Rare Shocks, Great Recessions

Vasco Cúrdia, Marco Del Negro, Daniel Greenwald
Federal Reserve Bank of New York and New York University

ESSIM; May 2012

Disclaimer: **The views expressed are mine and do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System**

Broad objectives of research agenda

- Show that shocks in linear models (VARs, DSGEs, factor models) estimated on macro variables have fat tails.
- Well known for financial variables, less for macro.

Broad objectives of research agenda

- Show that shocks in linear models (VARs, DSGEs, factor models) estimated on macro variables have fat tails.
- Well known for financial variables, less for macro.
- Why should we care?
  1. Inference using linear models turns out a bit different when allowing for fat tails.
  2. Can non-linearities explain away the fat tails?
  3. Move away from linear models? (For linear DSGEs, solution method is called into question)
  4. Implications for finance

The Great Moderation and its undoing (Great Recession)

- Main hypothesis so far:
  1. Time-variation in volatility
     - (Low frequency) changes in the volatility of shocks

The Great Moderation and its undoing (Great Recession)

- Main hypothesis so far:

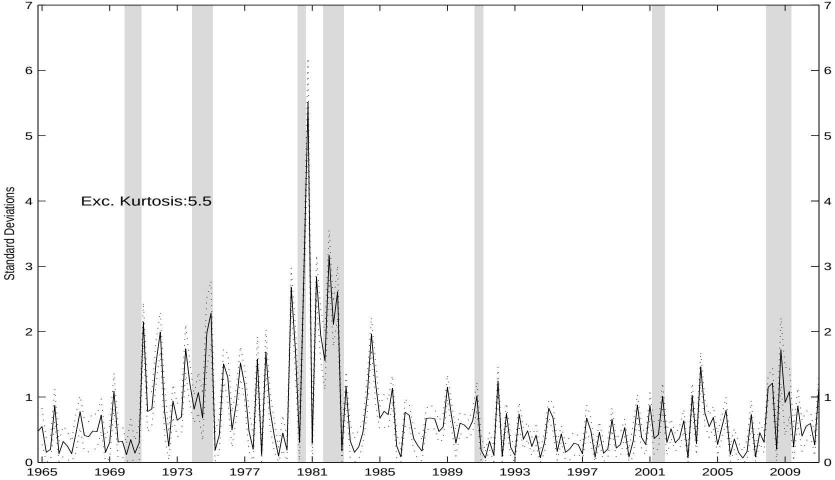
  1. Time-variation in volatility
     - (Low frequency) changes in the volatility of shocks

- Alternative:
  2. Rare large shocks
     - A time-invariant distribution that generates rare large events (high frequency bursts in volatility)

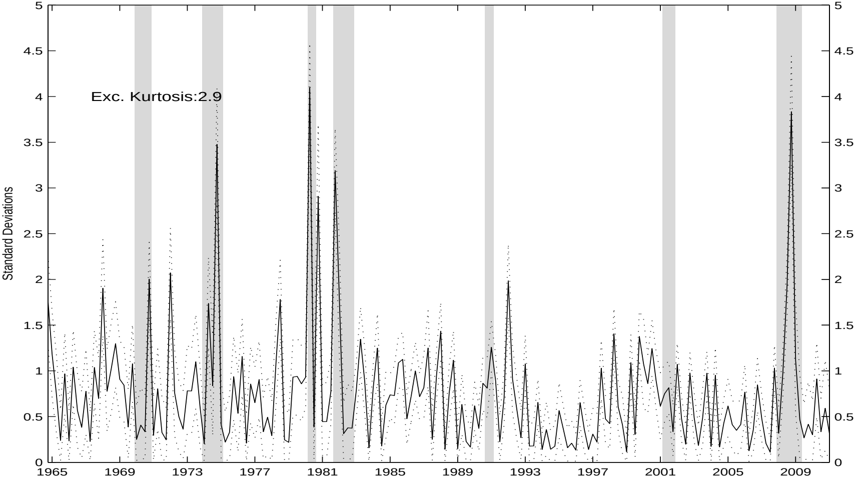
  We show that the two coexist: both low and high frequency changes in volatility

- The paper estimates a DSGE model with Student's t-distributed shocks . . . and time-varying volatility

