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Abstract

Countries' concerns about the value of their currency have been studied and documented extensively in the literature. Capital controls can be—and often are—used as a tool to manage exchange rate fluctuations. This paper investigates whether countries can benefit from using such a tool. We develop a welfare-based analysis of whether (or, in fact, how) countries should tax international borrowing. Our results suggest that restricting international capital flows through the use of these taxes can be beneficial for individual countries, although it would limit cross-border pooling of risk. The reason is because, while consumption risk-pooling is important, individual countries also care about domestic output fluctuations. Moreover, the results show that countries decide to restrict the international flow of capital exactly when this flow is crucial to ensure cross-border risk sharing. Our findings point to the possibility of costly "capital control wars" and thus to significant gains from international policy coordination.

Key words: capital controls, welfare, international asset markets

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1 Introduction

Countries' concerns with the value of their currency have been extensively studied and documented in the literature. As detailed in Fry et al. (2000), the majority of central banks around the world actually include the exchange rate as one of their main policy objectives – and the rationale for this has been the topic of a large literature on monetary policy in open economies (Corsetti et al. (2010) and references there in). But apart from traditional monetary policy, capital controls can be (and often are) used as a tool to manage exchange rate fluctuations (see survey by Edwards (1999) or, more recently, Schmitt-Grohe and Uribe (2012)). The aim of this paper is to shed light on whether countries can in fact benefit from using such tool.

We present a welfare based analysis of whether (or, in fact, how) countries may wish to intervene in the international flow of capital. To do so we lay out a simple two-country model with incomplete financial markets. In the proposed model, controlling capital flows may be beneficial for two reasons.

Imperfect risk-sharing across countries introduces a natural role for intervention in the international flow of capital. While movements in international prices can automatically ensure cross-border risk-sharing in special circumstances (Cole and Obstfeld (1991)), this is not generally the case. As shown in Corsetti et al. (2008), when domestic demand is not sensitive to changes in international relative prices (or the trade elasticity is low), movements in these prices are large and can create strong wealth effects that damage risk pooling among countries. Countries may also suffer from insufficient risk-sharing when shocks are persistent and the trade elasticity is large.

But apart from cross-border consumption risk-sharing, individual countries are also concerned with fluctuations in their own output. The incentive of such countries to strategically manage their terms of trade and manipulate domestic demand has been extensively studied in the monetary literature (e.g. Corsetti and Pesenti (2001), Tille (2001), Benigno and Benigno (2003), Sutherland (2006), Benigno (2009), De Paoli (2009a), Corsetti et al. (2012)). While in some cases countries might benefit from having an appreciated terms of trade as to improve domestic purchasing power vis-a-vis the rest of the world, in others a terms of trade depreciation may be a valuable way to promote domestic goods and enable domestic households to enjoy higher income and consumption.[1] Arguably, the latter case could be consistent, for example, with the Brazilian strategy of the 60's (see Edwards (2007)), when there was an explicit policy of "import substitution" via exchange rate management and extensive use of capital controls.

In our paper, it is the tug-of-war between these policy incentives that determines countries' desire to intervene in international capital flows. Our results suggest that restricting international capital movements is generally optimal from the individual country point of view, although it critically limits cross-border pooling of risk.

The following is an illustration of the results. After a fall in productivity, a subsidy to international borrowing can help domestic households share the burden of the shock with foreign households. This is particularly the case when domestic demand is too sensitive to changes in relative prices and the borrowing subsidy can enhance the, otherwise small, appreciation in domestic terms of trade. But individual countries actually find it optimal to tax, rather than subsidize, borrowing. With the goal of limiting fluctuations in domestic output and terms of trade, the country imposes restrictions on capital inflows that augment, rather than mitigate, the adverse effect of the shock on consumption.

Overall, our findings suggest that if capital controls are set in an uncoordinated fashion, they can have damaging implications for global risk-sharing and welfare. Ultimately, when countries simultaneously and independently engage in such interventions in the international flow of capital, not only global but individual welfare is adversely affected. Our results thus point to costly "capital control wars" and important gains from international coordination in the use of capital controls.

Other related literature:

Our work is directly related to the aforementioned papers assessing risk sharing in open economy models, as well as the literature evaluating the role of exchange rate stabilization in such models. Among the papers in this literature, our work is particularly linked to the

[1] This a result of what the literature calls "terms of trade externality".

ones evaluating the gains from monetary policy cooperation (see, for example, Benigno and Benigno (2006) and Rabitsch (2012)).

The analysis in the paper is also related to that of Costinot et al. (2011), who study the role of capital controls in a two-country endowment model with growth. Although in their framework capital controls can be used to manipulate intertemporal prices, the lack of labor supply decisions removes the policy incentives driven by the terms of trade externality described above. Another important strand of the normative literature on capital control include the recent contributions by Benigno et al. (2010), Korinek (2011), Bianchi (2011) and Bianchi and Mendoza (2010). Differently from our work, these studies evaluate the role of capital control as a prudential tool – or a tool to reduce the probability of financial crisis.

Our paper is structured as follows. Section 2 describes the model. Section 3 analyzes welfare under incomplete and complete markets – shedding light on the policy incentives of national and global policymakers. Section 4 presents the optimal coordinated and uncoordinated capital control policies and evaluates the gains from international cooperation. Section 5 concludes.

2 The Model

The framework consists of a two-country dynamic general equilibrium model featuring incomplete markets. The baseline model is a version of Benigno (2009) that abstracts from nominal rigidities and allows for home bias in consumption.[2] As shown in Corsetti et al. (2008) (or CDL, hereafter), introducing consumption home-bias and a non-unitary trade elasticity in incomplete markets models enables us to generate insufficient risk-sharing (and thus better match the empirical regularities documented in Backus and Smith (1993) and Kollmann (1995)). Finally, in order to consider a tool with which countries can control the international flow of capital, we assume that policymakers set taxes/subsidies on international borrowing/lending.

[2] Overall, our framework follows closely that of De Paoli (2009b), who focuses on the small open economy limit of the model presented in this paper.

2.1 Preferences

We consider two countries, H (Home) and F (Foreign). The world economy is populated with a continuum of agents of unit mass, where the population in the segment $[0, n)$ belongs to country H and the population in the segment $(n, 1]$ belongs to country F. The utility function of a consumer in country H is given by:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s) - V(N_s) \right], \tag{1}$$

where

$$U(C_s) = \frac{C_t^{1-\rho}}{1-\rho}, \; V(n_s^j) = \frac{(N_s)^{1+\eta}}{1+\eta}, \tag{2}$$

ρ is the coefficient of risk aversion and η is the inverse of the elasticity of labor supply.

Households obtain utility from consumption $U(C^j)$ and supply labor N^j, attaining disutility $V(N_s)$. The consumption index C is a C.E.S. (Constant Elasticity of Substitution) aggregate of home and foreign goods, defined by:

$$C = \left[v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

where $\theta > 0$ is the intratemporal elasticity of substitution between home and foreign-produced goods, C_H and C_F. As in Sutherland (2005), the parameter determining home consumers' preferences for foreign goods, $(1-v)$, is a function of the relative size of the foreign economy, $(1-n)$, and of the degree of openness, λ; more specifically, $(1-v) = (1-n)\lambda$.

Similar preferences are specified for the Foreign economy:

$$C^* = \left[v^{*\frac{1}{\theta}} C_H^{*\frac{\theta-1}{\theta}} + (1-v^*)^{\frac{1}{\theta}} C_F^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{4}$$

with $v^* = n\lambda$. That is, foreign consumers' preferences for home goods depend on the relative size of the home economy and the degree of openness. Note that the specification of v and v^* generates home bias in consumption. This bias only disappears when $\lambda = 1$.

The consumption-based price indices that correspond to the above specifications of preferences are given by:

$$P = \left[v P_H^{1-\theta} + (1-v)\left(P_F\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{5}$$

and

$$P^* = \left[v^* P_H^{*1-\theta} + (1-v^*)\left(P_F^*\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{6}$$

As Equations (5) and (6) illustrate, the home bias specification leads to deviations from purchasing power parity. We, thus, define the real exchange rate, Q_t, as the price of Foreign consumption basket in terms of Home consumption basket. We also define Home terms of trade, ToT_t, as the relative price of imports from the Foreign economy in terms of the price of Home goods.

