Financial Liberalization and Allocative Efficiency[*]

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Abstract

Is there a case for financial liberalization in order to foster growth and efficiency, despite the fact that it causes lending booms that are punctuated by severe crises and costly bailouts? In this paper, we present a two-sector model that integrates the empirical regularities associated with financial liberalization and that allows for decomposing the gains and costs of liberalization in the presence of systemic bailout guarantees. Under financial repression, borrowing constraints in the input sector lead to underinvestment, which causes bottlenecks throughout the economy and low growth. Liberalization allows for new financing instruments that relax the constraints. However, the use of new instruments generates new states of the world in which insolvencies occur, and thus a riskless economy is endogenously transformed into an economy prone to rare crises. A key result is that if only standard debt is allowed, then liberalization preserves financial discipline and brings resource allocation closer to the Pareto-efficient level, increasing average growth, total factor productivity and consumption possibilities. By contrast, when agents can also issue option-like liabilities without having to post collateral, financial discipline breaks down, unproductive projects are funded and allocative efficiency falls.

1   Introduction

Financial liberalization tends to enhance growth, but it also generates greater crisis-volatility, induced by systemic risk taking and lending booms. Here we analyze the gains and costs of financial liberalization in a setup that incorporates the growth crisis trade–off.

This paper makes three contributions. The first contribution is positive. Given the availability of new micro-level data sets, we now know much more about the key empirical regularities associated with financial liberalization, crises, and growth. This paper provides a theoretical framework for integrating these regularities. The second contribution is normative. Our framework allows us to decompose the effects of financial liberalization into gains from higher allocative efficiency and losses from a higher incidence of crises. Third, the paper contributes to the debate on financial regulatory design. It helps us to understand how, in a world with systemic bailout guarantees, the regulatory environment shapes the outcome of financial liberalization.

We show that, even when taking into account the costs of crises and the existence of bailout guarantees, there are net gains from liberalization provided that regulatory limits on the types of issuable liabilities ensure that borrowers risk their own capital. The micro-level risk-taking mechanism by which liberalization spurs growth—and that is motivated by firm-level evidence—generates aggregate booms that tend to be punctuated by rare crises.

In this paper, financial liberalization enhances growth and consumption possibilities because it improves allocative efficiency. This channel is important in economies where financial frictions hinder the growth of sectors that are more dependent on external finance. By allowing for new financing instruments and the undertaking of risk, liberalization relaxes the financing constraints. As a consequence, sectors more dependent on external finance invest more and grow faster. The rest of the economy benefits from this relaxation of the bottleneck via input–output linkages, and hence there is an increase in aggregate growth, production efficiency, and consumption possibilities. However, because financial liberalization induces systemic risk-taking, it generates financial fragility and so can lead to crises that, although rare, are severe.

We analyze the trade-off between risk taking, growth, and production efficiency in a two-sector model with financial frictions. Our model is designed to capture three prominent empirical regularities associated with financial liberalization. First, although crises have been costly, countries that have liberalized financially, and have experienced booms and busts have been, on average, growing faster than non liberalized countries.[1] Second, financial liberalization spurs aggregate growth more through gains in total factor productivity (TFP) rather than in aggregate capital

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[1] Bekaert, et.al. (2005), Bekaert, et.al. (2011), Ranciere et.al. (2008), and Henry (2007).

accumulation.[2] Such aggregate TFP gains are associated with a sectoral reallocation of resources. Following liberalization, sectors that are more dependent on external finance typically grow more —but then crash more severely during a crisis and suffer a greater decline during the subsequent credit crunch.[3] Third, implicit and explicit guarantees to bailout lenders during systemic crises have been widespread the world over.[4]

The argument relies on how, in the presence systemic bailout guarantees, the financial regulatory regime influences financing decisions and on how financing constraints in one sector affect the performance of the whole economy via input–output linkages. In a financially repressed economy, there is misallocation because the input-producing sector depends on external finance to fund its investment and faces borrowing constraints due to contract enforceability problems. These constraints generate a bottleneck that limits the supply of intermediate inputs for the final-goods sector, thereby compromising the overall economy's growth performance and production efficiency.

Both sectors compete every period for the available supply of inputs. Therefore, when contract enforceability problems are severe, the input producing sector has little leverage and commands only a small share of inputs for investment: there is a misallocation of inputs that results in low and socially inefficient aggregate growth. A central planner would increase the input sector investment share to attain the Pareto-optimal allocation. In a decentralized economy, the first best can be attained by reducing the enforceability problems that generate financing constraints. If such a judicial reform is not feasible, then financial liberalization can be seen as an alternative way to improve the allocation in spite of the financial fragility that may result.

Financial liberalization allows for new financing instruments, which relax the constraints and relieve the bottleneck. However—and this is key—the use of new instruments generates new states of the world in which insolvencies occur, and so a riskless economy is endogenously transformed into a risky one. Our framework provides an internally consistent mechanism that helps explain how that transformation can enhance long-run growth, production efficiency and consumption possibilities, even though occasional crises occur during which the input sector suffers the costs associated with widespread bankruptcies.

In order to analyze the link between financial regulation and production efficiency, we consider two regulatory regimes: financial repression and financial liberalization. Under financial repression, firms can issue only standard debt–under which a borrower must repay in all states or else face bankruptcy–and must denominate repayments in the good which it produces–i.e., cannot take on

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[2] Bonfiliogli (2008), Kose, et.al. (2009), and Bekaert, et.al. (2011).

[3] Galindo et al. (2002), Klingebiel, et.al. (2007), Dell'Arricia, et.al. (2008), Gupta et.al. (2009), and Levchenko, et.al. (2009).

[4] Jeanne and Zettelmeyer (2001), Ghandi and Lustig (2009), Ranciere, et. al.(2010), and Kelly et.al. (2011).

insolvency risk. In a liberalized regime firms can only issue standard debt, but can take on risk through a mismatch between the unit of the good they produce and the unit of the good on which they denominate their liabilities.[5]

Under financial repression there is only one equilibrium in which insolvencies and crises never occur. If there are significant contract enforceability problems then, along this 'safe' equilibrium, the input sector exhibits low growth because its investment is constrained by its cash flow. Financial liberalization allows the input sector to denominate debt in units of final goods.[6] The resulting currency mismatch is individually profitable if there is a small likelihood of a sharp decline in the input price that would bankrupt a critical mass of borrowers and trigger a bailout (i.e., a systemic crisis). This micro-level risk taking generates systemic risk when a critical mass of agents engages in it. Under such expectations, a currency mismatch reduces interest costs and relaxes borrowing constraints. However, it also generates financial fragility, because a shift in expectations can cause a sharp fall in the relative price of inputs, bankrupt input-producing firms, and land the economy in a crisis.

In order to address the growth–stability trade-off our model captures two costs typically associated with crises: bankruptcy and financial distress costs. Bankruptcy costs are static and derive from the severe input price decline that leads to firesales and bankrupts input sector firms with a mismatch on their balance sheets. Financial distress costs are dynamic and derive from the collapse in internal funds and the reduced risk taking in the aftermath of crisis, which depresses new credit and investment, and thus hampers growth.

Our first result is that a liberalized (financially fragile) economy will, on average, grow faster than a repressed (safe) economy even if bankruptcy costs are arbitrarily large—provided that the dynamic crisis costs are not too severe. This result follows because (i) crises must be rare in order for them to occur in equilibrium, and (ii) not all bankruptcy losses experienced by the input-producing sector during crises are aggregate deadweight losses. The financial distress costs of crises can be far more significant than bankruptcy costs because they spread dynamically: the decline in internal funds and the reduction in risk taking translate into depressed leverage and investment in the input sector, which in turn reduces aggregate growth.

Our second result is that, when contract enforceability problems are severe—so that there is a bottleneck in the input sector—systemic risk improves allocative efficiency and brings resource allocation nearer to the Pareto-optimal level. Systemic risk also increases the present value of consump-

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[5]In the context of emerging markets, this mechanism corresponds to the well-known currency mismatch by which firms in the nontradables sector issue liabilities denominated in a foreign currency.

[6]For micro-level evidence on currency mismatch see Bleakley and Cowan (2008), Ranciere, et.al. (2010), Berman and Hericourt (2011) and Kamil (2011).

tion that the economy can attain—even when we net out the fiscal costs of bailout guarantees—as long as crises are rare and the associated financial distress costs are not too large.

The efficiency benefits described so far rely on an increase in leverage that occurs without losing financial discipline. In our framework, two elements jointly ensure financial discipline. First, bailouts are systemic: they are granted only in the event of a systemic crisis, not if an idiosyncratic default occurs. Second, external finance is limited to standard debt contracts under which agents must repay in all states or else face bankruptcy. Because of contract enforceability problems, lenders impose borrowing constraints by requiring borrowers to risk their own equity. In this way the incentives of borrowers and creditors are aligned in selecting only projects with a high enough expected return, even though systemic bailout guarantees are present.

To make clear the disciplining role of standard debt, we consider an example where firms can issue (without collateral) catastrophe bonds paying zero in good states but promising a huge amount if a (rare) crisis occurs. We show that in the presence of systemic bailout guarantees, the introduction of these new financing instruments can overturn the gains from liberalization and reduce production efficiency by allowing large-scale funding of unprofitable projects.[7] This example emphasizes the importance of regulating the type of issuable securities in order to reap the benefits of financial liberalization. It also helps us analyze the recent US crisis, which featured the large scale issuance of option-like liabilities and a collapse of financial discipline.[8]

The rest of the paper is structured as follows. Section 2 relates our paper to the literature, after which Section 3 presents the model. Sections 4 and 5 analyze (respectively) long-run growth and production efficiency under financial repression and under financial liberalization. Section 6 discusses the effect of introducing new types of financing instruments on the efficiency gains of financial liberalization. Section 7 concludes.

2   Outline and Related Literature

In our model, the financially constrained input sector is the engine of growth. In a nutshell, the key determinant of aggregate growth and production efficiency is the share of intermediate goods

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[7]The reason is that such bonds allow for the funding of unproductive projects with a negative contribution to national income. These inferior projects are privately profitable because they exploit the subsidy implicit in the guarantee. A firm undertaking a non-profitable project could issue bonds that promise to repay only in a crisis state. Lenders would be willing to buy such bonds without requiring collateral because they would expect the promised repayment to be covered by the bailout. Thus the firm can fund inferior projects without risking its own equity, betting that the project turns out a large profit in good states.

[8]Landier et al.(2012), Levitin and Wachter (2010), Ranciere and Tornell (2011).

production that is used for investment in the intermediate goods sector:

$$\text{Investment share:} \quad \phi_t = \phi(\text{internal funds, financial regime, enforceability problems}). \quad (1)$$

The investment share ϕ_t determines the production of intermediate inputs and final goods through input–output linkages that, in equilibrium, take the following simple form:

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\begin{aligned}
\text{Intermediate good:} \quad & q_{t+1} = q(I_t), \qquad & I_t = q_t \cdot \phi_t; \\
\text{Final good:} \quad & y_t = y(d_t), \qquad & d_t = q_t \cdot [1 - \phi_t].
\end{aligned}
\qquad (2)
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In equilibrium, ϕ_t is determined by the interaction of bailout guarantees with contract enforceability problems. This interaction depends crucially on the regulatory regime. Under financial repression the ϕ_t-sequence is smooth, but it can be inefficiently low and result in slow aggregate growth. Under financial liberalization the ϕ_t-sequence has a higher mean, but it exhibits sharp and sudden contractions in response to crises. The underlying mechanism is this: when agents coordinate on systemic risk taking and, by doing so, exploit systemic bailout guarantees, they attain higher leverage, which increases investment and growth but also makes the economy vulnerable to crises.

In emphasizing the link between borrowing constraints and sectoral misallocation as well as input output linkages, this paper is related to Jones (2011a, 2011b), who emphasizes the consequence of resource misallocation in terms of intermediate inputs and its effects on aggregate productivity through input output linkages.[9] Analogously to the Jones steady-state input output multiplier, in our setup higher production efficiency results from a dynamic input output multiplier: an increase in today's investment in the intermediate input sector (ϕ_t) increases tomorrow's production in the final goods sector.

We show that, despite bailout and bankruptcy costs, shifting from a repressed regime to a liberalized regime—with regulatory limits—can increase aggregate growth, production efficiency, and the present value of consumption. These results are linked to a vast empirical literature on the growth effects of financial liberalization. Henry (2007) and Bekaert, et.al. (2005) find that it is generally growth enhancing, but earlier literature obtains more mixed results (Edison et.al., 2004). One reason for this is that financial liberalization has typically led both to higher growth and to more frequent crises. This dual effect is at the core of our theoretical mechanism. Ranciere, et.al. (2006) and Bonfiglioli (2008) find robust evidence for this dual effect of financial liberalization. The average growth gains in tranquil times dominate the output costs associated with a higher

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[9]A connected literature (see, e.g., Restuccia and Rogerson, 2007; Hsieh and Klenow, 2009) focuses on the aggregate TFP consequences of distortions that cause resource misallocation *between firms within sectors*.

propensity to crisis.[10]

Bonfiglioli (2008), Kose, et.al. (2009), and Bekaert, et.al. (2011) find that the growth gains from financial liberalization come from an increase in aggregate TFP rather than from an increase in aggregate capital accumulation. Our model predicts that financial liberalization promotes a more efficient allocation of intermediate inputs across sectors and therefore increases aggregate TFP. Galindo, et.al. (2007) construct indexes of efficiency in the allocation of investment based on sales or profits per unit of capital for listed firms in 12 developing countries and find that financial liberalization improves allocative efficiency. Abiad, et.al. (2008) provide similar evidence for such an allocative efficiency effect by comparing the dispersion of Tobin's Q among listed firms in five emerging markets before and after financial liberalization. While our model focuses on allocative efficiency across sectors, our summary measure of misallocation - the difference between the efficient investment share and the one implied by financing constraints - could also be used to discuss the effect of financial liberalization on the dispersion of investment rates and Tobin's q among firms within a sector.

Levchenko, et.al. (2008) use sector-level data to find that sectors more dependent on external finance grow more and become more volatile after financial liberalization.[11] In our model, the input sector depends on external finance to fund investment but the final goods sector does not. In a liberalized regime, the input sector grows faster than the final goods sector as long as a crisis does not occur. Hence, inputs become cheaper and more abundant, which in turn fosters more growth in the final goods sector. However, liberalization also generates crisis risk. During crises, the input sector suffers from severe financial distress costs and experiences a credit crunch that sharply reduces investment and output. Kroszner, et.al. (2007) and Dell'Arricia, et.al. (2008) find evidence that sectors more dependent on external finance suffer disproportionately more during financial crises. In our framework, crises cause financially constrained firms with a currency mismatch between income and liabilities to default. Using a representative panel of Korean firms around the 1998 financial crisis, Kim, Tesar and Zhang (2012) show that non-exporting small firms with more foreign currency debt are more likely to go bankrupt during the crisis. Based on a completely different setup, Buera, et.al. (2009) show how a relaxation of financial constraints can result in more efficient allocation

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[10]These results are related to those of Kaminsky and Schmukler (2008), who find that financial liberalization increases stock market volatility in the short run but reduces it in the long run. See also Loayza and Ranciere (2006), who show that financial development can reduce growth in the short run—through higher volatility and the incidence of crises—but increase it in the long run.

[11]Looking at the finance growth nexus at the sector level, Ilyina and Samaniego (2009, 2011) show that what really matters is the interaction between the ability to raise external finance and the need for such financing to fund growth-enhancing investment.

of capital and entrepreneurial talent across sectors.

Other theoretical papers emphasize the welfare gains from financial liberalization that are due to intertemporal consumption smoothing (Gourinchas and Jeanne, 2006), better international risk sharing (Obstfeld, 1994), and better domestic risk sharing (Townsend and Ueda, 2006). Gourinchas and Jeanne (2006) show that the welfare benefits associated with this mechanism are negligible in comparison to the increase in domestic productivity. The gains from risk sharing can be much larger: Obstfeld (1994) demonstrates that international risk sharing, by enabling a shift from safe to risky projects, strongly increases domestic productivity, production efficiency, and welfare. In our framework, the gains also stem from an increase in production efficiency but not from risk sharing. The gains derive from a reduction of the contract enforceability problem, not of the incomplete markets problem: efficiency gains are obtained by letting entrepreneurs take on more risk, not by having consumers face less risk. In Tirole and Pathak (2006), currency mismatch also results in social welfare gains, but through a discipline effect on government policy, not through a better allocation of resources.

Systemic bailout guarantees play a crucial role in our framework, like in Burnside et.al. (2004) and Schneider and Tornell (2004). By affecting collective risk taking and the set of fundable projects, they shape the growth and production efficiency effects of a regulatory regime. There is ample evidence of ex-post systemic bailouts, but evidence on bailout expectations is more difficult to obtain. By comparing the pricing of out-of-the money put options on a financial sector index with options pricing on the individual banks forming the index, Kelly, et.al. (2011) show that systemic bailouts–but not idiosyncratic bailouts–are expected.[12] Using firm-level data on loan pricing for a large sample of firms in Eastern Europe, Ranciere, et.al. (2010), find that some form of bailout expectation is necessary to rationalize differences in the pricing of foreign and domestic currency debt across firms.[13] Farhi and Tirole (2011) demonstrate how time-consistent bailout policies designed by optimizing governments can generate a collective moral hazard problem that explains the wide-scale maturity mismatch and high leverage observed in the US financial sector before the 2007-2008 crisis.

In our setup, standard debt is preferable to other types of state-contingent liabilities when systemic bailout guarantees are present. Gorton and Pennacchi (1990) and Dang, et.al. (2011)

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[12] In related research Ghandi and Lustig (2009) look at differences between the stock returns large and small US banks, provide evidence of an implicit guarantees on large banks but not on small ones.

[13] Bailout expectations are necessary to explain : (i) why firms in the non tradables sector with a currency mismatch on their book borrow at a cheaper rate than similar firms with no currency mismatch, and (ii) why the interest rate spread between debt denominated in foreign versus domestic currency is not significantly different for firms in the non radables sector and those in the tradables sector.

show that standard debt can mitigate the consequences of informational asymmetries. In a setup with moral hazard, Tirole (2003) shows that debt has good effects on government's incentives.

While belonging to the same line of research, this paper contrasts sharply with our previous papers (Tornell and Schneider, 2004; Ranciere, Tornell and Westermann, 2008) in important ways. Tornell and Schneider (2004) shares with this paper the credit market game but focuses on replicating a typical emerging market boom-bust cycle episode in a finite horizon economy. That paper is not designed to analyze long-run growth, to determine when is it that bailout guarantees are intertemporally financeable via taxation, nor to study the conditions under which risk-taking increases efficiency and consumption possibilities. In contrast, this paper presents an infinite horizon endogenous growth model allowing us to analyze these long-run issues. Ranciere, Tornell, Westermann, (2008) is mostly an empirical paper establishing that crises-prone economies grow faster than crisis-proof economies. The highly stylized growth model included in that paper features a one-sector economy subject to exogenous crisis-risk. In contrast this paper stresses the importance of having a two-sector framework to understand endogenous systemic risk and to discuss allocative efficiency.

Finally, the cycles generated by our model are much different from Schumpeter's (1934) cycles in which the adoption of new technologies plays a key role. Our cycles are more similar to Juglar's credit cycles (Juglar, 1862).[14]

3 The Model

We consider a model rich enough to reproduce the empirical facts just described, yet tractable enough that (i) the equilibria can be solved in closed form and (ii) we can characterize analytically the relationships among regulation, systemic risk taking, production efficiency, and growth. We embed—within a two-sector endogenous growth model—the credit market game of Schneider and Tornell (2004), in which systemic-risk results from the interaction of contract enforceability and bailout guarantees. A simpler, one-sector framework would not be able to capture the empirical link between the regulatory regime and sectoral misallocation, and neither could it explain systemic risk taking and the aggregate boom-bust cycles as an endogenous response to liberalization policies.

There are two goods: a final consumption good (T); and an intermediate good (N), which is used as an input in the production of both goods. We let the T-good be the numeraire and denote

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[14]Juglar characterizes asymmetric credit cycles as well as the periodic occurrence of crises in France, England, and the United States between 1794 and 1859. He concludes: "The regular development of wealth does not occur without pain and resistance. In crises everything stops for a while but it is only a temporary halt, prelude to the most beautiful destinies." (Juglar,1862, p. 13, our translation).

the relative price of N-goods by $p_t = p_t^N / p_t^T$.[15]

Agents. There are competitive, risk-neutral, international investors for whom the cost of funds is the world interest rate r. These investors lend any amount as long as they are promised an expected payoff of $1 + r$. They also issue a default-free T-bond that pays $1 + r$ in the next period.

There are overlapping generations of consumers who live for two periods and have linear preferences over consumption of T-goods: $c_t + \frac{1}{1+r} c_{t+1}$. Consumers are divided into two groups of measure 1: workers and entrepreneurs.

Workers are endowed with one unit of standard labor. In the first period of his life, a worker supplies inelastically his unit of labor ($l_t^T = 1$) and receives a wage income v_t^T. At the end of the first period, he retires and invests his wage income in the risk-free bonds.

Entrepreneurs are endowed with one unit of entrepreneurial labor. A "young" entrepreneur (i.e., one in the first period of her life) supplies inelastically one unit of entrepreneurial labor ($l_t = 1$) and receives a wage v_t. At the end of the first period, she starts running an N-firm and makes investment decisions. In the second period of her life, she receives the firm's profits, if any.

Production Technologies. There is a continuum of measure 1 of firms run by entrepreneurs who produce N-goods using entrepreneurial labor (l_t), and capital (k_t). Capital consists of N-goods invested during the previous period (I_{t-1}), and it fully depreciates after one period. The production function is

$$q_t = \Theta_t k_t^\beta l_t^{1-\beta}, \qquad \Theta_t =: \theta \overline{k_t}^{1-\beta}, \qquad k_t = I_{t-1}, \qquad \beta \in (0,1). \tag{3}$$

The technological parameter Θ_t, which each firm takes as given, embodies an external effect, for the average N-sector capital $\overline{k_t}$.

