Rare Shocks, Great Recessions

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Disclaimer: The views expressed are mine and do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System
Broad objectives of research agenda

• Show that shocks in linear models (VARs, DSGEs, factor models) estimated on macro variables have fat tails.

• Well known for financial variables, less for macro.
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• Show that shocks in linear models (VARs, DSGEs, factor models) estimated on macro variables have fat tails.

• Well known for financial variables, less for macro.

• Why should we care?

  ① Inference using linear models turns out a bit different when allowing for fat tails.

  ② Can non-linearities explain away the fat tails?

  ③ Move away from linear models? (For linear DSGEs, solution method is called into question)

  ④ Implications for finance
The Great Moderation and its undoing (Great Recession)

- Main hypothesis so far:
  1. Time-variation in volatility
     - (Low frequency) changes in the volatility of shocks

The paper estimates a DSGE model with Student's t-distributed shocks... and time-varying volatility.
The Great Moderation and its undoing (Great Recession)

- Main hypothesis so far:
  1. Time-variation in volatility
     - (Low frequency) changes in the volatility of shocks

- Alternative:
  2. Rare large shocks
     - A time-invariant distribution that generates rare large events (high frequency bursts in volatility)

We show that the two coexist: both low and high frequency changes in volatility

- The paper estimates a DSGE model with Student’s t-distributed shocks . . . and time-varying volatility
Policy Shocks
Discount rate shocks

Exc. Kurtosis: 2.9

Standard Deviations

Results

- Strong evidence that shocks are Student-$t$.

- These rare shocks matter in terms of business cycle fluctuations (rare shocks, great recessions)
Results
Results

- Strong evidence that shocks are Student-\(t\).

- These rare shocks matter in terms of business cycle fluctuations (rare shocks, great recessions).

- Slow-moving stochastic volatility less important in the presence of rare large shocks.
Results


Median: 1.87
Median: 2.18
Caveats

- Skewness (lack thereof)
- Non-linearities (lack thereof)
Literature

- Bayesian estimation with Student’s $t$-distributed shocks: Geweke (1993, 2005)

$$y_t = x_t'\beta + \varepsilon_t, \quad \varepsilon_t = \tilde{h}_t^{-1/2} \eta_t, \quad \eta_t \sim \mathcal{N}(0, \sigma^2)$$

$$\lambda \tilde{h}_t \sim \chi^2(\lambda)$$
Literature

- Bayesian estimation with Student’s $t$-distributed shocks: Geweke (1993, 2005)

\[
y_t = x_t' \beta + \varepsilon_t, \quad \varepsilon_t = \tilde{h}_t^{-1/2} \eta_t, \quad \eta_t \sim N(0, \sigma^2)
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- Stochastic volatility in DSGE models: Justiniano & Primiceri (2008)
Literature

• Bayesian estimation with Student’s $t$-distributed shocks: Geweke (1993, 2005)

$$y_t = x_t' \beta + \varepsilon_t, \quad \varepsilon_t = \tilde{h}_t^{-1/2} \eta_t, \quad \eta_t \sim \mathcal{N}(0, \sigma^2)$$

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• Stochastic volatility in DSGE models: Justiniano & Primiceri (2008)

• Many other relevant papers: great moderation ($\infty$), SV in DSGE (Fernandez-Villaverde and Rubio-Ramirez 2007)

• Chib and Ramamurthy (2011): Student-$t$ but no SV
... about $\tilde{h}_t$s and $\lambda$s (degrees of freedom)

- Student-t distributed shocks

$\varepsilon_t = \sigma_q \tilde{h}_t^{-1/2} \eta_t$, where $\eta_t \sim \mathcal{N}(0, 1)$ i.i.d., and

- Prior on $\tilde{h}_t$:

$\lambda_q \tilde{h}_t \sim \chi^2(\lambda_q)$
... about $\tilde{h}_t$s and $\lambda$s (degrees of freedom)

$$\tilde{h}_t \sim \chi^2(\lambda)/\lambda \Rightarrow E[\tilde{h}_t] = 1, \ Var[\tilde{h}_t] = 2/\lambda$$

Number of shocks larger (in abs. value) than $x$ standard deviations per 200 periods

