Fiscal Policy in an Unemployment Crisis

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Introduction

What is the size of the fiscal multiplier?

- I argue that it can be large (≈ 2) under three conditions:
  1. Low nominal interest rates (liquidity trap)
  2. High and...
  3. ... persistent unemployment
Introduction

What is the size of the fiscal multiplier?

- I argue that the can be large ($\approx 2$) under three conditions

  1. Low nominal interest rates (liquidity trap)
  2. High and . . .
  3. . . . persistent unemployment

- Key assumptions
  1. Zero lower bound
  2. Nominal wage rigidity (but flexible prices)
  3. Frictional labour market
A tale of recovery is a tale of a slump

- Disappointing news yields saving motive
- Can be averted by fall in nominal interest rate
- However, if news are really bad, the nominal interest rate will hit the zero lower bound
  - Still excess demand? ⇒ hoarding cash
- Yields a shortfall in nominal demand
A tale of recovery is a tale of a slump

- Will a fall in nominal demand imply a fall in real demand?
- Only if there are nominal rigidities
  - Sticky nominal wages
- Prices fall, and profits plummet
- The fall in profits lowers output and raises unemployment
- And unemployment exhibits inertia (frictional matching market)
A tale of recovery is a tale of a slump

- An increase in unemployment today leads to an increase in unemployment tomorrow
- Now the future looks even worse
  1. → more cash hoardings
  2. → larger drop in prices
  3. → less profits
  4. → less output
  5. → higher unemployment
  6. → back to step 1
Fiscal policy

- Is this **intertemporal** propagation mechanism important?
  - In a stylized framework, even very small drops in future productivity can push current output to **zero**

- So what can the government do?
  - By borrowing (or taxing) some of that hoarded cash and **spending it**, prices will increase
  - Profits will go up
  - Unemployment will fall

- By inertia, the future looks bright!
  - Government spending turns a vicious circle around
  - Hoardings are reduced, and private spendings increased
  - Unemployment falls even more, and on and on
The fiscal multiplier

\[ \frac{\partial y_t}{\partial g_0} \]

Period, \( t \)

Fiscal multiplier, \( \frac{\partial y_t}{\partial g_0} \)
Model

- Continuum of households of measure one
- Continuum of potential firms
- A government
- Two physical commodities
  - Cash, $m_t$, storable but not edible (numeraire)
  - Output, $y_t$, edible but not storable (trade at $p_t$)
- Cash in fixed supply $m_t = m$
- Time is discrete, $t = 0, 1, 2 \ldots$, and the horizon infinite
- Investments, but no capital
Households supply labor inelastically, $\ell_t = 1$

- Employment denoted $n_t$, so $u_t = 1 - n_t$
- Wage-rate is denoted $\tilde{w}_t$
- Total income, $w_t$, is labor income, $n_t \times \tilde{w}_t$, and dividends $q^t \times \tilde{d}_t$
- $q^t$ is the quantity of asset held in time $t$ (subscript) purchased in time $t$ (superscript)
Model: Households

- Only a fraction of the firms survive from one period to the next: $q_{t+1} = (1 - \lambda)q_t$

- Interpretation: $q_t$ is a diversified asset portfolio of which $\lambda$ firms go belly-up each period

  - $w_t$ paid out by the end of the period $t$
  - Thus, $w_t$ is disposable first in period $t + 1$
  - Need cash to go out shopping
Model: Households

- **Period budget constraint**

\[ b_t(1 + i_t) + p_t J_t(q_t^{t-1} - q_t^t) + (M_{t-1} - p_{t-1}c_{t-1}) \]
\[ + w_{t-1} - T_t = M_t + b_{t+1} \]

- **With CIA constraint**

\[ p_t c_t \leq M_t \]

- **For simplicity, define** \( x_{t+1} = M_t - p_t c_t \) (excess cash)
Model: Households

- Period budget constraint

\[ b_t(1 + i_t) + p_t J_t(q_{t-1}^t - q_t^t) + x_t \]