There is a continuum, of measure one, of competitive firms that produce the T-good combining standard labor (l_t^T) and the N-good (d_t) using a Cobb–Douglas technology: $y_t = a d_t^\alpha (l_t^T)^{1-\alpha}$. The representative T-firm maximizes profits taking as given the price of N-goods (p_t) and standard labor wage (v_t^T)

$$\max_{d_t, l_t} \left[y_t - p_t d_t - v_t^T l_t^T \right], \quad y_t = a d_t^\alpha (l_t^T)^{1-\alpha}, \quad \alpha \in (0,1). \tag{4}$$

Firm Financing. The investable funds of a firm consist of its internal funds w_t plus the liabilities B_t that it issues. These investable funds can be used to buy default-free bonds s_t or to buy N-goods $p_t I_t$ for the next period's production. It follows that the time-t budget constraint and time-$(t+1)$ profits of an N-firm are, respectively

$$p_t I_t + s_t = w_t + B_t \text{ and} \tag{5}$$

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[15] In an international setup, p_t is the inverse of the real exchange rate.

$$\pi(p_{t+1}) = p_{t+1}q_{t+1} + (1+r)s_t - v_{t+1}l_{t+1} - L_{t+1}, \tag{6}$$

where the cash flow of the firm equals the entrepreneur's wage ($w_t = v_t$), and L_{t+1} is the next period's promised debt repayment (described in the next paragraph). Because T-firms produce instantaneously by combining labor and intermediate inputs, they do not require financing.

A firm can issue two types of one-period standard bonds: N-bonds and T-bonds. N-bonds promise to repay in N-goods, while T-bonds promise to repay in T-goods. Since the respective interest rates are ρ_t and ρ_t^n, it follows that if the firm issues b_t T-bonds and b_t^n N-bonds, then the promised debt repayment is

$$L_{t+1} = (1+\rho_t)b_t \;+\; p_{t+1}(1+\rho_t^n)b_t^n. \tag{7}$$

If at $t+1$ the firm does not repay, then it must default. In Section 6 we expand the menu of issuable liabilities.

Credit Market Imperfections.   Firm financing is subject to three credit market imperfections. First, firms cannot commit to repay their liabilities. This imperfection might give rise to borrowing constraints in equilibrium.

Contract Enforceability Problems. If at time t the entrepreneur incurs a non-pecuniary cost $h[w_{i,t} + B_{i,t}]$, then at $t+1$ she will be able to divert all the returns *provided* the firm is solvent (i.e., provided $\pi_i(p_{t+1}) \geq 0$).

Second, there are systemic bailout guarantees that cover lenders against systemic crises but not against idiosyncratic default. This imperfection might induce N-firms to undertake insolvency risk by denominating their debt in T-goods rather than in N-goods.

Systemic Bailout Guarantees. If a majority of firms become insolvent, then a bailout agency pays lenders the outstanding liabilities of each defaulting firm.

Finally, there are bankruptcy costs. When a firm defaults, a share $1 - \mu - \mu_w$ of the insolvent firms' revenues is lost in bankruptcy procedures. In this case, the bailout agency can recoup only $\mu p_t q_t$ and the workers receive a wage of only $\mu_w p_t q_t$. The parameters μ and μ_w satisfy

$$\mu \in [0, \beta] \quad \text{and} \quad \mu_w \in (0, 1 - \beta). \tag{8}$$

Fiscal Solvency.   We impose the condition that bailout guarantees are domestically financed via taxation. We assume that the bailout agency is run by a government that has access to perfect

capital markets and can levy lump-sum taxes T_t. It follows that the intertemporal government budget constraint is

$$E_t \sum_{j=0}^{\infty} \delta^j \{[1 - \xi_{t+j}][L_{t+j} - \mu p_{t+j} q_{t+j}] - T_{t+j}\} = 0, \qquad \delta \equiv \frac{1}{1+r}, \tag{9}$$

where ξ_{t+j} is equal to 1 if no bailout is granted and to 0 otherwise.

Regulatory Regimes. The regulatory regime determines the set of liabilities that firms can issue. There are two regulatory regimes. First, a *financially repressed regime* is one under which a firm can issue only one-period standard bonds and must denominate debt in the good that it produces (i.e., firms cannot take on insolvency risk). Second, a *financially liberalized regime* under which a firm can issue only one-period standard bonds but is free to take on insolvency risk.

Equilibrium Concept. In this economy there is endogenous price risk: in equilibrium, p_{t+1} may equal either $\bar{p}_{t+1}$ with probability u_{t+1} or $\underline{p}_{t+1}$ with probability $1 - u_{t+1}$. The probability u_{t+1} may equal either 1 or u, and this is known at time t.

Since the only source of uncertainty is relative price risk, N-bonds constitute hedged debt. Meanwhile, T-bonds generate insolvency risk because there is a mismatch between the denomination of liabilities and the price that will determine future revenues. Thus, an N-firm's solvency will depend on tomorrow's price of N-goods.

A key feature of the mechanism is the existence of correlated risks across agents: since guarantees are systemic, the decisions of agents are interdependent. They are determined in the following credit market game, which is similar to that considered by Schneider and Tornell (2004). During each period t, each young entrepreneur takes prices as given and proposes a plan $P_t = (I_t, s_t, b_t, b_t^n, \rho_t, \rho_t^n)$ that satisfies budget constraint (5). Lenders then decide which of these plans to fund. Finally, funded young entrepreneurs make investment and diversion decisions.

Payoffs are determined at $t+1$. Consider first the plans that do not lead to funds being diverted. If the firm is solvent ($\pi(p_{t+1}) \geq 0$), then the old entrepreneur pays the equilibrium wage $v_{t+1} = [1 - \beta]p_{t+1}q_{t+1}$ to the young entrepreneur and pays L_{t+1} to lenders; she then collects the profit $\pi(p_{t+1})$. In contrast, if the firm is insolvent ($\pi(p_{t+1}) < 0$), then young entrepreneurs receive $\mu_w p_{t+1} q_{t+1}$ ($\mu_w < 1 - \beta$), lenders receive the bailout if any is granted, and old entrepreneurs get nothing. Now consider plans that do entail diversion. If the firm is solvent, then the young entrepreneur gets her wage $\beta p_{t+1} q_{t+1}$, the old entrepreneur gets the remainder $[1 - \beta]p_{t+1}q_{t+1}$, and lenders receive the bailout if any is granted. Under insolvency, entrepreneurs get nothing and lenders receive the bailout if any is granted. Therefore, the young entrepreneur's problem is to choose an investment

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plan P_t and diversion strategy η_t that solves

$$\max_{P_t, \eta_t} \quad E_t \left[\zeta_{t+1} \{ p_{t+1} q_{t+1} + (1+r)s_t - v_{t+1} l_{t+1} - (1-\eta_t) L_{t+1} \} - \eta_t h \cdot [w_t + B_t] \right]$$

subject to (5), where η_t is equal to 1 if the entrepreneur has set up a diversion scheme, and is equal to 0 otherwise and where ζ_{t+1} is equal to 1 if $\pi(p_{t+1}) \geq 0$, and is equal to 0 otherwise.

Definition. *A symmetric equilibrium is a collection of stochastic processes* $\{I_t, s_t, b_t, b_t^n, \rho_t, \rho_t^n, d_t, y_t, q_t, u_t, p_t, w_t, v_t^T, v_t\}$ *such that: (i) given current prices and the distribution of future prices, the plan $(I_t, s_t, b_t, b_t^n, \rho_t, \rho_t^n)$ is determined in a symmetric subgame-perfect equilibrium of the credit market game and d_t maximizes T-firms' profits; (ii) factor markets clear; and (iii) the market for intermediate goods clears*

$$d_t(p_t) + I_t(p_t, \underline{p}_{t+1}, \overline{p}_{t+1}, u_{t+1}) = q_t(I_{t-1}). \tag{10}$$

To close the model we assume that young date-0 entrepreneurs are endowed with $w_0 = (1 - \beta)p_0 q_0$ units of T-goods and that old date-0 entrepreneurs are endowed with q_0 units of N-goods and have no debt in the books.

3.1 Discussion of the Setup

Our framework is similar to a Rebelo-type two-sector, AK model. The source of endogenous growth is a production externality in the intermediate goods sector, which is also the investment sector. This N-sector uses its own goods as capital; as a result, the share ϕ of N-output commanded by the N-sector for investment is the key determinant of aggregate growth. Because the N-sector is subject to borrowing constraints, ϕ might be too small in equilibrium and so the economy as a whole might experience a *bottleneck* to growth. Our result about the gains from financial liberalization will derive from the fact that the undertaking of credit risk—by increasing the mean value of ϕ—may increase production efficiency and aggregate growth via linkages to the T-sector.[16] This modeling choice is consistent with the evidence provided by Harrison (2003) of robust positive externalities in the investment sector but not in the consumption goods sector. As shown by Febelmayr and Licandro (2005), the two-sector AK model is consistent with the time-series evidence of a fall in the price of the equipment sector relative to the final goods sector (Whelan, 2003). The fall in the price of investment is the consequence of the production externality in the investment goods sector, and it enables sustained growth in the aggregate economy.

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[16] In contrast, the assumptions that N-goods are not consumed and T-goods are not intermediate inputs are convenient but not essential. If N-goods were consumed, there would be a steeper fall in the demand of N-goods when N-firms become insolvent, accentuating the self-fulfilling depreciation that generates crises.

The agency problem and the representative entrepreneur who lives two periods is considered by Schneider and Tornell (2004). The advantage of this setup is that one can analyze financial decisions on a period-by-period basis. This will allow us to explicitly characterize the stochastic processes of prices and investment. These closed-form solutions are essential for deriving the limit distribution of growth rates and establishing our efficiency results.

Empirical evidence shows that the higher growth associated with financial liberalization is associated with greater crisis volatility. To capture this growth-volatility link, we consider a setup with no exogenous source of shocks. In equilibrium, endogenous insolvency risk arises from a self-reinforcing mechanism: N-firms find it profitable to issue T-debt in the presence of systemic guarantees and sufficient expected price variability. This variability, in turn, arises when N-firms issue enough T-debt: since N-goods are inputs in N-production, enough T-debt in the balance sheet of N-firms gives rise to the possibility of a crisis state characterized by the collapse of the N-good price and generalized bankruptcy.

To capture the dynamic and static effects of crises, we have allowed for two types of crisis costs: financial distress costs, which are indexed by $l^d \equiv 1 - \mu_w/(1 - \beta)$; and bankruptcy costs indexed by $l \equiv 1 - \mu/\beta$. All the equilibria we characterize exist for any $l^d \in (0, 1)$ and any $l \in (0, 1)$.

Financing constraints affect sectors asymmetrically. Contract enforceability problems give rise to financing constraints, which affect mainly the N-sector because it needs external financing to invest. In contrast, T-firms that use N-inputs do not require financing because they instantaneously transform inputs into final output. This simplification provides the same insight as would a more complex structure in which the N-sector is more financially constrained than the T-sector.

The assumption that bailouts are granted only during a systemic crisis is essential. If, instead, guarantees were granted whenever a single borrower defaulted, then the guarantees would neutralize the contract enforceability problems and borrowing constraints would not arise in equilibrium.

The two regulatory regimes we consider—repression and liberalization—are meant to capture, in a simple way, two regulatory environments. The first is one in which there is overregulation, credit policies are restrictive, and leverage is therefore low. The second is the case where agents are free to take on risk yet there is financial discipline that ensures lenders impose strict repayment criteria on their loans. In order to illustrate the role of standard debt in inducing financial discipline, in Section 6 we consider the example of an alternative regulatory environment where–in addition to standard bonds–agents can issue option-like liabilities.

Finally, throughout the paper we will impose the following restrictions on the degree of contract enforceability h, the crisis probability $1 - u$, and the entrepreneurs' profit share β :

$$h < 1 + r \equiv \delta^{-1}, \ u > h\delta, \ \beta > h\delta/u. \tag{11}$$

The restrictions $h < \delta^{-1}$ and $u > h\delta$ are necessary for borrowing constraints to arise in a safe equilibrium and a risky equilibrium, respectively. If h, the index of contract enforceability, were greater than the cost of capital, then it would always be cheaper to repay debt rather than to divert. The restriction $\beta > h\delta/u$ is necessary and sufficient for prices to be finite (it implies that the share of N-output commanded by the N-sector ϕ_t is always less than unity).

3.2   Symmetric Equilibria (SE)

We will construct two types of symmetric equilibria: safe and risky. The former exists in both the repressed and the liberalized regimes, whereas the latter exists only in the liberalized regime. The reader may skip this subsection and go directly to Sections 4 and 5, which contain the main results of the paper on growth, efficiency and consumption possibilities. The key results of this subsection are summarized in Propositions 3.1 and 3.2, that characterize the equilibrium paths and the parametric conditions that ensure the existence of internally consistent mechanisms whereby investment decisions generate the required equilibrium price paths–i.e., expected returns– that validate such decisions in safe and risky equilibria, respectively.

Consider first the final goods sector. The representative firm maximizes profits, taking the prices of goods and factors as given. Therefore, this firm sets $p_t d_t = \alpha y_t$ and $v_t^T l_t^T = (1 - \alpha)y_t$. Since consumers supply inelastically one unit of labor, it follows that the equilibrium T-output, consumers' income and the T-sector demand for N-goods are given by

$$y_t = d_t^\alpha, \qquad v_t^T = [1 - \alpha]y_t, \qquad d(p_t) = \left[\frac{\alpha}{p_t}\right]^{\frac{1}{1-\alpha}}. \tag{12}$$

Because $t + 1$ consumption is discounted using the riskless interest rate, consumers born in period t are indifferent between consumption in t and in $t + 1$. Thus,

$$c_{t+1} = [1 - \alpha]y_t. \tag{13}$$

Now consider the input sector. Given prices $(p_t, \overline{p}_{t+1}, \underline{p}_{t+1})$ and the likelihood of crisis $(1 - u_{t+1})$, each entrepreneur chooses how much to borrow, how to denominate debt, and whether or not to set up a diversion scheme. Prices and the likelihood of crisis are, in turn, endogenously determined by the entrepreneurs' choices. In a symmetric equilibrium, entrepreneurs' choices and the resulting prices validate each other. Propositions 3.1 and 3.2 characterize two such self-validating processes. The former characterizes a symmetric safe equilibrium in which all debt is hedged and crises never occur; the latter characterizes a risky equilibrium where all debt is unhedged, and where firms are solvent (resp., insolvent) in the high (resp., low) price state.

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We start by characterizing the transition equations, which are common to both symmetric equilibria. We then endogeneize $(p_t, \bar{p}_{t+1}, \underline{p}_{t+1})$ and u_{t+1}. Thus, suppose for a moment that expected returns are high enough that an entrepreneur finds it optimal to borrow up to the limit and invest all her funds in intermediate goods production. As Propositions 3.1 and 3.2 show, if crises are not frequent, then only those plans that do not involve diversion are funded. Hence, the borrowing limit is set by lenders so as to make diversion not profitable: $E(L_{t+1}) \leq h(w_t + B_t)$, where $B_t = b_t^n$ (N-debt) in a safe equilibrium and $B_t = b_t$ (T-debt) in a risky one. Combining the binding no-diversion condition with the budget constraint $(p_t I_t = w_t + B_t)$ generates the following borrowing constraint and investment equation:

$$B_t = [m_t - 1]w_t, \qquad I_t = m_t \frac{w_t}{p_t}. \tag{14}$$

The key to our results will be the value of the investment multiplier, which in turn depends on whether the equilibrium is risky or safe.

In any symmetric equilibrium, the representative N-firm's capital k_t is equal to average N-sector capital $\bar{k}_t$. Thus, (3) implies that equilibrium N-output is linear in investment:

$$q_t = \theta I_{t-1}. \tag{15}$$

If a firm is solvent, then the young entrepreneur's wage equals the marginal product of her labor; under insolvency, however, she receives only a share μ_w of revenues. Thus, in any SE the young entrepreneur's internal funds are

$$w_t = \begin{cases} [1 - \beta]p_t q_t & \text{if } \pi(p_t) \geq 0, \\ \mu_w p_t q_t & \text{if } \pi(p_t) < 0, \quad \text{where } \mu_w \in (0, 1 - \beta). \end{cases} \tag{16}$$

Under the assumption that expected returns are high enough that it is optimal to invest all funds in the production of N-goods, we can substitute (16) into (14) and so derive the following expression for N-sector investment

$$I_t = \phi_t q_t, \qquad \phi_t = \begin{cases} [1 - \beta]m_t & \text{if } \pi(p_t) \geq 0, \\ \mu_w m_t & \text{if } \pi(p_t) < 0. \end{cases} \tag{17}$$

Once we combine (12), (15), and (17) with market-clearing condition (10), it follows that in a symmetric equilibrium N-output, prices, and T-output evolve according to

$$q_t = \theta \phi_{t-1} q_{t-1}, \tag{18}$$

$$p_t = \alpha [q_t(1 - \phi_t)]^{\alpha - 1}, \tag{19}$$

$$y_t = [q_t(1 - \phi_t)]^\alpha = \frac{1 - \phi_t}{\alpha} p_t q_t. \tag{20}$$

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Equations (17)-(20) form an symmetric equilibrium provided that the implied returns validate the agents' expectations. The propositions presented next characterize two such equilibria. We assume throughout the rest of the paper that an entrepreneur denominates all debt in either N-goods or T-goods, but not in both.[17]

Proposition 3.1 (Safe Symmetric Equilibria (SSE)) *There exists an SSE if and only if (11) holds and the input sector productivity θ is greater than a threshold $\underline{\theta}^s$ given by (44). In an SSE, the following statements hold.*

1. *There is no currency mismatch ($b_t = 0$), and crises never occur ($u_{t+1} = 1$).*

2. *The interest rate satisfies $[1 + \rho_t^n]p_{t+1} = [1 + r]$.*

3. *The input sector investment share and leverage are, respectively*

$$\phi^s = \frac{1 - \beta}{1 - h\delta}, \quad and \quad \frac{w_t + b_t^n}{w_t} = \frac{1}{1 - h\delta} \equiv m^s. \tag{21}$$

4. *Input and final goods production are $q_t = \theta\phi^s q_{t-1}$ and $y_t = q_t^\alpha[1 - \phi^s]^\alpha$, respectively.*

5. *Prices evolve according to $\frac{p_{t+1}}{p_t} = (\theta\phi^s)^{\alpha-1}$.*

To grasp the intuition, observe that—given that all other entrepreneurs choose the safe equilibrium strategy—an entrepreneur and her lenders expect that no bailout will be granted next period. Thus, lenders will not fund any plan that leads to diversion. Furthermore, since lenders must break even, the entrepreneur must offer an interest rate of $1 + \rho_t^n = [1 + r]/p_{t+1}$. Since the expected debt repayment is $b_t^n[1+r]$, the no-diversion condition is $b_t^n[1+r] \leq h[w_t + b_t^n]$, which yields the borrowing constraint $b_t^n = [m^s - 1]w_t$ and investment $I_t = m_t^s w_t/p_t$. Notice that there are no incentives to denominate debt in T-goods because the expected interest payments are the same as those under N-debt.

An entrepreneur finds it profitable to borrow up to the limit and invest in the production of the intermediate input provided that her net return on equity is greater than the storage return. If the borrowing constraint is binding, so that $(1 + r)b_t = h(w_t + b_t)$, then the marginal net return per unit of investment is $\theta\beta p_{t+1}/p_t - h$. The entrepreneur's leverage $\frac{w_t+b_t^n}{w_t}$ equals m^s and so the return on equity is $[\theta\beta p_{t+1}/p_t - h]m^s w_t$, which is greater than $[1+r]w_t$ when $\theta\beta p_{t+1}/p_t > 1 + r$.[18]

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[17]It is possible to have a small share of T-debt in a safe equilibrium and a small share of N-debt in a risky equilibrium. Such a debt mix would not alter the main properties of the equilibria.

[18]Since the wage in an SSE is $\hat{v}_{t+1} = [1 - \beta]p_{t+1}\theta k_{t+1}$ and since $l_{t+1} = 1$, it follows that the net return is $\pi_{t+1}^s = \theta p_{t+1}k_{t+1} - \hat{v}_{t+1} - b_t^n[1 + r] = \beta\theta\frac{p_{t+1}}{p_t}[w_t + b_t^n] - b_t^n[1 + r]$. Replacing the borrowing limit, now yields $[\beta\theta\frac{p_{t+1}}{p_t} - h]m^s \geq 1 + r$, which is equivalent to $\beta\theta\frac{p_{t+1}}{p_t} \geq 1 + r$.

Because prices evolve according to $\frac{p_{t+1}}{p_t} = (\theta \phi^s)^{\alpha-1}$, the net return on equity is greater than the storage return if and only if $\theta > \underline{\theta}^s$. This parametric condition ensures the existence of an internally consistent mechanism whereby investment decisions generate the required expected returns.

Next we characterize risky symmetric equilibria (RSE), in which entrepreneurs choose unhedged T-debt. An entrepreneur finds it optimal to take on the implied insolvency risk only if: (i) $\bar{p}_{t+1}$ is high enough that expected returns are greater than the storage return $1 + r$; and (ii) $\underline{p}_{t+1}$ is low enough that all firms with T-debt become insolvent during the next period and a bailout is triggered. Our second proposition establishes the parameter conditions under which this self-validating mechanism arises: currency mismatch generates a large expected relative price variability, which in turn makes it optimal for entrepreneurs to denominate debt in T-goods.

