A Macroeconomic Model of Endogenous Systemic Risk Taking*

David Martinez-Miera       Javier Suarez
Universidad Carlos III    CEMFI and CEPR

First version: December 2010  
This version: 30 May 2012

Abstract

We analyze systemic risk taking in a simple dynamic general equilibrium model with banks. Banks collect funds from savers and make loans to firms, which use them to pay for capital and labor in advance. Banks are owned and managed by risk-neutral bankers who provide the equity needed to comply with capital requirements. Bankers decide their (unobservable) exposure to systemic shocks by trading off risk-shifting gains with the value of preserving their capital after a systemic shock. Capital requirements reduce credit and output in “normal times,” but also reduce banks’ systemic risk taking and, hence, the losses caused by systemic shocks. Under our calibration of the model, optimal capital requirements are quite high, have a sizeable negative impact on GDP, do not require counter-cyclical adjustment, and should be gradually introduced.

Keywords: Capital requirements, Risk shifting, Credit cycles, Systemic risk, Financial crises, Macroprudential policies.

JEL Classification: G21, G28, E44

* We would like to thank Matthieu Darracq-Paries, Giovanni Dell’Ariccia, Gianni De Nicolo, Martin Ellison, Carlos González-Aguado, Hans Gersbach, Claudio Michelacci, Enrico Perotti, Rafael Repullo, Diego Rodriguez-Palenzuela, Ruben Segura, Nicholas Trachter, Wolf Wagner, and seminar audiences at Banco de España, Bank of Canada, Bank of Japan, De Nederlandsche Bank, European Central Bank, HECER, International Monetary Fund, Kansas City Fed, New York Fed, Swiss National Bank, Universidad de Navarra, and the conferences on “Advances in Business Cycle Research – Directions Since the Crisis” in Brussels, “Financial Intermediation” in Funchal, “Financial Intermediation and the Real Economy” in Paris, “Macroprudential Policies, Regulatory Reform and Macroeconomic Modelling” in Rome, and RES 2012 in Cambridge for helpful comments and suggestions. We acknowledge financial support from Bank of Spain and Spanish government grants ECO2010-17158 and JCI-2011-08963 (Martinez-Miera) and from the European Central Bank and Spanish government grant ECO2011-26308 (Suarez). Address for correspondence: CEMFI, Casado del Alisal 5, 28014 Madrid, Spain. Phone: +34-914290551. E-mail: suarez@cemfi.es.
1 Introduction

This paper analyzes the role of banks in generating endogenous systemic risk. We consider a canonical problem of excessive risk-taking by banks in a dynamic general equilibrium model. In our economy banks are subject to capital requirements and the supply of equity funding to them is limited by wealth endogenously accumulated by bank owners. We find that systemic risk-taking can be reduced by increasing capital requirements. Yet doing so has negative implications for the levels of credit and output, producing trade-offs relevant for the socially optimal design of bank capital regulation.

Our economy is subject to systemic shocks which, like in Rancière et al. (2008), are small probability events in which certain investments fail in a highly correlated manner, provoking the default of the loans financing them. Importantly, banks make an unobservable decision on their degree of exposure to these shocks (i.e. the extent to which loans in their portfolio tend to default in a correlated manner). Banks’ temptation to undertake highly correlated investments is due to the risk-shifting incentives of levered firms, like in Jensen and Meckling (1976) and many other corporate finance and banking models.

In our model, banks collect deposits from final savers and are the exclusive providers of loans to perfectly competitive firms which need to pay for physical capital and labor in advance. Production processes are subject to failure risk and, depending on their degree of exposure to systemic shocks, may be systemic or non-systemic. For a given combination of physical capital and labor and conditional on their success or failure, all production processes yield the same output. However, non-systemic production processes have stochastically independent failures. In contrast, systemic production processes fail simultaneously if the negative systemic shock occurs (and in an independent fashion if it does not).

