

*Great Moderation or Great Mistake: Can  
overconfidence in low macro-risk explain the  
boom in asset prices?*

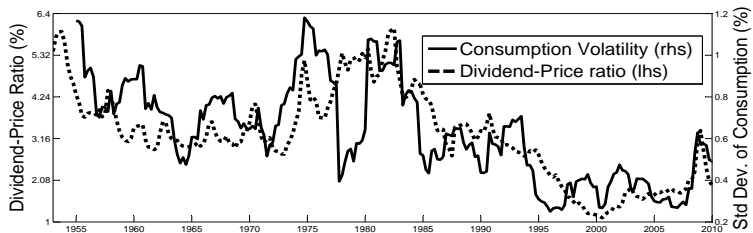
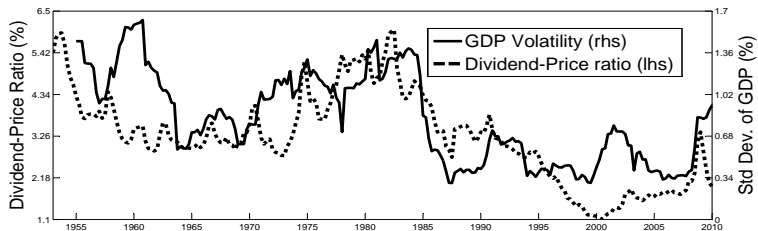
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Afroditi Kero, University of Cyprus

ESSIM 2012

# *Motivation*

# *1. Macroeconomic Volatility and Asset Prices*

## Macro-volatility and US PD ratio



## *2. Uncertainty about origin of GM*

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- But: uncertainty about origin and persistence of GM
  1. Academic literature: “Good Luck or Good Policy”, better inventory management, globalisation, financial liberalisation or development, etc.
  2. Market Participants:

*“The ongoing deterioration in surprise risk should be seen as one of the arguments behind the declining risk premium. Whether this is due to a more effective central bank policy, a major improvement in the forecast ability of economic observers around the globe, sheer luck or maybe a mix of all three factors can't finally be answered.”*

Unicredit (2006)

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*“But what matters is how market participants responded to these benign conditions. They are faced with what is, in essence, a complex signal-extraction problem. But whereas many such problems in economics involve learning about first moments of a distribution, this involves making inferences about higher moments. The longer such a period of low volatility lasts, the more reasonable it is to assume that it is permanent. But as tail events are necessarily rarely observed, there is always going to be a danger of underestimating tail risks.”*

Charles Bean, European Economic Association, 25 August 2009

### 3. *“Overconfidence” and the Great Mistake*

*“From the Great Moderation to the Great Conflagration: The decline in volatility led the financial institutions to underestimate the amount of risk they faced, thus essentially (though unintentionally) reintroducing a large measure of volatility into the market.”*

Thomas F. Cooley, Forbes.com, 11 December 2008

*The stress-tests required by the authorities over the past few years were too heavily influenced by behavior during the Golden Decade. [...] The sample in question was, with hindsight, most unusual from a macroeconomic perspective. The distribution of outcomes for both macroeconomic and financial variables during the Golden Decade differed very materially from historical distributions.”*

Andrew Haldane, Bank of England, 13 February 2009

## Motivation

- Late 80s / early 90s
  - Abrupt fall in macro-volatility (to  $\approx 40 - 45\%$  of pre-GM StDev) followed by strong gradual rise in PD ratio (75 – 200%)
  - GM noticed by academics and market-participants, but uncertainty about its origin and persistence
- Recent crisis
  - Abrupt rise in volatility and fall in PD ratio
  - Policymakers and academics blame *overconfidence* in benign macro-environment for *overvaluation* in asset prices

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- Research Questions
  1. Can learning significantly increase “confidence” in the Great Moderation?
  2. How large is the resulting boom in asset prices relative to that observed in the data?
  3. Is there “overvaluation” relative to full information / no learning?

