Hours and Participation with Job Heterogeneity

Claudio Michelacci Josep Pijoan-Mas

CEMFI and CEPR

ESSIM, Roda de Barà, May 2012

Introduction

Context

- Heterogeneity in jobs is an important feature of the labor market

  Akerlof (81): good jobs, like dam sites, are a scarce resource
- This leads to an assignment problem: worker allocation to jobs matters

- Labor earnings inequality larger than heterogeneity in worker abilities

  Sattinger (75), Jovanovic (98), Violante (02)
- A natural explanation for this friction:
  - costly new technology comes embedded into new capital goods
  - machines can only be used by a fixed number of workers

- Interesting labor allocation problem between
  a) number of jobs used in the economy
  b) time spent working in different jobs

We study this problem by incorporating a simple matching model with *frictionless assignment* into a neo-classical growth model with intensive and extensive margins of labor supply

Introduction

Theoretical Results

Two main theoretical results:

1. An increase in the speed of embodied technical change, which leads to an increase in the dispersion of job qualities, generates
   - Opposite movements of the intensive and extensive margins
   - Key: asymmetry of *income* and *substitution* effects across heterog. jobs

2. With heterogeneous workers.
   - Non-trivial *matching problem* with endogenous labor supply
   - Derive specific conditions for an *assortative equilibrium* to exist

Introduction

Quantitative Results

We bring our simple model to the **data**

1. We parameterize it to account for the dispersion of hours, employment, and labor income between education groups in the 70's
   - We find that *1/3 of college premium is due to heterogenity of jobs*

2. When we shift the source of growth from TFP to embodied technical change, as in *Greenwood, Hercowitz and Krusell (1997)*, it accounts for
   - The increase in hours and fall in participation between 1970 and 2000
   - The different change of hours and participation by education groups
   - 1/4 of the increase in the college premium

Heterogeneity in machines

(like Hornstein, Krusell and Violante, 2007)

- Time is continuous
- At every point in time, m new machines of quality $\tilde{k}_t^0 = e^{qt}$
- Quality at t of machine of age τ: $\tilde{k}_t^\tau = e^{q(t-\tau)}$
- Only the p best machines are operated
- Distribution of ages uniform in $[0, \tau^*]$, with $\tau^* = p/m$
- Detrended qualities:

$$k^\tau \equiv \frac{\tilde{k}_t^\tau}{e^{qt}} \quad \rightarrow \quad k^0 = 1; \quad k^\tau = e^{-q\tau}; \quad k^* = e^{-q\frac{p}{m}}$$

- Note that the quality gap to the frontier increases with q and τ
- Note that the quality gap of the marginal machine increases with the number of workers p

Social Planner Problem

Setup

- Continuum of **identical workers** of measure 1
- Utility of consumption

$$u(c) = \log c$$

- The disutility of working

$$v(n) = \begin{cases} \lambda_0 + \lambda_1 \frac{n^{1+\eta}}{1+\eta} & \text{if } n > 0 \\ 0 & \text{if } n = 0 \end{cases}$$

- Production of each machine given by

$$f(k, n) = k^{\alpha} n^{1-\alpha}$$

- Distribution of machine qualities k^{τ} as specified in previous slide

▷ Choose assignment, hours and consumption for each worker

Social Planner Problem

Solution

- We look at the BGP: there will be nothing dynamic about the problem
- Since workers have equal weight, they all get same consumption c
- Since skills are identical, assignment to machines undetermined
- Planner chooses:

$$\max_{c,p,n^{\tau}} \left\{ u\left(c\right) - \int_0^p v\left(n^{\tau}\right) d\tau \right\} \quad \text{s.t.} \quad c \leq \int_0^p f\left(k^{\tau}, n^{\tau}\right) d\tau$$

(we index workers by the age τ of the machine with which they are paired)
(we set $m = 1$, without loss of generality)

