Are banks too big to fail?

Measuring systemic importance of financial institutions

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Abstract We consider three measures on the systemic importance of a financial institution within a interconnected financial system. Based on the measures, we study the relation between the size of a financial institution and its systemic importance. From both theoretical model and empirical analysis, we find that when analyzing the systemic risk posed by one financial institution to the system, size should not be considered as a proxy of systemic importance. In other words, the "too big to fail" argument is not always valid, and measures on systemic importance should be considered. We provide the estimation methodology of systemic importance measures under the multivariate Extreme Value Theory (EVT) framework.

Keywords: Too big to fail; systemic risk; systemic importance; multivariate extreme value theory.

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1 Introduction

During financial crises, authorities have incentive to prevent the failure of a financial institution because such a failure would pose a significant risk to the financial system and consequently the broader economy. A bailout is usually supported by the argument that a financial firm is "too big to fail", i.e. larger banks exhibits higher systemic importance. A natural question arising from the debate on bailing out large financial firms is why particular large banks should be favored, i.e. are banks really too big to fail? An equivalent question is that "does the size of a bank really matter for its systemic impact if it fails?" The major difficulty in answering such a question is to design measures on the systemic importance of financial institutions. More specifically, we need to measure how much the failure of a particular bank will "contribute" to the systemic risk.

This paper deals with the problem in four steps. Firstly, we discuss the potential drawbacks of existing measures of systemic importance and propose alternatives that overcome these drawbacks. Secondly, we construct a theoretical model to assess whether larger banks correspond to higher systemic importance. Thirdly, we employ statistical method in estimating such measures within a constructed system consisting 28 US banks. Lastly, we use the estimated systemic importance measures and the size measures to empirically test the "too big to fail" statement.

Although the term "too big to fail" appears frequently in supporting bailout activities, its downside is well acknowledged in literature. Besides the distortion of the market discipline, the preference on large financial firms encourages excessive risk-taking behavior which potentially imposes more risk. Therefore, using "too big to fail" to support intervention will result in a moral hazard problem: large firms that the government is compelled to support were the greatest risk-takers during the boom period. Furthermore, such a moral hazard problem will provide incentive for firms to grow in order to be perceived as "too big to fail". We refer to Stern and Feldman (2004) for more discussions on the moral hazard problem.

Recently, both policy makers and academia start to distinguish size from the systemic importance by introducing new terms emphasizing on the potential systemic impact if a particular bank fails. For example, Bernanke (2009) addressed the problem of financial institutions that are deemed "too interconnected to fail"; Rajan (2009) used the term "too systemic to fail" to set the central focus of new regulation development. This urges the design of alternative measures on systemic importance. Measuring the systemic importance of financial institutions is particularly important for policy maker. It is the key issue in both financial stability assessment and macro-prudential supervision. On one hand, during crises, it is necessary to have such measures in order to justify bailout actions. On the other hand, it is crucial to supervise and monitor banks with higher systemic importance during regular period. Policy proposals for stabilizing financial system always rely on such measures. For instance, liquidity insurance is an alternative for defending systemic risk proposed by, e.g. Acharya et al. (2009) and Perotti and Suarez (2009). Systemic importance measures can serve as an indicator for pricing the corresponding insurance premium or taxation.

A few applicable measures on systemic importance appear in recent empirical studies. Adrian and Brunnermeier (2008) proposed the conditional Value-at-Risk (CoVaR) measure to measure risk spillover. Similar to the Value-at-Risk measure which quantifies the unconditional tail risk of a financial institution, the CoVaR quantifies how financial stress of one institution can increase the tail risk of the others. This measures provides a clear view on the bilateral relation between the tail risks of two financial institutions. From the setup of the CoVaR measure, it is designed for bilateral risk spillover. When applying CoVaR to assess the systemic importance of one financial institution to the system, it is necessary to construct a system indicator on the status of the system, and then analyze the bilateral relation between the system indicator and a specific bank. However, the complexity of the financial system is of a higher order than bilateral relations. Thus, a general indicator of the system is usually difficult to construct. Furthermore, the CoVaR measure is difficult to be generalized into a systemic context in order to analyze multiple financial institutions all together. As an alternative, Segoviano and Goodhart (2009) introduced the "probability that at least one bank becomes distressed" (PAO). Comparing these two measures, we observe that the CoVaR is a measure of conditional quantile, while PAO is a measure of conditional probability which acts as the counterpart of conditional quantile. Within a probability setup, generalization from bivariate to multivariate is possible. However, the PAO measure focuses on the probability of having a systemic impact-there exists at least one extra crisis-without specifying how severe the systemic impact is, for example, how many banks are influenced, when a particular bank fails. Therefore, it may not provide sufficient information on the systemic importance of a financial institution. Our empirical results partially reflects the less informative feature of PAO: the PAO measures remain in a constant level across different financial institutions and across time.

Extending the PAO measure while staying within the multivariate context, we propose the systemic impact index (SII) which measures the expected number of bank failures in the banking system given one particular bank fails. Clearly, the SII measure emphasizes more on the systemic impact. We also consider a reversed measure: the probability of a particular bank failure given that there exists at least one another failure in the system. We call it the vulnerability index (VI).

To test "too big to fail", we first consider a theoretical model from which both size and systemic importance measures–PAO, SII, VI, can be explicitly calculated. The model stems from the literature on systemic risk. In literature, two categories of models consider crises contagion and systemic risk in banking system: banks are systemically linked via either direct channels such as interbank market or indirect channels such as similar portfolio holdings in bank balance-sheets. Studies in the first category focus on the contagion effect, i.e. a crisis in one financial institution may cause crises in the others. The second category studies focus on modeling systemic risk, i.e. crises of different financial institutions may occur simultaneously. For the first category of studies, particularly on modeling interbank market, we refer to Allen and Gale (2000), Freixas et al. (2000), and Dasgupta (2004). Cifuentes et al. (2005) consider both two channels: similar portfolio holdings and mutual credit exposure.

They show that contagion is mainly driven by changes in asset prices, i.e. the indirect channel. For models focusing on the indirect channel, Lagunoff and Schreft (2001) assume that the return of one bank's portfolio depends on other banks' portfolio allocation and show that crises can either spread from one to another, or happen simultaneously due to forward-looking. de Vries (2005) starts from the fat tail property of the underlying assets shared by banks and argues that this creates the potential systemic breakdown. For an overview of contagion and systemic risk modeling, we refer to de Bandt and Hartmann (2001) and Allen et al. (2009).

Notice that the contagion literature mainly focuses on explaining the existence of a contagion effect, i.e. how a crisis in one financial institution leads to a crisis in the other. Thus, the models usually consider the risk spillover between only two banks. To address the financial system as a complex, network models combined with bilateral spillover are considered, e.g. Allen and Gale (2000). Following those theoretical studies, empirical analysis on systemic importance such as the CoVaR measure was designed for measuring bilateral relations. Differently, in order to address the systemic importance issue within a systemic context, we consider a multi-bank approach. Thus, we establish our theoretical model based on the indirect channel models, i.e. the second category. The fundamental intuition is that banks are interconnected due to the common exposures on their balance sheets. Thus, the systemic importance of a particularly bank is closely associated with how many different risky banking activities the bank is participating. This, in turn, may not be directly associated with the bank's size. Our model shows that banks concentrating on few specific activities can grow large without increasing their systemic importance.

This finding links the "too big to fail" problem to policy discussions on micro-level risk management and macro-level banking supervision. Since diversification is the usual tool in micro-level risk control. Financial institutions, particularly the large one, intend to take part in more banking activities in order to diversify away their individual risk. This may, on the other hand, increase its systemic importance. It is important to acknowledge the tradeoff between managing individual risk and keeping independency from the entire banking system. Reduction in individual risk can transfer the risk to systemic linkage and thus the systemic risk. Therefore, regulations on individual risk taking, such as Basel I and II type regulations, are not sufficient for maintaining stability of the entire financial system. A macro-prudential supervision is thus necessary for achieving financial stability. In a macro-prudential approach, it is necessary to consider both individual risk taking and the systemic importance of each individual financial institution.

Nextly, we demonstrate how to empirically estimate the proposed systemic importance measures. We adopt the multivariate Extreme Value Theory (EVT) framework for empirical estimation. When investigating crises, or rare events, the major difficulty is the scarcity of observations on crisis events. Since we intend to address the interconnectness of banking crises, we intend to observe joint crisis, the difficulty on lacking observations is further enhanced. A modern statistical instrument–EVT, fills the gap. The essential idea of EVT is to model the the intermediate level observations which are close to extreme, and extrapolate the observed properties into an extreme level. Therefore, the interconnectness of crises can be approximated by the interconnectness of tail events which are not necessarily at a crisis level. Univariate EVT has been applied in Value-at-Risk assessment for individual risks, see e.g. Embrechts et al. (1997). Recent development on multivariate EVT provides the opportunity to investigate extreme co-movements which serves our purpose. For instance, it has been applied to measure risk contagions across different financial markets in Poon et al. (2004) and Hartmann et al. (2004). An application of multivariate EVT in analyzing bilateral relations within banking system is in Hartmann et al. (2005). Beyond bivariate relations, the Global Financial Stability Report by IMF in April, 2009 (IMF (2009)) demonstrates the interconnection of financial distresses within a system consisting of three banks from an EVT analysis. In our empirical estimation of the systemic importance measures, we use multivariate EVT without restricting the number of banks under consideration.

We provide an empirical methodology on estimating the systemic importance measures–

PAO, SII, and VI–under multivariate EVT framework. An exercise for a constructed system consisting of 28 US banks is conducted. Then we test the correlation between the systemic importance measures and different measures on size. We find that, in general, systemic importance measures are not correlated with all bank size measures. Hence, the size of a financial institution should not be considered as a proxy of its systemic importance without careful justification. This agrees with our theoretical model. Overall, we conclude that it is necessary to have proper systemic importance measures for identifying the systemic important financial institutions.

2 Systemic importance measures

We consider a banking system containing d banks with their status indicated by $(X_1, \cdots, X_d)$: an extreme high value of X_i indicates a distress or crisis in bank i. Potential candidate for such a indicator can be the loss of equity returns, loss returns on balance sheet or Credit Default Swap (CDS) rates.

To define a crisis, it is necessary to consider proper high threshold. In our approach, we take Value-at-Risk as such a threshold. VaR at a tail probability level p is defined by

$$P(X > VaR(p)) = p.$$

Regulators consider the $p-$level as 1% or 0.1% in order to evaluate risk-taking behavior of a particular bank. We call a bank in crisis if $X > VaR(p)$ with a extremely low p. Here we do not specify the level p explicitly. Instead, we restrict that the $p-$level in the definition of banking crises is constant across all banks. Notice that banks may differ in their risk profiles which results in different endurability on risk-taking, i.e. different $VaR(p)$ level. Thus a unified level of loss for crisis definition may not fit the diversified situation of different financial institutions. Instead, an extreme event $X > VaP(p)$ corresponds to a return frequency as $1/p$. Fixing such a frequency for crisis definition takes the diversity of bank risk profiles into consideration. Furthermore, such a definition is aligned with the usual crisis description, for examples, with yearly data, a p equals to $1/50$ corresponds to "a crisis once per fifty years".