Proposition 3.2 (Risky Symmetric Equilibrium (RSE)) *There exists an RSE for any crisis' financial distress costs $l^d \in (0,1)$ and any bankruptcy costs $l \in (0,1)$ if and only if (11) holds, the input sector productivity satisfies $\theta \in (\underline{\theta}, \overline{\theta})$, the profit share of entrepreneurs satisfies $\beta < \overline{\beta}$, and crises are not frequent ($u > \underline{u}$). These bounds are defined by (45)-(47).*

1. *An RSE consists of lucky paths that are punctuated by crises. During a lucky period, input-producing firms take on systemic risk by denominating debt in final goods (i.e., currency mismatch). Systemic risk taking increases leverage.*

$$1 + \rho_t^r = 1 + r, \qquad \frac{w_t + b_t}{w_t} = \frac{1}{1 - h\delta/u} \equiv m^r > m^s. \tag{22}$$

2. *Currency mismatch generates systemic risk: there can be a sharp fall in the input price that bankrupts all input sector firms and generates a systemic crisis, during which creditors are bailed out.*

3. *Crises cannot occur in consecutive periods. In the RSE under which there is a reversion back to systemic risk taking in the period immediately after the crisis, the probability of a crisis and the input sector's investment share satisfy:*

$$1 - u_{t+1} = \begin{cases} 1 - u & \text{if } t \neq \tau_i; \\ 0 & \text{if } t = \tau_i; \end{cases} \qquad \phi_t = \begin{cases} \phi^l \equiv \frac{1-\beta}{1-h\delta u^{-1}} & \text{if } t \neq \tau_i; \\ \phi^c \equiv \frac{\mu_w}{1-h\delta} & \text{if } t = \tau_i. \end{cases} \tag{23}$$

Here τ_i denotes a crisis time.

4. *Input and final goods production are $q_t = \theta\phi_{t-1}q_{t-1}$ and $y_t = q_t^\alpha [1 - \phi_t]^\alpha$, respectively. If $t \neq \tau_i$, then next-period prices follow:*

$$p_{t+1} = \begin{cases} \bar{p}_{t+1} = (\theta\phi^l)^{\alpha-1} p_t & \text{with probability } u, \\ \underline{p}_{t+1} = (\theta\phi^l)^{\alpha-1} \left(\frac{1-\phi^l}{1-\phi^c}\right)^{1-\alpha} p_t & \text{with probability } 1 - u. \end{cases} \tag{24}$$

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If $t = \tau_i$, then next-period prices are $p_{t+1} = \left(\theta\phi^l\right)^{\alpha-1} \left(\frac{1-\phi^c}{1-\phi^l}\right)^{1-\alpha} p_t$.

5. *Bailout costs are financeable via domestic taxation if $\delta u \left[\theta\phi^l\right]^\alpha + \delta^2 [1-u] \left[\theta^2 \phi^l \phi^c\right]^\alpha < 1$.*

The proposition states that, if $\theta > \underline{\theta}$, then the marginal gross return per unit of investment $(u\theta\beta\overline{p}_{t+1}/p_t)$ is sufficiently high so as to make it profitable to borrow up to the limit. Furthermore, because crises are infrequent, diversion schemes are not optimal in spite of the guarantees. Thus, borrowing constraints bind.[19] Will the entrepreneur choose T-debt or N-debt? She knows that all other firms will go bust in the bad state (i.e., $\pi(\underline{p}_{t+1}) < 0$) provided there is *insolvency risk*–that is, if $\frac{\beta\theta\underline{p}_{t+1}}{p_t} < \frac{h}{u}$. However, the existence of systemic guarantees means that lenders will be repaid in full. Hence, the interest rate on T-debt that allows lenders to break even satisfies $1 + \rho_t = 1 + r$. It follows that the benefits of a risky plan derive from the fact that choosing T-debt over N-debt reduces the cost of capital from $1 + r$ to $[1 + r]u$. Lower expected debt repayments ease the borrowing constraint, as lenders will lend up to an amount that equates $u[1 + r]b_t$ to $h[w_t + b_t]$. Thus, investment is higher relative to a plan financed with N-debt. The downside of a risky plan is that it entails a probability $1 - u$ of insolvency. Will the two benefits of issuing T-debt–namely, more and cheaper funding–be large enough to compensate for the cost of bankruptcy in the bad state? If $u\beta\theta\overline{p}_{t+1}/p_t$ is high enough, then expected profits under a risky plan exceed those under a safe plan and under storage. High enough $u\beta\theta\overline{p}_{t+1}/p_t$ is assured by setting the productivity parameter $\theta > \underline{\theta}$.

To see how a crisis can occur consider a typical period t. Suppose that all inherited debt is denominated in T-goods and that agents expect a bailout at $t + 1$ if a majority of firms goes bust. Since debt repayment is independent of prices, there are two market-clearing prices; this is shown in Figure 1. In the "solvent" equilibrium (point A in Figure 1), the price is high enough that the N-sector can buy a large share of N-output. However, in the "crisis" equilibrium of point B, the price is so low that N-firms go bust: $\beta\underline{p}_t q_t < L_t$.

The key to having multiple equilibria is that part of the N-sector's demand comes from the N-sector itself. Thus, if the price fell below a cutoff level and N-firms went bust, the investment share of the N-sector would fall (from ϕ^l to ϕ^c). This, in turn, would reduce the demand for N-goods, validating the fall in price. The upper bound on θ ensures that the low price is low enough to bankrupt firms with T-debt, while the upper bound on β ensures that $\underline{\theta} < \overline{\theta}$. A low enough β (high $1 - \beta$) means that, when a crisis hits, the decline in cash flow of young entrepreneurs (from $[1-\beta]\overline{p}_{t-1}q_{t-1}$ to $\mu_w\underline{p}_t q_t$) leads to a large fall in input demand, validating the large fall in prices.

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[19] Diversion plans are not optimal when u is large because the interest rate entailed by such plans becomes too large, $1 + \rho^d = (1 + r)/(1 - u)$, and diversion requires the firm to be solvent.

Figure 1: Market Equilibrium for Inputs
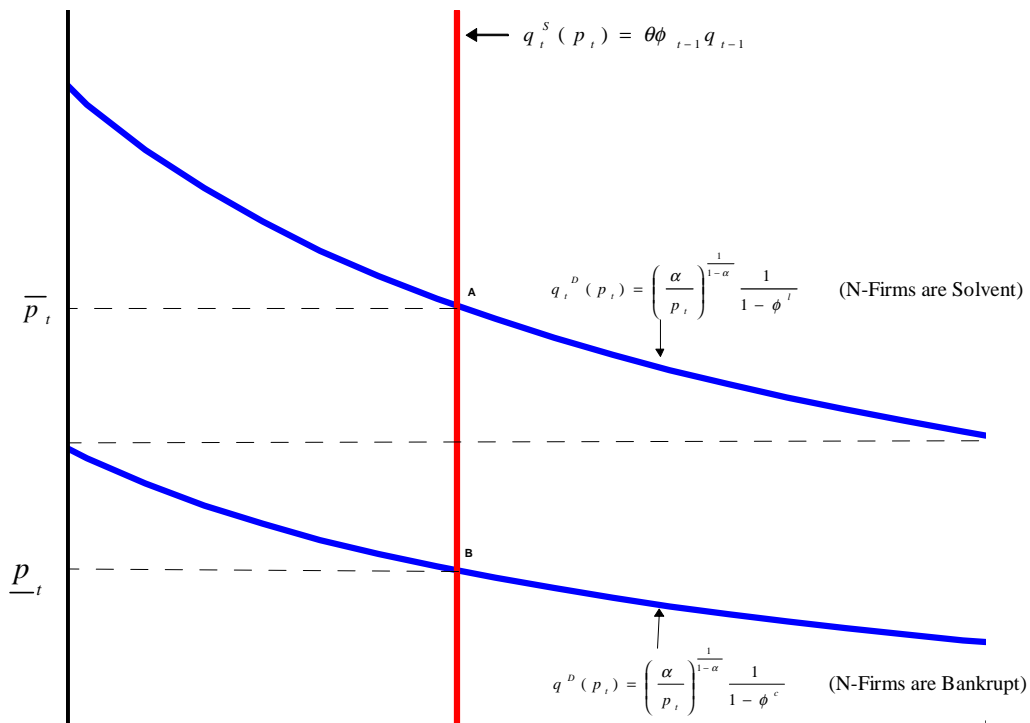
Three points are worth emphasizing. First, Proposition 3.2 holds for any $l = 1 - \frac{\mu}{\beta} \in (0,1)$ and any $l^d = 1 - \frac{\mu_w}{1-\beta} \in (0,1)$. That is, crisis costs are not *necessary* to trigger a crisis. A shift in expectations is sufficient: a crisis can occur whenever entrepreneurs expect that others will not undertake credit risk, resulting in reversion to the SSE characterized in Proposition 3.1. Second, two crises cannot occur consecutively. Because investment in the crisis period falls, the supply of N-goods during the post-crisis period will also fall. This has the effect of driving post-crisis prices up, which would prevent the occurrence of insolvencies even if all debt were T-debt. In other words, during the post-crisis period, a drop in prices large enough to generate insolvencies is impossible. Third, we focus in Proposition 3.2 on a RSE in which there is reversion back to a risky path in the period immediately after the crisis. In Appendix A, we relax this assumption and allow agents to choose safe strategies for multiple periods in the aftermath of crisis.

4 Long-Run Growth

In this section we compare the long-run growth rates of the financially repressed and liberalized regimes characterized in Propositions 3.1 and 3.2, respectively.

Since N-goods are intermediate inputs, whereas T-goods are final consumption goods, gross domestic product (GDP) equals the value of N-sector investment plus T-output: $gdp_t = p_t I_t + y_t$. It then follows from (17)-(20) that, in any symmetric equilibrium, GDP is given by

$$gdp_t = p_t \phi_t q_t + y_t = q_t^\alpha Z(\phi_t) = y_t \frac{Z(\phi_t)}{[1-\phi_t]} \quad \text{with } Z(\phi_t) = \frac{1-(1-\alpha)\phi_t}{[1-\phi_t]^{1-\alpha}}. \tag{25}$$

As this expression shows, the key determinant of GDP's evolution is the share of N-output commanded by the N-sector for investment: ϕ_t. This share is determined by the cash flow of young entrepreneurs and by the credit they can obtain. We emphasize that (25) also makes clear that, because of output-input linkages, measured aggregate total factor productivity $Z(\phi_t)$ is a function of the share of investment in the N-sector. This result is linked with the literature on input misallocation (e.g., Jones, 2010, 2011).[20]

4.1 Growth in a Financially Repressed Economy

In an SSE the investment share ϕ_t is constant and equal to ϕ^s. Thus, (25) implies that GDP and T-output grow at the same rate.

$$1 + \gamma^s := \frac{gdp_t}{gdp_{t-1}} = \frac{y_t}{y_{t-1}} = \left(\theta \frac{1-\beta}{1-h\delta}\right)^\alpha = (\theta \phi^s)^\alpha \tag{26}$$

Absent exogenous technological progress in the T-sector, the endogenous growth of the N-sector is the force driving growth in both sectors. As the N-sector expands, N-goods become more abundant and cheaper, allowing the T-sector to expand production. This expansion is possible if and only if N-sector productivity θ and the N-investment share ϕ^s are high enough, so that credit and N-output can grow over time: $\frac{b_t}{b_{t-1}} = \frac{q_t}{q_{t-1}} = \theta \phi^s > 1$. Observe that, for any positive growth rate of N-output, γ^s increases with the intensity α of the N-input in the production of T-goods.

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[20] Because, at time t, q_t^α is predetermined by past investment, the contemporaneous effect of investment share changes on aggregate TFP at t can be decomposed as

$$\frac{\partial gdp_t}{\partial \phi_t} = \frac{\partial Z(\phi_t)}{\partial \phi_t} = p_t q_t - \frac{\alpha y_t}{1-\phi_t} + q_t \phi_t \frac{\partial p_t}{\partial \phi_t} = q_t \phi_t \frac{\partial p_t}{\partial \phi_t} > 0$$

Market clearing in the N-goods market (i.e., $(1-\phi_t)p_t q_t = \alpha y_t$) implies that the induced changes in investment and final output cancel out. Since an increase in investment raises contemporaneously the price of N-goods, it follows that $q_t \phi_t \frac{\partial p_t}{\partial \phi_t} > 0$ and so measured aggregate TFP is increasing in ϕ.

4.2 Growth in a Financially Liberalized Economy

Proposition 3.2 shows that any RSE is composed of a succession of lucky paths punctuated by crisis episodes. In the RSE characterized by that proposition the economy is on a lucky path at time t if there has not been a crisis either at $t-1$ or at t. Since along a lucky path the investment share equals ϕ^l, (25) implies that the common growth rate of GDP and T-output is

$$1 + \gamma^l := \frac{gdp_t}{gdp_{t-1}} = \frac{y_t}{y_{t-1}} = \left(\theta \frac{1-\beta}{1-h\delta u^{-1}}\right)^\alpha = \left(\theta\phi^l\right)^\alpha. \tag{27}$$

A comparison of (26) and (27) reveals that, as long as a crisis does not occur, growth in a risky economy is higher than in a safe economy. Along the lucky path, the N-sector undertakes insolvency risk by issuing T-debt. Because there are systemic guarantees, financing costs fall and borrowing constraints are relaxed relative to a safe economy. These changes increase the N-sector's investment share $(\phi^l > \phi^s)$. Because there are sectorial linkages $(\alpha > 0)$, this increase in the N-sector's investment share benefits both the T-sector and the N-sector thereby fostering GDP growth.

However, in a risky economy a self-fulfilling crisis can occur with probability $1-u$, and during a crisis episode growth is lower than along a safe path. We have seen that any crisis episode consists of at least two periods. In the first period, the financial position of the N-sector is severely weakened and the investment share falls from ϕ^l to $\phi^c < \phi^s$; then, in the second period this share jumps back to ϕ^l. These transitions occur with certainty, so the mean crisis growth rate is given by

$$1 + \gamma^{cr} = \underbrace{\left(\left(\theta\phi^l\right)^\alpha \frac{Z(\phi^c)}{Z(\phi^l)}\right)^{1/2}}_{\text{Crisis period}} \underbrace{\left((\theta\phi^c)^\alpha \frac{Z(\phi^l)}{Z(\phi^c)}\right)^{1/2}}_{\text{Post-crisis period}} = \left(\theta(\phi^l\phi^c)^{1/2}\right)^\alpha. \tag{28}$$

The second equality in (28) shows that the average loss in GDP growth stems only from the fall in the N-sector's average investment share: $(\phi^l\phi^c)^{1/2}$. This reduction comes about through two channels: financial distress, indexed by $l^d = 1 - \frac{\mu_w}{1-\beta}$; and a reduction in leverage, indexed by $\frac{1-h\delta}{1-h\delta u^{-1}}$. Notice that the GDP growth variations generated by relative price changes at τ and $\tau+1$ cancel out (this result is derived in Appendix A).

A crisis has long-run effects because N-investment is the source of endogenous growth and so the level of GDP falls permanently. To determine under what conditions the mean long-run GDP growth in a liberalized economy is greater than in a repressed one (despite the occurrence of crises), we derive the limit distribution of GDP's compounded growth rate as $\log(gdp_t) - \log(gdp_{t-1})$.

Recall that, because internal funds collapse in a crisis, it is not possible to generate enough leverage to make it possible for another crisis to occur next period (Proposition 3.2). In any RSE, then: (i) everyone must choose a safe plan during a crisis and (ii) risky plans can be chosen in

any period after the crisis period. In the RSE in which the undertaking of credit risk resumes the period immediately after the crisis, the growth process is characterized by the following three-state Markov chain:[21]

$$\Gamma = \begin{pmatrix} \log\left((\theta\phi^l)^\alpha\right) \\ \log\left((\theta\phi^l)^\alpha \frac{Z(\phi^c)}{Z(\phi^l)}\right) \\ \log\left((\theta\phi^c)^\alpha \frac{Z(\phi^l)}{Z(\phi^c)}\right) \end{pmatrix}, \qquad T = \begin{pmatrix} u & 1-u & 0 \\ 0 & 0 & 1 \\ u & 1-u & 0 \end{pmatrix}.$$

The three elements of Γ are the growth rates in the lucky, crisis, and post-crisis states as given, respectively, by (23), (27), and (28). The element T_{ij} of the transition matrix is the transition probability from state i to state j. Because the transition matrix is irreducible, the growth process converges to a unique limit distribution over the three states that solves $T'\Pi = \Pi$. The solution is $\Pi = \left(\frac{u}{2-u}, \frac{1-u}{2-u}, \frac{1-u}{2-u}\right)'$, where the elements of Π can be interpreted as the shares of time that an economy spends in each state over the long run. It then follows that the mean long-run GDP growth rate is $E(1+\gamma^r) = \exp(\Pi'\Gamma)$.[22] That is:

$$E(1+\gamma^r) = (1+\gamma^l)^\omega (1+\gamma^{cr})^{1-\omega} = \theta^\alpha (\phi^l)^{\alpha\omega} (\phi^l \phi^c)^{\alpha\frac{1-\omega}{2}}, \quad \text{where} \ \ \omega = \frac{u}{2-u}. \tag{29}$$

A comparison of the long-run GDP growth rates in (26) and (29) reveals the trade-offs involved in following safe versus risky growth paths, allowing us to determine the conditions under which financial liberalization is growth enhancing.

Proposition 4.1 (Long-Run GDP Growth) *In an RSE, the mean long-run GDP growth rate is given by*

$$E(1+\gamma^r) = (1+\gamma^s)^\alpha \left(\frac{\phi^l}{\phi^s}\right)^{\frac{1}{2-u}} \left(\frac{\mu_w}{1-\beta}\right)^{\frac{1-u}{2-u}}.$$

Furthermore, mean growth is greater in a liberalized than in a repressed economy if and only if financial distress costs $\left(l^d = 1 - \frac{\mu_w}{1-\beta}\right)$ *are not too severe:*

$$l^d < \overline{l^d} = 1 - \left(\frac{1 - h\delta u^{-1}}{1 - h\delta u}\right)^{\frac{1}{1-u}}. \tag{30}$$

Rewriting (30) as $(1-u)\left[\log(1-\beta) - \log(\mu_w)\right] < \log(\phi^l) - \log(\phi^s)$ clarifies the costs and benefits associated with a risky path. A liberalized economy outperforms a repressed one if the benefits of higher leverage and investment in no-crisis times ($\phi^l > \phi^s$) compensate for the shortfall in internal funds and investment in crisis times ($\mu_w < 1 - \beta$) weighted by the frequency of crisis ($1 - u$). We

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[21] In Appendix B we consider alternative RSEs in which a crisis is followed by a "cooling off" phase of arbitrary length and during which only safe plans are undertaken.

[22] Here $E(1+\gamma^r)$ is the geometric mean of $1+\gamma^l$, $1+\gamma^{lc}$, and $1+\gamma^{cl}$.

remark that, the larger is h within the admissible range $(0, \delta^{-1})$, the larger are the growth benefits of systemic-risk taking and hence the less stringent is the condition on crisis costs $(l^d < \overline{l^d})$.[23]

Figure 2 illustrates Proposition 4.1 by plotting several risky growth paths associated with different degrees of crisis-induced financial distress.[24] As the figure shows, even if 90% of N-sector cash flow is lost during a crisis, a risky economy can outperform a safe economy over the long run. In other words, a risky economy can exhibit greater mean growth than a safe economy despite large financial distress costs.

Figure 3 illustrates the limit distribution of GDP growth rates by plotting different GDP paths corresponding to different realizations of the sunspot process. The risky paths outperform the safe path except for a few unlucky risky paths. If we increased the number of paths, the cross-section distribution would converge to the limit distribution.

# 5   Production Efficiency and Consumption Possibilities

We have considered an endogenous growth model where the financially constrained N-sector is the engine of growth because it produces the intermediate input used throughout the economy. Thus, the share ϕ_t of N-output invested in the N-sector is the key determinant of economic growth. When ϕ_t is too small, T-output is high in the short run, but long-run growth is slow; when ϕ_t is too high, there is inefficient accumulation of N-goods. In this section we ask three questions. First, what is the Pareto-optimal investment share sequence $\{\phi_t\}$? Second, can this Pareto-optimal investment sequence be replicated in a financially repressed economy? If not, can the average investment share be higher in a financially liberalized economy, where agents undertake credit risk and crises occur? Third, will the present value of consumption be greater in a liberalized economy after netting out the taxes necessary to finance the bailouts during crises?

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[23]The following table gives the upper bound for the financial distress costs $\overline{l^d}$ for different values of h.

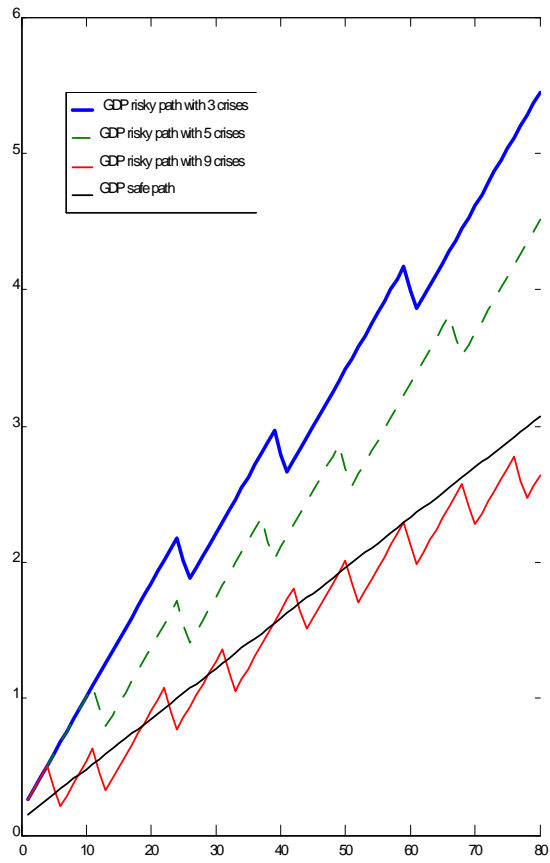|	$u = 0.95, \delta = 0.95$
$h = 0.4$	$\overline{l^d} = 48\%$
$h = 0.6$	$\overline{l^d} = 76.7\%$
$h = 0.8$	$\overline{l^d} = 96\%$

[24]See Appendix C for details of the model calibration. The simulations plotted in Figure 4 include four crises in 80 periods, which corresponds to a 5% probability of crisis.

