

# Optimal Sovereign Debt Default

Klaus Adam

Michael Grill

Mannheim University & CEPR

Deutsche Bundesbank

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- Provide a normative & quantitative analysis of sovereign default:

When is it optimal for a sovereign to default?

- To the best of our knowledge:

First paper determining fully optimal sovereign default

=> **Ramsey policy with full commitment**

- **Dominant view:** sovereign default (SD) is inefficient

## **Policy discussions:**

ex-post costs associated with SD

default 'inefficient'  $\Rightarrow$  private plans in disarray, fin. & ec. collapse

## **Academic view:**

limited commitment: SD ex-post efficient but ex-ante inefficient

Ramsey policy literature: full repayment assumed & no SD

- Important economic role for SD: resource transfer in times of scarcity  
  
SD *ex-ante efficient* if other insurance mechanisms more costly or unavailable
- Government bond markets incomplete:  
  
partial repayment can be optimal under commitment

- Ramsey policy literature:

Assumes market incompleteness, but also *assumes* full repayment

Aiyagari et al. (2002), Angeletos (2002), Chari et al. (1991), Sims (2001), Schmitt-Grohe & Uribe (2004), Adam (2011)

- Show that full repayment is inconsistent with optimality for empirically plausible levels of 'default costs'
- Sizable welfare gains possible: 1-2% cons. equiv.  
Even if default costs are large!
- Incompleteness arises endogenously from contracting frictions.

- Quantitative analysis:  
Default tends to be suboptimal following BC cycle-sized shocks, unless country close to maximally sustainable net foreign debt position.
- Full repayment assumption:  
'Reasonable' approx. to opt. repayment decisions  
An exact approximation only for very high levels of default costs
- Introduce economic disaster risk (Barro and Jin (2011):  
Default optimal following occurrence of a disaster shock  
Optimal even if far from maximal net foreign debt position  
& even if sufficient resources for repayment available!

# Outline of Remaining Talk

- 1 Introduce the model
- 2 Contracting frictions and optimal debt contract
- 3 Ramsey problem
- 4 Quantitative results & welfare analysis

# Model Setup: Households and Firms

- Small open economy with investment & shocks to domestic productivity
- Representative risk-averse consumer:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subsistence consumption:  $c_t \geq \bar{c} \geq 0$

- Domestic income risk:

$$y_t = z_t k_{t-1}^{\alpha},$$

$z_t \in Z = \{z^1, \dots, z^N\}$ , transition law  $\pi(z'|z)$

- Risk neutral foreign investors



# Model Setup: Government

- Fully committed government max'es utility of representative HH
- Seeks to smooth consumption implications of domestic income risk
- Can accumulate & decumulate foreign bonds/reserves:  
assumed risk free 1 period bonds (longer maturities possible)
- Internationally borrow by issuing own debt contracts:  
arbitrary repayment profiles allowed

# Model Setup: Government Debt Contracts

- Debt contract consists of an **explicit & implicit** component
  - explicit repayment profile: specified in the legal text of the contract
  - implicit repayment profile: not formalized, but commonly(!)  
understood
- **Default:** implicitly promised payment falls short of explicit one

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  - to protect the gov from such action: needs to reach an *explicit* legal settlement: costly => default costs.

# Model Setup: Government Debt Contracts

- Panizza, Sturzenegger, Zettelmeyer (JEL, 2009):  
'sovereign immunity' & 'act of state doctrine' not too much bite
- US Foreign Sovereign Immunities Act (FSIA) of 1976 allows to sue governments



# The Optimal Government Debt Contract

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Costs of non-contingent debt ( $I^n = 1$  for all  $n$ ) normalized to zero

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- **Implicit contracting dominates:**  
costs have to be paid only with certain probability (& later)!

