partial \hat{\theta} \partial \sqrt{\lambda_P}} = \frac{-R'(\hat{\theta}) (1 - \Phi(\sqrt{\lambda_P}\hat{\theta}))}{\int_{\hat{\theta}}^{\infty} R'(\theta) (1 - \Phi(\sqrt{\lambda_P}\theta)) d\theta} \left\{ \frac{-\hat{\theta} \phi(\sqrt{\lambda_P}\hat{\theta})}{1 - \Phi(\sqrt{\lambda_P}\hat{\theta})} + \frac{\int_{\hat{\theta}}^{\infty} R'(\theta) \theta \phi(\sqrt{\lambda_P}\theta) d\theta}{\int_{\hat{\theta}}^{\infty} R'(\theta) (1 - \Phi(\sqrt{\lambda_P}\theta)) d\theta} \right\}$$

which is negative whenever the term in brackets is positive. It's straight-forward to check that this is the case whenever $R(\cdot)$ is symmetric or dominated by upside risk, or $\sqrt{\lambda_P}\hat{\theta}$ not too negative.

(ii) As $\lambda_P \rightarrow 0$, $\rho\left(\frac{w}{k}; \hat{\theta}\right)$ converges to $\left(1/\omega(\hat{\theta})\right) \frac{1}{2} (\bar{R} - R(\hat{\theta}))$, where $\bar{R} = \lim_{\theta \rightarrow \infty} R(\theta)$. The incumbent shareholders will then choose to borrow if for some $\hat{\theta}$,

$$\frac{1}{2} (\bar{R} - R(\hat{\theta})) > 1 - \mathbb{E}(R(\theta)) + \varepsilon \int_{-\infty}^{\hat{\theta}} R(\theta) d\Phi(\sqrt{\lambda}\theta) + \int_{\hat{\theta}}^{\infty} (R(\theta) - R(\hat{\theta})) d\Phi(\sqrt{\lambda}\theta).$$

This is automatically satisfied if for some $\hat{\theta}$, $\frac{1}{2} (\bar{R} - R(\hat{\theta})) > 1 - R(\hat{\theta}) (1 - \Phi(\sqrt{\lambda}\hat{\theta}))$, or $\bar{R} > 2 - \max_{\hat{\theta}} R(\hat{\theta}) (1 - 2\Phi(\sqrt{\lambda}\hat{\theta}))$, which is strictly less than 2. But then if λ_P is sufficiently low, the incumbent shareholders will always prefer to borrow and take positive investment levels, even if unconditional returns are strictly less than 1.

Proof of Proposition 6:

We have $k^*(y) = e^{\frac{1}{\chi} \left(\frac{\kappa}{\lambda + \kappa} y + \frac{1}{2(\lambda + \kappa)} \right)}$, $V^*(y) = \frac{\chi}{1 + \chi} e^{\frac{1 + \chi}{\chi} \left(\frac{\kappa}{\lambda + \kappa} y + \frac{1}{2(\lambda + \kappa)} \right)}$, $V^* = \mathbb{E}(V^*(y)) = \frac{\chi}{1 + \chi} e^{\frac{1 + \chi}{\chi} \frac{1}{2\lambda} \left(1 + \frac{\kappa}{\lambda + \kappa} \frac{1}{\chi} \right)}$, and $\hat{k}(y)/k^*(y) = e^{\frac{1}{2\chi} \Psi(\kappa)}$, where $\Psi(\kappa) = 1/\lambda_P(\kappa) - 1/(\lambda + \kappa)$. Therefore,

$$\frac{\hat{V}(y)}{V^*(y)} = \frac{\hat{V}}{V^*} = \frac{1 + \chi}{\chi} \left(e^{\frac{1}{2\chi} \Psi(\kappa)} - \frac{1}{1 + \chi} e^{\frac{1 + \chi}{2\chi} \Psi(\kappa)} \right) = \frac{1 + \chi}{\chi} e^{\frac{1}{2\chi} \Psi(\kappa)} \left(1 - \frac{1}{1 + \chi} e^{\frac{1}{2} \Psi(\kappa)} \right).$$