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Figure 2: GDP Growth and Financial Distress Costs ($l^d = 1 - \frac{\mu_w}{1-\beta}$)
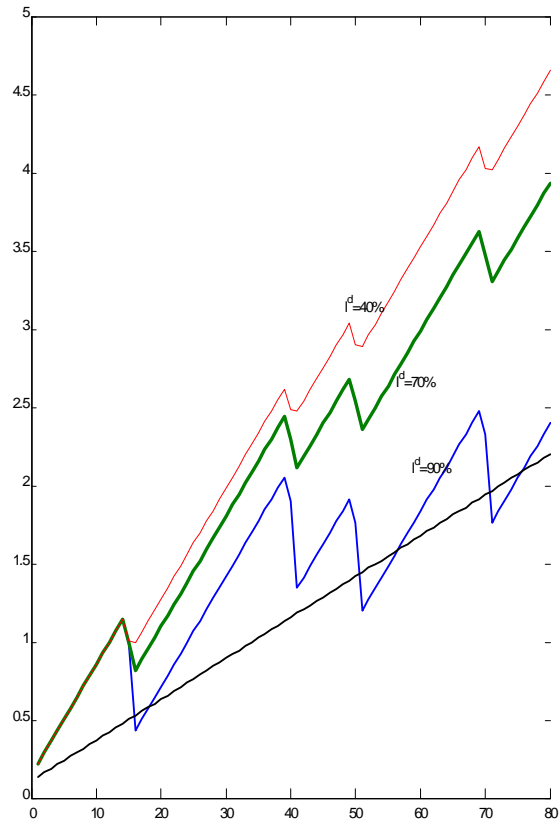
parameters : $\theta = 1.65$ $\alpha = 0.35$ $h = 0.76$ $1 - \beta = 0.2$ $1 - u = 5\%$

Figure 3: Limit Distribution of GDP

$parameters$ $: \theta = 1.65$ $\alpha = 0.35$ $h = 0.76$ $1 - \beta = 0.2$ $l^d = 70\%$ $1 - u = 5\%$

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5.1 Pareto Optimality

Consider a central planner who maximizes the present discounted value of the consumption of workers and entrepreneurs by allocating the supply of inputs to final goods production ($[1 - \phi_t]q_t$) and to input production ($\phi_t q_t$) as well as by assigning sequences of consumption goods to consumers and entrepreneurs for their consumption.

$$\max_{\{c_t, c_t^e, \phi_t\}_{t=0}^{\infty}} W^{po} = \sum_{t=0}^{\infty} \delta^t \left[c_t^e + c_t\right], \quad \text{s.t.} \quad \sum_{t=0}^{\infty} \delta^t \left[c_t + c_t^e - y_t\right] \leq 0,$$

$$y_t = [1 - \phi_t]^{\alpha} q_t^{\alpha}, \quad q_{t+1} = \theta \phi_t q_t. \tag{31}$$

Pareto optimality implies efficient accumulation of N-inputs: the planner should choose the investment sequence $\{\phi_t\}$ that maximizes the present value of final goods (T-)production ($\sum_{t=0}^{\infty} \delta^t y_t$). We show in Appendix D that the Pareto-optimal N-investment share is *constant* and equal to

$$\phi^{po} = (\theta^{\alpha}\delta)^{\frac{1}{1-\alpha}}, \quad \text{if} \quad \alpha < \log(\delta^{-1})/\log(\theta). \tag{32}$$

The Pareto-optimal share equalizes the discount rate δ^{-1} to the intertemporal rate of transformation. A marginal increase in the N-sector investment share ($\partial\phi$) reduces today's T-output by $\alpha \left[(1 - \phi)q_t\right]^{\alpha-1} \partial\phi$, but it also increases tomorrow's N-output by $\theta\partial\phi$ and tomorrow's T-output by $\alpha \left[(1 - \phi)\theta\phi q_t\right]^{\alpha-1} \theta\partial\phi$. Thus, at an optimum, $\theta^{\alpha}\phi^{\alpha-1} = \delta^{-1}$.

Can a decentralized economy replicate the Pareto-optimal allocation? The optimal investment share is determined by investment opportunities $\theta^{\alpha}\delta$. However, in a decentralized safe economy, the N-investment share ($\phi^s = \frac{1-\beta}{1-h\delta}$) is determined by credit market imperfections: the degree of contract enforceability (h) and the constrained sector's cash flow ($1-\beta$). If either h or $1-\beta$ are low, then clearly the N-sector investment share will be lower than the Pareto optimal share: $\phi^s < \phi^{po}$. That is, when the N-sector is severely credit constrained, low N-sector investment will keep the economy below production efficiency. For future reference we summarize matters in the following proposition.

Proposition 5.1 (Bottleneck) *Input production in a financially repressed economy is below the Pareto-optimal level (i.e., there is a "bottleneck") if and only if contract enforceability problems are severe:*

$$\phi^s < \phi^{po} \quad \Leftrightarrow \quad h < \left[1 - (1 - \beta)\theta \left(\theta\delta\right)^{-\frac{1}{1-\alpha}}\right]\frac{1}{\delta}.$$

When there is a bottleneck, the share of N-inputs allocated to T-production should be reduced and that allocated to N-production should be increased in order to bring the allocation nearer to the Pareto optimal level. Such reallocation reduces the initial level of T-output but also increases its growth rate and the present value of cumulative T-production.

Input-Output Linkages and the Dynamic Multiplier Effect. If there is a bottleneck (i.e., if $\phi^s < \phi^{po}$), then an increase in the investment share ϕ corresponds to a reduction of input misallocation. In the context of our two-sector endogenous growth model, that increase in ϕ leads to an increase in future final goods production. This dynamic input-multiplier effect is analogous to the steady-state approach proposed by Jones (2010, 2011) in the context of a neoclassical growth model.

A marginal increase in the N-sector investment share ($\partial\phi$) reduces today's T-output by $\alpha\left[(1-\phi)q_t\right]^{\alpha-1}\partial\phi$, but it increases tomorrow's N-output by $\theta\partial\phi$ and tomorrow's T-output by $\alpha\left[(1-\phi)\theta\phi q_t\right]^{\alpha-1}\theta\partial\phi$. The intertemporal multiplier effect is therefore

$$M = \frac{\alpha\left[(1-\phi)\theta\phi q_t\right]^{\alpha-1}\theta}{\alpha\left[(1-\phi)q_t\right]^{\alpha-1}} = \theta^\alpha\phi^{\alpha-1}.$$

It follows that the long-run dynamic gains in T-output resulting from a marginal increase in the investment rate in the N-sector are given by

$$M + M^2 + \cdots + M^j + \cdots = \sum_{j=1}^{\infty} M^j = \frac{1}{1-M} - 1.$$

These dynamic gains are maximized as M tends to 1 or, equivalently, as the investment share tends to $\phi^g \equiv \theta^{\frac{\alpha}{1-\alpha}}$. Notice that the value of ϕ^g is increasing in α, the *strength of the input-output linkage.*

To see the link between ϕ^g and ϕ^{po}, note that ϕ^g maximizes the sum of final goods production. If the objective were to maximize the discounted sum of final goods production, then we would obtain instead the Pareto-optimal investment share: $\phi^{po} = \delta^{\frac{1}{1-\alpha}}\theta^{\frac{\alpha}{1-\alpha}}$. Therefore, $\phi^{po} = \delta^{\frac{1}{1-\alpha}}\phi^g$.

5.2 Present Value of Consumption in a Decentralized Economy

Financial liberalization relaxes borrowing constraints and spurs growth. However, it also generates systemic risk that makes the economy vulnerable to crises, which entail deadweight losses as well as fiscal costs to cover bailouts. Here we determine conditions under which the present value of consumption in a financially liberalized economy is greater than in a repressed economy after netting out the crisis and bailout costs. We show that liberalization can improve consumption possibilities only if there is a bottleneck (i.e., only if $\phi^s < \phi^{po}$), so that growth is inefficiently low under financial repression.

The expected discounted value of workers' consumption and entrepreneurs' consumption in our decentralized economy may be written as

$$W^d = E_0\left(\sum_{t=0}^{\infty}\delta^t(c_t + c_t^e)\right) = E_0\left(\sum_{t=0}^{\infty}\delta^t[[1-\alpha]y_t + \pi_t - T_t]\right). \tag{33}$$

To derive the second equation in (33) notice that in equilibrium workers' income at t is $[1-\alpha]y_t$, entrepreneurs' income is equal to their profits π_t, and the fiscal cost of bailouts is financed with lump-sum taxes T_t.

In order to obtain a closed-form solution, notice that, at any $t \geq 1$, profits are equal to the old entrepreneurs' share in revenues minus debt repayments: $\pi_t = \beta p_t q_t - L_t = \frac{\alpha}{1-\phi^s}\beta y_t - \frac{\alpha}{1-\phi^s}\frac{h}{u}\phi^s y_{t-1}$. Since at $t = 0$ there is no debt burden, $\pi_0 = \frac{\alpha}{1-\phi^s}\beta y_0$. In a financially repressed economy, firms are always solvent and crises never occur. Hence, there are no bailouts and no taxes. It follows from (33) that the present value of consumption equals the present value of T-output:

$$W^s = \sum_{t=0}^{\infty} \delta^t y_t^s = \frac{1}{1 - \delta(\theta\phi^s)^\alpha} y_o^s = \frac{(1-\phi^s)^\alpha}{1 - \delta(\theta\phi^s)^\alpha} q_o^\alpha \qquad \text{if } \delta(\theta\phi^s)^\alpha < 1. \qquad (34)$$

Consider a liberalized economy. Along a lucky path, the investment share is greater than in a safe economy. Thus, if there is a bottleneck and crises are rare events, then the present value of T-output along the lucky path is greater than that along the safe path. However, along a lucky path a crisis can occur with probability $1 - u$, and a crisis involves three costs.

First, there is a fiscal cost. Lenders receive a bailout payment equal to the debt repayment they were promised: $L_\tau = u^{-1}h\phi^l p_{\tau-1}q_{\tau-1}$. Since the bailout agency recuperates only a share $\mu \leq \beta$ of firms' revenues $p_\tau q_\tau$ while the rest is dissipated in bankruptcy procedures, the fiscal cost of a crisis is $T(\tau) = L_\tau - \mu p_\tau q_\tau$. Second, there is a financial distress cost. In a crisis there is a decline in credit and investment, so the investment share of the input sector is $\phi^c = \frac{\mu_w}{1-h\delta}$ instead of ϕ^s as in a safe economy; here μ_w can be arbitrarily small. During a crisis, borrowing constraints are tighter than what they would be in a safe economy because (a) an N-firm's net worth is $\mu_w p_\tau q_\tau$ instead of $[1 - \beta]p_\tau q_\tau$ and (b) risk taking is curtailed; that is, only safe plans are financed. Third, since during a crisis all N-firms go bust, old entrepreneurs' profits are zero.

The deadweight loss of a crisis for the overall economy is less than the sum of these three costs. During a crisis there is a sharp *redistribution* from the N-sector to the T-sector generated by a large fall in the relative price of N-goods (a fire sale). Thus, some of the costs incurred in the N-sector show up as greater T-output and consumers' income. We show in the proof of Proposition 3.2 that, after netting out the costs and redistributions, a crisis involves two deadweight losses: (i) the revenues dissipated in bankruptcy procedures, $[\beta - \mu]p_\tau q_\tau$; and (ii) the fall in N-sector investment due to its weakened financial position, $[(1 - \beta) - \mu_w]p_\tau q_\tau$. We remark that (i) affects the fiscal burden of the bailouts but not future production. In contrast, (ii) affects future production and future investment, so it is propagated through the dynamic multiplier described in Section 5.1.

Using the market-clearing condition $\alpha y_t = [1 - \phi_t]p_t q_t$, we find that the sum of these two deadweight losses equals $\frac{\alpha}{1-\phi^c}[1 - \mu - \mu_w]y_\tau$ in terms of T-goods. Thus, in an RSE the present value of consumption is given by

$$W^r = E_0 \sum_{t=0}^{\infty} \delta^t \kappa_t y_t, \qquad \kappa_t = \begin{cases} \kappa^c := 1 - \frac{\alpha[1-\mu-\mu_w]}{1-\phi^c} & \text{if } t = \tau_i, \\ 1 & \text{otherwise;} \end{cases} \qquad (35)$$

as before, τ_i denotes a time of crisis. In order to compute this expectation, we need to calculate the limit distribution of $\kappa_t y_t$. This derivation is computed in the the proof of Proposition 3.2 and yields

$$W^r = \frac{\left[1 + \delta(1-u)\left(\theta\phi^l\frac{1-\phi^c}{1-\phi^l}\right)^\alpha \kappa^c\right][(1-\phi^l)q_0]^\alpha}{1 - \left[\theta\phi^l\right]^\alpha u\delta - \left[\theta^2\phi^l\phi^c\right]^\alpha [1-u]\delta^2} \text{ if } \left[\theta\phi^l\right]^\alpha u\delta + \left[\theta^2\phi^l\phi^c\right]^\alpha [1-u]\delta^2 < 1 \quad (36)$$

By comparing (34) and (36) we can determine the conditions under which the ex ante present value of consumption is greater in a financially liberalized economy.

Proposition 5.2 (Present Value of Consumption) *In an economy where crises are rare events and that satisfies parameter restrictions (63)-(68), the following statements hold.*

1. *Financial liberalization increases the present value of consumption only if the investment share ϕ in a financially repressed regime is less than the Pareto-optimal investment share ϕ^{po}.*

2. *When $\phi < \phi^{po}$, financial liberalization increases the present value of consumption for any level of bankruptcy costs (i.e., any $\mu \geq 0$) if financial distress costs are not too high ($l^d < \tilde{l}^d$) and the discount rate δ is not too low.*

This proposition establishes that the growth-enhancing effect of systemic risk taking in a liberalized regime translates into higher consumption possibilities for the economy as a whole only if there is a bottleneck—so that liberalization increases allocative efficiency—and, in addition, the dynamic costs of crises are not too severe.

Proposition 5.2 is proved in Appendix D by taking the derivative of W^r with respect to u and letting $u \to 1$. Since $W^r|_{u=1} = W_s$, it follows that financial liberalization, which enables systemic risk taking, increases the present value of consumption if and only if $\frac{\partial W^r}{\delta u}\big|_{u=1}$ is negative. We have

$$\frac{\partial W^r}{\partial u}\bigg|_{u=1} = \left\{ \underbrace{\alpha\phi'\left[\frac{D}{\phi} - 1\right]}_{\text{Efficiency gains}} + \underbrace{[1-D]\left[1 - \kappa_c\left[\frac{1-\phi^c}{1-\phi^l}\right][1-\phi]\right]}_{\text{Bankruptcy costs}} + \underbrace{\frac{[1-\phi]^\alpha D\delta\theta^\alpha [\phi^\alpha - (\phi^c)^\alpha]}{\text{Financial distress costs}}}_{} \right\} K, \quad (37)$$

where $D = \delta(\theta\phi)^\alpha = (\phi^{po})^{1-\alpha}\phi^\alpha$ and K is a strictly positive number.[25] Since the derivative is evaluated at $u = 1$, we have $\phi \equiv \phi^l = \phi^s$.

The first term in (37) captures the efficiency gains from financial liberalization. It can be rewritten as $\alpha\phi'((\frac{\phi^{po}}{\phi})^{1-\alpha} - 1)$, which is negative if and only if $\phi < \phi^{po}$. The second term captures the bankruptcy costs associated with crises. The third term reflects crisis-induced financial distress

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[25] It is given by $K = \left[\frac{q_0^\alpha (1-\phi)^{\alpha-1}}{1-[\theta\phi]^\alpha \delta u - [\theta^2\phi\phi^c]^\alpha \delta^2(1-u)}\right]^2 > 0.$

costs, which are increasing in the difference between the "tranquil times" investment share ϕ and the crisis investment share ϕ^c. When $\phi^c < \phi < \phi^{po}$, financial distress costs correspond to production efficiency losses since they move the allocation of intermediate inputs farther away from the Pareto-optimal level.

Because the second and third terms in (37) are positive, a necessary condition for $W^r > W^s$ is that the first term be negative; this occurs only if $\phi < \phi^{po}$. In other words, there are efficiency gains associated with financial liberalization only if there is a bottleneck. This is the first part of Proposition 5.2.

The second part of the proposition establishes conditions under which the crisis costs are outweighed by the efficiency gains. If the discount rate is high enough, then the bankruptcy costs, which are static in nature, become vanishingly small. In this case, the gains (or losses) from financial liberalization depend on the relative sizes of efficiency gains and financial distress costs, both of which are dynamic. That is, they propagate to future periods through the investment channel, and they affect future levels of T-production through input-output linkages. The efficiency gains depend on how much risk taking reduces the distortion in the allocation of intermediate inputs in tranquil times, while financial distress costs measure how the allocation of intermediate inputs becomes more distorted in times of crisis. On net there are positive gains from financial liberalization when financial distress costs are below a threshold: $l^d < \widetilde{l^d}$. Recall that these distress costs measure the severity of the credit crunch in the wake of crisis.

6   Financial Liberalization with a Larger Set of Securities: an Example.

We have established how, even if systemic bailout guarantees are present, financial liberalization can enhance growth and improve production efficiency in an environment where risk-taking is undertaken using standard debt contracts. In this section, we consider a simple example where the issuance of option-like liabilities without collateral is allowed, and show that financial discipline may break down. In this example, financial liberalization may enhance growth but hinders production efficiency and consumption possibilities.

The modified setup.   We add, to the setup of Section 3, an alternative (inferior) production technology and a new type of liabilities. The alternative technology for producing final T-goods

uses only T-goods as inputs according to:

$$y_{t+1} = \varepsilon_{t+1} I_t^\varepsilon, \quad \text{where } \varepsilon_{t+1} = \begin{cases} \bar{\bar{\varepsilon}} & \text{with probability} \quad \lambda, \\ 0 & \text{with probability} \quad 1 - \lambda; \end{cases} \qquad \bar{\bar{\varepsilon}} \leq 1 + r, \qquad (38)$$

and I_t^ε denotes the input of T-goods. This technology for producing final goods yields less than the risk-free return in all states, and so it is inferior to the θ-technology (4), which uses intermediate goods as inputs.

In this example, we expand the menu of issuable securities. In addition to standard debt, firms can issue *catastrophe bonds* with the following repayment schedule:

$$L_{t+1}^c = \begin{cases} 0 & \text{if } \varepsilon_{t+1} = \bar{\varepsilon}, \\ 1 + \rho_t^c & \text{if } \varepsilon_{t+1} = 0. \end{cases} \qquad (39)$$

With standard bonds a borrower must promise to repay the same nominal amount in all states. In contrast, with catastrophe bonds a debtor can promise to repay an arbitrarily large amount in the bad ($\varepsilon_{t+1} = 0$) state and zero in the good ($\varepsilon_{t+1} = \bar{\varepsilon}$) state.

We also introduce a new set (of measure 1) of entrepreneurs who have access to the ε-technology and live for two periods. When young, an ε-entrepreneur (who has zero internal funds) issues debt and uses the proceeds to buy T-goods (I_t^ε), which he invests to produce T-goods using production function (38). When old, the ε-entrepreneur consumes his profits.

Clearly, if standard bonds were the only class of issuable liabilities or bailout guarantees were absent, the inferior ε-technology would not be funded in equilibrium since, in both states, it yields a return inferior to the risk-free interest rate. The point of our example is that the combination of bailout guarantees and catastrophe bonds with no collateral is the key for the funding of this ε-technology.

Finally, since in this environment the equilibrium borrowing limit will depend on the expected bailout, we add an upper bound on the bailout.

Bailout Guarantees A bailout up to an amount $\Gamma_{i,t}$ is granted to lenders of a defaulting borrower if half of all borrowers default.

We parameterize $\Gamma_{i,t}$ as a share γ_i of final goods produced by the non-diverting part of the economy:

$$\Gamma_{i,t} = \gamma_i [y_t^{\theta,nd} + y_t^{\varepsilon,nd}], \qquad (40)$$

where $y_t^{\theta,nd}$ is the T-output produced using N-inputs from non-diverting N-firms, and $y_t^{\varepsilon,nd}$ is the T-output from non-diverting ε-firms. We set $\gamma_i = \gamma$ with $\gamma < \frac{1}{2}$ so that the total bailout granted Γ_t

is always lower than the value of the final good's production.[26] Bailouts are financed via lump-sum taxes on the non-diverting part of the economy.[27]

Equilibrium with Catastrophe Bonds.   Each lender observes whether the borrower is an ε- or a θ-entrepreneur, and then decides whether or not to buy the bonds. At time $t + 1$, lenders receive the promised repayment from non-defaulting borrowers or a bailout (if one is granted). The rest of the setup is the same as in Section 3.

The next proposition characterizes an equilibrium where the issuance of catastrophe bonds supports the funding of the inferior technology.

Proposition 6.1 (Equilibrium with Catastrophe Bonds) *Consider an economy where catastrophe bonds can be issued, (11) holds and N-sector productivity θ satisfies (44). Then the inferior ε-technology is funded in equilibrium if there are bailout guarantees, but they are not too generous (i.e., γ is below a threshold). In this equilibrium:*

1. *θ-agents issue standard bonds, hedge price risk, and do not divert. Input production is $q_t = \theta m^s w_t$;*

2. *ε-agents issue catastrophe bonds and default in the $\varepsilon = 0$ state with probability $1 - \lambda$; and*

3. *final goods production is $y_t^{nd} = y_t^{\theta,nd} + y_t^{\varepsilon,nd} = y_{t-1}^{nd}(\theta\phi^s)^\alpha(1 + \varepsilon_t\delta[1 - \lambda]\gamma)$.*

In the presence of bailout guarantees, the use of catastrophe bonds allows borrowers to shift all their liability repayments to the default state, where bailout payments are triggered. Therefore, the issuance of such securities implies that: (i) any positive return in the no-default state, even if lower than the risk-free interest rate, is enough to ensure positive profits in that state; and (ii) the solution to the borrower–lender agency problem does not require equity investment–the borrowing limit is determined by the expected generosity of the bailout rather than by internal funds. As a consequence, the inferior ε-technology is funded.

In contrast, in Section 3 the use of standard debt contracts restricts external finance to projects that return at least the risk-free rate in the no-default state. It also prevents borrowers from borrowing more than a given multiple of their own equity to eliminate incentives to divert. A consequence of these two factors—the lower bound on the project's return and requiring borrowers

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[26]If both ε-entrepreneurs and θ-entrepreneurs default, then the total bailout is $\Gamma_t = 2\gamma[y_t^{\theta,nd} + y_t^{\varepsilon,nd}] < y_t^{\theta,nd} + y_t^{\varepsilon,nd}$.