Policy Shocks



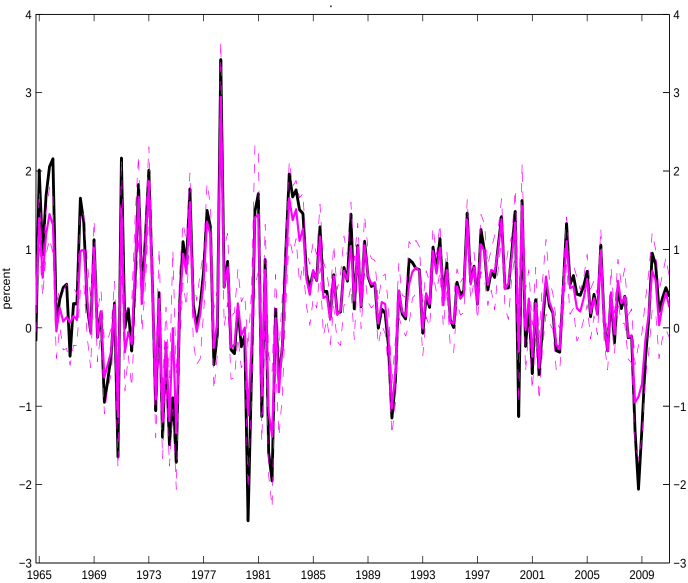
Discount rate shocks



Results

- Strong evidence that shock are Student-t.
- These rare shocks matter in terms of business cycle fluctuations (rare shocks, great recessions)

Results



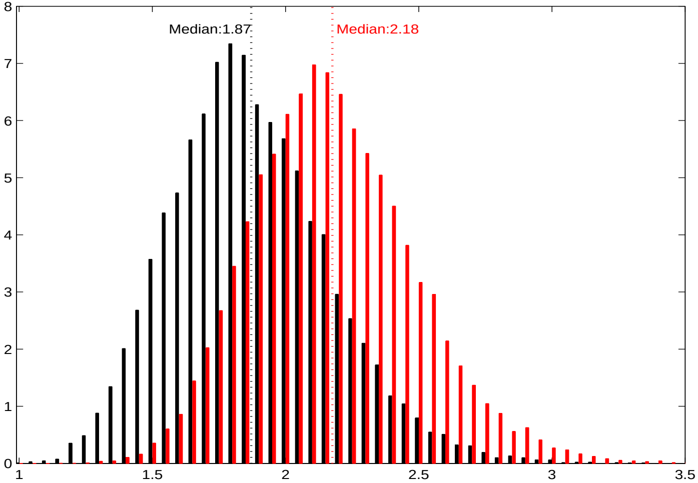
percent
4
3
2
1
0
−1
−2
−3
1965
1969
1973
1977
1981
1985
1989
1993
1997
2001
2005
2009

Results

- Strong evidence that shock are Student-t.
- These rare shocks matter in terms of business cycle fluctuations (rare shocks, great recessions)
- Slow-moving stochastic volatility less important in the presence of rare large shocks

Results

Great Moderation in output growth: St dev (1981) / St dev (1994)



Caveats

- Skewness (lack thereof)
- Non-linearities (lack thereof)

Literature

- Bayesian estimation with Student's t-distributed shocks: Geweke (1993, 2005)

$$y_t = x_t'\beta + \varepsilon_t,\ \varepsilon_t = \tilde{h}_t^{-1/2}\ \eta_t,\ \eta_t \sim \mathcal{N}(0,\sigma^2)$$

$$\lambda \tilde{h}_t \sim \chi^2(\lambda)$$

Literature

- Bayesian estimation with Student's t-distributed shocks: Geweke (1993, 2005)

$$y_t = x_t'\beta + \varepsilon_t,\ \varepsilon_t = \tilde{h}_t^{-1/2}\ \eta_t,\ \eta_t \sim \mathcal{N}(0, \sigma^2)$$

$$\lambda \tilde{h}_t \sim \chi^2(\lambda)$$

- Stochastic volatility in DSGE models: Justiniano & Primiceri (2008)