Now, $\frac{\partial V^*}{\partial \kappa} = V^* \frac{1 + \chi}{\chi^2} \frac{1}{2(\lambda + \kappa)^2}$ and $\frac{\partial (\hat{V}/V^*)}{\partial \kappa} = \frac{1 + \chi}{\chi^2} \frac{1}{2} \Psi'(\kappa) e^{\frac{1}{2\chi} \Psi(\kappa)} \left(1 - e^{\frac{1}{2} \Psi(\kappa)} \right)$, which is positive

since $\Psi(\kappa) = \frac{(\beta+\beta\delta)/\delta}{(\lambda+\kappa+\beta+\beta\delta)^2} > 0$ and $\Psi'(\kappa) = -2\frac{(\beta+\beta\delta)/\delta}{(\lambda+\kappa+\beta+\beta\delta)^3} < 0$. This gives

$$\begin{aligned}\frac{\partial \hat{V}}{\partial \kappa} &= \frac{\partial V^*}{\partial \kappa} \left(\frac{\hat{V}}{V^*} \right) + \frac{\partial (\hat{V}/V^*)}{\partial \kappa} V^* \\ &= V^* \frac{1+\chi}{\chi^2} \frac{1}{2(\lambda+\kappa)^2} \left\{ (1-\Delta) + (\lambda+\kappa)^2 \Psi'(\kappa) e^{\frac{1}{2\chi}\Psi(\kappa)} \left(1 - e^{\frac{1}{2}\Psi(\kappa)} \right) \right\} \\ &= V^* \frac{1+\chi}{\chi^2} \frac{1}{2(\lambda+\kappa)^2} e^{\frac{1}{2\chi}\Psi(\kappa)} \left\{ \frac{1+\chi}{\chi} \left(1 - \frac{1}{1+\chi} e^{\frac{1}{2}\Psi(\kappa)} \right) + (\lambda+\kappa)^2 \Psi'(\kappa) \left(1 - e^{\frac{1}{2}\Psi(\kappa)} \right) \right\}\end{aligned}$$

and $\frac{\partial \hat{V}}{\partial \kappa} \geq 0$ if and only if $1 + \chi - e^{\frac{1}{2}\Psi(\kappa)} + \chi(\lambda + \kappa)^2 \Psi'(\kappa) \left(1 - e^{\frac{1}{2}\Psi(\kappa)} \right) \geq 0$. Solving for χ we obtain that $\frac{\partial \hat{V}}{\partial \kappa} \geq 0$ if and only if

$$\chi \geq \hat{\chi}(\kappa) \equiv \frac{e^{\frac{1}{2}\Psi(\kappa)} - 1}{1 - (\lambda + \kappa)^2 \Psi'(\kappa) \left(e^{\frac{1}{2}\Psi(\kappa)} - 1 \right)}.$$

Clearly, $0 < \hat{\chi}(\kappa) < e^{\frac{1}{2}\Psi(\kappa)} - 1$. Therefore, if $\chi < \hat{\chi}(\kappa)$, \hat{V} is decreasing in κ , and since $\hat{\chi}(\kappa) < e^{\frac{1}{2}\Psi(\kappa)} - 1$, this also implies that $\hat{V} < 0$. Now, $\lim_{\kappa \rightarrow \infty} \hat{\chi}(\kappa) = 0$ and $\lim_{\kappa \rightarrow 0} \hat{\chi}(\kappa) = e^{\frac{1}{2}\Psi(0)} - 1$, so for given χ , \hat{V} is decreasing in κ when $\lambda + \kappa$ is sufficiently close to 0. To complete the proof, we check that $\frac{\partial \hat{\chi}}{\partial \kappa} < 0$, which implies that \hat{V} reaches a global minimum when $\lambda + \kappa$ is such that $\chi = \hat{\chi}(\kappa)$. After some algebra, we obtain:

$$\begin{aligned}\frac{\partial \hat{\chi}}{\partial \kappa} &= \frac{\frac{1}{2}\Psi'(\kappa) e^{\frac{1}{2}\Psi(\kappa)} + \left(e^{\frac{1}{2}\Psi(\kappa)} - 1 \right)^2 \left(2(\lambda + \kappa) \Psi'(\kappa) + (\lambda + \kappa)^2 \Psi''(\kappa) \right)}{\left[1 - (\lambda + \kappa)^2 \Psi'(\kappa) \left(e^{\frac{1}{2}\Psi(\kappa)} - 1 \right) \right]^2} \\ &= \frac{1}{2} \Psi'(\kappa) (\hat{\chi}(\kappa))^2 \left\{ \frac{e^{\frac{1}{2}\Psi(\kappa)}}{\left(e^{\frac{1}{2}\Psi(\kappa)} - 1 \right)^2} + 4(\lambda + \kappa) \left(1 + (\lambda + \kappa) \frac{\Psi''(\kappa)}{2\Psi'(\kappa)} \right) \right\}\end{aligned}$$

