Endogenous systemic liquidity risk ⋆

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Abstract

Traditionally, aggregate liquidity shocks are modelled as exogenous events. Extending our previous work (Cao & Illing, 2008), this paper analyzes the adequate policy response to endogenous systemic liquidity risk. We analyze the feedback between lender of last resort policy and incentives of private banks, determining the aggregate amount of liquidity available. We show that imposing minimum liquidity standards for banks ex ante are a crucial requirement for sensible lender of last resort policy. In addition, we analyze the impact of equity requirements and narrow banking, in the sense that banks are required to hold sufficient liquid funds so as to pay out in all contingencies. We show that both policies are strictly inferior to imposing minimum liquidity standards ex ante combined with lender of last resort policy.

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The events earlier this month leading up to the acquisition of Bear Stearns by JP Morgan Chase highlight the importance of liquidity management in meeting obligations during stressful market conditions. ... The fate of Bear Stearns was the result of a lack of confidence, not a lack of capital. ... At all times until its agreement to be acquired by JP Morgan Chase during the weekend, the firm had a capital cushion well above what is required to meet supervisory standards calculated using the Basel II standard.

— Chairman Cox, SEC, Letter to Basel Committee in Support of New Guidance on Liquidity Management, March 20th, 2008

Bear Stearns never ran short of capital. It just could not meet its obligations. At least that is the view from Washington, where regulators never stepped in to force the investment bank to reduce its high leverage even after it became clear Bear was struggling last summer. Instead, the regulators issued repeated reassurances that all was well. Does it sound a little like a doctor emerging from a funeral to proclaim that he did an excellent job of treating the late patient?

— Floyd Norris, New York Times, April 4th, 2008

1 Introduction

For a long time, presumably starting in 2004, financial markets seemed to have been awash with excessive liquidity. But suddenly, in August 2007, liquidity dried up nearly completely as a response to doubts about the quality of subprime mortgage-backed securities. Despite massive central bank interventions, the liquidity freeze did not melt away, but rather spread slowly to other markets such as those for auction rate bonds. On March 16th 2008, the investment bank Bear Sterns which — according to the SEC chairman — was adequately capitalized even a week before had to be rescued via a Fed-led takeover by JP Morgan Chase.

Following the turmoil on financial markets, there has been a strong debate about the adequate policy response. Some have warned that central bank actions may encourage dangerous moral hazard behaviour of market participants in the future.

Others instead criticized central banks of responding far too cautiously. The most prominent voice has been Willem Buiter who — jointly with Ann Sibert — right from the beginning of the crisis in August 2007 strongly pushed the idea that in times of crises, central banks should act as market maker of last resort. As an adaption of the Bagehot principles to modern times with globally integrated financial systems, central banks should actively purchase and sell illiquid private sector securities and so play a key role in assessing and pricing credit risk. In his FT blog "Maverecon", Willem Buiter stated the intellectual arguments behind such a policy very clearly on December 13th, 2007:

> *"Liquidity is a public good. It can be managed privately (by hoarding inherently liquid assets), but it would be socially inefficient for private banks and other financial institutions to hold liquid assets on their balance sheets in amounts sufficient to tide them over when markets become disorderly. They are meant to intermediate short maturity liabilities into long maturity assets and (normally) liquid liabilities into illiquid assets. Since central banks can create unquestioned liquidity at the drop of a hat, in any amount and at zero cost, they should be the liquidity providers of last resort, both as lender of last resort and as market maker of last resort. There is no moral hazards as long as central banks provide the liquidity against properly priced collateral, which is in addition subject to the usual 'liquidity haircuts' on this fair valuation. The private provision of the public good of emergency liquidity is wasteful. It's as simple as that."*

Buiter's statement represents the prevailing main stream view that there is no moral hazard risk as long as the Bagehot principles are followed as best practice in liquidity management.

According to Goodfriend & King (1988), a Lender of Last Resort Policy should target liquidity provision to the market, but not to specific banks. Central banks should "lend freely at a high rate against good collateral." This way, public liquidity support is supposed to be targeted towards solvent yet illiquid institutions, since insolvent financial institutions should be unable to provide adequate collateral to secure lending. This paper wants to challenge the view that a policy following the Bagehot principle does not create moral hazard. The key argument is that this view neglects the endogeneity of aggregate liquidity risk. Starting with Allen & Gale (1998) and Holmström & Tirole (1998), there have been quite a few models recently analyzing private and public provision of liquidity. But as far as we know, in all these models except our companion paper Cao & Illing (2008), aggregate systemic risk is assumed to be an exogenous probability event.

In Holmström & Tirole (1998), for instance, liquidity shortages arise when financial institutions and industrial companies scramble for and cannot find the cash required to meet their most urgent needs or undertake their most valuable projects. They show that credit lines from financial intermediaries are sufficient for implementing the socially optimal (second-best) allocation, as long as there is no aggregate uncertainty. In the case of aggregate uncertainty, however, the private sector cannot satisfy its own liquidity needs, so the existence of liquidity shortages vindicates the injection of liquidity by the government. In their model, the government can provide (outside) liquidity by committing future tax income to back up the reimbursements.

In the model of Holmström & Tirole (1998), the Lender of Last Resort indeed provides a free lunch: Public provision of liquidity in the presence of aggregate shocks is a pure public good, with no moral hazard involved. The reason is that aggregate liquidity shocks are modelled as exogenous events; there is no endogenous mechanism determining the aggregate amount of liquidity available. The same holds in Allen & Gale (1998), even though they analyze a quite different mechanism for public provision of liquidity: the adjustment of the price level in an economy with nominal contracts. We adopt Allen & Gale's mechanism. But we show that there is no longer a free lunch when private provision of liquidity affects the likelihood of an aggregate (systemic) event.

The basic idea of our model is fairly straightforward: Financial intermediaries can choose to invest in more or less (real) liquid assets. We model illiquidity in the following way: Some fraction of projects turns out to be realized late. The aggregate share of late projects is endogenous; it depends on the incentives of financial intermediaries to invest in risky, illiquid projects. This endogeneity allows us to capture the feedback from liquidity provision to risk taking incentives of financial intermediaries. We show that the anticipation of unconditional central bank liquidity provision will encourage excessive risk taking (moral hazard). It turns out that in the absence of liquidity requirements, there will be overinvestment in risky activities, creating excessive exposure to systemic risk.

In contrast to what the Bagehot principle suggests, unconditional provision of liquidity to the market (lending of central banks against good collateral) is exactly the wrong policy: It distorts incentives of banks to provide the efficient amount of private liquidity. In our model, we concentrate on pure illiquidity risk: There will never be insolvency unless triggered by illiquidity (by a bank run). Illiquid projects promise a higher, yet possibly delayed return. Relying on sufficient liquidity provided by the market (or by the central bank), financial intermediaries are inclined to invest more heavily in high yielding, but illiquid long term projects. Central bank's liquidity provision, helping to prevent bank runs with inefficient early liquidation, encourages banks to invest more in illiquid assets. At first sight, this seems to work fine, even if systemic risk increases: After all, public insurance against aggregate risks should allow agents to undertake more profitable activities with higher social return. As long as public insurance is a free lunch, there is nothing wrong with providing such a public good.

The problem, however, is that due to limited liability some banks will be encouraged to free-ride on liquidity provision. This competition will force other banks to reduce their efforts for liquidity provision, too. Chuck Prince, at that time chief executive of Citigroup, stated the dilemma posed in fairly poetic terms on July 10th 2007 in an infamous interview with Financial Times [1] :

[1] The key problem is best captured by the following remark about Citigroup in the New York Times report "Treasury Dept. Plan Would Give Fed Wide New Power" on March 29th, 2008: "*Mr. Frank said he realized the need for tighter regulation of Wall Street firms after a meeting with Charles O. Prince III, then chairman of Citigroup. When Mr. Frank asked why Citigroup had kept billions of dollars in 'structured investment vehicles' off the firm's balance sheet, he recalled, Mr. Prince responded that Citigroup, as a bank holding company, would have been at a disadvantage because investment firms can operate with higher debt and lower capital reserves*."

"*When the music stops, in terms of liquidity, things will be complicated. But as long as the music is playing, you've got to get up and dance. We're still dancing*."

The dancing banks simply enjoy liquidity provided in good states of the world and just disappear (go bankrupt) in bad states. The incentive of financial intermediaries to free-ride on liquidity in good states results in excessively low liquidity in bad states. Even worse: As long as they do not suffer run, "dancing" banks can always offer more attractive collateral in bad states — so they are able to outbid prudent banks in a liquidity crisis. For that reason, the Bagehot principle, rather than providing correct incentives, is the wrong medicine in modern times with a shadow banking system relying on liquidity being provided by other institutions.