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Model: Households

- Period budget constraint

\[ b_t(1 + i_t) + p_t J_t (q_t^{t-1} - q_t^t) + x_t + w_{t-1} - T_t = p_t c_t + x_{t+1} + b_{t+1} \]

- With CIA constraint

\[ 0 \leq x_{t+1} \]

- For simplicity, define \( x_{t+1} = M_t - p_t c_t \) (excess cash)
Model: Households

- Problem: Given prices and taxes pick feasible \(\{c_t, b_{t+1}, q_t^t, x_{t+1}\}\) to maximize

\[
U(\{c_t\}_{t=0}^{\infty}) = E \sum_{t=0}^{\infty} \beta^t u(c_t)
\]

- \(E\) denotes the (mathematical) expectations over future processes
Model: Households: FOC

First order condition with respect to

- Bonds, $b_{t+1}$

\[ u'(c_t) = \beta E_t[(1 + i_{t+1})\frac{p_t}{p_{t+1}} u'(c_{t+1})] \]

- Excess cash, $x_{t+1}$

\[ u'(c_t) - \mu_t = \beta E_t[\frac{p_t}{p_{t+1}} u'(c_{t+1})] \]

- $x_{t+1} \geq 0$, $\mu_t \geq 0$, $x_{t+1} \times \mu_t = 0$
Model: Households: FOC

- When is $\mu_t = 0$? When $i_{t+1} = 0$ (and vice versa)
- Bonds and cash are perfect substitutes; liquidity trap
- $x_{t+1} \geq 0, i_{t+1} \geq 0, x_{t+1} \times i_{t+1} = 0$
Model: Households: FOC

- When is $\mu_t = 0$? When $i_{t+1} = 0$ (and vice versa)
- Bonds and cash are perfect substitutes; liquidity trap
- $\Rightarrow x_{t+1} \geq 0, \ i_{t+1} \geq 0, \ x_{t+1} \times i_{t+1} = 0$
- It should be noted that the ZLB is a consequence of the model, and not a constraint per se
  - If $i_{t+1} < 0$, households would like to borrow indefinitely and save borrowed funds as excess cash
Model: Households: FOC

First order condition with respect to assets, $q_t^t$

$$J_t = \beta E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} \left( \frac{\tilde{d}_t}{p_{t+1}} + (1 - \lambda)J_{t+1} \right) \right]$$
Model: Government

- \( \{G_t, T_t, d_t\} \) is a fiscal plan satisfying

\[
T_t + d_{t+1} = G_t + (1 + i_t)d_t
\]

- and

\[
\lim_{t \to \infty} \frac{d_{t+1}/p_{t+1}}{\prod_{n=0}^{t}(1 + i_{n+1})p_n/p_{n+1}} \leq 0
\]
Model: Firms and Labour markets

- The labour market is frictional
- Matching takes the standard form

\[ H_t = H(v_t, u_t) \]

- With CRTS, the probability that a firm will match with a worker is given by

\[ \frac{H_t}{v_t} = \phi_t = \phi(\theta_t), \quad \theta_t = \frac{v_t}{u_t} \]
Model: Firms

- Analogously, an (unemployed) worker will meet a firm with probability

\[
\frac{H_t}{u_t} = \rho_t = \rho(\theta_t), \quad \rho(\theta_t) = \theta_t \phi(\theta_t)
\]

- If a firm and a worker are matched, they produce \( z_t \) units of the output good in period \( t \)
- The nominal wage, \( \tilde{w}_t \), is taken as given
- Dividends in period \( t \) is therefore given by \( \tilde{d}_t = p_t z_t - \tilde{w}_t \)
- With probability \( 1 - \lambda \) the firm and the worker separate
Model: Firms