[27]The government cannot tax the diverting part of the economy (i.e., the black market). This is a realistic assumption, and it is also important for the working of the model. If income in the diverting sector were taxable, then one could construct equilibria in which diversion is desirable because it would relax borrowing constraints (since lenders would not impose the no-diversion condition).

to risk their own equity—is that the ε-technology is not funded. Thus borrowers invest only in projects that have a private return (net of debt repayments) greater than the storage return $1 + r$.

The proposition states that for the inferior ε-technology to be funded in equilibrium, bailout guarantees must not be "too generous". If bailouts were too generous, then input-producing θ-entrepreneurs would have incentives to issue catastrophe bonds and implement diversion schemes, so there would be no tax base to fund the bailouts. In the equilibrium characterized by the proposition, the upper bound on the generosity of the bailout is tight enough to ensure that θ-entrepreneurs choose the safe, no-diversion plans characterized in Section 3; hence bailouts are fiscally viable.

Present Value of Consumption. From a growth perspective, an economy where catastrophe bonds can be issued outperforms the financially repressed regime: average growth of the final goods sector is $(1 + \bar{\varepsilon}\lambda\delta[1 - \lambda]\gamma)\,(\theta\phi^s)^\alpha$ in the former regime but only $(\theta\phi^s)^\alpha$ in the latter. However, this does not mean that the present value of consumption is greater that in the safe equilibrium (W^s), for we must net out the bailout costs of crises before a valid comparison can be made. In order to compute the expected present value of consumption (PVC) in the equilibrium with catastrophe bonds characterized by Proposition 6.1, we must add to equation (33) the terms corresponding to the consumption and profit of ε-entrepreneurs:

$$W^{CB} = E_0\left(\sum_{t=0}^\infty \delta^t(c_t + c_t^e + c_t^\varepsilon)\right) = E_0\left(\sum_{t=0}^\infty \delta^t[[1 - \alpha]y_t^t + \pi_t + \pi_t^\varepsilon - T_t]\right). \qquad (41)$$

This can be simplified as follows:

$$W^{CB} = \underbrace{W^s}_{\text{Safe economy's PVC}} + \underbrace{\sum_{t=1}^\infty \delta^t b_{t-1}^c(\bar{\varepsilon} - \frac{1+r}{1-\lambda}))}_{\varepsilon\text{-expected PVC} \ - \ \text{Expected bailout costs}}, \qquad (42)$$

where W^s is the present value of consumption in the safe equilibrium of Section 3.[28] Since the ε-technology has negative net present value (i.e., $(1 - \lambda)\bar{\varepsilon} < 1 + r$), it follows that $W^{CB} < W^s$.

This result implies that, even if average growth is higher in an economy where catastrophe bonds can be issued than in a repressed one, and production using the ε-technology is privately optimal, the losses it incurs during crisis times more than offset private profits. Therefore, the use of catastrophe bonds to fund the inferior technology generates net consumption losses for the overall economy.[29]

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[28] The calculation assumes that ε-agents start to borrow in the first period $(t = 0)$ and therefore $c_0^\varepsilon = 0$.

[29] We have not discussed the issuance of stocks. Note, however, that stocks are different from catastrophe bonds. Although stocks are liabilities that might promise very little in some states of the world, their issuance seldom involves political pressure for systemic bailout guarantees.

In sum, the simple example shows how in the presence of systemic bailout guarantees, regulatory limits on the set of issuable securities shapes the outcome of financial liberalization. In particular, it sheds light on the contrasting experience of emerging markets following financial liberalization and the recent US boom-bust cycle. Emerging markets' booms have featured mainly standard debt; while they have experienced crises (the so-called 'third-generation' or balance-sheet crises), systemic risk taking has been, on average, associated with higher long-run growth. In contrast, the recent US boom featured a proliferation of uncollateralized derivatives that supported large-scale funding of negative-NPV projects in the housing sector.[30]

7   Conclusions

We have shown that, in an economy with credit market imperfections, financial liberalization can help overcome obstacles to growth by improving allocative efficiency—that is, by easing financing constraints of bank-dependent sectors with no easy access to either stock markets or international capital markets. As a result, the whole economy can grow faster because it faces less severe bottlenecks as more abundant inputs are produced by the constrained sector. However, a side effect is that financial fragility ensues and so crises occur from time to time.

We have seen that, despite crises, financial liberalization can increase long-run growth, production efficiency and consumption possibilities. The key to this result is that, even though the liberalized regime induces systemic risk taking, it preserves financial discipline if regulation restricts liabilities to standard debt contracts. In such a liberalized regime, despite the presence of systemic bailout guarantees, lenders must screen out unprofitable projects and incentivize borrowers not to divert (e.g., by requiring them to risk their own equity to cover some fraction of the investment). Because there are bailout guarantees this discipline can break down in the absence of regulatory limits on the set of issuable securities. The possibility of issuing option-like instruments that concentrate all repayments in default states allows borrowers without any profitable investment opportunities to invest without putting equity down, thereby exploiting bailout guarantees.

Our results have important implications for financial regulation in a world with capital market imperfections. First, one should be cautious when interpreting the effects of financial liberalization. From the finding that liberalization has led to more crisis-induced volatility, one should not conclude that liberalization per se is bad for either growth or production efficiency. Furthermore, policies intended to eliminate all risk taking and financial fragility might have the unintended effect of blocking the forces that spur growth and allocative efficiency. Second, at the other extreme, the

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[30]Levitin and Wachter (2010); NPV = net present value.

gains from financial liberalization can be overturned in a regime with unfettered liberalization and in which option-like securities can be issued without collateral. In such an anything-goes regime, the presence of systemic bailout guarantees could well lead to excessive leverage, a lack of discipline in lending decisions, and misallocation of resources. Even though agents are optimizing and average growth might be higher under an anything-goes regime than under financial repression, the losses during crises more than offset private profits, resulting in net social losses.

Finally, although most of the current discussion regarding financial regulation concentrates on the asset side, our results suggest it is also important to regulate the liability side–the types of liabilities that can be issued–to maintain financial discipline and reap the benefits of financial liberalization.

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APPENDIX FOR ONLINE PUBLICATION ONLY

A The cost of Crises

During a crisis there are widespread bankruptcies, which generate deadweight losses as well as sectorial redistributions. Here, we net out these crises costs and show that the growth costs of crisis is simply equal to the fall in the N-sector's investment share, as expressed in (28).[31]

If a crisis occurs at some date, say τ, there is a firesale: there is a steep decline in the input price, and since there is currency mismatch, all N-firms default. As a result, the investment share falls from ϕ^l to ϕ^c.[32] The price of N-goods must fall to allow the T-sector to absorb a greater share of N-output, which is predetermined by $\tau - 1$ investment. At $\tau + 1$, N-output contracts due to the fall in investment at the time of the crisis. However, entrepreneurs adopt risky plans again, so the investment share increases from ϕ^s back to ϕ^l. Thus, the price of inputs increases. At $\tau + 2$, the economy is back on a lucky path, but the level of cash flow and N-output are below their pre-crisis trend.

Although GDP fluctuations are affected by changes in the input price, T-output and N-investment, GDP growth during a crisis episode is solely determined by the mean investment share $[\phi^l \phi^c]^{\frac{1}{2}}$ (by (28)). To understand why this is so note that GDP growth has two components: (i) relative price fluctuations (captured by $\frac{Z(\phi_t)}{Z(\phi_{t-1})}$) and (ii) output fluctuations (captured by $(\theta\phi_t)^\alpha$).[33] In the crisis period, GDP growth falls below trend because there is a decline in the input price ($\frac{Z(\phi^l)}{Z(\phi^c)} < 1$). In the post crisis period, there are two effects: (i) since investment contracted during the previous period, N-output falls below trend and depresses growth; but (ii) there is a rebound of the input

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[31] Although the main objective of the model is to address long-run issues, it is reassuring that it can account for key stylized facts of balance-sheet crises in emerging markets: a sharp real depreciation that coincides with a fall in credit growth, as well as the *asymmetric sectorial response* of N- and T-sectors.

[32] This is because young entrepreneurs income is only $\mu_w p_\tau q_\tau$ instead of $[1 - \beta]p_\tau q_\tau$, and at τ entrepreneurs can only choose safe plans in which there is no currency mismatch (by Proposition 3.2).

[33] To interpret (28) note that variations in the investment share ϕ_t have lagged and contemporaneous effects on GDP. The lagged effect comes about because a change in ϕ_t affects next period's GDP via its effect on N-output: $q_{t+1} = \theta I_t = \theta\phi_t q_t$. Using (25) and $y_t = ([1 - \phi_t]q_t)^\alpha$, the contemporoneous effect can be decomposed as:

$$\frac{\partial gdp_t}{\partial \phi_t} = -\frac{\alpha y_t}{1 - \phi_t} + p_t q_t + q_t \phi_t \frac{\partial p_t}{\partial \phi_t} = q_t \phi_t \frac{\partial p_t}{\partial \phi_t}$$

The first two terms capture variations in T-output and N-investment, while the third reflects input price fluctuations. Market clearing in the N–goods market –i.e., $(1 - \phi_t)p_t q_t = \alpha y_t$– implies that the induced changes in N-sector investment and T-output cancel out. Therefore, the contemporoneous changes in the investment share affect GDP contemporaneously only through its effect on the real exchange rate. Since $GDP_t = Z(\phi_t)q_t^\alpha$, we can express $q_t \phi_t \frac{\partial p_t}{\partial \phi_t}$ as $q_t^\alpha \frac{\partial Z_t}{\partial \phi_t}$. Thus, we can interpret $\frac{Z(\phi_t)}{Z(\phi_{t-1})}$ as the effect of real exchange rate fluctuations on GDP.

price as the investment share jumps from its crisis level $\left(\frac{Z(\phi^c)}{Z(\phi^l)} > 1 \right)$. As we can see, variations in GDP growth generated by input price changes at τ and $\tau + 1$ cancel out. Thus, the average loss in GDP growth stems only from the fall in the N-sector's average investment share.

In sum, a crisis has two distinct effects: sectorial redistribution and deadweight losses. At the time of the crisis the T-sector benefits from the financial collapse of the N-sector because it can buy N-output at firesale prices and expand production. The deadweight losses derive from the financial distress and the bankruptcy costs generated by crises. The former leads to a contraction in N-investment and thus has a long-run effect on output. In contrast, bankruptcy costs have only a static fiscal impact, which is the cost of the bailout.

B   Post-Crisis Cool-Off Phase and Growth.

Here, we show that form the perspective of long-run growth, nothing is gained by delaying the onset of the new risky phase.

In Proposition 3.2, we characterized a RSE where there is a reversion back to a risky path in the period immediately after the crisis. We then compared growth in such a risky economy to growth in a safe economy where risk-taking never occurs. The comparison of these polar cases makes the argument transparent, but opens the question of whether the growth results presented in Proposition 4.1 are applicable to recent experiences in which systemic crises have been followed by protracted periods of low leverage, low investment and low growth. In order to address this issue, we construct an alternative RSE under which a crisis is followed by a cool-off phase during which all agents choose safe plans. The cool-off phase can be interpreted either as a period in which agents believe that others are following safe strategies or as a period during which agents are prevented from taking on risk.[34]

To keep the model tractable, we assume that in the aftermath of a crisis, all agents follow safe plans with probability ζ. Hence, a crisis is followed by a cool-off phase of average length $1/(1-\zeta)$ before there is reversion to a risky path.[35] In this case, the mean long-run GDP growth rate is

$$E(1 + \gamma^r) = (\theta\phi^s)^\alpha \left(\frac{\phi^l}{\phi^s} \right)^{\frac{1-\zeta}{(1-\zeta)+(1-u)}} \left(\frac{\mu_w}{1-\beta} \right)^{\frac{u(1-\zeta)}{(1-\zeta)+(1-u)}}, \qquad (43)$$

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[34]Or alternatively as a period where agents revise downwards their bailout expectations because they perceive that the surge in public debt associated with prior bailouts makes future bailout less likely.

[35]The average lenght of the cooling off period is computed as:

$$\lambda = (1-\xi)\sum_{k=0}^{\infty} \xi^{k-1} k = \frac{1}{1-\xi}$$

which generalizes the growth rate of proposition 4.1. Comparing (43) with (26) we can prove the following Lemma.

Lemma B.1 *Consider an RSE where a crisis is followed by a cool-off period of average length $1/(1 - \zeta)$. Then:*

1. *The conditions under which mean long-run GDP growth is greater in a risky than in a safe equilibrium are independent of ζ, and are the same as those in Proposition 4.1.*

2. *The shorter the average cool-off period $1/(1 - \zeta)$, the higher the mean long-run GDP growth in a RSE.*

The reason why the growth-enhancing properties of risk taking–stated in Proposition 4.1–are independent of ζ is that during the cool-off phase the economy grows at the same rate as in a safe equilibrium. Part 2 makes the important point that the faster risk-taking resumes in the wake of crisis, the higher will be mean long-run growth.

C   Model Simulations.

The behavior of the model economy is determined by nine parameters: $u, \delta, \alpha, \theta, h, \beta, \mu_w, \mu$ and γ. We will set the probability of crisis $1 - u$, the world interest rate r and the share of N-inputs in T-production α equal to some empirical estimates. Then, given the values of u, δ and α, we determine the feasible set for the degree of contract enforceability h, the N-Sector's cash flow-to-sales ratio $(1 - \beta)$ and total factor productivity in the N-sector θ, such that both an RSE and an SSE exist. The values of the crisis costs μ_w and μ are irrelevant for the existence of equilibria. The generosity of the bailouts γ is relevant only in Section 6, on the anything-goes regime. The admissible parameter set is determined by the following conditions.

- $h < \delta^{-1}$: necessary for borrowing constraints to bind in the safe equilibrium.

- $u > h\delta$: necessary for borrowing constraints to bind in the risky equilibrium.

- $\beta > \underline{\beta} = h\delta/u$: necessary and sufficient for positive prices: $\phi^r < 1$, $\phi^s < 1$.

- $\theta > \underline{\theta}^s(\delta, h, \alpha, \beta)$: necessary and sufficient for the SSE's rate of return on equity to be larger than the risk-free rate

- $\theta < \overline{\theta}(\delta, h, u, \alpha, \mu_w, \beta)$: necessary and sufficient for default in low price state

- $\theta > \underline{\theta}(\delta, h, u, \alpha, \beta)$: necessary and sufficient for the RSE's rate of return on equity to be larger than the risk-free rate.

- $\beta < \overline{\beta}(\delta, h, u, \alpha, \mu_w)$: necessary and sufficient for $\overline{\theta} > \underline{\theta}$.

- $u > \underline{u}$: sufficient for no-diversion in a risky equilibrium.

- $\gamma < \overline{\gamma}$: necessary for a black-hole equilibrium. The bounds are:

$$\underline{\theta}^s \equiv \left[\frac{1}{\beta\delta}\right]^{\frac{1}{\alpha}} \left[\frac{1-\beta}{1-h\delta}\right]^{\frac{1-\alpha}{\alpha}} \tag{44}$$

$$\underline{\theta} \equiv \left[\left[\frac{1}{\delta}-\frac{h}{u}+h\right]\left[\frac{1-\beta}{1-h\delta/u}\right]^{1-\alpha}\frac{1}{u\beta}\right]^{\frac{1}{\alpha}}, \quad \overline{\theta} \equiv \left[\frac{1}{\beta}\left[\frac{1-\beta}{\beta-h\delta/u}\right]^{1-\alpha}\left[1-\frac{\mu_w}{1-h\delta}\right]^{1-\alpha}\frac{h}{u}\right]^{\frac{1}{\alpha}} \tag{45}$$

$$\overline{\beta} = \underline{\beta} + \left[1-\frac{\mu_w}{1-h\delta}\right]\left[\frac{1}{h\delta}-\frac{1}{u}+1\right]^{\frac{-1}{1-\alpha}}\left[\frac{1}{1-h\delta/u}\right]^{-1} \tag{46}$$

$$\max\left\{\beta\theta^\alpha\left[\frac{1-h\delta/u}{1-\beta}\right]^{1-\alpha}-\frac{h}{u[1-u]}, \ \beta\theta^\alpha\left[\frac{\beta-h\delta/u}{1-\beta}\right]^{1-\alpha}\left[1-\frac{\mu_w}{1-h\delta}\right]^{\alpha-1}-\frac{h}{1-u}\right\} < 0 \ \forall u > \underline{u} \tag{47}$$

$$\overline{\gamma} \equiv \max\left\{\frac{h\alpha}{[1-\lambda]}\frac{\phi}{1-\phi}\frac{1}{[\theta\phi]^\alpha}, \ \alpha\left[\theta^\alpha\phi^{\alpha-1}-h\right]\frac{\theta^\alpha\phi^{\alpha+1}}{1-\phi}, \ \frac{1}{[1-\lambda][1-\lambda\overline{\varepsilon}\delta]}\right\}, \quad \phi \equiv \frac{1-\beta}{1-h\delta} \tag{48}$$

In the simulations, the baseline crisis probability is set equal to 5%, slightly higher than 4.13%, the probability of a systemic crisis in emerging markets (Ranciere, et.al., 2008); α is calibrated in reference to the use of non-tradable goods as inputs in tradables production, using Mexico's input-output table. We choose h, β and θ so that: (i) both an RSE and an SSE exist for the range $u \in [0.91, 1]$, and (ii) we obtain plausible values for the growth rates along a safe economy and along a lucky path. In the baseline case: $h = 0.76$, $\theta = 1.65$, $\beta = 0.8$ and $u = 0.95$. These parameters imply a safe GDP growth rate of $(1+\gamma^s) = (1-\beta)^\alpha \frac{\theta}{1-h\delta} = 3.8\%$ and a lucky GDP growth rate of $(1+\gamma^l) = (1-\beta)^\alpha \left(\frac{\theta}{1-h\delta u^{-1}}\right)^\alpha = 8.7\%$. We choose the financial distress costs of crises $l^d = 1-\frac{\mu_w}{1-\beta}$ so that the cumulative decrease of GDP during a crisis episode is 13%, which is the mean value in the sample considered by Tornell and Westermann (2002). In the model, the cumulative decrease in GDP growth during a crisis episode is $(1+\gamma^{cr})^2 = \left[\frac{\mu_w}{1-\beta}\right]^\alpha (\theta^2\phi^l\phi^s)^\alpha$. Using the baseline case $h = 0.76$, $\theta = 1.65$, and $\alpha = 0.35$ we get that $(1+\gamma^{cr})^2 = (1-0.13)$ if $\left[\frac{\mu_w}{1-\beta}\right] = 0.45$. Thus, we set conservatively $l^d = 0.7$. In the baseline case, the level of bankruptcy costs is free. The discount rate $\delta = 1/(1+r)$ is set equal to 0.925.

Finally, the expected discounted sum of consumption is finite in the SSE provided $\delta[\theta\phi^s]^\alpha < 1$; and it is finite in the RSE provided $\left[\theta\phi^l\right]^\alpha u\delta + \left[\theta^2\phi^l\phi^c\right]^\alpha[1-u]\delta^2 < 1$. These two conditions impose

an upper bound on α.[36] Given the baseline parameter values we have chosen, these two conditions hold if $\alpha < 0.6$. Summing up:

Parameters	baseline value	range of variation	sources
Probability of crisis	$1 - u = 0.05$		Ranciere, et.al. (2008)
Intensity of N-inputs in T-production	$\alpha = 0.35$	$[0.2, 0.6]$	Input-ouput Table for Mexico $(Source: \; INEGI)$
Financial distress costs	$l^d = 70\%$	$[30\%, 99\%]$	Tornell-Westermann (2002)
Bankruptcy costs	$l^b = 100\%$	$[30\%, 100\%]$	
N-sector Productivity	$\theta = 1.6$		
Contract enforceability	$h = 0.76$		
Cash Flows/Sales in N-Sector	$1 - \beta = 20\%$		
Discount Rate	$\delta = 0.925$		

D   Proofs and Derivations.

Proof of Proposition 3.1.   First, we determine the conditions on returns $\frac{\beta \theta p^e_{t+1}}{p_t}$ that make the strategy of Proposition 3.1 optimal for an individual entrepreneur, given that all other entrepreneurs follow the equilibrium strategy: borrow up to the limit (i.e., $b_t p_{t+1}[1 + \rho_t] = h[w_t + b_t]$), invest all funds in the production of N-goods ($p_t k_t = w_t + b_t$, $s_t = 0$), and never default. We then determine the parameter conditions under which the price sequences that result if all entrepreneurs follow the equilibrium strategy, generate **a** high enough return to validate the strategy of an individual entrepreneur.

