Bank Leverage Cycles*

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Abstract

We study the cyclical fluctuations of leverage and assets of financial intermediaries and GDP in the United States. Leverage and assets are several times more volatile than GDP, and experience larger fluctuations for unregulated ('shadow') intermediaries than for regulated ones. While the leverage of regulated intermediaries is rather acyclical with respect to their assets and to GDP, the leverage of unregulated intermediaries is strongly procyclical in relation to their assets, and mildly procyclical in relation to GDP. We then build a general equilibrium model with both regulated and unregulated financial intermediaries. The latter borrow from investors in the form of short-term collateralized risky debt, and are subject to endogenous leverage constraints. We find that shocks to cross-sectional volatility are key to generate fluctuations and comovements similar to those found in the data. Also, in a scenario with lower cross-sectional volatility, output is higher on average but more volatile, due to higher leverage of unregulated banks.

Keywords: financial intermediaries, short-term collateralized debt, limited liability, call option, put option, moral hazard, leverage.

JEL codes: E20, G10, G21

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1 Introduction

The 2007-2009 financial crisis witnessed a severe disruption of financial intermediation in many industrialized economies. This has led to a surge in both empirical and theoretical research aimed at understanding the causes and consequences of the financial crisis, evaluating the policy measures put in place to tackle its effects, and proposing further policy actions and new regulatory frameworks.

A particularly influential strand of the literature has focused on the role played by the 'shadow banking' sector in the origin and propagation of the financial turmoil. The latter sector comprises all those financial intermediaries (investment banks, hedge funds, finance companies, off-balance-sheet investment vehicles, etc.) that have no access to central bank liquidity or public sector credit guarantees, and that are not subject to regulatory capital requirements.[1] Many of these financial intermediaries funded their asset purchases primarily by means of collateralized debt with very short maturity, such as sale and repurchase (*repo*) agreements or asset backed commercial paper (ABCP). As argued by Brunnermeier (2009), Gorton and Metrick (2010, 2011), Krishnamurthy et al. (2012) and others, the initial losses suffered by some of the assets that served as collateral in repo or ABCP transactions, together with the uncertainty surrounding individual exposures to such assets, led the holders of that short-term debt (mostly institutional investors, such as money market funds) to largely stop rolling over their lending. This funding freeze forced the shadow financial intermediaries to deleverage, with the resulting contraction in financing flows to the real economy.

In fact, the observed deleveraging of shadow intermediaries during the 2007-2009 financial crisis is not an isolated episode. As documented by Adrian and Shin (2010, 2011b), since the 1960s the leverage ratio of some financial intermediaries has exhibited a markedly procyclical pattern, in the sense that expansions in balance sheet size have gone hand in hand with increases in leverage. This procyclicality has been particularly strong in the case of security brokers and dealers, a category that used to include investment banks. Overall, these findings point to the importance of endogenous leverage fluctuations for the cyclical behavior of financial intermediation.

The aim of our paper is both empirical and theoretical. On the empirical front, we perform a systematic analysis of the cyclical fluctuations in the leverage ratio and the assets of US financial intermediaries, as well as GDP. Our analysis comprises all the subsectors in what Greenlaw et al. (2008) have termed the 'leveraged sector', which includes regulated intermediaries such as US-chartered commercial banks, savings institutions and credit unions, as well as unregulated ('shadow') intermediaries such as security brokers and dealers, finance companies and government-sponsored enterprises (GSEs). We focus both on the volatility of the series as well as on their correlations. This allows us to gauge the size of fluctuations in key financial aggregates, such as intermediary leverage and assets, in relation to real economic activity. It also allows us to study their cyclicality in relation to a standard measure of the business cycle such as GDP.

Our empirical findings can be summarized as follows. As regards the size of cyclical fluctuations, we find that financial intermediaries' leverage and total assets are several times more volatile than GDP. While there is a fair amount of heterogeneity across different types of financial intermediaries, overall unregulated intermediaries tend to experience larger fluctuations than regulated ones. This contrast is particularly visible for the two prominent subsectors within the regulated and the unregulated sectors: US-chartered commercial banks, and security broker/dealers, respectively.

Regarding the analysis of cyclicality, we find that leverage and total assets are positively correlated for all the different subsectors, with one important exception: US-chartered commercial banks. The latter are by far the largest group in terms of total assets within the regulated sector.

[1] See Pozsar et al. (2012) for an in-depth analysis of 'shadow banking' in the United States.

Once again, the starkest contrast is between commercial banks and security broker/dealers, as the latter display the highest correlation between leverage and assets.[2] Also, total assets are positively correlated with GDP in all cases, which is hardly surprising. Finally, there is substantial heterogeneity in the cyclicality of leverage with respect to GDP. While leverage is (marginally) procyclical for unregulated subsectors such as security brokers/dealers and finance companies, as well as for savings institutions, it is *a*cyclical for commercial banks and GSEs, and *counter*cyclical for credit unions. These findings suggest that the leverage of regulated intermediaries is rather acyclical, whereas it is mildly procyclical at best for unregulated intermediaries.

On the theoretical front, we construct a general equilibrium model of financial intermediation and endogenous leverage, and assess its ability to match the evidence discussed above. The model incorporates a two-tier financial intermediation sector consisting of regulated and unregulated ('shadow') banks. Both types of banks differ in two respects. First, regulated banks' liabilities are riskless, whereas those of unregulated banks are not. In particular, regulated banks borrow from households in the form of deposits that are insured by the government, whereas unregulated banks borrow from institutional investors in the form of short-term collateralized risky debt. The source of risk in unregulated banks' debt is the following. Both types of banks invest in the nonfinancial corporate (firm) sector. Banks and firms are segmented across islands, and firms are hit by island-specific shocks. Therefore, banks are exposed to island-specific risk, such that a fraction of them declare bankruptcy and default on their debt in each period.

Second, regulated banks are subject to a regulatory capital requirement, which is isomorphic to a maximum leverage constraint. By contrast, unregulated banks' leverage is endogenously determined by market forces. In particular, we assume the existence of a moral hazard problem based on the one developed by Adrian and Shin (2011a) in a partial equilibrium context.[3] Due to limited liability, the payoff structure of an unregulated bank resembles that of a call option on island-specific risk.[4] That is, unregulated banks enjoy the upside risk in their assets over and above the face value of their debt, leaving institutional investors to bear the downside risk. This provides banks with an incentive to engage in inefficiently risky lending practices. Such an incentive increases with the assumed debt commitment relative to the size of the bank's balance sheet. In order to induce each bank to invest efficiently, institutional investors restrict their lending to a certain ratio of the bank's net worth, i.e. they impose a leverage constraint.

We then calibrate our model to the US economy and analyze its dynamic properties.[5] In particular, we study the model economy's response to two exogenous driving forces: total factor productivity (TFP), and time-varying volatility of island-specific shocks. While TFP shocks are fairly standard in the real business cycle literature, changes in cross-sectional volatility have received considerable attention recently as a source of aggregate fluctuations.[6]

[2] Both the procyclicality of leverage with respect to assets for security broker/dealers and the lack of such procyclicality for commercial banks confirm the original findings by Adrian and Shin (2010). Our analysis of the data is somewhat different though. Whereas Adrian and Shin focus on the growth rates of leverage and assets, we focus on their cyclical components, calculated by means of a standard bandpass filter. We also consider real rather than nominal assets, given our interest in their comovements with real GDP and for consistency with our theoretical model.

[3] Adrian and Shin's (2011a) moral hazard problem is in turn inspired by earlier work by Holmström and Tirole (1997).

[4] For a pioneering analysis of the payoff structure of defaultable debt claims, equity stakes, and their relationship to option derivatives, see Merton (1974).

[5] For the purpose of calibrating the model, we take the commercial banking and security broker/dealer subsectors as representative of the regulated and the unregulated leveraged financial sectors, respectively.

[6] See e.g. Curdia (2007), Christiano et al. (2010), Gilchrist et al. (2010), Bloom (2009), and Bloom et al. (2011).

Our results show that TFP shocks by themselves are unable to generate fluctuations in assets and leverage comparable to those in the data. They also fail to produce a meaningful correlation between leverage, on the one hand, and assets or GDP on the other. On the contrary, shocks to cross-sectional volatility are able to produce significant fluctuations in assets and leverage, as well as a positive comovement between leverage, assets and GDP. The mechanism is as follows. Consider e.g. an increase in island-specific volatility. Higher uncertainty regarding asset returns makes it more attractive for unregulated banks to engage in inefficiently risky lending practices. In order to prevent them from doing so, institutional investors impose a tighter constraint on unregulated banks' leverage. For given net worth, this deleveraging forces unregulated banks to contract their balance sheets, thus producing a positive comovement between assets and leverage. At the same time, the reduction in unregulated banks' assets is not compensated by a similar increase in those of regulated banks, thus producing a fall in total intermediated assets. This leads to a fall in capital investment by firms, and in aggregate output. The consequence is a positive comovement between leverage and GDP. In fact, volatility shocks generate a procyclicality in leverage and assets well above the empirical ones. Combining the latter shocks with TFP shocks improves the model's performance, because the correlation of assets and leverage with GDP fall to levels that are comparable with those in the data.

Finally, we study how the steady-state level of cross-sectional volatility affects both the mean level and the volatility of economic activity in our model. We find that lower cross-sectional volatility raises the mean level of unregulated banks' leverage, through a channel very similar to the one described above. This produces an increase in the mean levels of intermediated assets (unregulated as well as total), and hence in the mean levels of capital investment and GDP. Perhaps more surprisingly, lower cross-sectional uncertainty *raises* the volatility of GDP. As unregulated banks become more leveraged, their relative size increases. But since their assets are more volatile than those of regulated intermediaries, the consequence is larger fluctuations in total intermediated assets and hence in aggregate output. This result is reminiscent of Minsky's (1992) 'financial instability hypothesis,' according to which a lower perception of uncertainty leads to riskier investment practices, thus creating the conditions for the emergence of a financial crisis. In our model, lower perceived risk leads financial intermediaries to raise their leverage ratios, thus making the economy more vulnerable to the effects of negative aggregate shocks.

Our paper contributes to the emerging literature on the macroeconomic effects of financial frictions in macroeconomics. On the one hand, a recent literature has provided theoretical explanations for the 'leverage cycles' discussed above, with contributions by Adrian and Shin (2011a), Ashcraft et al. (2011), Brunnermeier and Pedersen (2009), Brunnermeier and Sannikov (2011), Dang et al. (2011), Geanakoplos (2010) and Gorton and Ordoñez (2011), among others.[7] Most of these models consider some type of link between changes in 'uncertainty', typically defined as changes in the volatility of shocks, and the emergence of these leverage cycles. While these models provide important insights on the equilibrium behavior of leverage, they are primarily aimed at illustrating theoretical mechanisms and are thus mainly qualitative. In particular, most of these papers consider two- or three-period economies, or two-period-lived agents (i.e. an OLG structure). They also assume a partial equilibrium structure. We build on this literature by analyzing endogenous leverage cycles in a fully dynamic, general equilibrium model that can be compared to aggregate

Christiano et al. (2010) refer to such disturbances as 'risk shocks', whereas Bloom (2009) labels them 'uncertainty shocks'.

[7]Some of these authors focus on the behavior of 'margins' or 'haircuts' in short-term collateralized debt contracts, which are closely related to the concept of 'leverage'.

data and, more generally, be useful for quantitative analysis.

On the other hand, our paper is related to a growing literature about financial frictions in DSGE models. Early contributions, such as Carlstrom and Fuerst (1997), Bernanke et al. (1999) and Kiyotaki and Moore (1997), emphasized the importance of financial frictions for the macroeconomy, but largely obviated the role played by financial intermediaries. Recent contributions, such as Christiano et al. (2010), Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), focus on frictions arising in financial intermediation, but do not explicitly analyze the behavior of bank leverage. Building on this literature, we focus on the role of endogenous bank leverage cycles in the propagation of shocks.

The paper proceeds as follows. Section 2 presents empirical evidence on the cyclical behavior of GDP, assets and leverage of financial intermediaries in the US. Section 3 lays out the model. Section 4 calibrates and simulates the model, assessing its ability of replicate the data. Section 5 concludes.

2 Bank leverage cycles in the US economy

By definition, the size of a financial intermediary's balance sheet is the product of two components: its equity capital, and its leverage ratio, where the latter is the ratio between its total assets and its equity. Also by definition, the leverage ratio is the inverse of the capital ratio, understood as the ratio between the intermediary's equity capital and its assets. Financial intermediaries that are subject to certain regulatory capital requirements are likely to target a rather stable leverage ratio. On the contrary, intermediaries that are not subject to capital regulations are likely to manage their leverage ratio more actively as a means of expanding or contracting their balance sheets. In this regard, the recent financial crisis witnessed a process of intense deleveraging in certain segments of the US financial intermediation sector (Adrian and Shin, 2011b). More broadly, an interesting empirical question is to what extent different types of financial intermediaries adjust their leverage ratios as they expand or contract their lending activity. A related question is whether the leverage ratio of financial intermediaries comoves as well with aggregate economic activity, as represented by real GDP. Last but not least, the size of fluctuations in the leverage ratio and the balance sheets of financial intermediaries relative to those in real economic activity is itself a matter of empirical interest.

Figure 1 plots, for the six leveraged subsectors of the US financial intermediation sector, the joint comovement of the cyclical components of the leverage ratio (defined as the ratio between total assets and equity capital, both in dollars) and real total assets (defined as total assets in dollars divided by the GDP deflator) since the mid 80s. Figure 2 plots the comovement between the cyclical components of leverage and real GDP.[8] In both figures, the first column corresponds to subsectors that have access both to central bank liquidity and to public sector credit guarantees, and that are subject to regulatory capital requirements (US-chartered commercial banks, savings institutions and credit unions). We may refer to this group as the 'regulated' leveraged financial sector. The second column corresponds to subsectors that have no access to central bank liquidity or public sector credit guarantees, and that are not subject to capital regulations (security brokers and dealers, finance companies, GSEs). We may refer to the latter as the 'unregulated' leverage financial sector. Broadly speaking, this group belongs to what Pozsar et al. (2012) define as the

[8] The cyclical component is obtained by detrending each series with a band-pass filter that preserves cycles of 6 to 32 quarters and with lag length $K = 12$ (Baxter and King, 1999).

'shadow banking' sector. Finally, Table 1 displays a number of statistics regarding the business cycle fluctuations in leverage, real total assets and real GDP.

A first conclusion to extract is that the leverage ratios of the different subsectors are considerably more volatile than GDP, as is apparent from Figure 2. As shown in Table 1, the standard deviation of the leverage ratio of security broker/dealers and finance companies (both unregulated subsectors) is about 8 and 5 times larger than that of GDP, respectively. Somewhat surprisingly, the leverage of savings institutions (a regulated subsector) displays the largest fluctuations. For commercial banks, credit unions and GSEs, the leverage ratio fluctuates comparatively less, although their standard deviations are still about 3 times that of GDP. A similar conclusion holds for the fluctuations in total assets relative to those in GDP. Notice also that the leverage and total assets of the unregulated subsectors are generally more volatile than their counterparts in the regulated subsectors (with the exception of savings institutions).

A second lesson to draw is that assets and leverage tend to comove *positively* over the business cycle. This pattern is particular strong for security brokers and dealers. As shown in Table 1, for the latter subsector both variables have a contemporaneous correlation of 0.76 at business cycle frequencies. This observation confirms the original finding of Adrian and Shin (2010), albeit with a different treatment of the data.[9] As explained by these authors, such a strong comovement reveals an active management of leverage as a means of expanding and contracting the size of balance sheets. For the other subsectors, the correlation coefficients are smaller, but statistically significant in all cases. An important exception is the US-chartered commercial bank sector, for which little comovement seems to exist between assets and leverage. Their contemporaneous correlation of 0.21 is not statistically significant at the 5% confidence level. This finding was also emphasized by Adrian and Shin (2010). For instance, as shown in Figure 1, the early phase of the last recession witnessed a reduction in the cyclical component of commercial banks' leverage and an *increase* in the cyclical component of their total assets.[10] As argued by Adrian and Shin (2010), this acyclicality of leverage with respect to total assets would be consistent with commercial banks targeting a (roughly) constant leverage ratio. As we argued above, this in turn could be reflecting the effect of regulatory minimum capital requirements.