Consumers labor supply decision implies:

$$w_t = \frac{V_y\left(N_t\right)}{U_c(C_t)} \tag{7}$$

$$= N_t^{\eta} C, \tag{8}$$

where w_t is the real wage.

2.2 Firms

Given the consumers' preferences described above, the demand for domestic and foreign good can be written as:

$$Y_t^H = \left[\frac{P_{H,t}}{P_t}\right]^{-\theta} \left[nvC_t + (1-n)v^* \left(\frac{1}{Q_t}\right)^{-\theta} C_t^* \right], \tag{9}$$

$$Y_t^F = \left[\frac{P_{F,t}}{P_t}\right]^{-\theta} \left[n\left(1-v\right)C_t + (1-n)(1-v^*) \left(\frac{1}{Q_t}\right)^{-\theta} C_t^* \right]. \tag{10}$$

We assume that there is a continuum of n identical firms in the Home economy and $1-n$ in the Foreign economy. Each individual firm produces an equal share of total output in each country. We can then derive the demand for an individual good produced in country H, and the demand for a good produced in country F:

$$Y_t = \left[\frac{P_{H,t}}{P_t}\right]^{-\theta} \left[vC_t + \frac{v^*(1-n)}{n}\left(\frac{1}{Q_t}\right)^{-\theta} C_t^*\right], \tag{11}$$

$$Y_t^* = \left[\frac{P_{F,t}}{P_t}\right]^{-\theta} \left[\frac{(1-v)\,n}{1-n}C_t + (1-v^*)\left(\frac{1}{Q_t}\right)^{-\theta} C_t^*\right]. \tag{12}$$

In the case of no-home bias (where $\lambda = 1$, as in Benigno (2009)), these reduce to:

$$Y_t = \left[\frac{P_{H,t}}{P_t}\right]^{-\theta} \left[nC_t + (1-n)C_t^*\right], \tag{13}$$

$$Y_t^* = \left[\frac{P_{F,t}}{P_t}\right]^{-\theta} \left[nC_t + (1-n)C_t^*\right]. \tag{14}$$

Firms' production function is given by:

$$Y_t = \xi_t^{\frac{\eta}{\eta+1}} N_t,[3] \tag{15}$$

where productivity shocks are denoted by ξ. Labor demand in the Home economy is, thus, given by:

$$\frac{P_{H,t}}{P_t} = \xi_t^{-\frac{\eta}{\eta+1}} w_t. \tag{16}$$

Equating labor supply (8) and labor demand (16), we obtain the following labor-leisure relationship:

$$\frac{P_{H,t}}{P_t} C_t^{-\rho} = \xi_t^{-\eta} Y_t^{\eta}. \tag{17}$$

An analogous condition holds for the Foreign economy.

[3] The production function has the power $\frac{\eta}{\eta+1}$ on productivity ξ in order to be consistent with a Yeoman-farmer version of the model in Benigno (2009).

2.3 Asset Markets

We assume that households of both countries trade a real riskless bond paid in units of the Foreign consumption basket.[4] Moreover, we assume that households at Home face quadratic adjustment costs when changing their real asset position. Those are paid to Foreign households in the form of transfers. As in Benigno (2009), the introduction of portfolio adjustment costs enables us to pin down the steady state value of the foreign asset position. Finally, we assume that Home (Foreign) policymakers can impose taxes on international borrowing and that they are rebated back to Home (Foreign) households also in the form of transfers.

We can therefore write the household's budget constraint at Home as follows:

$$C_t + B_{F,t} \leq B_{F,t-1}\frac{Q_t}{Q_{t-1}}R^*_{t-1}(1+\tau_{t-1}) + \frac{P_{H,t}}{P_t}Y_t + \frac{P_{H,t}}{P_t}Tr_t - \frac{\delta}{2}B^2_{F,t}, \tag{18}$$

where $B_{F,t}$ denotes real bonds denominated in terms of the foreign consumption basket, R^*_t is a real rate of return on bond holdings, Tr_t are transfers made in the form of domestic goods and δ is a nonnegative parameter that measures the adjustment cost in terms of units of the consumption index. The variable τ_t is a tax on international bond holdings. Below we illustrate the role of this instrument:

- $B_{F,t} > 0$ and $\tau_t > 0$: Policy implies a subsidy on international lending or a subsidy on capital outflows,
- $B_{F,t} > 0$ and $\tau_t < 0$: Policy implies a tax on international lending or a tax on capital outflows,
- $B_{F,t} < 0$ and $\tau_t > 0$: Policy implies a tax on international borrowing or a tax on capital inflows,
- $B_{F,t} < 0$ and $\tau_t < 0$: Policy implies a subsidy on international borrowing or a subsidy on capital inflows.

[4] The present framework does not include a portfolio problem for households. For recent contributions on optimal international portfolios in incomplete markets settings, see, for example, Devereux and Sutherland (2011) and Evans and Hnatkovska (2005).

Similarly to Equation (18), the budget constraint of Foreign households can be written as follows:

$$C_t^* + B_{F,t}^* \leq B_{F,t-1}^* R_{t-1}^* (1+\tau_{t-1}^*) + \frac{P_{F,t}^*}{P_t^*} Y_t^* + \frac{P_{F,t}^*}{P_t^*} Tr_t^*. \tag{19}$$

where market clearing implies that $B_{F,t}^* = -B_{F,t}$.

Given the above specification, we can write the consumer's optimal intertemporal choice as:

$$U_C(C_t)(1+\delta B_{F,t}) = R_t^*(1+\tau_t)\beta E_t \left[U_C(C_{t+1}) \frac{Q_{t+1}}{Q_t} \right], \tag{20}$$

$$U_C(C_t^*) = R_t^*(1+\tau_t^*)\beta E_t \left[U_C\left(C_{t+1}^*\right) \right]. \tag{21}$$

3 Welfare

In this section we illustrate how different features of the model affect global and national welfare. Such analysis allows us to understand the incentives driving the policy decisions discussed in subsequent sections. Our conditional welfare measure is obtained using second-order perturbation methods – as described in Schmitt-Grohe and Uribe (2007) and Nam (2011).[5] National welfare is defined as the lifetime utility of each country (i.e. Home national welfare is given by Equation (1)). Global welfare is defined as the weighted average of these utilities, where the weights are given by country sizes. That is,

$$U_t^W = nU_t + (1-n)U_t^*,$$

and, thus, every household in the world receives the same weight when computing global welfare.[6]

We first illustrate the welfare implications of incomplete markets and the resulting inability of agents to fully share risk across-countries. As discussed in Cole and Obstfeld

[5] All our numerical simulations use perturbation methods. We use a second-order approximation procedure to obtain theoretical moments. For impulse responses we use a first-order approximation of the model.

[6] Clearly, there are a variety of ways of specifying welfare weights and analyzing policy (e.g. Negishi (1972)). Arguably, one can see the exercise in our paper as considering the implications of using alternative weights. That is, when we compare optimal national versus optimal global policy we are comparing a policy of equal weights with a policy in which only one utility is weighted.

(1991), under certain conditions, movements in international relative prices can automatically ensure such cross-border risk-sharing regardless of countries' ability to trade financial assets. Other early works in the literature (e.g. Baxter and Crucini (1995)) have shown that the level of risk sharing in incomplete market models can be quite large. But, as documented in the work by CDL, lack of risk sharing may be a significant feature of incomplete markets' models, even when agents are allowed to trade bonds. The authors show that both the degree of substitutability between domestic and foreign goods, as well as the degree of home bias, are important determinants of risk sharing in such models.

When asset markets are complete (i.e. when agents can trade, without any portfolio adjustment cost, a full set of contingent claims), adjusting for the real exchange rate, intertemporal marginal rates of substitution are equalized across borders. As a result, one can measure the lack of risk sharing based on the cross-country difference in such real exchange rate-adjusted marginal rates of substitutions. So, we define the "risk-sharing gap" as

$$\frac{U_C\left(C_{t+1}\right)}{U_C\left(C_t\right)}\frac{Q_{t+1}}{Q_t}-\frac{U_C\left(C_{t+1}^*\right)}{U_C\left(C_t^*\right)}. \tag{22}$$

Figure 1 presents the standard deviation of this gap (named "Risk-sharing inefficiency" as in Viani (2011)) for different values of the trade elasticity, θ, and for different degrees of home bias, λ. The calibration used to produce Figure 1 is shown in Table 1[7] and the exercise assumes no active tax policy (i.e. $\tau_t = \tau_t^* = 0$). Consistent with the results in CDL, the figure shows that such inefficiency is high as the trade elasticity deviates from unity. Intermediate levels of home bias also tend to deliver lower international risk-sharing.