The systemic importance measures consider the impact on other financial institutions when a particular one falls into crisis. We start from the measure proposed by Segoviano and Goodhart (2009): the conditional probability of having at least one extra bank failure given a particular bank fails (PAO). From our model, it considers the following probability

$$PAO_i(p) := P(\{\exists j \neq i, \quad \text{s.t.} \quad X_j > VaR_j(p)\} \,|X_i > VaR_i(p)). \tag{1}$$

We argue that the PAO measure may not provide sufficient information in identifying the systemically important banks. Consider the following example. Suppose we have a banking system with banks categorized into two separate groups. Banks within each group are strongly linked, while crises do not spillover between the two groups. One group contains only two banks X_1 and X_2, while the other segment contains more banks $X_3, \cdots, X_d$, $d > 4$. In other words, X_1 and X_2 are highly related, $X_3, \cdots, X_d$ are highly related, while X_i and X_j are independent for any $1 \leq i \leq 2$ and $3 \leq j \leq d$.

Then the PAO measure for X_1, PAO_1 will be close to 1 since a crisis of X_1 will be accompanied by a crisis of X_2. On the other hand, the PAO measure for X_3, PAO_3 will also be close to 1 because of similar reasoning. When d is high, for example, $d = 10$, it is clear that X_3 is more systemically important than X_1 because it is associated with a larger fraction of the entire banking system. However, this will not be reflected by the comparison between PAO_1 and PAO_3. In this example, PAO_1 and PAO_3 should be at a high, comparable level.

Generally speaking, the PAO measure only provides the probability of having a systemic impact–having an extra crisis in other financial institutions, without specifying the size of such an impact–the number of extra crises in the entire system, when one particular bank fails. Hence, in the case that every institution in the system is connected to a certain fraction of the system, their PAO measures should all stay at a high, comparable level. With indistinguishable PAO measures, it is not sufficient to identify the systemically important financial institutions.

A natural extension of the PAO measure is to consider the expected number of failures in the system given a particular bank fails. This is defined as our systemic impact index (SII). Using the notation above, it can be written as

$$SII_i(p) := E(\sum_{j=1}^{d} 1_{X_j > VaR_j(p)} | X_i > VaR_i(p)), \tag{2}$$

where 1_A is the indicator function which equals to 1 when A holds, 0 otherwise.

Since the PAO and SII measures characterize the outlook of the financial system when a particular bank fails, a reverse question is what is the probability of a particular bank fails when the system exhibits some distress. To characterize the system distress, we use the same term as in the PAO measure: there exists at least one another bank failure. Hence, we define a vulnerability index (VI) by swapping the two items in the PAO definition as follows.

$$VI_i(p) := P(X_i > VaR_i(p) | \{\exists j \neq i, \ \text{ s.t. } \ X_j > VaR_j(p)\}). \tag{3}$$

From the definitions, all three measures summarize specific information on the risk spillover in the banking system. It is necessary to consider all of them when assessing the systemic importance of financial institutions.

3 Extreme Value Theory and systemic importance measures

3.1 The setup of Extreme Value Theory

Consider our d-bank setup. Modeling crisis of a particular financial institution i corresponds to modeling the tail distribution of X_i. Moreover, modeling the systemic risk, i.e. the extreme co-movements among $(X_1, \cdots, X_d)$, corresponds to modeling the tail dependence structure of $(X_1, \cdots, X_d)$. Extreme Value Theory provides models for such a purpose.

To assess VaR with a low probability level p, univariate EVT was applied in modeling the tail behavior of the loss. Since we focus on systemic risk, we omit the details on univariate risk modeling(see e.g. Embrechts et al. (1997)). Multivariate EVT models considers not only the tail behavior of individual X_i, but also the extreme co-movements among them.

The fundamental setup of multivariate EVT is as follows. For any $x_1, x_2, \cdots, x_d > 0$, as $p \to 0$, we assume that

$$\frac{P\left(X_1 > VaR_1(x_1 p) \text{ or } \cdots \text{ or } X_d > VaR_d(x_d p)\right)}{p} \to L(x_1, x_2, \cdots, x_d). \tag{4}$$

where VaR_i denotes the Value-at-Risk of X_i, and L is a finite positive function.[1] The L function characterizes the co-movement of extreme events, i.e. X_i exceeds a high threshold $VaR_i(x_i p)$. $(x_1, \cdots, x_d)$ controls the level of high threshold, which in turn controls the direction of extreme co-movement. For the property on L function, see de Haan and Ferreira (2006).

The value of L at a specific point, $L(1, 1, \cdots, 1)$, is a measure on the systemic linkage of

[1]Notice that considering the union of the events, i.e. using "or" in (4) is simply due to the definition of distribution function. Define $F(x_1, \cdots, x_d) = P(X_1 \le x_1, \cdots, X_d \le x_d)$ as the distribution function of $(X_1, \cdots, X_d)$. In order to consider the tail property, the assumption is made on the tail part $1 - F$, which is the probability of the union of extremal events as in relation (4).

banking crises among the d banks. From the definition in (4), we have that

$$L(1,1,\cdots,1)=\lim_{p\to 0}\frac{P\left(X_1>VaR_1(p)\text{ or }\cdots\text{ or }X_d>VaR_d(p)\right)}{p}. \tag{5}$$

In our banking system context, when p is at a low level, it approximates the quotient ratio between the probability that there exists at least one bank in crises and the tail probability p used in the definition of individual crisis. For bivariate case, this was considered by Hartmann et al. (2004) in measuring systemic risk across different financial markets.

We remark that L function is connected with the modern instrument on dependence modeling–copula. Denote the joint distribution function of $(X_1,\cdots,X_d)$ as $F(x_1,\cdots,x_d)$ while the marginal distributions are denoted as $F_i(x_i)$ for $i=1,\cdots,d$. Then there exists a unique distribution function $C(x_1,\cdots,x_d)$ on $[0,1]^d$ such that

$$F(x_1,\cdots,x_d)=C(F_1(x_1),\cdots,F_d(x_d)),$$

where all marginal distributions of C are standard uniform distributions. C is called the *copula*. By decomposing F into marginal distributions and copula, we separate the marginal information from the dependence structure summarized in the copular C. Condition (4) is equivalent to the following relation. For any $x_1,x_2,\cdots,x_d>0$, as $p\to 0$,

$$\frac{1-C(1-px_1,\cdots,1-px_d)}{p}\to L(x_1,x_2,\cdots,x_d)$$

Hence L function characterize the limit behavior of the copula C at the corner point $(1,\cdots,1)\in[0,1]^d$. In other words, L function captures the tail behavior of the copula C.

Linking L function to the tail behavior of copula yields the two following views. Firstly, since it is connected to copula, L function does not contain any marginal information. Thus in modeling banking crises, L function is irrelevant to the individual bank risk profiles. Secondly, with characterizing the tail behavior of copula, L function does not contain dependence information at a moderate level as in the copula C. Instead, L only contains tail dependence information. To summarize, L function contains the minimal required information in modeling extreme co-movements. Therefore, models on L are flexible to accommodate all potential marginal risk profiles and potential moderate level dependence structures. Compared to Segoviano and Goodhart (2009) where the Consistent Information Multivariate Density Optimizing (CIMDO) approach on estimating the copula C is considered, since models on the copula C incorporate the interconnection of banking system in regular time, estimating a copula model may miss-specify the tail dependence structure. Because we intend to model the interconnection of banking crises, considering the L function in the multivariate EVT approach is sufficient and less restrictive.

3.2 Systemic importance measures under multivariate EVT

Under multivariate EVT setup, the limit of the three systemic importance measures– PAO, SII, VI, can be directly calculated from the L function. Notice that in the definitions of the systemic importance measures, the probability level p for defining crisis is considered. However, we prove that, as $p \to 0$, the systemic importance measures can be well approximated by their limits.

The following proposition shows the limit of the PAO measure. The proof is in Appendix A.

Proposition 1 *Suppose* $(X_1, X_2, \cdots X_d)$ *follows the multivariate EVT setup. With the definition of PAO in* (1)*, we have that*

$$PAO_i := \lim_{p \to 0} PAO_i(p) = L_{\neq i}(1, 1, \cdots, 1) + 1 - L(1, 1, \cdots, 1), \tag{6}$$

where L *is the* L *function characterizing the tail dependence of* $(X_1, \cdots, X_d)$ *and* $L_{\neq i}(1, 1, \cdots, 1)$ *is the* L *function characterizing the tail dependence of*

$(X_1, \cdots, X_{i-1}, X_{i+1}, \cdots X_d)$.

Notice that L is defined on $\mathbb{R}^d$ while $L_{\neq i}$ is defined on $\mathbb{R}^{d-1}$. Moreover,

$$L_{\neq i}(1, 1, \cdots, 1) = L(1, 1, \cdots, 1, 0, 1, \cdots, 1),$$

where 0 appears at the $i-$th dimension.

Proposition 1 shows that when considering a low level p, the measure $PAO_i(p)$ is close to its limit denoted by PAO_i. For calculating PAO_i, it is sufficient if the L function is known. Therefore, we could have a proxy of the PAO measure with low level p by estimating the L function. In a theoretical model, the L function can be explicitly calculated. For an empirical analysis, L function can be estimated from historical data. We provide a practical guide for estimating the L function in Appendix B. For more discussions, see de Haan and Ferreira (2006).

Analogous to that of PAO, the limit of $VI(p)$ exists under multivariate EVT setup. We present the result in the following proposition but omit the proof.

Proposition 2 *Suppose* $(X_1, X_2, \cdots X_d)$ *follows the multivariate-EVT setup. With the definition of VI in* (3)*, we have that*

$$VI_i := \lim_{p \to 0} VI_i(p) = \frac{L_{\neq i}(1, 1, \cdots, 1) + 1 - L(1, 1, \cdots, 1)}{L_{\neq i}(1, 1, \cdots, 1)}, \tag{7}$$

with the same notation defined in Proposition 1.

From Proposition 1 and 2, we get the following corollary.

Corollary 3 $PAO_i > PAO_j$ *holds if and only if* $VI_i > VI_j$*.*

Corollary 3 implies that when considering the ranking instead of the absolute level, the VI measure is in fact as informative as the PAO measure.

The following proposition shows how to calculate the limit of SII under multivariate EVT setup. The proof is again postponed to Appendix A.

Proposition 4 *Suppose* $(X_1, X_2, \cdots X_d)$ *follows the multivariate-EVT setup. With the definition of SII in* (2)*, we have that*

$$SII_i := \lim_{p \to 0} SII_i(p) = \sum_{j=1}^{d} (2 - L_{i,j}(1,1)), \tag{8}$$

where $L_{i,j}$ *is the* L *function characterizing the tail dependence of* (X_i, X_j)*.*

Notice that

$$L_{i,j}(1,1) = L(0, \cdots, 0, 1, 0, \cdots, 0, 1, 0, \cdots, 0),$$

where 1 appears only at the $i-$th and $j-$th dimensions. We remark that $2 - L_{i,j}(1,1)$ is in fact a measure of bilateral relation between the crises of banks X_i and X_j. Thus the SII measure is an aggregation of measures on bilateral relations. This is parallel to the spillover index studied in Diebold and Yilmaz (2009) when measuring volatility spillover in a multivariate system: with measuring the volatility spillover between each pairs, the spillover index is an aggregation of the measures on bilateral relations.

Again, Proposition 4 shows that SII_i is a good approximation of $SII_i(p)$ when p is at a low level. And the estimation of SII_i is only based on the estimation of the L function. From the calculation of PAO and SII, they provide different information on systemic importance. A ranking based on PAO does not necessarily imply the same ranking on SII. Thus, it is still necessary to look at both two measures in order to obtain a full picture on the systemic importance of a bank.

To summarize, the multivariate EVT setup provides the opportunity to evaluate all three systemic importance measures –PAO, SII, and VI when the L function is known. Since the L function characterizes the tail dependence structure in $(X_1, \cdots, X_d)$, all the systemic importance measures can be viewed as characterization of the tail dependence among banking crises.