- Optimality conditions,

$$\begin{aligned} u'\left(c\right) &= \mu \\ v'\left(n^{\tau}\right) &= \mu f_2\left(k^{\tau}, n^{\tau}\right) \\ v\left(n^{*}\right) &= \mu f\left(k^{*}, n^{*}\right) \end{aligned}$$

Optimal allocation of hours and bodies

- Unique allocation of hours to machine qualities:

$$v'(n^\tau) = \mu f_2(k^\tau, n^\tau) \quad \Rightarrow \quad n^\tau = \phi(k^\tau, \mu) = \phi\left(e^{-q\tau}, \mu\right)$$

  with $f_{12} > 0$

  - *workers matched with better machines work longer hours*: $\phi_1(k^\tau, \mu) > 0$
  - Hours dedcay with age τ at the exponential rate q

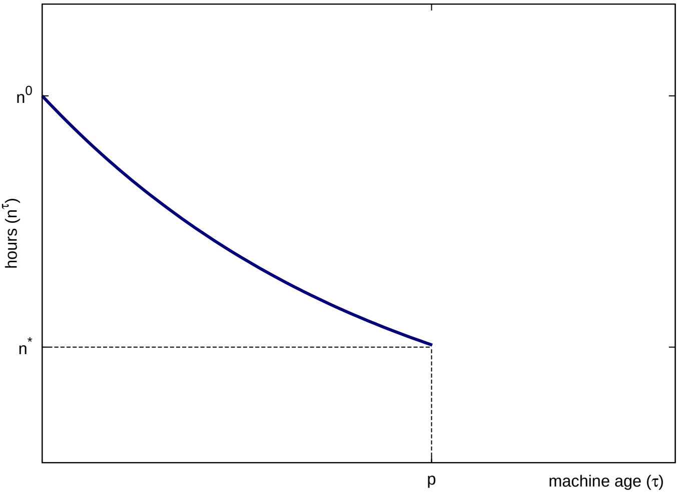
- *Utility cost of going to work determines* quality of marginal machine:

$$v(n^*) = \mu f(k^*, n^*) \quad \Rightarrow \quad v(n^*) = \mu f\left(e^{-qp}, n^*\right)$$

  and hence *the number of workers* p

▷ Note that the marginal utility of consumption μ raises both margins

Hours worked by machine age



Increase in the speed of technical change q

An increase in q has two effects in this economy

1. It increases the rate of growth along the BGP

   (Irrelevant because both preferences and technology are growth neutral)

2. It increases the dispersion of available technologies along the BGP

   - Average productivity falls relative to the frontier
   - Total output falls relative to output at the frontier machine
   - Detrended output falls

Increase in the speed of technical change q

Labor supply: income and substitution effects

i) Δq makes all machines $k^{\tau} = e^{-q\tau}$ worse relative to the frontier
   - More so the older ones

ii) Consumption c falls because of the *average* decrease in productivity
   - $\triangledown c$ raises the value of output in all machines the same

▷ Asymmetry of (i) substitution and (ii) income effects across machines:

- n^{τ} goes down in old machines and up in new ones
- p falls as the quality of the marginal machine falls more than c
- Average hours per worker n go up

▷ Job heterogeneity is key
   - otherwise substitution and income effects cancel out

Increase in the speed of technical change q

Three important properties

1. Hours n^0 in the top machine are independent of direct effect of q:

$$v'(n^\tau) = \mu\, f_2(k^\tau, n^\tau) \quad \Rightarrow \quad v'\left(n^0\right) = \mu\, f_2\left(e^{-q0}, n^0\right)$$

2. Quality k^τ and hence hours n^τ fall at the rate q with machine age τ:

$$v'(n^\tau) = \mu\, f_2(k^\tau, n^\tau) \quad \Rightarrow \quad n^\tau = \phi\left(e^{-q\tau}, \mu\right)$$