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- Non-linearity of prices in transition probabilities gives special role to uncertainty about persistence

## *Most related literature*

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### 2. Lettau et al (2008 - hf LLW) “The Declining Equity Premium: What Role Does Macroeconomic Risk Play?”

- Switch to a regime of low volatility as in US leads to asset price boom only if it is (essentially) permanent
- Uncertainty about current regime (posterior probabilities estimated in statistical model): boom muted, rel to full info

# *This paper*

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- Bayesian learning about persistence of volatility regimes in LLW asset pricing model

# *Outline*

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## *1.* Model



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2. Quantitative results

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3. Sensitivity analysis

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- Infinitely-lived representative agent
- Preferences over  $\{C_t\}_{t=0}^{\infty}$ :

$$U_t = [(1 - \beta)C_t^{\frac{1-\gamma}{\alpha}} + \beta(E_t U_{t+1}^{1-\gamma})^{\frac{1}{\alpha}}]^{\frac{\alpha}{1-\gamma}}$$

with  $\alpha = \frac{1-\gamma}{1-\frac{1}{\psi}}$  and  $\psi$  IES

## *HH problem*

$$\max_{C_t, S_t} U_t$$

$$s.t. S_t P_t + C_t = S_{t-1}(P_t + D_t)$$

$$S_0 \text{ given}$$

## *First order condition*

$$P_t = E_t[M_{t+1}(P_{t+1} + D_{t+1})] \quad (1)$$

$$\implies p_t \equiv \frac{P_t}{D_t} = E_t[M_{t+1}(p_{t+1} + 1)\frac{D_{t+1}}{D_t}] \quad (2)$$

$$\text{with } M_{t+1} = \left(\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\right)^\alpha R_{w,t+1}^{\alpha-1}$$

## *Consumption growth process with volatility regimes*

$$\Delta \ln C_t = \bar{g} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 \in \{\sigma_l^2, \sigma_h^2\}$$



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- $\Pr(\sigma_{t+1}^2 = \sigma_i^2 \mid \sigma_t^2 = \sigma_j^2)$  given by

$$\mathbf{F} = \begin{bmatrix} F_{ll} & 1 - F_{ll} \\ 1 - F_{hh} & F_{hh} \end{bmatrix}$$

## *“Leveraged” Dividend Process*

As in Campbell 1986, Abel 1999, LLW 2008, dividend growth is (potentially) more volatile than consumption

$$\Delta \ln D_t = \bar{g} + \lambda \varepsilon_t \quad \lambda \geq 1$$

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- Can calculate  $p_h, p_l$  as fixed points to (3)

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    - $F_{hh} = \widehat{F}_{hh}, F_{ll} = \widehat{F}_{ll}$  known
    - But small (conditional) prior probability  $\widehat{p}$  of structural break to permanent  $\sigma_l^2$

# *1. Learning about Transition Probabilities*

## *Beta Prior, Likelihood and Posterior*

- Beta distribution on  $[0, 1]$  summarises prior information  $\Sigma^0$

$$f(F_{hh} | \Sigma^0) = \text{beta}(n_0^{hh}, n_0^{hl}) \propto F_{hh}^{n_0^{hh}} (1 - F_{hh})^{n_0^{hl}}$$

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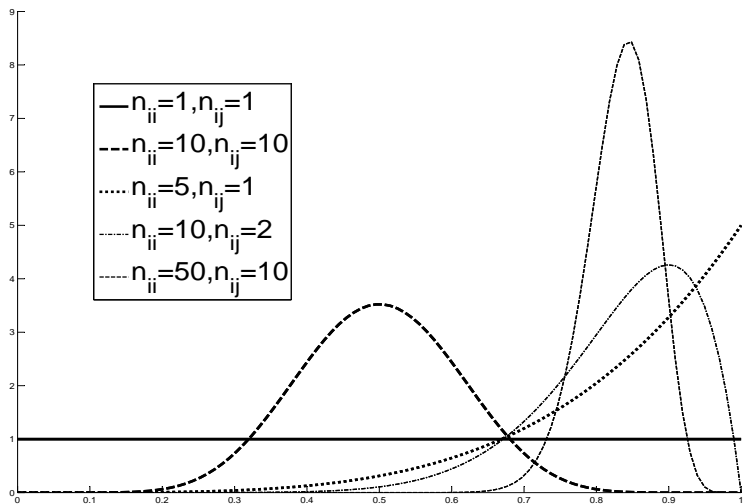
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- Yields beta posterior with updated “counters”  $\hat{n}_t^{ij} = n_t^{ij} + n_0^{ij}$ .