4 Are banks "too big to fail"? A theoretical model

We construct a simple model showing that large banks might have a lower level of systemic importance compared to small banks, i.e. banks are not necessarily too big to fail.

We start by reviewing a simple model in de Vries (2005) which explains the systemic risk within a two-bank system.

Consider two banks (X_1, X_2) holding exposures on two independent projects (Y_1, Y_2) as in the following affine portfolio model.

$$\begin{cases} X_1 = (1-\gamma)Y_1 + \gamma Y_2 \\ X_2 = \gamma Y_1 + (1-\gamma)Y_2, \end{cases} \tag{9}$$

where $0 < \gamma < 1$, (Y_1, Y_2) indicates the loss returns of the two projects, for example, two syndicated loans. To measure the systemic risk, de Vries (2005) considers the following measure,

$$\lim_{s\to\infty} E(\kappa|\kappa \geq 1) := \lim_{s\to\infty} \frac{P(X_1 > s) + P(X_2 > s)}{P(X_1 > s \text{ or } X_2 > s)}. \tag{10}$$

Intuitively, $E(\kappa|\kappa \geq 1)$ is the expected number of bank crises in the two-bank system given that at least one bank is in crisis. Here, the crisis of X_i is defined as $X_i > s$. It is proved that when Y_i, $i = 1, 2$, are normally distributed, $\lim_{s\to\infty} E(\kappa|\kappa \geq 1) = 1$. Thus, given that there exists at least one bank in crisis, the expected total number of crises is 1, i.e. there is no extra crisis except the existing one. This is called a weak fragility case. Hence, the systemic impact does not exist. On the contrary, suppose Y_i, $i = 1, 2$ follow a heavy-tailed distribution on the right tail. The result differs. The heavy-tail distribution is defined as

$$\begin{cases} P(Y_i > s) &= s^{-\alpha}K(s), \;\; i = 1, 2, \\ P(Y_i < -s) &= o(P(Y_i > s)), \end{cases} \tag{11}$$

where $\alpha > 0$ is called the *tail index* and $K(s)$ is a *slowly varying function* satisfying that

$$\lim_{t\to\infty} \frac{K(ts)}{K(s)} = 1,$$

for all $s > 0$. de Vries (2005) proved that for $\gamma \in [1/2, 1]$,

$$\lim_{s\to\infty} E(\kappa|\kappa \geq 1) = 1 + (1/\gamma - 1)^{\alpha} > 1.$$

This is called the strong fragility case because one existing crisis will be accompanied by potential extra crisis. Empirical literature extensively documented that the losses of asset returns follow heavy-tailed distributions. Therefore, the latter model based on heavy-tailed distributions reflects the empirical observations and explains the systemic risk existing in financial system.

We remark that when assuming the heavy-tailness of (Y_1, Y_2), and the affine portfolio model in (9), it is a direct consequence that (X_1, X_2) follows a two-dimensional EVT setup.[2] Moreover, if Y_1 and Y_2 are identically distributed, for a fixed tail probability p, the VaRs of X_1 and X_2 are equal, i.e. $VaR_1(p) = VaR_2(p)$. Replacing s by $VaR_i(p)$ in the definition of the systemic risk measure (10), and asking $p \to 0$, we get that

$$\lim_{p\to 0} E(\kappa|\kappa \geq 1) := \lim_{p\to 0} \frac{P(X_1 > VaR_1(p)) + P(X_2 > VaR_2(p))}{P(X_1 > VaR_1(p) \text{ or } X_2 > VaR_2(p))} = \frac{2}{L(1,1)}.$$

Therefore, the setup in de Vries (2005) imposes a multivariate-EVT setup, and the measure on the systemic risk is essentially based on $L(1, 1)$.

We point out that within this two-bank two-project model, it is not possible to differentiate the systemic importance of the two banks. From the model and Proposition 1, we get that

$$SII_i = 3 - L(1,1), \;\; i = 1, 2.$$

[2]For a formal proof, see Zhou (2008, Chapter 5).

Hence the systemic importance of the two banks measured by SII are equal. Similar results hold for the other two measures, PAO and VI. Intuitively, within a two-bank setup, the linkage of crises is a mutual bilateral relation. Hence, one could not distinguish the systemic importance of the two banks. In order to construct a model in which it is possible to compare the systemic importance at different levels, it is necessary to generalize de Vries (2005) model to a system consisting of at least three banks.

Next, we consider the size issue. In de Vries (2005) two-bank model, suppose that both the two banks X_1 and X_2 have total capital 1, and both the two projects Y_1 and Y_2 receive capital 1. According to the affine portfolio model (9), the capital market is clear. In this case the two banks have the same size in terms of total assets. In order to differentiate the sizes of the banks, a more complex affine portfolio model is necessary.

Addressing the above two points, we consider a model with three banks (X_1, X_2, X_3) and three independent projects (Y_1, Y_2, Y_3). Suppose X_1 holds capital 2 for investment, while X_2 and X_3 hold capital 1 each. Moreover, suppose the project Y_1 demands an investment 2, while Y_2 and Y_3 have a capital demand 1 each. Then the market is clear with the following affine portfolio model.

$$\begin{cases} X_1 &= (2-2\gamma)Y_1 + \gamma Y_2 + \gamma Y_3 \\ X_2 &= \gamma Y_1 + (1-\gamma-\mu)Y_2 + \mu Y_3, \\ X_3 &= \gamma Y_1 + \mu Y_2 + (1-\gamma-\mu)Y_3, \end{cases} \tag{12}$$

where $0 < \gamma, \mu < 1$ and $\gamma + \mu < 1$. Clearly, this is not the only possible allocation for market clearance. Nevertheless, it is sufficient to demonstrate our argument on "too big to fail" problem. Notice that X_1 is a larger bank compared to X_2 and X_3. Here the size refers to the total asset investing in the risky projects. We intend to compare the systemic importance of X_1 with those of X_2 and X_3.

The two parameters γ and μ are interpreted as the control of similarity in portfolio holdings across three banks. The parameter γ controls the similarity between the large bank and the small banks. When γ is close to 1, the strategy of the large bank is different from those of the two small banks, while the two small banks hold similar portfolios. When $\gamma = 1/2$, the large bank has exposures on all three projects proportional to their capital demands. Hence the large bank is involved in all projects. When γ is close to 0, the large bank is again different from the two small banks. In the latest case, the similarity of the two small banks is further controlled by the parameter μ: a μ lying in the middle of $(0, 1-\gamma)$ shows that the two small banks are similar in portfolio holding, while a μ lying close to the two corners of $(0, 1-\gamma)$ corresponds to different strategies between the two small banks.

Suppose all Y_i follow a heavy-tail distribution defined in (11) for $i = 1, 2, 3$. Then similar to the two-bank case, (X_1, X_2, X_3) follows a three-dimensional EVT setup. Instead of discussing all possible values on the parameters (γ, μ), we focus on three cases: γ is close to 1, $\gamma = 1/2$ and γ is close to 0. The results on comparing the SII measures are in the following theorem. The proof is again in Appendix A.

Theorem 5 *Consider a three-bank three-project model with the affine portfolio given in* (12)*. Suppose the losses of the three projects exhibit heavy-tail feature as in* (11) *with* $\alpha > 1$*. We have the following relations.*

Case I: $\frac{2}{3} \leq \gamma < 1$

$$SSI_1 < 1 = SSI_2 = SSI_3.$$

Case II: $\gamma = \frac{1}{2}$

$$SSI_1 \geq SSI_2 = SSI_3.$$

The equality holds if and only if $\mu = 1/4$*.*

Case III: $0 < \gamma < \frac{1}{3}$

There exists a $\mu^* < \frac{1-\gamma}{2}$*, such that for any* μ *satisfying* $\mu^* < \mu < 1 - \gamma - \mu^*$*,*

$$SSI_1 < SSI_2 = SSI_3.$$

On the other hand, for any μ satisfying $0 < \mu < \mu^*$ or $1 - \gamma - \mu^* < \mu < 1 - \gamma$, we have

$$SSI_1 > SSI_2 = SSI_3.$$

When $\mu = \mu^*$ or $\mu = 1 - \gamma - \mu^*$,

$$SII_1 = SII_2 = SII_3.$$

The following lemma shows that the comparison among the PAO measures follows the comparison among the SII measures in the three-bank model.

Lemma 6 *With the assumptions in Theorem 5, the order of PAO follows the order of SII, i.e. for any $i \neq j$, $PAO_i > PAO_j$ holds if and only if $SII_i > SII_j$.*

Combining Lemma 6 and Corollary 3, we have that the order of VI also follows the order of SII in the simple three-bank model. Notice that the three-bank model is a very specific and simple case. The results in Lemma 6 does not hold in a general context when the number of banks is more than three. Therefore, for empirical study within a multi-bank system, it is still necessary to estimate all three measure, which may provide different views.

We interpret the results in Theorem 5 as follows.

In the case that γ is close to 1, the large bank X_1 focuses on the two smaller project Y_2 and Y_3, while small banks X_2 and X_3 focus on the large project Y_1. In this case, the balance sheet of the large bank is quite different from the small banks, while the two small banks are holding similar portfolio. Therefore, the large bank has less systemic linkage to the other two small banks. We observed that it is less systemically important compared to the others, i.e. the large bank is not "too big to fail".

In the case $\gamma = 1/2$, the large bank X_1 invests $(1, 1/2, 1/2)$ at three projects. Hence it is involved in all three projects which creates the linkage to the other two small banks. In this case, it is "too big to fail". The inequality turns to be an equation if and only if $\mu = 1/4$. For

$\mu = 1/4$, the three banks all invest in three projects proportional to their capital demands. They have exactly the same strategy in managing their portfolios. A crisis in any of the three banks will be accompanied by the crises in the other two. Therefore, they are equally systemically important. Excluding $\mu = 1/4$, the large bank will be the most systemically important bank.

In the case that γ is close to 0, the large bank X_1 focuses on the large project Y_1, while still has exposures on Y_2 and Y_3. The small banks X_2, X_3 focuses on the two small projects Y_2 and Y_3. Now it matters how similar their portfolios are. In case μ is in the middle ($\mu^* < \mu < 1 - \gamma - \mu^*$), the two small banks have relatively similar composition of their balance sheets. Hence, they are more systemically important compared to the large bank. In case μ is close to the corner ($0 < \mu < \mu^*$ or $1 - \gamma - \mu^* < \mu < 1 - \gamma$), the two small banks differ in their balance sheets. Since the large bank still has exposures on Y_2 and Y_3 equally, it is the most systemically important bank. It is worth mentioning that the systemic importance of bank X_1 is determined not only by its own risk positions, but also the risk taking of the others. Even though the portfolio of bank X_1 is fixed by fixing γ, the change of the μ parameter which only changes the portolios of the other two banks can still result in a change of the systemic importance of X_1.

To summarize, we observe that "too big" is not necessarily the reason for being "too systemically important". Instead, having a balance sheet that expose to more risky projects would turn a bank to be more systemically important. Here, we regard Y_i, $i = 1, 2, 3$ as different risky projects. One may also regard them as different risky banking activities. Therefore, a bank that is more diversified in banking activities may turn to be "too big to fail".

Notice that having a diversified balance sheet is the usual way of managing individual risk. To obtain the diversification effect encourages banks, particularly large banks, to take part in more banking activities. The above discussion shows that this will result in a "too big to fail" problem at the same time. Conversely, a large bank specialized in a limited number of banking activities, might be risky as an individual, but less systemically important at the same time. There is a tradeoff between managing individual risk and keeping independency from the entire banking system. For maintaining the stability of the financial system, it is necessary to recognize such a tradeoff and impose proper regulations in order to balance the individual risk taking and the systemic importance.