This paper extends a model developed in Cao & Illing (2008). In that paper we did not allow for banks holding equity, so we could not analyze the impact of equity requirements. As we will show, imposing equity requirements can be inferior even to the outcome of a mixed strategy equilibrium with free-riding (dancing) banks. In contrast, imposing binding liquidity requirements ex ante combined with lender of last resort policy ex post is able to implement the second best outcome. In our model, it yields a strictly superior outcome compared to imposing equity requirements. We also prove that "narrow banking" (banks being required to hold sufficient equity so as to be able to pay out demand deposits in all states of the world) is inferior to ex ante liquidity regulation.

Allen & Gale (2007, p 213f) notice that the nature of market failure leading to systemic liquidity risk is not yet well understood. They argue that "a careful analysis of the costs and benefits of crises is necessary to understand when intervention is necessary." In this paper, we try to fill this gap, providing a cost / benefit analysis of different forms of banking regulation to better to understand what type of intervention is required. We explicitly compare the impact both of liquidity and capital requirements. To the best of our knowledge, this is the first paper providing such an analysis.

2 The structure of the model

2.1 The agents, time preferences, and technology

In the economy, there are three types of agents: investors, banks (run by bank managers) and entrepreneurs. All agents are risk neutral. The economy extends over 3 periods, $t = 0, 1, 2$, and the details of timing will be explained later. We assume that

(1) There is a continuum of investors each initially (at $t = 0$) endowed with one unit of resources. The resource can be either stored (with a gross return equal to 1) or invested in the form of bank deposits;

(2) There are a finite number of active banks engaged in Bertrand competition, competing for investors' deposits. Using the deposits, the banks as financial intermediaries can fund projects of entrepreneurs;

(3) There is a continuum of entrepreneurs. There are two types of them (denoted by i, $i = 1, 2$), characterized by their projects return R_i

- Projects of type 1 are realized early at period $t = 1$ with a safe return $R_1 > 1$;
- Projects of type 2 give a higher return $R_2 > R_1 > 1$. With probability p, these projects will also be realized at $t = 1$, but they may be delayed (with probability $1 - p$) until $t = 2$. Therefore, in the aggregate, the share p of type 2 projects will be realized early. The aggregate share p, however is not known at $t = 0$. It will be only revealed between 0 and 1 at some intermediate period, call it $t = \frac{1}{2}$. In the following, we are interested in the case of aggregate shocks. We model them in the simplest way: The aggregate share of type 2 projects realized early, p, can take on just two values: either p_H or p_L with $p_H > p_L$. The "good" state with a high share of early type 2 projects p_H, i.e., the state with plenty of liquidity, will be realized with probability π.

Investors are impatient: They want to consume early (at $t = 1$). In contrast, both entrepreneurs and bank managers are indifferent between consuming early ($t = 1$) or late ($t = 2$).

In addition, we assume that resources of investors are scarce in the sense that there are more projects of each type available than the aggregate endowment of investors. Thus, in the absence of commitment problems (to be explained in the next paragraph), total surplus would go to the investors, — investors would simply put all their funds in early projects and capture the full return. We take such frictionless market outcome as the reference point and analyze those equilibria coming closest to implement that market outcome. Since there is a market demand for liquidity only if investors' funds are the limiting factor, we concentrate on deviations from the frictionless market outcome and consider investors payoff as the relevant criterion.

We also assume that the contracts between banks and investors are fixed deposit contracts. Due to hold up problems as modelled in Hart & Moore (1994), entrepreneurs can only commit to pay a fraction $\gamma R_i > 1$ of their return. Banks as financial intermediaries can pool investment; they have superior collection skills (a higher γ, which justifies their role as intermediaries). Following Diamond & Rajan (2001), banks offer deposit contracts with a fixed payment d_0 payable at any time after $t = 0$ as a credible commitment device not to abuse their collection skills. The threat of a bank run disciplines bank managers to fully pay out all available resources pledged in the form of bank deposits. Deposit contracts, however, introduce a fragile structure into the economy: Whenever depositors have doubts about their bank's liquidity (the ability to pay depositors the promised amount d_0 at $t = 1$), they run on the bank at the intermediate date, forcing the bank to liquidate all its projects (even those funding safe early entrepreneurs) at high costs: Early liquidation of projects gives only the inferior return $c < 1$. In the following, we do not consider pure sunspot bank runs of the Diamond & Dybvig type. Instead, we concentrate on runs happening if liquid funds are not sufficient to payout depositors.

2.2 *Timing and events*

At date $t = 0$, banks competing for funds offer deposit contracts with payment d_0 which maximize expected consumption of investors at the given expected interest rates. Banks compete by choosing the share α of deposits invested in type 1 projects, taking their competitors choice as given. Investors have rational expectations about each banks default probability; they are able to monitor all banks investment. Remember that, at this stage, the share p of type 2 projects that will be realized early is not known.

At date $t = \frac{1}{2}$, the value of p reveals, so does the expected return of the banks at $t = 1$. A bank would experience a run if it cannot meet the investors demand. If this happens, all the assets — even the safe projects — have to be liquidated.

For those banks which didn't suffer from the bank run, they trade with early entrepreneurs in a perfect market for liquidity at $t = 1$, clearing at interest rate r. Note that because of the hold up problem, entrepreneurs retain a rent — their share $(1 - \gamma)R_i$. Since early entrepreneurs are indifferent between consuming at $t = 1$ or $t = 2$, they are willing to provide liquidity (using their rent to deposit at banks at $t = 1$ at the market rate r). Banks use the liquidity provided to pay out depositors. This way, impatient investors can profit indirectly from investment in high yielding long term projects. So banking allows transformation between liquid claims and illiquid projects.

At date $t = 2$, the banks collect the rent from the late projects and pay back the early entrepreneurs.

Note that the aggregate liquidity available at date $t = 1$ depends on the total share of funds invested in liquid type 1 projects at date $t = 0$. Let α be this share. As long as banks are liquid, the payoff structure is described as in Figure 1. But if α is so low that banks cannot honor deposits when p_L occurs, depositors will run at $t = \frac{1}{2}$. The payoff is captured in Figure 2.

2.3 The equilibrium

It has been shown in Cao & Illing (2008) that if the share p of type 2 projects realized early is known at $t = 0$, there is no aggregate uncertainty. The laissez-faire

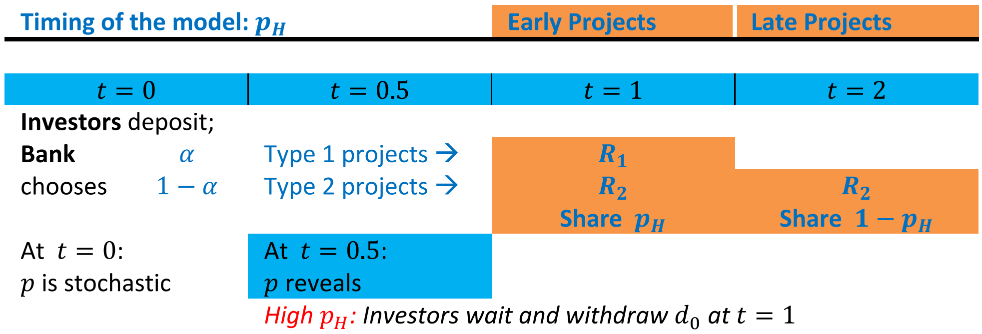

Fig. 1. Timing and payoff structure, when banks are liquid

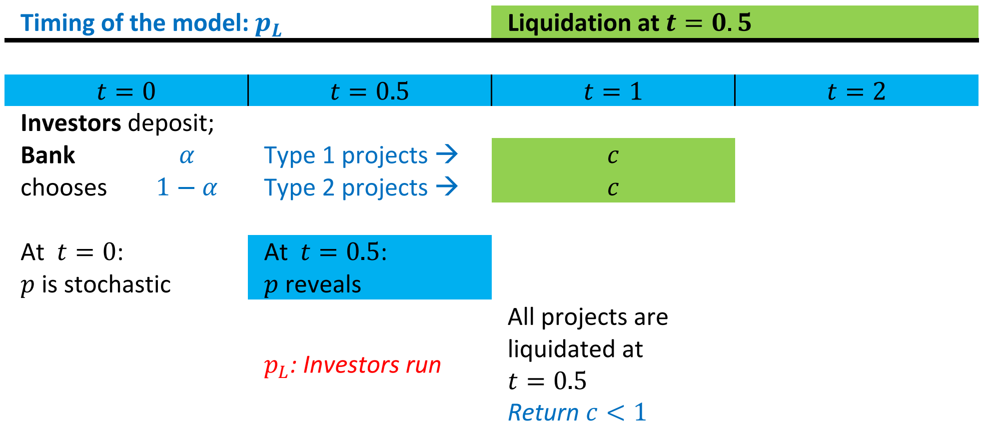

Fig. 2. Timing and payoff structure, when banks are illiquid

equilibrium is in line with the solution of the benevolent planner's problem, which is constrained-efficient: Banks will invest such that — on aggregate — they are able to fulfil depositors' claims in period 1, so there will be no run.