# Model Setup: Government's Policy Problem

- Debt optimally non-contingent in explicit terms (not in implicit terms)

Default gives rise to proportional default costs  $\lambda$

- From now on:

Model with non-contingent (explicit) bonds & default costs  $\lambda \geq 0$

# Optimal Policy Problem

- Ramsey allocation problem

$$\begin{aligned} & \max_{\{G_t^L \geq 0, G_t^S \geq 0, \Delta_t \in [0,1]^N, k_t \geq 0, c_t \geq \bar{c}\}} & E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \\ & \text{s.t. : } c_t + k_t + \frac{G_t^L}{1+r} = w_t + \frac{G_t^S}{1+R(z_t, \Delta_t)} \\ & & w_{t+1} \geq NBL(z_{t+1}) \quad \forall t \forall z_{t+1} \in Z \\ & & w_0 : \text{ given} \end{aligned}$$

- Beginning-of-period wealth:

$$w_t \equiv z_t k_{t-1}^\alpha + G_{t-1}^L - G_{t-1}^S \cdot (1 - (1 - \lambda^b) \delta^{l(z_t)}).$$

- Risk neutral international investors:

$$1 + r = \frac{1 - (1 + \lambda^I) \sum_{n=1}^N \delta^n \Pi(z^n | z_t)}{\frac{1}{1 + R(z_t, \Delta)}}$$

- Allows estimating lower bound on default costs  $\lambda^I$
- Combine data from Klingen, Weder, Zettelmeyer (2004) & Cruces and Trebesch (2011)

$$\lambda^I = 7.5\%$$

- $\lambda = \lambda^b + \lambda^I \Rightarrow$  quantitative analysis:  $\lambda = 10\%$  and  $\lambda = 20\%$



# Optimal Policy Problem

- Optimal policy surprisingly difficult to solve....
- Borrowing limits:
  - very loose  $\Rightarrow$  non-existence of optimal policies
  - too tight  $\Rightarrow$  exclude feasible policies
- Loosest constraints consistent w existence & non-explosive debt:
  - show how to compute & uniqueness
- Interest rate  $R(z_t, \Delta_t)$  depends on default policy: unclear if problem is concave & use of FOCs justified....
- Many occasionally binding inequality constraints  $G_t^L \geq 0$ ,  $G_t^S \geq 0$  and particular  $\Delta_t \in [0, 1]^N$  that are difficult to handle computationally
- Optimal default policies  $\Delta_t$  turn out to be non-continuous, complicating numerical solutions.

# Optimal Policy Problem

- Derive an equivalent formulation of problem:
  - concave (can use FOCs)
  - economizes on inequality constraints
  - continuous optimal policies

# Equivalent Problem

- Equivalent optimization problem:

$$\begin{aligned} \max_{\{b_t, a_t \geq 0, k_t \geq 0, c_t \geq \bar{c}\}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \\ \text{s.t. } \forall t : c_t = \quad & \tilde{w}_t - k_t - \frac{1}{1+r} b_t - p_t \cdot a_t \\ \tilde{w}_{t+1} \geq \quad & NBL(z_{t+1}) \quad \forall z_{t+1} \in Z \end{aligned}$$

$b \geq 0$  : riskless bond

$a$  : vector of Arrow securities

$p_t$  : price vector for Arrow securities: indep. of government policy!

$\tilde{w}_0 = w_0$  : initial condition

- Beginning-of-period wealth

$$\tilde{w}_t \equiv z_t k_{t-1}^\alpha + b_{t-1} + (1 - \lambda) a_{t-1}(z_t)$$

- Problem concave and economizes on inequality constraints

# Equivalent Problem

- Equivalence proof in paper.....
- $b$  has an interpretation as the net foreign asset position

$$b_t = G_t^L - G_t^S,$$

- Arrow securities capture state contingent default policies on own bonds
- In a setting with 2 productivity states:

$$a_t = \begin{pmatrix} G_t^S \delta^1 \\ G_t^S \delta^2 \end{pmatrix}$$

- For  $\lambda = 0$  default completes the market:  
**Proposition:** For  $\lambda = 0$ 
  - complete consumption smoothing optimal
  - default happens in all but the best productivity state
  - default proportional to news about PV of domestic value added
- Generalizes Grossman and van Huyck (1988, section II): optimal default endowment economy with iid income risk and  $\lambda = 0$
- Trade-off emerges only for  $\lambda > 0$ : no analytical results possible