- The (real) cost of posting a vacancy is given by $k$
- As a consequence, an entrepreneur will post a vacancy iff

$$k \leq \phi(\theta_t)J_t$$

- Employment evolves according to

$$n_t = H_t + (1 - \lambda)n_{t-1}$$
$$= (\lambda n_{t-1} + (1 - n_{t-1}))\rho(\theta_t) + (1 - \lambda)n_{t-1}$$
Equilibrium

Given a fiscal plan, a competitive equilibrium is a process of prices, \( \{p_t, p^A_t, i_{t+1}\} \), and quantities, \( \{c_t, q^t_t, b_{t+1}, x_{t+1}, \theta_t, y_t, n_t, l_t\} \), such that

1. Given prices, \( \{c_t, q^t_t, b_{t+1}, x_{t+1}\} \) solves the households’ problem
2. \( \{\theta_t\} \) satisfies the free-entry condition \( k = \phi(\theta_t) J_t \)
3. Employment, \( \{n_t\} \), satisfies the law of motion

\[
    n_t = (\lambda n_{t-1} + (1 - n_{t-1})) \rho(\theta_t) + (1 - \lambda) n_{t-1}
\]

4. Output, \( \{y_t\} \) is given by \( y_t = z_t \times n_t \)
5. Investment \( \{l_t\} \) is given by \( l_t = p_t v_t k \)
6. Bond markets clear; \( b_t = d_t \)
7. Equity markets clear; \( q^t_t = n_t \)
8. Goods markets clear; \( p_t y_t = p_t c_t + G_t + l_t \)
The equation of exchange

- **Budget constraints**

\[ b_t(1 + i_t) + x_t + p_t J_t(q_{t-1}^t - q_t^t) \]
\[ + w_{t-1} - T_t = p_t c_t + x_{t+1} + b_{t+1} \]

and

\[ -d_t(1 + i_t) + T_t = G_t + d_{t+1} \]

- **Can be combined to yield**

\[ (b_t - d_t)(1 + i_t) + x_t + p_t J_t(q_{t-1}^t - q_t^t) + w_{t-1} \]
\[ = p_t c_t + G_t + x_{t+1} + (b_{t+1} - d_{t+1}) \]
The equation of exchange

\[(b_t - d_t)(1 + i_t) + x_t + p_t J_t(q_{t-1}^t - q_t^t) + w_{t-1}\]

\[= p_t c_t + G_t + x_{t+1} + (b_{t+1} - d_{t+1})\]

- Bond market clearing, \(b_t = d_t\).
The equation of exchange

\[ x_t + p_tJ_t(q_{t-1}^t - q_t^t) + w_{t-1} = p_t c_t + G_t + x_{t+1} \]

- Bond market clearing, \( b_t = d_t \).
The equation of exchange

\[ x_t + p_t J_t (q_{t-1}^t - q_t^t) + w_{t-1} \]
\[ = p_t c_t + G_t + x_{t+1} \]

- Bond market clearing, \( b_t = d_t \).
- National accounting, \( p_t y_t \equiv p_t c_t + G_t + I_t \)
The equation of exchange

\[
x_t + p_t J_t (q_{t-1}^t - q_t^t) + w_{t-1} \\
= p_t y_t - l_t + x_{t+1}
\]

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- Bond market clearing, \( b_t = d_t \).
- National accounting, \( p_t y_t \equiv p_t c_t + G_t + l_t \).
- Equity markets and law of motion for employment infer

\[ p_t J_t (q_{t-1}^t - q_t^t) = -p_t J_t H_t \]
The equation of exchange

\[ x_t + p_t J_t (q_{t-1}^t - q_t^t) + w_{t-1} \]
\[ = p_t y_t - l_t + x_{t+1} \]

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\[ p_t J_t (q_{t-1}^t - q_t^t) = -p_t J_t H_t \]

- Free-entry of firms: \( k = \phi(\theta_t) J_t \)

\[ k = \phi J_t = \frac{H_t}{v_t} J_t \iff p_t^A H_t = kv_t p_t = l_t \]
The equation of exchange

\[ x_t + w_{t-1} = p_t y_t + x_{t+1} \]