It becomes tricky when there is aggregate uncertainty. In the following, let us briefly repeat the findings in Cao & Illing (2008). Given the structure as stated in the previous section, a bank seems to have just two options available from the planner's point of view: It may either invest so much in safe type 1 projects that it will be able to pay out its depositors all the time (that is, even if the bad state occurs). Let us call this share $\alpha(p_L)$. Alternatively, it may invest just enough, $\alpha(p_H)$, so as to pay out depositors in the good state. If so, the bank will experience a run in the bad state. Obviously, the optimal share depends on what other banks will do (since that determines aggregate liquidity available at $t = 1$ and so the interest rate for liquid funds between period 1 and 2), but also on the probability π for the good state. To gain some intuition, let us first assume that all banks behave the same — just like a representative bank. If so, it will not pay to take precautions against the bad state if the likelihood for that outcome is considered to be very low. Thus, if π is very high, the representative bank will obviously invest only a small share $\alpha(p_H)$ — just enough to pay out depositors in the good state. Alternatively, if π is very low (close to 0), it always pays to be prepared for the worst case, so the representative bank will invest a high share $\alpha(p_L) > \alpha(p_H)$ in safe projects. Since $\alpha(p_s)$, $s \in H, L$, is the share invested in safe projects with return R_1, the total payoff out of investment strategy $\alpha(p_s)$ is $E[R_s] = \alpha(p_s)R_1 + [1 - \alpha(p_s)]R_2$ with $E[R_H] > E[R_L]$.

With a high share $\alpha(p_L)$ of safe projects, the banks will be able to pay out depositors in all states. There will never be a bank run. So independent of π, the expected payoff for depositors is $\gamma E[R_L]$.

With $\alpha(p_H)$ there will be a bank run in the bad state, giving just the bankruptcy payoff c with probability $1-\pi$. So strategy $\alpha(p_H)$ gives $\pi\gamma E[R_H]+(1-\pi)c$, increasing in π. Depositors prefer $\alpha(p_H)$, if $\pi\gamma E[R_H] + (1 - \pi)c > \gamma E[R_L]$ or

$$\pi > \overline{\pi}_2 = \frac{\gamma E[R_L] - c}{\gamma E[R_H] - c}.$$

Obviously, for π below $\overline{\pi}_2$ depositors are better off with the safe strategy, so they prefer banks to choose $\alpha(p_L)$ rather than to exploit high profitability of type 2 entrepreneurs. The intuition is straightforward: When π is not high enough, the high return R_2 will come too late most of the time, triggering frequent bank runs in period 1. So depositors prefer banks to adopt the safe strategy in this range. In contrast, for $\pi > \overline{\pi}_2$ it would be inefficient for private banks to hold enough liquid assets on their balance sheets to prevent disaster when markets become disorderly. As long as all banks play according to the strategies outlined above, depositors' payoff is characterized by the dotted grey line in Figure 3.

However, such solution cannot survive in the free market: When all banks choose the strategy $\alpha(p_L)$, there will be excess liquidity at $t = 1$ if the good state occurs

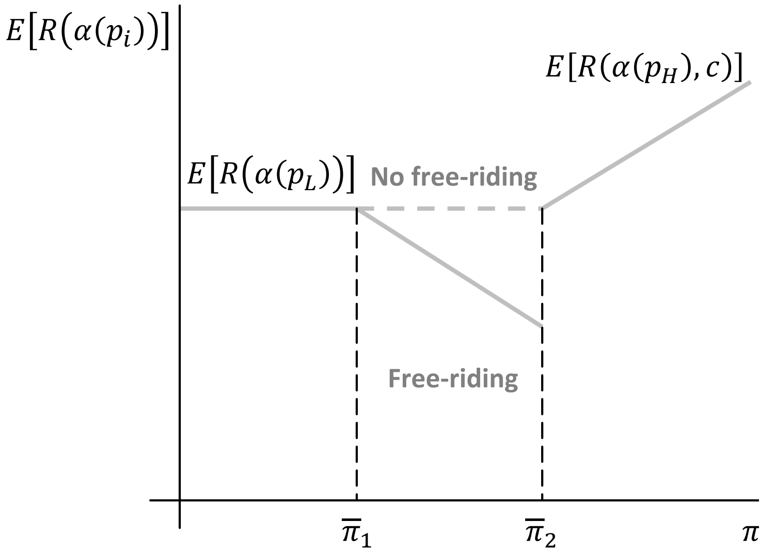

Fig. 3. Depositors' expected return

(with a large share of type 2 projects realized early). A bank anticipating this event has a strong incentive to invest all their funds in type 2 projects, reaping the benefit of excess liquidity in the good state. As long as the music is playing, such a deviating bank gets up and dances — Having invested only in high yielding projects, the free-riding bank can always credibly extract entrepreneurs' excess liquidity at $t = 1$, promising to pay back at $t = 2$ out of highly profitable projects. After all, at that stage, this bank, free-riding on liquidity, can offer a capital cushion with expected returns well above what prudent banks are able to promise. Of course, if the bad state happens, there is no excess liquidity. The free-riding banks would just bid up the interest rates, urgently trying to get funds. Rational depositors, anticipating that these banks won't succeed, will already trigger a bank run on these banks at $t = \frac{1}{2}$.

When the music stops, in terms of liquidity, things get complicated. As long as free-riding banks are not supported in the bad state, they are driven out of the market, providing just the return c. Nevertheless, a bank free-riding on liquidity in the good state can on average offer the attractive return $\pi\gamma R_2 + (1-\pi)c$ as expected payoff for depositors. Thus, a free-riding bank will always be able to outbid a prudent bank whenever the probability π for the good state is not too low. The condition is

$$\pi > \overline{\pi}_1 = \frac{\gamma E[R_L] - c}{\gamma R_2 - c}.$$

Since $R_2 > E[R_H]$, it pays to free-ride within the range $\overline{\pi}_1 \leq \pi < \overline{\pi}_2$.

Obviously, there cannot be equilibrium in pure strategies within that range. As long as the music is playing, all banks would like to "get up and dance". But then, there would be no prudent bank left providing the liquidity needed to be able to free-ride. In the resulting mixed strategy equilibrium, a proportion of banks behave prudently, investing some amount $\alpha_s < \alpha(p_L)$ in liquid assets, whereas the rest free-rides on liquidity in the good state, choosing $\alpha = 0$. Prudent banks reduce $\alpha_s < \alpha(p_L)$ in order to cut down the opportunity cost of investing in safe projects. Interest rates and α_s adjust so that depositors are indifferent between the two types of banks. At $t = 0$, both prudent and free-riding banks offer the same expected return to depositors. The proportion of free-riding banks is determined by aggregate market clearing conditions in both states. Free-riding banks experience a run for sure in the bad state, but the high return in the good state $R_2 > E[R_s]$ compensates depositors for that risk.

As shown in Proposition 2.1, free-riding drives down the return for investors (see Figure 3). They are definitely worse off than they would be if all banks coordinated on the prudent strategy $\alpha(p_L)$ — Here it is similar as the inefficient mixes strategy equilibrium in Allen & Gale (2004). As illustrated in Figure 3, the effective return on deposits for investors deteriorates in the range $\overline{\pi}_1 \leq \pi < \overline{\pi}_2$ as a result of free-riding behaviour.

Proposition 2.1 *In the mixed strategy equilibrium, investors are worse off than if all banks coordinate on the prudent strategy $\alpha(p_L)$.* □

Proof See Appendix A.1. □

3 Lender of Last Resort Policy

A lender of last resort cannot create real liquidity at period one. But a central bank can add nominal liquidity at the stroke of a pen. Following Allen & Gale (1998) and Diamond & Rajan (2006), assume from now on that deposit contracts are arranged in nominal terms. The mechanism is similar as that in Allen, Carletti, & Gale (2101): At $t = 1$, the liquidity injection with the banks' illiquid assets as collateral is done so that the banks are able to honor their nominal contracts, reducing the real value of deposits just to the amount of real resources available at that date. Then at $t = 2$ the banks repay the central bank by the return from the late projects, with gross nominal interest rate r^M. Such intervention raises the real payoff of depositors compared to inefficient liquidation, increasing the expected payoff of the risky strategy $\alpha(p_H)$.

As argued before, since the investors' funds is the limiting factor in this economy, the central bank's rate r^M is only optimal when the investors' return is maximized, i.e., the banks get liquidity injection at the lowest cost. Therefore, the central bank sets $r^M = 1$. With liquidity injection, bank runs are prevented when the bad state (with low payoffs at $t = 1$) occurs. Such a policy, preventing inefficient costly liquidation, raises investors' expected payoff and so definitely improves upon the allocation for high values $\pi > \pi_2$. Essentially, nominal deposits allow the central bank to implement state contingent payoffs. This argument seems to confirm the view that the lender of last resort indeed is a free lunch, providing a public good at no cost. It turns out, however, that the anticipation of these actions has an adverse impact on the amount of aggregate liquidity provided by the private sector, affecting endogenously the exposure to systemic risk.