We are also interested in the comovements between leverage and aggregate economic activity, as represented by real GDP. In this regard, Figure 2 and Table 1 reveal a heterogenous pattern across financial subsectors. On the one hand, the leverage of typically 'shadow' financial intermediaries such as security broker/dealers and finance companies display a mildly procyclical behavior. Their correlation with GDP, 0.22 and 0.24 respectively, are relatively small but are both statistically significant at the 5% confidence level. For instance, the recession starting in 2007 witnessed a sharp decline in the leverage ratio of security broker/dealers. A similar phenomenon occurred in the case of finance companies during the 1990-1991 recession. Interestingly, the same episode also witnessed a severe decline of the leverage ratio of a regulated subsector such as savings institutions. For the latter group, the correlation with GDP stands at 0.34. On the other hand, the leverage ratio of both commercial banks and GSEs seem to display little cyclicality, with correlation coefficients (-0.06 and -0.14) that are not statistically different from zero. Finally, the leverage of credit unions is actually *negatively* correlated with GDP (-0.57).

[9] Adrian and Shin (2010) focus on the comovement between the *growth rates* of leverage and *nominal* total assets. Here, we focus on the behavior of *real* total assets, due both to our interest in the comovement of financial variables with real GDP and for consistency with our subsequent theoretical model. Also, we use a standard band-pass filter so as to extract the cyclical component of assets and leverage. Our results show that Adrian and Shin's (2010) findings are robust to this different transformation of the data.

[10] Real total assets of commercial banks did not start to fall until 2009:Q1.

Given this heterogeneity across subsectors in the cyclicality of leverage with respect to GDP, it would be interesting to consolidate the assets and equity capital of the different subsectors (for instance, regulated subsectors on the one hand and unregulated ones on the other), and then study the cyclical properties of the resulting consolidated leverage ratios. Unfortunately, the Flow of Funds data does not allow this possibility, because asset and liability positions between the different subsectors are not netted out. As a result, simply adding assets and equity would lead to a double-counting of such cross positions. In this respect, it may be instructive to gauge the relative importance of each subsector in terms of their balance sheet size. Figure 3 plots the total assets of each subsector during our sample period. Within the regulated sector, US-chartered commercial banks are by far the larger subsector. With all the pertinent caveats, this means that the dynamics of the consolidated assets and equity of the regulated sector would be dominated by those of commercial banks. As a result, the resulting leverage ratio would probably display an acyclical behavior with respect to GDP. Similar implications hold regarding the correlation between leverage and total assets, which is also absent in the case of commercial banks. Regarding the unregulated sector, security brokers and dealers are clearly the larger group, although it is not as dominant as commercial banks are in the regulated sector. Given the relatively low (though statistically significant) positive correlation of leverage and GDP for security broker/dealers and finance companies, and given the lack of correlation in the case of GSEs, it is uncertain whether the leverage ratio of the consolidated unregulated sector would also display the same procyclical behavior.

To summarize, our empirical analysis reveals three main findings regarding the US leveraged financial sector. First, the leverage ratio of the different subsectors display large fluctuations, with standard deviations between 3 and 8 times as large as that of GDP. Second, the leverage of the unregulated subsectors tends to comove positively with total assets, whereas such comovement does not seem to exist for the dominant regulated subsector: US-chartered commercial banks. Finally, the leverage of unregulated financial intermediaries is at best mildly procyclical with respect to GDP, and acyclical for US-chartered commercial banks. In what follows, we present a general equilibrium model aimed at explaining the volatility and the comovement of financial intermediaries' leverage, assets and GDP in the United States.

3 Model

The model economy is composed by six types of agents: households, final good producers ('firms' for short), capital producers, institutional investors, regulated banks and unregulated banks. On the financial side, the model structure is as follows. Households lend to regulated banks in the form of deposits, and to institutional investors in the form of equity. Institutional investors use the latter funds to lend to unregulated banks in the form of short-term, collateralized debt. Both regulated and unregulated banks combine their external funding and their own accumulated net worth to invest in firms. We assume no frictions in the relationship between banks and firms, such that the Modigliani-Miller theorem applies to firm financing. For simplicity, following Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) we assume that firms issue perfectly state-contingent debt only, which can be interpreted as equity. Banks (regulated or not) and firms are segmented across islands, where the latter are subject to idiosyncratic shocks. Banks are thus exposed to island-specific risk, such that a fraction of them declare bankruptcy and default on their debt each period. Regulated banks enjoy deposit insurance, such that deposits are safe. However, unregulated banks' debt is not guaranteed, and is therefore risky. Institutional investors operate economy-wide and diversify perfectly across islands, thus insulating households from island risk.

The real side of the model is fairly standard. At the end of each period, after production has taken place, firms use borrowed funds to purchase physical capital from capital producers. At the beginning of the following period, firms combine their stock of capital and households' supply of labor to produce a final good. The latter is purchased by households for consumption purposes, and by capital producers. After production, firms sell their depreciated capital stock to capital producers, who use the latter and the final goods to produce new capital. The markets for labor, physical capital and the final good are all nation-wide.

We now analyze the behavior of each type of agent. All variables are expressed in real terms, with the final good acting as the numeraire.

3.1 Households

The representative household's utility is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t) - v(L_t) \right],$$

where C_t is consumption and L_t is labor supply. The budget constraint is

$$C_t + D_t + E_t = W_t L_t + R_{t-1} D_{t-1} + R_t^N N_{t-1}^{inv} + \Pi_t^b,$$

where D_t are deposits at regulated banks, R_t is the risk-free gross interest rate on deposits, N_t^{inv} are equity holdings at institutional investors, R_t^N is the return on institutional investor equity (to be defined later), W_t is the wage, and Π_t^b are lump-sum net dividend payments from the household's ownership of banks (regulated or not). As we will see later on, Π_t^b incorporates any equity injections by households into banks. The first order conditions are

$$\begin{aligned} 1 &= E_t \left[\Lambda_{t,t+1} R_t \right], \\ 1 &= E_t \left[\Lambda_{t,t+1} R_{t+1}^N \right], \\ W_t &= \frac{v'(L_t)}{u'(C_t)}, \end{aligned}$$

where

$$\Lambda_{t,t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}$$

is the stochastic discount factor.

3.2 Firms

The final good is produced by perfectly competitive firms. The latter are segmented across a continuum of 'islands' indexed by $j \in [0,1]$. These islands may be interpreted as regions, or alternatively as sectors. The representative firm in island j starts period t with a stock K_t^j of physical capital, purchased at the end of period $t-1$. The firm then receives an island-specific shock ω_t^j that changes the amount of effective capital to $\omega_t^j K_t^j$. The shock ω_t^j is iid over time and across islands. Let $F(\omega; \sigma_{t-1}) \equiv F_{t-1}(\omega)$ denote the cumulative distribution function of island-specific shocks at time t, where σ_{t-1} denotes the standard deviation of $\log \omega_t^j$. The latter standard deviation follows an exogenous process. Notice that the standard deviation of island-specific shocks in a given period is known one period in advance. We also assume that ω^j has a unit mean, $E\left[\omega^j\right] = 1$.

Effective capital is combined with labor to produce units of final good, Y_t^j, according to a Cobb-Douglas technology,

$$Y_t = Z_t(\omega^j K_t^j)^\alpha (L_t^j)^{1-\alpha}, \tag{1}$$

where Z_t is an exogenous aggregate total factor productivity (TFP) process. The firm maximizes operating profits, $Y_t^j - W_t L_t^j$, subject to (1). The first order condition is

$$W_t = (1-\alpha)\, Z_t \left(\frac{\omega^j K_t^j}{L_t^j}\right)^\alpha. \tag{2}$$

Therefore, the effective capital-labor ratio is equalized across islands: $\omega^j K_t^j / L_t^j = \left[W_t / (1-\alpha)\, Z_t\right]^{1/\alpha}$ for all j. The firm's profits are given by

$$Y_t^j - W_t L_t^j = \alpha Z_t (\omega^j K_t^j)^\alpha (L_t^j)^{1-\alpha} = R_t^k \omega^j K_t^j,$$

where

$$R_t^k \equiv \alpha Z_t \left[\frac{(1-\alpha)\, Z_t}{W_t}\right]^{(1-\alpha)/\alpha}$$

is the return on effective capital, which is equalized too across islands. After production, the firm sells the depreciated effective capital $(1-\delta)\, \omega^j K_t^j$ to capital producers at price one. The total cash flow from the firm's investment project, equal to the sum of operating profits and proceeds from the sale of depreciated capital, is given by

$$R_t^k \omega^j K_t^j + (1-\delta)\, \omega^j K_t^j = \left[R_t^k + (1-\delta)\right] \omega^j K_t^j. \tag{3}$$

The capital purchase in the previous period was financed entirely by state-contingent debt. In particular, the cash flow in (3) is paid off entirely to the lending banks.

At the end of period t, the firm buys K_{t+1}^j units of new capital at price one for production in $t+1$. In order to finance this purchase, the firm issues a number of claims on next period's cash flow equal to the number of capital units acquired, K_{t+1}^j. We assume that the firm can only borrow from banks located on the same island. This assumption may be justified on the basis that financial intermediaries tend to specialize in specific market segments; their resulting advantage at screening and monitoring investment projects in such segments would allow them to prevent competition from outsiders.[11] In particular, the firm sells A_t^j claims to unregulated banks on island j, and the rest, $A_t^{r,j}$, to regulated banks on the same island. The firm's balance sheet constraint is thus simply

$$K_{t+1}^j = A_t^j + A_t^{r,j}.$$

3.3 Capital producers

There is a representative, perfectly competitive capital producer. At the beginning of each period, after production of final goods has taken place, the capital producer purchases the stock of depreciated capital $(1-\delta)\, K_t$ from firms at price one. Used capital can be transformed into new capital

[11] In reality, financial intermediaries do diversify both geographically and sectorally. However, this diversification is far from perfect, especially for relatively small intermediaries or for highly specialized agents, such as hedge funds.

on a one-to-one basis at no cost. Capital producers also purchase final goods in the amount I_t, which are used to produce new capital goods on a one-to-one basis. At the end of the period, the new capital is sold to the firms at price one. In equilibrium, capital producers make zero profits.

3.4 Unregulated banks

In each island j there exists a representative unregulated bank. After production in period t, island j's firm pays the unregulated bank its share of the cash flow from the investment project, $\left[R_t^k + (1-\delta)\right]\omega^j A_{t-1}^j$. Therefore, the gross rate of return on the unregulated bank's assets is

$$\frac{\left[R_t^k + (1-\delta)\right]\omega^j A_{t-1}^j}{A_{t-1}^j} = \left[R_t^k + (1-\delta)\right]\omega^j \equiv R_t^A \omega^j.$$

Regarding the liabilities side of its balance sheet, the unregulated bank borrows from institutional investors by means of one-period collateralized risky debt contracts. The latter may be thought of as sale and repurchase (*repo*) agreements. Under the latter contract, at the end of period $t-1$ the bank sells its financial claims A_{t-1}^j (which serve as collateral) to the institutional investor at price B_{t-1}^j, and agrees to repurchase them at the beginning of time t at a non-state-contingent price $\bar{B}_{t-1}^j$. At the beginning of period t, the proceeds from the bank's assets, $R_t^A \omega^j A_{t-1}^j$, exceed the face value of its debt, $\bar{B}_{t-1}^j$, if and only if ω^j exceeds a threshold level $\bar{\omega}_t^j$ given by

$$\bar{\omega}_t^j \equiv \frac{\bar{B}_{t-1}^j}{R_t^A A_{t-1}^j}, \tag{4}$$

that is, the face value of debt normalized by the bank's assets times their aggregate return. If $\omega^j \geq \bar{\omega}_t^j$ the bank honors its debt, that is, it repurchases its assets at the pre-agreed price $\bar{B}_{t-1}^j$. If $\omega^j < \bar{\omega}_t^j$, the bank defaults and closes down, whereas the institutional investor simply keeps the collateral and cashes the resulting proceeds, $R_t^A \omega^j A_{t-1}^j$. Notice that the threshold $\bar{\omega}_t^j$ depends on R_t^A and is thus contingent on the aggregate state.

For non-defaulting banks, following Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) we assume that a random fraction $1-\theta$ of them close down for exogenous reasons each period, at which point the net worth accumulated in each bank is reverted to the household.[12] The remaining fraction θ of banks continue operating. For the latter, the flow of dividends distributed to the household is given by

$$\Pi_t^j = R_t^A \omega^j A_{t-1}^j - \bar{B}_{t-1}^j - N_t^j, \tag{5}$$

where N_t^j is net worth after dividends have been paid. As in Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), we assume that households inject equity in new banks, but cannot inject equity in continuing banks. Therefore, continuing banks are subject to a non-negativity constraint on dividends, $\Pi_t^j \geq 0$, or equivalently,

$$N_t^j \leq R_t^A \omega^j A_{t-1}^j - \bar{B}_{t-1}^j. \tag{6}$$

[12] As we show below, in equilibrium unregulated banks have no incentive to pay dividends. The assumption of an exogenous exit probability for non-defaulting banks should thus be viewed as a short-cut for motivating dividend payments by such banks, which would otherwise accumulate net worth indefinitely.

Once the bank has decided how much net worth to hold, it purchases claims on firm profits, A_t^j, subject to its balance sheet constraint,

$$A_t^j = N_t^j + B_t^j.$$

When borrowing from the institutional investor, the unregulated bank faces two constraints. First, a *participation constraint* requires that the institutional investor is willing to fund the bank. Indeed, the institutional investor may alternatively lend at the riskless rate R_t. The latter investment has a present discounted value of $E_t \Lambda_{t,t+1} R_t B_t^j = B_t^j = A_t^j - N_t^j$, where we have used the household's Euler equation and the bank's balance sheet constraint. Therefore, the participation constraint takes the form

$$E_t \Lambda_{t,t+1} \left\{ R_{t+1}^A A_t^j \int^{\bar{\omega}_{t+1}^j} \omega dF_t(\omega) + \bar{B}_t^j \left[1 - F_t\left(\bar{\omega}_{t+1}^j\right) \right] \right\} \geq A_t^j - N_t^j. \tag{7}$$

Second, in the spirit of Adrian and Shin (2011a) we assume that once the bank has received the funding it may choose to invest in either of two firm segments within its island: a 'standard' segment, and a 'substandard' segment. Both segments differ only in the distribution of island-specific returns, given by $F_t(\omega)$ and $\tilde{F}_t(\omega) \equiv \tilde{F}(\omega; \sigma_t)$ respectively. The substandard technology has lower average payoff, $\int \omega d\tilde{F}_t(\omega) < \int \omega dF_t(\omega) = 1$, and is thus inefficient. Furthermore, $F_t(\omega)$ is assumed to first-order stochastically dominate $\tilde{F}_t(\omega)$: $\tilde{F}_t(\omega) > F_t(\omega)$ for all $\omega > 0$. Therefore, the substandard technology has higher *downside* risk. In order to induce the bank to invest in the standard segment, the institutional investor imposes an *incentive compatibility* (IC) constraint. Let $V_{t+1}(\omega, A_t^j, \bar{B}_t^j)$ denote the value function at time $t+1$ of a continuing bank, to be defined below. Then the IC constraint takes the following form,

$$\begin{aligned} & E_t \Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^j} \left\{ \theta V_{t+1}\left(\omega, A_t^j, \bar{B}_t^j\right) + (1-\theta)\left[R_{t+1}^k A_t^j \omega - \bar{B}_t^j \right] \right\} dF_t(\omega) \\ \geq \quad & E_t \Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^j} \left\{ \theta V_{t+1}\left(\omega, A_t^j, \bar{B}_t^j\right) + (1-\theta)\left[R_{t+1}^k A_t^j \omega - \bar{B}_t^j \right] \right\} d\tilde{F}_t(\omega). \end{aligned} \tag{8}$$

To understand the bank's incentives to finance one firm segment or another, notice that its expected net payoff, conditional on a particular aggregate state at time $t+1$, can be expressed as

$$\int_{\bar{\omega}_{t+1}^j} \left(R_{t+1}^A A_t^j \omega - \bar{B}_t^j \right) dF_t(\omega) = R_{t+1}^A A_t^j \int_{\bar{\omega}_{t+1}^j} \left(\omega - \bar{\omega}_{t+1}^j \right) dF_t(\omega).$$