Therefore, since $\Psi''(\kappa) / (2\Psi'(\kappa)) = -\frac{3}{2(\lambda+\kappa+\beta+\beta\delta)}$ and $1 + (\lambda + \kappa) \frac{\Psi''(\kappa)}{2\Psi'(\kappa)} = \frac{\beta+\beta\delta-1/2\cdot(\lambda+\kappa)}{\lambda+\kappa+\beta+\beta\delta}$, $\frac{\partial \hat{\chi}}{\partial \kappa} < 0$, whenever

$$\frac{e^{\frac{1}{2}\Psi(\kappa)}}{\left(e^{\frac{1}{2}\Psi(\kappa)} - 1 \right)^2} > 2(\lambda + \kappa) \frac{\lambda + \kappa - 2(\beta + \beta\delta)}{\lambda + \kappa + \beta + \beta\delta}$$

This is automatically satisfied whenever $\lambda + \kappa \leq 2(\beta + \beta\delta)$. For $\lambda + \kappa > 2(\beta + \beta\delta)$, we restate this condition as

$$\begin{aligned}e^{\frac{1}{2}\Psi(\kappa)} + e^{-\frac{1}{2}\Psi(\kappa)} - 2 &< \frac{\lambda + \kappa + \beta + \beta\delta}{2(\lambda + \kappa)(\lambda + \kappa - 2(\beta + \beta\delta))}, \text{ or} \\ \frac{1}{\Psi(\kappa)} \left(e^{\frac{1}{2}\Psi(\kappa)} + e^{-\frac{1}{2}\Psi(\kappa)} - 2 \right) &< \frac{\delta(\lambda + \kappa + \beta + \beta\delta)^3}{2(\beta + \beta\delta)(\lambda + \kappa)(\lambda + \kappa - 2(\beta + \beta\delta))}.\end{aligned}$$

Since $e^{\frac{1}{2}\Psi} + e^{-\frac{1}{2}\Psi} - 2$ is positive, increasing and convex for $\Psi > 0$, and $\lim_{\Psi \rightarrow 0} e^{\frac{1}{2}\Psi} + e^{-\frac{1}{2}\Psi} - 2 = 0$, the LHS of this inequality is increasing in Ψ , or decreasing in κ , and converges to 0 as $\kappa \rightarrow \infty$, while the RHS is bounded away from 0. It follows that monotonicity of $\hat{\chi}(\kappa)$ also holds for sufficiently large κ . Now, the RHS can be rewritten as

$$\frac{\delta(\lambda + \kappa + \beta + \beta\delta)^3}{2(\beta + \beta\delta)(\lambda + \kappa)(\lambda + \kappa - 2(\beta + \beta\delta))} > \frac{1}{2} \sqrt{\frac{\delta}{\beta + \beta\delta}} \frac{1}{\sqrt{\Psi(\kappa)}}$$

This term is decreasing in $\Psi(\kappa)$, and increasing in κ and is thus bounded away from 0 for $\lambda + \kappa > 2(\beta + \beta\delta)$. $\lambda + \kappa > 2(\beta + \beta\delta)$ implies $\Psi(\kappa) < \hat{\Psi} \equiv \frac{1}{\delta(\beta + \beta\delta)} \frac{1}{9}$, and therefore it suffices to check that $\frac{1}{\hat{\Psi}} \left(e^{\frac{1}{2}\hat{\Psi}} + e^{-\frac{1}{2}\hat{\Psi}} - 2 \right) < \frac{3}{2}\delta$. Since $\hat{\Psi}$ is strictly decreasing in δ , the LHS is strictly decreasing in δ , and there exists $\bar{\delta} > 0$, such that monotonicity of $\hat{\chi}(\kappa)$ is satisfied for all κ whenever $\delta \geq \bar{\delta}$. For $\delta < \bar{\delta}$, the above argument doesn't rule out non-monotonicities, but these are confined to an intermediate set of realizations for $\lambda + \kappa$, and $\hat{\chi}(\kappa)$ is bounded away from 0 for any finite κ . Then there always exist a bound $\hat{\kappa}$, such that \hat{V} is strictly decreasing in κ whenever $\lambda + \kappa < \hat{\kappa}$, and $\hat{\kappa} \rightarrow \infty$ as $\chi \rightarrow 0$.