The incentive for free-riding prevalent in modern times of competitive financial markets complicates the picture dramatically. In the model presented, a lender of last resort, providing liquidity support to the market requesting good collateral as the only condition, will drive out all prudent banks — All banks are encouraged to free-ride and choose the risky strategy $\alpha(p_H)$, knowing that they can get liquidity support against good collateral. The public provision of emergency liquidity results in serious moral hazard. It is as simple as that.

Proposition 3.1 *Assume that* $\pi p_H R_2 + (1-\pi) p_L R_2 \geq 1$ *and that for* $\pi \in (\overline{\pi}_1, \overline{\pi}_2)$, $d_0^j = \gamma R_2 > \pi p_H R_2 + (1-\pi)c$. *If the central bank is willing to provide liquidity to the entire market in times of crisis, all banks have an incentive to free-ride, choosing* $\alpha_j = 0$. □

Proof See Appendix A.2. □

The reason for this surprising result is the following: Since we concentrate on the case of pure illiquidity risk, in our model, the liquidity shock just retards the realization of high yielding projects. In the end (at $t = 2$), all projects will certainly be realized. So there is no doubt about solvency of the projects, unless insolvency is triggered by illiquidity. Central bank support against allegedly good collateral, creating artificial liquidity at the drop of a hat, destroys all private incentives to care about ex ante liquidity provision. The key problem with the Bagehot principle here is that free-riding banks do invest in projects with higher returns, as long as they do not have to be terminated. In reality, there is no clear-cut distinction between insolvency and illiquidity. We leave it to future research to allow for the risk of insolvency. But we doubt that our basic argument will be affected.

So what policy options should be taken? One might argue that a central bank should provide liquidity support only to prudent banks (conditional on banks having invested sufficiently in liquid assets). As shown in Cao & Illing (2008), such a policy may improve the allocation at least to some extent. But we argued that such a commitment is simply not credible: As emphasised by Rochet (2004), there is a serious problem of dynamic consistency.

Rather than relying on an implausible commitment mechanism, the obvious solution would be a mix of two instruments: ex ante liquidity regulation combined with ex post lender of last resort policy. It seems to be rather surprising that conventional wisdom argues that central banks can pursue both price stability and financial stability using just one tool, interest rate policy. Instead, the second best outcome from the investors' point of view needs to be implemented by the following policy: In a first step, a banking regulator has to impose ex ante liquidity requirements. Requesting minimum investment in liquid type 1 assets of at least $\alpha(p_L)$ for $\pi < \overline{\pi}_2'$ and $\alpha(p_H)$ for $\pi > \overline{\pi}_2'$ would give investors the highest expected payoff as characterized in Figure 4. For $\pi < \overline{\pi}_2'$, playing safe gives investors the highest payoff. In contrast, for $\pi > \overline{\pi}_2'$ investors are better off if banks invest in liquid assets as low as $\alpha(p_H)$ as long as lender of last resort policy helps to prevent runs. Since such a rule would not allow banks to operate when liquidity holdings are less than required, it could get rid of incentives for free-riding. Given that the ex ante imposed liquidity requirements have been fulfilled, ex post the central bank can safely play its role as lender in the range $\pi > \overline{\pi}_2'$ whenever the bad state turns out to be realized. Note that this policy raises the expected payoff for investors, even though it increases the range of parameter values with systemic risk.

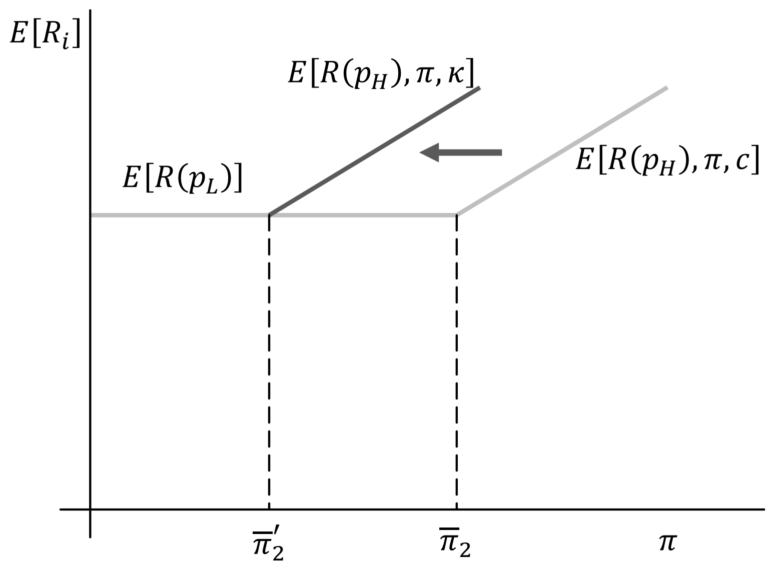

Fig. 4. Depositors' expected return with ex ante liquidity regulation and ex post lender of last resort policy

The key task for regulators and the central bank is to cope with free-riding incentives. As an alternative mechanism compared to ex ante liquidity regulation, the central bank might commit to try to mop up the excess liquidity available in the good state. If that can be done, potential free-riders would have no chance to survive. We doubt, however, that the central will be able to implement such a policy.

As a further alternative, one might impose narrow banking in the sense that banks are required to hold sufficient liquid funds so as to pay out in all contingencies. Finally, one might expect that imposing equity or capital requirements are sufficient to provide a cushion against liquidity shocks. As shown in the next section, both these options turn out to be strictly worse than imposing minimum liquidity standards ex ante combined with lender of last resort policy. They are even likely to be inferior to the outcome of a mixed strategy equilibrium with free-riding banks.

4 The role of equity and narrow banking

Let us now introduce equity requirements in the model, i.e. banks are required to hold some equity in their assets. We keep the same settings as before with the presence of aggregate uncertainty, except that instead of pure fixed deposit contracts, the banks issue a mixture of deposit contract and equity for the investors (Diamond & Rajan, 2000, 2005, 2006). To make it clear, equity is a claim that can be renegotiated so that the bank managers and the capital holders (here the investors) split the residual surplus after the deposit contract has been paid. The mixture of deposit contracts and equity seems to be a bit artificial setting at first sight. But actually it turns out to be a convenient modelling device. In particular, in the symmetric equilibria of the banks, such a mixture will be exactly the portfolio held by a representative agent out of the homogenous investors. In other words, whenever investors are homogenous, it is not necessary to separate equity holders from the depositors.

Equity can reduce the fragility, but it allows the bank manager to capture a rent. Being a renegotiatable claim, equity is always subject to the hold-up problem, i.e. equity holders can only get a share of ζ ($\zeta \in [0, 1]$) from the surplus. To make it simpler, in the following we simply assume that $\zeta = \frac{1}{2}$.

With $\zeta = \frac{1}{2}$ the bankers get a rent of $\frac{\gamma E[R] - d_0}{2}$, sharing the surplus over deposits equally with the equity holders. Suppose that all the banks have to meet the level of equity k which comes from the central bank's regulatory rules, then if a bank i

does not suffer a run k is defined as

$$k = \frac{\frac{\gamma E[R_{s,i}]-d_{0,i}}{2}}{\frac{\gamma E[R_{s,i}]-d_{0,i}}{2} + d_{0,i}}$$

in which $R_{s,i}$ is bank i's return achieved under state s.

The additional, but crucial assumptions concerning timing are that (1) the dividend of the equity is paid *after* the payment of $d_{0,i}$ and (2) capital requirement has to be met till the last minute before the dividend payment — This deters the bankers' incentive to transfer their dividend income to the investors *ex post*, which increases $d_{0,i}$ *ex ante*.

Solve for $d_{0,i}$ to get

$$d_{0,i} = \frac{1-k}{1+k}\gamma E[R_{s,i}].$$

Then one would ask: Under what conditions would it make sense to introduce equity requirements? It is easy to see that introducing equity will definitely reduce investors' payoff in the absence of aggregate risk. Somewhat counterintuitively, capital requirements even reduce the share α invested in the safe project in that case. The reason is that with equity, bank managers get a rent of $\frac{\gamma E[R]-d_0}{2}$, sharing the surplus over deposits equally with the equity holders. So investors providing funds in form of both deposits and equity to the banks will receive at $t = 1$ just $\frac{1}{1+k}\gamma E[R] < \gamma E[R]$. Since the return at $t = 2$ is higher than at $t = 1$, bank managers prefer to consume late, so the amount of resources needed at $t = 1$ is lower in the presence of equity. Consequently, the share α will be reduced. Of course, banks holding no equity provide more attractive conditions for investors, so equity could not survive. This at first sight counterintuitive result simply demonstrates that there is no role (or rather only a payoff reducing role) for costly equity in the absence of aggregate risk.