# Optimal Policies with Default Costs

- Calibrate the model at annual frequency  $z^h = 1.0133$ ,  $z^l = 0.9868$
- Transition matrix for the states, given by

$$\pi = \begin{pmatrix} 0.8077 & 0.1923 \\ 0.1923 & 0.8077 \end{pmatrix}$$

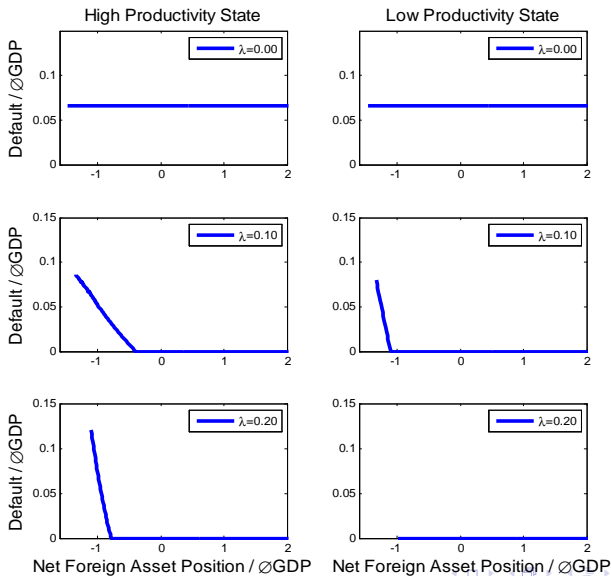
- Utility function is given by

$$u(c) = \frac{(c - \bar{c})^{1-\sigma}}{1-\sigma}$$

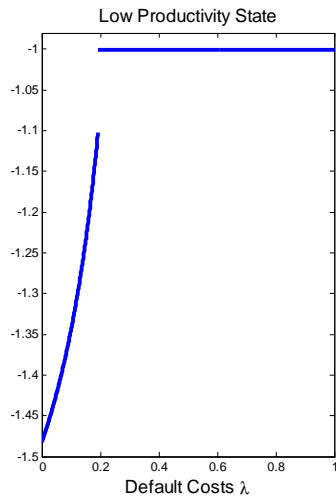
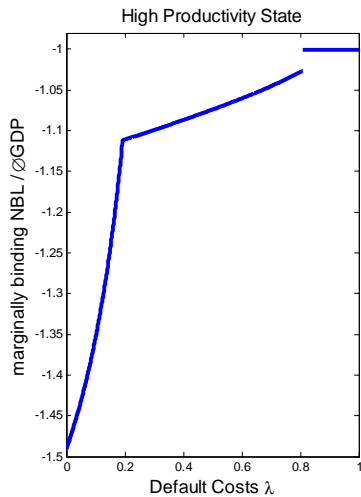
- $\bar{c}$  : if bonds must be repaid always, max sustainable NFA equals -100% of GDP (Lane and Milesi-Ferretti (2007))
- Remaining parameters:

$\alpha$	$\beta$	$\sigma$	$\bar{c}$	$1+r$
0.34	0.97	2	0.357	$1/\beta - 0.0005$

# Optimal Default Policies



# The Effect of Default Costs





- Calibrating Economic Disasters following Barro and Jin (2011):

shock process

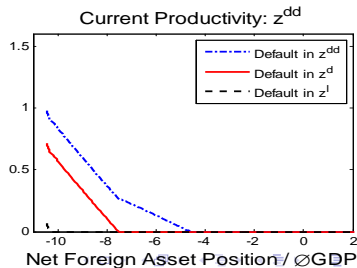
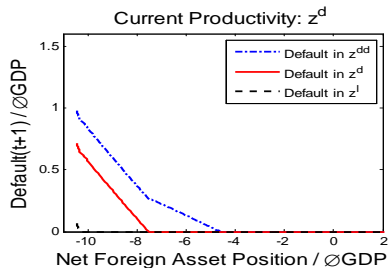
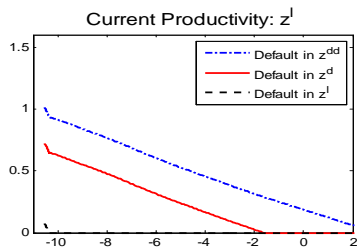
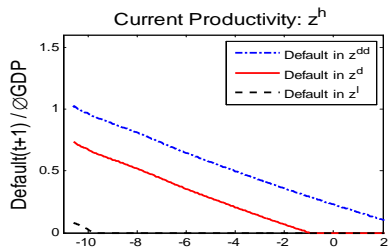
$$Z = \{z^h, z^l, z^d, z^{dd}\} = \{1.0133, 0.9868, 0.9224, 0.6696\}$$