Given that all other entrepreneurs follow the equilibrium strategy, crises never occur and prices are deterministic: $u_{t+1} = 1$ and $E_t(p_{t+1}) = p_{t+1}$. Thus no bailout is expected. First, since no bailout is expected, lenders will get repaid zero with any plan that leads to diversion. Hence, lenders only fund plans where the no-diversion condition holds. Second, an entrepreneur has no incentives to deviate an issue T-debt. Since competitive risk-neutral lenders have to break even: the interest rate offered on N-debt is $1 + \rho^N = [1 + r]/E_t(p_{t+1})$, while that on T-debt is $\rho^T = r$. Thus, the expected interest costs are the same under both types of debt: $[1 + \rho^N]E_t(p_{t+1}) = 1 + \rho^T = 1 + r$. Hence, the borrowing limits are the same under both types of debt, and so there is no incentive to issue T-debt. It follows that if all other firms choose a safe plan, the payoff of a safe plan is the

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[36]This result holds because the two boundness conditions are strictly satisfied for $\alpha = 0$ and their respective LHSs are strictly increasing in α.

solution to the following problem

$$\max_{b_{i,t},k_{i,t+1},l_{i,t+1}} Z_{i,t} = \left\{ E_t(p_{t+1})\Theta_{t+1}k_{i,t+1}^{\beta}l_{i,t+1}^{1-\beta} - b_{i,t}[1+r] - v_{t+1}l_{i,t+1} \right\}, \text{ subject to} \quad (49)$$

$$p_t k_{i,t} \leq w_{i,t} + b_{i,t} - s_{i,t}, \quad b_{i,t}[1+r] \leq h[w_t + b_t], \quad \pi_{i,t+1}^s \geq 0,$$

where prices and the wage are taken as given. Suppose for a moment that $\frac{\beta\theta E_t(p_{t+1})}{p_t}$ is high enough so that it is optimal to borrow up to the limit allowed by the no-diversion condition ($b_t[1+r] = h[w_{i,t} + b_{i,t}]$), and not store (so that $p_t k_{i,t+1} = w_{i,t} + b_{i,t}$). It follows that the first order conditions are

$$\frac{\partial Z_{i,t}}{\partial k_{i,t+1}} = E_t(p_{t+1})\Theta_{t+1}l_{i,t+1}^{1-\beta}k_{i,t+1}^{\beta-1}\beta - hp_t \geq 0, \quad \frac{\partial Z_{i,t}}{\partial l_{i,t+1}} = p_{t+1}\Theta_{t+1}l_{i,t+1}^{-\beta}k_{i,t+1}^{\beta}[1-\beta] - v_{t+1} \geq 0$$

$$(50)$$

Notice that π_{t+1}^s is concave in $k_{i,t+1}$ because $\beta < 1$. Since in a SSE all entrepreneurs choose the same investment level, $\Theta_{t+1}k_{i,t+1}^{\beta-1} = \theta\bar{k}_{t+1}^{1-\beta}k_{i,t+1}^{\beta-1} = \theta$. Furthermore, since labor is inelastically supplied ($l^s = 1$), the equilibrium wage is

$$\widehat{v}_{t+1} = p_{t+1}\theta k_{t+1}[1-\beta] \qquad (51)$$

Substituting the equilibrium wage in (50) we have that in an SSE, the marginal return of capital is:

$$\frac{\partial Z_{i,t}}{\partial k_{i,t+1}}\bigg|_{\widehat{v}_{t+1},k_{i,t+1}=\bar{k}_{t+1}} = E_t(p_{t+1})\theta\beta - hp_t$$

Thus, if $\beta\theta E_t(p_{t+1}) > p_t h$ the solution to (49) entails borrowing and investing as much as allowed by the no-diversion condition: $b_{i,t} = [\frac{1}{1-\delta h} - 1]w_{i,t} \equiv [m^s - 1]w_{i,t}$. It follows that the payoff associated with the equilibrium strategy is

$$\pi_{t+1}^s = \beta\theta p_{t+1}k_{t+1} - h[w_t + b_t] = \left[\frac{\beta\theta p_{t+1}}{p_t} - h\right][w_t + b_t] = \left[\frac{\beta\theta p_{t+1}}{p_t} - h\right]m^s w_t \qquad (52)$$

In order for the above solution to be optimal, this return on equity must be greater than the storage return: $[\theta\beta p_{t+1}/p_t - h]m^s > 1+r$, which is equivalent to $\theta\beta p_{t+1}/p_t > 1+r$. To determine whether this condition is satisfied we need to endogenize prices. To do so we use (18) and (19), and find that in a SSE $\frac{p_{t+1}}{p_t} = (\theta\phi^s)^{\alpha-1}$. Therefore,

$$\frac{\beta\theta p_{t+1}}{p_t} > \frac{1}{\delta} \Leftrightarrow \beta\theta^{\alpha}(\phi^s)^{\alpha-1} > \frac{1}{\delta} \Leftrightarrow \theta > \underline{\theta}^s \equiv \left[\frac{1}{\beta\delta}\right]^{\frac{1}{\alpha}}\left[\frac{1-\beta}{1-h\delta}\right]^{\frac{1-\alpha}{\alpha}} \qquad (53)$$

We conclude that the SSE characterized in Proposition 3.1 exists if and only if (53) holds, and $\beta < \bar{\beta}$ so that prices are positive (i.e., $\phi^s < 1$). Finally, notice that, because there is a production externality, there exists also a degenerate equilibrium in which nobody invests because everyone believes others will not invest: $\theta\bar{k}_{t+1}^{1-\beta} = 0$.

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Proof of Proposition 3.2. The proof is in three parts. In part A we construct an RSE where two crises do not occur in consecutive periods. Then, in part B we show that two crises cannot occur in consecutive periods. Finally in Part C we prove that the present value of income is large enough to finance the bailouts via domestic taxation.

Part A. Consider an RSE where, if there is no crisis at t, prices next period can take two values as in (24). Meanwhile, if there is a crisis at t, there is a unique p_{t+1}. In a no crisis period a firm can choose three types of plans: a "risky plan" where a firm denominates debt in T-goods (i.e., with currency mismatch), will default if $p_{t+1} = \underline{p}_{t+1}$, and does not divert; a "safe plan" where a firm denominates debt in N-goods, will never default and does not divert; finally a "diversion plan" where the firm will divert all funds. We will construct an RSE in which all entrepreneurs choose the risky plan during every period, except when a crisis erupts, in which case they choose the safe plan. In a first step we determine the conditions under which a risky plan is preferred to a safe plan and to storage. In a second step we determine the conditions under which diversion plans are not optimal.

Step 1. Suppose for a moment that $\underline{p}_{t+1}$ is low enough so as to bankrupt firms with T-debt (we will determine below the parameter set under which this holds). Since in an RSE every firm issues T-debt, a bailout will be granted next period in the low price state. In this case lenders will be repaid in all states (either by the borrowers or by the bailout agency) and so they break-even if the interest rate is $\rho^T = r$. It follows that if all other firms choose a risky plan, the payoff of a risky plan is the solution to the following problem:

$$\max_{b_{i,t},k_{i,t+1},l_{i,t+1}} u\pi^r_{i,t+1}(\bar{p}_{t+1}) = u\left\{\bar{p}_{t+1}\Theta_{t+1}k^{\beta}_{i,t+1}l^{1-\beta}_{i,t+1} - b_{i,t}[1+r] - v_{t+1}l_{i,t+1}\right\}, \text{ subject to}$$

$$p_t k_{i,t} \leq w_{i,t} + b_{i,t}, \quad u[1+r]b_{i,t} \leq h[w_t + b_{i,t}], \quad \pi^r_{i,t+1}(\bar{p}_{t+1}) \geq 0, \quad \pi^r_{i,t+1}(\underline{p}_{t+1}) \tag{54}$$

where $(p_t, \underline{p}_{t+1}, \bar{p}_{t+1}, v_{t+1}, \Theta_{t+1})$ are taken as given. The first order conditions are

$$\frac{\partial u\pi^r_{i,t+1}}{\partial k_{i,t+1}} = u\bar{p}_{t+1}\Theta_{t+1}l^{1-\beta}_{i,t+1}k^{\beta-1}_{i,t+1}\beta - hp_t \geq 0, \quad \frac{\partial u\pi^r_{i,t+1}}{\partial l_{i,t+1}} = p_{t+1}\Theta_{t+1}l^{-\beta}_{i,t+1}k^{\beta}_{i,t+1}[1-\beta] - v_{t+1} \geq 0$$
$$\tag{55}$$

Notice that π^r_{t+1} is concave in $k_{i,t+1}$ because $\beta < 1$. Since in an RSE all entrepreneurs choose the same investment level, $\Theta_{t+1}k^{\beta-1}_{i,t+1} = \theta\bar{k}^{1-\beta}_{t+1}k^{\beta-1}_{i,t+1} = \theta$. Furthermore, the equilibrium wage is given by (51). Following the same steps as in the proof of Proposition 3.1, we have that if $u\frac{\beta\theta\bar{p}_{t+1}}{p_t} > h$, the solution to (54) entails borrowing up to the limit allowed by the no-diversion constraint: $b_t = [m^r - 1]w_t = [m^r h\delta/u]w_t$. Thus, the payoff is[37]

$$E_t\pi^r_{t+1} = \left[u\frac{\beta\theta\bar{p}_{t+1}}{p_t} - h\right]m^r w_t \tag{56}$$

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[37]To simplify notation we will omit the subscripts that indentify individual agents.

In order for a firm to choose a risky plan the following conditions must be satisfied: (i)$\underline{p}_{t+1}$ must be low enough so as to bankrupt firms with T-debt, otherwise a bailout next period would not be expected and firms would not be able to take on systemic risk; (ii) $\bar{p}_{t+1}$ must be high enough so as make the risky plan preferred both to storage and a to safe plan:

$$\pi_{t+1}^r(\underline{p}_{t+1}) < 0, \quad E_t\pi_{t+1}^r > w_t[1+r], \quad E_t\pi_{t+1}^r > E_t\pi_{t+1}^s. \tag{57}$$

Next we derive equilibrium returns and determine the parameter conditions under which (57) holds. Using the equations for internal funds (16), N-output (18) and prices (19), and noting that in an RSE the investment share ϕ_{t+1} equals ϕ^l if N-firms are solvent, while $\phi_{t+1} = \phi^c$ if they are insolvent, it follows that equilibrium returns are

$$\overline{R} \equiv \beta\theta\frac{\bar{p}_{t+1}}{p_t} = \beta\theta^\alpha\left[\frac{1}{\phi^l}\right]^{1-\alpha}, \quad \underline{R} \equiv \beta\theta\frac{\underline{p}_{t+1}}{p_t} = \beta\theta^\alpha\left[\frac{1}{\phi^l}\right]^{1-\alpha}\left[\frac{1-\phi^l}{1-\phi^c}\right]^{1-\alpha} \tag{58}$$

First, substituting (58) in $\pi^r(\underline{p}_{t+1}) = \beta\underline{p}_{t+1}q_{t+1} - L_{t+1} = \frac{\beta\theta\underline{p}_{t+1}}{p_t}m^rw_t - \frac{h}{u}m^rw_t$, it follows that the risky plan defaults in the low price state if and only if $\left[\underline{R} - \frac{h}{u}\right]m^rw_t < 0$:

$$\pi^r(\underline{p}_{t+1}) < 0 \Leftrightarrow \frac{h}{u} > \underline{R} \Leftrightarrow \frac{h}{u} > \beta\theta^\alpha\left[\frac{1}{\phi^l} - 1\right]^{1-\alpha}\left[\frac{1}{1-\phi^c}\right]^{1-\alpha} \tag{59}$$

Second, the risky plan is preferred to storage if and only if $[u\overline{R} - h]m^rw_t \geq w_t/\delta$:

$$E_t\pi^r \geq \frac{w_t}{\delta} \Leftrightarrow u\overline{R} - h \geq \frac{1}{\delta} - \frac{h}{u} \Leftrightarrow u\beta\theta^\alpha\left[\frac{1}{\phi^l}\right]^{1-\alpha} - h \geq \frac{1}{\delta} - \frac{h}{u} \tag{60}$$

Third, to derive $E_t\pi_{t+1}^s$, note that if an entrepreneur were to deviate and choose a safe plan, the interest rate it would have to offer is $1 + \rho^s = [1+r]/p_{t+1}^e$, and her borrowing constraint would be $b_t^n[1+\rho^s]p_{t+1}^e \leq h[w_t + b_t^n]$. Following the same steps as in the proof of Proposition 3.1, we have that $b_t^n = [m^s - 1]w_t$, and the payoff would be

$$E_t\pi_{t+1}^s = \left[\frac{u\beta\theta\bar{p}_{t+1}}{p_t} + \frac{[1-u]\beta\theta\underline{p}_{t+1}}{p_t} - h\right]m^sw_t \tag{61}$$

Thus, a risky plan is preferred to a safe one if and only if $[u\overline{R} - h]m^rw_t \geq [u\overline{R} + (1-u)\underline{R} - h]m^sw_t$, which is equivalent to[38]

$$E_t\pi_{t+1}^r > E_t\pi_{t+1}^s \Leftrightarrow u\overline{R} - h \geq \underline{R}[1 - h\delta/u][h\delta/u]^{-1} \tag{62}$$

Next, we verify that (59), (60) and (62) can hold simultaneously. Notice that the LHS of (60) and (62) are the same. Thus, (60) implies (62) if and only if $\frac{1}{\delta} - \frac{h}{u} > \underline{R}[1 - (h\delta/u)](h\delta/u)^{-1}$. Rewriting

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[38]Rewrite $[u\overline{R}-h]m^rw_t \geq [u\overline{R}+(1-u)\underline{R}-h]m^sw_t$ as $[u\overline{R}-h][m^r - m^s] \geq (1-u)\underline{R}m^s \Leftrightarrow [u\overline{R}-h][(1-u)\frac{h\delta}{u}m^rm^s] \geq (1-u)\underline{R}m^s \Leftrightarrow [u\overline{R}-h][m^r-1] \geq \underline{R}$.

$\frac{1}{\delta} - \frac{h}{u}$ as $[1 - (h\delta/u)]\delta^{-1}$, the condition becomes $[1 - (h\delta/u)]\delta^{-1} > \underline{R}[1 - h\delta/u][h\delta/u]^{-1}$. Since an RSE exists only if $u > h\delta$, we get $\delta^{-1} > \underline{R}[h\delta/u]^{-1}$ or equivalently $h/u > \underline{R}$. Since $h/u > \underline{R}$ is (59), we have that (60) is stronger than (62) if and only if (59) holds. Hence, we conclude that that if (59) and (60) hold, then (62) must hold.

We next determine the parameter set such that (59) and (60) hold simultaneously. Condition (60) holds if and only if

$$\theta \geq \underline{\theta}(\delta, h, u, \alpha, \beta) \equiv \left(\left[\frac{1}{\delta} - \frac{h}{u} + h \right] \left[\frac{1 - \beta}{1 - h\delta/u} \right]^{1-\alpha} \frac{1}{u\beta} \right)^{1/\alpha} \tag{63}$$

Note that (58) implies that

$$\underline{R}\theta^{-\alpha} = \beta \left[\left(\frac{1}{\phi^l} - 1 \right) \left(\frac{1}{1 - \phi^c} \right) \right]^{1-\alpha} = \beta \left[\frac{\beta - h\delta u^{-1}}{1 - \beta} \right]^{1-\alpha} \left[1 - \frac{\mu_w}{1 - h\delta} \right]^{\alpha - 1}$$

Thus, condition (59) holds if and only if

$$\theta < \overline{\theta}(\delta, h, u, \alpha, \mu_w, \beta) \equiv \left[\frac{1}{\beta} \left[\frac{1 - \beta}{\beta - h\delta/u} \right]^{1-\alpha} \left[1 - \frac{\mu_w}{1 - h\delta} \right]^{1-\alpha} \frac{h}{u} \right]^{\frac{1}{\alpha}} \tag{64}$$

In order for (63) and (64) to hold simultaneously it is necessary that $\overline{\theta} > \underline{\theta}$:

$$\frac{1}{\beta} \left[\frac{1 - \beta}{\beta - h\delta/u} \right]^{1-\alpha} \left[1 - \frac{\mu_w}{1 - h\delta} \right]^{1-\alpha} \frac{h}{u} > \left[\frac{1}{\delta} - \frac{h}{u} + h \right] \left[\frac{1 - \beta}{1 - h\delta/u} \right]^{1-\alpha} \frac{1}{u\beta}$$

$$\left[\frac{1}{\beta - h\delta/u} \right]^{1-\alpha} \left[1 - \frac{\mu_w}{1 - h\delta} \right]^{1-\alpha} > \left[\frac{1}{h\delta} - \frac{1}{u} + 1 \right] \left[\frac{1}{1 - h\delta/u} \right]^{1-\alpha}$$

The LHS is decreasing in β. It ranges from infinity, for $\beta \to \underline{\beta} \equiv h\delta/u$, to $L(1) = \left[\frac{1}{1 - h\delta/u} \right]^{1-\alpha} \left[1 - \frac{\mu_w}{1 - h\delta} \right]^{1-\alpha}$ for $\beta = 1$. Since $L(1)$ is lower than the RHS (because $u > h\delta$), it follows that there is a unique threshold $\overline{\beta}$ such that $\overline{\theta} > \underline{\theta}$ if and only if $\beta < \overline{\beta}$. The above condition implies that this upper bound on β is

$$\overline{\beta}(\delta, h, u, \alpha, \mu_w) = \underline{\beta} + \left[1 - \frac{\mu_w}{1 - h\delta} \right] \left[\frac{1}{h\delta} - \frac{1}{u} + 1 \right]^{\frac{-1}{1-\alpha}} \left[\frac{1}{1 - h\delta/u} \right]^{-1} \tag{65}$$

Summing up, given that parameters (δ, h, u, β) satisfy (11), condition (59) holds if and only if $\theta < \overline{\theta}$, while condition (60) holds if and only if $\theta > \underline{\theta}$. Furthermore, $\overline{\theta} > \underline{\theta}$ if and only if $\beta < \overline{\beta}$. Thus, we conclude that during a no-crisis period, equilibrium expected returns are such that an entrepreneur prefers the equilibrium risky plan over both storage and a safe plan (i.e., the conditions in (58) hold) if and only if $\theta \in (\underline{\theta}, \overline{\theta})$ and $\beta < \overline{\beta}$.

Consider next a crisis period. Given that all other entrepreneurs choose a safe plan, there can be no crisis and no bailout in the post-crisis period. Thus, an entrepreneur faces the same problem

as that in a safe symmetric equilibrium. It follows from Proposition 3.1 that she will find it optimal to choose a safe investment plan if and only if $\beta\theta p_{t+1}/p_t \geq \delta^{-1}$. This condition is equivalent to $\beta\theta^\alpha(\phi^s)^{\alpha-1} \geq \delta^{-1}$, which is implied by (60) because $u > h\delta$.

Step 2. We show that the risky plan is preferred to a diversion plan if u is large. Consider first a "risky diversion plan" in which an entrepreneur deviates and chooses T-debt: she incurs a cost $h[w_t + b_t^d]$ at t, and will be able to divert all funds at $t+1$ provided she will be solvent. This plan maximizes

$$D_{t+1}^r = u\bar{p}_{t+1}\Theta_{t+1}l_{t+1}^{1-\beta}k_{t+1}^\beta - uv_{t+1}l_{t+1} - h[w_t + b_t], \text{ subject to}$$

$$1 + \rho^d = \frac{1+r}{1-u}, \quad \pi_{t+1}^{\text{div}}(\bar{p}_{t+1}) \geq 0, \quad \pi_{t+1}^{\text{div}}(\underline{p}_{t+1}) < 0, \quad h[w_t + b_t] < b_t[1 + \rho^d]$$

The interest rate is $\frac{1+r}{1-u}$ because lenders must break-even by cashing the bailout next period, with probability $1 - u$. We will construct an upper bound on D^r and show that it is lower than $E\pi_{t+1}^r$ if u is large enough. To do so we consider a fictitious auxiliary problem under which (i) only the first two conditions in the above problem are considered; and (ii) a diverting firm pays a wage $\tilde{v}_{t+1}(b_t^d) = [1 - \beta]\bar{p}_{t+1}\Theta_{t+1}k_{t+1}^\beta$. Under this fictitious wage, the diverting firm will find it optimal to set its labor demand equal to one: $l_{t+1} = 1$, and so its payoff, denoted by $\overline{D}$, is $\overline{D}(b_t^d) = u\beta\bar{p}_{t+1}\Theta_{t+1}k_{t+1}^\beta(b_t^d) - h[w_t + b_t^d]$.