5 Empirical results

5.1 Empirical setup and data

We apply the three proposed measures on systemic importance to an artificially constructed financial system consisting of 28 US banks. After estimating the three measures, we calculate the correlation coefficient between these measures and the measures of the size of the banks. From the test on correlation coefficients, we can empirically test whether larger banks exhibit larger systemic importance, i.e. testing the "too big to fail" argument. We also consider a moving window approach, which demonstrates the variation of the systemic importance measures across time.

The dataset for constructing the systemic importance measures consists of daily equity returns of 28 US banks listed on New York Stock Exchange (NYSE) from 1987 to 2009 (23 years).[3] The chosen banks are listed in Table 1 with the descriptive statistics on their stock returns.

The dataset regarding the size of the banks consists of various measures. We consider total assets, total equity, total debt, for the 28 banks. They are in a yearly frequency from 1987 to 2009. We present their end-of-2009 values as well as the average values across the 23 years in Table 2.

[3] The data are obtained from Datastream. Three selection criterions are applied: the financial institutions should be classified in the sector "Banks"; they should be traded primarily on the NYSE (DS code starting with "U:"); the time series should be active from the beginning of 1987 till the end of 2009. The selection procedure results in 28 banks.

From the descriptive statistics of the equity returns, we observe that all daily returns exhibit high kurtosis compared to the kurtosis from normal distribution, 3. It indicate that the daily stock returns may follow heavy-tailed distributions. Moreover, most of the skewness measures are negative or close to 0. This indicates that the heavy-tailness may come from the downside of the distribution, i.e. the losses. Hence, our heavy-tail assumption on the tail of losses are valid for the employed dataset.

From the descriptive statistics of the size measures, we observe a large variation on the size measures of the selected banks. The top three largest banks in the list, BANK OF AMERICA, JP MORGAN CHASE & CO. and CITIGROUP are approximately 1,000 times larger than the smallest bank in the list, STERLING BANC., in all aspects. Although the criterion that the selected banks have to be active in the stock market for 23 years may result in a sample selection bias: such banks are more likely to be large banks, the variation of the size measures show that the constructed banking system contains both large and small banks.

Using the stock returns is a natural choice for our approach in analyzing the systemic importance. The restriction imposed by the methodology of estimating the L function is that the sample size has to be sufficient, see Appendix B. Moreover, since we intend to perform an moving window analysis, the restriction on the length of the times series is further enhanced. Therefore, daily or higher frequency is necessary for a full non-parametric approach. This limits us to using financial market data. Equity returns are the most convenient choice. Other high frequency indicators such as Credit Default Swap (CDS) rates are also possible. The CDS data does not go back for sufficient long period, which limits us from performing a moving window analysis. It is also possible to apply the proposed methodology with low frequency data, such as bank balance sheet data. In that case, a full non-parametric estimate on the L function is not applicable. Instead, further modeling on the L function should be considered. In this study we intend to illustrate the methodology without modeling the L function. Hence, we stick to the equity return data.

5.2 Estimation of the systemic importance measures

By estimating the L function (for details, see Appendix B), we obtain the estimates of the three systemic importance measures–SII, PAO, and VI across the full sample period as shown in Table 3. We start with the PAO measure proposed by Segoviano and Goodhart (2009). A general observation is that all the estimates are quite high (above 60%). This agrees with our prediction that the PAO measures of all banks in a system lie in a relatively high level, and do not differ much from each other. Since the PAO measure is directly connected to the VI measure as shown in Corollary 3, similar feature is observed for the VI measures. In fact, the order of the VI measures follow the order of the PAO measures as proved in Corollary 3. To name a few with the highest PAO and VI measures, BANK OF AMERICA reports the highest PAO at 94.44% followed by MARSHALL & ILSLEY with 93.89% and KEYCORP with 93.33%. The VI measure corresponding to these three institutions are 10.28%, 10.23% and 10.18% respectively. At the bottom of the list ranked by the PAO measure, we have FNB, BANCORPSOUTH and CMTY.BK.SY.

The SII measure introduced in this paper gives a somewhat different outlook compared to the PAO measure. The three lowest SII banks are the same as those with the lowest three PAO, although with a different order. The lowest SII measure comes from CMTY.BK.SY., which is 6.53. It means that if CMTY.BK.SY. is experiencing a crisis, it will be accompanied by on average 5.53 extra failures in this system. Compared to the size of the system, 28 banks, this is not a high systemic impact. The highest estimated SII measure is 12.44 from KEYCORP, it is almost as twice as the lowest value. A crisis of KEYCORP will be accompanied by on average 11.44 extra crises in this system, as twice as the systemic impact of CMTY.BK.SY. Hence we observe a variation of the SII measure across different banks. To name a few with the highest SII measures, the top three are KEYCORP (12.44) SUNTRUST BANKS (12.11) and COMERICA (12.02). They are different from the banks with the top three highest PAO. In general, ranking the PAO measures is different from ranking the SII measures. For example, the bank with the highest PAO, BANK OF AMERICA, is only ranked at 10th place among all banks when considering the SII measure.

To summarize, the comparison between the three measures shows that although having different economic background, the PAO measure and the VI measure are equally informative in terms of ranking the systemic importance. On the contrary, the SII measure provides information on the size of the systemic impact corresponding to the failure of a particular bank. It provides a different view compared to the other two. Across different banks, the SII measure varies while the PAO measures remain in a high, comparable level. This agrees with our theoretical prediction. Therefore, the SII measure is more informative in distinguishing the systemic importance of financial institutions.

Besides estimating the three systemic importance measures from the full sample period, we consider subsamples for the estimation and perform a moving window approach. By moving the subsample window, we could obtain time varying estimation on the systemic importance measures. We consider the estimation window as 2,000 days (approximately 8 years), then move the estimation window month by month. The first possible window ends at September 1994. In other words, the first estimation considers data ending at September 30, 1994 and going back 2,000 days. From then on, we take the end of each month as the ending day and use the data back 2,000 days. By moving the estimation window month by month, we observe the estimates at the end of each month from September 1994 to December 2009. For simplicity we only plot the results for selected banks[4] as shown in Figure 1. The upper panel shows the moving window SII and the bottom panel shows the moving window PAO. The two vertical lines in the two figures correspond to the failures of two large investment banks: Bear Stearns (March 2008) and Lehman Brothers (September 2008).

From the moving window SII, we observe that the SII measures was gradually increased from 1998 to 2003, then remained in a relatively stable till end of 2006. From 2007 there was a sharp rising. The sharp rising of SII started before the failure of Bear Stearns, and

[4]We select the least systemic important bank, CMTY.BK.SY. and the two largest banks, BANK OF AMERICA and JP MORGAN CHASE & CO. in the plots. BANK OF AMERICA is the most systemic important bank in terms of PAO.

continued with the failure of Lehman Brothers, until early 2009. From mid 2009, the SII measures turned to be stable and with a slight downward slope. On the contrary, the PAO measures are stable across time, particularly for the large banks. Only for the least systemic important bank, some variation can be observed. This is due to the fact that the PAO measures of large banks was already in a high level in the early period of our sample. It is thus difficult to further rise to a higher level.

The observations from the moving window approach again confirms our theoretical prediction that the PAO measures always stay at a high, comparable level, while the SII measure varies across time and across institutions. The sharp rising of the SII measure addresses the crisis starting from 2008, hence, is more informative in analyzing the systemic risk in a financial system. Although we observe an early rising before the crisis, we do not emphasize that the SII measure is a predictor of the crisis. The sharp rising of SII might either be a predictor of the crisis, or an ex-post consequence caused by the crisis. The intuition for the latter possibility is as follows. Banks intend to take similar strategies, such as fire sales, during a crisis, which may result in more similar portfolio holdings across all banks. According to the theoretical model in Section 4, that leads to an increase of the SII measures on all banks. Hence, it is still an open question on analyzing the timing of the sharp rising of the SII measures.

There are a few other observations from the moving window analysis. Notice that the constructed financial system contains 28 banks. A SII measure at 15 means that given a certain bank fails, there will be on average extra 14 bank failures simultaneously. This is half of the entire system, which must be considered as a severe risk. Hence, the observed SII measures from end 2008 to mid 2009 indicate that the banking system suffers severe systemic risk during the crisis. Moreover, it is remarkable that CMTY.BK.SY., the least systemically important bank from the full sample analysis, also shows the least systemic importance during the crisis. Nevertheless, the absolute level of the SII reached a comparable level with the other large banks. This suggests that size may not be a good proxy of systemic importance, particularly during crisis period.

5.3 Test "too big to fail"

With the estimated systemic importance measure, we check whether they are correlated with the size measures. The correlation test is across different banks, thus, we need to have a unified value for each individual bank on each size measure. Since the sample period ends in year 2009, we firstly consider the end-of-2009 values of each size measure. The second approach is to take average of the size measures over the full sample period, i.e. from 1987 to 2009. Then, we calculate the Pearson correlation coefficients between each pair of size measure and systemic importance measure across 28 banks. Moreover, we test whether the correlation coefficient is significantly different from zero. The results are shown in Table 4.

Generally, the PAO and VI measures are positively correlated with the size measure with significance level 5% or 10%. Differently, the SII measure is not correlated to any size measure. We argued throughout the paper that the SII measure is a more informative measure on the systemic importance by considering the systemic impact of the failure of a particular bank. We conclude that the systemic impact of a bank failure is not correlated with the size measures. Therefore, "too big to fail" is not valid on the impact level, at least for the constructed banking system. Nevertheless, with the positive correlation between the PAO and VI measures, the big banks are more likely to cause extra crisis in the system. To summarize, it is not proper to use the size measures as a proxy of systemic importance. Instead, it is necessary to consider all the systemic importance measures when identifying systemic important financial institutions.

We have done extensive robustness check for the observed results. Firstly, with the full sample estimation on the systemic risk measure, we consider the size measure in other years (e.g. end of 2008, 2007, etc.). The results are comparable with that uses the end of 2009 value. We omit the details.

Secondly, with the moving window results on the systemic importance measures, we can get the end-of-year estimates on the systemic importance measures from 1994 to 2009 (16 years). We pool all the bank-year results together which results in $28 \cdot 16 = 448$ estimates for each systemic importance measure, and also 448 observations for each size measure. We then calculate the Pearson correlation coefficient for each pair, and repeat the test on the significancy. The results are in the first panel of Table 5. All three systemic importance measures are not correlated with any size measure.

Thirdly, since we have the 16 year data on the systemic importance and size for 28 banks, we also perform the correlation analysis at each year level. We observe that the significant positive correlation between size and the PAO measure (and the VI measure) is robust for year 1994-1999. From 2000 to 2009, the significance disappear. Interestingly, for the first period, the SII measure is also positively correlated with the three size measures. The (in)significant results are robust within each subperiod. To further explore these phenomena, we divide the period 1994-2009 into two subperiod: 1994-1999, 2000-2009, according to the individual year results. Then we pool bank-year data in each subperiod and repeat the correlation analysis. The results are reported in the second and third panels of Table 5. It confirms the results from the individual year analysis: in the first period, all three systemic importance measures are highly correlated with the size measures, while in the second period, the correlations disappear. It suggests that using size as a proxy for systemic importance was proper in 1990s, but the situation has been changed from the beginning of the new century. Therefore, it is particularly important to consider the measures on systemic importance within the current financial world.