[7] In Table 1 we denote $\log(\xi)$ as ε.

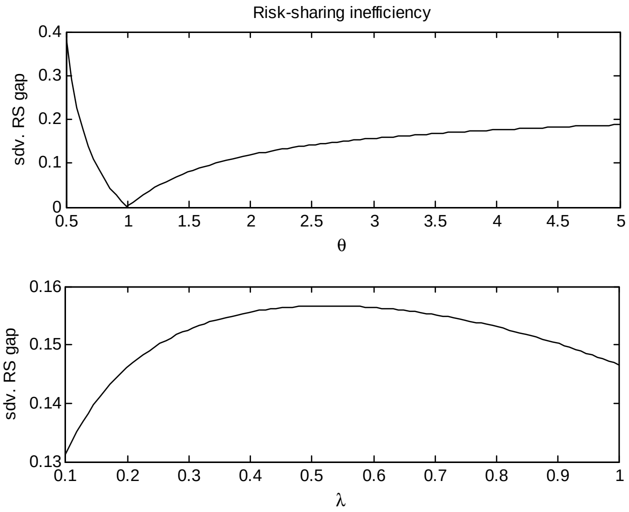

Figure 1: Standard deviation of risk-sharing gap (%), for different values of the trade elasticity, θ, and for different degrees of home bias, λ.

Parameter	Value	Notes:
β	0.99	Specifying a quarterly model with 4% steady-state real interest rate
η	0.47	Following Rotemberg and Woodford (1997)
ρ	1	Log utility
λ	$0.5; [0.1, 1]$	Benchmark 0.5, but other values considered
n	$0.5; [0.1, 0.9]$	Symmetric country sizes, but other values considered
θ	$3; [0.5, 3]$	Benchmark follows Obstfeld and Rogoff (1995), while range allows for complements and substitutes goods
δ	0.01	Following Benigno (2009)
$sdv(\varepsilon), sdv(\varepsilon^*)$	0.71%	Following Kehoe and Perri (2002)
$\kappa^{(\varepsilon)}$, $\kappa^{(\varepsilon^*)}$	0.95	Following Kehoe and Perri (2002)

Table 1: Parameter values used in the quantitative analysis

Figure 2 presents another metric of the size of the inefficiencies created by incomplete markets by showing the level of global welfare (measured as a percentage of steady state consumption) for our benchmark model and for a version of the model in which asset markets are complete. Although the level of risk sharing inefficiency shown in Figure 1 is still below the levels seen in the data (see Viani (2011)), welfare difference between complete and incomplete markets is not negligible – especially when considering that

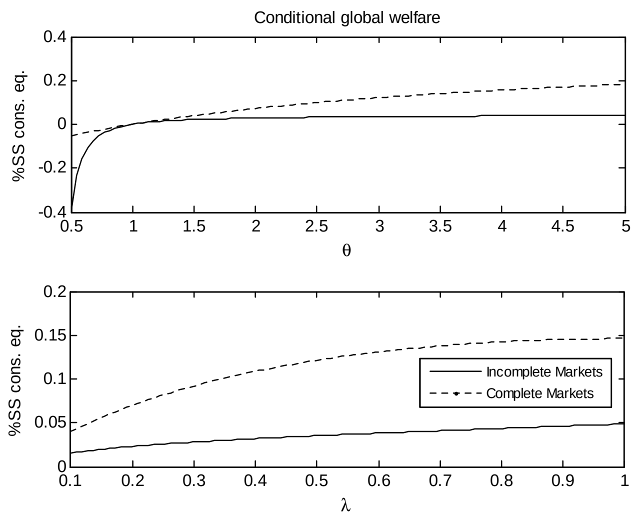

Figure 2: Conditional global welfare (measured as a percentage of steady state consumption) for different values of the trade elasticity, θ and for different degrees of home bias, λ.

welfare costs of economic fluctuations in consumptions based models of our kind tend to be small (Lucas (1987)). Note that the Cole and Obstfeld (1991) result of asset market irrelevance for welfare is replicated when $\theta = 1$.

In our model with endogenous labor supply, agents are not only concerned with cross-border consumption risk-sharing but also with fluctuations in their own output. As discussed in the introduction and documented in the monetary literature, open economies are affected by a terms of trade externality. That is, individual countries have an incentive to strategically manage the terms of trade in order to manipulate domestic demand towards or away-from foreign goods. In fact, as a result of this externality, welfare of *individual countries* may be larger under imperfect risk sharing (Figure 3 and 4).

When goods are substitutes in the utility (Figure 3), bigger domestic purchasing power under incomplete markets can allow agents to produce less without a proportional fall in consumption – as consumers switch to foreign goods. When the share of imports in consumers' baskets is sufficiently large, such equilibrium delivers higher welfare (for the Home country, this is the case when $1 - n$ is large, and for the Foreign economy this is the case when n is large). Figure 4 shows the case of complement goods. In this case, terms of trade depreciation may be associated with higher domestic welfare – as switching towards domestic goods can produce higher levels of consumption without an equivalent

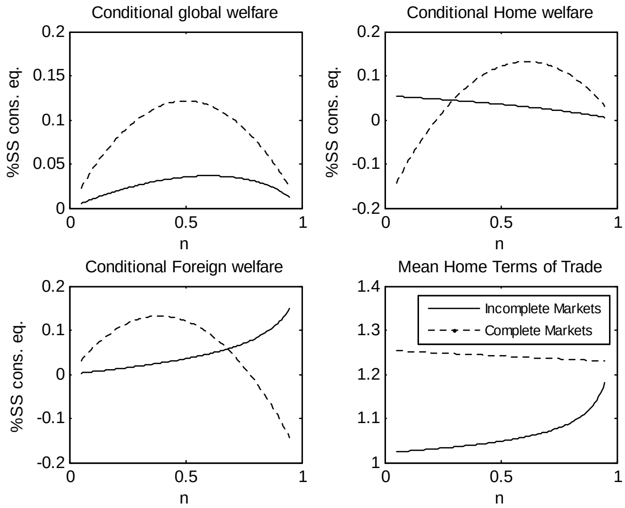

Figure 3: Conditional global and national welfare (measured in percentage deviations from steady state consumption) and unconditonal mean of Home terms of trade for different values of n and $\theta = 3$: complete versus incomplete markets.

increase in labor effort.

Kim and Kim (2003) highlight how spurious welfare reversals – i.e. false results showing that models with incomplete markets can deliver higher welfare than models with complete markets – may be a result of inaccurate welfare calculations. In our methodology such inaccuracy is not present given that welfare is properly obtained from a full second order approximation of the model (as in Schmitt-Grohe and Uribe (2007)). Nevertheless, genuine welfare reversals may occur due to the presence of endogenous labor supply and the associated terms of trade externality (note that such externality is absent in endowment models such as the one presented in Costinot et al. (2011)). Figure 5 exemplifies, in an asymmetric calibration ($n = 0.1$) of our model with elastic domestic demand ($\theta = 3$), how welfare reversals are present so long as labor is sufficiently elastic. In particular, in Figure 5, welfare reversals arise when, η, the inverse of the elasticity of labor supply is below 6 (a number well above our benchmark calibration of 0.47).

4 Optimal taxes under incomplete markets

We now analyze how policymakers would choose to tax international capital flows in light of the policy incentives described above. We consider different policy settings. First, we

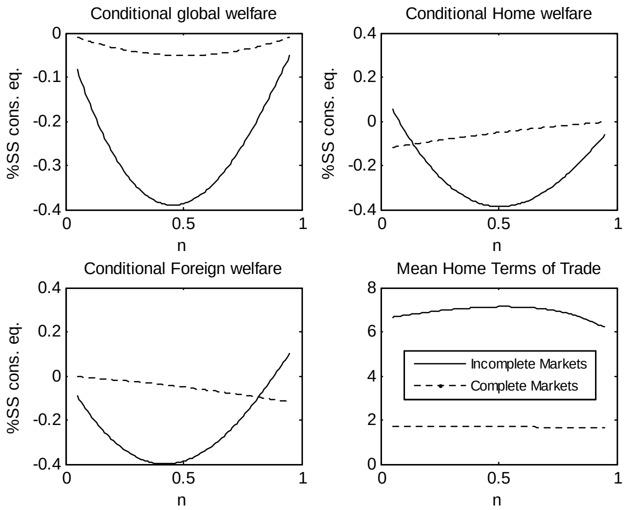

Figure 4: Conditional global and national welfare (measured in percentage deviations from steady state consumption) and unconditonal mean of Home terms of trade for different values of n and $\theta = 0.5$: complete versus incomplete markets.