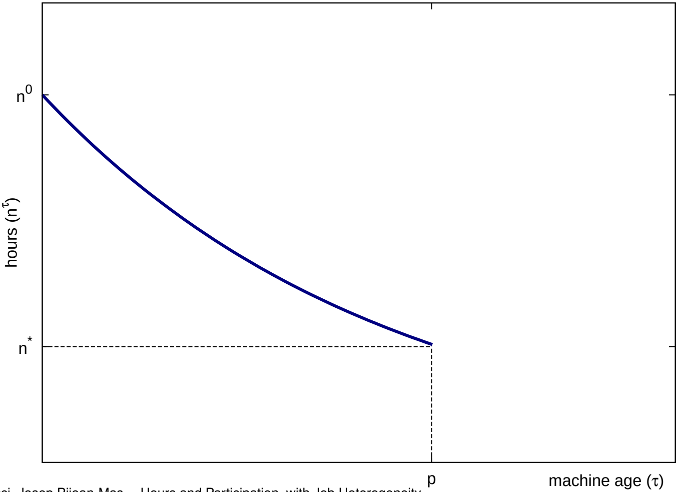
3. Hours n^* in the marginal machine are independent of q (and μ):

$$\left.\begin{array}{rcl} v'(n^*) & = & \mu f_2(k^*, n^*) \\ v(n^*) & = & \mu f(k^*, n^*) \end{array}\right\} \Rightarrow \frac{v'(n^*)}{v(n^*)} = \frac{1-\alpha}{n^*}$$

(Cobb-Douglas requirement of BGP)

Increase in the speed of technical change q

Graphical representation



Decentralization

- At any given period:
  - a fraction $1 - p$ of workers do not work
  - those working work different hours each
  - and all need to enjoy same consumption
- We opt for a setting with
  - spot market for labor services with price $w\,(n)$
  - workers are ∞-lived
  - firm profits are pooled and workers trade shares of a Lucas forest
- Worker i chooses:
  - Poisson rate p_i of going to work at every period
    (deterministic fraction of time going to work in a given time interval)
  - number of hours worked n_i when given a chance
  - consumption and saving

Balanced growth path equilibrium

- ▷ To characterize the equilibrium we need to determine jointly,
  - The assignment of workers to machines
  - The functional form for the wage function $w(n)$.

- ▷ We guess-and-verify the following equilibrium:

a) Workers are freely assigned to different machines every period

$$\varphi_i(k) : R_+ \times [0,1] \to [0,1]$$

- with the constrain that their permanent incomes are identical
- with the constrain that all machines with $\tau < p$ are in use

$$\int_{[0,1]} \varphi_i(k)\, di = \int_{k^*}^{k} g(\kappa)\, d\kappa \quad \forall k \geq k^*$$

b) The wage function is,

$$w(n) = \left(a_0 + a_1 \frac{n^{1+\eta}}{1+\eta}\right) \quad \text{if } n > 0$$

This guarantees that work effort is higher in better machines

Household choices

- At any given period the choice of hours and the participation decisions are indeterminate at the individual level
  - The intratemporal condition for hours gives,

$$w'(n_t)\frac{1}{c} = v'(n_t) \quad \Rightarrow \quad a_1 n_t^{\eta}\frac{1}{c} = \lambda_1 n_t^{\eta} \quad \Rightarrow \quad \frac{a_1}{c} = \lambda_1$$

  - The condition for participation at t is

$$w(n_t)\frac{1}{c} \geq v(n_t) \quad \Rightarrow \quad \frac{a_0}{c} \geq \lambda_0$$

- But the lifetime labor supply is determined:

$$c = \rho\left[b_0 + p\int_0^{\infty} e^{-\rho t} w(n_t)\, dt\right]$$

(This is analogous to Prescott, Rogeson and Wallenius, 2009)

