

# Heterogeneity, Selection and Labor Market Disparities\*

Alessandra Bonfiglioli<sup>†</sup>

Gino Gancia<sup>‡</sup>

UPF and CEPR

CREI and CEPR

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## Abstract

We study the incentives to improve ability in a model where heterogeneous firms and workers interact in a labor market characterized by matching frictions and costly screening. When effort in improving ability raises both the mean and the variance of the resulting ability distribution, multiple equilibria may arise. In the high-effort equilibrium, heterogeneity in ability is sufficiently large to induce firms to select the best workers, thereby confirming the belief that effort is important for finding good jobs. In the low-effort equilibrium, ability is not sufficiently dispersed to justify screening, thereby confirming the belief that effort is not so important. The model has implications for wage inequality, the distribution of firm characteristics, sorting patterns between firms and workers, and unemployment rates that can help explaining observed cross-country variation in socio-economic and labor market outcomes.

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<sup>†</sup>Universitat Pompeu Fabra and Barcelona GSE, Ramon Trias Fargas, 25-27, 08005, Barcelona, SPAIN. E-mail: alessandra.bonfiglioli@upf.edu

<sup>‡</sup>CREI and Barcelona GSE, Ramon Trias Fargas, 25-27, 08005, Barcelona, SPAIN. E-mail: ggancia@crei.cat

## 1 INTRODUCTION

Developed countries differ markedly in a number of social and economic indicators. Wage inequality, labor productivity, school attainment and employment rates are all higher in the United States than in Southern Europe. The population of active firms differs too, with a relatively larger number of small and less productive firms in the latter group of countries. While understanding these differences is important both from a positive and a normative standpoint, their origin remains largely an open question. One strand of literature attributes them to distortions, but typically does not explain how they arose in the first place.<sup>1</sup> Another strand of literature emphasizes the role of cultural values, but does not explain exactly how they translate into economic outcomes and why they have diverged, even in places that started with similar conditions such as the Southern and Northern regions in Italy or Spain.<sup>2</sup>

The objective of this paper is to show that large differences in socio-economic and labor market outcomes can emerge as alternative equilibria sustained by different, and yet rational, beliefs on the role played by ability and effort in determining individual economic success. We will argue that the mechanism we identify has implications for wage inequality, the distribution of firm characteristics, sorting patterns between firms and workers, and unemployment rates that can help to explain the cross-country variation observed in the data.

To this end, we study the incentives to invest in ability in a model where heterogeneous firms and workers interact in a labor market with matching frictions. Ability is unobservable, but firms can use a screening technology to select the best workers. As in Helpman, Itskhoki and Redding (2010), the combination of these ingredients yields realistic distributions of firms and wages. We then allow workers to invest costly effort to improve their ability. Our key assumption is that effort increases average ability, but also its dispersion in the population.<sup>3</sup> This introduces a complementarity between firms' and workers' strategies. On the one hand, the returns to screening are higher when ability is more dispersed, i.e., when workers have put effort. On the other hand, investing effort pays out more when firms screen workers.

This complementarity can give rise to two equilibria. In the high-effort equilibrium,

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<sup>1</sup>See for example Bartelsman, Haltiwanger and Scarpetta (2013), Bartelsman, Gautier and de Wind (2011), Restuccia and Rogerson (2008), and Wasmer (2006).

<sup>2</sup>See, for example, Guiso, Sapienza, and Zingales (2006) and Tabellini (2010).

<sup>3</sup>This assumption captures the notion that investing in education is risky and/or that it amplifies any pre-existing differences in raw talent (e.g., see Levhari and Weiss, 1974). It is consistent with standard human capital accumulation functions (e.g., Heckman, Lochner and Taber, 1998) and with the observation that wage inequality is higher for more educated workers (Lemieux, 2006).

heterogeneity in ability is sufficiently large to induce firms to select the best workers. In turn, this confirms the initial belief that effort is important for finding good jobs. In the low-effort equilibrium, instead, ability is not sufficiently dispersed to justify screening so that the probability of finding jobs depends more on luck rather than merit, thereby confirming the initial belief on the low value of effort. Relative to the alternative scenario, in the high-effort equilibrium ability is higher and more dispersed, firms are more productive, and a stronger sorting pattern between firms and workers generates more inequality, both across firms and workers. Our aim is to show that this mechanism can indeed replicate several salient differences observed between countries such as the United States, Italy and Spain.<sup>4</sup>

First, regarding perceptions, the existing evidence suggests that Americans believe in individual merit, work ethic and competition more than Southern Europeans. For instance, according to the 1981-2000 World Values Survey, 26.4% of Americans strongly agree with the statement that “hard work brings success”, against a share of 14.6% in Italy and 12.2% in Spain. Those who instead strongly believe that success “is a matter of luck and connections” represent 2.3%, 8.9% and 7.8% of respondents in the three countries respectively. Similarly, 43.3% of Americans think that “hard work is an important quality that a child should learn”, against 26.8% in Italy. More broadly, 29.6% of Americans strongly believe that “competition is good”, as opposed to 19.2% of Italians and 15.6% of Spaniards.

Second, these beliefs come together with significant differences in investment in education. Available data on the quality and quantity of schooling indicate that Americans attach a higher value to education than people from Southern Europe. For instance, in 2010 the working-age population with tertiary schooling was 42% in the United States against 15% in Italy and 32% in Spain (OECD, 2013).<sup>5</sup> Investment in education, both private and total, is also higher in the United States. For instance, total expenditure on tertiary education as a percentage of GDP is 2.8%, 1% and 1.3% in the three countries respectively. Regarding outcomes, U.S. students outperform those from Italy and Spain in all major international comparisons, but also exhibit more dispersion in the results (see Brown et al. 2007). Finally, the United States also score higher than Southern European countries in reported measures of discipline at school, which may be a proxy for effort (OECD, 2010a).

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<sup>4</sup>We refer mostly to these three countries because they seem the most natural and economically important examples of the equilibria we have in mind. More generally, our model can be useful to understand differences between Anglo-Saxon and Southern European countries. Nordic countries, where governments play a particularly important role, are outside the scope of the paper.

<sup>5</sup>The same figures for the cohort 25-34 are 43%, 21% and 39%, respectively.

Third, the differential value attached to education and effort is also reflected in measures of wage inequality and other labor market outcomes. In particular, the college premium relative to the earnings of workers with secondary education is around 1.8 in the United States against 1.5 in Italy and 1.4 in Spain (OECD, 2013). Broader measures of wage inequality display similar patterns: as reported in Krueger et al. (2010), for instance, the total variance of the logarithm of U.S. wages is above 0.4 and around 0.2 in the other two countries. On the contrary, the unemployment rate is consistently higher in Southern Europe.

Fourth, there are also large cross-country differences in firm-level outcomes. Available data suggest U.S. firms to be on average bigger and more productive, and their size distribution to be more dispersed than their European counterparts. Interestingly, there is also evidence that American markets are more selective: for example, the survival rate for new firms is about 10% lower in the United States than in Italy (Bartelsman, Haltiwanger and Scarpetta, 2009). Finally, regarding the covariance between size and productivity, Bartelsman, Haltiwanger and Scarpetta (2013) find that, within the typical U.S. manufacturing industry, labor productivity is almost 50% higher than it would be if employment was allocated randomly and that this measure of allocative efficiency is much lower on average in European countries.

Fifth, there is also evidence that American firms value selecting talent more. From their survey on managerial practices around the world, Bloom and Van Reenen (2010) build a synthetic measure of how strongly firms value selection, based on the answers to questions on the importance of attracting and keeping talented people to the company. In their sample of 17 countries, U.S. firms have the highest average score, while Italian firms have the lowest one.<sup>6</sup>

To our knowledge, our theory is the first to match all these observations without referring to exogenous differences in preferences and/or institutions. Despite this being a remarkable result, it is important to stress that we do not believe the multiplicity of equilibria identified in our paper to be the only or even the most important source of the socio-economic differences across developed countries. Rather, our theory illustrates a simple and yet powerful mechanism through which large differences in economic outcomes can be generated even when countries have access to the same technologies and share similar market and political institutions. The success at replicating qualitatively some of the salient differences between the two sides of the Atlantic makes us more confident that the model is capturing real world phenomena. Moreover, given that labor markets are often segmented regionally, we believe that our model can be useful for understanding

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<sup>6</sup>Unfortunately, Spain is not in the sample.

disparities in firm- and labor-market outcomes between regions of the same country (such as the North and South of Italy) which share the same broad institutions and policies.<sup>7</sup>

Our paper is related to several lines of research. First, it contributes to a set of papers that study the role of social beliefs in explaining the main differences in economic performance and inequality observed between the United States, Europe and other developed countries. Several important contributions show how alternative sets of beliefs can sustain equilibria with high and low levels of inequality. In Benabou (2000), Alesina and Angeletos (2005), and Hassler et al. (2005) this happens through the endogenous determination of the political support for redistributive policies; in Piketty (1998) through a status motive, in other papers through endogenous preference formation (e.g., Doepke and Zilibotti, 2013). Differently from these works, we focus on a complementarity between workers' effort decisions and the hiring strategies of heterogeneous firms. This approach seems well-suited for our aim of studying especially differences in the distribution of wages, workers and firms, which are believed to be of first-order importance.

The paper is also related to the large literature on the role of human capital, broadly defined, for economic development. Several contributions have shown how multiple equilibria and poverty traps can arise in the presence of increasing returns due to human capital externalities (e.g., Azariadis and Drazen, 1990), non-convexities coupled with credit frictions (e.g., Galor and Zeira, 1993), or a complementarity between talent and technological change (e.g., Hassler and Rodriguez Mora, 2000). Differently from these works, increasing returns, technological externalities or credit frictions are not needed in our approach to generate multiple equilibria. Moreover, none of the above mentioned papers examines the interaction between workers and firm heterogeneity.

Closer to our spirit, Acemoglu (1996) shows that social increasing returns to human capital may arise naturally when labor markets are characterized by search frictions. Similarly to our model, agents choose schooling depending on the type of jobs available and firms choose jobs depending on the average education of the workforce.<sup>8</sup> Differently from our framework, however, he abstracts from firm heterogeneity and selection through screening. The importance of the allocation of talent is stressed by many papers, including Acemoglu (1995), Bonfiglioli and Gancia (2014) and Hsieh et al. (2012).<sup>9</sup> None of them, however, studies its interplay with the hiring strategies of heterogeneous firms which is at

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<sup>7</sup>We do not push this interpretation further because it is more difficult to obtain the relevant data. Yet, a cursory look at existing evidence suggests regional disparities within Italy and Spain to be broadly consistent with the two equilibria described in this paper.

<sup>8</sup>A similar source of equilibrium multiplicity is present in models of statistical discrimination. See, for example, Samuelson, Mailath and Shaked (2000). Fang and Moro (2010) provide a recent survey. Our application is however very different.

<sup>9</sup>See also, Bhattacharya, Guner and Ventura (2013) and Caselli and Gennaioli (2013).

the core of our theory. Contrary to the classical contribution by Spence (1973) and the papers that followed, we abstract from the signaling role of education.