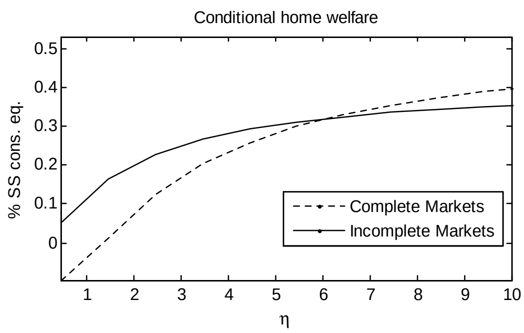

Figure 5: Conditional home welfare, measured as a percentage of steady state consumption, for different values of the inverse of the elasticity of labour supply, η: complete versus incomplete markets in an asymmetric calibration ($n = 0.1$) of the model with elastic domestic demand ($\theta = 3$).

assume that the Home policymaker chooses taxes as to minimize domestic social losses, while the Foreign country does not have access to a tool to control capital flows. We then analyze the case in which taxes are determined by a global social planner who minimizes global social losses. Finally, we consider the case in which both countries decide how to set taxes on international bonds, arriving at a Nash equilibrium.

4.1 National optimal policy

In this section, we assume that only the Home policymaker has an active policy instrument. That is, while the Foreign policymaker keeps taxes constant ($\tau_t^* = 0$), the domestic policymaker decides on the evolutions of taxes, τ_t, that maximizes domestic welfare. The Ramsey policy problem and first order conditions are shown in Section 6.1 of the Appendix.

First, we analyze economic dynamics following a negative Home productivity shock under the assumption that $\theta = 3$, i.e. home and foreign goods are imperfect substitutes. As we can see in Figure 6, in response to the shock, home output and home consumption decrease while the terms of trade appreciate. Domestic households, in order to smooth consumption, would like to borrow from foreign agents. However, the domestic social planner increases taxes on international borrowing. Higher taxes effectively increase an interest rate paid on foreign bond holdings and discourage domestic households from borrowing. The result is an even stronger fall in consumption and a larger deviation from complete risk sharing. The policymaker's action reduces fluctuations in domestic labor supply – or lowers output volatility – at the expense of financial integration among countries.

Figure 7 considers the case in which domestic and foreign goods are complements. Under this specification, the strong appreciation in the terms of trade actually introduces a large positive wealth effect at home (as described in CDL), which implies that domestic agents become net lenders to foreign households. The optimal policy implies a tax on capital outflow (by reducing the effective returns to domestic lenders) that, again, limits international risk-sharing. The policy, however, allows for a smaller drop in consumption

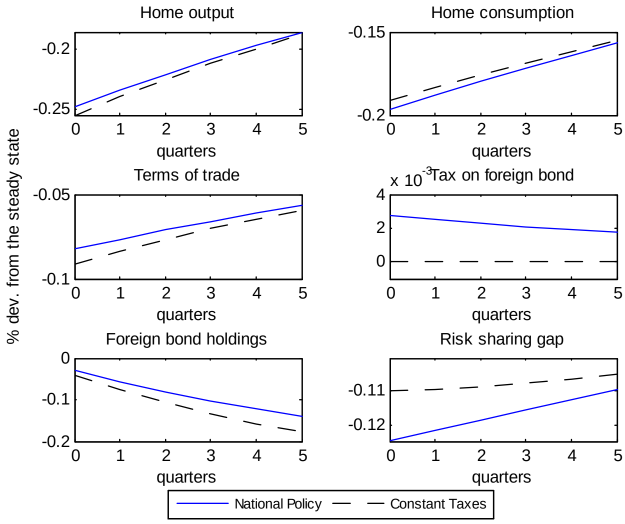

Figure 6: Optimal national policy following a negative Home productivity shock with $\theta = 3$: comparison with the case in which there is no active tax policy.

without a significant change in domestic labor supply.

4.2 Global optimal policy

We now consider the case in which a global policymaker sets the same instrument, τ_t, in order to maximize global welfare. Details of the optimal policy problem and first order conditions can be found in Section 6.2 in the Appendix.

As shown in Figure 8, the optimal policy that maximizes global welfare has opposing tax prescriptions when compared to the policy designed to maximize national policy. After a negative shock to home productivity, when domestic and foreign goods are substitutes in the utility, the global social planner lowers taxes in order to promote international borrowing, increase capital flows and enhance cross border risk-sharing. In fact, global policy eliminates any fluctuations in our measure of the risk-sharing gap after the initial period. In particular, under the global policy, taxes actually rise permanently as to minimize distortions in agents intertemporal decisions, while the risk-sharing gap follows a random walk as to minimize inefficiencies created by market incompleteness.

But, as Figure 9 shows, while global policy increases global welfare and improves cross-border risk-sharing, it may reduce welfare of the Home economy.[8] As the effect of

[8]Note that for the global policy, the standard deviation of the risk sharing gap was calculated using

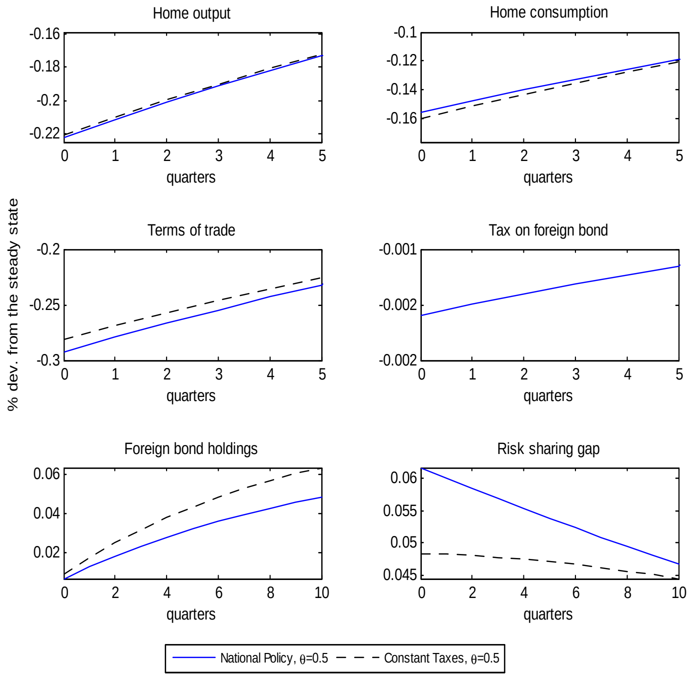

Figure 7: Optimal national policy following a negative Home productivity shock with $\theta = 0.5$: comparison with the case in which there is no active tax policy.

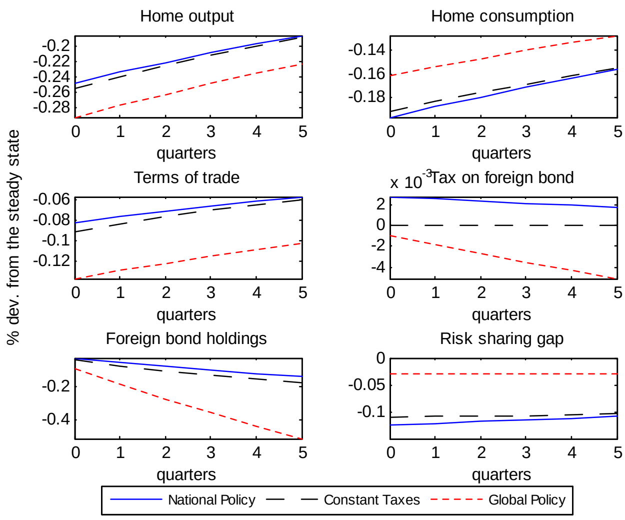

Figure 8: Optimal global and national policy following a negative Home productivity shock with $\theta = 3$: comparison with the case in which there is no active tax policy.