But when there is aggregate risk, equity helps to absorb the aggregate shock. In the simple 2-state set up, equity holdings need to be just sufficient to cushion the bad state. So with equity, the bank will chose $\alpha^* = \alpha(p_H)$. The level of equity k needs to be so high that, given $\alpha^* = \alpha(p_H)$, the bank just stays solvent in the bad state — it is just able to pay out the fixed claims of depositors, whereas all equity will be wiped out.

With equity k, the total amount that can be pledged to both depositors and equity in the good state is $\frac{1}{1+k}\gamma E[R_H]$ with claims of depositors being $d_0 = \frac{1-k}{1+k}\gamma E[R_H]$ and equity $EQ = \frac{k}{1+k}\gamma E[R_H]$. In the bad state, a marginally solvent bank can pay out to depositors $d_0 = \alpha(p_H) R_1 + (1 - \alpha(p_H)) p_L R_2$. So k is determined by the condition:

$$\frac{1-k}{1+k}\gamma E[R_H] = \alpha(p_H) R_1 + (1 - \alpha(p_H)) p_L R_2,$$

and we solve to get

$$k = \frac{\gamma E[R_H] - d_0}{\gamma E[R_H] + d_0}. \tag{1}$$

It is observed that k is decreasing in p_L: The higher p_L, the lower the equity k is needed to stay solvent in the bad state. The necessary level of equity k is zero for $p_L = p_H$, and for p_L close to p_H equity holding is superior to the strategy $\alpha^* = \alpha(p_H)$. That is if

$$d_0 \geq \gamma E[R_H]\pi + (1 - \pi)c.$$

So (d_0, k) is the equilibrium for the banks. The reason is easy to see: First, no banks are willing to set k_i higher — because equity holding is costly and the bank is not able to compete with the other banks for $(d_{0,i}, k_i)$; second, no banks are able to set higher $d_{0,i}$ given (d_0, k) set by all the other banks — because k has to be met when $d_{0,i}$ is paid, the only thing the deviator can do is to bid up interest rate and this leads to bank runs across the whole banking industry — the deviation is not profitable.

From the regulator's point of view, the unique optimal equity requirement k it imposes is exactly the k determined by condition (1), which is so high that the bank just stays solvent in the bad state — it is just able to payout the fixed claims of depositors, whereas all equity will be wiped out. The reason is simple: Since holding equity is costly, the only reason for the central bank to make it sensible is to eliminate the costly bank run. Therefore neither too low k (which is purely a cost and doesn't prevent any bank run) nor too high k (which prevents bank runs, but incurs too high a cost of holding capital) is optimal. Thus from now on we can concentrate on such level of k without loss of generality.

Now the interesting question is: Can capital requirement improve the allocation in this economy, in comparison to the *laissez-faire* outcome we studied before?

Definition Define a representative depositor's expected return function without equity requirements as $\Pi(\pi, \cdot)$, such that

$$\Pi(\pi, \cdot) = \begin{cases} \gamma E[R_L], & \text{if } \pi \in [0, \overline{\pi}_1]\,; \\ \alpha_s^* R_1 + (1 - \alpha_s^*)\, p_L R_2, & \text{if } \pi \in (\overline{\pi}_1, \overline{\pi}_2)\,; \\ \gamma E[R_H]\pi + (1 - \pi)c, & \text{if } \pi \in [\overline{\pi}_2, 1] \end{cases}$$

and her expected return function under equity requirements as $\Pi_e(\pi, \cdot)$, as well as the set S in which the investor's payoff is improved under equity requirement, such that

$$S := \{\hat{\pi} | \Pi_e(\hat{\pi}, \cdot) \geq \Pi(\hat{\pi}, \cdot)\}\,. \qquad \square$$

The black lines of Figure 5 describe the laissez-faire outcome $\Pi(\pi, \cdot)$, and the grey line shows the depositors expected return $\Pi_e(\pi, \cdot) = d_0 + \frac{\Pi}{2}\pi$ under capital requirement, which consists of two terms:

- The deposit payment d_0;
- The dividend of equity holdings $\frac{\Pi}{2}$, which is only achieved in the good state, and its value is determined by

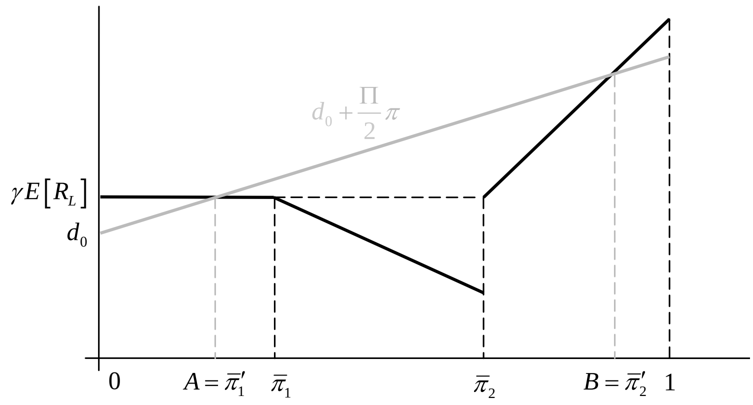

Fig. 5. Expected return with / without equity — Case 1

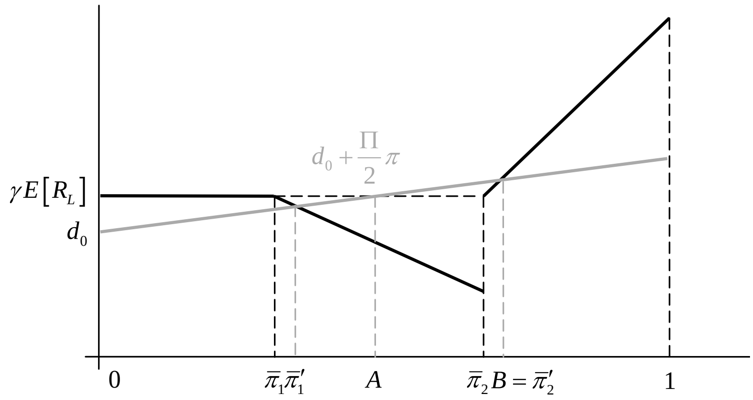

Fig. 6. Expected return with / without equity — Case 2

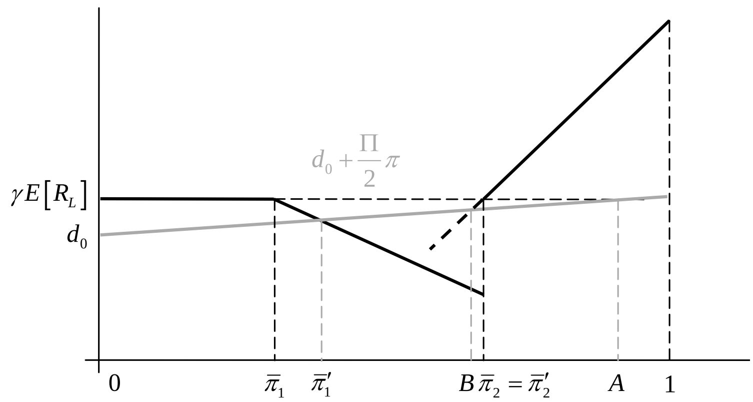

Fig. 7. Expected return with / without equity — Case 3

$$\frac{\Pi}{2} = \frac{\gamma E[R_H] - d_0}{2} = \frac{\gamma E[R_H] - \frac{1-k}{1+k}\gamma E[R_H]}{2} = \frac{k}{1+k}\gamma E[R_H].$$

Denote the intersection of $\Pi_e(\pi, \cdot) = d_0 + \frac{\Pi}{2}$ and $\gamma E[R_L]$ by A, which is equal to (see Appendix A.4 for detail)

$$A = \frac{2(R_1 - p_L R_2)}{(1-\gamma)R_1 + (\gamma - p_L)R_2},$$

as well as the intersection of $\Pi_e(\pi, \cdot) = d_0 + \frac{\Pi}{2}$ and $\gamma E[R_H]\pi + (1-\pi)c$ by B, which is equal to (see Appendix A.4 for detail)

$$B = \frac{2\left[(1-\gamma)(cR_1 - p_L R_1 R_2) + (\gamma - p_H)(cR_2 - R_1 R_2)\right]}{2(1-\gamma)cR_1 + 2(\gamma - p_H)cR_2 + \left[\gamma(p_H - 1) - (\gamma - p_H) - (1-\gamma)p_L\right] R_1 R_2}.$$

Now it is straightforward to compare investor's payoff under equity requirements with the laissez faire free-riding equilibrium for some extreme values:

Lemma 4.1 *The depositors' expected return under the equity requirement is lower than the laissez-faire outcome when $\pi = 0$ or $\pi = 1$.* □

Proof See Appendix A.3. □

The intuition of Lemma 4.1 is straightforward: There is no uncertainty when $\pi = 0$ or $\pi = 1$, so it is inferior to hold costly equities as we already explained before.