1 Systemic shocks resemble the rare economic disasters considered in Rietz (1988) and Barro (2009), among others. We focus on banks’ endogenous exposure to exogenous systemic shocks to avoid the complexity associated with modeling the mechanisms that generate correlated losses (bubbles, negative spillovers caused by fire sales, interbank linkages, bank panics, etc.). A systemic shock may empirically correspond to phenomena such as the bust of the US housing market (and its implications for subprime mortgages, securitization markets, and money markets) around the Summer of 2007.

2 A similar correlation decision has been analyzed in the microeconomic banking literature by Acharya and Yorulmazer (2007).

3 See Section 2 for additional references.
Following the literature on risk shifting, we assume that systemic firms have probabilities of failure that, conditional on not suffering the systemic shock, are lower than those of non-systemic firms. However, unconditionally systemic firms are more likely to fail and, thus, generate lower expected net present value. Yet they may be a tempting investment opportunity for highly levered institutions such as banks.

Realistically like in many models in the literature on bank risk-taking, we assume that (i) bank deposits are fully insured by the government and (ii) banks are subject to capital requirements. We model the dynamics of bank capital along the same lines as Gertler and Kiyotaki (2010), Gertler and Karadi (2011), and Meh and Moran (2010). This implies thinking of bank capital as the funds provided by the special class of agents who own and manage the bank (the “bankers”). The maximum available amount of this funding is determined by bankers’ wealth, whose endogenous dynamics is affected by the profits and losses made by the banks in each period.4

Bankers may be interested in making systemic loans because they are protected by limited liability and, if their bank fails, part of the losses go to the government as the provider of deposit insurance.5 Bank capital requirements influence bankers’ incentives in regards to the adoption of systemic risk in two ways. First, the conventional leverage-reduction effect diminishes the static gains from risk-taking. Second, higher capital requirements increase the relative scarcity of bank capital in each state of the economy, altering bankers’ dynamic incentives in an interesting manner.

Indeed, the anticipated value of a unit of bank capital in the future is key to bankers’ systemic risk-taking decisions. When a banker’s capital is devoted to make systemic loans, the banker obtains higher returns (and accumulates more capital) insofar as the systemic shock does not occur. But if the systemic shock occurs, the invested equity is lost. In contrast, if the banker invests in non-systemic loans, he receives a lower return in “normal times” (i.e. if the systemic shock does not realize), but preserves his capital when the

---

4Gertler et al. (2010) consider a setup where bankers’ inside equity can be complemented with outside equity but an agency problem limits the access to the latter to a certain multiple of the former.

5A similar effect would occur vis-a-vis bank depositors if the deposits were uninsured. What is really important for the distortion is limited liability and the unobservability of the banks’ risk-taking decisions.
systemic shock occurs. The destruction of bank capital after a shock allows surviving capital to earn higher scarcity rents and produces a “last bank standing effect” like in Perotti and Suarez (2002). This effect reduces bankers’ inclination for systemic lending.\textsuperscript{6}

Systemic risk taking has negative static and dynamic implications. First, even from a single-period investment horizon perspective, systemic firms generate less overall expected net present value than non-systemic firms. Second, when the systemic shock realizes, the economy suffers a loss of aggregate bank capital which in turn produces a credit crunch, and some output and net consumption losses during the transition periods it takes to recover the pre-crisis levels of bank capital and output.

Strengthening capital requirements reduces bankers’ systemic risk taking through the two channels mentioned above. Higher capital requirements reduce the proportion of resources going into inefficient systemic investments as well as the contractions in credit supply that follow the realization of systemic shocks. However, these gains come at the cost of reducing credit and output in “normal times.” Measuring welfare as the expected present value of total net consumption flows in the economy, we find that there is a unique interior social welfare maximizing level of capital requirements.

Under our calibration, social welfare is maximized under positive and relatively large capital requirement: it is optimal to require banks to finance 14% of their loans with bankers’ wealth. To fix ideas, we compare the scenario with a 14% capital requirement with another with a 7% capital requirement. We find that the unconditional mean of the fraction of bank equity devoted to support systemic lending is 71% under the low capital requirement and 24% under the optimal capital requirement. The social welfare gain from having the requirement of 14% rather than 7% is equivalent to a perpetual increase of 0.9% in aggregate net consumption.