# *pdf of Beta Distribution*



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- $\text{Prob}(\sigma_{t+1}^2 = \sigma_i^2 | \Sigma_t) = \int F_{ij} f(F | \Sigma_t) dF$ , when  $\sigma_t^2 = \sigma_j^2$  so

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- True transition probabilities play no role
- Expectations over beta distribution give role to non-linearity of full-info asset prices in  $F_{ll}, F_{hh}$

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- Posterior

$$\begin{aligned} P(F_{ll} = 1 | \sigma_l^N) &= \frac{P(F_{ll} = 1 \wedge \sigma_l^N)}{P(F_{ll} = 1 \wedge \sigma_l^N) + P(F_{ll} = \widehat{F}_{ll} \wedge \sigma_l^N)} \\ &= \frac{\widehat{p}}{\widehat{p} + \widehat{F}_{ll}^N (1 - \widehat{p})} \end{aligned} \quad (5)$$

## 2. Learning about a structural break to permanent GM

- Agents know “normal” transition probabilities  $\widehat{F}_{ll}, \widehat{F}_{hh}$
- When  $\sigma_h^2 \rightarrow \sigma_l^2$ , prior probability of permanent low volatility  
 $P(F_{ll} = 1 | \Sigma_0) = \widehat{p}$
- Likelihood of low-volatility sequence in “normal” times

$$L(\{\sigma_s^2 = \sigma_l^2, s = t, t + 1, \dots, t + N | \sigma_t^2 = \sigma_l^2, F_{ll} = \widehat{F}_{ll}\}) = \widehat{F}_{ll}^N$$

- Posterior

$$\begin{aligned} P(F_{ll} = 1 | \sigma_l^N) &= \frac{P(F_{ll} = 1 \wedge \sigma_l^N)}{P(F_{ll} = 1 \wedge \sigma_l^N) + P(F_{ll} = \widehat{F}_{ll} \wedge \sigma_l^N)} \\ &= \frac{\widehat{p}}{\widehat{p} + \widehat{F}_{ll}^N (1 - \widehat{p})} \end{aligned} \quad (5)$$

- Bayesian Approach:  $p_t$  function of  
 $\{p_{ls}, p_{hs}, P(F_{ll} = 1 | \Sigma_s)\}_{s=t}^{\infty}$ .

2. *Learning about a structural break to permanent GM -  
Discussion*



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- Likelihood  $\widehat{F}_{II}^t$  falls geometrically

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- Likelihood  $\widehat{F}_{II}^t$  falls geometrically
- True transition probability  $\widehat{F}_{II}$  is important
- First switch to high volatility reveals “no structural break”