<table>
<thead>
<tr>
<th>$\lambda$, $x$:</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>.54</td>
<td>.012</td>
<td>1e^{-4}</td>
</tr>
<tr>
<td>15</td>
<td>1.79</td>
<td>.23</td>
<td>.03</td>
</tr>
<tr>
<td>9</td>
<td>2.99</td>
<td>.62</td>
<td>.15</td>
</tr>
<tr>
<td>6</td>
<td>4.80</td>
<td>1.42</td>
<td>.49</td>
</tr>
</tbody>
</table>
... about $\tilde{h}_t$s and $\lambda$s (degrees of freedom)

- Student-t distributed shocks

$$\varepsilon_t = \sigma_q \tilde{h}_t^{-1/2} \eta_t, \text{ where } \eta_t \sim \mathcal{N}(0, 1) \text{ i.i.d., and}$$

- Prior on $\tilde{h}_t$:

$$\lambda_q \tilde{h}_t \sim \chi^2(\lambda_q)$$

- Posterior on $\tilde{h}_t$:

$$\left[ \lambda_q + \varepsilon_{q,t}^2 / \sigma_q^2 \right] \tilde{h}_{q,t} \sim \chi^2(\lambda_q + 1).$$
Estimating DSGEs with Student’s $t$ shocks + SV

• Measurement:
  \[ y_t = Z(\theta)s_t \]

• Transition:
  \[ s_{t+1} = T(\theta)s_t + R(\theta)\varepsilon_t \]
  where $\theta$ are the DSGE parameters

• Shocks
  \[ \varepsilon_{q,t} = \sigma_q \sigma_{q,t} \tilde{h}_{q,t}^{-1/2} \eta_{q,t} \]
  \[ \eta_{q,t} \sim \mathcal{N}(0, 1), \text{i.i.d. across } q, t. \]
  \[ \log \sigma_{q,t} = \log \sigma_{q,t-1} + \zeta_{q,t}, \sigma_{q,0} = 1, \zeta_{q,t} \sim \mathcal{N}(0, \omega_q^2) \]
  \[ \lambda_q \tilde{h}_{q,t} \sim \chi^2(\lambda_q) \]
Fat tails vs. Time-variation in volatility: some intuition

- (persistent) stochastic volatility

\[ \varepsilon_t = \sigma_t \eta_t, \text{ where } \eta_t \sim \mathcal{N}(0, 1) \text{ i.i.d., and} \]

**Transition**

\[ \log \sigma_t = \log \sigma_{t-1} + \zeta_t, \sigma_{q,0} = 1, \zeta_t \sim \mathcal{N}(0, \omega_q^2) \]

**Measurement**

\[ \log(\varepsilon_t^2) = 2 \log \sigma_t + \eta_t^*, \eta_t^* \sim \log(\chi_1^2) \]
Fat tails vs. Time-variation in volatility: some intuition

- (persistent) stochastic volatility + Student-t

\[ \varepsilon_t = \sigma_t \, \tilde{h}_t^{-1/2} \eta_t, \text{ where } \eta_t \sim \mathcal{N}(0, 1) \text{ i.i.d., and} \]

**Transition**

\[ \log \sigma_t = \log \sigma_{t-1} + \zeta_t, \quad \sigma_{q,0} = 1, \quad \zeta_t \sim \mathcal{N}(0, \omega_q^2) \]

**Measurement**

\[ \log(\tilde{h}_t \varepsilon_t^2) = 2 \log \sigma_t + \eta_t^*, \quad \eta_t^* \sim \log(\chi_1^2) \]
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**Measurement**

\[ \log(\varepsilon_t^2) = 2 \log \sigma_t + \eta_t^*, \ \eta_t^* \sim \log(\chi^2_1) \]

• Stochastic volatility affects inference about fat tails:

\[ [\lambda_q + \varepsilon_{q,t}^2 / \sigma_{q,t}^2] \tilde{h}_{q,t} \sim \chi^2(\lambda_q + 1). \]
Measurement

- Measurement equation:

Output growth = \( LN((GDPC)/LNSINDEX) \times 100 \)