Importantly, common macroeconomic aggregates such as GDP and bank credit have

\textsuperscript{6}As we further discuss in Section 6.3, in order for this mechanism to have the highest impact, it is convenient to resolve systemic crisis with the maximum dilution of the pre-existing equity of failed banks; partial dilution would lower the effectiveness of this mechanism. Full dilution is, in principle, compatible with resolution practices in which the failed banks continue as a going concern (but in hands of new owners). In our model, however, banks have no going-concern value beyond the value of the equity of their owners.
lower unconditional expected values (6.5% and 21% lower, respectively) in the economy with a capital requirement of 14% than with one of 7%. However, the fall in aggregate net consumption, GDP, and bank credit in the year that follows a systemic shock is much lower with the 14% capital requirement.\footnote{In the presence of systemic risk, GDP is a bad proxy of social welfare since it neglects important losses associated with the realization of such risk. Specifically, the value added measured in GDP is gross of capital depreciation and, hence, does not properly account for the depreciation of physical capital associated with firm failure (which occurs with a larger unconditional probability under the systemic production mode).}

The model is suitable for the analysis of the transition from a regime with a low capital requirement (say, 7%) to another with a higher capital requirement. It allows to explicitly take into account transitional dynamics and the welfare losses implied by the credit crunch suffered when the requirements are raised but the economy has not yet accumulated the levels of bank capital that will characterize the new regime. We find that it is socially optimal to implement the higher requirements in a gradual way (over 7 to 10 years under our calibration) and to establish a more modest long-term goal than if transitional cost were neglected (12% or 13%, depending on the desired speed of convergence to the target, rather than 14%).

The rest of the paper is organized as follows. Section 2 places the contribution of the paper in the context of the existing literature. Section 3 describes the model. Section 4 derives the conditions relevant for the definition of equilibrium. Section 5 describes our calibration exercise and the main quantitative results. Section 6 contains the results on optimal gradualism in the introduction of capital requirements and several other extensions and discussions. Section 7 concludes. The appendices contain proofs, derive our measure of social welfare, and describe the numerical method used to solve for equilibrium.

2 Related literature

Our paper is related to recent efforts to incorporate banks and their (endogenous) contribution to systemic risk into core macroeconomic analysis. Dynamic stochastic general equilibrium (DSGE) models in use by central banks prior to the beginning of the crisis (e.g. in the tradition of Smets and Wouters, 2007) paid no or very limited attention to financial...
frictions. Several models considered idiosyncratic default risk and endogenous credit spreads using the framework provided by Bernanke et al. (1999) but very few were explicit about banks.\(^8\) More recently, various authors have extended models in the DSGE tradition with the explicit goal of capturing banking frictions.\(^9\) However, the reduced-form approach typically leaves aside an explicitly microfounded role for the introduced regulatory ingredients and impedes a fully-fledged welfare analysis.

Because of our modeling of the dynamics of bank capital and the explicit connection between bank capital and bankers' incentives, the papers most closely related to ours are Gertler and Kiyotaki (2010), Gertler and Karadi (2011), and Meh and Moran (2010). These papers prescribe for bankers' wealth the same type of dynamics previously postulated for entrepreneurial net worth in the models with financial constraints (e.g. Carlstrom and Fuerst, 1997, and Kiyotaki and Moore, 1997). In Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) requiring bankers to fund some minimal fraction of their banks with their own funds prevents a potential fund diversion problem, like in Hart and Moore (1994). Meh and Moran (2010) look at the type of market-imposed capital requirements of Holmström and Tirole (1997), i.e. those that emerge in a setup in which banks’ outside financiers are not protected by government guarantees and bankers make costly unobservable decisions regarding the effectiveness of their monitoring.