The integral represents the value of a *call option* on island-specific returns with strike price equal to the default threshold, $\bar{\omega}_{t+1}^j$, or equivalently to the (normalized) face value of debt, $\bar{B}_t^j / R_{t+1}^A A_t^j$. Intuitively, limited liability implies that the bank enjoys the upside risk in asset returns over and above the face value of its debt, but does not bear the downside risk, which is transferred to the institutional investor. Furthermore, the value of the call option on island-specific risk may be expressed as

$$\int_{\bar{\omega}_{t+1}^j} \left(\omega - \bar{\omega}_{t+1}^j \right) dF_t(\omega) = \int \omega dF_t(\omega) + \int^{\bar{\omega}_{t+1}^j} \left(\bar{\omega}_{t+1}^j - \omega \right) dF_t(\omega) - \bar{\omega}_{t+1}^j.$$

Therefore, *given* the (normalized) face value of its debt, the bank's expected net payoff increases with the mean island-specific return, $\int \omega dF_t(\omega)$, but also with the value of the *put option* on island-specific returns with strike price $\bar{\omega}^j_{t+1}$,[13]

$$\int^{\bar{\omega}^j_{t+1}} (\bar{\omega}^j_{t+1} - \omega) dF_t(\omega) \equiv \pi_t(\bar{\omega}^j_{t+1}) \equiv \pi(\bar{\omega}^j_{t+1}; \sigma_t).$$

The put option value under the substandard technology, which we denote by $\tilde{\pi}_t(\bar{\omega}^j_{t+1})$, is defined analogously, with $\tilde{F}_t$ replacing F_t. Given our assumptions on both distributions, it can be shown that $\tilde{\pi}_t(\bar{\omega}^j_{t+1}) > \pi_t(\bar{\omega}^j_{t+1})$.[14] Therefore, when choosing between investment strategies, the bank trades off the *higher mean return* of investing in the standard firm segment against the *lower put option value*. Furthermore, letting $\Delta\pi_t(\bar{\omega}^j_{t+1}) \equiv \tilde{\pi}_t(\bar{\omega}^j_{t+1}) - \pi_t(\bar{\omega}^j_{t+1})$ denote the difference in put option values, we have that $\Delta\pi'_t(\bar{\omega}^j_{t+1}) = \tilde{F}_t(\bar{\omega}^j_{t+1}) - F_t(\bar{\omega}^j_{t+1}) > 0$: the incentive to invest in the riskier firm segment increases with the (normalized) debt commitment.

We are ready to spell out the unregulated bank's maximization problem. Let $V_t(\omega, A^j_{t-1}, \bar{B}^j_{t-1})$ denote the value function of a non-defaulting unregulated bank at time t before paying out dividends, and let $\bar{V}_t(N^j_t)$ denote the bank's value function after paying out dividends and at the time of borrowing from the institutional investor. We then have the following Bellman equations:

$$V_t\left(\omega, A^j_{t-1}, \bar{B}^j_{t-1}\right) = \max_{N^j_t} \left\{\Pi^j_t + \bar{V}_t\left(N^j_t\right)\right\},$$

subject to (5) and (6); and

$$\bar{V}_t\left(N^j_t\right) = \max_{A^j_t, \bar{B}^j_t} E_t \Lambda_{t,t+1} \int_{\bar{\omega}^j_{t+1}} \left[\theta V_{t+1}\left(\omega, A^j_t, \bar{B}^j_t\right) + (1-\theta)\left(R^A_{t+1} A^j_t \omega - \bar{B}^j_t\right)\right] dF_t(\omega),$$

subject to (4), (7) and (8). Let $\bar{b}^j_t \equiv \bar{B}^j_t / A^j_t$ denote the face value of debt normalized by the bank's assets. This allows us to express the default threshold as $\bar{\omega}^j_t = \bar{b}^j_{t-1}/R^A_t$. The appendix proves the following result.

Proposition 1 (solution to unregulated bank's problem) *Assume the model parameters satisfy*

$$0 < \beta R^A - 1 < (1-\theta)\beta R^A \int_{\bar{\omega}^j} \left(\omega - \bar{\omega}^j\right) dF(\omega),$$

where R^A and $\bar{\omega}^j$ are the steady-state values of R^A_t and $\bar{\omega}^j_t$, respectively. Then the equilibrium dynamics of unregulated bank j in a neighborhood of the deterministic steady state are characterized by the following features:

1. *The bank optimally retains all earnings,*

$$N^j_t = \left(\omega^j - \frac{\bar{b}_{t-1}}{R^A_t}\right) R^A_t A^j_{t-1}, \tag{9}$$

where $\bar{b}_{t-1}$ is equalized across islands.

[13] The relationship between the values of a European call option and a European put option is usually referred to as the 'put-call parity'.

[14] Using integration by parts, it is possible to show that $\pi_t(\bar{\omega}^j_{t+1}) = \int^{\bar{\omega}^j_{t+1}} F_t(\omega) d\omega$. First-order stochastic dominance of $F_t(\omega)$ over $\tilde{F}_t(\omega)$ implies second-order dominance: $\int^x \tilde{F}_t(\omega) d\omega > \int^x F_t(\omega) d\omega$ for all $x > 0$. It thus follows that $\tilde{\pi}_t(\bar{\omega}^j_{t+1}) > \pi_t(\bar{\omega}^j_{t+1})$ for all $\bar{\omega}^j_{t+1} > 0$.

2. The IC constraint holds with equality. In equilibrium, the latter can be expressed as

$$1-\int \omega d\tilde{F}_t(\omega) = E_t\left\{\frac{\Lambda_{t,t+1}R^A_{t+1}(\theta\lambda_{t+1}+1-\theta)}{E_t\Lambda_{t,t+1}R^A_{t+1}(\theta\lambda_{t+1}+1-\theta)}\left[\tilde{\pi}(\bar{\omega}_{t+1};\sigma_t)-\pi(\bar{\omega}_{t+1};\sigma_t)\right]\right\}, \quad (10)$$

where $\bar{\omega}_{t+1} = \bar{b}_t/R^A_{t+1}$ and λ_{t+1} is the Lagrange multiplier associated to the participation constraint. Both $\bar{\omega}_{t+1}$ and λ_{t+1} are equalized across islands.

3. The participation constraint holds with equality,

$$A^j_t = \frac{1}{1-E_t\Lambda_{t,t+1}R^A_{t+1}\left[\bar{\omega}_{t+1}-\pi(\bar{\omega}_{t+1};\sigma_t)\right]}N^j_t \equiv \phi_t N^j_t. \quad (11)$$

According to (10), the (normalized) repurchase price $\bar{b}_t$ is set such that the gain in mean return from investing in the standard firm segment exactly compensates the bank for the loss in the put option value. According to (11), the bank's demand for assets equals its net worth times a *leverage ratio* ϕ_t which is equalized across islands. Notice that leverage decreases with the left tail risk of the bank's portfolio, as captured by the put option value $\pi(\bar{\omega}_{t+1};\sigma_t)$. Intuitively, since all the downside risk in the bank's assets is born by the institutional investor, a higher perception of such risk leads the latter to impose a tighter leverage constraint.

Once $\bar{b}_t$ and ϕ_t have been determined, it is straightforward to obtain the actual loan size, $B^j_t = (\phi_t - 1)N^j_t$; its face value, $\bar{B}^j_t = \bar{b}_t A^j_t = \bar{b}_t\phi_t N^j_t$; and the implicit gross 'repo' rate, $\bar{B}^j_t/B^j_t = \bar{b}_t\phi_t/(\phi_t-1)$. The loan-to-value ratio is then $B^j_t/A^i_t = (\phi_t-1)/\phi_t$, and the 'repo' haircut or margin is $1 - B^j_t/A^i_t = 1/\phi_t$.

3.5 Regulated banks

Regulated banks are segmented across islands too, and are thus indexed by $j \in [0,1]$. Regulated banks differ from unregulated ones in three aspects. First, regulated banks borrow in the form of deposits, D^j_{t-1}, which are remunerated at the perfectly competitive gross rate R_{t-1}, and the face value of which is protected by deposit insurance should the bank default. In particular, the representative regulated bank on island j defaults at the beginning of period t if and only if

$$R^A_t\omega^j A^{r,j}_{t-1} < R_{t-1}D^j_{t-1} \Leftrightarrow \omega^j < \frac{R_{t-1}D^j_{t-1}}{R^A_t A^{r,j}_{t-1}} \equiv \bar{\omega}^{r,j}_t,$$

where $A^{r,j}_{t-1}$ are the regulated bank's assets purchased at the end of period $t-1$. Deposit insurance implies that regulated banks are not subject to a participation constraint, because households are always willing to invest in safe bank deposits.

Second, regulated banks are subject to a regulatory capital requirement. In particular, the bank's net worth, $N^{r,j}_t$, must be at least a fraction $1/\phi^r$ of its assets: $N^{r,j}_t \geq (1/\phi^r)A^{r,j}_t$. This is equivalent to establishing an upper bound on its leverage ratio: $A^{r,j}_t/N^{r,j}_t \leq \phi^r$. Third, we assume for simplicity that regulated banks do not have access to the substandard firm segment, which eliminates the possibility of moral hazard issues. Analogously to unregulated banks, regulated banks are subject to a balance sheet constraint, $D^j_t + N^{r,j}_t = A^{r,j}_t$, and to a non-negativity constraint on dividends, $N^{r,j}_t \leq R^A_t\omega^j A^{r,j}_{t-1} - R_{t-1}D^j_{t-1}$.

The appendix lays out and solves the regulated bank's maximization problem. Here we summarize the main results. Assume the model parameters satisfy

$$0 < \beta R^A \phi^r \int_{\bar{\omega}^r} \left(\omega - \bar{\omega}^{r,j}\right) dF(\omega) - 1 < \beta R^A \phi^r \left(1 - \theta^r\right) \int_{\bar{\omega}^{r,j}} \left(\omega - \bar{\omega}^{r,j}\right) dF(\omega),$$

where $\bar{\omega}^{r,j}$ is the steady-state value of $\bar{\omega}_t^{r,j}$. Then both the non-negativity constraint on dividends and the leverage constraint bind in equilibrium,

$$\begin{aligned} N_t^{r,j} &= R_t^A A_{t-1}^{r,j} \left(\omega^j - \bar{\omega}_t^r\right), \\ A_t^{r,j} &= \phi^r N_t^{r,j}, \end{aligned}$$

where the default threshold equals

$$\bar{\omega}_t^r = \frac{R_{t-1}}{R_t^A} \frac{\phi^r - 1}{\phi^r}$$

and is thus equalized across islands.

3.6 Institutional investors

A representative institutional investor collects funds from households in the form of equity, and lends these funds to unregulated banks through short-term collateralized debt. Its balance sheet is simply $N_t^{inv} = B_t$, where $B_t = \int_0^1 B_t^j dj$. The institutional investor operates economy-wide and hence perfectly diversifies its portfolio across islands. The institutional investor's return from financing the island-j unregulated bank is

$$\min\left\{R_t^A \omega^j A_{t-1}^j, \bar{B}_{t-1}^j\right\} = R_t^A A_{t-1}^j \min\left\{\omega^j, \frac{\bar{b}_{t-1}}{R_t^A}\right\} = R_t^A \phi_{t-1} N_{t-1}^j \min\left\{\omega^j, \bar{\omega}_t\right\}.$$

Aggregating across islands and substracting gross interest payments on deposits, we obtain the return on the institutional investor's equity,

$$\begin{aligned} R_t^N N_{t-1}^{inv} &= R_t^A \phi_{t-1} \int_0^1 N_{t-1}^j \min\left\{\omega^j, \bar{\omega}_t\right\} dj - R_{t-1} B_{t-1} \\ &= R_t^A \phi_{t-1} N_{t-1} \left\{\left[1 - F_{t-1}\left(\bar{\omega}_t\right)\right] \bar{\omega}_t + \int^{\bar{\omega}_t} \omega dF_{t-1}(\omega)\right\} - R_{t-1} B_{t-1}, \end{aligned}$$

where in the second equality we have used the fact ω^j is distributed independently from N_{t-1}^j, and where $N_{t-1} \equiv \int_0^1 N_{t-1}^j dj$ is aggregate net worth of unregulated banks.

3.7 Aggregation and market clearing

Aggregate net worth of unregulated banks at the *end* of period t, N_t, is the sum of the net worth of continuing banks, N_t^{cont}, and that of new banks, N_t^{new},

$$N_t = N_t^{cont} + N_t^{new}.$$

From (9), $\bar{b}_{t-1}/R_t^A = \bar{\omega}_t$ and $A_{t-1}^j = \phi_{t-1} N_{t-1}^j$, we have that $N_t^j = R_t^A \left(\omega^j - \bar{\omega}_t\right) \phi_{t-1} N_{t-1}^j$. Aggregating across islands, we obtain the total net worth of continuing unregulated banks,

$$N_t^{cont} = \theta R_t^A \int_{\bar{\omega}_t} \left(\omega - \bar{\omega}_t\right) dF_{t-1}\left(\omega\right) \phi_{t-1} N_{t-1},$$

where we have used the fact that ω^j is distributed independently from N_{t-1}^j. Banks that default or exit the market exogenously are replaced by an equal number of new banks, $F_{t-1}\left(\bar{\omega}_t\right) + \left[1 - F_{t-1}\left(\bar{\omega}_t\right)\right]\left(1-\theta\right) = 1 - \theta\left[1 - F_{t-1}\left(\bar{\omega}_t\right)\right]$. We assume that new unregulated banks are endowed by households with a fraction τ of total assets at the beginning of the period, $A_{t-1}^T \equiv A_{t-1} + A_{t-1}^r$, where $A_t \equiv \int_0^1 A_t^j dj$ and $A_t^r \equiv \int_0^1 A_t^j dj$ are total assets of unregulated and regulated banks, respectively. Therefore,

$$N_t^{new} = \left\{1 - \theta\left[1 - F_{t-1}\left(\bar{\omega}_t\right)\right]\right\} \tau A_{t-1}^T.$$

We thus have

$$N_t = \theta R_t^A \int_{\bar{\omega}_t} \left(\omega - \bar{\omega}_t\right) dF_{t-1}\left(\omega\right) \phi_{t-1} N_{t-1} + \left\{1 - \theta\left[1 - F_{t-1}\left(\bar{\omega}_t\right)\right]\right\} \tau A_{t-1}^T. \qquad (12)$$

New banks leverage their starting net worth with the same ratio as continuing banks. We thus have

$$A_t = \phi_t \left(N_t^{cont} + N_t^{new}\right) = \phi_t N_t.$$

Aggregation for regulated banks is performed analogously. Their total net worth, N_t^r, and their total assets evolve as follows

$$N_t^r = \theta^r R_t^A \int_{\bar{\omega}_t^r} \left(\omega - \bar{\omega}_t^r\right) dF_{t-1}\left(\omega\right) \phi^r N_{t-1}^r + \left\{1 - \theta^r\left[1 - F_{t-1}\left(\bar{\omega}_t^r\right)\right]\right\} \tau^r A_{t-1}^T, \qquad (13)$$

$$A_t^r = \phi^r N_t,$$

where $\left(\theta^r, \tau^r\right)$ may differ from $\left(\theta, \tau\right)$. Aggregate net dividends to households from unregulated and regulated banks are given, respectively, by

$$\begin{aligned}
\Pi_t &= \left(1-\theta\right) R_t^A \int_{\bar{\omega}_t} \left(\omega - \bar{\omega}_t\right) dF_{t-1}\left(\omega\right) \phi_{t-1} N_{t-1} - N_t^{new}, \\
\Pi_t^r &= \left(1-\theta^r\right) R_t^A \int_{\bar{\omega}_t^r} \left(\omega - \bar{\omega}_t^r\right) dF_{t-1}\left(\omega\right) \phi^r N_{t-1}^r - N_t^{new,r},
\end{aligned}$$

where $N_t^{new,r}$ is defined analogously to N_t^{new}. Total net dividends from banks to households are $\Pi_t^b = \Pi_t + \Pi_t^r - T_t$, where $T_t \equiv R_t^A \int^{\bar{\omega}_t^r} \left(\bar{\omega}_t^r - \omega\right) dF_{t-1}\left(\omega\right) \phi^r N_{t-1}^r$ are lump-sum payments to the deposit insurance fund, equivalent to the amount needed to cover the current period's gap between the face value of deposits and asset returns in defaulting regulated banks.[15]

Market clearing for capital requires that total demand by firms equals total supply by capital producers, $\int_0^1 K_t^j dj = K_t$. The aggregate capital stock evolves as follows,

$$K_{t+1} = I_t + \left(1-\delta\right) K_t.$$

[15] The deposit insurance fund may be interpreted as being financed by households by means of lump-sum taxes, which would require us to include a (trivial) fiscal authority in the model. Alternatively, the fund may be financed by compulsory contributions by non-defaulting regulated banks, imposed by a hypothetical regulator.