Literature

- Bayesian estimation with Student's t-distributed shocks: Geweke (1993, 2005)

$$y_t = x_t'\beta + \varepsilon_t,\ \varepsilon_t = \tilde{h}_t^{-1/2}\ \eta_t,\ \eta_t \sim \mathcal{N}(0, \sigma^2)$$

$$\lambda \tilde{h}_t \sim \chi^2(\lambda)$$

- Stochastic volatility in DSGE models: Justiniano & Primiceri (2008)
- Many other relevant papers: great moderation (∞), SV in DSGE (Fernandez-Villaverde and Rubio-Ramirez 2007)
- Chib and Ramamurthy (2011): Student-t but no SV

. . . about $\tilde{h}_t$s and λs (degrees of freedom)

- Student-t distributed shocks

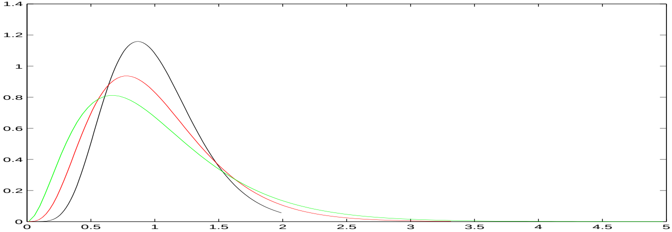
$$\varepsilon_t = \sigma_q \ \tilde{h}_t^{-1/2} \ \ \eta_t, \text{ where } \eta_t \sim \mathcal{N}(0,1) \text{ i.i.d., and}$$

- Prior on $\tilde{h}_t$:

$$\lambda_q \tilde{h}_t \sim \chi^2(\lambda_q)$$

... about $\tilde{h}_t$s and λs (degrees of freedom)

$$\tilde{h}_t \sim \chi^2(\lambda)/\lambda \;\Rightarrow E[\tilde{h}_t] = 1,\; Var[\tilde{h}_t] = 2/\lambda$$



Number of shocks larger (in abs. value) than x standard deviations per 200 periods

λ, x:	3	4	5
∞	.54	.012	$1e^{-4}$
15	1.79	.23	.03
9	2.99	.62	.15
6	4.80	1.42	.49

. . . about $\tilde{h}_t$s and λs (degrees of freedom)

- Student-t distributed shocks

$$\varepsilon_t = \sigma_q \ \tilde{h}_t^{-1/2} \ \ \eta_t, \text{ where } \eta_t \sim \mathcal{N}(0,1) \text{ i.i.d., and}$$

- Prior on $\tilde{h}_t$:

$$\lambda_q \tilde{h}_t \sim \chi^2(\lambda_q)$$

- Posterior on $\tilde{h}_t$:

$$\left[\lambda_q + \varepsilon_{q,t}^2/\sigma_q^2\right] \tilde{h}_{q,t} \ \sim \ \chi^2(\lambda_q + 1).$$

Estimating DSGEs with Student's t shocks + SV

- Measurement:

$$y_t = Z(\theta)s_t$$

- Transition:

$$s_{t+1} = T(\theta)s_t + R(\theta)\varepsilon_t$$

where θ are the DSGE parameters

- Shocks

$$\varepsilon_{q,t} = \sigma_q \;\; \sigma_{q,t} \;\; \tilde{h}_{q,t}^{-1/2} \; \eta_{q,t}$$

$$\eta_{q,t} \sim \mathcal{N}(0,1), \text{ i.i.d. across } q,\ t.$$

$$\log \sigma_{q,t} = \log \sigma_{q,t-1} + \zeta_{q,t},\ \sigma_{q,0} = 1,\ \zeta_{q,t} \sim \mathcal{N}(0,\omega_q^2)$$

$$\lambda_q \tilde{h}_{q,t} \sim \chi^2(\lambda_q)$$

Fat tails vs. Time-variation in volatility: some intuition

- (persistent) stochastic volatility

$$\varepsilon_t = \sigma_t \ \eta_t, \text{ where } \eta_t \sim \mathcal{N}(0,1) \text{ i.i.d., and}$$

Transition

$$\log \sigma_t = \log \sigma_{t-1} + \zeta_t, \ \sigma_{q,0} = 1, \ \zeta_t \sim \mathcal{N}(0, \omega_q^2)$$

Measurement

$$\log(\varepsilon_t^2) = 2 \log \sigma_t + \eta_t^*, \ \eta_t^* \sim \log(\chi_1^2)$$

Fat tails vs. Time-variation in volatility: some intuition

- (persistent) stochastic volatility $+$ Student-t

$$\varepsilon_t = \sigma_t \quad \tilde{h}_t^{-1/2} \, \eta_t, \text{ where } \eta_t \sim \mathcal{N}(0,1) \text{ i.i.d., and}$$