Similar bank capital dynamics is postulated by Brunnermeier and Sannikov (2011), who also model crises as the result of large but infrequent shocks. However, they put the emphasis on identifying channels through which fire sales are a source of pecuniary externalities. In their model, the interaction of asset price volatility with value-at-risk based capital requirements plays a key role in the transmission of the shocks, but optimal capital regulation is not discussed.\(^{10}\)

Our explicit focus on risk taking decisions and optimal capital regulation connects our

---

\(^8\) An exception is Van den Heuvel (2008), who introduces banks’ liquidity provision function in a macroeconomic setup and assesses the welfare cost of capital requirements. Yet, in his model, capital requirements are an ad hoc piece of regulation with no explicit impact on risk-taking incentives or bank solvency.

\(^9\) See, for instance, Agénor et al. (2009), Christiano et al. (2010), Darraaq-Pariès et al. (2010), and Gerali et al. (2010).

\(^{10}\) They focus on the polar case in which capital requirements guarantee that banks never fail.
contribution to long traditions in the corporate finance and banking literatures whose review exceeds the scope of this section (see Bhattacharya at al., 1998, and Freixas and Rochet, 2008, for excellent surveys). The seminal references on risk-shifting include Jensen and Meckling (1976) in a corporate finance context, and Stiglitz and Weiss (1981) in a credit market equilibrium context.

Excessive risk-taking by banks is identified by Kareken and Wallace (1978) as an important side effect of deposit insurance, and by Allen and Gale (2000) as the origin of credit booms and bubbles.11 The role of capital requirements in ameliorating this problem and their interaction with the incentives coming from banks’ franchise values is a central theme in Hellmman at al. (2000) and Repullo (2004), where banks earn rents due to market power.12 The dynamic incentives for prudence associated with the rise in the franchise value of surviving banks after a systemic crisis appear in Perotti and Suarez (2002) and Acharya and Yorulmazer (2007, 2008).

The shadow value of bank capital in our context plays an incentive role similar to that of franchise value in the previous literature. However, differently from the prior tradition, the banks in our model are perfectly competitive and the relevant continuation value is attached to bank capital, which is solely provided by bankers and earns scarcity rents because bankers’ endogenously accumulated wealth is limited.13

---

11 When some relevant dimension of risk taking is unobservable, equilibrium risk-taking may be excessive even without government guarantees. Yet the underpricing of those guarantees (or their flat pricing) may worsen the problem. Dewatripont and Tirole (1994) describe safety net guarantees as part of a social contract whereby depositors delegate the task of controlling banks’ risk taking on the supervisory authorities who provide deposit insurance in exchange.

12 We abstract, for simplicity, from the entrepreneurial-incentives channel emphasized by Boyd and De Nicoló (2005) in their reexamination of the link between market power and bank solvency (see also Martinez-Miera and Repullo, 2010). Extending our framework for the explicit consideration of entrepreneurs’ incentives (and the dynamics of their net worth) would require a two-tier formulation similar to Holmström and Tirole (1997) which, in its simpler formulation, would add a second aggregate state variable in the model, making its solution computationally more demanding.

13 As already pointed out, these features are shared by the models of Gertler and Kiyotaki (2010), Gertler and Karadi (2011), and Gertler et al. (2010), which do not consider the implications for bank risk taking.
3 The model

We consider a perfect competition, infinite horizon model in discrete time \( t = 0, 1, \ldots \) in which all agents are risk neutral, production takes time and is subject to failure risk, banks intermediate between savers and firms so as to allow the latter to pay for their factors of production in advance, and banks are own by bankers who provide them with the equity needed to satisfy some regulatory capital requirement. The next subsections describe and motivate each of these ingredients in detail.

3.1 Agents

The economy is populated by two classes of risk-neutral agents: patient agents, who essentially act as providers of funding to the rest of the economy, and impatient agents, who include pure workers, bankers, and entrepreneurs. Additionally, there is a government which provides deposit insurance and imposes a capital requirement to banks.