Finally, the paper builds on the literature on wage inequality in models with imperfect labor markets and firm heterogeneity. Acemoglu (1997) shows how search frictions *à la* Mortensen and Pissarides (1994) can generate and shape wage inequality. Helpman, Itskhoki and Redding (2008, 2010) combine search frictions, firm heterogeneity (as in Melitz, 2003, and Hopenhayn, 1992) and worker heterogeneity to study wage dispersion, wage-size premia and unemployment. Our model builds on these frameworks by adding an endogenous ability distribution and by exploring how the novel equilibrium multiplicity that arises can help explain some of the observed cross-country differences in the distribution of wages, firm characteristics and unemployment rates.

The rest of the paper is organized as follows. In Section 2 we lay down the model and derive the conditions for equilibrium multiplicity. In Section 3 we compare labor market outcomes, firms and welfare across equilibria. In Section 4 we work out some extensions to add realism and show the robustness of the main findings. We explore the quantitative implications of the model in Section 5, where we compare numerical simulations to some data for the United States, Italy and Spain. Section 6 concludes.

## 2 THE MODEL

We build a static model where heterogeneous firms and workers meet in a labor market characterized by search frictions along the lines of Helpman, Itskhoki and Redding (2010). For ease of comparison, we borrow their notation, whenever possible. Firms are matched randomly with workers of unknown ability although they can use a screening technology to select them. Screening is profitable only if heterogeneity among workers is sufficiently high. Moreover, ability is relatively more beneficial for more productive firms, which have an incentive to screen more intensively. Within this framework, we make the ability distribution endogenous by adding a stage in which workers can invest costly effort to improve their ability. We then show that when investing effort raises both the mean and the variance of the resulting ability distribution, multiple equilibria may arise due to a complementarity between effort and screening decisions. We derive the implications of the model for the equilibrium distribution of firms, wages, sorting patterns, the unemployment rate and welfare.

## 2.1 PREFERENCES AND TECHNOLOGY

The economy is populated by a unit measure of identical households of size  $\bar{L}$  with quasi-linear preferences over the consumption of two homogenous goods,  $q$  and  $Q$ :<sup>10</sup>

$$U = q + \frac{Q^\zeta}{\zeta}, \quad \zeta \in (0, 1).$$

The demand for  $Q$ , which we refer to as the advanced good, is

$$Q = P^{-\frac{1}{1-\zeta}}$$

where  $P$  is its price. The demand for  $q$  is residual and its price is taken as the numeraire ( $p = 1$ ). We call  $q$  the residual good. The indirect utility is

$$W = E + \frac{1-\zeta}{\zeta} Q^\zeta \tag{1}$$

with  $E$  denoting expenditure.

The residual good is produced by employing one unit of labor per unit of output, and is sold in a perfectly competitive market. Hence the wage in the residual sector equals the price, which is set to one.<sup>11</sup>

The advanced good is also homogeneous, but it is produced by heterogeneous firms employing labor subject to decreasing returns to scale. Firms entering the market incur a fixed cost,  $f_e > 0$ , expressed in terms of the numeraire. Once the firm has sunk the entry cost, it observes its productivity  $\theta$ , which is independently distributed and drawn from a Pareto distribution with support on  $[1, \infty)$ , shape parameter  $z > 1$ , and c.d.f.  $G(\theta) = 1 - (\theta)^{-z}$ . After observing  $\theta$ , the firm can decide whether to exit or to produce. Exit does not require any additional cost, while production entails a fixed cost of  $f_d > 0$  units of the residual good. The mass of entering firms,  $M$ , is endogenously determined by free entry.

Output of a firm with productivity  $\theta$ , employing a measure  $h$  of workers with average ability  $\bar{a}$  is given by:

$$y(\theta) = \theta h^\gamma \bar{a}, \quad \gamma \in (0, 1).$$

This technology has the following important features. First,  $\gamma < 1$  implies that there are decreasing returns to hiring more workers as, for example, in Lucas' (1978) span

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<sup>10</sup>Note that households are infinitesimal, but any idiosyncratic risk faced by individual workers is diversified at the household level. Alternatively, we could have assumed complete insurance markets.

<sup>11</sup>We assume that parameters are such to guarantee positive demand for the residual good in equilibrium.

of control model. Second, there is a complementarity in workers' ability such that the productivity of a worker depends on the average ability of the entire team. Third, there is a complementarity between firms' productivity and workers' ability. As we will see, these assumptions imply that firms face a trade-off between the quantity and quality of hired workers and that ability matters relatively more for more productive firms.<sup>12</sup>

Workers' ability is assumed to be independently distributed and drawn from a Pareto distribution with support on  $[1, \infty)$ , shape parameter  $k > 1$  and c.d.f.  $I(a) = 1 - (a)^{-k}$ . For now, we take  $k$  as given (it will be endogenized in Section 2.4) and assume it to be common knowledge. The labor market is characterized by search and matching frictions *à la* Diamond-Mortensen-Pissarides. A firm has to pay  $bn$  units of the residual good to be matched randomly with a measure  $n$  of workers. In turn, the search cost  $b$  is endogenously determined by the labor market tightness, as derived in Section 2.4.

We assume that ability is unknown both to the firm and to the worker.<sup>13</sup> However, once the match is formed, the firm has access to a screening technology which allows it to identify workers with ability below  $a^*$  at the cost of  $c[(a^*)^\delta - 1]/\delta$  units of the numeraire, with  $c > 0$ ,  $\delta > 1$ .<sup>14</sup> Given the distribution of ability,  $I(a)$ , a firm matched with  $n$  workers and screening at the cutoff  $a^*$  will hire a measure  $h$  of workers, where

$$h = n(1/a^*)^k, \quad (2)$$

with an average ability of  $\bar{a} = a^*k/(k-1)$ . With these results, the production function can be rewritten as a function of  $n$  and  $a^*$ :

$$y(\theta) = \frac{k}{k-1} \theta n^\gamma (a^*)^{1-\gamma k}.$$

Note that if  $\gamma < 1/k$ , output of a firm is increasing in the ability cutoff,  $a^*$ . When this condition is satisfied, there are sufficiently strong diminishing returns relative to the dispersion of ability that a firm can increase its output by not hiring the least productive workers. When  $\gamma > 1/k$ , instead, no firm wants to screen because employing even the least productive worker raises the firm's output and revenue, while screening is costly.

Wages in the advanced sector are determined through strategic bargaining between

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<sup>12</sup>See Helpman, Itskhoki and Redding (2008) for possible microfoundations of this production function. Eeckhout and Kircher (2012) study the trade-off between quantity and quality of workers in a more general setting.

<sup>13</sup>Ability can be interpreted either as firm specific or worker specific. Interpretation aside, this makes no difference in our static model.

<sup>14</sup>Differently from Helpman, Itskhoki and Redding (2010), we assume that the cost of screening is zero at the minimum  $a^* = 1$ . We do this to avoid asymmetries in fixed costs between firms that do or do not screen.



the firm and workers (as in Stole and Zwiebel, 1996a,b). Since in the bargaining stage only average ability is known, the firm retains a fraction of revenues equal to the Shapley value,  $1/(1+\gamma)$ , and pays the rest to the workers.<sup>15</sup> Using  $P = Q^{\zeta-1}$ , we can express revenue as:

$$r(\theta) = Q^{\zeta-1} \frac{k\theta n^\gamma (a^*)^{1-\gamma k}}{k-1}. \quad (3)$$

## 2.2 THE FIRM'S PROBLEM

Firms choose how many workers to sample,  $n$ , and the cutoff ability for hired workers,  $a^*$ , so as to maximize profits:

$$\pi(\theta) = \max_{n>0, a^* \geq 1} \left\{ \frac{r(\theta)}{1+\gamma} - bn - c \frac{(a^*)^\delta - 1}{\delta} - f_d \right\} \quad (4)$$

where  $r(\theta)$  is given by (3).

The first-order conditions are:

$$\frac{\gamma}{1+\gamma} r(\theta) = bn(\theta) \quad (5)$$

$$\frac{1-\gamma k}{1+\gamma} r(\theta) = ca^*(\theta)^\delta \quad \text{if } k < 1/\gamma \quad (6)$$

and  $a^*(\theta) = 1$  if  $k \geq 1/\gamma$  in which case all sampled workers are hired,  $n(\theta) = h(\theta)$ . Combining the first order conditions, it is immediate to show that firms that sample more workers (higher  $n$ ) screen more intensively (higher  $a^*$ ) and therefore hire workers with higher average ability. Moreover, if  $\delta > k$ , as we will assume, firms that screen to a higher ability cutoff also hire more workers.

Substituting equation (2) into (5), and using the definition of wages as a share of revenue per hired worker, we obtain the equilibrium wage as

$$w(\theta) \equiv \frac{\gamma}{1+\gamma} \frac{r(\theta)}{h(\theta)} = ba^*(\theta)^k,$$

which is equal to the replacement cost of a worker. In turn, this is constant and equal to the search cost  $b$  if firms do not screen and higher than  $b$  and increasing in the cutoff  $a^*(\theta)$  otherwise.<sup>16</sup> Recall that  $a^*(\theta)$  increases in  $h(\theta)$  when  $\delta > k$ . Thus, under this

<sup>15</sup>See Helpman, Itskhoki and Redding (2008) for a formal derivation.

<sup>16</sup>Note that the expected wage conditional on being matched with a firm is constant:  $w(\theta)h(\theta)/n(\theta) = b$ . This also implies that workers have no incentives to direct their search.

assumption, firms hiring more workers also pay higher wages:

$$\frac{\partial \ln w(\theta)}{\partial \ln h(\theta)} = \frac{k}{\delta - k}. \quad (7)$$

This makes the model with screening ( $a^*(\theta) > 1$ ) consistent with a positive employer-size wage premium, as commonly found in the data (see for example, Oi and Idson 1999 and Troske, 1999). To match this empirical regularity, from now on we restrict the analysis to the case  $\delta > k$ . Note that the model yields wage variation across firms, but the assumption of unobservable worker heterogeneity implies that the wages are the same across all workers within a firm. Still, average wages conditional on ability vary across workers, because high-ability workers are more likely to be hired by firms paying high wages.

Profit can be rewritten as a fraction of revenue minus a constant cost by replacing  $n$  and  $a^*$  from (5) and (6) into (4):

$$\pi(\theta) = \frac{\Gamma}{1 + \gamma} r(\theta) - f \quad (8)$$

with

$$f \equiv f_d - \mathbb{I}_s \frac{c}{\delta} \quad (9)$$

$$\Gamma \equiv 1 - \gamma - \mathbb{I}_s \frac{1 - \gamma k}{\delta} > 0. \quad (10)$$

where  $\mathbb{I}_s$  is an indicator taking value 1 when  $a^*$  is interior (greater than one) and 0 in case of a corner solution (i.e., when firms prefer not to screen).