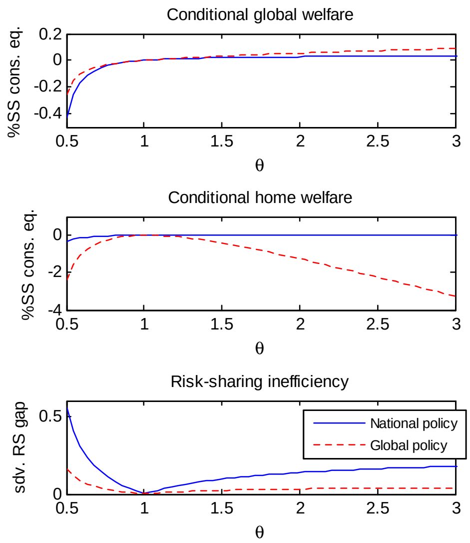

Figure 9: Risk-sharing, national and global welfare under the national policy and global policy for different values of θ.

changes in the terms of trade on the composition of demand increases (or as θ moves away from unity), raising the strength of the terms of trade externality, Home welfare losses under the global optimal policy also increase.

4.3 Nash equilibrium

Finally, we consider a Nash equilibrium in which the Home policymaker chooses the optimal path for domestic borrowing taxes, τ_t, while the Foreign policymaker controls the evolution of τ_t^*.[9] Again, the details of such policy problem and set of first order conditions can be found in Section 6.3 of the Appendix.

Figure 10 compares the Nash equilibrium with the case in which a global central planner sets taxes optimally and the case of constant taxes.[10] Following a negative pro-

simulated moments – given its non-stationary property.

[9] In particular we consider an open-loop Nash equilibrium (see, for example, Coenen et al. (2008)) where each policy authority choses the optimal allocation taking as given the evolution of the other authority's policy instrument.

[10] Note that when illustrating the global optimal policy, we assume that there is only one policy instrument available to the global social planner. Adding τ_t^* as an additional would be of little help since it would affect the same margin – namely the cross-border risk-sharing condition (or a combination of Equations 20 and 21) – and, with only one instrument, optimal global policy already implies zero volatility in the variable measuring deviations from full risk sharing after the initial period (see Figures 10).

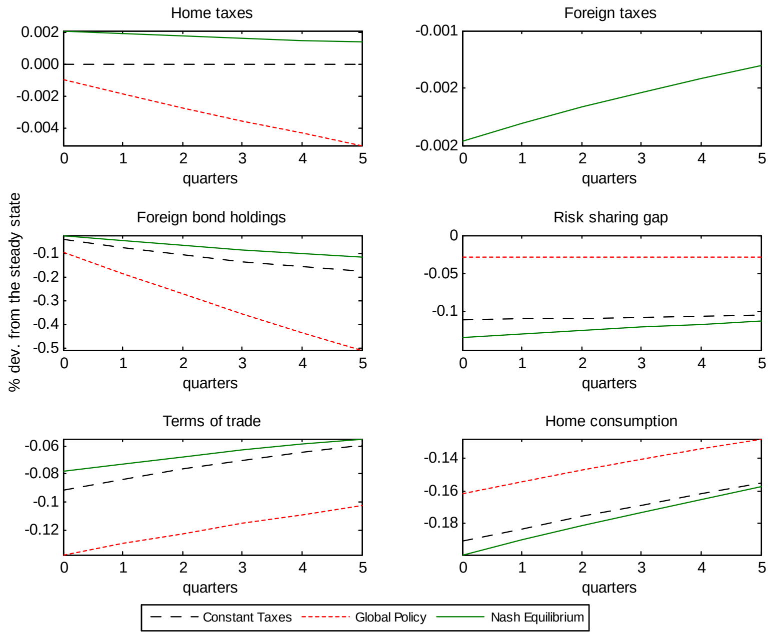

Figure 10: Optimal global and national policy following a negative Home productivity shock with $\theta = 3$: comparison with the Nash equilibrium.

ductivity shock, capital flows from Foreign to Home ($B_F < 0$). But instead of subsidizing such flow, Home taxes the capital inflow (as it reduces the domestic incentive to borrow). At the same time, the negative taxes in the Foreign country decrease returns to lenders, working as a tax on capital outflows from the Foreign country. Both policies, at home and abroad, contribute to reducing the flow of capital between countries. Home terms of trade are weaker under the Nash equilibrium in a period of low domestic productivity – consistent with lower cross-border risk-sharing.

Figure 9 already showed that the incentives of the Home economy to deviate from the socially optimal policy (i.e. the difference between Home welfare under the national policy and under the global policy) are the largest exactly when the losses from unilateral decision making (i.e. the difference between global welfare under the national policy and under the global policy) are the biggest. At the same time, chart 1 of Figure 11 shows that if countries simultaneously and independently engage in such interventions in the international flow of capital, individual as well as global welfare would be adversely affected – as illustrated by the fact that Home welfare is smaller in the Nash equilibrium when compared with the constant tax policy. This chart illustrates the costs for the

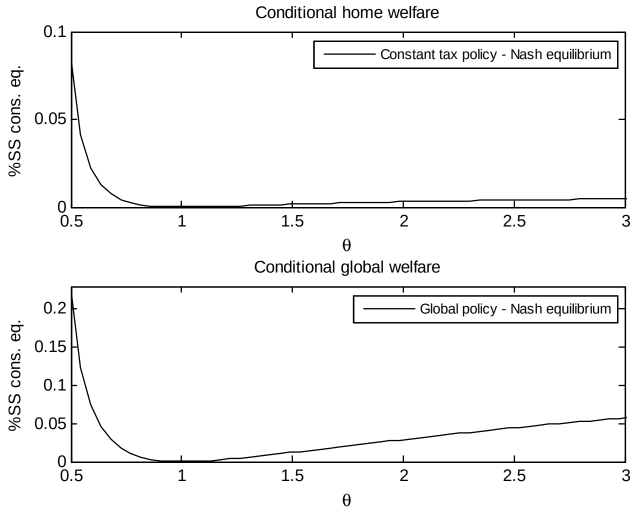

Figure 11: Difference in conditional Home welfare delivered under constant taxes and the Nash equilibrium for different values of θ.

Home economy of what one could call a "capital control war".

Finally, chart 2 of Figure 11 highlights explicitly that there is an important role for international coordination in how capital controls are set in different countries. The gains from international coordination, measured as the difference in conditional global welfare delivered under the global optimal policy and the Nash equilibrium may be higher than 0.2%. This is because the incentives of individual countries and the global policy makers are completely orthogonal when it comes to interventions in capital flows.

5 Concluding remarks

In this paper, we analyze the effect of capital controls on domestic and world welfare. We show that countries incentive to limit cross-border flow of capital damages international risk sharing. Such uncoordinated use of capital controls is beggar-thy-neighbor and, thus, there is a clear role for international coordination.

Our proposed model is stylized. This allows us to keep the welfare and policy analysis parsimonious and transparent. Nevertheless, to quantify the real gains from international coordination, a richer model may be required. Early works in the literature have shown that the level of risk sharing in incomplete market models (where agents can trade bonds) can be quite large. As shown in CDL, frameworks like ours may need to feature near-permanent shocks and possibly a distribution sector (that introduces significant deviations from the law of one price) in order to generate an insufficient level of risk-sharing that matches the data. A fruitful avenue for this research may be to enrich the model in these directions and re-evaluate the quantitative effects of capital controls and the associated benefits from international cooperation.