Production

- Each firm owns a machine of quality k^τ:

$$\max_{n^\tau} \left\{ f\left(k^\tau, n^\tau\right) - w\left(n^\tau\right) \right\}$$

- Hence, workers matched to different machines work different hours

$$f_2\left(k^\tau, n^\tau\right) = w'\left(n^\tau\right) \quad \Rightarrow \quad n^\tau = \phi\left(k^\tau\right)$$

- The oldest machines are scrapped and free entry implies zero profits at the marginal machine

$$\pi^* = 0 \quad \Rightarrow \quad f\left(k^*, \phi\left(k^*\right)\right) = w\left(\phi\left(k^*\right)\right)$$

Balanced growth path equilibrium

▷ A **BGP equilibrium** for this economy is characterized by a wage function $w(n)$, an assignment function $\varphi(k)$, an aggregate participation rate p, an interest rate r and individual consumption, saving and working plans and firm labor demands $\phi(k)$ such that

- Workers and firms solve their optimization problems
- Free entry condition in production is satisfied
- The labor market clears

$$\int_{[0,1]} p_i di = p \quad \text{and} \quad \int_{[0,1]} p_i \varphi_i(k)\, di = p \int_{k^*}^{k} g(\kappa)\, d\kappa \quad \forall k \geq k^*$$

- The capital market clears - Lucas forest
- By Walras law, the goods market clears
- Output grows at a constant rate x
  (This implies that $x = \alpha q$ and $r = \rho + x$)

A model with types

Sketch

- I education types w/ human capital h_i, mass z_i, and initial wealth $b_{0,i}$
- Each worker supplies $e = h^{1-\theta} n^{\theta}$ efficiency units of labor
- Final output is given by $y(k, e) = k^{\alpha} e^{1-\alpha}$
- The conjectured equilibrium
  - Assortative matching
  - Within worker type the assignment is like the one-type model
  - The functional form for the wage function is type-dependent:

$$w_i(n) = \left(a_{0,i} + a_{1,i} \frac{n^{1+\eta}}{1+\eta} \right) \quad \text{if } n > 0$$

A model with types

Assortative matching

- Assortative matching requires:
  - *skill gap* large enough compared to *consumption gap*

$$\left(\frac{h_i}{h_{i+1}}\right)^{(1-\alpha)(1-\theta)} > \left(\frac{c_i}{c_{i+1}}\right)^{\frac{A-1}{A}}$$

  - this prevents income effect of low skilled to make cheap labor

- In terms of model fundamentals we need
  a) *skill gap* large enough compared to *machine quality gap*
  b) Redistribution: non-labor income is a larger fraction of total income for low skilled

A model with types

The key equations

- Firms: similar FOC for hours demand

$$f_2\left(k, h_i^{1-\theta} n^{\theta}\right) \theta \left(\frac{h_i}{n}\right)^{1-\theta} = w_i'(n)$$

- New free entry conditions,

$$\pi\left(k_i^*, h_i\right) = \pi\left(k_i^*, h_{i+1}\right) \quad \text{and} \quad \pi\left(k_I^*, h_I\right) = 0$$

- Same FOC for households
- And the new household budget constraints in equilibrium

$$c_i = \rho b_{0,i} + p_i \int_{k_i^*}^{k_{i-1}^*} w\left(h_i, \phi\left(\kappa, h_i\right)\right) g\left(\kappa\right) d\kappa$$

Conclusions

We emphasize a relatively new friction that we incorporate into a standard neoclassical growth model with participation and hours decision

- An acceleration of investment-specific technical change generates
  - a fall in participation
  - an increase in hours per worker
  - an increase in labor income inequality

- Absent job heterogeneity none of this would happen

- In addition, with worker heterogeneity:
  - Workers assigned to best machines increase hours more and decrease participation less
  - Workers assigned to worst machines increase hours less and decrease participation more
  - Average income gap between worker types increases
  - ▷ These three patterns are consistent with data by education

Thanks for your attention !