Next, we determine the values of u for which the solvency constraint $(\pi_{t+1}^{\text{div}}(\bar{p}_{t+1}) = \bar{p}_{t+1}\Theta_{t+1}l_{t+1}^{1-\beta}k_{t+1}^\beta - \tilde{v}_{t+1}(b_t^d)l_{t+1} - b_t^d\frac{1+r}{1-u} \geq 0)$ is violated if $b_t^d = b_t^r$

$$\pi_{t+1}^{\text{div}} = \frac{\beta\bar{p}_{t+1}\theta}{p_t}[w_t + b_t^r] - b_t^r\frac{1+r}{1-u} = \left[\frac{\beta\bar{p}_{t+1}\theta}{p_t}m^r - \frac{m^r - 1}{[1-u]\delta}\right]w_t = \left[\frac{\beta\bar{p}_{t+1}\theta}{p_t} - \frac{h}{u[1-u]}\right]m^r w_t$$

Using (58), we know that in an RSE $\beta\theta\frac{\bar{p}_{t+1}}{p_t} = \beta\theta^\alpha\left[\frac{1}{\phi^l}\right]^{1-\alpha}$. Since $1/\phi^l = \frac{1-h\delta/u}{1-\beta}$,

$$\pi_{t+1}^{\text{div}}(\bar{p}_{t+1}) < 0 \iff 0 > \beta\theta^\alpha\left[\frac{1-h\delta/u}{1-\beta}\right]^{1-\alpha} - \frac{h}{u[1-u]} \equiv G^r(u)$$

Recall that an RSE exists only if $u \in (h\delta, 1)$. Since $G^r(u)$ is continuous and $\lim_{u\to 1}G^r(u) = -\infty$, there exist a lower bound $\underline{u}^r > h\delta$ so that $G^r(u) < 0$ for all $u > \underline{u}^r$. It follows that if $u > \underline{u}^r$ the solvency constraint of the auxiliary diversion problem $(\pi_{t+1}^{\text{div}}(\bar{p}_{t+1}) \geq 0)$ is violated at the equilibrium debt level b_t^r. Since $\pi_{t+1}^{\text{div}}(\bar{p}_{t+1}) = \frac{\beta\bar{p}_{t+1}\theta}{p_t}[w_t + b_t^d] - \frac{1+r}{1-u}b_t^d$ is decreasing in b_t^d (because $\frac{\beta\bar{p}_{t+1}\theta}{p_t} < \frac{h}{u[1-u]} < \frac{1+r}{1-u}$ if $u > \underline{u}^r$), diversion must entail $b_t^d < b_t^r$. Lower debt in turn implies that the fictitious wage is lower than the actual equilibrium wage (i.e., the diverting firm gets a subsidy under the auxiliary problem), and so the payoff of the auxiliary problem $\overline{D}(b_t^d)$ is an upper bound on the diversion payoff. Finally, notice that if the diverting firm could borrow b_t^r, its payoff would be $\overline{D}(b_t^r) = [u\frac{\beta\bar{p}_{t+1}\theta}{p_t} - h]m^r w_t$, which equals the equilibrium payoff $E_t\pi_{t+1}^{r,nd}$. Since

$\overline{D}(b_t^d) < \overline{D}(b_t^r) = E_t \pi_{t+1}^{r,nd}$, a firm has no incentives to choose a risky diversion plan if $u > \underline{u}^r$

$$\underline{u}^r \text{ such that } \beta\theta^\alpha \left[\frac{1 - h\delta/u}{1 - \beta}\right]^{1-\alpha} - \frac{h}{u\left[1 - u\right]} < 0, \quad \forall u > \underline{u}^r. \tag{66}$$

Next, consider a "safe diversion plan" in which an entrepreneur chooses N-debt and will be solvent in both states next period. Under such plan the interest rate is $\frac{1+r}{1-u}$. Using the same argument we can show that there is a $\underline{u}^s$, such that if $u > \underline{u}^s$, the solvency constraint associated with the auxiliary problem (i.e., $0 \le \pi_{t+1}^{\text{div}}(\underline{p}_{t+1}) = \frac{\beta\underline{p}_{t+1}\theta}{p_t}[w_t + b_t^d] - b_t^d \frac{1+r}{1-u}$) does not hold at the debt level of a safe plan $b_t^s = [m^s - 1]w_t$. Thus, in an RSE, the payoff of a safe diversion plan is bounded above by the payoff of a safe no-diversion plan $E_t \pi_{t+1}^{s,nd}$. Since we have shown in Part A that $E_t \pi_{t+1}^{s,nd} < E_t \pi_{t+1}^{r,nd}$, it follows that a firm has no incentives to choose a safe diversion plan if $0 > \pi_{t+1}^{\text{div}}(\underline{p}_{t+1}) = \left[\frac{\beta\underline{p}_{t+1}\theta}{p_t}m^s - \frac{m^s - 1}{[1-u]\delta}\right]w_t = \left[\frac{\beta\underline{p}_{t+1}\theta}{p_t} - \frac{h\delta}{[1-u]\delta}\right]m^s w_t$. Using (58) we have that there are no incentives to choose a safe diversion plan if $u > \underline{u}^s$:

$$\underline{u}^s \text{ such that } \beta\theta^\alpha \left[\frac{1 - \phi^l}{\phi^l}\right]^{1-\alpha} \left[\frac{1}{1 - \phi^c}\right]^{1-\alpha} - \frac{h}{1 - u} < 0, \quad \forall u > \underline{u}^s. \tag{67}$$

We conclude that the risky equilibrium plan is preferred to a diversion plan if crisis are not frequent: $u > \max\{\underline{u}^r, \underline{u}^s\}$

$$\underline{u} \text{ s.t. } \max\left\{\beta\theta^\alpha \left[\frac{1 - \frac{h\delta}{u}}{1 - \beta}\right]^{1-\alpha} - \frac{h}{u\left[1 - u\right]}, \ \beta\theta^\alpha \left[\frac{\beta - \frac{h\delta}{u}}{1 - \beta}\right]^{1-\alpha} \left[1 - \frac{\mu_w}{1 - h\delta}\right]^{\alpha-1} - \frac{h}{1 - u}\right\} < 0 \ \forall u > \underline{u} \tag{68}$$

Part B. We show that two crises cannot occur in consecutive periods. Suppose to the contrary that if a crisis occurs at τ, firms choose risky plans at τ. We will show that it is not possible for firms to become insolvent in the low price state at $\tau + 1$ (i.e., $\pi(\underline{p}_{\tau+1}) < 0$), and so a bailout cannot be expected. It suffices to consider the case in which firms internal funds at $\tau + 1$ equal μ_w, and they undertake safe plans at $\tau + 1$, as this generates the lowest possible price $\underline{p}_{\tau+1}$. We will show that even in these extreme case it is not possible to generate $\pi(\underline{p}_{\tau+1}) < 0$. Along this path the N-investment share is $\phi_\tau = \tilde{\phi}^c := \mu_w m^r$ and $\phi_{\tau+1} = \phi^c := \mu_w m^s$. Thus,

$$\tilde{\pi}(\underline{p}_{\tau+1}) = \beta\underline{p}_{\tau+1}q_{\tau+1} - L_{\tau+1} = \left[\frac{\beta\theta\underline{p}_{\tau+1}}{p_\tau} - \frac{h}{u}\right]m^r \mu_w = \left[\beta\theta\frac{\alpha[1 - \phi^c]^{\alpha-1}[\theta\tilde{\phi}^c q_\tau]^{\alpha-1}}{\alpha[1 - \tilde{\phi}^c]^{\alpha-1}[q_\tau]^{\alpha-1}} - \frac{h}{u}\right]\tilde{\phi}^c$$

In order to get $\tilde{\pi}(\underline{p}_{\tau+1}) < 0$ it is necessary that

$$u\beta\theta^\alpha \frac{[1 - \phi^c]^{\alpha-1}[\tilde{\phi}^c]^{\alpha-1}}{[1 - \tilde{\phi}^c]^{\alpha-1}} < h \quad \Leftrightarrow \quad u\beta\theta^\alpha \left[\frac{1}{\tilde{\phi}^c}\right]^{1-\alpha} < h \left[\frac{1 - \phi^c}{1 - \tilde{\phi}^c}\right]^{1-\alpha} \tag{69}$$

Recall that a RSE requires that a risky plan be preferred to storage, i.e., condition (60): $u\beta\theta^\alpha \left[\frac{1}{\phi^l}\right]^{1-\alpha} \ge h + \frac{1}{\delta} - \frac{h}{u}$. Since $\frac{1}{\delta} - \frac{h}{u} > 0$ (because a necessary condition for an RSE is $u > h\delta$), condition (60)

implies $u\beta\theta^\alpha\left[\frac{1}{\phi^l}\right]^{1-\alpha} \geq h$. Notice also that $\phi^l \equiv [1-\beta]m^r > \mu_w m^r \equiv \tilde{\phi}^c$ because we have imposed financial distress costs of crisis: $w_{crisis} = \mu_w < 1 - \beta$. Combining these facts we have

$$u\beta\theta^\alpha\left[\frac{1}{\tilde{\phi}^c}\right]^{1-\alpha} > u\beta\theta^\alpha\left[\frac{1}{\phi^l}\right]^{1-\alpha} > h \tag{70}$$

Finally, notice that (70) contradicts (69) because $\left[\frac{1-\phi^c}{1-\tilde{\phi}^c}\right] > 1$. Hence, in an RSE it is not possible for agents to choose a risky plan during a crisis period.

Part C. Finally, we prove that bailout costs can be financed via domestic taxation (i.e., that (9) holds). We do so by showing that the expected discounted sum of entrepreneurs' profit and workers' income is greater than the bailout costs:

$$W^r = E_0\left(\sum_{t=0}^\infty \delta^t[[1-\alpha]y_t + \pi_t - T_t]\right) \geq 0$$

To simplify notation we assume, temporarily, that there is only one crisis (at time τ). It follows that profits and the bailout cost are:

$$\pi_t = \frac{\alpha}{1-\phi^l}\beta y_t - \frac{\alpha\phi^l}{1-\phi^l}\frac{h}{u}y_{t-1}, \qquad t \neq \{0, \tau, \tau+1\}$$
$$\pi_0 = \frac{\alpha}{1-\phi^l}\beta y_0, \quad \pi_\tau = 0, \quad \pi_{\tau+1} = \frac{\alpha}{1-\phi^l}\beta y_{\tau+1} - \frac{\alpha\phi^c}{1-\phi^c}hy_\tau \tag{71}$$

$$T(\tau) = L_{\tau-1} - \mu p_\tau q_\tau = \frac{\alpha}{1-\phi^l}\frac{h}{u}\phi^l y_{\tau-1} - \mu p_\tau q_\tau = \frac{\alpha}{1-\phi^l}\frac{h}{u}\phi^l y_{\tau-1} - \mu\frac{\alpha}{1-\phi^c}y_\tau \tag{72}$$

Replacing these expressions in (33) and using the market clearing condition $p_t q_t[1 - \phi_t] = \alpha y_t$, we get

$$
\begin{aligned}
W^r(\tau) &= (1-\alpha)y_o + \frac{\alpha\beta y_o}{1-\phi^l} + \sum_{t=1}^{\tau-1}\delta^t\left[[(1-\alpha)y_t + \frac{\alpha\beta y_t}{1-\phi^l} - \frac{\alpha\phi^l y_{t-1}}{1-\phi^l}\frac{h}{u}\right] + \delta^\tau\left[(1-\alpha)y_\tau + \frac{\mu\alpha y_\tau}{1-\phi^c} - \frac{\alpha\phi^l y_{\tau-1}}{1-\phi^l}\frac{h}{u}\right] \\
&\quad + \delta^{\tau+1}\left[(1-\alpha)y_{\tau+1} + \frac{\alpha}{1-\phi^l}\beta y_{\tau+1} - \frac{\alpha h\phi^c}{1-\phi^c}y_\tau\right] + \sum_{t=\tau+2}^\infty\delta^t\left[(1-\alpha)y_t + \frac{\alpha\beta}{1-\phi^l}y_t - \frac{\alpha\phi^l}{1-\phi^l}\frac{h}{u}y_{t-1}\right] \\
&= \sum_{t\neq\tau}\delta^t\left[(1-\alpha)y_t + \frac{\alpha}{1-\phi^l}\beta y_t - \frac{\alpha}{1-\phi^l}\frac{\delta h}{u}\phi^l y_t\right] + \delta^\tau\left[(1-\alpha)y_\tau + \mu\frac{\alpha}{1-\phi^c}y_\tau - \frac{\alpha\phi^c}{1-\phi^c}\delta h y_\tau\right] \\
&= \sum_{t\neq\tau}\delta^t y_t + \kappa^c y_\tau, \qquad \kappa^c := 1 - \alpha + \mu\frac{\alpha}{1-\phi^c} - \frac{\alpha}{1-\phi^c}\delta h\phi^c = 1 - \frac{\alpha[1-(\mu+\mu_w)]}{1-\phi^c}
\end{aligned}
$$

Notice that κ^c can be simplified as follows

$$\kappa^c = \alpha + \frac{\alpha}{1-\phi^c}(\mu - (1-\mu_w) + (1-\mu_w) - \delta h\phi^c) = \alpha + \frac{\alpha}{1-\phi^c}((1-\mu_w) - \delta h\phi^c) - \frac{\alpha[1-(\mu+\mu_w)]}{1-\phi^c}$$

Since $\frac{1}{1-\phi^c}((1-\mu_w) - \delta h\phi^c) = \frac{(1-\mu_w)(1-h\delta) - h\delta\mu_w}{1-h\delta-\mu_w} = \frac{1-h\delta-\mu_w}{1-h\delta-\mu_w} = 1$, it follows that $\kappa^c = 1 - \frac{\alpha[1-(\mu+\mu_w)]}{1-\phi^c}$.

If we allow for the possibility of multiple crises, then W^r is given by

$$W^r = E_0\sum_{t=0}^\infty \delta^t\kappa_t y_t, \qquad \kappa_t = \begin{cases} \kappa^c := 1 - \frac{\alpha[1-\mu-\mu_w]}{1-\phi^c} & \text{if } t = \tau_i, \\ 1 & \text{otherwise}; \end{cases} \tag{73}$$

where, τ_i denotes a time of crisis. In order to compute this expectation, we need to calculate the limit distribution of $\kappa_t y_t \equiv \tilde{y}_t$, which corresponds to T-output net of bankruptcy costs. To derive this limit distribution notice that $\frac{\tilde{y}_t}{\tilde{y}_{t-1}}$ follows a three-state Markov chain defined by:

$$\widetilde{T} = \begin{pmatrix} u & 1-u & 0 \\ 0 & 0 & 1 \\ u & 1-u & 0 \end{pmatrix}, \qquad \widetilde{G} = \begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} = \begin{pmatrix} (\theta\phi^l)^\alpha \\ \left[\theta\phi^l \frac{1-\phi^c}{1-\phi^l}\right]^\alpha \kappa_c \\ \left[\theta\phi^c \frac{1-\phi^l}{1-\phi^c}\right]^\alpha \frac{1}{\kappa_c} \end{pmatrix} \tag{74}$$

To derive W^r in closed form consider the following recursion

$$\begin{aligned} V(\tilde{y}_0, g_0) &= E_0 \sum_{t=0}^\infty \delta^t \tilde{y}_t = \tilde{y}_0 + \delta E_0 V(\tilde{y}_1, g_1) \\ V(\tilde{y}_t, g_t) &= y_t + \beta E_t V(\tilde{y}_{t+1}, g_{t+1}) \end{aligned} \tag{75}$$

Suppose that the function V is linear: $V(\tilde{y}_t, g_t) = \tilde{y}_t w(g_t)$, with $w(g_t)$ an undetermined coefficient. Substituting this guess into (75), we get $w(g_t) = 1 + \delta E_t g_{t+1} w(g_{t+1})$. Combining this condition with (74), it follows that $w(g_{t+1})$ satisfies

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \delta \begin{pmatrix} u & 1-u & 0 \\ 0 & 0 & 1 \\ u & 1-u & 0 \end{pmatrix} \begin{pmatrix} g_1 w_1 \\ g_2 w_2 \\ g_3 w_3 \end{pmatrix} \Rightarrow \begin{matrix} w_1 = \frac{1+(1-u)\delta g_2}{1-(1-u)\delta^2 g_2 g_3 - u\delta g_1} \\ w_2 = \frac{1+\delta g_3 - u\delta g_1}{1-(1-u)\delta^2 g_2 g_3 - u\delta g_1} \\ w_3 = \frac{1+(1-u)\delta g_2}{1-(1-u)\delta^2 g_2 g_3 - u\delta g_1} \end{matrix}$$

This solution exists and is unique if and only if $1 > g_1 \delta u + g_2 g_3 \delta^2 (1-u)$, or equivalently $1 > \left[\theta\phi^l\right]^\alpha \delta u + \left[\theta^2 \phi^l \phi^c\right]^\alpha \delta^2 (1-u)$.[39] Since at time 0 the economy is in the lucky state (i.e., $V(y_0, g_0) = w_1 y_0^l$) and since $g_2 g_3 = \left(\theta\phi^l\right)^\alpha (\theta\phi^c)^\alpha$, it follows that W^r is given by

$$W^r = \frac{1 + \delta(1-u)\left[\theta\phi^l \frac{1-\phi^c}{1-\phi^l}\right]^\alpha \kappa^c}{1 - \left[\theta\phi^l\right]^\alpha \delta u - \left[\theta^2 \phi^l \phi^c\right]^\alpha \delta^2 (1-u)} [(1-\phi^l)q_0]^\alpha \tag{76}$$

Finally, we show that W^r is positive. We have seen that if W^r converges, the denominator must be positive. The numerator is positive because $\phi^l < 1$ and $\phi^c < 1$. Hence, if the boundedness condition holds (i.e., $1 > \left[\theta\phi^l\right]^\alpha \delta u + \left[\theta^2 \phi^l \phi^c\right]^\alpha \delta^2 (1-u)$), then bailouts are financeable via domestic taxation.$\square$

Proof of Proposition 4.1. *Growth Limit Distribution.* Here, we derive the limit distribution of GDP's compounded growth rate $(\log(gdp_t) - \log(gdp_{t-1}))$ along the RSE characterized in Proposition 3.2. In this RSE, firms choose safe plans in a crisis period and resume risk-taking the period immediately after the crisis. It follows from (23), (27) and (28) that the growth process follows a

---

[39] Since the RHS is strictly increasing in α and $\delta u + \delta^2(1-u) < 1$, this boundedness condition holds if α is below a threshold.

three-state Markov chain characterized by

$$\Gamma = \begin{pmatrix} \log\left((\theta\phi^l)^\alpha\right) \\ \log\left((\theta\phi^l)^\alpha \frac{Z(\phi^c)}{Z(\phi^l)}\right) \\ \log\left((\theta\phi^c)^\alpha \frac{Z(\phi^l)}{Z(\phi^c)}\right) \end{pmatrix}, \qquad T = \begin{pmatrix} u & 1-u & 0 \\ 0 & 0 & 1 \\ u & 1-u & 0 \end{pmatrix} \tag{77}$$

The three elements of Γ are the growth rates in the lucky, crisis and post-crisis states, respectively. The element T_{ij} of the transition matrix is the transition probability from state i to state j. Since the transition matrix is irreducible, the growth process converges to a unique limit distribution over the three states that solves $T'\Pi = \Pi$. Thus, $\Pi = \left(\frac{u}{2-u}, \frac{1-u}{2-u}, \frac{1-u}{2-u}\right)'$, where the elements of Π are the shares of time that an economy spends in each state over the long-run. It then follows that the mean long-run GDP growth rate is $E(1+\gamma^r) = \exp(\Pi'\Gamma)$. It then follows from (26) and (29) that

$$\gamma^r > \gamma^s \Leftrightarrow \left(\frac{\mu_w}{1-\beta}\right) > \left(\frac{1 - h\delta u^{-1}}{1 - h\delta u}\right)^{1/(1-u)}$$

$$\Leftrightarrow l^d \equiv 1 - \frac{\mu_w}{1-\beta} < 1 - \left(\frac{1 - h\delta u^{-1}}{1 - h\delta u}\right)^{1/(1-u)}$$

Derivation of (31). Any solution to the Pareto problem is characterized by the optimal accumulation of N-goods that maximizes the discounted sum of T-production

$$\max_{\{d_t\}\in\mathcal{C}^1} \sum_{t=0}^\infty \delta^t d_t^\alpha, \quad \text{s.t.} \quad k_{t+1} = \begin{cases} \theta k_t - d_t & \text{if } t \geq 1 \\ q_0 - d_0 & \text{if } t = 0 \end{cases}, \quad d_t \geq 0, \ q_o \text{ given}$$

The Hamiltonian associated with this problem is $H_t = \delta^t[d_t]^\alpha + \lambda_t[\theta k_t - d_t]$. Since $\alpha \in (0,1)$, the necessary and sufficient conditions for an optimum are

$$0 = H_d = \delta^t\alpha[d_t]^{\alpha-1} - \lambda_t, \quad \lambda_{t-1} = H_k = \theta\lambda_t, \quad \lim_{t\to\infty} \lambda_t k_t = 0 \tag{78}$$

Thus, the Euler equation is

$$\hat{d}_{t+1} = [\delta\theta]^{\frac{1}{1-\alpha}} d_t = \theta\hat{\phi}d_t, \quad \hat{\phi} := [\delta\theta^\alpha]^{\frac{1}{1-\alpha}} \quad t \geq 1 \tag{79}$$

To get a closed form solution for d_t we replace (79) in the accumulation equation:

$$k_t = \theta^{t-1}k_1 - d_0 \sum_{s=0}^{t-2} \theta^{t-s-2}[\delta\theta]^{\frac{s+1}{1-\alpha}} = \theta^{t-1}\left[k_1 - d_0\hat{\phi}\frac{1-\hat{\phi}^{t-1}}{1-\hat{\phi}}\right] = \theta^{t-1}\left[k_1 - \frac{d_1}{\theta}\frac{1-\hat{\phi}^{t-1}}{1-\hat{\phi}}\right] \tag{80}$$

Replacing (79) and (80) in the transversality condition we get

$$0 = \lim_{t\to\infty} \delta^t\alpha[d_t]^{\alpha-1}k_t = \lim_{t\to\infty} \delta^t\alpha\left[[\delta\theta]^{\frac{t}{1-\alpha}}d_0\right]^{\alpha-1}\left[\theta^{t-1}k_1 - d_0\hat{\phi}\frac{1-\hat{\phi}^{t-1}}{1-\hat{\phi}}\right]$$

$$= \frac{\alpha d_0^{\alpha-1}}{\theta}\left[k_1 - d_0\hat{\phi}\frac{1}{1-\hat{\phi}}\right] \quad \text{iff} \quad \hat{\phi} < 1$$

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Since $k_1 = q_0 - d_0$, the bracketed term equals zero if and only if $\hat{d}_0 = [1 - \hat{\phi}]q_0$. The accumulation equation then implies that the unique optimal solution is $\hat{d}_t = [1 - \hat{\phi}]q_t$. $\square$

Proof of Proposition 5.2

Consider the value functions W^s and W^r given by (34) and (35), respectively, and notice that if $u = 1$, both are equal. Since W^s does not depend on u, we will prove the proposition by determining conditions under which $W^r_u := \partial W^r / \partial u|_{u=1}$ is negative. That is, an increase in crisis-risk improves the present value of consumption along a risky path. Let's denote

$$
L = 1 - \left[\theta\phi^l\right]^\alpha \delta u - \left[\theta^2 \phi^l \phi^s\right]^\alpha \delta^2 (1 - u), \quad T = \left(1 + \delta(1 - u)\left[\theta\phi^l \frac{1 - \phi^c}{1 - \phi^l}\right]^\alpha k_c\right)(1 - \phi^l)^\alpha,
$$

so that

$$
W^r = \frac{T}{L} q_0^\alpha, \quad \text{and} \quad W^r_u := \frac{\partial W^r}{\partial u}\bigg|_{u=1} = \frac{LT_u - L_u T}{L^2} q_0^\alpha. \tag{81}
$$