Consumption growth = \( LN((PCEC/GDPDEF)/LNSINDEX) \times 100 \)

Investment growth = \( LN((FPI/GDPDEF)/LNSINDEX) \times 100 \)

Real Wage growth = \( LN(PRS85006103/GDPDEF) \times 100 \)

Hours = \( LN((PRS85006023 \times CE16OV/100)/LNSINDEX) \times 100 \)

Inflation = \( LN(GDPDEF/GDPDEF(-1)) \times 100 \)

FFR = \( FEDERAL\ FUNDS\ RATE/4 \)

Sample 1964:Q1–2011:Q1

- Same prior on \( \theta \) as SW.
Incorporating 10-yrs inflation expectations from surveys

- SW forecasts inflation relatively well but ... somewhat tight prior on $\pi^* \sim Gamma(.62, .10)$.

- No need of such a prior: Use a loose prior ($\pi^* \sim Gamma(.75, .40)$) and survey data as an observable:

$$
\pi_{t}^{O_{40}, 40} = \pi^* + E_t \left[ \frac{1}{40} \sum_{k=1}^{40} \pi_{t+k} \right]
$$

$$
= \pi^* + \frac{1}{40} Z(\theta)(\pi, \cdot) (I - T(\theta))^{-1} (T(\theta) - T(\theta)^{41}) s_t,
$$

- ... and change the model to be able to explain it:

$$
R_t = \rho_R R_{t-1} + (1 - \rho_R) \left( \psi_1(\pi_t - \pi^*_t) + \psi_2(y_t - y^f_t) \right) + \psi_3 \left( (y_t - y^f_t) - (y_{t-1} - y^f_{t-1}) \right) + r^m_t,
$$

where $\pi^*_t = \rho_{\pi^*} \pi^*_{t-1} + \sigma_{\pi^*} \epsilon_{\pi^*, t}$. 
Fit

<table>
<thead>
<tr>
<th>Without Stochastic Volatility</th>
<th>With Stochastic Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gaussian shocks</strong></td>
<td></td>
</tr>
<tr>
<td>-973.3</td>
<td>-812.1</td>
</tr>
<tr>
<td><strong>Student-t distributed shocks (ν = 4)</strong></td>
<td></td>
</tr>
<tr>
<td>( \lambda = 15 )</td>
<td>-810.5</td>
</tr>
<tr>
<td>( \lambda = 9 )</td>
<td>-796.7</td>
</tr>
<tr>
<td>( \lambda = 6 )</td>
<td>-780.3</td>
</tr>
</tbody>
</table>
# Fit

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<tr>
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<tr>
<td></td>
<td>-973.3</td>
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<tr>
<td><strong>Student-t distributed shocks</strong> ( \nu = 4 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda = 15 )</td>
<td>-810.5</td>
<td>-762.0</td>
</tr>
<tr>
<td>( \lambda = 9 )</td>
<td>-796.7</td>
<td>-765.1</td>
</tr>
<tr>
<td>( \lambda = 6 )</td>
<td>-780.3</td>
<td>-753.0</td>
</tr>
</tbody>
</table>
Fit – Loose prior on degrees of freedom

<table>
<thead>
<tr>
<th></th>
<th>Gaussian shocks</th>
<th>Student-t distributed shocks</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>-973.3</td>
<td>-812.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student-t distributed shocks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu = 4$ (tighter prior)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda = 15$</td>
<td>-762.0</td>
<td>-759.9</td>
</tr>
<tr>
<td>$\lambda = 9$</td>
<td>-765.1</td>
<td>-754.5</td>
</tr>
<tr>
<td>$\lambda = 6$</td>
<td>-753.0</td>
<td>-751.7</td>
</tr>
</tbody>
</table>

Prior on degrees of freedom $\lambda$ is Gamma($\lambda/\nu, \nu$):

$$p(\lambda_q | \lambda, \nu) = \frac{(\lambda/\nu)^{-\nu}}{\Gamma(\nu)} \lambda_q^{\nu-1} \exp(-\nu \frac{\lambda_q}{\lambda}), \quad q = 1, \ldots, Q.$$
# Estimated degrees of freedom, Student-\( t \)