Last but not least, using daily equity returns may suffer a potential problem due to the heteroscedasticity in high frequency financial returns. Diebold et al. (2000) argues that the fundamental assumption of EVT that the observations are independent and identical distributed (i.i.d.) is usually violated when dealing with high frequency financial returns. They provide two potential solutions: firstly, one may investigate low frequency block maxima which reduces the dependency across time; secondly, one may apply a conditional extreme value model. Chavez-Demoulin et al. (2005) considers a point process approach to de-cluster the financial return data. This is in line with the second type solution proposed by Diebold et al. (2000). Notice that the non-i.i.d. problem is mainly for evaluating the univariate tail events, while our study is under a multivariate framework which models cross-sectional dependence of tail events. The non-i.i.d. problem on the time dimension is of a less concern. Nevertheless, we conduct a robustness check in line with the first type solution proposed by Diebold et al. (2000). Instead of daily returns, we consider monthly returns. This reduces the dependency on the time dimension as taking block maxima. Due to the low frequency, we have to use the full sample (276 months) for the analysis. The estimation of the three systemic risk measures are similar to that uses daily returns, at least in terms of ranking the systemic importance. The results are in Table 6. We repeat the correlation analysis as shown in Table 7. Again, we confirmed that the systemic importance measures are not correlated with the size measures.

6 Conclusion

In this paper, we consider three measures on the systemic importance of financial institutions in a financial system. Since we regard the system as the combination of individual institutions, it is a multivariate relation rather than bilateral. We consider the PAO measure proposed by Segoviano and Goodhart (2009) as well as new measures: the SII measure which measures the size of the systemic impact if one bank fails, and the VI measure which measures the impact on a particular bank when the other part of the system is in distress.

We use a theoretical model based on affine portfolio holdings to show that a large bank is not necessarily more systemically important in terms of exhibiting high levels of the three proposed systemic importance measures. Only with diversified banking activities, a large bank may become systemically important. On the contrary, an isolated large bank will not be harmful to the system. The discussion can be extended to regulation issues. With acknowledging the tradeoff between micro-level risk management and systemic importance, we conclude the necessity of the macro-prudential approaches on financial regulation and supervision. Moreover, measuring systemic importance is the key to identify systemic important institutions when imposing macro-prudential regulations.

Besides the theoretical model, we conduct an empirical analysis estimating the systemic importance measures. Multivariate EVT is employed in estimating the systemic importance measures. The empirical observation confirms that the PAO measure is not as informative as the SII measure in terms of distinguishing the systemically important banks. A moving window analysis shows similar results. Moreover, the VI measure is shown to be as informative as the PAO measure in terms of identifying systemically important banks.

With the estimated systemic importance measures, we test whether they are correlated with the measures on the bank size. Regarding the systemic impact of bank failure measured by the SII measure, there is no empirical evidence supporting the "too big to fail" arguments. On the contrary, the other two systemic importance measures PAO and VI are positively correlated to the size measures. We identify that in more recent period, the correlations disappear, which suggests that the systemic importance measures should be particularly considered in recent years.

The empirical analysis in this paper is based on an artificial bank system. Therefore, the evidence from the empirical analysis should not be regarded as a strong disproof on the "too big to fail" argument. The bottom line is that we show the possibility of having a banking system in which the size measures are not good proxy of the systemic importance. Therefore, size should not be automatically regarded as a proxy of systemic importance.

Although in the current empirical analysis, our proposed SII measure is shown to be more informative than the PAO measure proposed by Segoviano and Goodhart (2009), we address one potential drawback of the SII measure: it is a simple counting measure without taking into account the differences between potential losses when different financial institutions fall in crisis. In other words, when calculating the expected number of failures in the system, whether it is a failure of a big bank or a small bank is not distinguished in the SII measure. This can be improved by considering the expected total loss in the system given one bank fails, i.e. calculating expected shortfall conditional on a certain bank failure and incorporating with the sizes of all banks. Acharya et al. (2009) designs systemic importance measures in this manner, while taking the heavy-tailness of individual returns into consideration. To model the dependence structure, they apply the dynamic conditional correlation (DCC) models which has an EVT flavor but deviate from the multivariate EVT framework. A systemic importance measure addressing expected shortfall under the multivariate EVT framework may overcome the drawback of the proposed SII measure. This is left for future research.

References

V.V. Acharya, J.A.C. Santos, and T. Yorulmazer. Systemic Risk and Deposit Insurance Premiums. *Economic Policy Review, Federal Reserve Bank of New York*, forthcoming, 2009.

T. Adrian and M. Brunnermeier. CoVaR. *Federal Reserve Bank of New York Staff Report*, 348, 2008.

F. Allen and D. Gale. Financial contagion. *Journal of Political Economy*, 108:1–33, 2000.

F. Allen, E. Carletti, and A. Babus. Financial Crises: Theory and Evidence. *Annual Review of Financial Economics*, 1(1), 2009.

B. Bernanke. Financial Reform to Address Systemic Risk. *Speech at the Council on Foreign Relations*, March 10, 2009.

V. Chavez-Demoulin, A.C. Davison, and A.J. McNeil. Estimating value-at-risk: a point process approach. *Quantitative Finance*, 5(2):227–234, 2005.

R. Cifuentes, H.S. Shin, and G. Ferrucci. Liquidity risk and contagion. *Journal of the European Economic Association*, 3(2-3):556–566, 2005.

A. Dasgupta. Financial contagion through capital connections: a model of the origin and spread of bank panics. *Journal of the European Economic Association*, 2(6):1049–1084, 2004.

O. de Bandt and P. Hartmann. Systemic risk in banking: a survey. In C. Goodhart and G. Illing, editors, *Financial Crisis, Contagion, and the Lender of Last Resort: A Reader*, pages 249–298. Oxford University Press, 2001.

L. de Haan and A. Ferreira. *Extreme Value Theory: An Introduction*. Springer, 2006.

C.G. de Vries. The simple economics of bank fragility. *Journal of Banking and Finance*, 29 (4):803–825, 2005.

F.X. Diebold and K. Yilmaz. Measuring Financial Asset Return and Volatility Spillovers, with Application to Global Equity Markets*. *Economic Journal*, 119(534):158–171, 2009.

F.X. Diebold, T. Schuermann, and J.D. Stroughair. Pitfalls and opportunities in the use of extreme value theory in risk management. *Journal of Risk Finance*, 1(2):30–35, 2000.

P. Embrechts, C. Klüppelberg, and T. Mikosch. *Modelling extremal events: for insurance and finance*. Springer, 1997.

X. Freixas, B.M. Parigi, and J.C. Rochet. Systemic risk, interbank relations, and liquidity provision by the central bank. *Journal of Money, Credit and Banking*, 32:611–638, 2000.

P. Hartmann, S. Straetmans, and C.G. de Vries. Asset market linkages in crisis periods. *Review of Economics and Statistics*, 86:313–326, 2004.

P. Hartmann, S. Straetmans, and C.G. de Vries. Banking System Stability: A Cross-Atlantic Perspective. *NBER Working Papers*, 2005.

IMF. Global financial stability report. *International Monetary Fund*, April, 2009.

R. Lagunoff and S.L. Schreft. A model of financial fragility. *Journal of Economic Theory*, 99(1):220–264, 2001.

E. Perotti and J. Suarez. Liquidity Insurance for Systemic Crises. *CEPR Policy Insight*, 31, 2009.

S. Poon, M. Rockinger, and J. Tawn. Extreme value dependence in financial markets: Diagnostics, models, and financial implications. *The Review of Financial Studies*, 17, No.2:581–610, 2004.

R.G. Rajan. Too systemic to fail: consequences, causes and potential remedies. *Written statement to the Senate Banking Committee Hearings*, May 6, 2009.

M. Segoviano and C. Goodhart. Banking Stability Measures. Technical report, IMF Working Paper 09/04 (Washington: International Monetary Fund), 2009.

G.H. Stern and R.J. Feldman. *Too big to fail: The hazards of bank bailouts*. Brookings Institution Press, 2004.

C. Zhou. *On Extreme Value Statistics*. PhD thesis, Tinbergen Institute, 2008.

Appendix A Proofs

Proof of Proposition 1.

Recall the definition of the PAO measure in (1). We have that

$$\begin{aligned}
PAO_i(p) = & \frac{P(\{\exists j \neq i, \quad \text{s.t.} \quad X_j > VaR_j(p)\} \cap \{X_i > VaR_i(p)\})}{P(X_i > VaR_i(p))} \\
& = \frac{1}{p} P(\{\exists j \neq i, \quad \text{s.t.} \quad X_j > VaR_j(p)\}) + 1 \\
& \quad - \frac{1}{p} P(\{\exists j \neq i, \quad \text{s.t.} \quad X_j > VaR_j(p)\} \cup \{X_i > VaR_i(p)\}) \\
& = \frac{1}{p} P(\{\exists j \neq i, \quad \text{s.t.} \quad X_j > VaR_j(p)\}) + 1 - \frac{1}{p} P(\{\exists j, \quad \text{s.t.} \quad X_j > VaR_j(p)\}) \\
& =: I_1 + 1 - I_2
\end{aligned}$$

From the definition of the L function in (4), as $p \to 0$, $I_1 \to L_{\neq i}(1, 1, \cdots, 1)$ and $I_2 \to L(1, 1, \cdots, 1)$, which implies (6). ■

Proof of Proposition 4.

Recall the definition of the SII measure in (2). We have that

$$\begin{aligned}
SII_i(p) &= \sum_{j=1}^{d} E\left(1_{X_j > VaR_j(p)} | X_i > VaR_i(p)\right) \\
&= \sum_{j=1}^{d} P\left(X_j > VaR_j(p) | X_i > VaR_i(p)\right) \\
&= \sum_{j=1}^{d} \frac{P(X_j > VaR_j(p), X_i > VaR_i(p))}{P(X_i > VaR_i(p))} \\
&= \sum_{j=1}^{d} \frac{P(X_j > VaR_j(p)) + P(X_i > VaR_i(p)) - P(X_j > VaR_j(p) \text{ or } X_i > VaR_i(p))}{p} \\
&= \sum_{j=1}^{d} 2 - \frac{P(X_j > VaR_j(p) \text{ or } X_i > VaR_i(p))}{p}
\end{aligned}$$

From the definition of the L function in (4), as $p \to 0$,

$$\frac{P(X_j > VaR_j(p) \text{ or } X_i > VaR_i(p))}{p} \to L_{i,j}(1,1).$$

The relation (8) is thus proved. ■

Proof of Corollary 3.

Since $L(1,1,\cdots,1)-1<0$, the relation (7) implies that a higher value of the VI measure corresponds to a higher level of $L_{\neq i}(1,1,\cdots,1)$. Together with (6). The corollary follows.

■

Proof of Theorem 5.

Firstly, since the heavy-tail feature in (11) assumes that the right tail of Y_i dominates its left tail, it is sufficient to assume that Y_i are all positive random variables for $i=1,2,3$, i.e. without left tail. We adopt this assumption in the rest of the proof.

We use the Feller convolution theorem to deal with the sum of independently heavy-tailed distributed random variables as in the following lemma.

Lemma 7 *Suppose positive random variables U and V are independent. Assume that they are both heavy-tailed distributed with the same tail index α. Then, as $s \to \infty$,*

$$P(U+V>s) \sim P(U>s)+P(V>s).$$

Notice that the heavy-tailed feature implies $P(U>s)P(V>s)=o(P(U>s)+P(V>s))$, as $s \to \infty$. Hence, the Feller convolution theorem is equivalent to

$$P(U+V>s) \sim P(\max(U,V)>s),$$

i.e., the sum and the maximum of two independently heavy-tailed distributed random variables are tail equivalent. A proof using sets manipulation can be found in Embrechts et al. (1997). With an analogous proof under multivariate framework, a multivariate version of the Feller theorem can be obtained. In multivariate case, the tail equivalence between two random vectors is defined as the combination of having tail equivalence for each marginal distribution and having the same L function for the tail dependence structure. We present the result in a 2-d context in the following lemma without providing the proof.