The total issuance of state-contingent claims by firms must equal total demand by banks (regulated and unregulated),

$$K_{t+1} = A_t + A_t^r.$$

From (2), firm j's labor demand is $L_t^j = \left[(1-\alpha) Z_t / W_t\right]^{1/\alpha} \omega^j K_t^j$. Aggregating across islands and imposing labor market clearing, we have

$$\int_0^1 L_t^j dj = \left(\frac{(1-\alpha) Z_t}{W_t}\right)^{1/\alpha} \int_0^1 \omega^j K_t^j dj = \left(\frac{(1-\alpha) Z_t}{W_t}\right)^{1/\alpha} K_t = L_t, \tag{14}$$

where we have used the fact that ω^j and K_t^j are distributed independently and the fact that ω^j has unit mean. Equations (2) and (14) then imply that $\omega^j K_t^j / L_t^j = K_t / L_t$. Using the latter and (1), aggregate supply of the final good by firms equals

$$Y_t = \int_0^1 Y_t^j dj = Z_t \left(\frac{L_t}{K_t}\right)^{1-\alpha} \int_0^1 \omega^j K_t^j dj = Z_t K_t^\alpha L_t^{1-\alpha}.$$

Finally, total supply of the final good must equal consumption demand by households and investment demand by capital producers,

$$Y_t = C_t + I_t.$$

4 Quantitative Analysis

4.1 Calibration and steady state

We calibrate our model to the US economy for the period 1984:Q1-2011:Q3. The parameters are shown in Table 2. We may divide the parameters between those that are standard in the real business cycle (RBC) literature, and those that are particular to the financial structure of the model. From now onwards, we let variables without time subscripts denote steady-state values.

We set the RBC parameters to standard values. In particular, we set $\beta = 0.99 = 1/R$, $\alpha = 0.36 = 1 - WL/Y$, $\delta = 0.025 = I/K$, which are broadly consistent with long-run averages for the real interest rate, the labor share, and the investment to capital ratio. For future use, we note that the steady-state return on banks' assets is $R^A = \alpha\,(Y/K) + 1 - \delta$. We target a capital-output ratio of $K/Y = 8$, with is consistent with a ratio of investment over GDP of 20 percent, roughly in line with the historical evidence. We then have $R^A = 1.02$. Our functional forms for preferences are standard: $u(x) = \log(x)$, $v(L) = L^{1+\varphi}/(1+\varphi)$. We set $\varphi = 1$, in line with other macroeconomic studies (see e.g. Comin and Gertler, 2006). We assume an AR(1) process for the natural log of TFP,

$$\log(Z_t/Z) = \rho_z \log(Z_{t-1}/Z) + \varepsilon_t^z,$$

where $\varepsilon_t^z \overset{iid}{\sim} N(0, \sigma_z)$. Our empirical counterpart for $\log\left(Z_t/\bar{Z}\right)$ is the quarterly TFP series constructed by the CSIP at the Federal Reserve Bank of San Francisco, after being logged and linearly detrended.[16] We then choose ρ_z and σ_z so as to match their empirical counterparts. Z is chosen such that steady-state output is normalized to one.

Regarding the parameters related to the financial side of the model, our calibration strategy is as follows. In the model there are two types of leveraged financial intermediaries: regulated and

[16] See the data appendix for more information.

unregulated banks. We identify the regulated banking sector in the model with the US-chartered commercial banks sector in the data, and the unregulated sector with the security broker/dealers sector.[17] We then set the leverage ratio of regulated banks to match the average leverage ratio of commercial banks during our sample period, $\phi^r = 10.66$. We also use the average leverage ratio of security broker/dealers in our sample, 29.30, as the target for the steady-state leverage ratio of unregulated banks, ϕ. The latter implies a repo loan-to-value ratio of $b = B/A = (\phi - 1)/\phi = 0.9659$, or equivalently a repo haircut of 3.41%; the latter is in line with average pre-crisis haircuts for repos backed by corporate debt and private-label ABS, as documented by Krishnamurthy et al. (2012). The same authors show that the spread between the repo rates for the same collateral categories and the Fed funds rate was close to zero in the pre-crisis period. Based on this, we target a spread in short-term collateralized debt contracts of 25 annualized basis points. The repo rate then equals $\bar{R} = R(1.0025)^{1/4} = 1.0107$. The face value of repo debt (normalized by assets) is then $\bar{b} = \bar{R}b = 0.9762$. This implies a default threshold for unregulated banks of $\bar{\omega} = \bar{b}/R^A = 0.9571$. For regulated banks, the default threshold is $\bar{\omega}^r = \left(R/R^A\right)(\phi^r - 1)/\phi^r = 0.8974$.

Island-specific shocks are assumed to be lognormally distributed. In particular, the distribution of island-specific shocks to the standard and the substandard firm segment is given by

$$\log \omega \overset{iid}{\sim} N\left(\frac{-\sigma_t^2}{2}, \sigma_t\right),$$
$$\log \tilde{\omega} \overset{iid}{\sim} N\left(\frac{-\eta\sigma_t^2 - \psi}{2}, \sqrt{\eta}\sigma_t\right),$$

respectively. Therefore, $F(\omega; \sigma_t) = \Phi\left(\frac{\log(\omega) + \sigma_t^2/2}{\sigma_t}\right)$, where $\Phi(\cdot)$ is the standard normal cdf. The parameters $\psi > 0$ and $\eta > 1$ control, respectively, the mean and the volatility of the substandard technology relative to the standard one. Notice in particular that

$$E[\omega] = 1 > E[\tilde{\omega}] = e^{-\psi/2}.$$

These distributional assumptions imply the following expressions for the values of the unit put options on island-specific risk,[18]

$$\pi(\bar{\omega}_t; \sigma_{t-1}) = \bar{\omega}_t \Phi\left(\frac{\log(\bar{\omega}_t) + \sigma_{t-1}^2/2}{\sigma_{t-1}}\right) - \Phi\left(\frac{\log(\bar{\omega}_t) - \sigma_{t-1}^2/2}{\sigma_{t-1}}\right), \tag{15}$$
$$\tilde{\pi}(\bar{\omega}_t; \sigma_{t-1}) = \bar{\omega}_t \Phi\left(\frac{\log(\bar{\omega}_t) + \left(\psi + \eta\sigma_{t-1}^2\right)/2}{\sqrt{\eta}\sigma_{t-1}}\right) - e^{-\psi/2}\Phi\left(\frac{\log(\bar{\omega}_t) + \left(\psi - \eta\sigma_{t-1}^2\right)/2}{\sqrt{\eta}\sigma_{t-1}}\right) \tag{16}$$

The standard deviation of island-specific shocks is assumed to follow an AR(1) process in logs,

$$\log(\sigma_t/\sigma) = \rho_\sigma \log(\sigma_{t-1}/\sigma) + \varepsilon_t^\sigma,$$

[17] Ideally, one would construct empirical counterparts of both sectors by consolidating the different subsectors described in section 2. The regulated banking sector would be the result of consolidating the balance sheets of US-chartered commercial banks, savings institutions and credit unions, whereas the unregulated one would be composed of security brokers and dealers, finance companies and GSEs. As explained in section 2, this consolidation is however not feasible, due to the existence of cross-positions among financial subsectors and the need to avoid double-counting. For this reason, we choose to identify the regulated and unregulated banking sectors in the model with the US-chartered commercial bank and security broker/dealer subsectors, respectively, which are the largest in terms of total assets within each sector (see Figure 3).

[18] The proof is available upon request.

where $\varepsilon_t^{\sigma} \overset{iid}{\sim} N(0, \sigma_\sigma)$. In order to calibrate σ, we notice that the participation constraint (eq. 11) in the steady state implies $\pi(\bar{\omega}; \sigma) = \bar{\omega} - (1 - 1/\phi) / \beta R^A = 0.0006$. Using the steady-state counterpart of (15), we can then solve for $\sigma = 0.0272$. The default rates of unregulated and regulated banks in the steady state then equal $F(\bar{\omega}; \sigma) = 0.0547$ and $F(\bar{\omega}; \sigma) = 0.00004$, respectively. In order to calibrate the parameters governing the dynamics of island-specific volatility $(\rho_\sigma, \sigma_\sigma)$, we use the TFP series for all 4-digit SIC manufacturing industries constructed by the NBER and the US Census Bureau's Center for Economic Studies (CES). We then construct a time series for σ_t by calculating the cross-sectional standard deviation of the industry-level TFP series (in log deviations from a linear trend) at each point in time. Fitting an autoregressive process to the resulting series, we obtain $\rho_\sigma = 0.9457$ and $\sigma_\sigma = 0.0465$.[19]

Regarding the parameters of the substandard technology, ψ and η, we make use of the IC constraint in the steady state,

$$1 - e^{-\psi/2} = \tilde{\pi}(\bar{\omega}; \sigma) - \pi(\bar{\omega}; \sigma),$$

where $\tilde{\pi}(\bar{\omega}; \sigma)$ is given by expression (16) in the steady state. We thus have one equation for two unknowns, ψ and η. We choose to set ψ to 0.01 for illustrative purposes, and use the IC constraint to solve for $\eta = 3.1442$. This implies that shocks to the substandard firm segment are $\sqrt{\eta} = 1.77$ times more volatile than the standard one.

Finally, the exogenous bank continuation rates (θ, θ^r) and the bank equity injection parameters (τ, τ^r) are calibrated as follows. We start by targeting the size of the unregulated banking sector relative to the regulated one, A/A^r. As noted above, for the purpose of calibrating the unregulated leverage ratio, we focused on security broker/dealers as representative of the entire unregulated sector. While this approach avoids the problem of double-counting of cross positions, it is also likely to considerably underestimate the size of the consolidated unregulated sector, because both finance companies and GSEs are relatively large in terms of assets (see again Figure 3). For this reason, as a rough approximation we assume $A/A^r = 1$, that is, we consider both sectors to be of the same size. In the steady state, the law of motion of unregulated bank net worth (eq. 12) becomes

$$\frac{1}{\phi} = \theta R^A \int_{\bar{\omega}} (\omega - \bar{\omega}) \, dF(\omega; \sigma) + \{1 - \theta [1 - F(\bar{\omega}; \sigma)]\} \tau \left(1 + \frac{A^r}{A}\right), \tag{17}$$

where we have normalized by A. We set θ to 0.75, and then use (17) to solve for $\tau = 0.0015$.[20] We proceed analogously for regulated banks. In particular, using the steady-state counterpart of eq. (13), rescaling by A^r, and assuming $\theta^r = \theta$ for symmetry, we obtain $\tau^r = 0.0306$.

4.2 The response to TFP shocks

We follow the lead of the traditional RBC literature by exploring how well a TFP shock can explain the unconditional patterns found in the data. Table 3 displays the second-order moments of interests. They include the standard deviations of GDP, assets and leverage, as well as the correlations of assets and leverage with GDP, and the correlation between assets and leverage of the unregulated banks. As commented above, we use the US-chartered commercial banks and the security brokers/dealers as the empirical counterparts of the regulated and unregulated banking

[19] See data appendix for details.

[20] Equation (17) implies that τ is a decreasing function of θ, given the other parameters and steady state values. In the choice of θ, we are restricted by the requirement that $\tau \geq 0$, which holds for $\theta \leq 0.77$.

sectors in the model, respectively. Model moments are based on simulated series. In order to make the model moments comparable with the empirical ones, we first log the simulated series and filter them using the same bandpass filter as the one applied to the data.[21] The table also includes the correlation coefficients based on *un*filtered simulated series, which allows us to verify whether the existence of correlations (or lack thereof) is indeed the result of the model's endogenous propagation mechanism, or is just an artifact of the filtering procedure.

As shown by the second column of Table 3, conditional on TFP shocks the model replicates fairly well the standard deviation of GDP, as well as the correlations of the assets of both regulated and unregulated banks with GDP. However, the model fails dramatically at reproducing the volatility of assets and leverage. It also fails to produce any meaningful procyclicality in the leverage ratio of unregulated banks. Finally, TFP shocks seem to produce a correlation between assets and leverage of unregulated banks (0.64) similar to the empirical one (0.76). However, the correlation based on unfiltered simulated series (0.08) clearly indicates that such comovement is not inherent to the model's dynamics, but is merely induced by the filtering procedure.

To understand these results, Figure 4 displays the (unfiltered) impulse response to a negative TFP shock (dashed line). On impact, the fall in TFP produces a sharp fall in the return on assets, which increases the number of bankruptcies both in the unregulated and regulated banking sectors (the latter not displayed). The fall in the profitability of banks' investments reduces their equity. In the case of unregulated banks, the leverage ratio barely reacts; indeed, the latter responds mainly to *expected* changes in the default threshold (see eq. 11), which is virtually back to baseline after the impact period. This explains the low volatility of leverage and its lack of correlation with assets or output. Since their leverage remains stable, unregulated banks' assets basically reproduce the response of their net worth; i.e. the effects of TFP shocks on unregulated bank credit operate mainly through the equity channel.[22] Since net worth responds relatively little, so do assets, hence their low volatility.

4.3 The volatility-leverage channel

A recent financially oriented literature shows how an increase in the volatility of asset returns reduces borrowers' leverage. For example, Brunnermeier and Pedersen (2009) analyze how an increase in the volatility of asset prices leads investors to demand higher margins, thus forcing borrowers to deleverage. Similarly, Geanakoplos (2010) or Fostel and Geanakoplos (2008), consider shocks that not only decrease the expected asset returns but also their volatility. Such shocks, which the authors refer to as 'scary bad news', lead to tighter margins as lenders protect themselves against increased uncertainty. From a more macro perspective, recent work suggests that exogenous changes in volatility may be an important driving force behind business cycle fluctuations (see e.g. Bloom, 2009; Bloom et al., 2011; Christiano et al., 2010; Gilchrist et al., 2010).

In our model, an increase in the standard deviation of island-specific shocks, σ_t, induces a reduction in the leverage of unregulated banks, via a mechanism close to the one described in Adrian and Shin (2011a) and sketched in Figure 5. The upper subplot represents the steady-state counterpart of the IC constraint (eq. 10). The blue line is the gain in left tail risk from investing in the substandard firm segment, $\Delta\pi\left(\bar{\omega};\sigma\right)=\tilde{\pi}\left(\bar{\omega};\sigma\right)-\pi\left(\bar{\omega};\sigma\right)$, which under our distributional

[21] In particular, we simulate the model for 5,000 periods and discard the first 500 observations to eliminate the effect of initial conditions. The model is solved by means of a first-order Taylor approximation (in levels). The code has been implemented in Dynare.

[22] In the case of regulated bank credit, the equity channel is the only one by construction, as leverage (ϕ^r) is exogenous.

assumptions is an increasing function of the (normalized) face value of debt, $\bar{\omega} = \bar{B}^j / (R^A A^j) = \bar{b}/R^A$. The horizontal line is the loss in mean return, $E(\omega) - \tilde{E}(\omega) = 1 - \int \omega d\tilde{F}(\omega, \sigma)$. The IC constraint requires $\bar{\omega}$ to be such that the gain in left tail risk from investing in the substandard technology does not exceed the loss in mean return. Since the constraint is binding in equilibrium, $\bar{\omega}$ is determined by the intersection of both lines. Consider now an increase in cross-sectional volatility, σ. Provided $\Delta\pi$ is increasing in σ (which holds under our distributional assumptions), then *ceteris paribus* the $\Delta\pi(\bar{\omega}, \cdot)$ schedule shifts upwards and $\bar{\omega}$ goes down. Intuitively, since higher volatility makes it more attractive for the bank to invest inefficiently, the institutional investor reduces the (normalized) face value of debt so as to induce the former to invest efficiently.