<table>
<thead>
<tr>
<th></th>
<th>Without Stochastic Volatility</th>
<th>With Stochastic Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g )</td>
<td>6.1</td>
<td>5.6</td>
</tr>
<tr>
<td></td>
<td>(2.5,9.9)</td>
<td>(2.1,9.2)</td>
</tr>
<tr>
<td>( b )</td>
<td>4.7</td>
<td>5.4</td>
</tr>
<tr>
<td></td>
<td>(2.5,7.0)</td>
<td>(2.4,8.4)</td>
</tr>
<tr>
<td>( \mu )</td>
<td>5.4</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td>(2.5,8.5)</td>
<td>(2.4,10.7)</td>
</tr>
<tr>
<td>( z )</td>
<td>3.7</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td>(1.8,5.6)</td>
<td>(1.7,7.2)</td>
</tr>
<tr>
<td>( \lambda_f )</td>
<td>6.8</td>
<td>12.7</td>
</tr>
<tr>
<td></td>
<td>(2.5,11.3)</td>
<td>(4.2,21.2)</td>
</tr>
<tr>
<td>( \lambda_w )</td>
<td>5.5</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>(2.5,8.5)</td>
<td>(2.4,7.1)</td>
</tr>
<tr>
<td>( r^m )</td>
<td>2.7</td>
<td>10.9</td>
</tr>
<tr>
<td></td>
<td>(1.7,3.7)</td>
<td>(3.7,18.0)</td>
</tr>
<tr>
<td>( \pi^* )</td>
<td>1.7</td>
<td>9.0</td>
</tr>
<tr>
<td></td>
<td>(1.2,2.2)</td>
<td>(2.9,15.6)</td>
</tr>
</tbody>
</table>
## Fit – pre-Great Recession sample (2004:Q4)

<table>
<thead>
<tr>
<th></th>
<th>Constant Volatility</th>
<th>Stochastic Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gaussian shocks</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-864.7</td>
<td>-742.1</td>
</tr>
<tr>
<td><strong>Student-t distributed shocks, prior with 1 degree of freedom</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda = 15$</td>
<td>-731.9</td>
<td>-709.9</td>
</tr>
<tr>
<td>$\lambda = 9$</td>
<td>-731.8</td>
<td>-705.3</td>
</tr>
<tr>
<td>$\lambda = 6$</td>
<td>-729.58</td>
<td>-700.9</td>
</tr>
</tbody>
</table>
Shocks vs “tamed” shocks

Discount rate shocks

shocks ($|\varepsilon_t|$)

“tamed” shocks ($|\tilde{h}_t^{1/2}\varepsilon_t|$)
Shocks vs “tamed” shocks

Policy shocks

shocks ($|\varepsilon_t|$)

“tamed” shocks ($|\tilde{h}_t^{1/2}\varepsilon_t|$)
Output growth w/o fat tails

### Chart Description

The chart illustrates the output growth without fat tails from 1965 to 2009. The y-axis represents percentage output growth, ranging from -3% to 4%, while the x-axis shows years from 1965 to 2009. The data is depicted with a series of peaks and troughs, indicating fluctuations in output growth over time.
Output growth w/o fat tails

Rolling Window Standard Deviation

Output Growth

percent

Consumption growth w/o fat tails
Consumption growth w/o fat tails

Rolling Window Standard Deviation
Hours w/o fat tails
Inference about $\sigma_t$, w/o and with fat tails

Discount shocks

SV

SV+Stud-$t$
Inference about $\sigma_t$, w/o and with fat tails

Policy shocks

SV

SV+Stud-$t$
Time-variation in the unconditional variance of output

SV

SV + Stud-t

Median: 1.87
Median: 2.18

Cúrdia, Del Negro, Greenwald

Rare Shocks

ESSIM; May 2012 29 / 30
Conclusions

• Strong evidence in favor of fat tails in linear DSGEs

  • Even when considering slow-moving stochastic volatility

  • Slow-moving stochastic volatility less important in the presence of rare large shocks

• Rare shocks matter

  • In the absence of rare shocks the evolution of GDP, hours, ... would have been similar to “just” a run-of-the-mill recession