Transition

$$\log \sigma_t = \log \sigma_{t-1} + \zeta_t, \ \sigma_{q,0} = 1, \ \zeta_t \sim \mathcal{N}(0, \omega_q^2)$$

Measurement

$$\log(\ \tilde{h}_t \varepsilon_t^2) = 2 \log \sigma_t + \eta_t^*, \ \eta_t^* \ \sim \ \log(\chi_1^2)$$

Fat tails vs. Time-variation in volatility: some intuition

- (persistent) stochastic volatility + Student-t

$$\varepsilon_t = \sigma_t \ \tilde{h}_t^{-1/2}\eta_t, \text{ where } \eta_t \sim \mathcal{N}(0,1) \text{ i.i.d., and}$$

Transition

$$\log \sigma_t = \log \sigma_{t-1} + \zeta_t,\ \sigma_{q,0} = 1,\ \zeta_t \sim \mathcal{N}(0, \omega_q^2)$$

Measurement

$$\log(\varepsilon_t^2) = 2\log \sigma_t + \eta_t^*,\ \eta_t^* \ \sim \ \log(\chi_1^2)$$

- Stochastic volatility affects inference about fat tails:

$$\left[\lambda_q + \varepsilon_{q,t}^2/\sigma_{q,t}^2\right] \tilde{h}_{q,t} \sim \chi^2(\lambda_q + 1).$$

Measurement

- Measurement equation:

$$
\begin{array}{lcl}
\textit{Output growth} & = & LN((GDPC)/LNSINDEX) * 100 \\
\textit{Consumption growth} & = & LN((PCEC/GDPDEF)/LNSINDEX) * 100 \\
\textit{Investment growth} & = & LN((FPI/GDPDEF)/LNSINDEX) * 100 \\
\textit{Real Wage growth} & = & LN(PRS85006103/GDPDEF) * 100 \\
\textit{Hours} & = & LN((PRS85006023 * CE16OV/100)/LNSINDEX) \\
 & & *100 \\
\textit{Inflation} & = & LN(GDPDEF/GDPDEF(-1)) * 100 \\
\textit{FFR} & = & FEDERAL\ FUNDS\ RATE/4
\end{array}
$$

Sample 1964:Q1–2011:Q1

- Same prior on θ as SW.

Incorporating 10-yrs inflation expectations from surveys

- SW forecasts inflation relatively well but ... somewhat tight prior on π^*:$\sim$ *Gamma*$(.62, .10)$.
- No need of such a prior: Use a loose prior ($\pi^* \sim$ *Gamma*$(.75, .40)$) and survey data as an observable:

$$\begin{aligned}\pi_t^{O,40} &= \pi_* + \boldsymbol{E}_t\left[\frac{1}{40}\sum_{k=1}^{40}\pi_{t+k}\right] \\ &= \pi_* + \frac{1}{40}Z(\theta)_{(\pi,.)}(I - T(\theta))^{-1}\left(T(\theta) - T(\theta)^{41}\right)s_t,\end{aligned}$$

- ... and change the model to be able to explain it:

$$\begin{aligned}R_t &= \rho_R R_{t-1} + (1-\rho_R)\left(\psi_1(\pi_t - \pi_t^*) + \psi_2(y_t - y_t^f)\right) \\ &\quad +\psi_3\left((y_t - y_t^f) - (y_{t-1} - y_{t-1}^f)\right) + r_t^m,\end{aligned}$$

where $\pi_t^* = \rho_{\pi^*}\pi_{t-1}^* + \sigma_{\pi^*}\epsilon_{\pi^*,t}$.