with transition matrix

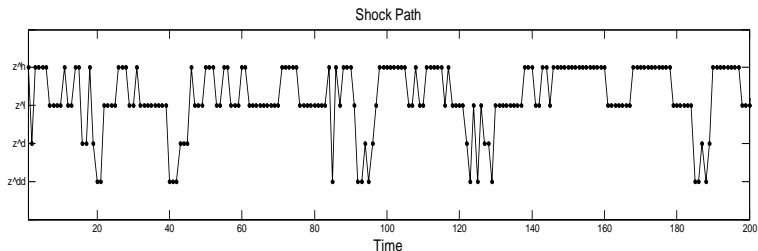
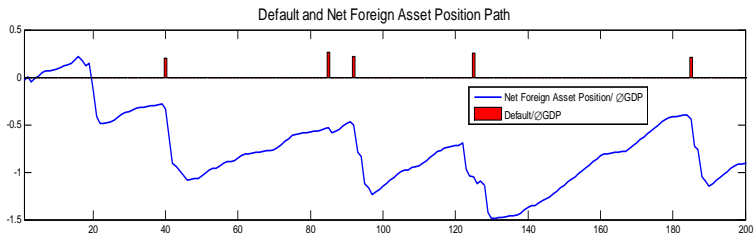
$$\pi = \begin{pmatrix} 0.7770 & 0.1850 & 0.019 & 0.019 \\ 0.1850 & 0.7770 & 0.019 & 0.019 \\ 0.1429 & 0.1429 & 0.3571 & 0.3571 \\ 0.1429 & 0.1429 & 0.3571 & 0.3571 \end{pmatrix}.$$

- Recalibrate the subsistence level of consumption to  $\bar{c} = 0.198$ .

# Optimal Default with Disaster Risk



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# Welfare Implications of Optimal Default

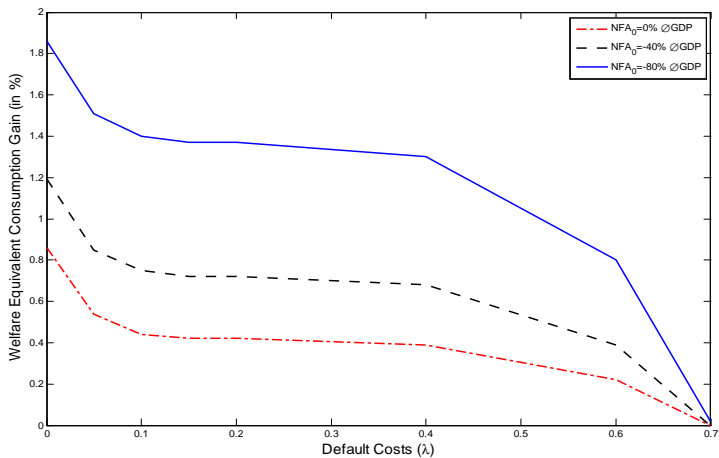
- Welfare equivalent consumption gain from default (first 500 years)
- Compute consumption change  $\omega$  solving

$$E_0 \left[ \sum_{t=0}^{500} \beta^t \frac{((c_t^1(1 + \omega) - \bar{c}))^{1-\gamma}}{1 - \gamma} \right] = E_0 \left[ \sum_{t=0}^{500} \beta^t \frac{(c_t^2 - \bar{c})^{1-\gamma}}{1 - \gamma} \right]$$

$c_t^1$  : optimal consumption path in the no-default economy (repayment assumed)

$c_t^2$  : the corresponding consumption path with optimal (costly) default.

# Welfare Implications of Optimal Default



- No difference from introducing long foreign bonds: no value for insurance
- No difference from long domestic bonds if repayment is assumed (unlike in Angeletos(2001))
- Long domestic bonds with default option:  
  
(partial) default *in the future* after bad event *today*  $\Rightarrow$  bonds fall in value  
  
repurchase at depreciated value & realize a capital gain
- Improvements possible: if repurchase has lower costs than default....

# Conclusion

- Sovereign default is optimal under commitment if bond markets incomplete
- Relaxes borrowing limits, increases welfare & optimal after bad output realizations
- Welfare gains large (1-2% of cons.) & not very sensitive to default costs
- Long bonds coupled with buyback potentially even more efficient

Commitment view on SD of normative interest...

... upon closer inspection also reasonable from a positive perspective



- Can rationalize high outstanding debt levels (default option *relaxes* the borrowing limits)
- Optimal default can look like a 'willingness to pay problem' sufficient resources for repayment available!
- Default optimal following negative shocks to domestic income.