The lower subplot of Figure 5 represents the steady-state counterpart of the participation constraint, $\phi = 1/\{1 - \beta R^A [\bar{\omega} - \pi(\bar{\omega}; \sigma)]\}$. The latter represents an upward-sloping relationship between leverage, $\phi = (B^j + N^j)/N^j$, and the normalized face value of debt, $\bar{\omega}$.[23] *Ceteris paribus*, the increase in σ has a double effect on leverage. First, the leverage schedule shifts down, which reduces equilibrium leverage for a given $\bar{\omega}$. Intuitively, higher volatility of island-specific shocks increases the downside risk $\pi(\bar{\omega}; \sigma)$ of the assets that serve as collateral, which reduces the investor's expected payoff; in order to induce the investor to lend, the bank reduces its demand for funds as a fraction of its net worth. Second, the reduction in $\bar{\omega}$ through the IC constraint produces a leftwards movement *along* the leverage schedule, thus further reducing equilibrium leverage. Both effects are mutually reinforcing.

How does this volatility-leverage channel operate in general equilibrium? To analyze this, we simulate the model conditional on shocks to cross-sectional volatility. The results are shown in the third column of Table 3. The model generates now large fluctuations in the leverage ratio of unregulated banks, comparable to those in the data. It also produces larger fluctuations in the assets of both regulated and unregulated banks than those generated by TFP shocks. The fluctuations in output are however relatively modest. In terms of correlations, volatility shocks produce a strong procyclicality in the assets and leverage of unregulated banks relative to GDP, well above the empirical correlations. It also produces a strong positive comovement between assets and leverage, similar to that found in the data. A pitfall of volatility shocks is that they generate countercyclical fluctuations in regulated banks' assets, which is clearly at odds with the data. Finally, the correlations based on unfiltered simulated series are very close to the baseline ones, which clearly indicates that such correlations are indeed inherent to the model structure.

To understand these results, the solid line in Figure 4 displays the responses to an increase in cross-sectional volatility. The shock produces a sharp reduction in the (normalized) face value of debt of unregulated banks, $\bar{\omega}_t = \bar{b}_t / R^A_{t+1}$, right after the impact period. This fall in the debt commitment, together with the increase in uncertainty, produce a drastic reduction in the leverage ratio of unregulated banks, of about 2 and a half percentage points. Unregulated banks' net worth increases after the impact period, due to the reduction in the default threshold $\bar{\omega}_t$ and hence in the number of defaulting banks. However, the drop in leverage dominates the increase in net worth, as evidenced by the large fall in unregulated banks' assets. The shock also generates a substitution effect, in the sense that regulated banks' assets rise. The latter follows from the increase in regulated banks' net worth (not shown) together with a constant leverage ratio.[24] However, the

[23] The investor's expected payoff is $\beta R^A [\bar{\omega} - \pi(\bar{\omega})]$. That is, the investor's exposition to island-specific risk is equivalent to holding cash in the amount $\bar{\omega}$ and a short position in a put option with strike price $\bar{\omega}$ (Merton, 1974; Adrian and Shin, 2011a). Since $\pi'(\bar{\omega}) = F(\bar{\omega}) < 1$, the investor's expected payoff from lending to the bank *increases* with $\bar{\omega}$. As a result, the bank can borrow more (as a fraction of its net worth) while still persuading the investor to lend the funds.

[24] We may interpret this substitution effect in terms of a *flight to quality* following an increase in uncertainty.

latter effect is dominated by the fall in unregulated banks' assets, with the resulting contraction in total intermediated assets, the capital stock, and aggregate output.[25] This volatility-leverage channel provides an alternative mechanism to the ones presented by Bloom et al. (2011) or Gilchrist et al. (2010), through which changes in cross-sectional uncertainty may generate aggregate business cycles.

Finally, the last column in Table 3 shows the combined effects of both TFP and volatility shocks in the model. This specification improves upon the previous ones mostly in terms of correlations. In particular, the existence of two uncorrelated sources of fluctuations reduces the procyclicality of assets and leverage of unregulated banks to levels comparable to those in the data, whereas it preserves the high correlation between assets and leverage. The unconditional correlation between regulated banks' assets and GDP is (slightly) negative, indicating that the substitution effect between regulated and unregulated bank assets due to volatility shocks is strong enough to overcome the effect of TFP shocks.[26] Regarding the standard deviations, the unconditional volatility of aggregate output is dominated by TFP shocks, while that of assets and leverage is mostly determined by volatility shocks. In particular, the model overpredicts the volatility of regulated banks' assets and underpredicts that of unregulated banks' assets, while capturing fairly well the size of fluctuations in unregulated banks' leverage.

4.4 The risk diversification paradox

The exercises presented above seem to indicate that the model is able to roughly replicate the data in a number of dimensions. In particular, it can explain the bank leverage cycles observed in the data as the result of exogenous changes in cross-sectional volatility. Given these results, this section analyzes which is the macroeconomic impact of different levels of *average* volatility. We may indeed consider a scenario in which financial innovation allows banks, regulated and unregulated, to better diversify their risks. In terms of the model, this amounts to a reduction in the steady-state volatility of island-specific shocks, σ. The question then is: what is the effect of this financial innovation both on the mean level *and* the volatility of output.

To answer this question, we study the behavior of the model as we lower σ from its baseline value of 0.027 to 0.015. For the purpose of this exercise, we simulate the model with both TFP and volatility shocks. Figure 6 displays the results. The upper panels display the mean values of unregulated bank leverage (ϕ) and output (Y), as well as the mean relative size of the unregulated bank sector. The lower panels display the standard deviations of leverage and output. In this case the data have not been filtered, as we need to preserve the means and we do not compare model results with data.

As shown in the figure, a reduction in cross-sectional uncertainty allows unregulated banks to increase their leverage on average, through a mechanism very similar to the one explained before. For a given net worth, higher leverage allows unregulated banks to expand the size of their balance-sheets. This in turn leads to an increase in the stock of capital, and hence in the average level

Indeed, contrary to unregulated banks' liabilities, regulated banks' deposits are protected by deposit insurance and thus their expected value is not affected by changes in volatility.

[25] Aggregate output falls by less than in the case of TFP shocks, due to a smaller reduction in private consumption (not shown).

[26] This negative correlation can be reduced by assuming a smaller relative size of the unregulated banking sector compared to the regulated one, as it reduces the impact of the substitution effect on the total volume of the regulated banks assets. Nevertheless, we prefer to maintain an equal size of both sectors for clarity reasons and leave this issue for future research.

of output. Therefore, financial innovations that improve risk diversification induce an economic expansion on average via an increase in capital accumulation. This results is not controversial and has been confirmed by historical evidence, as discussed in Kindleberger (1986).

The effects on the volatilities are more striking. A reduction in cross-island volatility generates an *increase* in the volatility of output. For lack of a better name, we have named this effect 'the risk diversification' paradox, even though such a paradox is only apparent. A reduction in cross-island volatility increases the relative size of the unregulated banking sector (upper right panel). As discussed in the previous sections, the assets of unregulated banks are more volatile than those of regulated ones, due to the endogenous fluctuations in the leverage ratio of the former. In addition, these leverage cycles are larger as a consequence of the increase in the mean value of leverage. The consequence is that a reduction in cross-island volatility leads to larger fluctuations in total intermediated assets. This in turn results in larger fluctuations in the capital stock, and hence in aggregate output. This unconditional result holds also conditionally on TFP shocks and volatility shocks.[27]

The conclusion is that risk diversification has both a positive level effect on economic activity, and a negative effect through an increase in aggregate volatility, where the latter is due to higher leverage and thus a larger size of the unregulated banking sector. The optimal size of risk diversification will depend on the degree risk aversion of the households, a point that we leave for further research.

5 Conclusions

We have presented empirical evidence regarding the comovements between the assets and leverage of financial intermediaries and GDP in the United States. We have found that leverage and assets are several times more volatile than GDP, and that they are more volatile for unregulated ('shadow') intermediaries such as security brokers and dealers or finance companies, than for regulated intermediaries such as commercial banks. We have also found that the leverage of regulated intermediaries is rather acyclical with respect to both assets and GDP, whereas the leverage of unregulated intermediaries is strongly procyclical with respect to assets, and marginally procyclical at best with respect to GDP. These findings suggest the need to consider endogenous leverage within the context of macroeconomic models with financial intermediaries.

We have then built a general equilibrium model with financial intermediaries and endogenous leverage, and assessed its ability to match the evidence. The model incorporates a two-tier financial intermediation sector, regulated and unregulated. The leverage ratio of unregulated intermediaries is endogenously determined as the result of a contracting problem between the latter and a sector of institutional investors. Due to moral hazard on the part of unregulated banks, institutional investors restrict their lending to a certain ratio of the former's net worth. In the model, TFP shocks produce rather small fluctuations in leverage and assets, and fail to produce any meaningful comovement between leverage and either assets of GDP. Shocks to cross-sectional volatility do generate large fluctuations in assets and leverage, as well as a positive (albeit excessively so) comovement between leverage and assets or GDP. Combining TFP and volatility shocks allows the model to produce cyclical comovements roughly similar to those in the data.

Finally, we have shown that, in the context of our model, an economy with lower average cross-sectional volatility has a higher average stock of capital and higher average output. However, it also has a higher output volatility. This stems from the fact a lower perception of risk in asset

[27] Results are available upon request.

returns leads to an increase in the leverage of unregulated financial intermediaries and to larger fluctuations in their lending activity.

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Appendix

Data appendix

Data on equity capital and total assets of the six leveraged financial subsectors (US-chartered commercial banks, savings institutions, credit unions, security brokers and dealers, finance companies, and GSEs) are from the Z.1 files of the US Flow of Funds.[28] The series corresponding to savings institutions are the sum of OTS and FDIC reporters. Data on *levels* in the Z.1 files (denoted by 'FL' in the series identifier) suffer from discontinuities that are caused by changes in the definition of the series. The Flow of Funds accounts correct for such changes by constructing *discontinuities* series (denoted by 'FD').[29] In particular, for each series the *flow* (denoted by 'FU') is equal to the change in level outstanding less any discontinuity. That is: $\text{FU}_t = \text{FL}_t - \text{FL}_{t-1} - \text{FD}_t$. Therefore, the flow data are free from such discontinuities. In order to construct discontinuity-free level series, we take the value of the level in the first period of the sample and then accumulate the flows onwards.

For each subsector, the leverage ratio is the ratio between total assets and equity capital, both in dollars. In the tables and figures (except figure 3), 'total assets' refer to real total assets, which are total assets (in dollars) divided by the GDP Implicit Price Deflator. The latter and Real GDP are both from the Bureau of Economic Analysis. Both series are readily available at the Federal Reserve Bank of St. Louis FRED database.[30]

The sample period is 1984:Q1-2011:Q3, except for data on GSEs which run through 2008:Q2. The reason is that the equity capital of GSEs plummeted from \$113bn to \$8bn between 2008:Q2 and 2008:Q4, whereas total assets remained fairly constant. As a result, the consolidated leverage ratio rose in the same time span from around 29 (the highest up to that moment but still comparable in magnitude with the historical series) to 405. Coincidentally with these developments, both Freddie Mac and Fannie Mae, two agencies which jointly accounted for most total assets of GSEs throughout our sample, were placed in conservatorship in September 2008 by the US federal government. For this reason, we restrict our sample for GSEs to the 2nd quarter of 2008.

In order to obtain an empirical proxy for aggregate TFP, we use the quarterly Business sector TFP growth series (labelled 'dtfp') constructed by the Center for the Study of Income and Productivity (CSIP) at the Federal Reserve Bank of San Francisco.[31] We then accumulate growth rates to obtain the level series.

Finally, in order to construct a proxy for island-specific volatility, we use the annual TFP series for all 4-digit SIC manufacturing industries constructed by the National Bureau of Economic Research (NBER) and the US Census Bureau's Center for Economic Studies (CES).[32] The data run through 2005, so our sample period in this case is 1984-2005. We discard those industries that exit the sample in the mid-nineties due to the change in industry classification from SIC to NAICS. We then log and linearly detrend each industry TFP series. Our proxy for the time series of (annual) island-specific volatility is the cross-sectional standard deviation of all industry TFP series in each year. We may denote the latter by σ_τ^a, where τ is the year subscript. Assuming that the underlying quarterly process is $\log \sigma_t = (1-\rho_\sigma)\log\sigma + \rho_\sigma \log \sigma_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim iid\,(0, \sigma_\sigma)$, and that each annual observation corresponds to the last quarter in the year, then the annual process satisfies

[28] Website: http://www.federalreserve.gov/datadownload/Choose.aspx?rel=Z1

[29] For instance, changes to regulatory report forms and/or accounting rules typically trigger 'FD' entries for the affected series.

[30] Website: http://research.stlouisfed.org/fred2/

[31] Website: http://www.frbsf.org/csip/tfp.php

[32] Website: http://www.nber.org/data/nbprod2005.html

$corr(\log\sigma^a_\tau, \log\sigma^a_{\tau-1}) = \rho^4_\sigma$, and $var(\log\sigma^a_\tau) = \frac{1+\rho^2_\sigma+\rho^4_\sigma+\rho^6_\sigma}{1-\rho^8_\sigma}\sigma^2_\sigma$. The sample autocorrelation and variance of $\log\sigma^a_\tau$ are 0.7997 and 0.0205, respectively, which imply $\rho_\sigma = 0.9457$ and $\sigma_\sigma = 0.0465$.

The unregulated bank's problem

We start by defining the ratio $\bar{b}^j_{t-1} \equiv \bar{B}^j_{t-1}/A^j_{t-1}$ and using the latter to substitute for $\bar{B}^j_{t-1} = \bar{b}^j_{t-1}A^j_{t-1}$. Given the choice of investment size A^j_t, the bank then chooses the ratio $\bar{b}^j_t$. With this transformation, and abusing somewhat the notation V_t and $\bar{V}_t$ in the main text, the bank's maximization problem can be expressed as

$$V_t\left(\omega, A^j_{t-1}, \bar{b}^j_{t-1}\right) = \max_{N^j_t}\left\{\left(\omega - \frac{\bar{b}^j_{t-1}}{R^A_t}\right)R^A_t A^j_{t-1} - N^j_t + \bar{V}_t\left(N^j_t\right) + \mu^j_t\left[\left(\omega - \frac{\bar{b}^j_{t-1}}{R^A_t}\right)R^A_t A^j_{t-1} - N^j_t\right]\right\}, \tag{18}$$

$$\bar{V}_t\left(N^j_t\right) = \max_{A^j_t, \bar{b}^j_t} E_t\Lambda_{t,t+1}\int_{\bar{b}^j_t/R^A_{t+1}}\left[\theta V_{t+1}\left(\omega, A^j_t, \bar{b}^j_t\right) + (1-\theta)\left(\omega - \bar{b}^j_t/R^A_{t+1}\right)R^A_{t+1}A^j_t\right]dF_t(\omega)$$

subject to the participation constraint,

$$E_t\Lambda_{t,t+1}R^A_{t+1}A^j_t\left\{\int^{\bar{b}^j_t/R^A_{t+1}}\omega dF_t(\omega) + \frac{\bar{b}^j_t}{R^A_{t+1}}\left[1 - F_t\left(\frac{\bar{b}^j_t}{R^A_{t+1}}\right)\right]\right\} \geq A^j_t - N^j_t,$$

and the IC constraint

$$\begin{aligned} & E_t\Lambda_{t,t+1}\int_{\bar{b}^j_t/R^A_{t+1}}\left\{\theta V_{t+1}\left(\omega, A^j_t, \bar{b}^j_t\right) + (1-\theta)R^A_{t+1}A^j_t\left(\omega - \frac{\bar{b}^j_t}{R^A_{t+1}}\right)\right\}dF_t(\omega) \\ \geq\quad & E_t\Lambda_{t,t+1}\int_{\bar{b}^j_t/R^A_{t+1}}\left\{\theta V_{t+1}\left(\omega, A^j_t, \bar{b}^j_t\right) + (1-\theta)R^A_{t+1}A^j_t\left(\omega - \frac{\bar{b}^j_t}{R^A_{t+1}}\right)\right\}d\tilde{F}_t(\omega). \end{aligned}$$