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The equation of exchange

\[ x_t + w_{t-1} = p_t y_t + x_{t+1} \]

- Suppose \( x_t + w_{t-1} = m \)
  \[ \Rightarrow m = p_t y_t + x_{t+1} \]

- \( p_t y_t \) is dividends and labour income tomorrow, \( w_t \)
- \( x_{t+1} \) is cash tomorrow
- Thus \( x_{t+1} + w_t = m \)
The equation of exchange

- Initial condition

\[ x_0 + w_{-1} = m \]

- So

\[ m = p_t y_t + x_{t+1} \quad t = 0, 1, \ldots \]

- Or define \( v_t = \frac{m-x_{t+1}}{m} \)

\[ m v_t = p_t y_t \quad t = 0, 1, \ldots \]
Nominal wage rigidity

**Assumption 1**
Nominal wages are rigid; $\tilde{w}_t = \tilde{w}$
Calibration

- The matching function is given by

\[ H_t = v_t (1 - e^{-\frac{\eta}{\theta_t}}) \]

which displays CRTS, and \( \rho_t \) and \( \phi_t \) are in \((0, 1)\) for \( \theta \in [0, \infty) \).

- The parameters to calibrate are therefore: \( \sigma, \beta, k, \lambda, \tilde{w}, \) and \( \eta \).
Calibration

- Monthly frequency: $\beta = 0.95^{\frac{1}{12}}$
- The coefficient of relative risk aversion, $\sigma$, is set to 3.
- The separation rate, $\lambda$, is set to 3.4% per month (JOLTS).
- Average labor market tightness 2000-2010 is 0.45 (JOLTS), set $\eta$ such that $u = 0.95$.
- The steady state level of $z$ is normalised to one, and cash, $m$, to 0.95.
- $\Rightarrow$ steady state prices, $p$, equals one.
Calibration

- Government spending is constant at: \( \bar{g} = 0.2y \) (NIPA)
- The wage rate, \( \tilde{w} \), is set to 0.971 (cf. Hall: 0.965, Shimer: 0.993)
- The steady state level of (real) asset prices can be calculated as
  \[
  J = \frac{z - \tilde{w}}{1 - \beta(1 - \lambda)}
  \]
  and \( k \) is therefore set such that
  \[
  k = q(\theta)J
  \]
Experiment: News shock

- The economy is initialized at its steady-state in period $t = -1$
- In time zero, agents are informed that $z_1 = 0.93$, but $z_t = 1$ for $t = 2, 3, \ldots$
- Evaluate the effect on the economy
- Evaluate the effect of a discretionary one-shot increase in government spending in period zero
“Business Cycle Properties”

- **Output**
- **Consumption**
- **Investments**
- **Asset prices**
“Business Cycle Properties”

Period, \( t \) vs. Unemployment
Effect of government spending

Unemployment

Period, $t$
Effect of government spending

Graphs showing the effects of government spending on output, consumption, investments, and asset prices over different periods.
Effect of government spending

Fiscal multiplier, $\frac{\partial y_t}{\partial g_0}$

Period, $t$
Effect of government spending

Fiscal multiplier, \( \partial y_t/\partial g_0 \)

Period, \( t \)
Effect of government spending

Fiscal multiplier, $\frac{\partial y_t}{\partial g_0}$
Effect of government spending

![Graph showing cash hoarding over periods.](image-url)
Effect of government spending
Effect of government spending

Fiscal multiplier, $\frac{\partial y_t}{\partial g_0}$

- $z_1 = 1$
- $z_1 = 0.97$
- $z_1 = 0.95$
- $z_1 = 0.93$

Period, $t$
Effect of government spending

Cumulative multiplier, $\sum_t \frac{\partial y_t}{\partial g_0}$

- $z_1=0.93$
- $z_1=0.95$
- $z_1=0.97$
- $z_1=1$