The derivatives $L_u := \partial L/\partial u|_{u=1}$ and $T_u := \partial T/\partial u|_{u=1}$ are

$$
L_u = -\delta(\theta\phi)^\alpha - \alpha\phi'\delta(\theta\phi)^{\alpha-1} + \left[\theta\phi^l\phi^c\right]^\alpha \delta^2
$$

$$
T_u = -\alpha\phi'[(1 - \phi)]^{\alpha-1} - \delta\left[\theta\phi\right]^\alpha (1 - \phi)^\alpha = (1 - \phi)^{\alpha-1}\left[-\alpha\phi' - \delta\left[\theta\phi\right]^\alpha \kappa_c(1 - \phi)\left(\frac{1 - \phi^c}{1 - \phi}\right)^\alpha\right],
$$

where $\phi = \phi^s = \phi^l|_{u=1}$ and $\phi' = \partial\phi^l/\partial u|_{u=1}$. It then follows from (81) that:

$$
\frac{L^2}{q_0^\alpha} W^r_u = (D - 1)(1 - \phi)^{\alpha-1}(\alpha\phi' + D(1 - \phi)\kappa_c\left(\frac{1 - \phi^c}{1 - \phi}\right)^\alpha) + (1 - \phi)^\alpha(D + \alpha\phi'\frac{D}{\phi} - D\delta(\theta\phi^c)^\alpha)
$$

$$
\frac{L^2}{q_0^\alpha(1-\phi)^{\alpha-1}} W^r_u = (D - 1)(1 - \phi)^{\alpha-1}(\alpha\phi' + D(1 - \phi)\kappa_c\left(\frac{1 - \phi^c}{1 - \phi}\right)^\alpha) + (1 - \phi)^\alpha(D + \alpha\phi'\frac{D}{\phi} - D\delta(\theta\phi^c)^\alpha),
$$

with $D = \delta(\theta\phi)^\alpha$. Note that $D < 1$ because $\delta < \delta_{\max} := (\theta\phi)^{-\alpha}$ is necessary for W^s in (34) to be well defined. After some algebraic manipulations, the expression above can be expressed as follows:

$$
\frac{L^2 W^r_u}{q_0^\alpha(1-\phi)^{\alpha-1}} = \underbrace{\alpha\phi'(\frac{D}{\phi} - 1)}_{\text{Pareto gains}} + \underbrace{(1 - D)(1 - \kappa_c(\frac{1 - \phi^c}{1 - \phi^l})(1 - \phi))}_{\text{Bankruptcy costs}} + \underbrace{(1 - \phi)^\alpha D\delta(\theta)^\alpha((\phi)^\alpha - (\phi^c)^\alpha)}_{\text{Financial distress costs}}
$$

$$\tag{82}$$

Since $D = \delta(\theta\phi)^\alpha = (\phi^{po})^{1-\alpha}\phi^\alpha$, the first term can be rewritten as $\alpha\phi'((\frac{\phi^{po}}{\phi})^{1-\alpha} - 1)$, which is negative if and only if $\phi < \phi^{po}$ because ϕ' is negative (a reduction in u increases leverage). Since the two other terms are positive, a necessary condition for $W^r > W^s$ is $\phi < \phi^{po}$, where ϕ^{po} is the Pareto optimal investment share. This establishes part (i) of the proposition. To prove part (ii), consider first the case in which financial distress costs are small ($\mu_w \to 1 - \beta$). In this case the last

term in (82) is zero as $\phi^c = \phi$. Thus, W_u^r is negative if and only if:

$$\frac{T^2 W_u^r}{q_0^\alpha (1-\phi)^{\alpha-1}} = \alpha\phi'\left(\left[\frac{\phi^{po}}{\phi}\right]^{1-\alpha} - 1\right) + (1-D)\left(\frac{\alpha[\beta-\mu]}{1-\phi}\right)(1-\phi) < 0$$

$$\Leftrightarrow \quad \frac{\mu}{\beta} > 1 + \beta^{-1}\phi'\left(\left[\frac{\phi^{po}}{\phi}\right]^{1-\alpha} - 1\right)(1-D)^{-1} \tag{83}$$

A necessary and sufficient condition for (83) is:

$$\phi < \phi^{po} \text{ and } \mu > \mu^* := \max\left\{0, \ 1 + \frac{\phi'}{\beta(1-D)}\left(\left[\frac{\phi^{po}}{\phi}\right]^{1-\alpha} - 1\right)\right\}\beta. \tag{84}$$

If in addition δ is large enough, this condition holds for any $\mu \geq 0$. To see this observe that $\lim_{\delta \to \delta_{\max}} D = 1$, and D is continuous and increasing in δ.

Second, consider the case $\mu_w < 1 - \beta$, but let the discount factor $\delta \to \delta_{\max}$, so that $D \to 1$. In this case the second term in (82) converges to zero. Therefore, $W_u^r < 0$ is equivalent to

$$(1-\phi)^\alpha(1 - (\frac{\phi^c}{\phi})^\alpha) < -\alpha\phi'\left(\left[\frac{1}{\phi}\right]^{1-\alpha} - 1\right). \tag{85}$$

A necessary and sufficient condition for (85) is:

$$\phi < \phi^{po} \text{ and } l^d > \widetilde{l^d} \equiv 1 - \left(1 + \alpha\phi'((\frac{1}{\phi})^{1-\alpha} - 1)(1-\phi)^{-\alpha}\right)^{1/\alpha}$$

To see this, develop (85):

$$(1 - (\frac{\phi^c}{\phi})^\alpha) < -\alpha\phi'((\frac{1}{\phi})^{1-\alpha} - 1)(1-\phi)^{-\alpha}$$

$$\left(\frac{\mu_w}{1-\beta}\right)^\alpha < 1 + \alpha\phi'((\frac{1}{\phi})^{1-\alpha} - 1)(1-\phi)^{-\alpha}$$

$$l^d > 1 - \left(1 + \alpha\phi'((\frac{1}{\phi})^{1-\alpha} - 1)(1-\phi)^{-\alpha}\right)^{1/\alpha}$$

Intuition for Proposition 6.1. To see why guarantees are necessary, observe that if they were absent, then the negative-NPV ε-technology would not be funded. Since the ε-technology yields less than the riskless return in all states (because $\bar\varepsilon < 1 + r$), any profitable strategy for an ε-entrepreneur involves the issuance of catastrophe bonds. However, no lender is willing to buy catastrophe bonds from an ε-entrepreneur because they will never be repaid: in the good state the lender is promised zero, and in the bad state the ε-entrepreneur will go bust. Only θ-agents will be funded and, as in Section 3, they will choose a safe plan with no insolvency risk. Thus, in the absence of bailout guarantees, neither the catastrophe bonds nor the inferior ε-technology are used in equilibrium. They are irrelevant for the allocation of resources.

With bailout guarantees, the no-diversion condition does not generate a borrowing constraint for an ε-entrepreneur who issues catastrophe bonds. Since an ε-entrepreneur promises to repay zero in the good state and will go bust in the bad state, the no-diversion constraint is $0 < hb_t^{c,\varepsilon}$; this constraint does not require equity investment (i.e., $w_t > 0$) and is satisfied for any borrowing level. Instead, the borrowing limit to an ε-entrepreneur equals the expected present value of the bailout. A bailout next period will be granted to the creditors of an ε-entrepreneur if $\varepsilon_{t+1} = 0$. In that state we have $y_{t+1}^{\varepsilon,nd} = 0$ and $y_{t+1}^{\theta,nd} = y_t^\theta(\theta\phi^s)^\alpha$, so total lending to the ε-sector is $b_t^{c,\varepsilon} = \delta(1-\lambda)\gamma y_t^\theta(\theta\phi^s)^\alpha$. Therefore, the equilibrium final goods production when using the ε-technology is $y_{t+1}^{\varepsilon,nd} = \varepsilon_{t+1}I_t^\varepsilon = \varepsilon_{t+1}\delta(1-\lambda)\gamma y_t^\theta(\theta\phi^s)^\alpha$. Because the ε-entrepreneur will make zero debt repayments in all states, his expected payoff is positive.

A θ-entrepreneur believes that all other θ-entrepreneurs will not default next period and that ε-entrepreneurs will default if $\varepsilon_{t+1} = 0$. Thus, she expects that the next period price p_{t+1} will be unique and that a bailout will be granted only if $\varepsilon_{t+1} = 0$. Given these expectations, she chooses whether to issue standard bonds or catastrophe bonds as well as whether or not to implement a diversion scheme. The proof shows that if conditions (44) and (48) hold, then a θ-entrepreneur finds it optimal to issue standard debt and not to implement a diversion scheme. To see the intuition note that a θ-agent with no diversion plan is indifferent between both types of bonds. Under standard bonds, the best plan of a θ-agent is the same as that in the safe equilibrium characterized by Proposition 3.1: all debt is indexed to p_{t+1}, and borrowing constraints bind with $b_t^s = [m^s - 1]w_t$. If the θ-agent issues catastrophe bonds and will repay in the $\varepsilon = 0$ state, then lenders will require an interest rate no smaller than ρ_t^c, defined by

$$1 + \rho^c = \frac{1+r}{1-\lambda}. \tag{86}$$

The borrowing limit is then determined by the no-diversion condition

$$[1-\lambda][1+\rho^c]b_t^{c,\theta} \le h[w_t + b_t^{c,\theta}]. \tag{87}$$

Since $[1-\lambda][1+\rho^c] = 1 + r$ (by (86)), condition (87) implies that the borrowing constraint with catastrophe bonds is $b_t^{c,\theta} \le \frac{h\delta}{1-h\delta}w_t$, which is the same as with standard debt: $b_t^s = [m^s - 1]w_t = \left[\frac{1}{1-h\delta} - 1\right]w_t$. Furthermore, expected debt repayments are the same: $[1-\lambda][1+\rho^c]b_t^{c,\theta} = [1+r]b_t^s$. Thus, in the absence of default, a θ-entrepreneur is indifferent between both types of bonds—a result similar to the Modigliani–Miller theorem.

Now consider plans with catastrophe bonds that lead to default next period (these include diversion plans and no-diversion plans with excessive promised repayment). Under such plans, lenders will receive nothing if $\varepsilon_{t+1} = \bar\varepsilon$ or the bailout if $\varepsilon_{t+1} = 0$. Thus, they lend up to the present

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value of the bailout: $b_t^{c,\theta,def} = \delta[1-\lambda]\gamma y_{t+1}^\theta$. Here is where the restriction on the bailout's generosity kicks in: if $\gamma < \overline{\gamma}$, the borrowing limit under a default plan is lower than under a no-default plan and therefore generates less expected profit than the equilibrium plan, as shown in the proof of Proposition 6.1.

Finally, we verify the fiscal solvency of the bailout agency. A bailout occurs with probability $1 - \lambda$, and so if we assume that, starting at $t = 1$, the non-diverting part of the economy can be taxed in a lump-sum way, then fiscal solvency requires:

$$\sum_{t=0}^{\infty} \delta^{t+1}[(1-\alpha)y_{t+1}^{\theta,nd} + \pi_{t+1}^{\theta,nd} + \pi_{t+1}^{\varepsilon,nd}] \geq E\left(\sum_{t=0}^{\infty}\delta^{t+1}\Gamma_{t+1}\right). \tag{88}$$

The proof shows that this inequality holds if and only if γ is low enough.

Proof of Proposition 6.1. Throughout we assume that the returns condition (53) is satisfied. *θ-entrepreneurs.* In the black-hole equilibrium (FBE) we construct, all θ-entrepreneurs issue standard bonds and never default. Meanwhile, ε-entrepreneurs default if $\varepsilon_{t+1} = 0$. Thus, each θ-entrepreneur expects next period a unique price p_{t+1} and that a bailout will be granted if and only if $\varepsilon_{t+1} = 0$. Given these expectations, a θ-entrepreneur's problem is to choose whether to issue standard bonds or catastrophe bonds, and whether to implement a diversion scheme or not. We will show that if the bailout is not too generous, a θ-entrepreneur has no incentives to deviate from the equilibrium.

First, if a θ-entrepreneur issues standard bonds and will never default, her borrowing limit is $b_t^s = [m^s - 1]w_t$ and and her expected profits are the same as those of the equilibrium safe plan of Proposition 3.1, given by (52). Second, consider plans with catastrophe bonds that will not default. Lenders require an interest rate no smaller than

$$1 + \rho^c = 1/[1 - \lambda]\delta. \tag{89}$$

To prevent diversion lenders lend up to an amount that satisfies the no-diversion condition

$$[1 - \lambda][1 + \rho^c]b_t^{c,nd} \leq h[w_t + b_t^{c,nd}] \tag{90}$$

Since $[1 - \lambda][1 + \rho^c] = 1 + r$ (by (89)), the no-diversion condition (90) implies that the borrowing constraint with catastrophe bonds is $b_t^{c,nd} \leq \frac{h\delta}{1-h\delta}w_t$. Notice that this borrowing limit is the same as the one under the equilibrium strategy with standard debt: $b_t^s = [m^s - 1]w_t$. Furthermore, the expected debt repayments are the same under both types of debt (i.e., $b_t^s[1+\rho^s] = b_t^{c,nd}[1-\lambda][1+\rho^c]$). Thus, the expected profits are the same under both types of debt. Hence, conditional on no default, the θ-entrepreneur has no incentives to deviate from the FBE.

Third, consider plans where the θ-entrepreneur issues catastrophe bonds and will default next period (in the $\varepsilon_{t+1} = 0$ state). Since catastrophe bonds promise to repay only in the $\varepsilon_{t+1} = 0$ state, these plans include both diversion plans and no-diversion plans with an excessive promised repayment that will make the firm insolvent. Under both plans, lenders are willing to lend up to the present value of the bailout that they will receive in the $\varepsilon_{t+1} = 0$ state

$$b_{i,t}^{c,def} = \delta[1 - \lambda]\Gamma_{i,t+1} \tag{91}$$

Since the bailout will be $\Gamma_{i,t+1} = \gamma y_{t+1}^\theta$, condition $\gamma < \overline{\gamma}'$ in Proposition **??** implies that the borrowing limit for plans that lead to default is lower than the limit for non-defaulting plans

$$
\begin{aligned}
b_{i,t}^{c,def} &= \delta[1-\lambda]\gamma y_{t+1}^\theta < [m^s - 1]w_{i,t} = b_{i,t}^s \\
&\Leftrightarrow \quad \gamma < \overline{\gamma}' = \frac{[m^s - 1]}{\delta[1-\lambda]}\frac{w_t}{y_{t+1}^\theta} = \frac{[m^s-1]}{\delta[1-\lambda]}\frac{[1-\beta]\alpha}{1-\phi}\frac{1}{[\theta\phi]^\alpha}
\end{aligned}
\tag{92}
$$

This bound is time-invariant because along the equilibrium path $\frac{w_t}{y_{t+1}^\theta}$ is constant:

$$\frac{w_t}{y_{t+1}^\theta} = \frac{w_t}{y_t}\frac{y_t}{y_{t+1}^\theta} = \frac{(1-\beta)p_t q_t}{\frac{(1-\phi)p_t q_t}{\alpha}}\cdot\frac{1}{[\theta\phi]^\alpha} = \frac{[1-\beta]\alpha}{1-\phi}\cdot\frac{1}{[\theta\phi]^\alpha} \tag{93}$$

Consider a no-diversion plan with catastrophe bonds that leads to default. Under such plan a θ-entrepreneur borrows up to $b_{i,t}^{c,def}$, promises an interest rate $1/[1-\lambda]\delta$, and will become insolvent if $\varepsilon_{t+1} = 0$. Under this deviation a θ-entrepreneur avoids repaying debt altogether, but it sacrifices profits in the $\varepsilon_{t+1} = 0$ state. The requirement that the firm be insolvent in the $\varepsilon_{t+1} = 0$ state, implies that the maximum payoff under this deviation is $\lambda\Gamma_{i,t+1}$ (because the highest revenue consistent with insolvency in the $\varepsilon_{t+1} = 0$ state is $b_{i,t}^{c,def}[1+\rho^c] = \Gamma_{i,t+1}$).

$$E\overline{\pi}_{i,t+1}^{c,def} \le \lambda\Gamma_{i,t+1} = \lambda\gamma y_{t+1}^\theta = \lambda\gamma\frac{y_{t+1}^\theta}{y_t^\theta}y_t^\theta = \lambda\gamma\frac{y_t^\theta}{[\theta\phi]^\alpha} = \lambda\gamma\frac{1}{[\theta\phi]^\alpha}\frac{1-\phi}{[1-\beta]\alpha}w_t$$

The last equality follows from (93). Comparing this upper bound with the equilibrium payoff in (52), we find that this deviation is not profitable provided the generosity of the guarantee is below $\overline{\gamma}''$

$$
\begin{aligned}
E\pi_{i,t+1}(b_{i,t}^{c,def}) \quad &< \quad \pi_{i,t+1}(b_{i,t}^s) \iff \frac{\lambda\gamma}{[\theta\phi]^\alpha}\frac{[1-\phi]w_t}{[1-\beta]\alpha} < \left[\theta^\alpha\phi^{\alpha-1} - h\right]\frac{\phi w_t}{1-\beta} \\
&\iff \quad \gamma < \overline{\gamma}'' \equiv \alpha\left[\theta^\alpha\phi^{\alpha-1} - h\right]\frac{\theta^\alpha\phi^{\alpha+1}}{1-\phi}
\end{aligned}
\tag{94}
$$

Consider next diversion plans with catastrophe bonds. In a diversion plan the entrepreneur incurs a cost $h[w_t + b_{i,t}^c]$ at t, and is able to divert funds at $t+1$ provided she is solvent. Under such plan her borrowing limit is $b_{i,t}^{c,def}$ in (91). Using the same argument as the one in the proof of

Proposition 3.2, one can show that this deviation is not profitable because the debt ceiling under diversion is lower than under the equilibrium strategy $(b_{i,t}^{c,def} < b_{i,t}^{s})$, which is implied by $\gamma_i < \overline{\gamma}'_i$. In sum, $\theta-$entrepreneurs issue standard debt, do not divert and invest according to Proposition 3.1 if the bailout is not too generous: $\gamma < \min\{\overline{\gamma}', \overline{\gamma}''\}$.

$\varepsilon-entrepreneurs.$ Since the ε-technology has negative NPV, ε-agents find it profitable only to issue catastrophe bonds. In the presence of bailout guarantees, lenders are willing to buy these catastrophe bonds. Given the expected bailout $\Gamma_{i,t+1}$, lenders are willing to lend to each ε-agent up to an amount $b_{it}^c = \delta[1 - \lambda]\Gamma_{i,t+1}$ at a rate ρ_t^c (in (89)). At $t + 1$, if the good state realizes ($\varepsilon_{t+1} = \overline{\varepsilon}$), lenders will get zero–as promised–while if $\varepsilon_{t+1} = 0$ lenders will get the bailout $\Gamma_{it} = b_{it}^c[1 + \rho_t^c]$. It follows that an ε-agent will de-facto repay zero in all states of the world, and so he does not gain anything by implementing a diversion scheme. His expected payoff is $E\pi_{i,t+1}^\varepsilon = \lambda\overline{\varepsilon}b_{it}^c = \lambda\overline{\varepsilon}\frac{1}{2}\delta[1 - \lambda]\Gamma_{i,t+1}$. Since he does not need to risk his own capital, the ε-agent finds this project profitable.

Fiscal Solvency. To prove that bailouts are financeable via taxation, we show that condition (88) holds if and only if $\gamma \leq \overline{\gamma}'''$. Using the derivation of (34) and setting $\pi_{t+1}^{\varepsilon,nd} = y_{t+1}^{\varepsilon,nd}$, it follows that the LHS of (88) is equal to the discounted sum of T-production in the no-diverting part of the economy:

$$E\sum_{t=0}^{\infty}\delta^{t+1}[(1-\alpha)y_{t+1}^{\theta,nd} + \pi_{t+1}^{\theta,nd} + \pi_{t+1}^{\varepsilon,nd}] = E\left(\sum_{t=0}^{\infty}\delta^{t+1}[y_{t+1}^{\theta,nd} + y_{t+1}^{\varepsilon,nd}]\right)$$

Therefore, (88) becomes:

$$E\left(\sum_{t=0}^{\infty}\delta^{t+1}[y_{t+1}^{\theta,nd} + y_{t+1}^{\varepsilon,nd}]\right) \geq \sum_{t=0}^{\infty}\delta^{t+1}\gamma[1-\lambda]y_{t+1}^{\theta,nd} \tag{95}$$

Since bailouts are granted only in the $\varepsilon_{t+1} = 0$ state, and in this state all θ-firms are solvent, while all ε-firms go bust $(y_{t+1}^{\varepsilon,nd} = \varepsilon_{t+1}I_t^\varepsilon = 0)$, the bailout payment if $\varepsilon_{t+1} = 0$ is $\Gamma_{t+1} = \gamma y_{t+1}^{\theta,nd}$. Therefore, (95) can be re-expressed as:

$$\sum_{t=0}^{\infty}\delta^{t+1}y_{t+1}^{\theta,nd}\left[1 + \lambda\overline{\varepsilon}\delta[1-\lambda]\gamma - \gamma[1-\lambda]\right] \geq 0$$

$$\sum_{t=0}^{\infty}\delta^{t+1}y_{t+1}^{\theta,nd}\left[1 + \gamma[1-\lambda][\lambda\overline{\varepsilon}\delta - 1]\right] \geq 0$$

$$\frac{y_o^s}{1 - \delta(\theta\phi^s)^\alpha}\left[1 + \gamma[1-\lambda][\lambda\overline{\varepsilon}\delta - 1]\right] \geq 0 \qquad \text{if } \delta(\theta\phi^s)^\alpha < 1$$

$$\frac{(1-\phi^s)^\alpha}{1 - \delta(\theta\phi^s)^\alpha}q_o^\alpha\left[1 + \gamma[1-\lambda][\lambda\overline{\varepsilon}\delta - 1]\right] \geq 0 \Leftrightarrow \gamma \leq \overline{\gamma}''' := \frac{1}{[1-\lambda][1 - \lambda\overline{\varepsilon}\delta]} \tag{96}$$

Since $\phi^s < 1$ and $\delta(\theta\phi^s)^\alpha < 1$, the LHS is non-negative iff $\gamma \leq \overline{\gamma}'''$. Putting together the three bounds in (92), (94) and (96) we conclude that a financial black-hole equilibrium exists if $\gamma \leq \overline{\gamma}$,

with

$$\overline{\gamma} \equiv \max\left\{\frac{h\alpha}{[1-\lambda]}\frac{\phi}{1-\phi}\frac{1}{[\theta\phi]^\alpha}, \alpha\left[\theta^\alpha\phi^{\alpha-1}-h\right]\frac{\theta^\alpha\phi^{\alpha+1}}{1-\phi}, \frac{1}{[1-\lambda][1-\lambda\overline{\varepsilon}\delta]}\right\}, \quad \phi = \phi^s = \frac{1-\beta}{1-h\delta} \quad (97)$$