Revenue, in turn, is an increasing function of productivity and parameters. To see this, substitute (5) and (6) into (3):

$$r(\theta) = (1 + \gamma)^{1 - \frac{1}{\Gamma}} \left( \theta \frac{kQ^{\zeta-1} \gamma^\gamma}{k-1} \frac{1}{b^\gamma} \right)^{\frac{1}{\Gamma}} \left( \frac{1 - \gamma k}{c} \right)^{\mathbb{I}_s \frac{1 - \gamma k}{\delta \Gamma}} \quad (11)$$

Since revenue is continuously increasing in productivity and there is a fixed production cost, the least productive firms make negative profits and hence exit the market. The cutoff productivity  $\theta^*$  below which firms exit is defined by the condition:

$$\pi(\theta^*) = \frac{\Gamma}{1 + \gamma} r(\theta^*) - f = 0. \quad (12)$$

Note that the relative revenue of any two firms only depends on their relative productivity,  $r(\theta)/r(\theta^*) = (\theta/\theta^*)^{1/\Gamma}$ . This result, combined with (8) and (12), allows us to express

profits of firms with productivity  $\theta$  as

$$\pi(\theta) = f \left[ \left( \frac{\theta}{\theta^*} \right)^{\frac{1}{\Gamma}} - 1 \right]. \quad (13)$$

As in Helpman, Itskhoki and Redding (2010), we can obtain all firm-level equilibrium variables as a function of productivity relative to the exit cutoff:<sup>17</sup>

$$r(\theta) = \frac{1 + \gamma}{\Gamma} f \left( \frac{\theta}{\theta^*} \right)^{\frac{1}{\Gamma}} \quad (14)$$

$$n(\theta) = \frac{\gamma}{\Gamma} \frac{f}{b} \left( \frac{\theta}{\theta^*} \right)^{\frac{1}{\Gamma}} \quad (15)$$

$$a^*(\theta) = \left( \frac{1 - \gamma k}{\Gamma} \frac{f}{c} \right)^{\frac{\Gamma_s}{\delta}} \left( \frac{\theta}{\theta^*} \right)^{\frac{\Gamma_s}{\delta \Gamma}} \quad (16)$$

$$h(\theta) = \frac{\gamma}{\Gamma} \frac{f}{b} \left( \frac{\theta}{\theta^*} \right)^{\frac{1}{\Gamma}} a^*(\theta)^{-k} \quad (17)$$

$$w(\theta) = b a^*(\theta)^k \quad (18)$$

Hence, more productive firms are larger in terms of revenues, sampled and hired workers, are more selective (screen at a higher ability cutoff) and pay higher wages.

### 2.3 INDUSTRY EQUILIBRIUM

To determine the equilibrium of the advanced sector, we need to solve for the cutoff productivity,  $\theta^*$ , the overall consumption of advanced good,  $Q$ , and the measure of entering firms,  $M$ .

First, we pin down the cutoff productivity,  $\theta^*$ , from the free-entry condition, requiring expected profits to equal the entry cost:

$$f_e = \int_{\theta^*}^{\infty} \pi(\theta) dG(\theta) = f \int_{\theta^*}^{\infty} \left[ \left( \frac{\theta}{\theta^*} \right)^{1/\Gamma} - 1 \right] dG(\theta), \quad (19)$$

where the second equality uses (13). After replacing  $G(\theta) = 1 - \theta^{-z}$  and  $dG(\theta) =$

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<sup>17</sup>To derive the expression for profit, we use condition (12) to obtain

$$\left( \frac{Q^{-(1-\zeta)}}{1 + \gamma} \frac{k}{k-1} \right)^{\frac{1}{\Gamma}} \left( \frac{\gamma}{b} \right)^{\frac{\gamma}{\Gamma}} \left( \frac{1 - \gamma k}{c} \right)^{\frac{\Gamma_s}{\delta} \frac{1 - \gamma k}{\Gamma}} \Gamma = f (\theta^*)^{-\frac{1}{\Gamma}}$$

and replace it in (4). As regards the other variables, we used the definition of profit to obtain  $r(\theta)$ , (5) and (6) to express  $n(\theta)$  and  $a^*(\theta)$  as functions of  $r(\theta)$ . Given  $n(\theta)$  and  $a^*(\theta)$ , we obtain  $h(\theta)$  from (2). We use the definition of wages to obtain  $w(\theta)$ .

$(z\theta^{-z-1})d\theta$  into (19), we obtain the equilibrium value for  $\theta^*$ :

$$\theta^* = \left[ \left( \frac{1}{z\Gamma - 1} \right) \frac{f}{f_e} \right]^{1/z}. \quad (20)$$

We assume that parameters are such that firm selection occurs in equilibrium, i.e.  $\theta^* > 1$ . This is equivalent to requiring  $f$  to be large enough relative to the entry cost  $f_e$ , and  $z\Gamma$  to be higher than one.<sup>18</sup> We also assume that all surviving firms do select workers in the interior equilibrium, i.e.  $a^*(\theta^*) > 1$ .

Next, we obtain  $Q$  by substituting  $\theta^*$  into (11) and (12):

$$Q = \left[ \frac{1}{1 + \gamma} \frac{k}{k - 1} \left( \frac{\Gamma}{f} \right)^\Gamma \left( \frac{\gamma}{b} \right)^\gamma \left( \frac{1 - \gamma k}{c} \right)^{\mathbb{I}_s \frac{1 - \gamma k}{\delta}} \theta^* \right]^{1/(1 - \zeta)} \quad (21)$$

We derive the equilibrium relationship between consumption of the advanced good  $Q$  and the measure of entrants  $M$  by imposing market clearing, i.e. that expenditure in the advanced good be equal to total revenues of the sector, and substituting for  $PQ = Q^\zeta$ :

$$Q^\zeta = M \int_{\theta^*}^{\infty} r(\theta) dG(\theta) = f \frac{1 + \gamma}{\Gamma} M \int_{\theta^*}^{\infty} \left( \frac{\theta}{\theta^*} \right)^{1/\Gamma} dG(\theta),$$

where we used  $r(\theta) = f(\theta/\theta^*)^{1/\Gamma} (1 + \gamma)/\Gamma$ .<sup>19</sup> Substituting  $dG(\theta)$  as above delivers the equilibrium mass of entrants,

$$M = \frac{1}{1 + \gamma} \frac{Q^\zeta}{z f_e},$$

and hence the measure of surviving firms,  $M[1 - G(\theta^*)] = M(1/\theta^*)^z$ .

#### 2.4 LABOR MARKET, ABILITY AND MULTIPLE EQUILIBRIA

The equilibrium of the advanced sector was derived for given search cost,  $b$ , and shape parameter of the ability distribution,  $k$ . In this section, we solve for the equilibrium  $b$ , the allocation of workers between the two sectors and we endogenize the ability distribution. This will allow us to close the model.

We start with the problem of workers. Agents must first choose the sector of occupation. They can decide to work in the residual sector, in which case they will be hired with certainty at the wage of one. This follows from the assumptions that ability is irrelevant

<sup>18</sup>All the exact parameter restrictions are summarized in the Appendix.

<sup>19</sup>The market-clearing condition can be analogously expressed in terms of real quantities, i.e.,  $M \int_{\theta^*}^{\infty} y(\theta) dG(\theta) = Q$ , with  $y(\theta)$  replaced from the expression for equilibrium revenue,  $r(\theta) = Q^{-(1-\zeta)} y(\theta)$ .

in the residual sector and that there are no search frictions (we relax this assumption in Section 4.1). Alternatively, they can seek employment in the advanced sector, where ability matters and there is unemployment risk. We assume that working in the advanced sector requires formal education and for now we normalize the cost of acquiring it to zero (we relax this assumption in Section 4.2). Hence, we denote with  $L \leq \bar{L}$  the mass of job seekers in the advanced sector and we identify them as the measure of educated workers. The distribution from which workers draw their ability depends however on the effort they put in acquiring education. Without effort, ability is drawn from a Pareto distribution with minimum of one and shape parameter  $k_0 > 1/\gamma$ . Putting effort costs  $\eta$  units of the residual good and allows workers to draw ability from a distribution with the same minimum, but with a lower shape parameter,  $k_1 < 1/\gamma$ . This implies that effort raises both average ability ( $k_1/(k_1 - 1) > k_0/(k_0 - 1)$ ) and the variance of its realizations.<sup>20</sup> This assumption captures the notion that investing in education is risky and/or that studying hard amplifies any pre-existing differences in raw ability.

We restrict attention to pure-strategy equilibria, in which all  $L$  workers make the same effort decision.<sup>21</sup> We denote the effort choice with the indicator function  $\mathbb{I}_\eta \in \{0, 1\}$ , taking value one if workers put effort (i.e., invest  $\eta$ ) and zero otherwise. In sum, the equilibrium effort choice (yet to be solved for) pins down the ability distribution of the population of workers, which is Pareto with shape parameter:

$$k = \begin{cases} k_0 > 1/\gamma & \text{if } \mathbb{I}_\eta = 0 \\ k_1 < 1/\gamma & \text{if } \mathbb{I}_\eta = 1. \end{cases}$$

We also assume that the effort level exerted by a single worker is unobservable, although firms know the overall ability distribution.

To solve for the equilibrium, we still need to specify the labor market frictions. To this end, we follow Blanchard and Gali (2008) and Helpman and Itskhoki (2009) in modeling the search cost,  $b$ , as an increasing function of the labor-market tightness,  $N/L$ :<sup>22</sup>

$$b = \alpha \left( \frac{N}{L} \right)^\beta, \quad \alpha > 1 + \eta \quad \beta > 0. \quad (22)$$

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<sup>20</sup>Note that putting effort allows workers to draw ability from a distribution that first-order stochastically dominates the ability distribution in the no-effort case.

<sup>21</sup>We prove that no single worker has an incentive to deviate. However, we do not consider explicitly equilibria in which only a fraction of workers chooses effort. We do this for simplicity, since the distribution of the sum of Pareto distributions is intractable. In any case, simulations suggest that equilibria where only a fraction of workers put effort, when they exist, are unstable.

<sup>22</sup>As shown in these papers, this relationship can be derived from a Cobb-Douglas matching function and a fixed cost of posting vacancies.

