Inattention to Rare Events

Bartosz Maćkowiak, *European Central Bank and CEPR*

Mirko Wiederholt, *Northwestern University*

ESSIM May 22, 2012

The views expressed here are solely those of the authors and do not necessarily reflect the views of the ECB

Introduction

- Recently the world experienced several events with disastrous consequences:
  - the global financial crisis,
  - the European sovereign debt crisis,
  - the Fukushima nuclear accident.
- These events have in common: People were unprepared for them, which made them more disastrous.

Introduction

- We propose a model to answer the following questions:
  - What determines the degree of preparation for (the quality of actions taken in) contingent events?
  - Would society be ex ante better off if agents thought more carefully about optimal actions in unusual times?

Main features of the model

- Decision-making takes time, and once the regime realizes agents have to act quickly. Therefore, agents have to plan ahead.

- Agents make state-contingent plans with a limited ability to process information.

- There are payoff externalities.

Model

- There is a continuum of agents indexed by $i \in [0, 1]$.

- Time is discrete and indexed by $t = 0, 1, 2, ...$

- Each period the economy is in one of two regimes. The regime follows a two-state Markov chain. For simplicity, the regime is i.i.d. over time.

- Let $p_n > 0$ denote the probability of regime n.

Model

- Each agent commits to a state-contingent plan for the next period.

- The plan agent i commits to in period $t-1$ for period t:

$$a_{i,t} = \begin{pmatrix} a_{i,t,1} \\ a_{i,t,2} \end{pmatrix}$$

Model

- The payoff of agent i in regime n:

$$U^n\left(a_{i,t,n}, a_{t,n}, z_{t,n}\right) = U^n\left(a^*_{i,t,n}, a_{t,n}, z_{t,n}\right) - \delta_n\left(a_{i,t,n} - a^*_{i,t,n}\right)^2$$

where

$$a^*_{i,t,n} = \varphi_n + \gamma_n a_{t,n} + (1-\gamma_n)\, z_{t,n}$$

$$\delta_n \equiv -\frac{U^n_{a_i a_i}}{2} > 0$$

$$\gamma_n \equiv -\frac{U^n_{a_i a}}{U^n_{a_i a_i}} \in (-1, 1)$$

Model

- Agents' common prior belief concerning fundamentals:

$$z_t = \begin{pmatrix} z_{t,1} \\ z_{t,2} \end{pmatrix} \sim i.i.d.N\left(0, \Sigma\right)$$

- Each agent can process information before committing to a plan. This activity is modeled as receiving a signal:

$$s_{i,t-1} = \begin{pmatrix} z_{t,1} \\ z_{t,2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{i,t-1,1} \\ \varepsilon_{i,t-1,2} \end{pmatrix}$$

where

$$\begin{pmatrix} \varepsilon_{i,t-1,1} \\ \varepsilon_{i,t-1,2} \end{pmatrix} \sim i.i.d.N\left(0, \Lambda\right)$$

Model

- Each agent decides how carefully to think about the optimal actions in the regimes:

$$\min_{\Lambda} \sum_{n=1}^{2} p_n \delta_n E\left[\left(a_{i,t,n} - a^*_{i,t,n}\right)^2\right]$$

subject to

$$a_{i,t,n} = E\left[\varphi_n + \gamma_n a_{t,n} + (1-\gamma_n) z_{t,n} | s_{i,t-1}\right]$$

$$s_{i,t-1} = \begin{pmatrix} z_{t,1} \\ z_{t,2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{i,t-1,1} \\ \varepsilon_{i,t-1,2} \end{pmatrix}$$

and

$$\frac{1}{2}\log_2(\det \Sigma) - \frac{1}{2}\log_2(\det \Omega) \leq \kappa$$

The case of independence of optimal actions across regimes

- Assume $\Sigma_{12} = 0$.
- It is optimal to receive independent signals about the regimes, $\Lambda_{12} = 0$.
- The information-processing constraint reduces to:

$$\underbrace{\frac{1}{2}\log_2\left(\frac{\Sigma_{11}}{\Omega_{11}}\right)}_{\kappa_1} + \underbrace{\frac{1}{2}\log_2\left(\frac{\Sigma_{22}}{\Omega_{22}}\right)}_{\kappa_2} \leq \kappa$$