Fit

	Without Stochastic Volatility	With Stochastic Volatility
Gaussian shocks		
	-973.3	-812.1
Student-t distributed shocks ($\underline{\nu} = 4$)		
$\underline{\lambda} = 15$	-810.5	-762.0
$\underline{\lambda} = 9$	-796.7	-765.1
$\underline{\lambda} = 6$	-780.3	-753.0

Fit

	Without Stochastic Volatility	With Stochastic Volatility
Gaussian shocks		
	-973.3	-812.1
Student-t distributed shocks ($\underline{\nu} = 4$)		
$\underline{\lambda} = 15$	-810.5	-762.0
$\underline{\lambda} = 9$	-796.7	-765.1
$\underline{\lambda} = 6$	-780.3	-753.0

Fit – Loose prior on degrees of freedom

Gaussian shocks		
	-973.3	-812.1
Student-t distributed shocks		
	$\underline{\nu} = 4$ *(tighter prior)*	$\underline{\nu} = 1$ *(flat prior)*
$\underline{\lambda} = 15$	-762.0	-759.9
$\underline{\lambda} = 9$	-765.1	-754.5
$\underline{\lambda} = 6$	-753.0	-751.7

Prior on degrees of freedom λ is Gamma($\underline{\lambda}/\underline{\nu}$,$\underline{\nu}$):

$$p(\lambda_q|\underline{\lambda},\underline{\nu}) = \frac{(\underline{\lambda}/\underline{\nu})^{-\underline{\nu}}}{\Gamma(\underline{\nu})}\lambda_q^{\underline{\nu}-1} exp(-\underline{\nu}\frac{\lambda_q}{\underline{\lambda}}),\ \ q = 1, .., \mathrm{q}.$$

Estimated degrees of freedom, Student-t

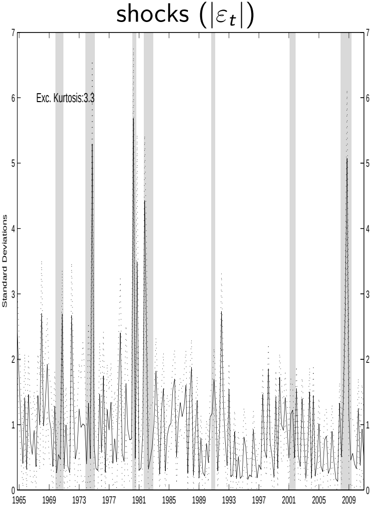
	Without Stochastic Volatility	*With Stochastic Volatility*
g	6.1 (2.5,9.9)	5.6 (2.1,9.2)
b	4.7 (2.5,7.0)	5.4 (2.4,8.4)
μ	5.4 (2.5,8.5)	6.5 (2.4,10.7)
z	3.7 (1.8,5.6)	4.3 (1.7,7.2)
λ_f	6.8 (2.5,11.3)	12.7 (4.2,21.2)
λ_w	5.5 (2.5,8.5)	4.8 (2.4,7.1)
r^m	2.7 (1.7,3.7)	10.9 (3.7,18.0)
π^*	1.7 (1.2,2.2)	9.0 (2.9,15.6)

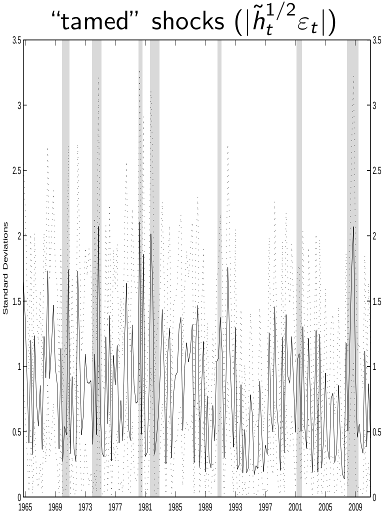
Fit – pre-Great Recession sample (2004:Q4)

	Constant Volatility	Stochastic Volatility
Gaussian shocks		
	-864.7	-742.1
Student-t distributed shocks, prior with 1 degree of freedom		
$\lambda = 15$	-731.9	-709.9
$\lambda = 9$	-731.8	-705.3
$\lambda = 6$	-729.58	-700.9