# *Quantitative results*

# *The exercise*

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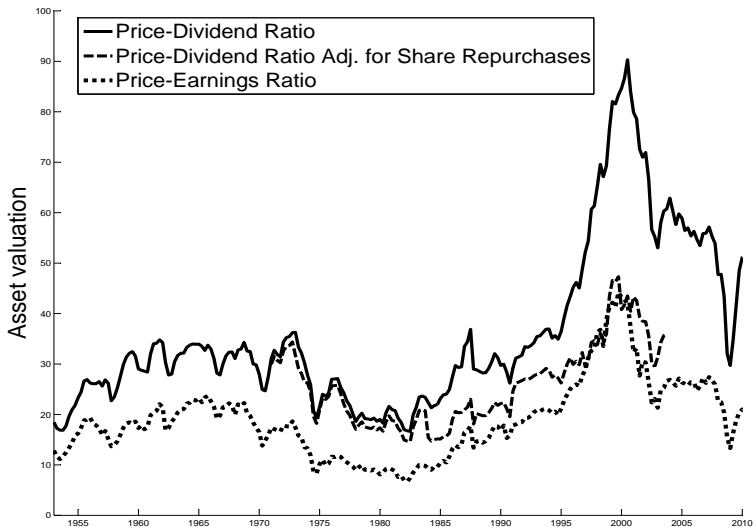
## *The exercise*

- Analyse scenario of volatility regimes similar to post-WW II US experience
  1. High volatility regime 1952Q2 - 1984Q4
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  3. Assume: Crisis marks return to high volatility regime

## *The exercise*

- Analyse scenario of volatility regimes similar to post-WW II US experience
  1. High volatility regime 1952Q2 - 1984Q4
  2. Great Moderation 1985Q1 - 2006Q4
  3. Assume: Crisis marks return to high volatility regime
- Compare price-dividend ratios across
  - Data
  - 2 learning schemes
  - Full information price based on ex-post estimate of  $F$

## *Different Valuation Measures*



## *Calibration of preferences*

$\beta$	0.9935	Discount Factor	$R = 1.95\%$ pre-GM
$\gamma$	30	Risk Aversion	$\approx$ PD-ratio pre-GM
$\psi$	1.5	IES	$> 1$ to get $\frac{dp}{d\sigma^2} < 0$

## Calibration of consumption and dividend process

$\bar{g}$	0.0059		Mean $\Delta C$	US data 1952 - 2010
$\sigma_l$	0.0037		Low Vol	US data 1985 - 2006
$\sigma_h$	0.0082		High Vol	US data 1952 - 1984
$\lambda$	4.5		Leverage	LLW
Ex-post $\mathbf{F}$	0.989	$1 - 0.989$	Trans Prob	$E[T_{\sigma_h}] = 1984 - 1952$
	$1 - 0.992$	0.992		$E[T_{\sigma_l}] = 2006 - 1985$
	<b>Comparison</b>			
$\mathbf{F}^{LLW}$	0.991	$1 - 0.991$	LLW (2008)	Est Reg Switch model
	$1 - 0.994$	0.994		

# *Results I*

## *Learning about Transition Probabilities*

# *Calibration of learning parameters $n_0^{ij}$*

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1. Benchmark: Uninformative uniform prior  $n_0^{ij} = 1, i, j \in \{h, l\}$



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## *Calibration of learning parameters $n_0^{ij}$*

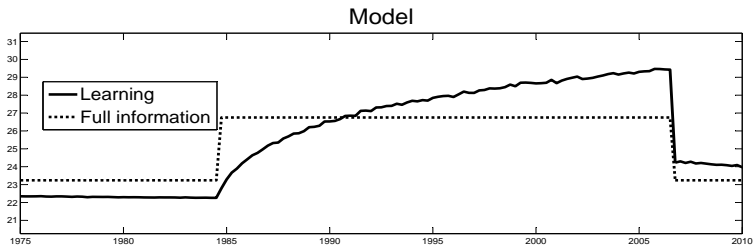
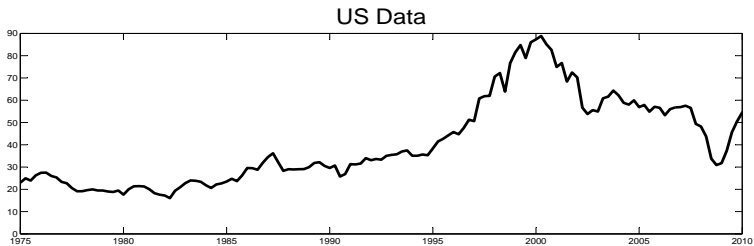
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  - Uninformative prior after WWII:  $n_0^{hh} = n_0^{hl} = 1$