Patient agents have deep pockets. Their required expected rate of return is \( r \) per period, and can be interpreted as the exogenous return on some risk-free technology. Patient savers provide a perfectly elastic supply of funds to banks in the form of deposits but, due to unmodeled informational and agency frictions, cannot directly lend to the final borrowers.\(^{14}\)

Impatient agents, of whom there is a continuum of measure one, are infinitely lived, have a discount factor \( \beta < 1/(1 + r) \), and inelastically supply a unit of labor per period at the prevailing wage rate \( w_t \). Most impatient agents are mere workers. Each worker has a small independent probability \( \phi \psi/(1 - \phi) \) of learning in each date \( t \) that he will become a banker (i.e. posses the skills needed to own and manage a bank) at date \( t + 1 \). In parallel, each banker active at date \( t \) has a small independent probability \( \psi \) of becoming a mere worker again at date \( t + 1 \). This produces a stationary size \( \phi \) for the population of active bankers.

Finally, a tiny fraction \( \mu \) of the patient agents who do not act as bankers in each given date receive the opportunity to act as entrepreneurs (i.e. owning and managing a firm) during the imminent period. We focus on parameterizations under which patient agents find

\(^{14}\)In an open economy interpretation, one can think of patient agents as international capital market investors and \( r \) as the international risk-free rate.
it optimal to act as bankers or entrepreneurs if the occasion arises.\textsuperscript{15} We also assume that the probabilities $\phi\psi$ and $\mu$ are small enough for the accumulation of wealth by workers not to be worthy prior to learning about their conversion into bankers or entrepreneurs.

### 3.2 Firms

The entrepreneurs active in every period run a continuum of perfectly competitive firms indexed by $i \in [0, \mu]$. Each firm operates a constant returns to scale technology that transforms the capital $k_{it}$ and labor $n_{it}$ employed at $t$ into

$$y_{it+1} = (1 - z_{it+1})[AF(k_{it}, n_{it}) + (1 - \delta)k_{it}] + z_{it+1}(1 - \lambda)k_{it}$$

units of the consumption good (which is the numeraire) at $t + 1$. The binary random variable $z_{it+1} \in \{0, 1\}$, realized at $t + 1$, indicates whether the firm’s production process succeeds ($z_{it+1} = 0$) or fails ($z_{it+1} = 1$). The parameters $\delta$ and $\lambda \geq \delta$ are the rates at which capital depreciates when the firm succeeds and when it fails, respectively.\textsuperscript{16} Net output in case of success is the product of total factor productivity $A$ and the function

$$F(k_{i}, n_{i}) = k_i^{\alpha}n_i^{1-\alpha},$$

with $\alpha \in (0, 1)$.\textsuperscript{17} In case of failure, firms do not produce any output on top of depreciated capital.

The possible correlation of $z_{it+1}$ across firms is due to the exposure of firms to a common \textit{systemic} shock $\varepsilon_{t+1} \in \{0, 1\}$, whose bad realization $\varepsilon_{t+1} = 1$ is assumed to occur with a constant independent small probability $\varepsilon$ at the end of each period. The production technology can be operated in two modes that differ in their degree of exposure to the systemic shock: one is not exposed to it or \textit{non-systemic} ($\xi_{it} = 0$) and the other is highly exposed to it or \textit{systemic} ($\xi_{it} = 1$). For firms operating in the non-systemic mode, $z_{it+1}$ is

\textsuperscript{15}In equilibrium, entrepreneurs eventually receive a competitive profit of zero at all dates.

\textsuperscript{16}In order to be able to summarize all the aggregate dynamics of the model through the evolution of a single state variable (bankers’ wealth), we assume that physical capital can be transformed into the consumption good at all dates on a one-to-one basis.

\textsuperscript{17}Notice that $A$ is presented as a constant, so we abstract from the type of productivity shocks emphasized in the real business cycle literature.
independently and identically distributed across firms with

\[ \Pr[z_{it+1} = 1 \mid \varepsilon_{t+1} = 0, \xi_{it} = 0] = \Pr[z_{it+1} = 1 \mid \varepsilon_{t+1} = 1, \xi_{it} = 0] = p_0. \]

So, by the law of large numbers, the failure rate associated to any positive measure of non-systemic firms is constant and equal to \( p_0 \).