Proof of Proposition 7:

The ex ante expectation of $\Omega(z, k^*(z))$ is

$$\begin{aligned} \mathbb{E} \{ \Omega(z, k^*(z)) \} &= k^*(\hat{z}) \int (\mathbb{E}(R(\theta) | x = z, z) - \mathbb{E}(R(\theta) | z)) \frac{k^*(z)}{k^*(\hat{z})} d\Phi \left(\sqrt{\lambda \frac{\beta\delta}{\lambda + \beta\delta}} z \right) \\ &> k^*(\hat{z}) (\mathbb{E}(\mathbb{E}(R(\theta) | x = z, z)) - \mathbb{E}(R(\theta))) = \mathbb{E} \{ \Omega(z, k^*(\hat{z})) \} \end{aligned}$$

Moreover, as $\chi \rightarrow 0$, $k^*(z)/k^*(\hat{z}) = (\mathbb{E}(R(\theta) | z) / \mathbb{E}(R(\theta) | \hat{z}))^{1/\chi}$ converges to ∞ for $z > \hat{z}$ and 0 for $z < \hat{z}$. Thus, $\lim_{\chi \rightarrow 0} \mathbb{E} \{ \Omega(z, k^*(z)) \} = \infty$, and $\mathbb{E} \{ \Omega(z, k^*(z)) \} > 0$ for χ sufficiently small.

Proof of Proposition 8:

Part (i) follows from monotonicity of $\mathbb{E}(R(\theta) | x = z, z) / \mathbb{E}(R(\theta) | z)$ w.r.t. z . For parts (ii) and (iii), fix $\beta > 0$, and consider the limiting case where $\delta \rightarrow 0$.

Part (ii) The volatility of investment satisfies $Var(\hat{k}(z)) = Var(\mathbb{E}(R(\theta) | x = z, z)^{1/\chi})$, which converges to $Var(\mathbb{E}(R(\theta) | x = z)^{1/\chi})$, as $\delta \rightarrow 0$. But $\mathbb{E}(R(\theta) | x = z)$ is strictly increasing in z and unbounded above, and $Var(z) \rightarrow \infty$ as $\delta \rightarrow 0$, so $Var(\mathbb{E}(R(\theta) | x = z)^{1/\chi}) \rightarrow \infty$. At the same time, $cov(\hat{k}(z), R(\theta)) = cov(\hat{k}(z), \mathbb{E}(R(\theta) | z))$, and therefore

$$\left| \text{correl}(\hat{k}(z), R(\theta)) \right| = \frac{\left| cov(\hat{k}(z), \mathbb{E}(R(\theta) | z)) \right|}{\left(Var(\hat{k}(z)) Var(R(\theta)) \right)^{1/2}} \leq \sqrt{\frac{Var(\mathbb{E}(R(\theta) | z))}{Var(R(\theta))}}.$$

But $Var(\mathbb{E}(R(\theta)|z))/Var(R(\theta)) \rightarrow 0$, as $\delta \rightarrow 0$ and the signal z becomes perfectly uninformative.

(iii) The expected wedge is

$$\mathbb{E}\left\{\Omega\left(z, \hat{k}(z)\right)\right\} = \hat{k}(\hat{z}) \int (\mathbb{E}(R(\theta)|x=z, z) - \mathbb{E}(R(\theta)|z)) \frac{\hat{k}(z)}{\hat{k}(\hat{z})} d\Phi\left(\sqrt{\lambda \frac{\beta\delta}{\lambda + \beta\delta}} z\right).$$

As $\delta \rightarrow 0$, $\hat{k}(\hat{z}) \rightarrow \mathbb{E}(R(\theta))^{1/\chi}$, $\mathbb{E}(R(\theta)|x=z, z) - \mathbb{E}(R(\theta)|z) \rightarrow \mathbb{E}(R(\theta)|x=z) - \mathbb{E}(R(\theta))$, and $\hat{k}(z)$ is strictly increasing in z and unbounded. Since $Var(z) \rightarrow \infty$, $\mathbb{E}\left\{\Omega\left(z, \hat{k}(z)\right)\right\} \rightarrow \infty$.