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6 Appendix: Optimal policy problem

6.1 Derivation of first order conditions: National optimal policy, two-country model

Period utility function

$$W = \ln C_t - A_t^{-\eta} \frac{Y_t^{\eta+1}}{\eta+1} \tag{23}$$

Structural equations:

1. Home demand equation

$$Y_t p_{H,t}^{\theta} = \nu C_t + \frac{(1-n)\nu^*}{n} C_t^* Q_t^{\theta} \tag{24}$$

   where $p_{H,t} \equiv P_{H,t}/P_t$

2. Foreign demand equation

$$Y_t^* p_{F,t}^{\theta} = \frac{n(1-\nu)}{1-n} C_t + (1-\nu^*) C_t^* Q_t^{\theta} \tag{25}$$

   where $p_{F,t} \equiv P_{F,t}/P_t$

3. Home labor supply

$$p_{H,t}C_t^{-\rho} = \left(\frac{Y_t}{A_t}\right)^{\eta} \tag{26}$$

4. Foreign labor supply

$$\frac{p_{F,t}}{Q_t}C_t^{*-\rho} = \left(\frac{Y_t^*}{A_t^*}\right)^{\eta} \tag{27}$$

5. Relative prices (1)

$$p_{H,t}^{\theta-1} = \nu + (1-\nu)\left(\frac{p_{F,t}}{p_{H,t}}\right)^{1-\theta} \tag{28}$$

6. Relative prices (2)

$$\left(\frac{p_{H,t}}{Q_t}\right)^{\theta-1} = \nu^* + (1-\nu^*)\left(\frac{p_{F,t}}{p_{H,t}}\right)^{1-\theta} \tag{29}$$

7. Euler equation (1)

$$R_t^* = \frac{1}{\beta}E_t\left(\frac{C_{t+1}^{*}{}^{\rho}}{C_t^{*\rho}}\right) \tag{30}$$

8. Euler equation (2)

$$R_t^*(1+\tau_t) = \frac{1}{\beta}E_t\left(\frac{C_{t+1}{}^{\rho}Q_t}{C_t{}^{\rho}Q_{t+1}}\right)(1+\delta_f Bf_{h,t}) \tag{31}$$

9. Budget constraint

$$p_{H,t}Y_t + Bf_{h,t-1}R_{t-1}^*\frac{Q_t}{Q_{t-1}} = C_t + Bf_{h,t} + \frac{1}{2}\delta_f Bf_{h,t}^2 \tag{32}$$

First order conditions:

- wrt Y_t

$$-A_t^{-\eta}Y_t^{\eta} + p_{H,t}^{\theta}\gamma_{1,t} - \eta\gamma_{3,t}A_t^{-\eta}Y_t^{\eta-1} + \gamma_{9,t}p_{H,t} = 0$$

- wrt Y_t^*

$$p_{F,t}^{\theta}\gamma_{2,t} - \eta\gamma_{4,t}A_t^{*-\eta}Y_t^{*\eta-1} = 0$$

- wrt $p_{H,t}$

$$\theta p_{H,t}^{\theta-1} Y_t \gamma_{1,t} + C_t^{-\rho}\gamma_{3,t} + \gamma_{5,t}(\theta-1)p_{H,t}^{\theta-2} - (1-\nu)(\theta-1)p_{F,t}^{1-\theta}p_{H,t}^{\theta-2}\gamma_{5,t}$$
$$+ (\theta-1)p_{H,t}^{\theta-2}Q_t^{1-\theta}\gamma_{6,t} - (1-\nu^*)(\theta-1)p_{F,t}^{1-\theta}p_{H,t}^{\theta-2}\gamma_{6,t} + \gamma_9 Y_t$$
$$= 0$$

- wrt $p_{F,t}$

$$\gamma_{2,t}Y_t^*\theta p_{F,t}^{\theta-1} + \frac{C_t^{*-\rho}}{Q_t}\gamma_{4,t} - (1-\nu)p_{H,t}^{\theta-1}(1-\theta)p_{F,t}^{-\theta}\gamma_{5,t} - (1-\nu^*)p_{H,t}^{\theta-1}(1-\theta)p_{F,t}^{-\theta}\gamma_{6,t} = 0$$

- wrt Q_t

$$- \theta\frac{(1-n)\nu^*}{n}C_t^*Q_t^{\theta-1}\gamma_{1,t}$$
$$- \theta(1-\nu^*)C_t^*Q_t^{\theta-1}\gamma_{2,t} - p_{F,t}C_t^{*-\rho}Q_t^{-2}\gamma_{4,t}$$
$$+ \gamma_{6,t}p_{H,t}^{\theta-1}(1-\theta)Q_t^{-\theta}$$
$$- E_t\left(\frac{1}{\beta}\frac{{C_{t+1}}^{\rho}}{{C_t}^{\rho}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t}$$
$$+ \frac{1}{\beta^2}\frac{{C_t}^{\rho}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t^2}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1} + \gamma_{9,t}Bf_{h,t-1}R_{t-1}^*\frac{1}{Q_{t-1}} - \beta E_t(\gamma_{9,t+1}Q_{t+1})Bf_{h,t}R_t^*\frac{1}{Q_t^2}$$
$$= 0$$

- wrt C_t

$$\frac{1}{C_t} - \nu\gamma_{1,t} - \frac{n(1-\nu)}{1-n}\gamma_{2,t}$$
$$- \rho\gamma_{3,t}p_{H,t}C_t^{-\rho-1}$$
$$+ \rho E_t\left(\frac{1}{\beta}\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho+1}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t}$$
$$- \rho\frac{1}{\beta^2}\frac{{C_t}^{\rho-1}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1} - \gamma_{9,t}$$
$$= 0$$

- C_t^*

$$
\begin{aligned}
&-(1-n)\frac{\nu^*}{n}Q_t^{\theta}\gamma_{1,t} - (1-\nu^*)Q_t^{\theta}\gamma_{2,t} - \rho\gamma_{4,t}\frac{p_{F,t}}{Q_t}C_t^{*-\rho-1} \\
&+\rho\frac{1}{\beta}E_t\left(C_{t+1}^*\right)^{\rho}C_t^{*-\rho-1}\gamma_{7,t} - \frac{1}{\beta^2}\rho C_t^{*\rho-1}{C_{t-1}^*}^{-\rho}\gamma_{7,t-1} \\
&= 0
\end{aligned}
$$

- wrt R_t^*

$$(1+\tau_t)\gamma_{8,t} + \gamma_{7,t} + \beta Bf_{h,t}E_t\left(\frac{Q_{t+1}}{Q_t}\gamma_{9,t+1}\right) = 0$$

- wrt $Bf_{h,t}$

$$-\gamma_{8,t}E_t\left(\frac{1}{\beta}\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho}Q_{t+1}}\right)\delta_f + \beta E_t\left(\gamma_{9,t+1}R_t^*\frac{Q_{t+1}}{Q_t}\right) - \gamma_{9,t}(1+\delta_f Bf_{h,t}) = 0$$

- wrt τ_t

$$\gamma_{8,t}R_t^* = 0$$

6.2 Derivation of first order conditions: Global optimal policy, two-country model

Period utility function

$$W_g = n\left(\ln C_t - A_t^{-\eta}\frac{Y_t^{\eta+1}}{\eta+1}\right) + (1-n)\left(\ln C_t^* - A_t^{*-\eta}\frac{Y_t^{*\eta+1}}{\eta+1}\right) \tag{33}$$

Structural equations:

1. Home demand equation

$$Y_t p_{H,t}^{\theta} = \nu C_t + \frac{(1-n)\nu^*}{n}C_t^* Q_t^{\theta} \tag{34}$$

2. Foreign demand equation

$$Y_t^* p_{F,t}^{\theta} = \frac{n(1-\nu)}{1-n} C_t + (1-\nu^*) C_t^* Q_t^{\theta} \tag{35}$$

3. Home labor supply

$$p_{H,t} C_t^{-\rho} = \left(\frac{Y_t}{A_t}\right)^{\eta} \tag{36}$$

4. Foreign labor supply

$$\frac{p_{F,t}}{Q_t} C_t^{*-\rho} = \left(\frac{Y_t^*}{A_t^*}\right)^{\eta} \tag{37}$$

5. Relative prices (1)

$$p_{H,t}^{\theta-1} = \nu + (1-\nu)\left(\frac{p_{F,t}}{p_{H,t}}\right)^{1-\theta} \tag{38}$$

6. Relative prices (2)

$$\left(\frac{p_{H,t}}{Q_t}\right)^{\theta-1} = \nu^* + (1-\nu^*)\left(\frac{p_{F,t}}{p_{H,t}}\right)^{1-\theta} \tag{39}$$

7. Euler equation (1)

$$R_t^* = \frac{1}{\beta} E_t \left(\frac{{C_{t+1}^*}^{\rho}}{C_t^{*\rho}}\right) \tag{40}$$