In contrast, we assume that all firms operating in the systemic mode have

\[ \Pr[z_{it+1} = 1 \mid \varepsilon_{t+1} = 0, \xi_{it} = 1 ] = p_1 < \Pr[z_{it+1} = 1 \mid \varepsilon_{t+1} = 1, \xi_{it} = 1 ] = 1, \]

where failure in case of no shock (\( \varepsilon_{t+1} = 0 \)) is independently distributed across firms. Hence, the failure rate among systemic firms can be described as:

\[ z_{t+1} = \begin{cases} 
  p_1 & \text{if } \varepsilon_{t+1} = 0, \\
  1 & \text{if } \varepsilon_{t+1} = 1,
\end{cases} \tag{3} \]

since systemic firms fail independently (with probability \( p_1 \)) if the negative systemic shock does not occur and simultaneously if it occurs.

Finally, following the risk-shifting literature, we assume that:

**A1.** \( p_1 < p_0 \).

**A2.** \( E(z_{it+1} \mid \xi_{it} = 1) = (1 - \varepsilon)p_1 + \varepsilon > E(z_{it+1} \mid \xi_{it} = 0) = p_0. \)

Assumption A1 implies that bankers protected by limited liability may find it attractive to lend to systemic firms. However, assumption A2 means that systemic firms are overall less efficient (yield lower total expected returns) than non-systemic ones, so systemic risk taking is socially undesirable.

The entrepreneurs, who run the firms, are penniless, enjoy limited liability, and maximize their expected payoffs at the end of the production period, when they become mere workers again.\(^{18}\) Each firm requires a bank loan of size \( l_{it} = k_{it} + w_t n_{it} \) to pay in advance for the capital \( k_{it} \) and labor \( n_{it} \) used at date \( t \). The loan involves the promise to repay the amount

\(^{18}\)Limited liability may be interpreted as an exogenous institutional constraint or an implication of anonymity, implying that entrepreneurs’ contemporaneous or future wages cannot be used as collateral for entrepreneurial activities.
$B_{it} \leq AF(k_{it}, n_{it}) + (1 - \delta)k_{it}$ at $t + 1$. This debt contract implies an effective repayment $B_{it}$ if the firm does not fail, and $\min\{B_{it}, (1 - \lambda)k_{it}\} = (1 - \lambda)k_{it}$ if the firm fails. The tuple $(\xi_{it}, k_{it}, n_{it}, l_{it}, B_{it})$ is determined in the contracting between each firm and its bank at date $t$, where, to capture bank competition, entrepreneurs have all the bargaining power. Importantly, a firm’s systemic orientation $\xi_{it}$ is private information of the firm and its bank, ruling out regulations directly contingent on it.

### 3.3 Banks

Regulation obliges banks to finance a fraction $\gamma_t$ of their one-period loans to firms with equity capital i.e. funds coming from bankers’ accumulated wealth. Banks complement their funding with fully-insured one-period deposits taken from patient agents (and the would-be bankers who save their labor income so as to invest in bank capital in the next date). The deposit insurance scheme is paid for with contemporaneous non-distortionary taxes levied on impatient agents.

We assume that banks hold non-granular loan portfolios, that is, extend infinitesimal loans to a continuum of firms, thus fully diversifying away firms’ idiosyncratic failure risk. Diversification, however, does not eliminate the systemic risk associated with the lending to systemic firms. In fact, due to convexities induced by limited liability, bankers find it optimal to specialize their banks in either non-systemic or systemic loans. Since banks are perfectly competitive and operate under constant returns to scale, we can refer w.l.o.g. to a representative non-systemic bank ($\xi = 0$) and a representative systemic bank ($\xi = 1$). Each

---

19 With non-negative loan rates and wages, we necessarily have $B_{it} \geq l_{it} = k_{it} + w_t n_{it} \geq k_{it} \geq (1 - \lambda)k_{it}$.