The expected firm value is $V\left(z, \hat{k}(z)\right) = \mathbb{E}(R(\theta)|z) \hat{k}(z) - C\left(\hat{k}(z)\right)$. As $\delta \rightarrow 0$, $\mathbb{E}(R(\theta)|z) \rightarrow \mathbb{E}(R(\theta))$ and $Var\left(\hat{k}(z)\right) \rightarrow \infty$. Since V is strictly concave in $\hat{k}(z)$, and unbounded below, it follows that $\lim_{\delta \rightarrow 0} \mathbb{E}\left\{V\left(z, \hat{k}(z)\right)\right\} = -\infty$.

Proof of Proposition 9:

The final shareholders' expected payoff is $\mathbb{E}\left[(R(\theta) - \mathbb{E}(R(\theta)|x=z, z)) k^\sigma l(z)^{1-\sigma}\right]$. Therefore any perturbation of k and $l(z)$, such that $k^\sigma l(z)^{1-\sigma}$ remains unchanged from their equilibrium levels also leaves the final shareholders' expected payoffs unchanged. Consider therefore any variation of the form $l(z) = \hat{l} \cdot \mathbb{E}(e^\theta|z)^{\frac{1}{\sigma}}$ and \hat{k} of pairs (\hat{k}, \hat{l}) , such that $\hat{k}^\sigma \hat{l}^{1-\sigma}$ is kept at its equilibrium level: $\hat{k}^\sigma \hat{l}^{1-\sigma} = \tilde{k}(1-\sigma)^{\frac{1-\sigma}{\sigma}}$. The pair (\hat{k}, \hat{l}) that maximizes incumbent shareholders' expected payoffs then satisfies the first-order condition

$$C'\left(\hat{k}\right) = \sigma(1-\sigma)^{\frac{1-\sigma}{\sigma}} \left(\frac{\hat{k}}{\tilde{k}}\right)^{\frac{1}{1-\sigma}} \mathbb{E}\left(\mathbb{E}\left(e^\theta|z\right)^{\frac{1}{\sigma}}\right)$$

or equivalently $C'\left(\hat{k}\right)/C'\left(k^*\right) = \left(\hat{k}/k^*\right)^{\frac{1}{1-\sigma}}$. It follows that unless $\tilde{k} = k^*$, the solution to this FOC is strictly between \tilde{k} and k^* .

Proof of Proposition 10:

k^* is implemented if and only if

$$C'\left(k^*\right) = \mathbb{E}(R(\theta)) = \frac{\mathbb{E}\{(1-\tau(z)) \mathbb{E}(R(\theta)|x=z, z)\}}{\mathbb{E}(1-\tau(z))} = \mathbb{E}\{(1-\hat{\tau}(z)) \mathbb{E}(R(\theta)|x=z, z)\},$$

given the definition of $\hat{\tau}(\cdot)$. Now, notice that

$$0 = \mathbb{E}(\hat{\tau}(z)) = \int_{-\infty}^{\infty} \hat{\tau}(z) \sqrt{\lambda_z} \phi\left(\sqrt{\lambda_z} z\right) dz = \lim_{z \rightarrow -\infty} \hat{\tau}(z) + \int_{-\infty}^{\infty} \left(1 - \Phi\left(\sqrt{\lambda_z} z\right)\right) d\hat{\tau}(z),$$

where $\lambda_z = \frac{\lambda\beta\delta}{\lambda+\beta\delta}$, and therefore

$$\begin{aligned}
& \mathbb{E} \{ \hat{\tau}(z) \mathbb{E}(R(\theta) | x = z, z) \} \\
&= \int_{-\infty}^{\infty} \hat{\tau}(z) \mathbb{E}(R(\theta) | x = z, z) \sqrt{\lambda_z} \phi(\sqrt{\lambda_z} z) dz \\
&= \lim_{z \rightarrow -\infty} \hat{\tau}(z) \mathbb{E}(\mathbb{E}(R(\theta) | x = z, z)) + \int_{-\infty}^{\infty} \int_z^{\infty} \mathbb{E}(R(\theta) | x = z', z') \sqrt{\lambda_z} \phi(\sqrt{\lambda_z} z') dz' d\hat{\tau}(z) \\
&= -\mathbb{E}(\mathbb{E}(R(\theta) | x = z, z)) \int_{-\infty}^{\infty} \left(1 - \Phi(\sqrt{\lambda_z} z)\right) d\hat{\tau}(z) \\
&+ \int_{-\infty}^{\infty} \left(1 - \Phi(\sqrt{\lambda_z} z)\right) \mathbb{E}(\mathbb{E}(R(\theta) | x = z', z') | z' \geq z) d\hat{\tau}(z),
\end{aligned}$$

from which condition (8) follows immediately. For the converse, notice that if $\hat{\tau}(z)$ satisfies (8), then $\mathbb{E} \{ (1 - \hat{\tau}(z)) \mathbb{E}(R(\theta) | x = z, z) \} = \mathbb{E}(R(\theta))$, and by construction the same is true for any tax schedule $\tau(z)$, s.t. $1 - \tau(z) = b(1 - \hat{\tau}(z))$, with $b > 0$.