8. Euler equation (2)

$$R_t^*(1+\tau_t) = \frac{1}{\beta} E_t \left(\frac{{C_{t+1}}^{\rho} Q_t}{{C_t}^{\rho} Q_{t+1}}\right)(1+\delta_f Bf_{h,t}) \tag{41}$$

9. Budget constraint

$$p_{H,t} Y_t + Bf_{h,t-1} R_{t-1}^* \frac{Q_t}{Q_{t-1}} = C_t + Bf_{h,t} + \frac{1}{2}\delta_f Bf_{h,t}^2 \tag{42}$$

First order conditions (global policy):

- wrt Y_t

$$-nA_t^{-\eta} Y_t^{\eta} + p_{H,t}^{\theta}\gamma_{1,t} - \eta\gamma_{3,t} A_t^{-\eta} Y_t^{\eta-1} + \gamma_{9,t} p_{H,t} = 0$$

- wrt Y_t^*

$$-(1-n)A_t^{*-\eta}Y_t^{*\eta} + p_{F,t}^{\theta}\gamma_{2,t} - \eta\gamma_{4,t}A_t^{*-\eta}Y_t^{*\eta-1} = 0$$

- wrt $p_{H,t}$

$$\begin{aligned}
&\theta p_{H,t}^{\theta-1}Y_t\gamma_{1,t} + C_t^{-\rho}\gamma_{3,t} + \gamma_{5,t}(\theta-1)p_{H,t}^{\theta-2} - (1-\nu)(\theta-1)p_{F,t}^{1-\theta}p_{H,t}^{\theta-2}\gamma_{5,t}\\
&+(\theta-1)p_{H,t}^{\theta-2}Q_t^{1-\theta}\gamma_{6,t} - (1-\nu^*)(\theta-1)p_{F,t}^{1-\theta}p_{H,t}^{\theta-2}\gamma_{6,t} + \gamma_9 Y_t\\
&=0
\end{aligned}$$

- wrt $p_{F,t}$

$$\gamma_{2,t}Y_t^*\theta p_{F,t}^{\theta-1} + \frac{C_t^{*-\rho}}{Q_t}\gamma_{4,t} - (1-\nu)p_{H,t}^{\theta-1}(1-\theta)p_{F,t}^{-\theta}\gamma_{5,t} - (1-\nu^*)p_{H,t}^{\theta-1}(1-\theta)p_{F,t}^{-\theta}\gamma_{6,t} = 0$$

- wrt Q_t

$$\begin{aligned}
&-\theta\frac{(1-n)\nu^*}{n}C_t^*Q_t^{\theta-1}\gamma_{1,t}\\
&-\theta(1-\nu^*)C_t^*Q_t^{\theta-1}\gamma_{2,t} - p_{F,t}C_t^{*-\rho}Q_t^{-2}\gamma_{4,t} + \gamma_{6,t}p_{H,t}^{\theta-1}(1-\theta)Q_t^{-\theta}\\
&-\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}}{{C_t}^{\rho}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t}\\
&+\frac{1}{\beta^2}\frac{{C_t}^{\rho}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t^2}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1} + \gamma_{9,t}Bf_{h,t-1}R_{t-1}^*\frac{1}{Q_{t-1}} - \beta E_t\left(\gamma_{9,t+1}Bf_{h,t}R_t^*\frac{Q_{t+1}}{Q_t^2}\right)\\
&=0
\end{aligned}$$

- wrt C_t

$$\begin{aligned}
&\frac{n}{C_t} - \nu\gamma_{1,t} - \frac{n(1-\nu)}{1-n}\gamma_{2,t} - \rho\gamma_{3,t}p_{H,t}C_t^{-\rho-1}\\
&+\rho\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho+1}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t}\\
&-\rho\frac{1}{\beta^2}\frac{{C_t}^{\rho-1}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1} - \gamma_{9,t}\\
&=0
\end{aligned}$$

- C_t^*

$$\frac{1-n}{C_t^*} - (1-n)\frac{\nu^*}{n}Q_t^\theta\gamma_{1,t} - (1-\nu^*)Q_t^\theta\gamma_{2,t} - \rho\gamma_{4,t}\frac{p_{F,t}}{Q_t}C_t^{*-\rho-1}$$
$$+\rho\frac{1}{\beta}E_t\left(C_{t+1}^*\right)^\rho C_t^{*-\rho-1}\gamma_{7,t} - \frac{1}{\beta^2}\rho C_t^{*\rho-1}C_{t-1}^{*}{}^{-\rho}\gamma_{7,t-1}$$
$$=0$$

- wrt R_t^*

$$(1+\tau_t)\gamma_{8,t} + \gamma_{7,t} + \beta Bf_{h,t}E_t\left(\frac{Q_{t+1}}{Q_t}\gamma_{9,t+1}\right) = 0$$

- wrt $Bf_{h,t}$

$$-\gamma_{8,t}\frac{1}{\beta}E_t\left(\frac{C_{t+1}{}^\rho Q_t}{C_t{}^\rho Q_{t+1}}\right)\delta_f + \beta E_t\left(\gamma_{9,t+1}R_t^*\frac{Q_{t+1}}{Q_t}\right) - \gamma_{9,t}(1+\delta_f Bf_{h,t}) = 0$$

- wrt τ_t

$$\gamma_{8,t}R_t^* = 0$$

6.3 Nash equilibrium in a two-country world

Structural equations:

1. Home demand equation

$$Y_t p_{H,t}^\theta = \nu C_t + \frac{(1-n)\nu^*}{n}C_t^*Q_t^\theta \tag{43}$$

2. Foreign demand equation

$$Y_t^* p_{F,t}^\theta = \frac{n(1-\nu)}{1-n}C_t + (1-\nu^*)C_t^*Q_t^\theta \tag{44}$$

3. Home labor supply

$$p_{H,t}C_t^{-\rho} = \left(\frac{Y_t}{A_t}\right)^\eta \tag{45}$$

4. Foreign labor supply

$$\frac{p_{F,t}}{Q_t} C_t^{*-\rho} = \left(\frac{Y_t^*}{A_t^*}\right)^{\eta} \tag{46}$$

5. Relative prices (1)

$$p_{H,t}^{\theta-1} = \nu + (1-\nu)\left(\frac{p_{F,t}}{p_{H,t}}\right)^{1-\theta} \tag{47}$$

6. Relative prices (2)

$$\left(\frac{p_{H,t}}{Q_t}\right)^{\theta-1} = \nu^* + (1-\nu^*)\left(\frac{p_{F,t}}{p_{H,t}}\right)^{1-\theta} \tag{48}$$

7. Euler equation (1)

$$R_t^*(1+\tau_t^*) = E_t\left(\frac{1}{\beta}\frac{{C_{t+1}^*}^{\rho}}{C_t^{*\rho}}\right) \tag{49}$$

8. Euler equation (2)

$$R_t^*(1+\tau_t) = \frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho}Q_{t+1}}\right)(1+\delta_f Bf_{h,t}) \tag{50}$$

9. Budget constraint

$$p_{H,t}Y_t + Bf_{h,t-1}R_{t-1}^*\frac{Q_t}{Q_{t-1}} = C_t + Bf_{h,t} + \frac{1}{2}\delta_f Bf_{h,t}^2 \tag{51}$$

Home first order conditions (almost the same as national policy):

Period utility function

$$W = \ln C_t - A_t^{-\eta}\frac{Y_t^{\eta+1}}{\eta+1} \tag{52}$$

First order conditions:

- wrt Y_t

$$-A_t^{-\eta}Y_t^{\eta} + p_{H,t}^{\theta}\gamma_{1,t} - \eta\gamma_{3,t}A_t^{-\eta}Y_t^{\eta-1} + \gamma_{9,t}p_{H,t} = 0$$

- wrt Y_t^*

$$p_{F,t}^{\theta}\gamma_{2,t} - \eta\gamma_{4,t}A_t^{*-\eta}Y_t^{*\eta-1} = 0$$

- wrt $p_{H,t}$

$$\theta p_{H,t}^{\theta-1} Y_t \gamma_{1,t} + C_t^{-\rho}\gamma_{3,t} + \gamma_{5,t}(\theta-1)p$$

$$+(\theta-1)p_{H,t}^{\theta-2}Q_t^{1-\theta}\gamma_{6,t} - (1-\nu^*)(\theta-1)p_{F,t}^{1-\theta}p_{H,t}^{\theta-2}\gamma_{6,t} + \gamma_9 Y_t = 0$$