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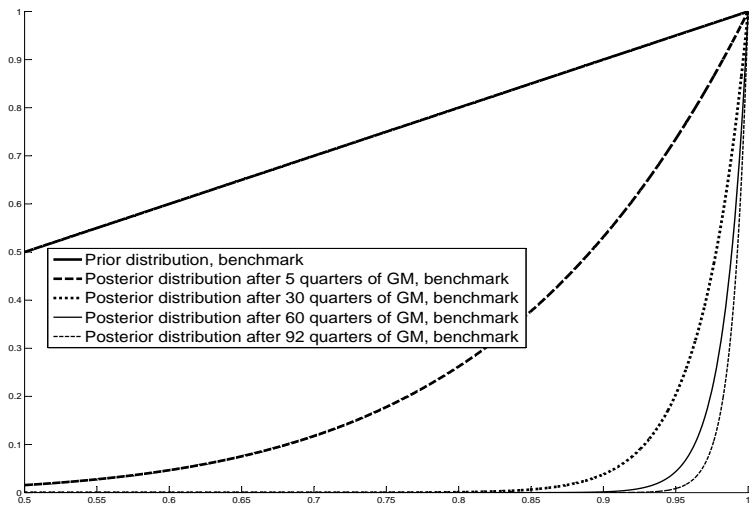
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2. Alternative Assumption: Observed persistence of high-volatility regime increases prior persistence of low volatility
  - Uninformative prior after WWII:  $n_0^{hh} = n_0^{hl} = 1$
  - But persistent prior for GM with mean  $n_0^{ll}/(n_0^{ll} + n_0^{lh}) = 0.9$

# *Results*

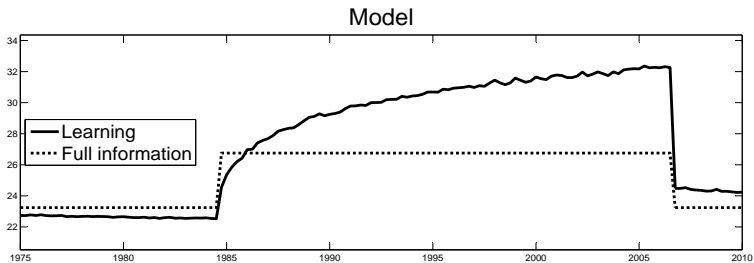
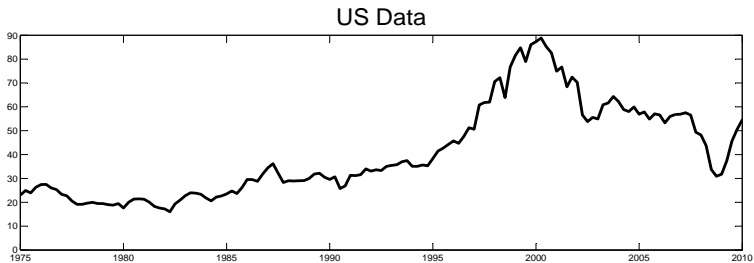
## *Dividend Ratios: Benchmark Model*



## Posterior CDFs: Benchmark Model



## *Dividend Ratios: Persistent prior for GM*



## Results

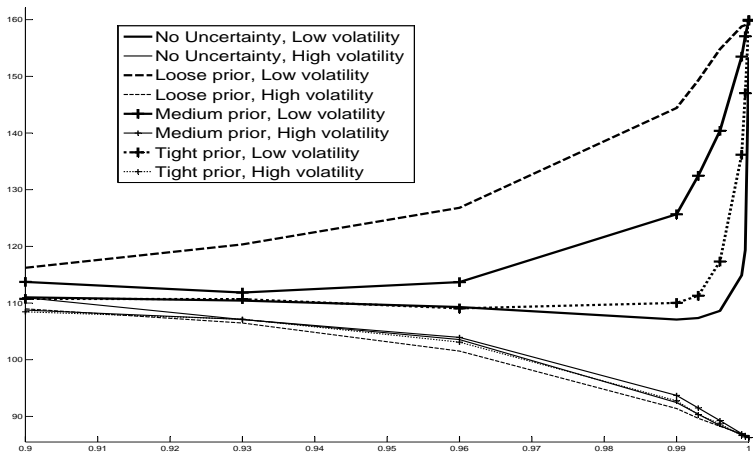
### Asset Prices - Learning about Transition Probabilities