In equilibrium, workers must be indifferent between being employed in the residual sector and looking for a job in the advanced one. This requires the probability of being matched (equal to  $N/L$ ) times the expected wage conditional on being matched ( $w(\theta)h(\theta)/n(\theta) = b$ ) to be equal to the certain wage of one in the residual sector, plus the effort cost  $\eta$  if  $\mathbb{I}_\eta = 1$ :

$$1 + \mathbb{I}_\eta \eta = \frac{N}{L} b.$$

Using this into (22), we can solve for:

$$b = \alpha^{\frac{1}{1+\beta}} (1 + \mathbb{I}_\eta \eta)^{\frac{\beta}{1+\beta}} > 1 + \eta \quad (23)$$

and

$$\frac{N}{L} = \frac{1 + \mathbb{I}_\eta \eta}{b} = \left( \frac{1 + \mathbb{I}_\eta \eta}{\alpha} \right)^{\frac{1}{1+\beta}}. \quad (24)$$

Equations (23) and (24) imply that  $b$  (the equilibrium search cost) and  $N/L$  (the equilibrium market tightness) should be higher when  $\mathbb{I}_\eta = 1$ , because workers have to be compensated for their effort by higher wages and matching probabilities. Note also that the restriction  $\alpha > 1 + \eta$  makes sure that job seekers always exceed vacancies in the advanced sector, i.e.,  $N/L < 1$ . Finally, the measure of job seekers,  $L$ , is found imposing their *ex-ante* expected wage,  $L(1 + \mathbb{I}_\eta \eta)$ , to be equal to the wage bill:

$$L(1 + \mathbb{I}_\eta \eta) = \frac{\gamma}{1 + \gamma} Q^\zeta. \quad (25)$$

We can now show that, under the conditions  $k_1 < 1/\gamma < k_0$  and

$$\eta < \frac{\delta z \Gamma_{\mathbb{I}_s=1} + k_0 - k_1}{\delta z \Gamma_{\mathbb{I}_s=1}} a^* (\theta_{\mathbb{I}_s=1}^*)^{k_0 - k_1} - 1, \quad (26)$$

two (only two) pure-strategy equilibria exist, sustained by different beliefs on the screening strategy of firms in the advanced sector.

**Proposition 1** *Assume  $k_1 < 1/\gamma < k_0$  and that (26) is satisfied. Then there exist only two pure-strategy equilibria sustained by different workers' beliefs on firms' screening decisions. In the screening equilibrium all job seekers in the advanced sectors exert effort ( $\mathbb{I}_\eta = 1$ ) and all firms screen workers ( $\mathbb{I}_s = 1$ ). In the no-screening equilibrium, investment in effort is zero ( $\mathbb{I}_\eta = 0$ ), and firms do not screen workers ( $\mathbb{I}_s = 0$ ).*

**Proof.** *in the Appendix* ■

The intuition for the equilibrium multiplicity is simple and is based on a complementarity between the effort decision of workers and the hiring strategy of firms. If workers

invest in effort, the distribution of ability will be sufficiently dispersed ( $k_1 < 1/\gamma$ ) so as to induce firms to screen. In turn, given that firms screen out workers with low ability, putting effort is a way to increase the job-finding probability. If condition (26) is satisfied, then this effect is strong enough to justify the cost of the investment,  $\eta$ . On the contrary, in the low-effort equilibrium, ability is not sufficiently dispersed to justify screening ( $k_0 > 1/\gamma$ ). But then workers have indeed no incentives to put effort, because they would incur in a cost without this affecting their expected wage, since in the no-screening equilibrium all workers have the same job-finding probability and wage, irrespective of ability. Thus, the belief that effort is or is not important in the labor market turns out to be self-fulfilling.

When condition (26) is not satisfied, instead, only the no-effort equilibrium survives. In this case, the effort cost is so high that deviating from the high-effort equilibrium is profitable, despite the lower job-finding probability. Note however that condition (26) will always be satisfied for  $k_0$  sufficiently high. To see this, notice that as  $k_0 \rightarrow \infty$  workers who do not invest in effort will have ability equal to one and will be unemployed for sure (recall that  $a^*(\theta_{\mathbb{I}_s=1}^*) > 1$ ).

Proposition 1, establishing the coincidence of effort and screening in equilibrium, allows us to denote both choices with the same indicator variable  $\mathbb{I}_s$ . We therefore refer to the case  $\mathbb{I}_s = 1$  as the “high-effort” or “screening” equilibrium interchangeably. With this notation, we are now able to compute the unemployment rate in the advanced sector, which is given by:

$$u = 1 - \frac{N}{L} \frac{H}{N}.$$

Note that there are two potential sources of unemployment. First, due to the matching friction, only a fraction  $N/L$  of job seekers is interviewed by firms. Second, in the equilibrium with screening, only a fraction  $H/N$  of the workers matched with firms is actually hired. To find  $u$ , we first solve for  $H/N$  by integrating  $h(\theta)$  and  $n(\theta)$ , from (17) and (15) respectively, across active firms:

$$\frac{H}{N} = \frac{\int_{\theta^*}^{\infty} h(\theta) dG(\theta)}{\int_{\theta^*}^{\infty} n(\theta) dG(\theta)} = \frac{a^*(\theta^*)^{-k} (z\Gamma - 1)}{z\Gamma - (1 - \mathbb{I}_s k/\delta)}. \quad (27)$$

Substituting this expression and  $N/L$  from (24) into  $u$ , we obtain:

$$u = 1 - \frac{a^*(\theta^*)^{-k} (z\Gamma - 1)}{z\Gamma - (1 - \mathbb{I}_s k/\delta)} \frac{1 + \mathbb{I}_s \eta}{b}. \quad (28)$$

The overall unemployment rate is positive and increasing in matching frictions, which are proportional to  $b$ , and in the degree of selection chosen by firms.

In sum, given  $\mathbb{I}_s \in \{0, 1\}$ , the equilibrium values of  $\theta^*$ ,  $Q$ ,  $b$ ,  $L$  and  $u$  are given by (20), (21), (23), (25) and (28), respectively. Given  $\theta^*$  and  $b$ , all firm-level outcomes are given by (14)-(18).

## 2.5 DISTRIBUTIONS

We now derive the distribution of revenue and employment across firms, and wages across workers in the advanced sector. Given that revenue and employment are power functions of productivity, which is Pareto distributed, they will also inherit the same type of distribution. In particular, using (14) and (17) and the properties of the Pareto distribution we obtain:<sup>23</sup>

$$F_r(r) = 1 - \left(\frac{r^*}{r}\right)^{\Gamma z} \quad \text{for } r \geq r^* = \frac{1+\gamma}{\Gamma} f, \quad (29)$$

$$F_h(h) = 1 - \left(\frac{h^*}{h}\right)^{\frac{\Gamma z}{1-\mathbb{I}_s k/\delta}} \quad \text{for } h \geq h^* = \frac{\gamma}{\Gamma} \frac{f}{b} a^* (\theta^*)^{-k}. \quad (30)$$

We show in the Appendix that the equilibrium distribution of wages in the advanced sector is also Pareto, with the following c.d.f.:

$$F_w(w) = 1 - \left(\frac{w^*}{w}\right)^{1+\frac{\delta}{\mathbb{I}_s k}(\Gamma z-1)} \quad \text{for } w \geq w^* = b a^* (\theta^*)^k. \quad (31)$$

Note that in the equilibrium with no screening the distribution of wages is degenerate, with  $w = b$  for all firms.

## 3 COMPARING EQUILIBRIA

In this section, we compare the predictions of the model for a number of variables of interest in the two equilibria. In what follows, we use subindexes 1 and 0 to denote the equilibrium with and without screening, respectively.

### 3.1 LABOR MARKET OUTCOMES

We first compare the main labor-market indicators: wage inequality, both between and within groups, and the unemployment rate.

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<sup>23</sup>If  $\theta$  follows a Pareto( $\theta^*$ ,  $z$ ), then  $x \equiv \log(\theta/\theta^*)$  is distributed as an exponential with parameter  $z$ . Then, any power function of  $\theta$  of the type  $A\theta^B$ , with  $A$  and  $B$  constant, is distributed as a Pareto( $A(\theta^*)^B$ ,  $z/B$ ), since  $A\theta^B = A(\theta^*)^B e^{Bx}$  with  $Bx \sim \text{Exp}(z/B)$ , by the properties of the exponential distribution.



### 3.1.1 Wage Inequality

We consider two measures of wage inequality: the skill premium, defined as the average wage of workers employed in the advanced sector (which requires formal education) relative to workers in the residual sector (which does not require any education), and the dispersion of wages within the advanced sector.

The skill premium is higher in the equilibrium with screening. To see this, note that, since the wage in the residual sector is one, the skill premium is just the average wage in the advanced sector. This is constant and equal to  $b_0$ , i.e., the replacement cost, in the equilibrium without screening. For  $\mathbb{I}_s = 1$ , instead, the wage in the advanced sector is higher for two reasons. First, the wage must exceed  $b_1$ , because workers must be compensated for the risk of being discarded by firms. Second,  $b_1$  is higher than  $b_0$  to compensate workers for their costly effort. More precisely, the lowest wage paid by the marginal firm in the screening equilibrium is:

$$w_1(\theta^*) = b_1 a^* (\theta_1^*)^{k_1} > b_1 > b_0 = w_0,$$

moreover, recall that  $w_1(\theta)$  increases with  $\theta$ .

It is also useful to write the relative skill premium in the two equilibria as:

$$\frac{\bar{w}_1}{w_0} = (1 + \eta) \frac{1 - u_0}{1 - u_1}, \quad (32)$$

where  $\bar{w}_1$  denotes the average wage, and  $u_0$  and  $u_1$  the unemployment rates in the advanced sector in the alternative equilibria. This relationship highlights the fact that the wage of the advanced sector must compensate workers for any costly investment in effort and unemployment risk.

In the screening equilibrium there is also another type of wage inequality: when  $\mathbb{I}_s = 1$ ,  $w$  varies across firms and is increasing in productivity, while it is constant when  $\mathbb{I}_s = 0$ . Thus, screening generates wage dispersion within workers with education. To measure it, we can use the expression in (31) to compute the standard deviation of the logarithm of wages in the advanced sector:<sup>24</sup>

$$SD(\log w_1) = \frac{k_1}{k_1 + \delta(\Gamma_1 z - 1)} \quad \text{and} \quad SD(\log w_0) = 0,$$

where  $SD$  denotes the standard deviation.

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<sup>24</sup>Since wages in the advanced sector are distributed as a Pareto, their log follows an exponential distribution with rate equal to the shape parameter of the Pareto, i.e.,  $1 + (\Gamma z - 1) \delta / (\mathbb{I}_s k)$ .

### 3.1.2 Unemployment

Next, we compare the unemployment rate in the advanced sector,  $u$ , across the two equilibria by using the expression in (28). Note that, in principle,  $u$  can be lower or higher in the screening equilibrium. The reason for this ambiguity is that there are two forces working in opposite directions. On the one hand, screening is a direct source of unemployment, because workers that were matched with firms but discarded remain unemployed. On the other hand, to compensate workers for the effort cost, the labor market must be tighter (higher  $N/L$ ), thereby lowering frictional unemployment.

Using (28) we find that unemployment is lower with screening, i.e.,  $u_1 < u_0$  if and only if:

$$(1 + \eta)^{\frac{1}{1+\beta}} > a^* (\theta_1^*)^k \frac{z\Gamma_1 - 1 + k_1/\delta}{z\Gamma_1 - 1} \quad (33)$$

The right-hand side of this condition measures how selective firms are and is higher than one. Thus, for the condition to be satisfied,  $\eta$  must be sufficiently high. Recall however that  $\eta < \alpha - 1$ , because at this upper bound every worker is interviewed by some firm ( $L_1 = N_1$ ). What is the role of the other labor market parameters? Higher search frictions, measured by a fall in  $\beta$ , make (33) more likely, because they raise unemployment relatively more in the no-screening equilibrium. Changes in labor market frictions due to changes in  $\alpha$ , instead, affect employment similarly in both equilibria. However, a higher  $\alpha$  relaxes the constraint on  $\eta$ , thereby increasing the parameter space under which  $u_1 < u_0$ .