Lemma 8 *Suppose positive random variables U_1 and U_2 are independent. Assume that they are both heavy-tailed distributed with the same tail index α. Then for any positive constants m_{ij}, $1 \leq i, j \leq 2$, we have that the distribution functions of $(m_{11}U_1 + m_{12}U_2, m_{21}U_1 + m_{22}U_2)$ and $(m_{11}U_1 \vee m_{12}U_2, m_{21}U_1 \vee m_{22}U_2)$ are tail equivalent.*

To prove Theorem 5, it is necessary to calculate the L function of (X_1, X_2, X_3) on points $(1, 1, 0)$, $(1, 0, 1)$ and $(0, 1, 1)$. The main instrument in the calculation is Lemma 8. We start by comparing the individual risks taken by the three banks.

Proposition 9 *For the three-bank model on (X_1, X_2, X_3), as $p \to 0$,*

$$VaR_2(p) = VaR_3(p) \sim cVaR_1(p), \tag{13}$$

where

$$c := \left(\frac{(2-2\gamma)^\alpha + 2\gamma^\alpha}{\gamma^\alpha + \mu^\alpha + (1-\gamma-\mu)^\alpha} \right)^{-1/\alpha}.$$

Proof of Proposition 9.

From Lemma 7, we get that, as $s \to \infty$,

$$\begin{aligned} P(X_1 > s) &\sim P((2-2\gamma)Y_1 > s) + P(\gamma Y_2 > s) + P(\gamma Y_3 > s) \\ &= ((2-2\gamma)^\alpha + 2\gamma^\alpha)\, s^{-\alpha} K(s) \end{aligned} \tag{14}$$

Similarly, we get that

$$P(X_2 > s) = P(X_3 > s) \sim (\gamma^\alpha + \mu^\alpha + (1-\gamma-\mu)^\alpha)\, s^{-\alpha} K(s) \tag{15}$$

By comparing (14) and (15), the relation (13) is a direct consequence. ■

We remark that $c < 1$. This agrees with the fact that the large bank X_1 can take more risks than the small banks. On the other hand, when $\gamma > 1/2$, we get $c > 1/2$. In this case, the comparison between bank risk taking are not proportional to their sizes. The small banks are taking relatively more risks. In other words, the large bank X_1 benefits from diversification.

Next, we calculate $L(1,1,0)$. Denote $v(p) := VaR_1(p)$. From Lemma 8, we get that, as $p \to 0$

$$\begin{aligned}
&P(X_1 > VaR_1(p) \text{ or } X_2 > VaR_2(p)) \\
\sim &P((2-2\gamma)Y_1 \vee \gamma Y_2 \vee \gamma Y_3 > v(p) \text{ or } \gamma Y_1 \vee (1-\gamma-\mu)Y_2 \vee \mu Y_3 > cv(p)) \\
= &P\left(Y_1 > \frac{v(p)}{(2-2\gamma)\vee\frac{\gamma}{c}} \text{ or } Y_2 > \frac{v(p)}{\gamma \vee \frac{1-\gamma-\mu}{c}} \text{ or } Y_3 > \frac{v(p)}{\gamma\vee\frac{\mu}{c}}\right) \\
\sim &P\left(Y_1 > \frac{v(p)}{(2-2\gamma)\vee\frac{\gamma}{c}}\right) + P\left(Y_2 > \frac{v(p)}{\gamma \vee \frac{1-\gamma-\mu}{c}}\right) + P\left(Y_3 > \frac{v(p)}{\gamma\vee\frac{\mu}{c}}\right) \\
\sim &\left[\left((2-2\gamma)\vee\frac{\gamma}{c}\right)^\alpha + \left(\gamma\vee\frac{\mu}{c}\right)^\alpha + \left(\gamma\vee\frac{1-\gamma-\mu}{c}\right)^\alpha\right] P(Y_1 > v(p)) \qquad (16)
\end{aligned}$$

From (14), we get that $p \sim ((2-2\gamma)^\alpha + 2\gamma^\alpha)P(Y_1 > v(p))$ as $p \to 0$. Together with (16) and (5), we have that

$$\begin{aligned}
L(1,1,0) &= \lim_{p\to 0} \frac{P(X_1 > VaR_1(p) \text{ or } X_2 > VaR_2(p))}{p} \\
&= \frac{\left((2-2\gamma)\vee\frac{\gamma}{c}\right)^\alpha + \left(\gamma\vee\frac{\mu}{c}\right)^\alpha + \left(\gamma\vee\frac{1-\gamma-\mu}{c}\right)^\alpha}{(2-2\gamma)^\alpha + 2\gamma^\alpha}. \qquad (17)
\end{aligned}$$

Due to symmetry, we have that $L(1,0,1) = L(1,1,0)$. Following similar calculation, it is obtained that

$$L(0,1,1) = \frac{\left(\frac{\gamma}{c}\right)^\alpha + 2\left(\frac{\mu\vee(1-\gamma-\mu)}{c}\right)^\alpha}{(2-2\gamma)^\alpha + 2\gamma^\alpha}. \qquad (18)$$

Proposition 4 implies that $SII_2 = SII_3$. Moreover, $SII_1 > SII_2$ if and only if $L(1,1,0) <$

$L(0,1,1)$. Hence it is only necessary to compare the two values of L function.

Denote

$$Q := ((2-2\gamma)^{\alpha} + 2\gamma^{\alpha})(L(1,1,0) - L(0,1,1)).$$

We study the sign of Q in order to compare the systemic importance measures SII_i, $i = 1,2,3$.

Case I: $\frac{2}{3} \leq \gamma < 1$

Since $\gamma > 1/2$, we have $c > 1/2$. Thus, $\frac{\gamma}{1-\gamma} > 2 > \frac{1}{c}$, which implies that

$$\gamma > \frac{1-\gamma}{c} > \max\left(\frac{1-\gamma-\mu}{c}, \frac{\mu}{c}\right).$$

Therefore

$$Q = \left(\left((2-2\gamma) \vee \frac{\gamma}{c}\right)^{\alpha} + 2\gamma^{\alpha}\right) - \left(\left(\frac{\gamma}{c}\right)^{\alpha} + 2\left(\frac{\mu \vee (1-\gamma-\mu)}{c}\right)^{\alpha}\right) > 0.$$

Hence, $SII_1 < SII_2 = SII_3$.

Case II: $\gamma = 1/2$

In this case, we still have $c \geq 1/2$. The equality holds if and only if $\mu = 1/4$. Due to the symmetric position between X_2 and X_3, without loss of generality, we assume that $\mu \leq 1/4$. Then we get $\mu \leq 1-\gamma-\mu$. Moreover, the inequality $\mu/\gamma \leq 1/2 \leq c$ implies that $\gamma \geq \frac{\mu}{c}$ and it is not difficult to obtain that $\gamma \leq \frac{1-\gamma-\mu}{c}$. Hence,

$$\begin{aligned}
Q &= \left(1+2^{-\alpha}+\left(\frac{1/2-\mu}{c}\right)^{\alpha}\right)-\left(\left(\frac{\gamma}{c}\right)^{\alpha}+2\left(\frac{1/2-\mu}{c}\right)^{\alpha}\right) \\
&= 1+2^{-\alpha}-\left(\frac{1}{2^{\alpha}}+\left(\frac{1}{2}-\mu\right)^{\alpha}\right)c^{-\alpha} \\
&= 1+\frac{1}{2^{\alpha}}-\frac{1+(1-2\mu)^{\alpha}}{2^{\alpha}}\frac{2^{\alpha}+2}{1+(2\mu)^{\alpha}+(1-2\mu)^{\alpha}} \\
&= 1+\frac{1}{2^{\alpha}}-\left(1+\frac{2}{2^{\alpha}}\right)\left(1-\frac{(2\mu)^{\alpha}}{1+(2\mu)^{\alpha}+(1-2\mu)^{\alpha}}\right) \\
&\leq 1+\frac{1}{2^{\alpha}}-\left(1+\frac{2}{2^{\alpha}}\right)\left(1-\frac{(1/2)^{\alpha}}{1+(1/2)^{\alpha}+(1/2)^{\alpha}}\right) \\
&= 0
\end{aligned}$$

Here we used the facts that $\frac{(2\mu)^{\alpha}}{1+(2\mu)^{\alpha}+(1-2\mu)^{\alpha}}$ is an increasing function with respect to μ and $\mu \leq 1/4$. The equality holds if and only if $\mu = 1/4$. Hence, in case $\gamma = 1/2$, we conclude that $SII_1 \geq SII_2 = SII_3$, with the equality holds if and only if $\mu = 1/4$.

Case III: $0 < \gamma \leq 1/3$

In this case, it is not difficult to verify that $2-2\gamma > \frac{\gamma}{c}$ and $\gamma < \frac{1-\gamma}{2c}$. Due to symmetry, we only consider the case $0 < \mu \leq \frac{1-\gamma}{2}$. Then we have that $\mu \leq 1-\gamma-\mu$ and $\gamma < \frac{1-\gamma-\mu}{c}$. Therefore

$$Q = (2-2\gamma)^{\alpha}+\left(\gamma \vee \frac{\mu}{c}\right)^{\alpha}-\frac{\gamma^{\alpha}+(1-\gamma-\mu)^{\alpha}}{c^{\alpha}}. \tag{19}$$

For any fixed γ, c is a function of μ denoted by $c(\mu)$. For $0 < \mu < \frac{1-\gamma}{2}$, $c(\mu)$ is a strictly decreasing function. Thus $g(\mu) := \frac{\mu}{c(\mu)}$ is a continuous, strictly increasing function. Notice that $g(0) = 0 < \gamma$ and $g((1-\gamma)/2) > \gamma$. There must exists a unique $\mu^* \in (0, (1-\gamma)/2)$ such that $g(\mu^*) = \gamma$.

Denote $c^* := c(\mu^*)$. From $\frac{\mu^*}{c^*} = \gamma$, we get that

$$\frac{\gamma^{\alpha}}{\mu^{*\alpha}} = (c^*)^{-\alpha} = \frac{(2-2\gamma)^{\alpha}+2\gamma^{\alpha}}{\gamma^{\alpha}+\mu^{*\alpha}+(1-\gamma-\mu^*)^{\alpha}}.$$

It implies that

$$\frac{\gamma^\alpha}{\mu^{*\alpha}} = (c^*)^{-\alpha} = \frac{(2-2\gamma)^\alpha + \gamma^\alpha}{\gamma^\alpha + (1-\gamma-\mu^*)^\alpha}.$$

Continuing from equation (19), we get that

$$Q(\mu^*) = (2-2\gamma)^\alpha + \gamma^\alpha - (\gamma^\alpha + (1-\gamma-\mu^*)^\alpha)\,(c^*)^{-\alpha} = 0.$$

Hence, we conclude that for $\mu = \mu^*$, $SII_1 = SII_2 = SII_3$.