	Boom	Overvaluation	Bust
Full Info	15%	0	15%
Uninformed prior	32%	11%	22%
Moderately persistent prior	42%	20%	32%



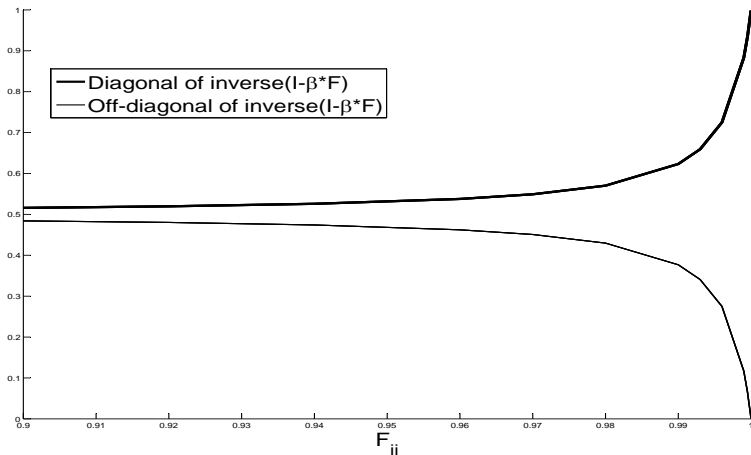
*Discussion: Mean and Variance Effects of Learning  
about Transition Probabilities*

# *Dividend Ratios as a Function of (symmetric) mean Persistence and Prior Tightness*



Persistence  $F_{ll}, F_{hh}$ , Posterior means

# Entries of $(I - \beta\mathbb{F})^{-1}$ for symmetric $\mathbb{F}$



## *Results II*

*Learning about a structural break to permanent GM*

## *Calibration of learning parameters*

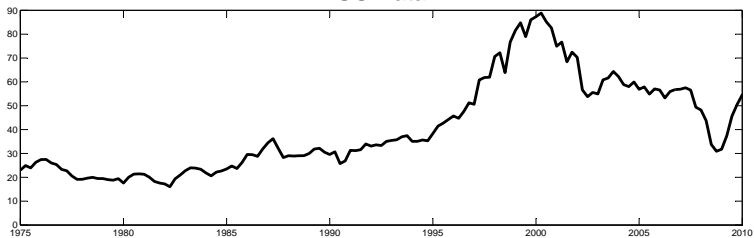
- Need:
  1. Prior probability of structural break  $\hat{p}$
  2. Trans probabilities  $\hat{F}_{ll}, \hat{F}_{hh}$  in “normal times”

## *Calibration of learning parameters*

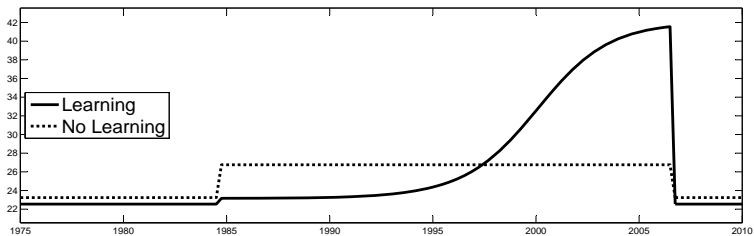
- Conditional prior probability of structural break: 1 percent
- Transition probabilities in “normal” times:
  1. Take ex-post estimate of  $F_{hh}$ .
  2. Choose  $\widehat{F}_{ll}$  to imply “suspicion” about structural break in mid-1990s:  
$$\text{Prob}(\sigma_t^2 = \sigma_l^2, t = 1, \dots, 48 | \widehat{F}_{ll}) = 0.1$$
  3. Yields  $\widehat{F}_{ll} = 0.87$