## 3.2 FIRM-LEVEL OUTCOMES

We now compare the main firm-level outcomes: productivity, the ability of hired workers, revenue and employment.

### 3.2.1 Productivity

Productivity in the advanced sector is higher in the screening equilibrium for three reasons. First, firms are more productive because the cutoff for exit is higher. From (20):

$$\frac{\theta_1^*}{\theta_0^*} = \left[ \frac{z\Gamma_0 - 1}{z\Gamma_1 - 1} \frac{f_1}{f_0} \right]^{1/z} > 1$$

where the inequality follows from the condition for  $a^*(\theta_1^*) > 1$ , which requires  $\Gamma_0/\Gamma_1 > f_0/f_1 > 1$ . This is due to the fact that screening allows the more productive firms to hire more able workers, which makes them even more profitable, thereby increasing entry and making it harder for the less productive firms to remain in the market. This can be seen

from (13): profit,  $\pi_1(\theta)$ , is a steeper function of productivity than  $\pi_0(\theta)$ , because more productive firms benefit relatively more from screening. Given the Pareto distribution of  $\theta$ , a higher cutoff implies immediately a higher average productivity:  $\bar{\theta}_1/\bar{\theta}_0 = \theta_1^*/\theta_0^*$ , where  $\bar{\theta}$  denote the average across surviving firms.

Second, the average ability of workers seeking a job in the advanced sector is higher due to the direct effect of effort:

$$\mathbb{E}[a|\mathbb{I}_s = 1] = \frac{k_1}{k_1 - 1} > \frac{k_0}{k_0 - 1} = \mathbb{E}[a|\mathbb{I}_s = 0].$$

Third, the average ability of hired workers,  $\bar{a}$ , is even higher due to screening:

$$\mathbb{E}[\bar{a}|\mathbb{I}_s = 1] = \frac{k_1 a^*(\theta_1^*)}{k_1 - 1} \frac{k_1 + \delta(\Gamma_1 z - 1)}{k_1 + \delta(\Gamma_1 z - 1) - 1},$$

which is greater than  $\mathbb{E}[\bar{a}|\mathbb{I}_s = 0] = k_0/(k_0 - 1)$ .<sup>25</sup> Given that more productive firms are more selective, their overall productivity advantage is relatively larger in the high-effort equilibrium, in that they hire better workers.

### 3.2.2 Firm Size: Revenue and Employment

In the screening equilibrium firms are larger in terms of revenue. To see this, consider the revenue of the smallest surviving firm. Using (29):

$$\frac{r_1(\theta_1^*)}{r_0(\theta_0^*)} = \frac{\Gamma_0 f_1}{\Gamma_1 f_0} > 1,$$

since  $a^*(\theta_1^*) > 1$  requires  $\Gamma_0/\Gamma_1 > f_0/f_1$ . Moreover, screening makes revenue a steeper function of productivity. Thus, average revenue,  $\bar{r} = r(\theta^*) z\Gamma/(z\Gamma - 1)$ , is even higher:

$$\frac{\bar{r}_1}{\bar{r}_0} = \frac{z\Gamma_0 - 1}{z\Gamma_1 - 1} \frac{f_1}{f_0}.$$

Revenues are also more dispersed across firms when  $\mathbb{I}_s = 1$ . This can be shown computing the standard deviation of log-revenue from (29) and using the properties of the Pareto distribution:

$$\frac{SD(\log r_1)}{SD(\log r_0)} = \frac{\Gamma_0}{\Gamma_1} > 1.$$

<sup>25</sup>To derive the expression for  $\mathbb{E}[\bar{a}|\mathbb{I}_s = 1]$ , we solved the integral:

$$\mathbb{E}[\bar{a}|\mathbb{I}_s = 1] = \frac{k_1}{k_1 - 1} \frac{\int_{\theta_1^*}^{\infty} a^*(\theta) h_1(\theta) dG(\theta)}{\int_{\theta_1^*}^{\infty} h_1(\theta) dG(\theta)}.$$

Despite being larger in terms of revenue, firms that screen out workers may actually be smaller in terms of employment. In particular, from (30), the relative employment of the smallest firm in the two equilibria is:

$$\frac{h_1(\theta_1^*)}{h_0(\theta_0^*)} = \frac{\Gamma_0 f_1 b_0}{\Gamma_1 f_0 b_1} a^* (\theta_1^*)^{-k}.$$

This expression shows that screening tends to increase  $h_1/h_0$  by raising profitability ( $\Gamma_0/\Gamma_1 > f_0/f_1$ ). There are, however, two forces working in the opposite direction. First, the higher market tightness ( $b_0/b_1 < 1$ ) makes hiring relatively more costly. Second, screening lowers the probability of hiring conditional on sampling workers ( $a^*(\theta_1^*) > 1$ ). Given the distribution of  $h$  in (30), we can compute the relative average firm size, in terms of employment, as:

$$\frac{\bar{h}_1}{\bar{h}_0} = a^* (\theta_1^*)^{-k} \frac{z\Gamma_0 - 1}{z\Gamma_1 - 1 + k/\delta} \frac{f_1}{f_0} (1 + \eta)^{-\frac{\beta}{1+\beta}}$$

which may be smaller or greater than one, depending on parameters.

### 3.3 LABOR ALLOCATION AND WELFARE

We now compare the size of the advanced sector, the allocation of workers and welfare.

#### 3.3.1 Size of the Advanced Sector and Labor Allocation

Using (21) and (23), we derive the relative output of the advanced sector under the two equilibria:

$$\left(\frac{Q_1}{Q_0}\right)^{1-\zeta} = \frac{k_1(k_0 - 1)}{(k_1 - 1)k_0} a^* (\theta_1^*)^{1-\gamma k_1} \frac{\theta_1^*}{\theta_0^*} \left(\frac{\Gamma_1 f_0}{\Gamma_0 f_1}\right)^{1-\gamma} (1 + \eta)^{\frac{-\gamma\beta}{1+\beta}}.$$

With screening,  $Q$  tends to be larger because workers and firms are more productive. On the other hand, the cost of effort discourages workers from entering the advanced sector. If effort has no cost ( $\eta = 0$ ), then it can be shown that  $Q_1$  is necessarily higher than  $Q_0$ .<sup>26</sup> However, if  $\eta$  is sufficiently high and the benefit of screening sufficiently low, then screening can potentially shrink the size of the advanced sector. As we will see shortly, in this case the cost of effort is so high that welfare is actually lower in the screening equilibrium.

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<sup>26</sup>The factor  $(\Gamma_1/\Gamma_0)^{1-\gamma} < 1$  accounts for the resources invested by firms in screening. However, as we formally prove in the Appendix, this cost is more than compensated by the benefit of screening, captured by the preceding factors. This is intuitive, since screening is chosen optimally by firms.

We can also solve for the allocation of workers between the two sectors, which in our interpretation of the model gives also the fraction of the population with high education. Equation (25) implies that  $L$  is an increasing function of the output of the advanced sector:

$$\frac{L_1}{L_0} = \frac{1}{1+\eta} \left( \frac{Q_1}{Q_0} \right)^\zeta > 1 \quad \text{iff} \quad Q_1/Q_0 > (1+\eta)^{1/\zeta}.$$

Thus, the determinants of  $L_1/L_0$  are the same as those of  $Q_1/Q_0$ , although the negative effect of the effort cost is stronger. For this reason, it is possible for the advanced sector in the screening equilibrium to be bigger in terms of output, but smaller in terms of employment.

### 3.3.2 Welfare

Taking indirect utility as our measure of welfare, it is easy to show that it behaves like output of the advanced good,  $Q$ . To see this, note first that expenditure  $E$  is equal to the sum of the wages of the  $\bar{L}$  agents in the economy minus their investment in human capital:

$$E = (\bar{L} - L) + (1 + \mathbb{I}_s \eta) L - \mathbb{I}_s \eta = \bar{L}.$$

This is intuitive, since the *ex-ante* average wage must be one. Using (1), we obtain relative per capita utility as:

$$\frac{W_1}{W_0} = \frac{1 + \frac{1-\zeta}{\zeta} Q_1^\zeta}{1 + \frac{1-\zeta}{\zeta} Q_0^\zeta} = \frac{1 + (1+\eta) \frac{1-\zeta}{\zeta} \frac{1+\gamma}{\gamma} \frac{L_1}{\bar{L}}}{1 + \frac{1-\zeta}{\zeta} \frac{1+\gamma}{\gamma} \frac{L_0}{\bar{L}}}. \quad (34)$$

Thus,  $Q_1/Q_0 > 1$  is a necessary and sufficient condition for welfare to be higher with screening. Interestingly, however, if the cost of effort is too high, the high-effort equilibrium may turn out to be inefficient.

## 4 EXTENSIONS

We now consider three extensions of the model. First, we add unemployment in the residual sector. Beyond being more realistic, this version of the model yields richer results on the aggregate unemployment rate through compositional effects. Second, we show how to introduce a cost of acquiring the formal education required to enter the advanced sector. This extension breaks the one-to-one mapping between the unemployment rate and the skill premium in the no-screening equilibrium. Finally, we study what happens when the matching cost depends also on the mass of unemployed worker generated by selection.

#### 4.1 UNEMPLOYMENT IN THE RESIDUAL SECTOR

Following Helpman and Itskhoki (2009), we capture search frictions in the labor market of the residual sector with a Cobb-Douglas matching function that gives the mass of hired workers,  $N_q$ , as a function of vacancies posted by firms,  $V_q$ , and the measure of job seekers,  $L_q = \bar{L} - L$ :

$$N_q = V_q^\chi L_q^{1-\chi},$$

where  $\chi \in (0, 1)$ . This implies that the probability that a firm fills a vacancy is  $N_q/V_q = (N_q/L_q)^{(\chi-1)/\chi}$ . We assume that firms can freely enter by paying the cost of posting a vacancy,  $v$ , and they exit if not matched with any worker. A firm employing  $n_q$  workers produces and has revenue equal to  $n_q$ , which is split with the workers through Nash bargaining. Hence, the wage is equal to the bargaining power of workers  $w_q = 1 - \Gamma_q$  and profit is  $\Gamma_q n_q$ . Free entry, driving expected profits to zero, requires  $(N_q/L_q)^{(\chi-1)/\chi} \Gamma_q = v$ . This pins down the tightness of the labor market:

$$\frac{N_q}{L_q} = \left( \frac{v}{\Gamma_q} \right)^{\frac{-\chi}{1-\chi}}.$$

Note that we need  $v > \Gamma_q$  in order to have  $N_q/L_q < 1$ . The unemployment rate in the residual sector is just  $u_q = 1 - N_q/L_q$ .