For $0 < \mu < \mu^*$, it is clear that

$$\begin{aligned} Q(\mu) &= (2-2\gamma)^\alpha + \gamma^\alpha - (\gamma^\alpha + (1-\gamma-\mu)^\alpha)\,c^{-\alpha} \\ &= (2-2\gamma)^\alpha + \gamma^\alpha - (\gamma^\alpha + (1-\gamma-\mu)^\alpha)\,\frac{(2-2\gamma)^\alpha + 2\gamma^\alpha}{\gamma^\alpha + \mu^\alpha + (1-\gamma-\mu)^\alpha} \\ &= \frac{((2-2\gamma)^\alpha + \gamma^\alpha)\,\mu^\alpha - \gamma^\alpha\,(\gamma^\alpha + (1-\gamma-\mu)^\alpha)}{\gamma^\alpha + \mu^\alpha + (1-\gamma-\mu)^\alpha} \end{aligned}$$

is an strictly increasing function with respect to μ. Moreover, for $\mu^* < \mu < \frac{1-\gamma}{2}$, Q is calculated as

$$\begin{aligned} Q(\mu) &= (2-2\gamma)^\alpha + (\mu^\alpha - \gamma^\alpha - (1-\gamma-\mu)^\alpha)\,c^{-\alpha} \\ &= (2-2\gamma)^\alpha + (\mu^\alpha - \gamma^\alpha - (1-\gamma-\mu)^\alpha)\,\frac{(2-2\gamma)^\alpha + 2\gamma^\alpha}{\gamma^\alpha + \mu^\alpha + (1-\gamma-\mu)^\alpha} \\ &= \frac{2(2-2\gamma)^\alpha\mu^\alpha + 2\gamma^\alpha\,(\mu^\alpha - \gamma^\alpha - (1-\gamma-\mu)^\alpha)}{\gamma^\alpha + \mu^\alpha + (1-\gamma-\mu)^\alpha} \end{aligned}$$

which is still an strictly increasing function with respect to μ. Therefore, for $0 < \mu < \mu^*$, $Q < 0$ while for $\mu^* < \mu < \frac{1-\gamma}{2}$, $Q > 0$. Correspondingly, we have that $SII_1 > SII_2 = SII_3$ in the former case, and $SII_1 < SII_2 = SII_3$ in the latter case.

Due to symmetry, similar result holds when $\frac{1-\gamma}{2} < \mu < 1-\gamma$, and the switch point is then $1-\gamma-\mu^*$. More specifically, for $\frac{1-\gamma}{2} < \mu < 1-\gamma-\mu^*$, $SII_1 < SII_2 = SII_3$; for $\mu = 1-\gamma-\mu^*$, $SII_1 = SII_2 = SII_3$; for $1-\gamma-\mu^* < \mu < 1-\gamma$, $SII_1 > SII_2 = SII_3$. The theorem is thus proved for Case III. ■

Proof of Lemma 6.

Along the lines of the proof of Theorem 5, we obtained that in a three-bank model.

$$\begin{aligned}SII_1 - SII_2 &= \{1 + (2 - L(1,1,0)) + (2 - L(1,0,1))\} - \{(2 - L(1,1,0)) + 1 + (2 - L(0,1,1))\}\\ &= L(0,1,1) - L(1,0,1).\end{aligned}$$

Therefore, $SII_i > SII_j$ if and only if $L_{\neq i}(1,1,\cdots,1) > L_{\neq j}(1,1,\cdots,1)$. Together with (6), the lemma is proved.

We remark that the above calculation is not based on the affine portfolio model. Hence, the result is valid in a general three-bank model. However, one can not obtain similar relations when the dimensions exceed three. Hence, Lemma 6 is not valid for assessing a banking system with more than three banks. ■

Appendix B Statistical Estimation on the L function

Consider independently and identically distributed (i.i.d.) observations from the random vector $(X_1,\cdots,X_d)$ denoted by

$$\{(X_{1s}, X_{2s}, \cdots, X_{ds}) | 1 \leq s \leq n\}.$$

The sample size is n. The non-parametric approach of estimating the L function starts from the assumption (4). Roughly speaking, the estimation takes a certain p value for which the VaR for each dimension can be estimated by the order statistics. Then the probability in the numerator of (4) is estimated by a counting measure. To ensure that $p \to 0$, theoretically we take a sequence $k := k(n)$, such that $k(n) \to \infty$ and $k(n)/n \to 0$ as $n \to \infty$. We get an empirical estimation of the L function from replacing p by k/n and using the empirical estimation on the distribution function of $(X_1, X_2 \cdots, X_d)$. The explicit estimator is given as

$$\hat{L}(x_1, \cdots, x_d) := \frac{1}{k}\sum_{s=1}^{n} 1_{\exists 1\leq i\leq d, \text{ s.t. } X_{is}>X_{i,n-[kx_i]}},$$

where $X_{i,1} \leq X_{i,2} \leq \cdots \leq X_{i,n}$ are the order statistics of the $i-th$ dimension of the sample, $X_{i1}, \cdots, X_{in}$, for $1 \leq i \leq d$. Particularly, $L(1, 1, \cdots, 1)$ is estimated by

$$\hat{L}(1, 1, \cdots, 1) := \frac{1}{k}\sum_{s=1}^{n} 1_{\exists 1\leq i\leq d, \text{ s.t. } X_{is}>X_{i,n-k}}.$$

For the estimator of the L function, usual statistical properties, such as consistency and asymptotic normality, have been proved, see, e.g. de Haan and Ferreira (2006).

Practically, since n is always finite, the theoretical conditions on k is not relevant for a finite sample analysis. Thus it is a major issue on how to choose a proper k in the estimator. Instead of taking an arbitrary k, a usual procedure is to calculate the estimator of $L(1, 1, \cdots, 1)$ under different k values and draw a line plot against the k values. With a low k value, the estimation exhibits a large variance, while for a high k value, since the estimation uses too many observations in the moderate level, it bears a potential bias. Therefore, k is usually chosen by picking the first stable part of the line plot which balances the tradeoff between the variance and the bias. The estimates follow from the k choice. Because k is chosen from a stable part of the line plot, a small variation of the k value does not change the estimates. Thus, the exact k value is not sensitive for the estimation on the L function. Such a procedure has been applied in univariate EVT for tail index estimation.

With the chosen k, we in fact consider a tail event as the loss return exceeds a VaR with tail probability level k/n. In our empirical application, the chosen k value differs according to the sample size. For the full sample analysis (sample size 6,000), we choose $k = 180$, which corresponds to a p level at 3%; for the moving window analysis (sample size 2,000), we choose $k = 100$, which corresponds to $p = 5\%$; for the monthly data analysis (sample size 276), we choose $k = 20$, which corresponds to $p = 7.2\%$. With lower number of observations, we choose a slightly higher level of p, albeit a low absolute level.

Appendix C Tables and Figures

Table 1: Descriptive statistics on daily stock returns of 28 US banks

BANKS	Mean	Std. Dev.	Min.	Max.	Skew.	Kurtosis
BANCORPSOUTH	0.040	2.28	-15.12	19.71	0.23	8.63
BANK OF AMERICA	0.031	2.70	-34.21	30.21	-0.35	32.80
BANK OF HAWAII	0.045	1.78	-25.51	12.95	-0.63	19.77
BB&T	0.041	2.09	-26.61	21.20	0.12	19.53
CENTRAL PAC.FINL.	0.009	3.44	-66.87	69.31	0.28	68.85
CITIGROUP	0.020	2.98	-49.47	45.63	-0.50	42.92
CITY NATIONAL	0.029	2.29	-18.92	20.21	0.13	11.15
CMTY.BK.SY.	0.041	2.30	-14.22	16.55	0.37	9.11
COMERICA	0.038	2.13	-22.69	18.81	-0.16	18.68
CULLEN FO.BANKERS	0.054	2.17	-21.46	19.78	0.17	13.52
FIRST HORIZON NATIONAL	0.033	2.36	-44.11	25.54	-1.18	45.41
FNB	0.024	2.52	-25.53	19.57	-0.19	14.23
JP MORGAN CHASE & CO.	0.033	2.55	-32.46	22.39	-0.12	17.58
KEYCORP	0.011	2.56	-40.55	43.34	-0.51	50.04
M&T BK.	0.054	1.75	-17.59	22.83	0.33	24.60
MARSHALL & ILSLEY	0.017	2.53	-30.15	32.93	-0.23	40.12
OLD NATIONAL BANCORP	0.019	1.74	-19.47	15.84	0.31	16.55
PNC FINL.SVS.GP.	0.031	2.28	-53.44	31.55	-1.30	68.26
REGIONS FINL.NEW	0.001	2.76	-52.88	39.48	-0.62	56.56
STERLING BANC.	0.017	2.27	-21.26	19.39	0.27	12.36
SUNTRUST BANKS	0.023	2.40	-31.71	26.67	-0.39	29.95
SYNOVUS FINL.	0.017	2.78	-30.07	24.86	-0.02	17.68
TCF FINANCIAL	0.047	2.35	-17.65	23.53	0.49	13.73
US BANCORP	0.054	2.17	-20.05	25.76	0.22	18.69
VALLEY NATIONAL BANCORP	0.034	2.22	-17.48	21.71	0.29	11.39
WEBSTER FINANCIAL	0.024	2.56	-23.51	31.03	0.07	20.03
WELLS FARGO & CO	0.060	2.35	-27.21	28.34	0.66	27.47
WILMINGTON TRUST	0.026	2.11	-23.12	27.62	0.05	20.85

Note: The sample period runs from Jan 2 1987 to Dec 31 2009 (sample size 6000). All values except the skewness and kurtosis are in percentage. The list consists of all banks that are preliminarily traded on NYSE during the sample period.

Table 2: Descriptive statistics on yearly size measures of 28 US banks

BANKS	Total Asset		Total Equity		Total Debt	
	2009	average	2009	average	2009	average
BANCORPSOUTH	13.2	6.5	12.2	6.0	1.0	0.5
BANK OF AMERICA	2223.3	648.4	1458.6	449.2	764.7	199.3
BANK OF HAWAII	12.4	11.3	10.7	9.1	1.7	2.2
BB&T	165.2	55.0	135.7	43.7	29.5	11.3
CENTRAL PAC.FINL.	4.9	2.4	4.0	2.1	0.9	0.3
CITIGROUP	1856.6	1145.3	1295.3	798.5	561.3	346.8
CITY NATIONAL	20.9	8.6	19.5	7.9	1.4	0.8
CMTY.BK.SY.	5.4	2.3	4.5	2.0	0.9	0.3
COMERICA	59.2	38.4	47.7	31.5	11.5	6.9
CULLEN FO.BANKERS	16.3	7.4	15.4	6.8	0.9	0.6
FIRST HORIZON NATIONAL	26.1	18.4	19.2	13.9	6.8	4.6
FNB	8.7	3.9	7.5	3.4	1.2	0.5
JP MORGAN CHASE & CO.	2032.0	655.8	1406.7	482.3	625.3	173.5
KEYCORP	92.7	69.7	79.1	53.1	13.6	16.6
M&T BK.	68.9	28.5	56.2	23.0	12.7	5.5
MARSHALL & ILSLEY	56.2	26.0	48.6	20.7	7.5	5.3
OLD NATIONAL BANCORP	7.9	6.0	6.9	5.0	1.0	1.0
PNC FINL.SVS.GP.	269.9	87.3	230.6	68.8	39.3	18.4
REGIONS FINL.NEW	142.3	48.8	120.2	40.8	22.1	8.0
STERLING BANC.	2.1	1.2	1.9	1.0	0.3	0.2
SUNTRUST BANKS	174.2	92.8	151.3	75.6	22.9	17.3
SYNOVUS FINL.	32.8	14.6	30.6	13.2	2.2	1.4
TCF FINANCIAL	17.9	9.7	13.1	7.4	4.8	2.4
US BANCORP	281.2	109.6	217.3	82.7	63.9	26.9
VALLEY NATIONAL BANCORP	14.3	6.9	10.9	5.7	3.3	1.1
WEBSTER FINANCIAL	17.6	9.0	15.6	6.9	2.0	2.1
WELLS FARGO & CO	1240.4	295.0	997.6	225.2	242.8	69.8
WILMINGTON TRUST	10.9	6.9	9.9	5.7	1.0	1.2

Note: The sample period is from 1987 to Dec 2009 (23 years). The list consists of all banks that are preliminarily traded in NYSE during the sample period.