The role of probabilities

- Assume $\gamma_1 = \gamma_2 = 0$. The unique equilibrium allocation of attention is:

$$\kappa_1 = \begin{cases} \kappa & \text{if } \sqrt{\frac{p_1}{p_2}\frac{\Sigma_{11}}{\Sigma_{22}}\frac{\delta_1}{\delta_2}} \geq 2^{\kappa} \\ \frac{1}{2}\kappa + \frac{1}{4}\log_2\left(\frac{p_1}{p_2}\frac{\Sigma_{11}}{\Sigma_{22}}\frac{\delta_1}{\delta_2}\right) & \text{otherwise} \\ 0 & \text{if } \sqrt{\frac{p_1}{p_2}\frac{\Sigma_{11}}{\Sigma_{22}}\frac{\delta_1}{\delta_2}} \leq 2^{-\kappa} \end{cases}$$

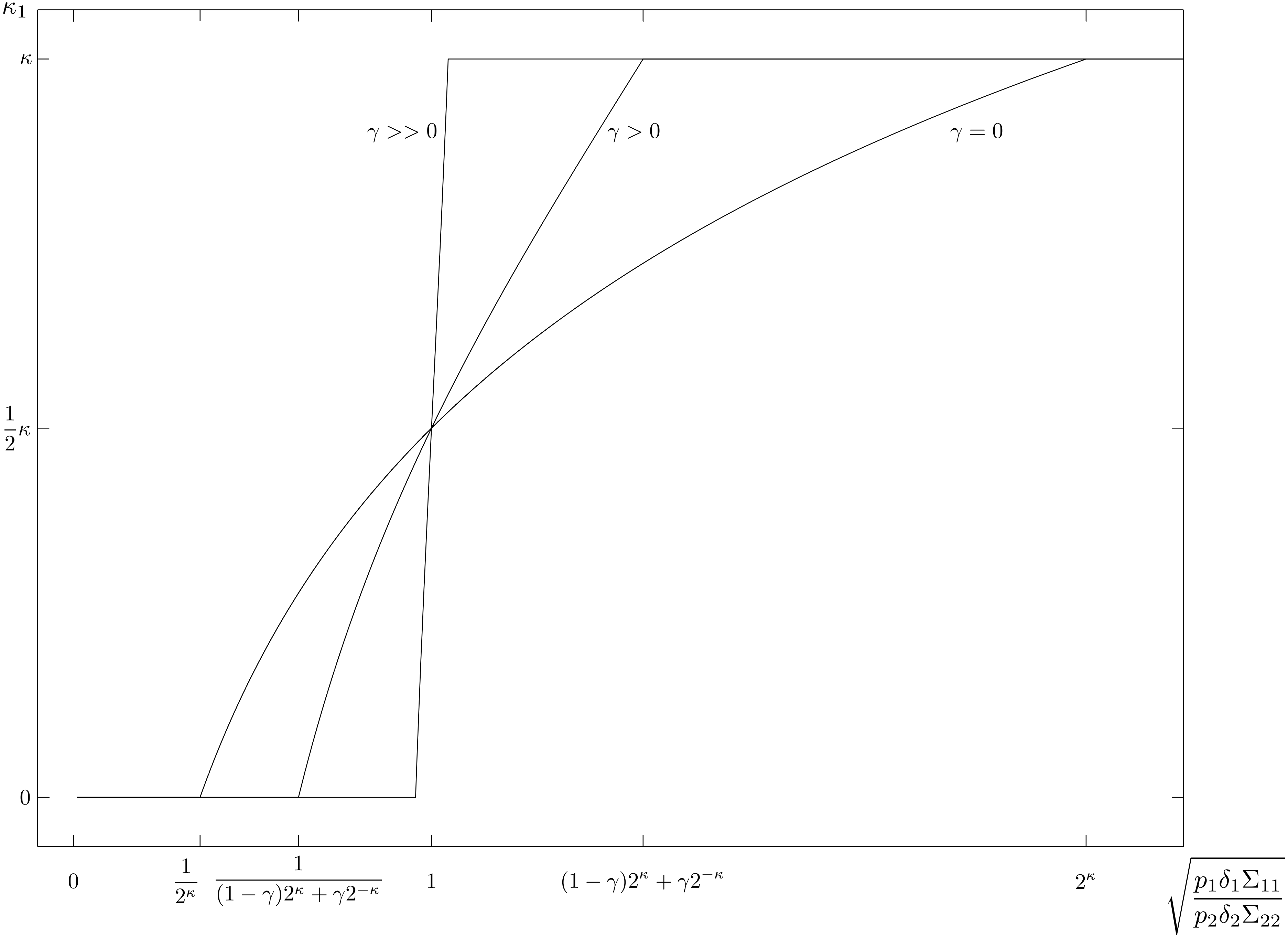
- Agents equate the probability-weighted expected loss across regimes:

$$p_1\delta_1\Omega_{11} = p_2\delta_2\Omega_{22} \quad \Longrightarrow \quad \frac{\delta_1\Omega_{11}}{\delta_2\Omega_{22}} = \frac{1}{\frac{p_1}{p_2}}$$

The role of strategic complementarity in actions

- Assume $\gamma_1 = \gamma_2 > 0$.
- The allocation of attention becomes more extreme.
- The fact that other agents are not thinking about a regime reduces the incentive for each agent to think about that regime.
  - This effect is stronger for the regime that agents are thinking less about.

Figure 3: Attention to state one as function of relative likelihood



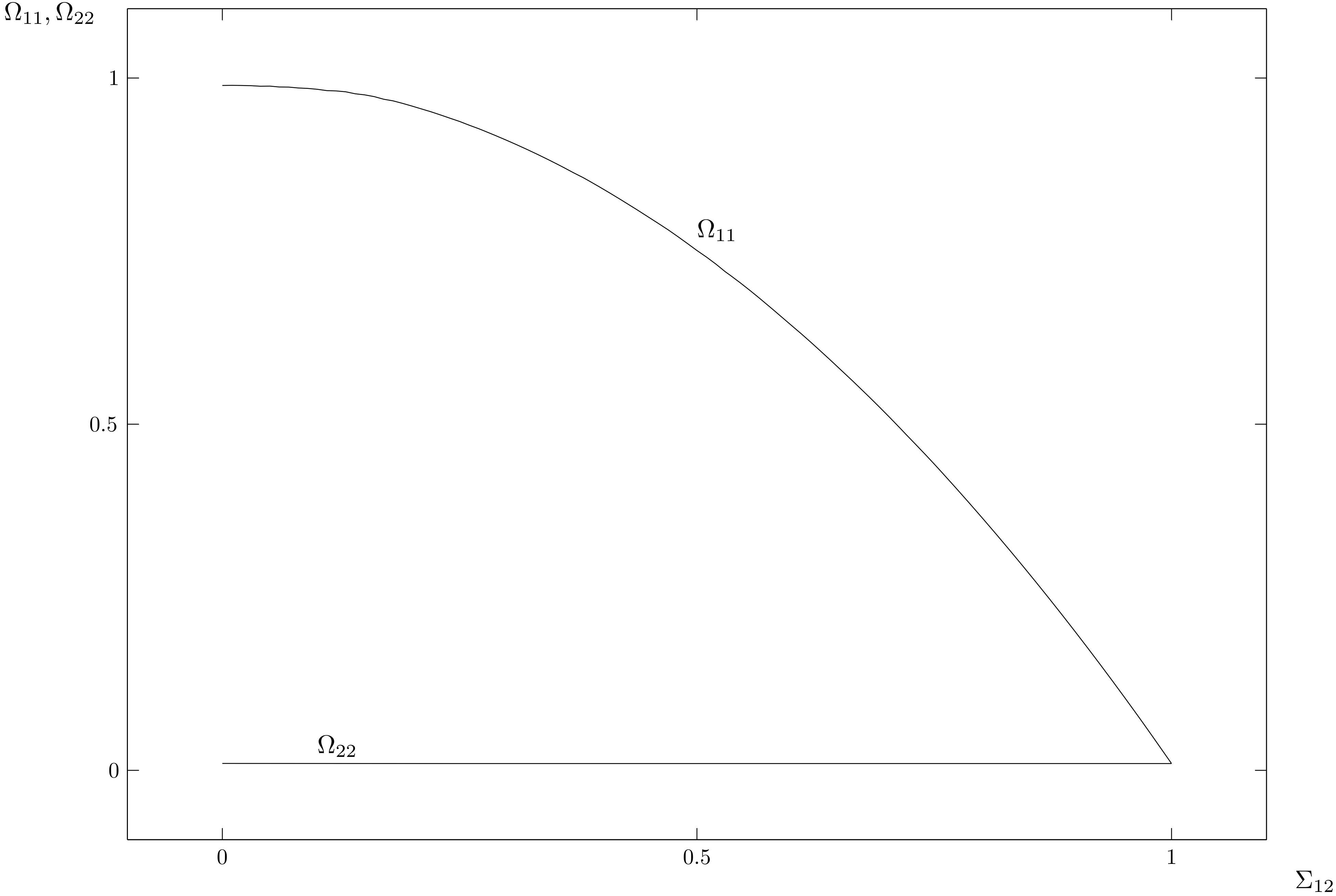
The role of correlation of optimal actions across regimes