Differently from the baseline case, the expected income of a worker seeking a job in the residual sector, denoted by  $\omega_q$ , is no longer one. Instead, it is equal to  $1 - \Gamma_q$  times the probability of finding a job:

$$\omega_q = (1 - \Gamma_q) \frac{N_q}{L_q}.$$

The condition for being indifferent between seeking jobs in the two sectors becomes:

$$\omega_q + \mathbb{I}_s \eta = \frac{N}{L} b,$$

which implies:

$$b = \alpha^{\frac{1}{1+\beta}} (\omega_q + \mathbb{I}_s \eta)^{\frac{\beta}{1+\beta}} \quad \text{and} \quad \frac{N}{L} = \left( \frac{\omega_q + \mathbb{I}_s \eta}{\alpha} \right)^{\frac{1}{1+\beta}}, \quad (35)$$

where  $\alpha > \omega_q + \eta$  is needed to guarantee that  $N/L < 1$ . Finally, the mass of workers who choose the advanced sector is now:

$$L (\omega_q + \mathbb{I}_s \eta) = \frac{\gamma}{1 + \gamma} Q^\zeta.$$

All the equilibrium values in (20), (21), (25) and (28) and all firm-level outcomes are still valid using the new expression for  $b$  given in (35).<sup>27</sup>

The aggregate unemployment rate becomes

$$\bar{u} = \frac{L}{\bar{L}}u + \frac{\bar{L} - L}{\bar{L}}u_q,$$

which depends on the allocation of labor and on the unemployment rate in both sectors. In this version of the model, it is now possible for the aggregate unemployment rate to be lower in the high-effort equilibrium even if screening generates more unemployment in the advanced sector ( $u_1 > u_0$ ), provided that the unemployment rate of the residual sector is higher ( $u_q > u_1$ ) and that the advanced sector expands sufficiently ( $L_1 > L_0$ ). This is an interesting possibility, because the unemployment rate is typically lower among workers with high education ( $L$ ). Thus, compositional effects may help generating lower unemployment in the screening equilibrium.

#### 4.2 COSTLY ENTRY IN THE ADVANCED-SECTOR LABOR MARKET

In the baseline model, we normalized the cost of seeking employment in the advanced sector, which we interpreted as the cost of education, to zero. We now generalize the model to the case in which the education required to apply for a job in the advanced sector costs  $\varepsilon \geq 0$  units of the numeraire. After paying this cost and without additional effort, the worker can draw ability from a Pareto distribution with support on  $[1, \infty)$  and shape parameter  $k_0$ . The cost and effect of effort are as in the baseline model. Following the same steps as in section 2.4, the equilibrium search cost and tightness become:

$$b = \alpha^{\frac{1}{1+\beta}} (1 + \varepsilon + \mathbb{I}_s \eta)^{\frac{\beta}{1+\beta}} \quad \text{and} \quad \frac{N}{L} = \frac{1 + \varepsilon + \mathbb{I}_s \eta}{b} = \left( \frac{1 + \varepsilon + \mathbb{I}_s \eta}{\alpha} \right)^{\frac{1}{1+\beta}} < 1,$$

where  $\alpha > 1 + \varepsilon + \eta$  is needed to guarantee  $N/L < 1$ . As in the previous case, all the equilibrium conditions apply with the new  $b$ .<sup>28</sup>

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<sup>27</sup>Note that in this case condition (26) for Proposition 1 becomes:

$$\eta < \omega_q \left[ \frac{\delta z \Gamma_{\mathbb{I}_s=1} + k_0 - k_1}{\delta z \Gamma_{\mathbb{I}_s=1}} a^* (\theta_{\mathbb{I}_s=1}^*)^{k_0 - k_1} - 1 \right].$$

<sup>28</sup>Note that in this case condition (26) for Proposition 1 becomes:

$$\eta < (1 + \varepsilon) \left[ \frac{\delta z \Gamma_{\mathbb{I}_s=1} + k_0 - k_1}{\delta z \Gamma_{\mathbb{I}_s=1}} a^* (\theta_{\mathbb{I}_s=1}^*)^{k_0 - k_1} - 1 \right].$$

The advantage of this version of the model is that it breaks the tight relationship between the skill premium ( $w$ ) and the unemployment rate ( $u$ ) in the equilibrium without screening. In particular, when  $\mathbb{I}_s = 0$ , the workers' indifference condition becomes:

$$1 + \varepsilon = (1 - u) w.$$

Thus, the skill premium, (i.e., the wage of the advanced sector  $w$ ) must be high enough to compensate both the unemployment risk and the cost of education  $\varepsilon$ . Together with the previous extension, the modified model can be made consistent with a positive skill premium and a relatively lower unemployment rate in the advanced sector, which seems the most realistic case.

### 4.3 SEARCH COST AS A FUNCTION OF THE UNEMPLOYMENT RATE

In the benchmark case, the search cost is assumed to be an increasing function of labor market tightness as measured by the ratio of sampled to available workers. In search models where all sampled workers are hired this induces a perfect negative correlation between search cost and the unemployment rate, capturing the fact that firms find it harder to hire a worker if there are relatively few unemployed. In our framework, however, the cost of screening is imperfectly correlated to unemployment due to the fact that screening increases the mass of unemployed workers without affecting the search cost.

In this section, we reintroduce the perfect negative correlation between search cost and unemployment by assuming that the matching function takes the following form:

$$b = \alpha \left( \frac{N}{L} \frac{H}{N} \right)^\beta = \alpha (1 - u)^\beta, \quad \beta > 0.$$

The condition for workers to be indifferent between the residual and the advanced sector remains:

$$\frac{1 + \mathbb{I}_s \eta}{b} = \frac{N}{L},$$

which implies that the equilibrium search cost becomes:

$$b = \alpha^{\frac{1}{1+\beta}} \left[ (1 + \mathbb{I}_s \eta) \frac{H}{N} \right]^{\frac{\beta}{1+\beta}}.$$

This can be expressed as a function of the model parameters by replacing  $H/N$  from equation (27):

$$b = \alpha^{\frac{1}{1+\beta}} (1 + \mathbb{I}_s \eta)^{\frac{\beta}{1+\beta}} \left[ \frac{a^* (\theta^*)^{-k} (z\Gamma - 1)}{z\Gamma - (1 - \mathbb{I}_s k/\delta)} \right]^{\frac{\beta}{1+\beta}},$$



which is the same as in the baseline model in the equilibrium without screening, and lower for  $\mathbb{I}_s = 1$  since the higher unemployment generated through screening directly reduces the search cost. For given parameters, this version of the model tends to generate lower unemployment.

Note that the ratio of interviews to candidates,  $N/L$ , is necessarily smaller than one as long as  $1 + \eta < b$  or, using  $b$ , when:

$$1 + \eta < \alpha \left[ \frac{a^* (\theta^*)^{-k} (z\Gamma - 1)}{z\Gamma - (1 - k/\delta)} \right]^\beta, \quad (36)$$

which implies a smaller upper bound on  $\eta$  for given  $\alpha$  than in the baseline case.<sup>29</sup>

## 5 NUMERICAL EXAMPLES

In this section we complement the qualitative comparison between equilibria presented in Section 3 with some numerical examples. The goal is twofold. First, given that the model predictions for some outcomes are potentially ambiguous (e.g., whether the unemployment rate is higher in the screening equilibrium), it is useful to explore them using plausible parameter values. Second, we would like to have a sense of how much of the observed cross-country differences in economic outcomes can be accounted for by our theory. To this end, we need to map our model to the data. We choose the United States as an example of a country in the high-effort equilibrium and Spain or Italy as representative of the other scenario.<sup>30</sup> To choose parameters, we rely whenever possible on existing empirical studies or choose them so as to match roughly key observations in Spain or Italy under the no-screening equilibrium. For parameters that cannot be identified easily, we consider a range of plausible values. We then compute the predicted differences across equilibria and compare them with available data.

First, we set the parameters of the labor market. To calibrate  $\beta$ , we exploit the mapping between our model and standard search and matching models where the expected cost per match for a firm is proportional to the expected duration of a vacancy. In steady state, this duration is  $V/N$ , where  $V$  denotes the number of vacancies. Assuming a Cobb-Douglas matching function,  $N = V^\chi L^{1-\chi}$ , we can solve for  $V/N = (N/L)^{(1-\chi)/\chi}$ . Thus, the parameter  $\beta$  in (22) corresponds to  $(1 - \chi)/\chi$  and is proportional to the elasticity of the matching function to vacancies. Since estimates of  $\chi$  are typically close to  $1/2$ ,

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<sup>29</sup>In this case condition (26) for Proposition 1 is unchanged since  $bN/L$  does not change relative to baseline case.

<sup>30</sup>For most of the variables of interest, Italy and Spain look very similar. In a few instances, however, we lack data for one of the two countries. We therefore keep both of them as our benchmark.

we assume  $\beta = 1$  (as in Blanchard and Galí, 2010). Next, we set  $\alpha$  so as to have an unemployment rate of 10% in the no-screening equilibrium. Using  $u_0 = 1 - (1/\alpha)^{1/(1+\beta)}$  yields  $\alpha = 1.23$ .

The cost of effort,  $\eta$ , cannot be observed directly. Note however that equation (32) gives a close relationship between  $\eta$ , the skill premium and the unemployment rate. If unemployment is lower in the screening equilibrium, which seems to be the empirically relevant case, then the cost of effort should be greater than  $\bar{w}_1/\bar{w}_0 - 1$ . Identifying workers in the advanced sector as those with some college degree,  $\bar{w}$  is the college premium. Looking at the data, the relative earnings of workers with tertiary education is around 1.8 in the United States and around 1.45 in Italy and Spain (OECD, 2013), which would suggest  $\eta > 0.23$ . At the same time, recall that in order to guarantee  $H/N \leq 1$ ,  $\eta$  can be no greater than  $\alpha - 1$ , which happens to be equal to 0.23. We therefore set  $\eta = 0.23$  (we will also consider lower values).

We now turn to the parameters of technology ( $\gamma$ ,  $\delta$ ,  $c$ ) and of the distributions of productivity and ability ( $z$ ,  $k$ ). As shown in Section 3, some important differences between the two equilibria depend on the difference between  $\Gamma_1$  and  $\Gamma_0$ , which is given by  $(1 - \gamma k)/\delta$ . Thus, for the model to produce significant disparities in the distribution of firms,  $\delta$  and  $\gamma k$  must be sufficiently low. To put discipline on these parameters, notice from (7) that the employer-size wage premium depends on  $\delta/k$ . Available estimates for the elasticity of wages to scale are around 10% (Oi and Idson, 1999, Troske, 1999), which implies  $\delta = 11 * k$ .<sup>31</sup> We calibrate  $z$  so as to match the firm size distribution in Italy and Spain. In particular, we compute the standard deviation of the log of value added in Italy, Spain and, for comparison, the United States using data from the U.S. Census and the SDBS Structural Business Statistics (OECD, 2010b). Since the data are aggregated into size categories, we follow Helpman, Melitz and Yeaple (2004) in assuming that all establishments falling within the same bin have the same value added as the group mean and using the number of firms in each size category as weights.<sup>32</sup> By doing so, we find that the standard deviation of log value added in 2007 is around 0.5 for Italy and Spain, and 0.66 for the United States. Since the standard deviation of the log of revenue in the no-screening equilibrium is  $[z(1 - \gamma)]^{-1}$ , we assume  $z = 2/(1 - \gamma)$ .<sup>33</sup>

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<sup>31</sup>Controlling for compositional effects, empirical estimates of the size premium tend to become somewhat smaller. However, in order to identify  $\delta$  in our model one should probably not condition on worker characteristics.