# Dividend Ratios

## US Data



## Model



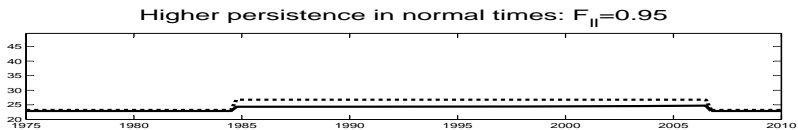
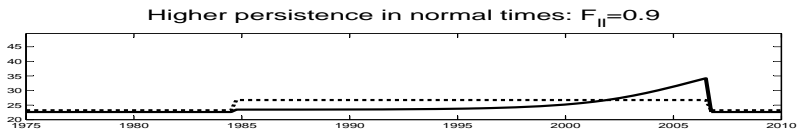
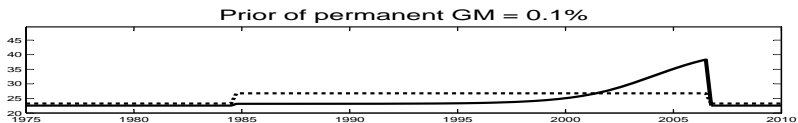
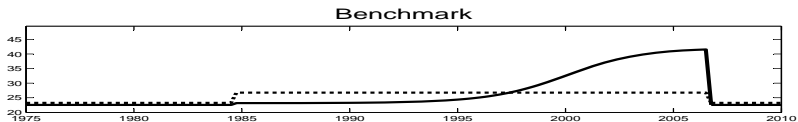
## *Results*

### **Asset prices - learning about structural break**

	Boom	Overvaluation	Bust
Full Info	3%	0	3%
Struct Break	77%	79%	84%



## *Struct-break learning: Alternative priors*



# *Sensitivity Analysis*

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## 1. Non-Bayesian, ad-hoc Learning

# *Sensitivity Analysis*

1. Non-Bayesian, ad-hoc Learning
2. Risk Aversion and Dividend Volatility

# *Ad-hoc / recursive learning*

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- Agents ignore 2-regime nature of dividend process, use simple rules to infer “average” volatility of dividends

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## *Ad-hoc / recursive learning*

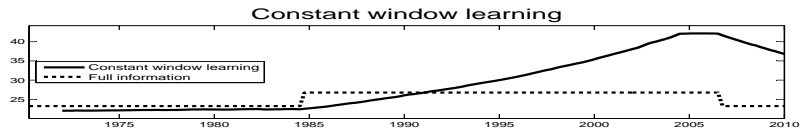
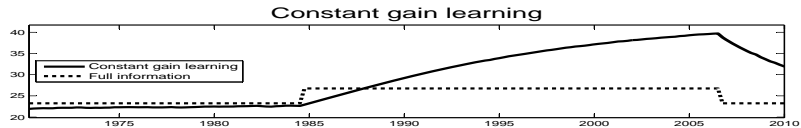
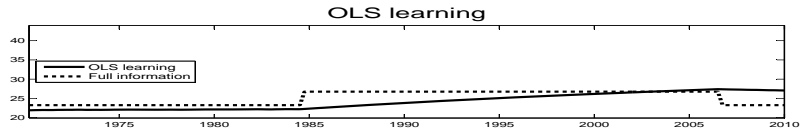
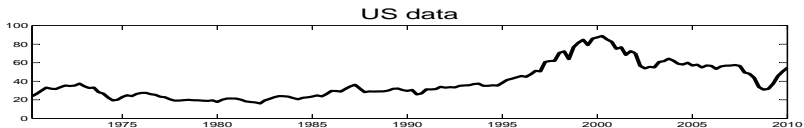
- Agents ignore 2-regime nature of dividend process, use simple rules to infer “average” volatility of dividends
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  2. OLS estimate on history of constant length