- Assume $\Sigma_{12} > 0$.

- Actions improve in both regimes.
  - Thinking about one regime, agents learn about the other regime.

- So long as the prior correlation of optimal actions across regimes is low or moderate, agents do much worse in unusual times than in normal times.

Figure 2a: Posterior covariance matrix of optimal actions as function of prior correlation of optimal actions



This figure assumes: $\gamma_1 = \gamma_2 = 0, \delta_1 = \delta_2, \Sigma_{11} = \Sigma_{22} = 1, p_1 = 0.01$

The model vs. the recent events

- Why were people so unprepared for the global financial crisis, the European sovereign debt crisis, and the Fukushima nuclear accident?

- Humans make state-contingent plans with a limited ability to process information.

- The recent events had the following features:
  - A low-probability regime realized.
  - Actions were strategic complements.
  - The correlation of optimal actions across regimes was low.

Efficient allocation of attention

- Would society be ex ante better off if agents allocated their attention differently than they do in equilibrium?

- To answer this question, we study the following planner problem. The planner:
  - can tell agents how to allocate attention,
  - has to respect the agents' information-processing constraint,
  - maximizes ex ante utility of agents.

Planner problem: assumptions

- The economy is efficient under perfect information.

- Analytical solution: $\mathbf{\Sigma}_{12} = 0$.

- Some symmetry across regimes: $\gamma_n \equiv -\frac{U^n_{a_i a}}{U^n_{a_i a_i}}$ and $\frac{U^n_{aa}}{U^n_{a_i a_i}}$ are independent of n.

- Equilibrium is unique.

- Planner problem is convex.

Planner problem: results

- If
$$-\frac{U_{a_i a}}{U_{a_i a_i}} - \frac{U_{aa}}{U_{a_i a_i}} = 0$$
the equilibrium allocation of attention *equals* the efficient allocation of attention.

- If this equality fails to hold, $\kappa_1 \neq \frac{1}{2}\kappa$, and $\kappa_1^{equ} \in (0, \kappa)$, the equilibrium allocation of attention *differs* from the efficient allocation of attention.
  - In particular, when $U_{a_i a} + U_{aa} < 0$, agents think too little about the optimal action in unusual times.

Conclusions

- People are unprepared for contingent events when:
  - a low-probability regime realizes,
  - actions are strategic complements,
  - and the correlation of optimal actions across regimes is low.

- A simple condition on the payoff function governs the socially optimal allocation of attention.

Extensions

- Learning the probability of unusual times.
- How does limited liability affect preparation for contingent events?
- The planner has a different objective than agents.