<sup>32</sup>We use five size categories: 0-9, 10-19, 20-49, 50-250, over 250 employees. Helpman, Melitz and Yeaple (2004) show that this methodology to compute dispersion yields results that are highly correlated with direct measures based on the entire population (when available).

<sup>33</sup>This value is in between the typical estimates of firm heterogeneity in the macro literature (such as Axtell, 2001) and those based on trade data (such as Eaton, Kortum and Kramarz, 2011, and Bernard et al., 2003). This is not surprising, given that estimates of firm dispersion vary somewhat with the class

In the absence of direct evidence on the cost of screening, we set the parameter  $c$  of the screening technology so that the marginal firm hires all the workers it meets (i.e., we impose  $a^*(\theta_1^*) = 1$  in equation (16)), which seems a natural benchmark. Regarding the remaining parameters,  $\gamma$  and  $k$ , we experiment with different values. For the span-of-control parameter (or diminishing returns) in the production function,  $\gamma$ , we consider three possibilities: 0.2, 0.5 and 0.8.<sup>34</sup> Note that the choice of  $\gamma$  may limit severely the set of admissible values for  $k_1$ , the shape parameter of the ability distribution, because  $k_1 < 1/\gamma$  is needed to sustain the screening equilibrium (and  $k_1 > 1$ ). Consistent with this restriction, we experiment with  $k_1 \in \{1.1, 1.5, 2\}$  for  $\gamma = 0.2$ ,  $k_1 \in \{1.1, 1.5\}$  for  $\gamma = 0.5$  and  $k_1 = 1.1$  for  $\gamma = 0.8$ . In this way, we span most of the admissible parameter space for  $\gamma$  and  $k$ . The shape parameter of the ability distribution under no effort,  $k_0$ , must satisfy  $k_0 > 1/\gamma$ . In order to minimize the effect of effort on the ability distribution, we set  $k_0$  at its limit,  $k_0 \rightarrow 1/\gamma$ . It turns out, however, that the exact choice of  $k_0$  is unimportant for most of the results. Given this calibration, we compare the predictions of the model for wages, unemployment rates and firms characteristics across the two equilibria.<sup>35</sup>

The results are reported in Table 1, Panel A, where, for notational convenience, we denote by  $\Delta$  the percentage difference between the equilibrium with and without screening. With screening, the skill premium ( $\Delta\bar{w}$ ) is around 22% higher, replicating closely the observed difference between the United States and Italy or Spain. This is not surprising, given that the cost of education was chosen to roughly match this observation. More interestingly, the model with screening generates wage dispersion around the mean  $\bar{w}_1$ . In particular, the standard deviation of the log of wages in the advanced sector,  $SD(\log w_1)$ , is close to 0.1, against zero in the other equilibrium. Although in absolute terms this number may be rather low, the difference across equilibria may help explain the significantly higher wage dispersion in the United States, where the overall standard deviation of log wages is 0.66, relative to Spain, where the corresponding number is 0.48.<sup>36</sup>

We next focus on the unemployment rate in the advanced sector. Recall that the parameters are chosen so as to generate an unemployment rate of 10% in the no-screening equilibrium. As Table 1 shows,  $u_1$  is consistently lower, varying from 8.8% to 9.8%. Although these differences are not large, their sign is nonetheless remarkable, because it was

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of firms considered. We therefore decided to match the dispersion measure that we could compute in our data.

<sup>34</sup>A span-of-control parameter of 0.8 is often used in quantitative models. The other values may appear low. However, note that  $\gamma$  captures the overall curvature of the revenue function, which, in a more general model, would also depend on product differentiation (as in Melitz, 2003).

<sup>35</sup>The remaining parameters are not needed to obtain predictions on the variables of interest.

<sup>36</sup>To match more closely the level of variability in data, some additional source of wage dispersion (even just noise) should be added in both equilibria.

not targeted in the calibration (recall that screening can potentially generate higher unemployment rates). Perhaps even more importantly, the model suggests that the origin of unemployment is radically different across countries: while it may be due to (involuntary) matching frictions in the Southern European equilibrium, it could mostly be driven by deliberate hiring strategies of firms in the American equilibrium. Indeed, in these simulations, frictional unemployment is close to zero in the screening equilibrium. Note also that the model could account for a larger variation in  $u$  through compositional effect if we had a (realistically) higher unemployment rate in the residual (low-skill) sector, as in the extension described in Section 4.1. At any rate, we do believe that adding differences in labor market rigidities would be important to fully explain the data.

We then consider firm-level variables. Average revenue per firm is between 3.2% and 9.6% higher in the screening equilibrium ( $\Delta\bar{r}$ ). Moreover, selection is tougher. To see this, note that the survival rate upon entry in the model is inversely proportional to average revenue. Thus,  $\Delta\bar{r}$  also captures the higher survival probability in the no-screening equilibrium (recall that  $\bar{r}_1/\bar{r}_0 = \Pr(\theta > \theta_0^*) / \Pr(\theta > \theta_1^*)$ ). For comparison with the data, Bartelsmann, Haltiwanger and Scarpetta (2009) report that the survival rate of firms at four years of age is about 10% higher in Italy than in the United States (the difference is very similar also at age two). If we take survival rates of new firms as a proxy for the probability of successful entry, then the model does a good job at replicating cross-country differences along this dimension.<sup>37</sup> The model also predicts more dispersion across firms in the screening equilibrium, where the standard deviation of log revenue is between 3.1% and 8.8% higher ( $\Delta SD(\log r)$ ). In the data, the standard deviation of value added in the United States is about 30% higher than in Italy and Spain. Thus, the prediction of the model goes in the right direction, but accounts for 10%-30% of the observed variation in firm heterogeneity. The only dimension along which the model fails qualitatively is average employment: while firms in the screening equilibrium are predicted to be bigger in terms of revenue, they are smaller in terms of average number of employees ( $\Delta\bar{h}$ ). This is due to the fact that firms are more selective and workers that are screened out are not rehired. As we will see shortly, the extension of the model presented in Section 4.3 alleviates this problem.<sup>38</sup> As a general pattern, we find that all differences across equilibria tend to be larger when  $k_1$  and  $\gamma$  are low and when  $k_1$  is significantly smaller than  $1/\gamma$ . This is as expected, because in these cases the value of selection is higher.

Before concluding, we replicate the simulations using the version of the model where

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<sup>37</sup>Regarding the level, the entry cost can always be calibrated to match the desired surviving probability in one equilibrium.

<sup>38</sup>The small differences in average firm size partly depend on the special properties of Pareto distributions.

|                      | (1)            | (2)         | (3)            | (4)         | (5)         | (6)            |
|----------------------|----------------|-------------|----------------|-------------|-------------|----------------|
|                      | $\gamma = 0.2$ |             | $\gamma = 0.5$ |             |             | $\gamma = 0.8$ |
|                      | $k_1 = 1.1$    | $k_1 = 1.5$ | $k_1 = 2$      | $k_1 = 1.1$ | $k_1 = 1.5$ | $k_1 = 1.1$    |
| Panel A: Benchmark   |                |             |                |             |             |                |
| $\Delta \bar{w}$     | 23.1%          | 22.4%       | 21.9%          | 22.9%       | 21.8%       | 22.3%          |
| $SD(\log w_1)$       | 0.098          | 0.092       | 0.089          | 0.096       | 0.088       | 0.092          |
| $u_1$                | 9.7%           | 9.2%        | 8.9%           | 9.6%        | 8.8%        | 9.1%           |
| $\Delta \bar{r}$     | 9.6%           | 5.9%        | 3.7%           | 8.7%        | 3.2%        | 5.5%           |
| $\Delta SD(\log r)$  | 8.8%           | 5.6%        | 3.5%           | 8.0%        | 3.1%        | 5.2%           |
| $\Delta \bar{h}$     | -11%           | -13.4%      | -15%           | -11.6%      | -15.3%      | -13.7%         |
| Panel B: Section 4.3 |                |             |                |             |             |                |
| $\Delta \bar{w}$     | 11.4%          | 11.1%       | 10.9%          | 11.3%       | 10.8%       | 11.0%          |
| $SD(\log w_1)$       | 0.098          | 0.092       | 0.089          | 0.096       | 0.088       | 0.092          |
| $u_1$                | 9.5%           | 9.2%        | 9.1%           | 9.4%        | 9.0%        | 9.2%           |
| $\Delta \bar{r}$     | 9.6%           | 5.9%        | 3.7%           | 8.7%        | 3.2%        | 5.5%           |
| $\Delta SD(\log r)$  | 8.8%           | 5.6%        | 3.5%           | 8.0%        | 3.1%        | 5.2%           |
| $\Delta \bar{h}$     | -1.7%          | -4.7%       | -6.5%          | -2.3%       | -6.9%       | -5%            |

Note:  $\Delta$  denotes percentage differences between the equilibrium with/without screening.

Table 1: Comparing Labor Market Outcomes

the hiring cost is a function of the overall unemployment rate (including workers discarded by firms). In this case, however, the constraint on the admissible values of the cost of effort,  $\eta$ , is tighter. In particular, given our choices of parameters, we find  $\eta = 0.12$  to be the highest value always compatible with (36). We therefore lower the cost of effort to 0.12, knowing that, by doing so, the model will not be able to replicate fully the differences in skill premia. The results are reported in Panel B of Table 1. As expected, the model can now explain only about 50% of the observed difference in the college premium. More interestingly, firms in the screening equilibrium are now larger in terms of employment, although they are still marginally smaller than in the other equilibrium. The remaining outcomes do not differ significantly from the predictions of the baseline model in Panel A.<sup>39</sup>

Overall, our numerical exercises yield results that are broadly consistent across specifications and parametrizations. Despite being very stylized, the model can replicate reasonably well differences in wages, wage inequality and firm productivity. The simulations suggest that, despite the more selective hiring strategies of firms, the unemployment rate can be lower in the high-effort equilibrium. The model can also replicate some of the differences in the distribution of firm. These results, albeit suggestive, are remarkable

<sup>39</sup>This version of the model would deliver a significantly lower  $u_1$  if  $\eta$  were chosen so as to match the skill premium. This however would imply  $N/L > 1$ , that is, workers receiving on average more than one interview. While not unrealistic, this implies some technical complications that we prefer to avoid.

considering that they are obtained without imposing any exogenous difference in rigidities, entry costs or any other structural parameter, which are certainly important in the real world.