The first order condition with respect to N^j_t is given by

$$\mu^j_t = \bar{V}'_t\left(N^j_t\right) - 1.$$

We can now guess that $\bar{V}'_t(N^j_t) > 1$. Then $\mu^j_t > 0$ and the non-negativity constraint on dividends is binding, such that a continuing bank optimally decides to retain all earnings,

$$N^j_t = \left(\omega - \frac{\bar{b}^j_{t-1}}{R^A_t}\right)R^A_t A^j_{t-1}. \tag{19}$$

From (18), we then have $V_t(\omega, A^j_{t-1}, \bar{b}^j_{t-1}) = \bar{V}_t((\omega - \bar{b}^j_{t-1}/R^A_t)R^A_t A^j_{t-1})$. Using the latter, we can express the Bellman equation for $\bar{V}_t(N^j_t)$ as

$$\bar{V}_t\left(N^j_t\right) = \max_{A^j_t, \bar{b}^j_t}\left\{\begin{array}{c} E_t\Lambda_{t,t+1}\int_{\bar{b}^j_t/R^A_{t+1}}\left[\theta\bar{V}_{t+1}\left(\left(\omega - \frac{\bar{b}^j_t}{R^A_{t+1}}\right)R^A_{t+1}A^j_t\right) + (1-\theta)\left(\omega - \frac{\bar{b}^j_t}{R^A_{t+1}}\right)R^A_{t+1}A^j_t\right]dF_t(\omega) \\ +\lambda^j_t\left\{E_t\Lambda_{t,t+1}R^A_{t+1}A^j_t\left[\int^{\bar{b}^j_t/R^A_{t+1}}\omega dF_t(\omega) + \frac{\bar{b}^j_t}{R^A_{t+1}}\left[1 - F_t\left(\frac{\bar{b}^j_t}{R^A_{t+1}}\right)\right]\right] - \left(A^j_t - N^j_t\right)\right\} \\ +\xi^j_t E_t\Lambda_{t,t+1}\int_{\bar{b}^j_t/R^A_{t+1}}\left\{\theta\bar{V}_{t+1}\left(\left(\omega - \frac{\bar{b}^j_t}{R^A_{t+1}}\right)R^A_{t+1}A^j_t\right) + (1-\theta)R^A_{t+1}A^j_t\left(\omega - \frac{\bar{b}^j_t}{R^A_{t+1}}\right)\right\}dF_t(\omega) \\ -\xi^j_t E_t\Lambda_{t,t+1}\int_{\bar{b}^j_t/R^A_{t+1}}\left\{\theta\bar{V}_{t+1}\left(\left(\omega - \frac{\bar{b}^j_t}{R^A_{t+1}}\right)R^A_{t+1}A^j_t\right) + (1-\theta)R^A_{t+1}A^j_t\left(\omega - \frac{\bar{b}^j_t}{R^A_{t+1}}\right)\right\}d\tilde{F}_t(\omega) \end{array}\right\}$$

where λ_t^j and ξ_t^j are the Lagrange multipliers associated to the participation and IC constraints, respectively. The first order conditions with respect to A_t^j and $\bar{b}_t^j$ are given by

$$\begin{aligned} 0 &= E_t\Lambda_{t,t+1}R_{t+1}^A \int_{\bar{\omega}_{t+1}^j} \left[\theta \bar{V}_{t+1}'\left(N_{t+1}^j\right) + 1 - \theta\right]\left(\omega - \bar{\omega}_{t+1}^j\right) dF_t(\omega) \\ &\quad + \lambda_t^j \left\{E_t\Lambda_{t,t+1}R_{t+1}^A \left[\int^{\bar{\omega}_{t+1}^j} \omega dF_t(\omega) + \bar{\omega}_{t+1}^j\left[1 - F_t\left(\bar{\omega}_{t+1}^j\right)\right]\right] - 1\right\} \\ &\quad + \xi_t^j E_t\Lambda_{t,t+1}R_{t+1}^A \int_{\bar{\omega}_{t+1}^j} \left\{\theta \bar{V}_{t+1}'\left(N_{t+1}^j\right) + 1 - \theta\right\}\left(\omega - \bar{\omega}_{t+1}^j\right) dF_t(\omega) \\ &\quad - \xi_t^j E_t\Lambda_{t,t+1}R_{t+1}^A \int_{\bar{\omega}_{t+1}^j} \left\{\theta \bar{V}_{t+1}'\left(N_{t+1}^j\right) + 1 - \theta\right\}\left(\omega - \bar{\omega}_{t+1}^j\right) d\tilde{F}_t(\omega), \end{aligned}$$

$$\begin{aligned} 0 &= -E_t\Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^j} \left[\theta \bar{V}_{t+1}'\left(N_{t+1}^j\right) + (1 - \theta)\right] dF_t(\omega) - E_t\Lambda_{t,t+1}\theta \frac{\bar{V}_{t+1}(0)}{R_{t+1}^A A_t^j} f_t\left(\bar{\omega}_{t+1}^j\right) \\ &\quad + \lambda_t^j E_t\Lambda_{t,t+1}\left[1 - F_t\left(\bar{\omega}_{t+1}^j\right)\right] \\ &\quad - \xi_t^j E_t\Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^j} \left\{\theta \bar{V}_{t+1}'\left(N_{t+1}^j\right) + (1 - \theta)\right\} dF_t(\omega) - \xi_t^j E_t\Lambda_{t,t+1}\theta \frac{\bar{V}_{t+1}(0)}{R_{t+1}^A A_t^j} f_t\left(\bar{\omega}_{t+1}^j\right) \\ &\quad + \xi_t^j E_t\Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^j} \left\{\theta \bar{V}_{t+1}'\left(N_{t+1}^j\right) + (1 - \theta)\right\} d\tilde{F}_t(\omega) + \xi_t^j E_t\Lambda_{t,t+1}\theta \frac{\bar{V}_{t+1}(0)}{R_{t+1}^A A_t^j} \tilde{f}_t\left(\bar{\omega}_{t+1}^j\right), \end{aligned}$$

respectively, where we have used $\bar{b}_t^j / R_{t+1}^A = \bar{\omega}_{t+1}^j$. We also have the envelope condition

$$\bar{V}_t'\left(N_t^j\right) = \lambda_t^j.$$

At this point, we guess that in equilibrium $\bar{V}_t(N_t^j) = \lambda_t^j N_t^j$, and that the multipliers λ_t^j and ξ_t^j are equalized across islands: $\lambda_t^j = \lambda_t$ and $\xi_t^j = \xi_t$ for all j. Using this, the IC constraint simplifies to

$$E_t\Lambda_{t,t+1}R_{t+1}^A \left\{\theta\lambda_{t+1} + (1 - \theta)\right\}\left[\int_{\bar{\omega}_{t+1}^j} \left(\omega - \bar{\omega}_{t+1}^j\right) dF_t(\omega) - \int_{\bar{\omega}_{t+1}^j} \left(\omega - \bar{\omega}_{t+1}^j\right) d\tilde{F}_t(\omega)\right] \geq 0. \quad (20)$$

The first order conditions then become

$$\begin{aligned} 0 &= E_t\Lambda_{t,t+1}R_{t+1}^A \left[\theta\lambda_{t+1} + 1 - \theta\right] \int_{\bar{\omega}_{t+1}^j} \left(\omega - \bar{\omega}_{t+1}^j\right) dF_t(\omega) \\ &\quad + \lambda_t \left\{E_t\Lambda_{t,t+1}R_{t+1}^A \left[\int^{\bar{\omega}_{t+1}^j} \omega dF_t(\omega) + \bar{\omega}_{t+1}^j\left[1 - F_t\left(\bar{\omega}_{t+1}^j\right)\right]\right] - 1\right\}, \end{aligned} \quad (21)$$

$$\begin{aligned} 0 &= \lambda_t E_t\Lambda_{t,t+1}\left[1 - F_t\left(\bar{\omega}_{t+1}^j\right)\right] - E_t\Lambda_{t,t+1}\left[\theta\lambda_{t+1} + 1 - \theta\right]\left[1 - F_t\left(\bar{\omega}_{t+1}^j\right)\right] \\ &\quad + \xi_t E_t\Lambda_{t,t+1}\left\{\theta\lambda_{t+1} + 1 - \theta\right\}\left[F_t\left(\bar{\omega}_{t+1}^j\right) - \tilde{F}_t\left(\bar{\omega}_{t+1}^j\right)\right], \end{aligned} \quad (22)$$

where in (21) we have used the fact that ξ_t^j times the left-hand side of (20) must be zero as required by the Kuhn-Tucker conditions, and in (22) we have used the fact that, according to our guess, $\bar{V}_{t+1}(0) = 0$. Solving for the Lagrange multipliers, we obtain

$$\lambda_t = \frac{E_t \Lambda_{t,t+1} R_{t+1}^A \left[\theta \lambda_{t+1} + 1 - \theta\right] \int_{\bar{\omega}_{t+1}^j} \left(\omega - \bar{\omega}_{t+1}^j\right) dF_t(\omega)}{1 - E_t \Lambda_{t,t+1} R_{t+1}^A \left[\int^{\bar{\omega}_{t+1}^j} \omega dF_t(\omega) + \bar{\omega}_{t+1}^j \left[1 - F_t\left(\bar{\omega}_{t+1}^j\right)\right]\right]}, \tag{23}$$

$$\xi_t = \frac{\lambda_t E_t \Lambda_{t,t+1} \left[1 - F_t\left(\bar{\omega}_{t+1}^j\right)\right] - E_t \Lambda_{t,t+1} \left[\theta \lambda_{t+1} + 1 - \theta\right] \left[1 - F_t\left(\bar{\omega}_{t+1}^j\right)\right]}{E_t \Lambda_{t,t+1} \left\{\theta \lambda_{t+1} + 1 - \theta\right\} \left[\tilde{F}_t\left(\bar{\omega}_{t+1}^j\right) - F_t\left(\bar{\omega}_{t+1}^j\right)\right]}. \tag{24}$$

In the steady state, the Lagrange multipliers are

$$\lambda = \frac{\beta R^A (1-\theta) \int_{\bar{\omega}^j} \left(\omega - \bar{\omega}^j\right) dF(\omega)}{1 - \beta R^A + (1-\theta) \beta R^A \int_{\bar{\omega}^j} \left(\omega - \bar{\omega}^j\right) dF(\omega)},$$

$$\xi = \frac{(\lambda - 1)(1-\theta)}{\theta \lambda + 1 - \theta} \frac{\left[1 - F\left(\bar{\omega}^j\right)\right]}{\tilde{F}\left(\bar{\omega}^j\right) - F\left(\bar{\omega}^j\right)},$$

where we have used $\int \left(\omega - \bar{\omega}^j\right) dF(\omega) = 1 - \bar{\omega}^j$. Provided the parameter values are such that

$$0 < \beta R^A - 1 < (1-\theta) \beta R^A \int_{\bar{\omega}^j} \left(\omega - \bar{\omega}^j\right) dF(\omega),$$

then $\lambda > 1$, which in turn implies $\xi > 0$. That is, both the participation and IC constraints hold in the steady state.[33] Provided aggregate shocks are sufficiently small, we will also have $\lambda_t > 1$ and $\xi_t > 0$ along the cycle. But if $\lambda_t > 1$, then our guess that $\bar{V}_t'(N_t^j) > 1$ is verified. Also, given that $\bar{\omega}_{t+1}^j = \bar{b}_t^j / R_{t+1}$, the ratio $\bar{b}_t^j$ is then pinned down by the IC constraint (equation 20) holding with equality. Since we have guessed that the multiplier λ_t is equalized across islands, so are $\bar{b}_t^j = \bar{b}_t$ and $\bar{\omega}_{t+1}^j = \bar{\omega}_{t+1} = \bar{b}_t / R_{t+1}$. But if $\bar{\omega}_{t+1}$ is equalized, then from (23) and (24) our guess that λ_t and ξ_t are symmetric across islands is verified too.

The participation constraint (holding with equality) is given by

$$E_t \Lambda_{t,t+1} R_{t+1}^A A_t^j \left\{\int^{\bar{\omega}_{t+1}} \omega dF_t(\omega) + \bar{\omega}_{t+1} \left[1 - F_t(\bar{\omega}_{t+1})\right]\right\} = A_t^j - N_t^j.$$

Using the latter to solve for A_t^j, we obtain

$$A_t^j = \frac{1}{1 - E_t \Lambda_{t,t+1} R_{t+1}^A \left\{\bar{\omega}_{t+1} - \pi_{t+1}(\bar{\omega}_{t+1})\right\}} N_t^j \equiv \phi_t N_t^j,$$

where we have also used the definition of the put option value, $\pi_t(\bar{\omega}_{t+1}) = \int^{\bar{\omega}_{t+1}} (\bar{\omega}_{t+1} - \omega) dF_t(\omega)$. Therefore, the leverage ratio $A_t^j / N_t^j = \phi_t$ is equalized across firms too. Finally, using $\bar{V}_{t+1}(N_{t+1}^j) = \lambda_{t+1} N_{t+1}^j$, $N_{t+1}^j = (\omega - \bar{\omega}_{t+1}) R_{t+1}^A A_t^j$ and $A_t^j = \phi_t N_t^j$, the value function $\bar{V}_t\left(N_t^j\right)$ can be expressed as

$$\bar{V}_t\left(N_t^j\right) = \phi_t N_t^j E_t \Lambda_{t,t+1} R_{t+1}^A \left[\theta \lambda_{t+1} + 1 - \theta\right] \int_{\bar{\omega}_{t+1}} (\omega - \bar{\omega}_{t+1}) dF_t(\omega),$$

[33] Our calibration in Table 2 implies $\lambda = 9.2936$ and $\xi = 1.6731$.

which is consistent with our guess that $\bar{V}_t(N_t^j) = \lambda_t N_t^j$ only if

$$\begin{aligned}\lambda_t &= \phi_t E_t \Lambda_{t,t+1} R_{t+1}^A \left[\theta \lambda_{t+1} + 1 - \theta\right] \int_{\bar{\omega}_{t+1}} \left(\omega - \bar{\omega}_{t+1}\right) dF_t\left(\omega\right) \\ &= \frac{E_t \Lambda_{t,t+1} R_{t+1}^A \left[\theta \lambda_{t+1} + 1 - \theta\right] \left\{1 - \bar{\omega}_{t+1} + \pi_t\left(\bar{\omega}_{t+1}\right)\right\}}{1 - E_t \Lambda_{t,t+1} R_{t+1}^A \left\{\bar{\omega}_{t+1} - \pi_t\left(\bar{\omega}_{t+1}\right)\right\}}.\end{aligned}$$

But the latter corresponds exactly with (23) without j subscripts, once we use the definition of $\pi_t\left(\bar{\omega}_{t+1}\right)$. Our guess is therefore verified.