Then Proposition 4.2 characterizes the improvement in investor's payoff achievable by introducing equity requirements.

Proposition 4.2 *Given the equity requirement k imposed by the regulator,*

- *When $A \in (0, \overline{\pi}_1]$, i.e.*

  $$(2\gamma R_2 - \gamma E[R_H] - d_0)(\gamma E[R_L] - d_0) + (2\gamma E[R_L] - \gamma E[R_H] - d_0)(d_0 - c) \leq 0,$$

  then $S = [A, B] \supseteq [\overline{\pi}_1, \overline{\pi}_2]$;
- *When $A \in (\overline{\pi}_1, \overline{\pi}_2]$, i.e.*

  $$(2\gamma R_2 - \gamma E[R_H] - d_0)(\gamma E[R_L] - d_0) + (2\gamma E[R_L] - \gamma E[R_H] - d_0)(d_0 - c) > 0,$$

  and

$$\gamma\,(E[R_H] - E[R_L])\,(d_0 - c) \geq (\gamma E[R_H] - c)\,(\gamma E[R_L] - d_0)\,,$$

then $S = [\tilde{\pi}, B]$ *in which* $\tilde{\pi} \in (\overline{\pi}_1, \overline{\pi}_2]$ *and* $S \bigcap [\overline{\pi}_1, \overline{\pi}_2] = [\tilde{\pi}, \overline{\pi}_2]$*;*

- *When* $A \in (\overline{\pi}_2, 1]$*, i.e.*

$$2\,(\gamma E[R_L] - d_0)\,(\gamma E[R_H] - c) \geq (\gamma E[R_H] - d_0)\,(\gamma E[R_L] - c)\,,$$

then $S \subseteq [\tilde{\pi}, B]$ *in which* $\tilde{\pi} \in (\overline{\pi}_1, \overline{\pi}_2]$ *and* $S \bigcap [\overline{\pi}_1, \overline{\pi}_2] = [\tilde{\pi}, \overline{\pi}_2]$. □

Proof See Appendix A.4. □

The three possible cases are characterized in Figures 5, 6 and 7, respectively. Numerical examples simulating these cases are presented in Appendix B.

Equity requirements give investors a higher payoff than the laissez-faire market outcome whenever their payoff with a safe bank holding sufficient equity exceeds the payoff of the mixed strategy equilibrium with free-riding banks for all parameter values. This case is captured as case 1, shown in Figure 5. Since free-riding partly destroys the value of deposits held by prudent banks (forcing them to hold a riskier portfolio), it seems obvious that imposing equity requirements will always dominate the laissez-faire outcome with mixed strategies. Unfortunately, this need not be the case. It is quite likely that equity requirements result in inferior payoffs for some range of parameter values (as shown in case 2 — see Figure 6). It might even be that imposing equity requirements makes investors worse off than laissez-faire for all parameter values. This is shown in Figure 7, representing case 3.

The intuition behind this at first surprising result is that holding equity can be quite costly; if so, it may be superior to accept the fact that systemic risk is a price to be paid for higher returns on average.

The mix of ex ante liquidity requirements with ex post lender of last resort policy is always dominating equity requirements. See Figure 8. The reason is as follows: Consider that the banks are required to hold $\underline{\alpha} = \alpha(p_H)$ when π is high. Then when p_H reveals, the investors' real return is $\gamma E[R_H]$; and when p_L reveals, the investors' real return is $\alpha(p_H)R_1 + (1 - \alpha(p_H))p_L R_2$. Therefore the investors' overall expected return turns out to be

$$\Pi_m = \gamma E[R_H]\pi + (1-\pi)\left[\alpha(p_H)R_1 + (1-\alpha(p_H))p_L R_2\right],$$

which is linear in π, as the chain line of FIGURE 8 shows. Note that when $\pi = 1$, $\Pi_m = \gamma E[R_H] > d_0 + \frac{\Pi}{2}$; and when $\pi = 0$, $\Pi_m = \alpha(p_H)R_1 + (1-\alpha(p_H))p_L R_2 = d_0$. Therefore, Π_m line is above $d_0 + \frac{\Pi}{2}\pi$, $\forall \pi \in (0,1]$, i.e. the mix of liquidity requirements with lender of last resort policy is always dominating equity requirements when aggregate uncertainty exists.

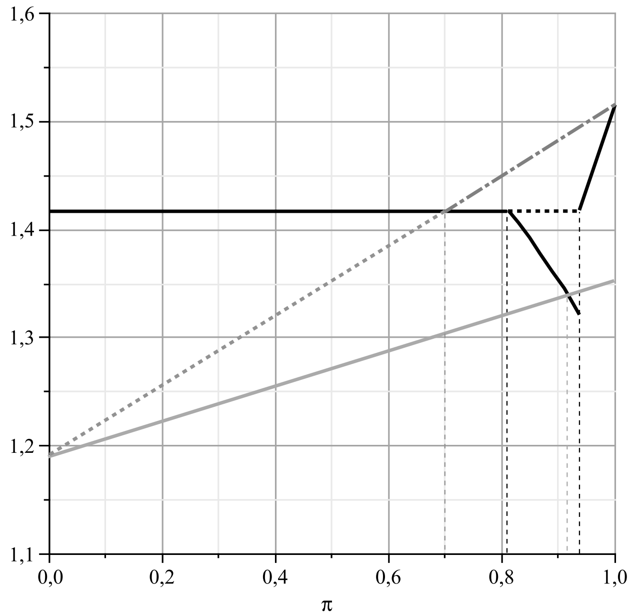

Fig. 8. Expected return with credible liquidity injections (for the case of FIGURE B.3)

In times of crises, frequently there are calls to go back to narrow banking in order to avoid the risk of runs. Under narrow banking, institutions with deposits would be required to hold as assets only the most liquid instruments so as to be always able to meet any deposit withdrawal by selling its assets. Obviously, narrow banking can be extremely costly. In our model, banks would be required to hold sufficient liquid funds to pay out in all contingencies: $\alpha > \alpha(p_L)$. As FIGURE 9 illustrates, under narrow banking an investor's payoff can be much lower for high π compared to ex ante liquidity regulation combined with ex post lender of last resort policy. Just as with equity requirements, narrow banking (imposing the requirement that banks hold sufficient equity so as to be able to pay out demand deposits in all states of the world) can be quite inferior: If the bad state is a rare probability event, it simply makes no sense to dispense with all the efficiency gains from investing in high yielding illiquid assets despite its impact on systemic risk.

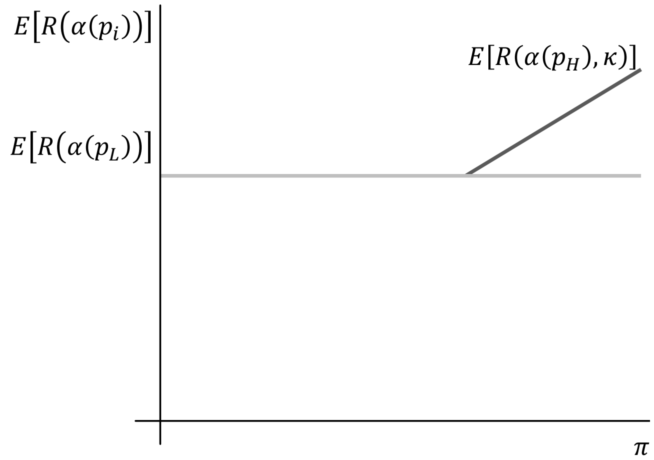

Fig. 9. Expected return with narrow banking compared to ex ante liquidity regulation

5 Conclusion

Traditionally, aggregate liquidity shocks have been modelled as exogenous events. In this paper, we derive the aggregate share of liquid projects endogenously. It depends on the incentives of financial intermediaries to invest in risky, illiquid projects. This endogeneity allows us to capture the feedback between financial market regulation and incentives of private banks, determining the aggregate amount of liquidity available. As a consequence of limited liability, banks are encouraged to free-ride on liquidity provision. Relying on sufficient liquidity provided by the market, they are inclined to invest excessively in illiquid long term projects.

Liquidity provision by central banks can help to prevent bank runs with inefficient early liquidation. However, the anticipation of unconditional liquidity provision results in overinvestment in risky activities (moral hazard), creating excessive exposure to systemic risk. We show that it is crucial for efficient lender of last resort policy to impose ex ante minimum liquidity standards for banks. In addition, we analyze the impact of equity requirements. We show that it is even likely to be inferior to the outcome of a mixed strategy equilibrium with free-riding banks. For similar reasons, imposing narrow banking (require banks to hold sufficient liquid funds to pay out in all contingencies) turns out to be strictly inferior relative to the combination of liquidity requirements with lender of last resort policy.