## 6 CONCLUSIONS

We have proposed a model that explains disparities in several economic and labor market outcomes across developed countries based on multiple equilibria sustained by different beliefs on the value of effort and ability. In particular, when effort raises the dispersion of workers' ability and firms have access to a costly screening technology, two equilibria arise: in the "American" equilibrium, workers expect firms to be selective and hence invest effort to improve their job prospects. This raises the dispersion in workers' ability and induces firms to be selective. The opposite occurs in the "Southern European" equilibrium, where workers believe effort to be less important and do not invest, thereby inducing firms not to screen. In an economy with labor market frictions and heterogeneous firms, the screening equilibrium is characterized by higher productivity of both firms and workers, higher wage inequality, higher and more dispersed firm-level revenues and possibly lower unemployment. A numerical exercise shows that the model is able to replicate some significant differences in these outcomes across developed countries such as the United States, Italy and Spain, even when they all share the same structural parameters.

In addition to these positive results, the model yields useful policy insights. It suggests that governments in countries like Italy and Spain could play an important role in trying to transition the economy towards the high-effort equilibrium. If these countries were trapped in a no-screening equilibrium, for instance because investing in human capital is too costly or not effective enough, a first policy intervention should aim at improving the technology for human capital formation, so as to make the screening equilibrium possible. Moreover, measures could be taken to strengthen the social perception of effort and meritocracy in such a way to coordinate workers and firms on the desirable equilibrium.

Although we kept the model simple to obtain clear analytical results, it could be extended in a number of interesting directions. As it is common in this class of models, we left the problem of equilibrium selection entirely outside the analysis. Allowing for learning dynamics (for instance, as in Blume, 2006) may help understand persistence and the role of policy in equilibrium selection. Introducing dynamics and shocks in the model may also have novel implications for the cyclical properties of labor market outcomes across different equilibria.<sup>40</sup> Endogenizing the degree of distortions and rigidities (exogenous and equal

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<sup>40</sup> Adding explicit dynamics, as in the standard Dimond-Mortensen-Pissarides framework, is possible,

across countries in the model) may also help explain why countries in the “American” equilibrium also tend to have more flexible labor and product market regulations. Finally, opening the economy in our model may provide interesting insights on the effects of trade and migration between countries in different equilibria. We leave all these questions open for future research.

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although we have preferred to use a static version to preserve transparency.

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## 7 APPENDIX

### 7.1 PARAMETER RESTRICTIONS

Parameters have to satisfy the following restrictions:

1.  $k_1 < 1/\gamma < k_0$
2.  $\delta > k_1 > 1$
3.  $z\Gamma_1 > 1$
4.  $\frac{f_d}{f_e} > z(1 - \gamma) - 1$
5.  $\frac{\Gamma_0}{\Gamma_1} > \frac{\delta f_d}{\delta f_d - c}$
6.  $\alpha > 1 + \eta$
7.  $0 < \zeta < 1$

where  $\Gamma_0 = 1 - \gamma$  and  $\Gamma_1 = 1 - \gamma - (1 - \gamma k)/\delta$ . Restriction 1. guarantees that firms choose to screen ( $\mathbb{I}_s = 1$ ) only if workers acquire human capital ( $\mathbb{I}_\eta = 1$ ). The first inequality in restriction 2. guarantees that firms that sample more workers also hire more and pay higher wages, while the second one is needed for the mean of the ability distribution to be finite. Restriction 3. is needed for the mean of firm size distributions to be finite. The fourth restriction makes sure that there is firm selection even in the equilibrium without screening ( $\theta_0^* > 1$ ). Restriction 5. guarantees that all firms effectively screen in the equilibrium with  $\mathbb{I}_s = 1$  ( $a^*(\theta_1^*) > 1$ ). The sixth restriction makes sure that labor market tightness in the advanced sector is always lower than one ( $N/L < 1$ ).

### 7.2 PROOF OF PROPOSITION 1

We first notice that, since firms observe the ability distribution before choosing whether to screen workers or not, there are only two rational equilibria in pure strategies: either (i) all workers expect firms not to screen ( $\mathbb{E}[\mathbb{I}_s] = 0$ ) and hence do not invest effort ( $\mathbb{I}_\eta = 0$ ), thereby inducing firms not to screen ( $\mathbb{I}_s = 0$ ) since  $k_0 > 1/\gamma$ ; or (ii) all workers expect

firms to screen ( $\mathbb{E}[\mathbb{I}_s] = 1$ ) and hence invest effort ( $\mathbb{I}_\eta = 1$ ), thereby inducing firms to screen ( $\mathbb{I}_s = 1$ ) since  $k_1 < 1/\gamma$ .

Next, we prove that both equilibria are deviation proof when condition (26) is satisfied. If  $\mathbb{E}[\mathbb{I}_s] = 0$  and nobody puts effort ( $\mathbb{I}_\eta = 0$ ), which gives a payoff of  $b_0 N_0/L_0$ , an individual choosing  $\mathbb{I}_\eta = 1$  would face a payoff of  $b_0 N_0/L_0 - \eta$ . This is due to the fact that effort costs  $\eta$ , but employment probability remains equal to the market tightness ( $N_0/L_0$ ) in the absence of screening, and the expected wage remains  $b_0$  because wages are proportional to average ability of all workers employed by a firm and cannot be changed by an individual agent. Hence, there is no profitable deviation from this equilibrium.

If  $\mathbb{E}[\mathbb{I}_s] = 1$  and all workers invest effort ( $\mathbb{I}_\eta = 1$ ), which gives a payoff of  $b_1 N_1/L_1 - \eta$ , an individual choosing  $\mathbb{I}_\eta = 0$  faces the same wage as the others if hired, but a lower probability of being hired. This is not profitable as long as

$$b_1 \frac{N_1}{L_1} - \eta > \frac{\int_{\theta^*}^{\infty} w_1(\theta) a^*(\theta)^{-k_0} dG(\theta) N_1}{1 - G(\theta_1^*)} \frac{N_1}{L_1}.$$

We replace  $w_1$  from (18) with  $\mathbb{I}_s = 1$ ,  $1 - G(\theta_1^*) = (\theta_1^*)^{-z}$ ,  $dG(\theta) = (z\theta^{-z-1})d\theta$ , and compute the integral, to obtain the following condition:

$$\eta < b_1 \frac{N_1}{L_1} \left[ 1 - \frac{a^*(\theta_1^*)^{-(k_0-k_1)} \delta\Gamma_1 z}{\delta\Gamma_1 z + (k_0 - k_1)} \right],$$

which, after substituting for  $b_1 N_1/L_1$  from (23) and (24), becomes:

$$\eta < \frac{\delta\Gamma_1 z + k_0 - k_1}{\delta\Gamma_1 z} a^*(\theta_1^*)^{k_0-k_1} - 1$$

Intuitively, this requires the cost of effort to be low enough relative to its return, which increases with the difference between  $k_0$  and  $k_1$ .

### 7.3 WAGE DISTRIBUTION

To find the equilibrium distribution of wages, notice that a measure  $h(\theta)$  of workers in each firm with productivity  $\theta$  receive the same wage. Hence, the cumulated density of  $w$  is

$$F_w(w) = \frac{\int_{\theta^*}^{\theta_w(w)} h(\theta) dG(\theta)}{\int_{\theta^*}^{\infty} h(\theta) dG(\theta)} = 1 - \frac{\int_{\theta_w(w)}^{\infty} h(\theta) dG(\theta)}{\int_{\theta^*}^{\infty} h(\theta) dG(\theta)} \text{ for } w > w(\theta^*),$$

where  $\theta_w(w)$  is the productivity of firms paying wages  $w$ ,  $\theta_w(w) = \theta^* [w/w(\theta^*)]^{\delta\Gamma/\mathbb{I}_s k}$  and  $w(\theta^*) = ba^*(\theta^*)^k$ . This expression can be simplified by replacing  $h(\theta)$  from (17) and

$dG(\theta) = (z\theta^{-z-1})d\theta$  to yield

$$F_w(w) = 1 - \left[ \frac{w(\theta^*)}{w} \right]^{1 + \frac{\delta}{\Gamma_s k} (\Gamma z - 1)} \quad \text{for } w > ba^*(\theta^*)^k.$$

#### 7.4 COMPARING EMPLOYMENT DISTRIBUTIONS

To prove that  $h(\theta)$  is less dispersed in the equilibrium with screening, we show that the shape parameter of its distribution is higher in this case. The ratio of the shape parameters for  $\mathbb{I}_s = 1$  and  $\mathbb{I}_s = 0$  is:

$$\frac{\delta \Gamma_1 z}{\delta - k_1} \frac{1}{\Gamma_0 z} = \frac{\delta(1 - \gamma) - (1 - \gamma k_1)}{(\delta - k_1)(1 - \gamma)}.$$

The assumptions that  $k_1 > 1$  and  $\delta > k_1$  imply that

$$\frac{\delta \Gamma_1 z}{\delta - k_1} \frac{1}{\Gamma_0 z} > \frac{\delta(1 - \gamma) - (1 - \gamma)}{\delta - k_1} \frac{1}{1 - \gamma} = \frac{\delta - 1}{\delta - k_1} > 1,$$

which means that  $h(\theta)$  is less dispersed in the equilibrium with screening.

#### 7.5 COMPARING THE ADVANCED SECTOR SIZE

The term capturing the net benefit of screening and effort in the expression for the relative size of the advanced sector,  $Q_1^{1-\zeta}/Q_0^{1-\zeta}$ , is:

$$\frac{k_1}{k_1 - 1} \frac{k_0 - 1}{k_0} \left( \frac{\Gamma_1}{\Gamma_0} \right)^{1-\gamma} \left( \frac{f_0}{f_1} \right)^{1-\gamma} a^*(\theta_1^*)^{1-\gamma k_1} \frac{\theta_1^*}{\theta_0^*}.$$

This is positive, since  $a^*(\theta_1^*) > 1$ ,  $f_0 > f_1$ , and  $\theta_1^* > \theta_0^*$ , as shown in section 3, and  $\frac{k_1}{k_1 - 1} \frac{k_0 - 1}{k_0} \left( \frac{\Gamma_1}{\Gamma_0} \right)^{1-\gamma} > 1$ . To prove the latter, we first notice that

$$\frac{k_1}{k_1 - 1} \frac{k_0 - 1}{k_0} \left( \frac{\Gamma_1}{\Gamma_0} \right)^{1-\gamma} > \frac{k_1}{k_1 - 1} \frac{k_0 - 1}{k_0} \frac{\Gamma_1}{\Gamma_0}$$

since  $\gamma \in (0, 1)$ . Next,  $\delta > k_1$  implies  $\Gamma_1 > (k_1 - 1)/k_1$ , hence:

$$\frac{k_1}{k_1 - 1} \frac{k_0 - 1}{k_0} \left( \frac{\Gamma_1}{\Gamma_0} \right)^{1-\gamma} > \frac{k_0 - 1}{k_0} \frac{1}{\Gamma_0} = \frac{k_0 - 1}{k_0} \frac{1}{1 - \gamma} > 1,$$

because  $k_0 > 1/\gamma$ .