Proof of Proposition 11:

(i) Condition (9) holds with $\hat{k} = k^*$ if and only if $\hat{z} = z^*$. (ii) For each share bought, the policy maker earns a dividend $\Pi(\theta, \hat{k}) - \bar{R}\hat{k} - C(\hat{k}) = (R(\theta) - \bar{R})\hat{k}$. Given θ and u , the measure of shares purchased is $\Phi(\sqrt{\beta}(\hat{x}(\bar{P}) - \theta)) - \Phi(u) = \Phi(\sqrt{\beta}(\hat{x}(\bar{P}) - \theta)) - \Phi(\sqrt{\beta}(z - \theta)) = \Pr(x \in [z, \hat{x}(\bar{P})] | \theta)$, where $\hat{x}(\bar{P})$ is the threshold signal when the support price is active, i.e. whenever $z < \hat{z}$, where $\bar{R} = \mathbb{E}(R(\theta) | x = \hat{z}, \hat{z})$. It follows that $\bar{R} = \mathbb{E}(R(\theta) | \hat{x}(\bar{P}), z \leq \hat{z})$ and therefore $\hat{x}(\bar{P}) > \hat{z}$. The expected revenue is then

$$\begin{aligned}
& \hat{k} \int_{-\infty}^{\infty} \int_{-\infty}^{\hat{z}} (R(\theta) - \bar{R}) \Pr(x \in [z, \hat{x}(\bar{P})] | \theta) d\Phi(\sqrt{\beta\delta}(z - \theta)) d\Phi(\sqrt{\lambda}\theta) \\
&= \hat{k} (\mathbb{E}(R(\theta) | x \in [z, \hat{x}(\bar{P})], z \leq \hat{z}) - \bar{R}) \cdot \Pr(x \in [z, \hat{x}(\bar{P})], z \leq \hat{z}),
\end{aligned}$$

which is negative since $\mathbb{E}(R(\theta) | x \in [z, \hat{x}(\bar{P})], z \leq \hat{z}) < \mathbb{E}(R(\theta) | \hat{x}(\bar{P}), z \leq \hat{z}) = \bar{R}$.

Proof of Proposition 12:

A continuous, differentiable contract $W(\Pi)$ implements investment level k if and only if it satisfies the manager's first-order condition:

$$\mathbb{E} \{ W'(\Pi(\theta, k)) (R(\theta) - C'(k)) \} = 0 \text{ or } C'(k) = \frac{\mathbb{E} \{ W'(\Pi(\theta, k)) R(\theta) \}}{\mathbb{E} \{ W'(\Pi(\theta, k)) \}}.$$

Clearly, if $W'(\Pi)$ is constant, the implemented investment level is k^* . Now, for each $\hat{\theta}$, there exists a unique $k(\hat{\theta}) > k^*$ such that $C'(k(\hat{\theta})) = \mathbb{E}(R(\theta) | \theta > \hat{\theta})$, and a unique $\underline{W}(\hat{\theta}) = R(\hat{\theta})k(\hat{\theta}) - C(k(\hat{\theta}))$. By construction, $W(\Pi) = \max \{ \underline{W}(\hat{\theta}), \Pi \}$ implements effort level $k(\hat{\theta})$. As $\hat{\theta} \rightarrow \infty$,