In modern economies, a significant part of intermediation is provided by the shadow banking sector. These institutions (like hedge funds and investment banks) are not financed via deposits, but they are highly leveraged. Incentives to dance (to free-ride on liquidity provision) seem to be even stronger for the shadow banking industry. So imposing liquidity requirements only for the banking sector will not be sufficient to cope with free-riding. In future work, we plan to analyze incentives for leveraged institutions within our framework.

Appendix

A Proofs

A.1 Proof of Proposition *2.1*

The mixed strategy equilibrium is characterized as Proposition 2 of Cao & Illing (2008). By chooseing α_s^* a prudent bank should have equal return at both states, $d_0^s = d_0^s(p_H) = d_0^s(p_L)$, i.e.

$$\gamma\left[\alpha_s^* R_1 + (1-\alpha_s^*)p_H R_2 + \frac{(1-\alpha_s^*)(1-p_H)R_2}{r_H}\right]$$
$$=\gamma\left[\alpha_s^* R_1 + (1-\alpha_s^*)p_L R_2 + \frac{(1-\alpha_s^*)(1-p_L)R_2}{r_L}\right].$$

With some simple algebra this is equivalent to

$$\frac{1}{r_H} = \frac{1-p_L}{1-p_H}\frac{1}{r_L} - \frac{p_H - p_L}{1-p_H}.$$

Plot $\frac{1}{r_H}$ as a function of $\frac{1}{r_L}$ as the black line in Figure A.1 shows.

The slope $\frac{1-p_L}{1-p_H} > 1$ and intercept $-\frac{p_H-p_L}{1-p_H} < 0$, and the line goes through $(1,1)$. But $r_H = r_L = 1$ cannot be equilibrium outcome here, because $\alpha(p_L)$ is dominant strategy in this case and subject to deviation. So whenever $r_H > 1$ (suppose $\frac{1}{r_H} = A$ in the graph), there must be $r_H > r_L > 1$ (because $\frac{1}{r_H} < \frac{1}{r_L} = B < 1$).

At p_L, given that $r_L > 1$ the prudent bank's return is equal to $d_0^s = \kappa(\alpha_s^*(p_L, r_L)) < \kappa(\alpha(p_L))$, since the latter maximizes the bank's expected return with $r^* = 1$ by Lemma 2 of Cao & Illing (2008). Therefore in the mixed strategy equilibrium, investors are worse off than if all banks would coordinate on the prudent strategy $\alpha(p_L)$. $\square$

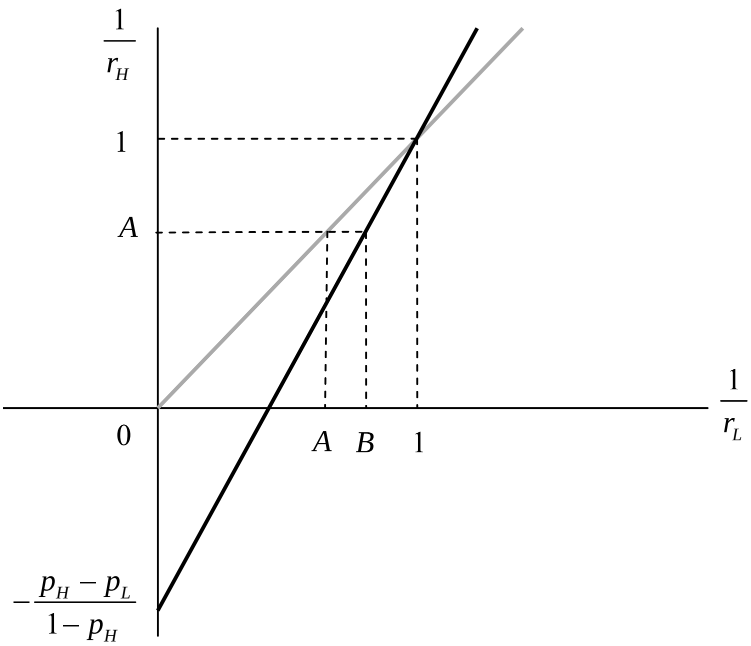

Fig. A.1. Higher interest rates in the mixed strategy equilibrium

A.2 Proof of Proposition *3.1*

Suppose that a representative bank chooses to be prudent with $\alpha_i = \underline{\alpha}$, and promises a nominal deposit contract $d_0^i = \gamma\left[\underline{\alpha}R_1 + (1 - \underline{\alpha})R_2\right]$ in order to maximize its investors return. Then when the bad state with high liquidity needs is realized, the central bank has to inject enough liquidity into the market to keep interest rate at $r = 1$ in order to ensure bank i's survival. However, given $r = 1$, a naughty bank j can always profit from setting $\alpha_j = 0$, promising the nominal return $d_0^j = \gamma R_2 > d_0^i$ to its investors. Thus, surely the banks prefer to play naughty.

For those parameter values such that $\pi p_H R_2 + (1 - \pi) p_L R_2 < 1$ there exists no equilibrium with liquidity injection. The reason is the following:

(1) Any symmetric strategic profile cannot be equilibrium, because
   (a) If there is no trade under such strategic profile, i.e. α is so small that the real return is less than 1, one bank can deviate by setting $\alpha = 1$ and trading with investors;

(b) If there is trade under such strategic profile, i.e. $\alpha > 0$ for all the banks, then one bank can deviate by setting $\alpha = 0$ and getting higher nominal return than the other banks.

(2) Any asymmetric strategic profile, or profile of mixed strategies, cannot be equilibrium, because

(a) If there is no trade under such strategic profile, then the argument of 1 a) applies here;

(b) If there is trade under such strategic profile, then one bank can deviate by choosing a pure strategy, $\alpha = 0$, and get better off — there is no reason to mix with the other dominated strategies. $\square$

A.3 Proof of Lemma *4.1*

When $\pi = 0$,

$$\begin{aligned} d_0 + \frac{\Pi}{2} \cdot 0 &= \alpha\left(p_H\right) R_1 + \left(1 - \alpha\left(p_H\right)\right) p_L R_2 \\ &< \alpha\left(p_L\right) R_1 + \left(1 - \alpha\left(p_L\right)\right) p_L R_2 \\ &= \gamma E[R_L]; \end{aligned}$$

When $\pi = 1$,

$$\begin{aligned} d_0 + \frac{\Pi}{2} &= \frac{\alpha\left(p_H\right) R_1 + \left(1 - \alpha\left(p_H\right)\right) p_L R_2 + \alpha\left(p_H\right) R_1 + \left(1 - \alpha\left(p_H\right)\right) p_H R_2}{2} \\ &< \alpha\left(p_H\right) R_1 + \left(1 - \alpha\left(p_H\right)\right) p_H R_2 \\ &= \gamma E[R_H]. \qquad \square \end{aligned}$$

A.4 Proof of Proposition *4.2*

Generically, there are three cases concerning the relative positions of $\Pi(\pi, \cdot)$ and $\Pi_e(\pi, \cdot)$:

(1) As Figure 5 shows, the intersection A lies between 0 and $\overline{\pi}_1$;

(2) As FIGURE 6 shows, the intersection A lies between $\overline{\pi}_1$ and $\overline{\pi}_2$;

(3) As FIGURE 7 shows, the intersection A lies between $\overline{\pi}_2$ and 1.

The intersection A takes the value of π, such that

$$\gamma E[R_L] = d_0 + \frac{\Pi}{2}\pi.$$

Solve to get

$$A = \frac{2\left(\gamma E[R_L] - d_0\right)}{\gamma E[R_H] - d_0} = \frac{2(R_1 - p_L R_2)}{(1-\gamma)R_1 + (\gamma - p_L)R_2}.$$

The intersection B takes the value of π, such that

$$\gamma E[R_H]\pi + (1-\pi)c = d_0 + \frac{\Pi}{2}\pi.$$

Solve to get

$$\begin{aligned} B &= \frac{d_0 - c}{\frac{\gamma E[R_H] + d_0}{2} - c} \\ &= \frac{2\left[(1-\gamma)(cR_1 - p_L R_1 R_2) + (\gamma - p_H)(cR_2 - R_1 R_2)\right]}{2(1-\gamma)cR_1 + 2(\gamma - p_H)cR_2 + \left[\gamma(p_H - 1) - (\gamma - p_H) - (1-\gamma)p_L\right] R_1 R_2}. \end{aligned}$$

Then the set S can be determined in each case:

(1) As FIGURE 5 shows, when $A \in (0, \overline{\pi}_1]$,

$$\frac{2\left(\gamma E[R_L] - d_0\right)}{\gamma E[R_H] - d_0} \leq \overline{\pi}_1 = \frac{\gamma E[R_L] - c}{\gamma R_2 - c},$$

rearrange to get

$$\begin{aligned} &(2\gamma R_2 - \gamma E[R_H] - d_0)\left(\gamma E[R_L] - d_0\right) + \left(2\gamma E[R_L] - \gamma E[R_H] - d_0\right)(d_0 - c) \\ &\leq 0. \end{aligned}$$