The regulated bank's problem

The maximization problem of a regulated bank differs from that of an unregulated in that (i) the bank is subject to a maximum leverage ratio ($A_t^{r,j}/N_t^{r,j} \leq \phi^r$), and (ii) it is not subject to the IC or participation constraints. Otherwise, the problem is analogous to that of an unregulated bank. Let $d_{t-1}^j \equiv D_{t-1}^j / A_{t-1}^{r,j}$ denote the loan-to-value ratio, such that $\bar{\omega}_t^{r,j} = R_{t-1} d_{t-1}^j / R_t^A$. Also, the bank's balance sheet constraint can be written as $A_t^{r,j}(1 - d_t^j) = N_t^{r,j}$. The regulated bank's maximization problem is

$$V_t^r\left(\omega, A_{t-1}^{r,j}, d_{t-1}^j\right) = \max_{N_t^{r,j}} \left\{\Pi_t^{r,j} + \bar{V}_t\left(N_t^{r,j}\right) + \mu_t^{r,j}\left[\left(\omega - \frac{R_{t-1} d_{t-1}^j}{R_t^A}\right) R_t^A A_{t-1}^{r,j} - N_t^{r,j}\right]\right\}, \quad (25)$$

subject to the expression for dividends, $\Pi_t^{r,j} = (\omega - R_{t-1} d_{t-1}^j / R_t^A) R_t^A A_{t-1}^{r,j} - N_t^{r,j}$, and

$$\bar{V}_t^r\left(N_t^{r,j}\right) = \max_{A_t^{r,j}, d_t^j} \left\{\begin{array}{c} E_t \Lambda_{t,t+1} \int_{R_t d_t^j / R_{t+1}^A} \left[\theta^r V_{t+1}\left(\omega, A_t^{r,j}, d_t^j\right) + (1 - \theta^r)\left(\omega - \frac{R_t d_t^j}{R_{t+1}^A}\right) R_{t+1}^A A_t^{r,j}\right] dF_t\left(\omega\right) \\ + \lambda_t^{r,j}\left[\phi^r N_t^{r,j} - A_t^{r,j}\right] + \xi_t^{r,j}\left[N_t^{r,j} - A_t^{r,j}\left(1 - d_t^j\right)\right] \end{array}\right\},$$

where now λ_t^j and ξ_t^j are the Lagrange multipliers associated to the leverage and balance-sheet constraints, respectively. The first order condition with respect to $N_t^{r,j}$ is given by

$$\mu_t^{r,j} = \bar{V}_t^{r\prime}(N_t^{r,j}) - 1.$$

We now guess that $\bar{V}_t^{r\prime}(N_t^{r,j}) > 1$. Then $\mu_t^{r,j} > 0$ and the non-negativity constraint on dividends is binding, such that a continuing bank optimally decides to retain all earnings, $\Pi_t^{r,j} = 0$, or

$$N_t^{r,j} = \left(\omega - \frac{R_{t-1} d_{t-1}^j}{R_t^A}\right) R_t^A A_{t-1}^{r,j}. \quad (26)$$

From (25), we then have $V_t^r(\omega, A_{t-1}^{r,j}, d_{t-1}^j) = \bar{V}_t^r((\omega - R_{t-1} d_{t-1}^j / R_t^A) R_t^A A_{t-1}^{r,j})$. Using the latter, we can express the Bellman equation for $\bar{V}_t^r(N_t^{r,j})$ as

$$\bar{V}_t^r\left(N_t^{r,j}\right) = \max_{A_t^{r,j}, d_t^j} \left\{\begin{array}{c} E_t \Lambda_{t,t+1} \int_{R_t d_t^j / R_{t+1}^A} \left[\theta^r \bar{V}_{t+1}^r\left(\left(\omega - \frac{R_t d_t^j}{R_{t+1}^A}\right) R_{t+1}^A A_t^{r,j}\right) + (1 - \theta^r)\left(\omega - \frac{R_t d_t^j}{R_{t+1}^A}\right) R_{t+1}^A A_t^{r,j}\right] dF_t\left(\omega\right) \\ + \lambda_t^{r,j}\left[\phi^r N_t^{r,j} - A_t^{r,j}\right] + \xi_t^{r,j}\left[N_t^{r,j} - A_t^{r,j}\left(1 - d_t^j\right)\right] \end{array}\right\}$$

The first order conditions with respect to $A_t^{r,j}$ and d_t^j are given by

$$0 = E_t \Lambda_{t,t+1} R_{t+1}^A \int_{\bar{\omega}_{t+1}^{r,j}} \left[\theta^r \bar{V}_{t+1}^{r\prime} \left(N_{t+1}^{r,j} \right) + (1 - \theta^r) \right] \left(\omega - \bar{\omega}_{t+1}^{r,j} \right) dF_t(\omega) - \lambda_t^{r,j} - \xi_t^{r,j} \left(1 - d_t^j \right).$$

$$0 = -E_t \Lambda_{t,t+1} R_t \int_{\bar{\omega}_{t+1}^{r,j}} \left[\theta^r \bar{V}_{t+1}^{r\prime} \left(N_{t+1}^{r,j} \right) + (1 - \theta^r) \right] dF_t(\omega) - E_t \Lambda_{t,t+1} \theta^r \frac{R_t \bar{V}_{t+1}^r(0)}{R_{t+1}^A A_t^{r,j}} f_t \left(\bar{\omega}_{t+1}^{r,j} \right) + \xi_t^{r,j},$$

respectively, where we have used $R_t d_t^j / R_{t+1}^A = \bar{\omega}_{t+1}^{r,j}$. We also have the envelope condition

$$\bar{V}_t^{r\prime} \left(N_t^{r,j} \right) = \lambda_t^{r,j} \phi^r + \xi_t^{r,j}.$$

At this point, we guess that in equilibrium $\bar{V}_t^r(N_t^{r,j}) = (\lambda_t^{r,j} \phi^r + \xi_t^{r,j}) N_t^{r,j}$, and that the multipliers $\lambda_t^{r,j}$ and $\xi_t^{r,j}$ are equalized across islands: $\lambda_t^{r,j} = \lambda_t^r$, $\xi_t^{r,j} = \xi_t^r$ for all j. The first order conditions then become

$$0 = E_t \Lambda_{t,t+1} R_{t+1}^A \left[\theta^r \left(\lambda_{t+1}^r \phi^r + \xi_{t+1}^r \right) + (1 - \theta^r) \right] \int_{\bar{\omega}_{t+1}^{r,j}} \left(\omega - \bar{\omega}_{t+1}^{r,j} \right) dF_t(\omega) - \lambda_t^r - \xi_t^r \left(1 - d_t^j \right). \tag{27}$$

$$0 = -E_t \Lambda_{t,t+1} R_t \left[\theta^r \left(\lambda_{t+1}^r \phi^r + \xi_{t+1}^r \right) + (1 - \theta^r) \right] \left[1 - F_t \left(\bar{\omega}_{t+1}^{r,j} \right) \right] + \xi_t^r, \tag{28}$$

where in (28) we have used the fact that, according to our guess, $\bar{V}_{t+1}^r(0) = 0$. Equations (27) and (28) jointly determine the path of the two Lagrange multipliers, λ_t^r and ξ_t^r. We now guess that the leverage constraint binds in equilibrium ($\lambda_t^r > 0$), such that $\phi^r N_t^{r,j} = A_t^{r,j}$. This, together with the balance sheet constraint, implies $d_t^j = 1 - 1/\phi^r$ for all j, which in turn implies that $\bar{\omega}_{t+1}^{r,j} = R_t (1 - 1/\phi^r) / R_{t+1}^A = \bar{\omega}_{t+1}^r$ for all j. But then, from (27) and (28), we have that both Lagrange multipliers are equalized across islands, thus verifying our previous guess. Using the fact that $1 - d_t^j = 1/\phi^r$ in (27), we have that

$$\lambda_t^r \phi^r + \xi_t^r = E_t \Lambda_{t,t+1} R_{t+1}^A \phi^r \left[\theta^r \left(\lambda_{t+1}^r \phi^r + \xi_{t+1}^r \right) + (1 - \theta^r) \right] \int_{\bar{\omega}_{t+1}^r} \left(\omega - \bar{\omega}_{t+1}^r \right) dF_t(\omega).$$

In the steady state,

$$\lambda^r \phi^r + \xi^r = \frac{\beta R^A \phi^r (1 - \theta^r) \int_{\bar{\omega}^r} (\omega - \bar{\omega}^r) dF(\omega)}{1 - \beta R^A \phi^r \int_{\bar{\omega}^r} (\omega - \bar{\omega}^r) dF(\omega) + (1 - \theta^r) \beta R^A \phi^r \int_{\bar{\omega}^r} (\omega - \bar{\omega}^r) dF(\omega)} \equiv \Xi, \tag{29}$$

Provided the parameter values satisfy

$$0 < \beta R^A \phi^r \int_{\bar{\omega}^r} (\omega - \bar{\omega}^r) dF(\omega) - 1 < \beta R^A \phi^r (1 - \theta^r) \int_{\bar{\omega}^r} (\omega - \bar{\omega}^r) dF(\omega),$$

then $\lambda^r \phi^r + \xi^r > 1$, which implies that the non-negativity constraint binds in the steady state. Provided aggregate shocks are sufficiently small, we will also have $\lambda_t^r \phi^r + \xi_t^r > 1$ along the equilibrium path, thus verifying our guess that $\bar{V}_t^{r\prime}(N_t^{r,j}) > 1$. Once we have solved for $\lambda^r \phi^r + \xi^r$, we can use the steady-state counterpart of (28) to calculate

$$\xi^r = \left[\theta^r \Xi + (1 - \theta^r) \right] \left[1 - F(\bar{\omega}^r) \right] > 0.$$

Using the latter in (29), we obtain

$$\lambda^r = \frac{\Xi - \xi^r}{\phi^r} = \frac{\left[1-\theta^r + \theta^r F\left(\bar{\omega}^r\right)\right]\Xi - \left(1-\theta^r\right)\left[1-F\left(\bar{\omega}^r\right)\right]}{\phi^r} > 0,$$

where the inequality follows from $\Xi > 1$. Therefore, the leverage constraint binds in the steady state. For small enough shocks, we also have $\lambda_t^r > 0$ out of the steady state, thus verifying our previous guess.[34]

Complete set of equations (not for publication)

The complete set of equations is

$$\begin{aligned}
1 &= E_t\left[\beta\frac{u'(C_{t+1})}{u'(C_t)}\right]R_t,\\
\frac{v'(L_t)}{u'(C_t)} &= (1-\alpha)\frac{Y_t}{L_t},\\
K_{t+1} &= A_t + A_t^r,\\
A_t &= \phi_t N_t,\\
A_t^r &= \phi^r N_t^r,\\
R_t^A &= (1-\delta) + \alpha\frac{Y_t}{K_t},\\
1 - \int \omega d\tilde{F}_t(\omega) &= E_t\left\{\frac{u'(C_{t+1})R_{t+1}^A\left(\theta\lambda_{t+1}+1-\theta\right)}{E_t u'(C_{t+1})R_{t+1}^A\left(\theta\lambda_{t+1}+1-\theta\right)}\left[\tilde{\pi}_t\left(\frac{\bar{b}_t}{R_{t+1}^A}\right) - \pi_t\left(\frac{\bar{b}_t}{R_{t+1}^A}\right)\right]\right\}\\
\bar{\omega}_t &= \bar{b}_{t-1}/R_t^A,\\
C_t + I_t &= Y_t,\\
Y_t &= Z_t L_t^{1-\alpha}K_t^{\alpha},\\
K_{t+1} &= I_t + (1-\delta)K_t,\\
N_t &= \theta R_t^A\left[1-\bar{\omega}_t + \pi_{t-1}\left(\bar{\omega}_t\right)\right]A_{t-1} + \left\{1-\theta\left[1-F_{t-1}\left(\bar{\omega}_t\right)\right]\right\}\tau K_t,\\
N_t^r &= \theta^r R_t^A\left[1-\bar{\omega}_t^r + \pi_{t-1}\left(\bar{\omega}_t^r\right)\right]A_{t-1}^r + \left\{1-\theta^r\left[1-F_{t-1}\left(\bar{\omega}_t^r\right)\right]\right\}\tau^r K_t,\\
\bar{\omega}_t &= \frac{R_{t-1}}{R_t^A}\frac{\phi^r-1}{\phi^r},\\
\phi_t &= \frac{1}{1-E_t\left[\beta\frac{u'(C_{t+1})}{u'(C_t)}R_{t+1}^A\left[\bar{\omega}_{t+1} - \pi_t\left(\bar{\omega}_{t+1}\right)\right]\right]},\\
\lambda_t &= \frac{E_t\beta\frac{u'(C_{t+1})}{u'(C_t)}R_{t+1}^A\left[\theta\lambda_{t+1}+1-\theta\right]\left\{1-\bar{\omega}_{t+1}+\pi_t\left(\bar{\omega}_{t+1}\right)\right\}}{1-E_t\beta\frac{u'(C_{t+1})}{u'(C_t)}R_{t+1}^A\left\{\bar{\omega}_{t+1}-\pi_t\left(\bar{\omega}_{t+1}\right)\right\}},
\end{aligned}$$

for a vector of endogenous variables $\left[C_t, R_t, \bar{b}_t, R_t^A, L_t, K_t, A_t, A_t^r, I_t, N_t, N_t^r, Y_t, , \bar{\omega}_t, \bar{\omega}_t^r, \lambda_t, \phi_t\right]$. There are 16 endogenous variables and 16 equations.

[34] In particular, our calibration in Table 2 implies $\lambda^r = 0.0143$, $\xi^r = 1.4567$ and $\lambda^r\phi^r + \xi^r = 1.6090$.

Tables and Figures

Table 1. Business cycle statistics, US data

	Standard deviations (%)		
	Total assets	Leverage	GDP
			1.03
Regulated intermediaries			
US-chartered commercial banks	1.30	3.12	
Savings institutions	4.59	8.61	
Credit unions	2.34	2.75	
Unregulated intermediaries			
Security brokers and dealers	7.57	7.62	
Finance companies	3.05	5.34	
GSEs	3.85	2.90	

	Correlation with GDP		Correlation
	Total assets	Leverage	assets & leverage
Regulated intermediaries			
US-chartered commercial banks	0.46 **	−0.06	0.21
	(0.0000)	(0.5942)	(0.0518)
Savings institutions	0.73 **	0.34 **	0.32 **
	(0.0000)	(0.0014)	(0.0023)
Credit unions	−0.36 **	−0.57 **	0.70 **
	(0.0007)	(0.0000)	(0.0000)
Unregulated intermediaries			
Security brokers and dealers	0.47 **	0.22 *	0.76 **
	(0.0000)	(0.0444)	(0.0000)
Finance companies	0.41 **	0.24 *	0.52 **
	(0.0001)	(0.0252)	(0.0000)
GSEs	0.33 **	−0.14	0.32 **
	(0.0045)	(0.2376)	(0.0048)

Note: Leverage is total assets divided by equity capital (both in dollars). 'Total assets' in the table refer to real total assets, which are total assets (in dollars) divided by the GDP deflator. All series are from the US Flow of Funds, except real GDP and the GDP deflator which are from the Bureau of Economic Analysis. The sample period is 1984:Q1-2011:Q3, except for assets and leverage of GSEs which run through 2008:Q2. See Data Appendix for details. Leverage, real total assets and real GDP have been logged and detrended with a band-pass filter that preserves cycles of 6 to 32 quarters (lag length $K = 12$). P-values of the test of no correlation against the alternative of non-zero correlation are reported in parenthesis. Asterisks denote statistical significance of non-zero correlation at the 1% (**) and 5% (*) confidence level.