## *Ad-hoc / recursive learning*

- Agents ignore 2-regime nature of dividend process, use simple rules to infer “average” volatility of dividends
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  2. OLS estimate on history of constant length
  3. Constant gain learning: higher weight on recent observations

# Ad-Hoc Learning: Results



*Sensitivity: lower leverage and risk aversion*

## *Sensitivity: lower leverage and risk aversion*

### **Asset prices - Lower Risk Aversion and Dividend Volatility**

	Boom	Overvaluation	Bust
Full Info (benchmark)	15%	0	15%
Full Info ( $\gamma = 20$ )	12%	0	12%
Full Info ( $\lambda = 2.5$ )	8%	0	7%
Trans Probabilities (benchmark)	32%	11%	22%
Trans Probabilities ( $\gamma = 20$ )	30%	9%	18%
Trans Probabilities ( $\lambda = 2.5$ )	17%	6%	11%
Perm vs Trans (benchmark)	77%	79%	84%
Perm vs Trans ( $\gamma = 20$ )	59%	56%	59%
Perm vs Trans ( $\lambda = 2.5$ )	45%	43%	45%

## Conclusion

- Standard asset pricing model with Bayesian learning responds with strong boom and bust to GM
- Much of this is due to *overconfidence*, as effect of GM on full information prices is small
- Non-linearity of prices in regime-persistence gives special role to uncertainty about transition probabilities
- Bayesian assumption crucial for strong bust

*Great Moderation or Great Mistake: Can  
overconfidence in low macro-risk explain the  
boom in asset prices?*

Tobias Broer, IIES Stockholm University and CEPR  
Afroditi Kero, University of Cyprus

ESSIM 2012

## *Moments of Consumption Growth*

<i>Date</i>	<i>Mean</i>	<i>StDev</i>
1952Q2 : 1991Q4	0.57%	0.82%
1992Q1 : 2006Q4	0.61%	0.36%
2007Q1 : 2010Q2	-0.19%	0.50%

## *Moments of GDP growth*

<i>Date</i>	<i>Mean</i>	<i>StDev</i>
1952Q2 : 1983Q4	0.53%	1.1%
1984Q1 : 2006Q4	0.51%	0.51%
2007Q1 : 2010Q2	-0.16%	0.90%



## Full information asset prices

$$\begin{aligned} \rho_{\sigma_t^2 = \sigma_i^2} &= \rho_i^{1-a} \beta^a e^{\left(-\frac{a}{\psi} + a\right) \bar{g}} \\ &\{F_{ii} e^{\frac{\left(-\frac{a}{\psi} + a - 1 + \lambda\right)^2}{2}} \sigma_i^2 (1 + \rho_i)^{a-1} (1 + p_i) \\ &+ F_{ij} e^{\frac{\left(-\frac{a}{\psi} + a - 1 + \lambda\right)^2}{2}} \sigma_j^2 (1 + \rho_j)^{a-1} (1 + p_j)\} \end{aligned}$$

with

$$\begin{aligned} \rho_{\sigma_t^2 = \sigma_i^2}^a &= \left(\frac{P_{it}^C}{C_{it}}\right)^a = \rho_i^{1-a} \beta^a e^{\left(-\frac{a}{\psi} + a\right) \bar{g}} \\ &\left(F_{ii} e^{\frac{\left(-\frac{a}{\psi} + a\right)^2}{2}} \sigma_i^2 (1 + \rho_i)^a + F_{ij} e^{\frac{\left(-\frac{a}{\psi} + a\right)^2}{2}} \sigma_j^2 (1 + \rho_j)^a\right) \end{aligned}$$

# *Ad-hoc learning: Lower gain and longer windows*