Since $\Pi_e(\pi, \cdot)$ is strictly increasing in π, then

$$\Pi_e(\pi,\cdot)|_{\pi=B} > \Pi_e(\pi,\cdot)|_{\pi=A} \geq \gamma E[R_L]|_{\pi=\overline{\pi}_1} = (\gamma E[R_H]\pi + (1-\pi)c)\,|_{\pi=\overline{\pi}_2}$$
$$\geq \Pi(\pi,\cdot)|_{\pi\in[\overline{\pi}_1,\overline{\pi}_2]},$$

which implies $S = [A, B] \supseteq [\overline{\pi}_1, \overline{\pi}_2]$;

(2) As FIGURE 6 shows, when $A \in (\overline{\pi}_1, \overline{\pi}_2]$,

$$\overline{\pi}_1 = \frac{\gamma E[R_L] - c}{\gamma R_2 - c} < \frac{2\,(\gamma E[R_L] - d_0)}{\gamma E[R_H] - d_0},$$

rearrange to get

$$(2\gamma R_2 - \gamma E[R_H] - d_0)\,(\gamma E[R_L] - d_0) + (2\gamma E[R_L] - \gamma E[R_H] - d_0)\,(d_0 - c)$$
$$> 0.$$

What's more, in this case $B \in [\overline{\pi}_2, 1]$, and this is equivalent to

$$\frac{\gamma E[R_L] - c}{\gamma E[R_H] - c} = \overline{\pi}_2 < \frac{d_0 - c}{\frac{\gamma E[R_H] + d_0}{2} - c},$$

rearrange to get

$$\gamma\,(E[R_H] - E[R_L])\,(d_0 - c) \geq (\gamma E[R_H] - c)\,(\gamma E[R_L] - d_0)\,.$$

Similarly,

$$\Pi_e(\pi,\cdot)|_{\pi\leq A} \leq \gamma E[R_L]|_{\pi=\overline{\pi}_1} = (\gamma E[R_H]\pi + (1-\pi)c)\,|_{\pi=\overline{\pi}_2} \leq \Pi(\pi,\cdot)|_{\pi\in[\overline{\pi}_2,B]}$$
$$\leq \Pi_e(\pi,\cdot)|_{\pi\geq B},$$

which implies $S = [\tilde{\pi}, B]$ in which $\tilde{\pi} \in (\overline{\pi}_1, \overline{\pi}_2]$ and $S \bigcap [\overline{\pi}_1, \overline{\pi}_2] = [\tilde{\pi}, \overline{\pi}_2]$;

(3) As FIGURE 7 shows, when $A \in (\overline{\pi}_2, 1]$,

$$\overline{\pi}_2 = \frac{\gamma E[R_L] - c}{\gamma E[R_H] - c} < \frac{2\,(\gamma E[R_L] - d_0)}{\gamma E[R_H] - d_0},$$

rearrange to get

$$2\,(\gamma E[R_L] - d_0)\,(\gamma E[R_H] - c) \geq (\gamma E[R_H] - d_0)\,(\gamma E[R_L] - c)\,.$$

Similarly,

$$\Pi_e(\pi,\cdot)|_{\pi\leq B} < \Pi_e(\pi,\cdot)|_{\pi\geq A} \leq \gamma E[R_L]|_{\pi=\overline{\pi}_1} = (\gamma E[R_H]\pi + (1-\pi)c)\,|_{\pi=\overline{\pi}_2},$$

which implies $S \subseteq [\tilde{\pi}, B]$ in which $\tilde{\pi} \in (\overline{\pi}_1, \overline{\pi}_2]$ and $S \bigcap [\overline{\pi}_1, \overline{\pi}_2] = [\tilde{\pi}, \overline{\pi}_2]$. $\square$

B Results of numerical simulations

The following figures present numerical simulations representing the three different cases.

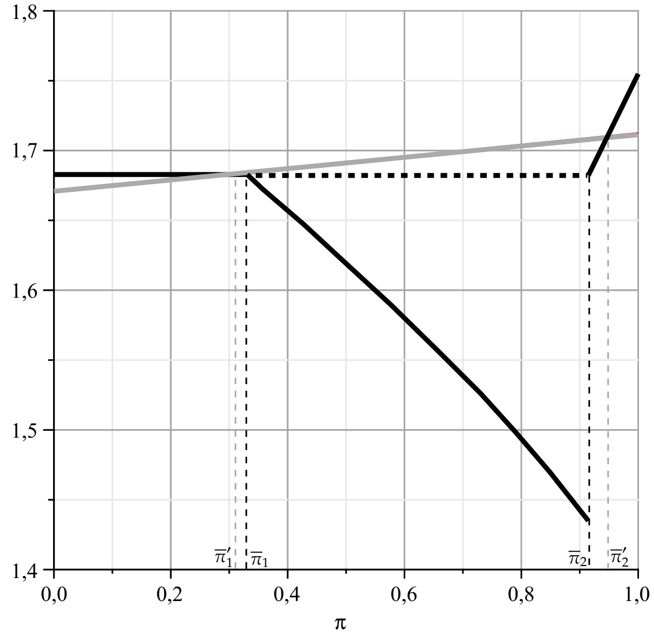

Fig. B.1. Expected return with / without equity, with $p_H = 0.3$, $p_L = 0.25$, $\gamma = 0.6$, $R_1 = 1.8$, $R_2 = 5.5$, $c = 0.9$

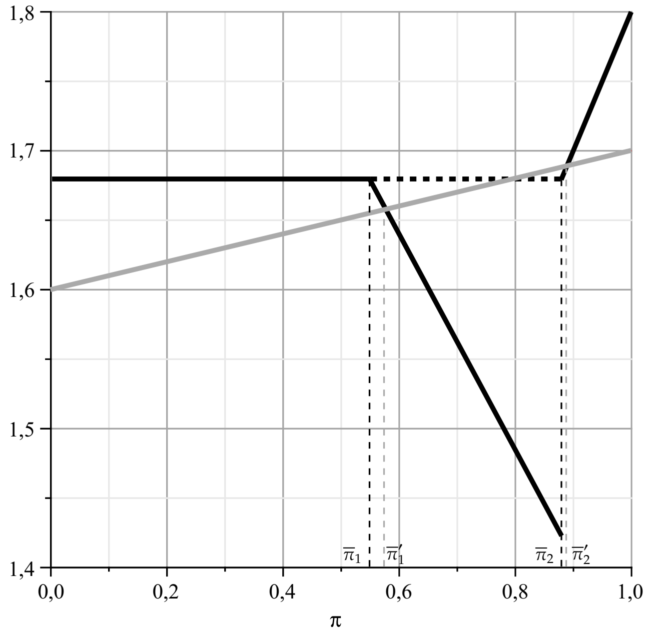

Fig. B.2. Expected return with / without equity, with $p_H = 0.4$, $p_L = 0.3$, $\gamma = 0.6$, $R_1 = 2$, $R_2 = 4$, $c = 0.8$

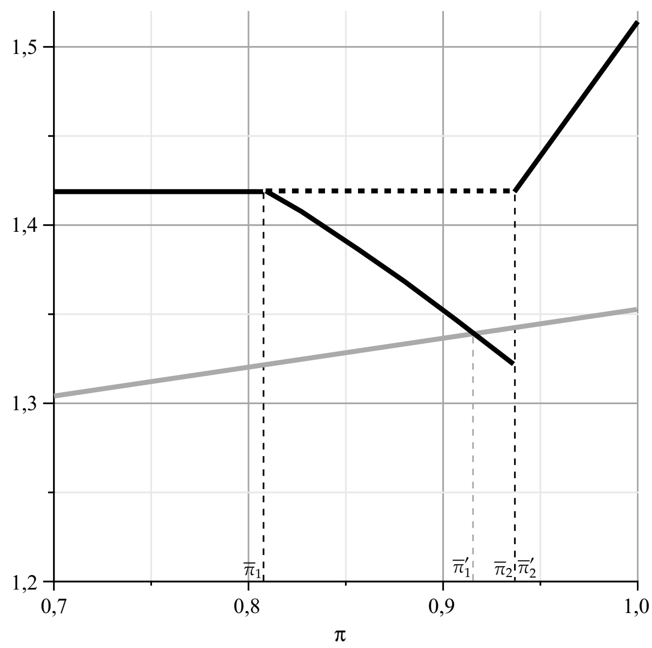

Fig. B.3. Expected return with / without equity, with $p_H = 0.5$, $p_L = 0.25$, $\gamma = 0.7$, $R_1 = 1.8$, $R_2 = 2.5$, $c = 0$

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