$k(\hat{\theta}) \rightarrow \bar{k}$, while as $\hat{\theta} \rightarrow -\infty$, $k(\hat{\theta}) \rightarrow k^*$. Likewise, for each $\tilde{\theta}$, there exists a unique $k(\tilde{\theta}) < k^*$ such that $C'(k(\tilde{\theta})) = \mathbb{E}(R(\theta) | \theta < \tilde{\theta})$, and a unique $\bar{W}(\tilde{\theta}) = R(\tilde{\theta})k(\tilde{\theta}) - C(k(\tilde{\theta}))$. By construction, $W(\Pi) = \min\{\Pi, \bar{W}(\tilde{\theta})\}$ implements effort level $k(\tilde{\theta})$. As $\tilde{\theta} \rightarrow \infty$, $k(\tilde{\theta}) \rightarrow k^*$, while as $\tilde{\theta} \rightarrow \infty$, $k(\tilde{\theta}) \rightarrow \underline{k}$. Finally, $\Pi(\theta, k)$ is strictly increasing in k for $k < \underline{k}$, for all θ , so that for any non-decreasing contract $W(\cdot)$, $\mathbb{E}\{W(\Pi(\theta, k))\} \leq \mathbb{E}\{W(\Pi(\theta, \underline{k}))\}$ for $k < \underline{k}$, and the inequality is strict if $W(\cdot)$ is strictly increasing for some Π . Likewise since $\Pi(\theta, k)$ is strictly decreasing in k for $k > \bar{k}$, for all θ , it follows that $\mathbb{E}\{W(\Pi(\theta, k))\} \leq \mathbb{E}\{W(\Pi(\theta, \bar{k}))\}$ for $k > \bar{k}$, for any non-decreasing contract $W(\cdot)$, that is strictly increasing for some θ .

Proof of Proposition 13:

We need to show that whenever $(\hat{k} - k^*)T'_n(k^*) > 0$ for some $T_n(\cdot)$, the initial shareholders can use $T_n(\cdot)$ to distort investment in their desired direction. By construction, for any $\eta > 0$, there exist $\varepsilon_1 > 0$, and $\varepsilon_2 > 0$, such that $T_0(k^*) - \eta = T_0(k^* - \varepsilon_1) = T_0(k^* + \varepsilon_2)$, and $T_0(k^*) - \eta \leq T_0(k)$ iff $k \in [k^* - \varepsilon_1, k^* + \varepsilon_2]$, and $T'_0(k) > 0$ for $k \in [k^* - \varepsilon_1, k^*]$, and $T'_0(k) < 0$ for $k \in [k^*, k^* + \varepsilon_2]$.

Suppose now that $\hat{k} > k^*$ and there exists $T_n(\cdot)$, such that $T'_n(k^*) > 0$. Choose η sufficiently small, so that $T'_n(k) > 0$ for all $k \in [k^* - \varepsilon_1, k^* + \varepsilon_2]$, and $T_n(\cdot)$ is bounded, so we can choose $\xi \in (0, \eta / (2 \max_k \|T_n(k)\|))$. Then, for $k \notin [k^* - \varepsilon_1, k^* + \varepsilon_2]$, $T_0(k) + \xi T_n(k) - T_0(k^*) - \xi T_n(k^*) < -\eta + 2\xi \max_k \|T_n(k)\| \leq 0$, so a contract paying $T_0(k) + \xi T_n(k)$ must implement an investment level $k \in [k^* - \varepsilon_1, k^* + \varepsilon_2]$. Moreover, $T'_0(k) + \xi T'_n(k) > 0$ for $k \in [k^* - \varepsilon_1, k^*]$, implying that the contract $T_0(k) + \xi T_n(k)$ is maximized at $k \in (k^*, k^* + \varepsilon_2)$.

By the same argument, if $\hat{k} < k^*$ and there exists $T_n(\cdot)$, such that $T'_n(k^*) < 0$, then the contract $T_0(k) + \xi T_n(k)$ implements $k < k^*$, for ξ sufficiently small.

Proof of Proposition 14:

The first-order condition for \hat{k} is $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} = C'(\hat{k}) + \eta W'(\hat{k})$. From this, it follows that $\hat{k}'(\eta) = -W'(\hat{k}) / (C''(\hat{k}) + \eta W''(\hat{k}))$. Therefore, social welfare is increasing in η , whenever

$$\frac{d}{d\eta} \left(\mathbb{E}(R(\theta))\hat{k} - C(\hat{k}) - \eta W(\hat{k}) \right) = -W(\hat{k}) + \hat{k}'(\eta) \left(\mathbb{E}(R(\theta)) - C'(\hat{k}) - \eta W'(\hat{k}) \right) \geq 0,$$

or equivalently $\frac{\hat{k}'(\eta)}{W(\hat{k})} (\mathbb{E}(R(\theta)) - \mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\}) \geq 1$. The result then follows from substituting and re-arranging the terms.