Table 3: Estimated systemic importance measures: full sample analysis

BANKS	SII	PAO	VI
BANCORPSOUTH	6.72	59.44%	6.73%
BANK OF AMERICA	10.84	94.44%	10.28%
BANK OF HAWAII	10.44	84.44%	9.30%
BB&T	10.88	86.11%	9.46%
CENTRAL PAC.FINL.	8.68	65.00%	7.31%
CITIGROUP	10.59	90.56%	9.90%
CITY NATIONAL	9.30	76.11%	8.46%
CMTY.BK.SY.	6.53	59.44%	6.73%
COMERICA	12.02	92.78%	10.12%
CULLEN FO.BANKERS	8.05	73.89%	8.23%
FIRST HORIZON NATIONAL	10.84	83.33%	9.19%
FNB	7.41	57.78%	6.55%
JP MORGAN CHASE & CO.	9.76	86.67%	9.52%
KEYCORP	12.44	93.33%	10.18%
M&T BK.	11.10	86.67%	9.52%
MARSHALL & ILSLEY	11.92	93.89%	10.23%
OLD NATIONAL BANCORP	9.36	77.78%	8.63%
PNC FINL.SVS.GP.	10.73	86.11%	9.46%
REGIONS FINL.NEW	11.91	90.56%	9.90%
STERLING BANC.	8.69	65.56%	7.37%
SUNTRUST BANKS	12.11	92.78%	10.12%
SYNOVUS FINL.	10.11	83.89%	9.24%
TCF FINANCIAL	10.57	81.11%	8.96%
US BANCORP	10.32	78.89%	8.74%
VALLEY NATIONAL BANCORP	7.82	65.00%	7.31%
WEBSTER FINANCIAL	9.87	82.22%	9.07%
WELLS FARGO & CO	11.25	90.00%	9.85%
WILMINGTON TRUST	10.91	83.33%	9.19%

Note: SII is the systemic importance index defined as the number of expected bank failures given a particular bank fails, see (2); PAO is the probability of causing at least one extra bank failure when a particular bank fails defined in (1); VI is the vulnerability index defined as the probability of failure given there exists at least one other bank failure in the system, see (3).

Table 4: Correlation coefficients: full sample analysis

		SII	PAO	VI
	Total Asset	0.1790	0.3968**	0.3943**
		(0.3622)	(0.0366)	(0.0379)
End of 2009	Total Equity	0.1954	0.4085**	0.4061**
		(0.3191)	(0.0309)	(0.0320)
	Total Debt	0.1399	0.3640*	0.3615
		(0.4777)	(0.0569)	(0.0588)*
	Total Asset	0.1733	0.3746**	0.3723*
		(0.3779)	(0.0495)	(0.0510)
Average	Total Equity	0.1811	0.3824**	0.3802**
		(0.3565)	(0.0446)	(0.0460)
	Total Debt	0.1542	0.3546*	0.3523*
		(0.4334)	(0.0641)	(0.0660)

Note: SII is the systemic importance index defined as the number of expected bank failures given a particular bank fails, see (2); PAO is the probability of causing at least one extra bank failure when a particular bank fails defined in (1); VI is the vulnerability index defined as the probability of failure given there exists at least one other bank failure in the system, see (3). The numbers in parentheses are the p-value for testing whether the correlation coefficient is significantly different from zero. The first panel reports the results based on using end-of-2009 value of the size measure, while the second panel reports the results based on using the average of the size measure across the full sample period.
Significance level: 1%–***, 5%–**, 10%–*.

Table 5: Correlation coefficients: moving window analysis

		SII	PAO	VI
	Total Asset	0.0263	0.2073	0.2069
		(0.8943)	(0.2900)	0.2908
Full sample	Total Equity	0.0418	0.2242	0.2236
		(0.8329)	(0.2514)	(0.2527)
	Total Debt	-0.0080	0.1665	0.1667
		(0.9678)	(0.3971)	(0.3966)
	Total Asset	0.4541**	0.4950***	0.4911***
		(0.0152)	(0.0074)	(0.0080)
Period 1:	Total Equity	0.4490**	0.4913***	0.4875***
1994-1999		(0.0165)	(0.0079)	(0.0085)
	Total Debt	0.4648**	0.5014***	0.4973***
		(0.0127)	(0.0066)	(0.0071)
	Total Asset	0.0263	0.2073	0.2069
		(0.8943)	(0.2900)	(0.2908)
Period 2:	Total Equity	0.0418	0.2242	0.2236
2000-2009		(0.8329)	(0.2514)	(0.2527)
	Total Debt	-0.0080	0.1665	0.1667
		(0.9678)	(0.3971)	(0.3966)

Note: SII is the systemic importance index defined as the number of expected bank failures given a particular bank fails, see (2); PAO is the probability of causing at least one extra bank failure when a particular bank fails defined in (1); VI is the vulnerability index defined as the probability of failure given there exists at least one other bank failure in the system, see (3). The numbers in parentheses are the p-value for testing whether the correlation coefficient is significantly different from zero. The first panel reports the result based on pooling all bank-year observations (448 observations). The second and third panels report the result based on pooling bank-year observations in two periods: 1994-2000, and 2001-2009.
Significance level: 1%–***, 5%–**, 10%–*.

Table 6: Systemic importance measures: monthly data

BANK	SII	PAO	VI
BANCORPSOUTH	10.15	75.00%	11.28%
BANK OF AMERICA	11.70	85.00%	12.59%
BANK OF HAWAII	11.60	100.00%	14.49%
BB&T	13.50	90.00%	13.24%
CENTRAL PAC.FINL.	8.65	90.00%	13.24%
CITIGROUP	12.00	95.00%	13.87%
CITY NATIONAL	10.20	80.00%	11.94%
CMTY.BK.SY.	9.10	75.00%	11.28%
COMERICA	13.40	100.00%	14.49%
CULLEN FO.BANKERS	10.20	95.00%	13.87%
FIRST HORIZON NATIONAL	9.85	75.00%	11.28%
FNB	9.45	75.00%	11.28%
JP MORGAN CHASE & CO.	10.80	90.00%	13.24%
KEYCORP	12.45	90.00%	13.24%
M&T BK.	13.10	95.00%	13.87%
MARSHALL & ILSLEY	13.15	100.00%	14.49%
OLD NATIONAL BANCORP	8.25	70.00%	10.61%
PNC FINL.SVS.GP.	12.20	95.00%	13.87%
REGIONS FINL.NEW	11.85	95.00%	13.87%
STERLING BANC.	9.80	65.00%	9.92%
SUNTRUST BANKS	12.85	100.00%	14.49%
SYNOVUS FINL.	11.25	95.00%	13.87%
TCF FINANCIAL	10.60	90.00%	13.24%
US BANCORP	12.80	100.00%	14.49%
VALLEY NATIONAL BANCORP	10.80	85.00%	12.59%
WEBSTER FINANCIAL	11.35	90.00%	13.24%
WELLS FARGO & CO	13.50	90.00%	13.24%
WILMINGTON TRUST	11.35	95.00%	13.87%

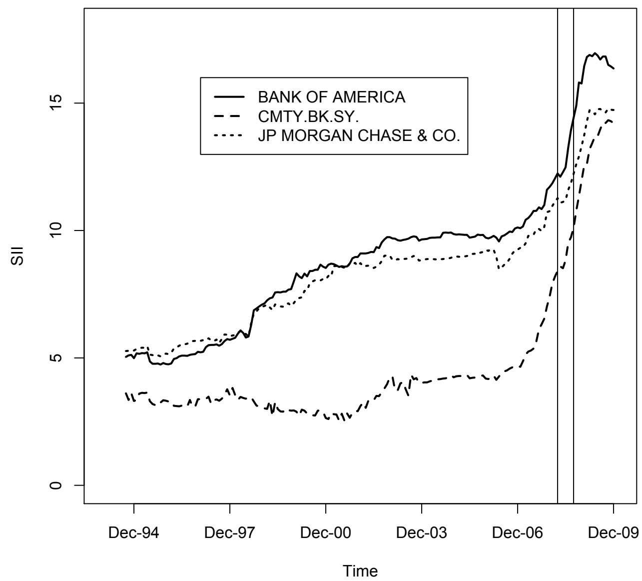
Note: SII is the systemic importance index defined as the number of expected bank failures given a particular bank fails, see (2); PAO is the probability of causing at least one extra bank failure when a particular bank fails defined in (1); VI is the vulnerability index defined as the probability of failure given there exists at least one other bank failure in the system, see (3).

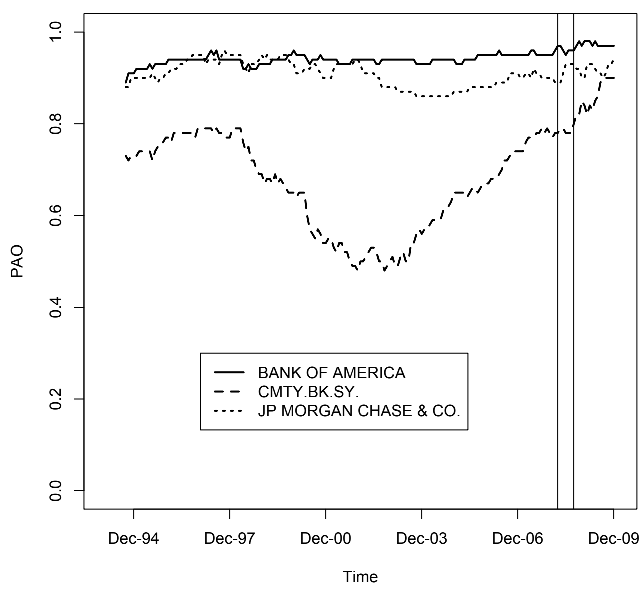
Table 7: Correlation coefficients: monthly data

		SII	PAO	VI
	Total Asset	0.2137	0.1028	0.1068
		(0.2749)	(0.6027)	(0.5884)
End of 2009	Total Equity	0.2416	0.1182	0.1222
		(0.2155)	(0.5493)	(0.5356)
	Total Debt	0.1489	0.0673	0.0714
		(0.4497)	(0.7335)	(0.7182)
	Total Asset	0.1981	0.1460	0.1488
		(0.3121)	(0.4585)	(0.4498)
Average	Total Equity	0.2086	0.1537	0.1565
		(0.2869)	(0.4348)	(0.4263)
	Total Debt	0.1728	0.1272	0.1300
		(0.3792)	(0.5189)	(0.5098)

Note: SII is the systemic importance index defined as the number of expected bank failures given a particular bank fails, see (2); PAO is the probability of causing at least one extra bank failure when a particular bank fails defined in (1); VI is the vulnerability index defined as the probability of failure given there exists at least one other bank failure in the system, see (3). The numbers in parentheses are the p-value for testing whether the correlation coefficient is significantly different from zero. The first panel reports the results based on using end-of-2009 value of the size measure, while the second panel reports the results based on using the average of the size measure across the full sample period.
Significance level: 1%–***, 5%–**, 10%–*.

Figure 1: Moving window results on systemic importance measures





Note: The moving window measures are estimated from 2,000-day subsample ending at the end of each month year. The upper panel presents the results for SII, which is the systemic importance index defined as the number of expected bank failures given a particular bank fails in (2), while the bottom panel presents the results for PAO which is the probability of causing at least one extra bank failure when a particular bank fails defined in (1). The two vertical lines refer to the failures of Bear Stearns (March 2008) and Lehman Brothers (September 2008).