Table 2. Model parameters

Parameter	Value	Description	Source/Target
RBC parameters			
β	0.99	discount factor	$R^4 = 1.04$
α	0.36	capital share	$WL/Y = 0.64$
δ	0.025	depreciation rate	$I/K = 0.025$
φ	1	inverse labor supply elasticity	macro literature
$\bar{Z}$	0.5080	steady-state TFP	$Y = 1$
ρ_z	0.9297	autocorrelation TFP	FRB San Francisco-CSIP TFP series
σ_z	0.0067	standard deviation TFP	FRB San Francisco-CSIP TFP series
Non-standard parameters			
ϕ^r	10.66	leverage of regulated banks	leverage commercial banks
σ	0.0272	steady-state island-specific volatility	leverage security broker/dealers (ϕ=29.30)
ρ_σ	0.9457	autocorr. island-specific volatility	NBER-CES manufacturing industry TFP
σ_σ	0.0465	standard dev. island-specific volatility	NBER-CES manufacturing industry TFP
η	3.1442	variance substandard technology	$\left(\bar{R}/R\right)^4 - 1 = 0.25\%$
ψ	0.01	mean substandard technology	illustrative
τ	0.0015	equity injections new unreg. banks	$A = A^r$, law of motion N
τ^r	0.030	equity injections new regulated banks	$A = A^r$, law of motion N^r
θ	0.75	continuation prob. unregulated banks	$\tau > 0$
θ^r	0.75	continuation prob. regulated banks	$\theta^r = \theta$

Table 3. Business cycle statistics: data and model

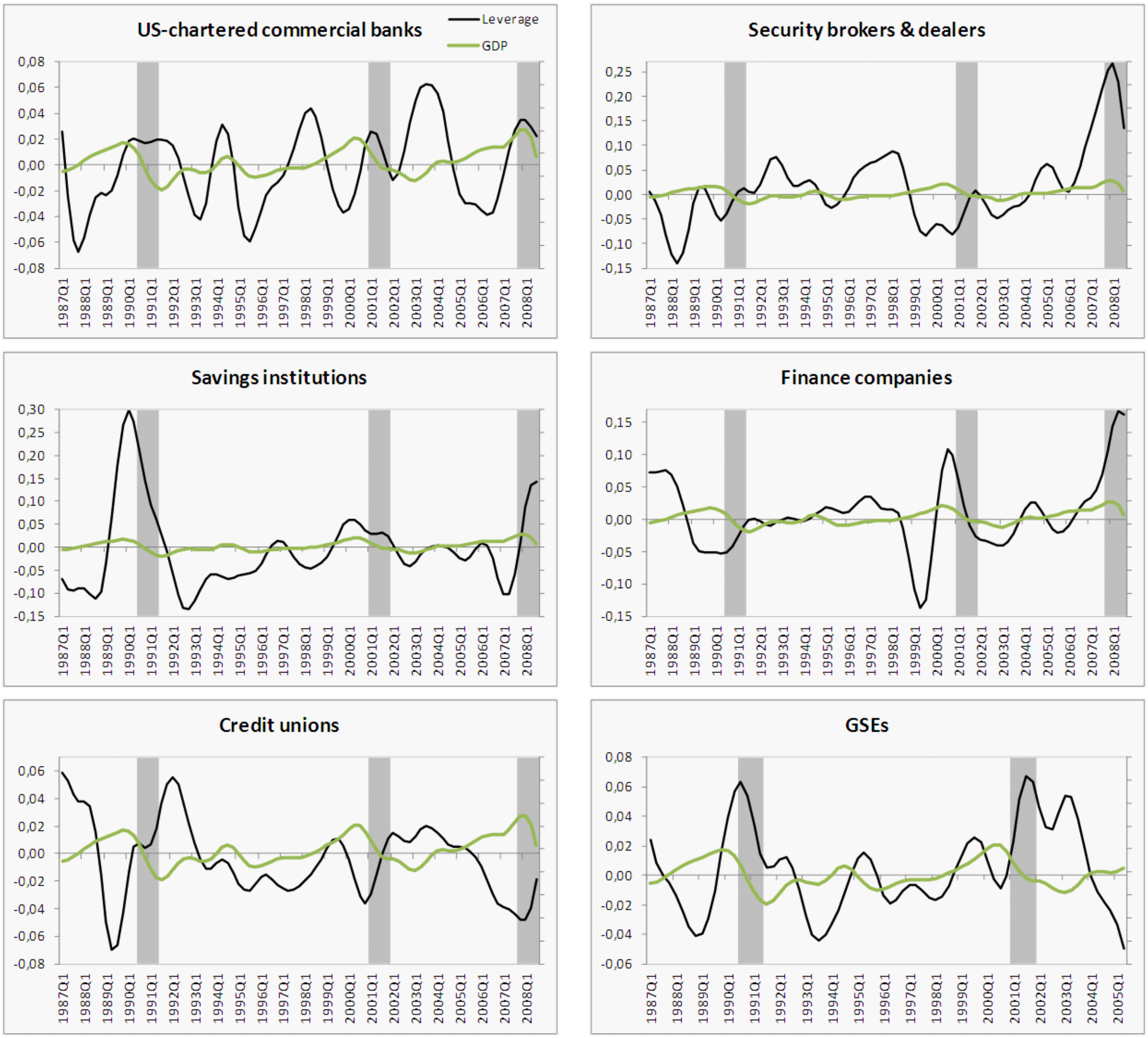
	Data	Model: TFP	Model: Volatility	Model: Both
Standard deviations (%)				
GDP	1.03	1.02	0.27	1.06
Assets regulated banks	1.30	0.26	2.40	2.46
Assets unregulated banks	7.57	0.50	2.98	3.02
Leverage unregulated banks	7.62	0.40	9.27	9.12
Correlations				
Assets regulated - GDP	0.46	0.46	−0.89	−0.19
Assets unregulated - GDP	0.47	0.36	0.87	0.29
Leverage unregulated - GDP	0.22	−0.04	0.90	0.25
Assets -leverage (unregulated)	0.76	0.64	0.91	0.89
Correlations (unfiltered)				
Assets regulated - GDP		0.79	−0.86	−0.03
Assets unregulated - GDP		0.82	0.96	0.54
Leverage unregulated - GDP		−0.14	0.86	0.31
Assets -leverage (unregulated)		0.08	0.92	0.90

Note: Model statistics are obtained by simulating the model for 5,000 periods and discarding the first 500 observations. The model is solved using a first-order perturbation method. Both data and model-simulated series have been logged and detrended with a band-pass filter that preserves cycles of 6 to 32 quarters (lag length $K = 12$), except indicated otherwise.

Figure 1. Cyclical fluctuations in leverage and real total assets in the US financial sector

Note: Leverage is total assets divided by equity capital (both in dollars). 'Total assets' in the figure refer to real total assets, which are total asset (in dollars) divided by the GDP deflator. All series are from the US Flow of Funds, except the GDP deflator which is from the Bureau of Economic Analysis. See Data Appendix for details. Leverage and real total assets have been logged and detrended with a band-pass filter that preserves cycles of 6 to 32 quarters (lag length $K = 12$). Grey areas represent NBER-dated recessions.

Figure 2. Cyclical fluctuations in leverage and real GDP



Note: Leverage is total assets divided by equity capital (both in dollars). All series are from the US Flow of Funds, except real GDP which is from the Bureau of Economic Analysis. See Data Appendix for details. Leverage and real GDP have been logged and detrended with a band-pass filter that preserves cycles of 6 to 32 quarters (lag length $K = 12$). Grey areas represent NBER-dated recessions.

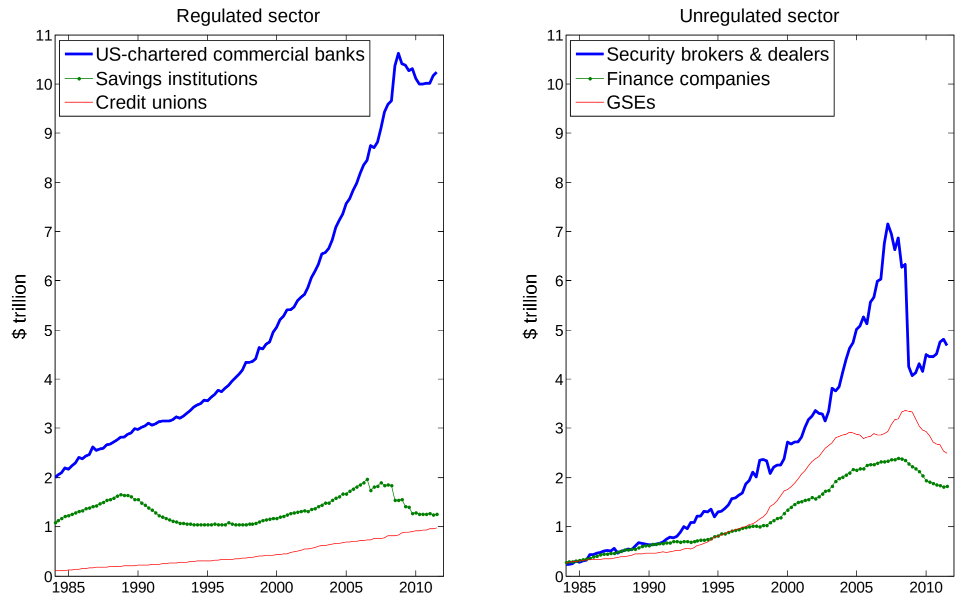


Figure 3. Total assets of the leveraged financial subsectors

Note: All series from the US Flow of Funds. See data appendix for details.

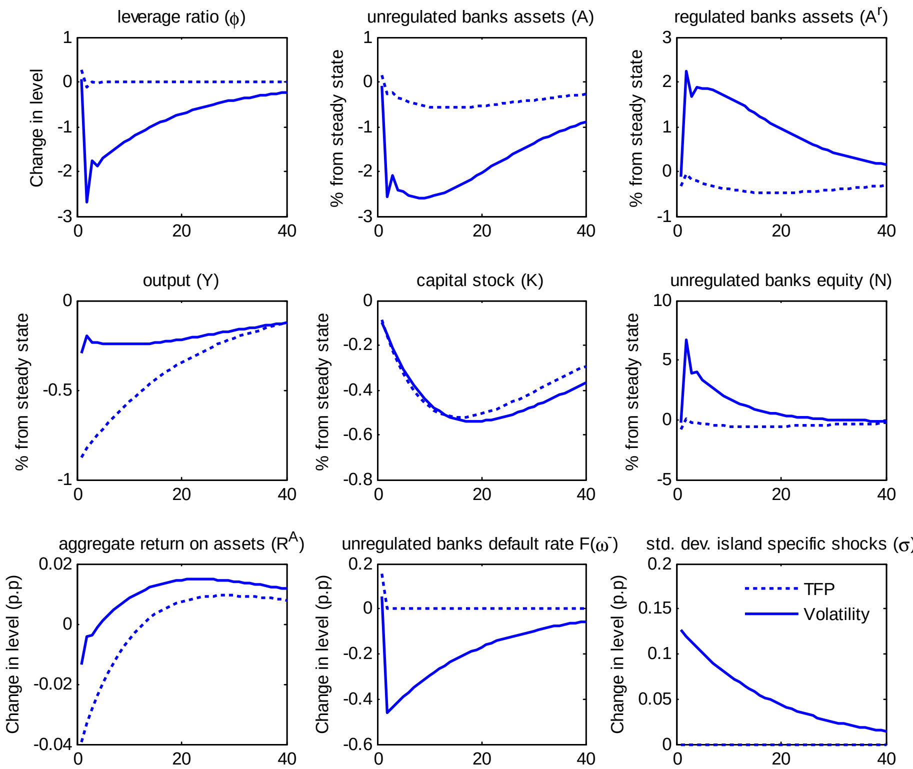
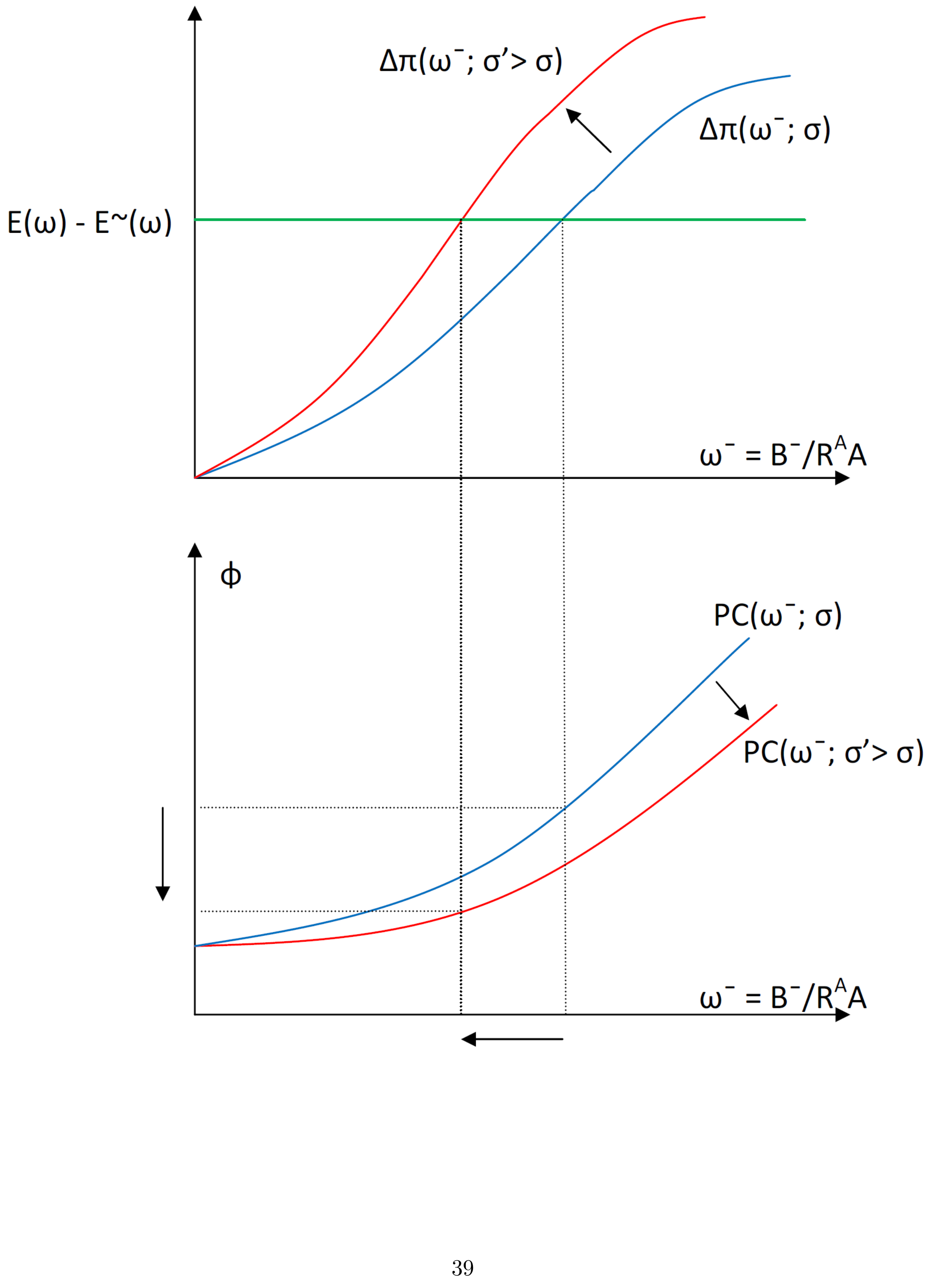


Figure 4. Impulse responses

Figure 5: The volatility-leverage channel



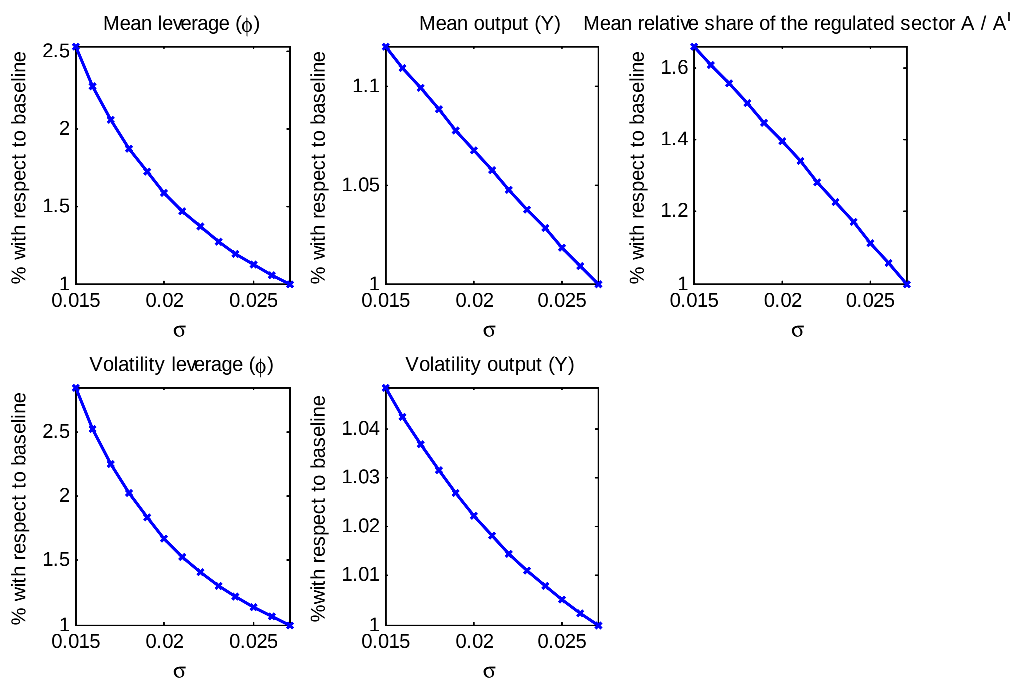


Figure 6: The effect of changes in average cross-sectional volatility