Shocks vs "tamed" shocks

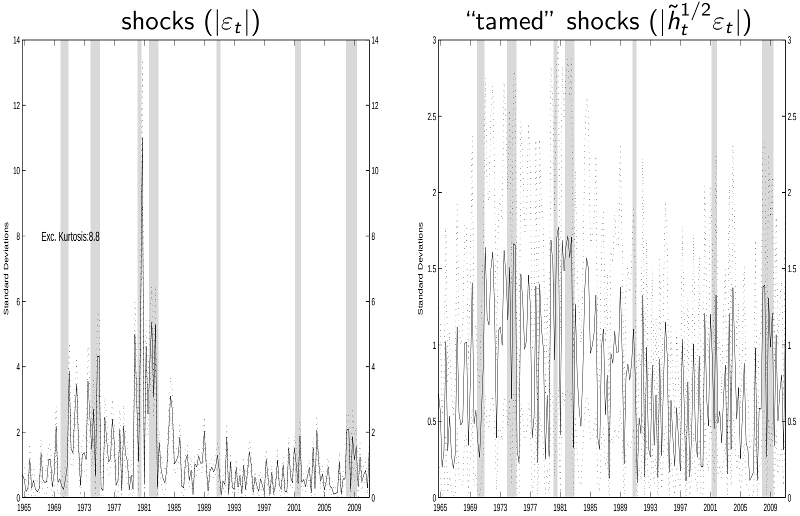
Discount rate shocks



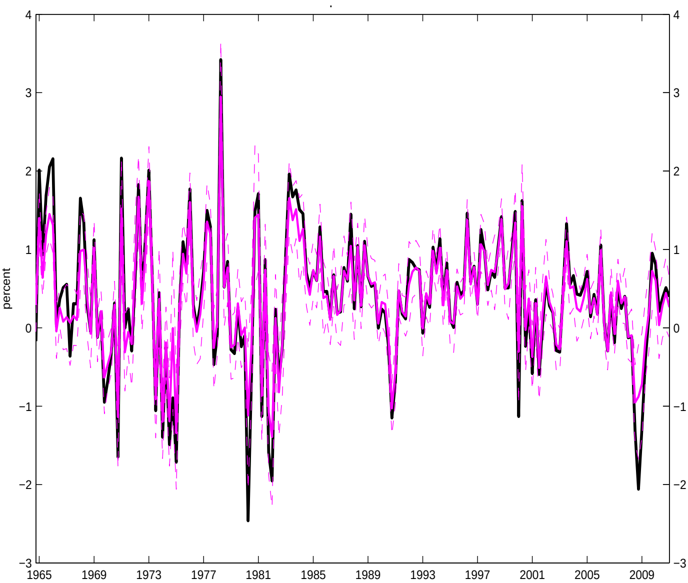


Shocks vs "tamed" shocks

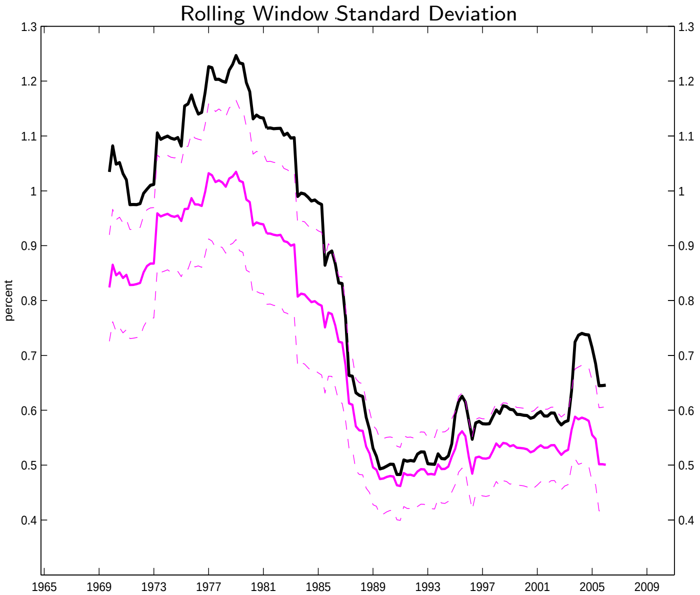
Policy shocks



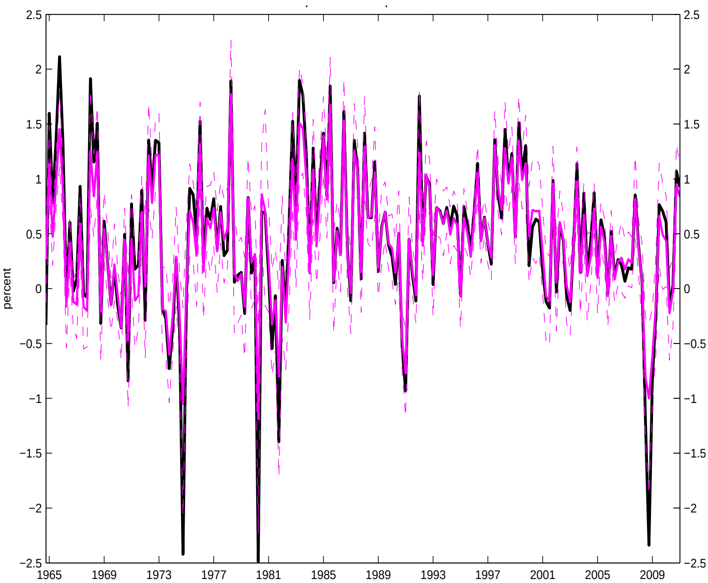
Output growth w/o fat tails



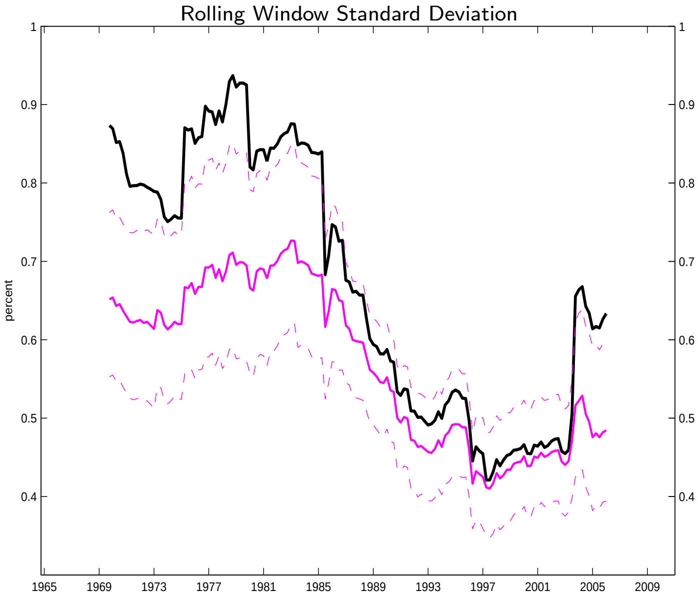
Output growth w/o fat tails



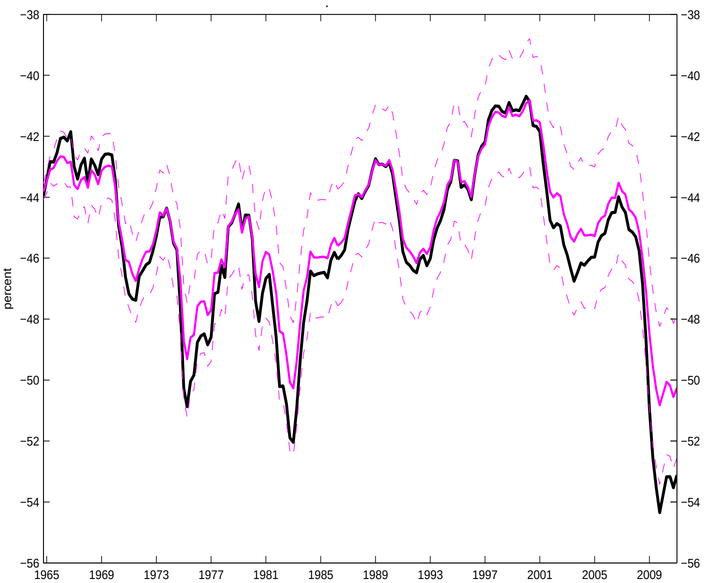
Consumption growth w/o fat tails