- wrt $p_{F,t}$

$$\gamma_{2,t}Y_t^*\theta p_{F,t}^{\theta-1} + \frac{C_t^{*-\rho}}{Q_t}\gamma_{4,t} - (1-\nu)p_{H,t}^{\theta-1}(1-\theta)p_{F,t}^{-\theta}\gamma_{5,t} - (1-\nu^*)p_{H,t}^{\theta-1}(1-\theta)p_{F,t}^{-\theta}\gamma_{6,t} = 0$$

- wrt Q_t

$$\begin{aligned}
&-\theta\frac{(1-n)\nu^*}{n}C_t^*Q_t^{\theta-1}\gamma_{1,t}\\
&-\theta(1-\nu^*)C_t^*Q_t^{\theta-1}\gamma_{2,t} - p_{F,t}C_t^{*-\rho}Q_t^{-2}\gamma_{4,t}\\
&+\gamma_{6,t}p_{H,t}^{\theta-1}(1-\theta)Q_t^{-\theta}\\
&-\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}}{{C_t}^{\rho}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t}\\
&+\frac{1}{\beta^2}\frac{{C_t}^{\rho}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t^2}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1} + \gamma_{9,t}Bf_{h,t-1}R_{t-1}^*\frac{1}{Q_{t-1}} - \beta E_t\left(\gamma_{9,t+1}Bf_{h,t}R_t^*\frac{Q_{t+1}}{Q_t^2}\right)\\
&=0
\end{aligned}$$

- wrt C_t

$$\begin{aligned}
&\frac{1}{C_t} - \nu\gamma_{1,t} - \frac{n(1-\nu)}{1-n}\gamma_{2,t}\\
&-\rho\gamma_{3,t}p_{H,t}C_t^{-\rho-1}\\
&+\rho\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho+1}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t}\\
&-\rho\frac{1}{\beta^2}\frac{{C_t}^{\rho-1}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1} - \gamma_{9,t}\\
&=0
\end{aligned}$$

- C_t^*

$$
\begin{aligned}
&-(1-n)\frac{\nu^*}{n}Q_t^\theta\gamma_{1,t}-(1-\nu^*)Q_t^\theta\gamma_{2,t}-\rho\gamma_{4,t}\frac{p_{F,t}}{Q_t}C_t^{*-\rho-1}\\
&+\rho\frac{1}{\beta}RE_t({C_{t+1}^*}^\rho)C_t^{*-\rho-1}\gamma_{7,t}-\frac{1}{\beta^2}\rho C_t^{*\rho-1}{C_{t-1}^*}^{-\rho}\gamma_{7,t-1}\\
&=0
\end{aligned}
$$

- wrt R_t^*

$$
(1+\tau_t)\gamma_{8,t}+(1+\tau_t^*)\gamma_{7,t}+\beta E_t\left(Bf_{h,t}\frac{Q_{t+1}}{Q_t}\gamma_{9,t+1}\right)=0
$$

- wrt $Bf_{h,t}$

$$
-\gamma_{8,t}\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^\rho Q_t}{{C_t}^\rho Q_{t+1}}\right)\delta_f+\beta E_t\left(\gamma_{9,t+1}R_t^*\frac{Q_{t+1}}{Q_t}\right)-\gamma_{9,t}(1+\delta_f Bf_{h,t})=0
$$

- wrt τ_t

$$
\gamma_{8,t}R_t^*=0
$$

First order conditions (foreign policy)

Period utility function

$$
W=\ln C_t^*-A_t^{*-\eta}\frac{Y_t^{*\eta+1}}{\eta+1} \tag{53}
$$

First order conditions:

- wrt Y_t

$$
p_{H,t}^\theta\gamma_{1,t}^*-\eta\gamma_{3,t}^*A_t^{-\eta}Y_t^{\eta-1}+\gamma_{9,t}^*p_{H,t}=0
$$

- wrt Y_t^*

$$
-A_t^{*-\eta}Y_t^{*\eta}+p_{F,t}^\theta\gamma_{2,t}^*-\eta\gamma_{4,t}^*A_t^{*-\eta}Y_t^{*\eta-1}=0
$$

- wrt $p_{H,t}$

$$\theta p_{H,t}^{\theta-1} Y_t \gamma_{1,t}^* + C_t^{-\rho}\gamma_{3,t}^* + \gamma_{5,t}^*(\theta-1)p_{H,t}^{\theta-2} - (1-\nu)(\theta-1)p_{F,t}^{1-\theta}p_{H,t}^{\theta-2}\gamma_{5,t}^*$$
$$+ (\theta-1)p_{H,t}^{\theta-2}Q_t^{1-\theta}\gamma_{6,t}^* - (1-\nu^*)(\theta-1)p_{F,t}^{1-\theta}p_{H,t}^{\theta-2}\gamma_{6,t}^* + \gamma_{9,t}^* Y_t$$
$$= 0$$

- wrt $p_{F,t}$

$$\gamma_{2,t}^* Y_t^* \theta p_{F,t}^{\theta-1} + \frac{C_t^{*-\rho}}{Q_t}\gamma_{4,t}^* - (1-\nu)p_{H,t}^{\theta-1}(1-\theta)p_{F,t}^{-\theta}\gamma_{5,t}^* - (1-\nu^*)p_{H,t}^{\theta-1}(1-\theta)p_{F,t}^{-\theta}\gamma_{6,t}^* = 0$$

- wrt Q_t

$$- \theta\frac{(1-n)\nu^*}{n}C_t^* Q_t^{\theta-1}\gamma_{1,t}^*$$
$$- \theta(1-\nu^*)C_t^* Q_t^{\theta-1}\gamma_{2,t}^* - p_{F,t}C_t^{*-\rho}Q_t^{-2}\gamma_{4,t}^*$$
$$+ \gamma_{6,t}^* p_{H,t}^{\theta-1}(1-\theta)Q_t^{-\theta}$$
$$- \frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}}{{C_t}^{\rho}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t}^*$$
$$+ \frac{1}{\beta^2}\frac{{C_t}^{\rho}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t^2}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1}^* + \gamma_{9,t}^* Bf_{h,t-1}R_{t-1}^*\frac{1}{Q_{t-1}} - \beta E_t\left(\gamma_{9,t+1}^* Bf_{h,t}R_t^*\frac{Q_{t+1}}{Q_t^2}\right)$$
$$= 0$$

- wrt C_t

$$- \nu\gamma_{1,t}^* - \frac{n(1-\nu)}{1-n}\gamma_{2,t}^*$$
$$- \rho\gamma_{3,t}^* p_{H,t}C_t^{-\rho-1}$$
$$+ \rho\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho+1}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t}^*$$
$$- \rho\frac{1}{\beta^2}\frac{{C_t}^{\rho-1}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1}^* - \gamma_{9,t}^*$$
$$= 0$$

- C_t^*

$$\frac{1}{C_t^*} - (1-n)\frac{\nu^*}{n}Q_t^\theta\gamma_{1,t}^* - (1-\nu^*)Q_t^\theta\gamma_{2,t}^* - \rho\gamma_{4,t}^*\frac{p_{F,t}}{Q_t}C_t^{*-\rho-1}$$
$$+\rho\frac{1}{\beta}E_t({C_{t+1}^*}^\rho)C_t^{*-\rho-1}\gamma_{7,t}^* - \frac{1}{\beta^2}\rho C_t^{*\rho-1}{C_{t-1}^*}^{-\rho}\gamma_{7,t-1}^*$$
$$=0$$

- wrt R_t^*
$$(1+\tau_t)\gamma_{8,t}^* + (1+\tau_t^*)\gamma_{7,t}^* + \beta Bf_{h,t}E_t\left(\frac{Q_{t+1}}{Q_t}\gamma_{9,t+1}^*\right) = 0$$

- wrt $Bf_{h,t}$

$$-\gamma_{8,t}^*\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^\rho Q_t}{{C_t}^\rho Q_{t+1}}\right)\delta_f + \beta E_t\left(\gamma_{9,t+1}^* R_t^*\frac{Q_{t+1}}{Q_t}\right) - \gamma_{9,t}^*(1+\delta_f Bf_{h,t}) = 0$$

- wrt τ_t^*
$$\gamma_{7,t}^* R_t^* = 0$$