Consumption growth w/o fat tails



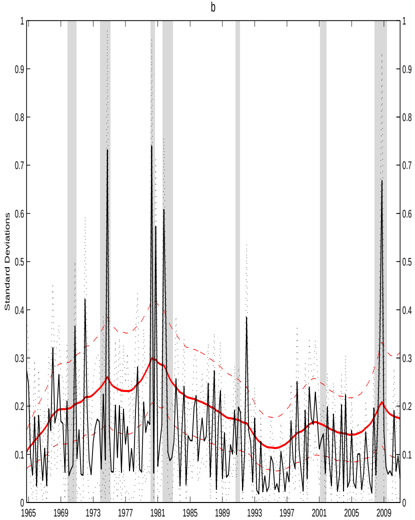
Hours w/o fat tails



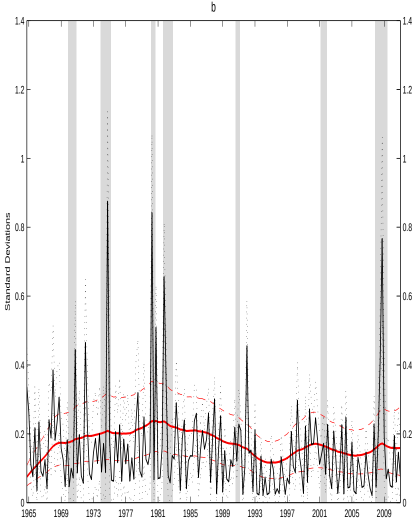
Inference about σ_t, w/o and with fat tails

Discount shocks

SV



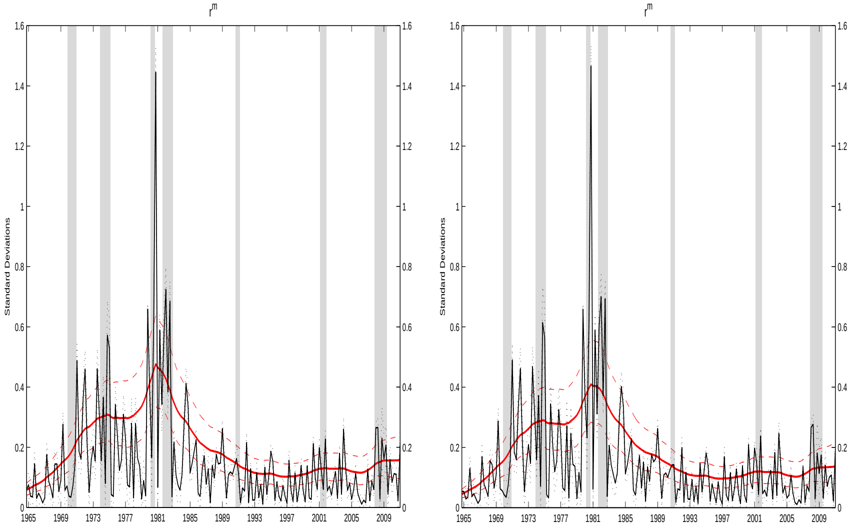
SV+Stud-t



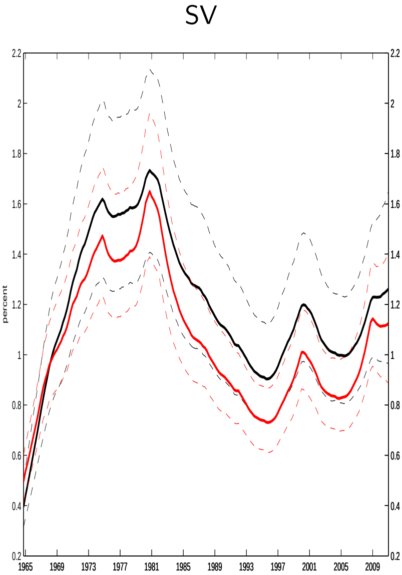
Inference about σ_t, w/o and with fat tails

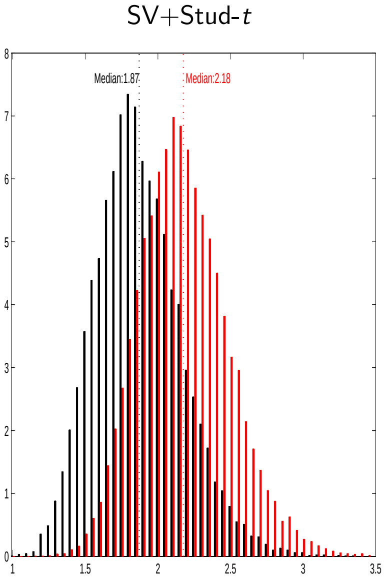
Policy shocks

SV | SV+Stud-*t*



Time-variation in the unconditional variance of output





Conclusions

- Strong evidence in favor of fat tails in linear DSGEs
  - Even when considering slow-moving stochastic volatility
  - Slow-moving stochastic volatility less important in the presence of rare large shocks
- Rare shocks matter
  - In the absence of rare shocks the evolution of GDP, hours, ... would have been similar to “just” a run-of-the-mill recession