20 We assume that impatient agents cannot borrow for consumption purposes. This could be due to the impossibility of pledging future income because of e.g. intertemporal anonymity. One could argue that banks can borrow from the patient agents and firms from the banks because they own assets at the end of each period (depreciated physical capital and net output) that are pledgeable.

21 E.g. a tax on workers’ consumption. Imposing this cost on impatient agents prevents the possibility of using deposit insurance as a means of redistribution of wealth from patient agents to impatient ones. Deposit insurance simplifies the analysis of risk-shifting but is not essential for our results insofar as the pricing of uninsured deposits cannot be contingent on the unobservable systemic orientation of each bank.

22 We can think of this diversification as an easy-to-enforce regulatory imposition.

23 For a formal argument, see Repullo and Suarez (2004).
bank’s balance sheet constraint imposes

\[ l_{\xi t} = d_{\xi t} + e_{\xi t}, \quad (4) \]

for \( \xi = 0, 1 \), where \( l_{\xi t} \) denotes the loans made by the bank at date \( t \), \( d_{\xi t} \) are its deposits, and \( e_{\xi t} \) is equity provided by bankers.\(^{24}\)

The allocation of bank capital to each bank takes place in a perfectly competitive fashion. Bankers can invest their wealth at any date \( t \) as capital of the non-systemic bank, capital of the systemic bank or insured deposits, and they can also consume all or part of their wealth.\(^{25}\) If they contribute \( e_{\xi t} \) to bank \( \xi \), they receive the free cash flow of the bank at \( t + 1 \) (i.e. the difference between payments from loans and payments to deposits) if it is positive, and zero otherwise. Bankers allocate their wealth based on their expectation about bank equity returns (and the valuation of the resulting wealth) across different possible states at \( t + 1 \).

Banks take as given bankers’ valuation of wealth across possible states at \( t + 1 \) and their value-weighted required return on wealth (see section 4.1). Based on this, they formulate the participation constraint that guarantees that bankers are willing to provide the equity funding \( e_{\xi t} \) needed by each bank at \( t \). As explained below, this constraint is taken into account when setting the terms of the lending contracts \((\xi_{it}, k_{it}, n_{it}, l_{it}, B_{it})\) with each of the entrepreneurs.

4 Equilibrium analysis

In our economy, bankers solve the genuinely dynamic optimization problems that determine the investment of (all or part of) their wealth as equity of the non-systemic bank \( e_{0t} \) or equity of the systemic bank \( e_{1t} \). Banks instead are the perfectly competitive one-period ventures in which the bankers invest. In order to facilitate the exposition, we will focus the

\(^{24}\)Given that both classes of banks have access to unlimited deposit funding at a common rate, we can abstract from interbank lending and borrowing.

\(^{25}\)Bankers can opt for any mixture of these four options so they can, in particular, invest simultaneously in equity of the non-systemic and the systemic banks, although their risk-neutrality provides no especial incentive for (or against) the diversification of their personal portfolios.
presentation of equilibrium conditions in the main text on the case in which bankers invest their whole wealth as equity of the existing banks (full reinvestment equilibrium), relegating to Appendix C the generalization of these conditions to equilibria in which bankers consume part of their wealth or save part of it in the form of bank deposits. The fraction of total bank capital invested in systemic banks will be denoted by $x_t \equiv e_{1t}/e_t \in [0,1]$.

We will assume that banks play a pooling equilibrium in which the non-systemic bank solves its individual maximization problem when contracting with firms, while the systemic bank prevents being identified as such (which would imply its dissolution by the regulator) by mimicking the non-systemic bank in every aspect except the unobservable systemic orientation of the firms receiving its loans ($\xi_{it} = 1$).\(^{26}\)

Importantly, when the systemic shock does not occur, the realized return of the systemic bank, $R_{1t+1}$, will tend to be higher than the return on equity of the non-systemic bank, $R_{0t+1}$, but we assume that bank accounts and managerial compensation practices are opaque enough to allow its owners to appropriate the excess return without being discovered.\(^{27}\)

### 4.1 Bankers’ portfolio problem

Continuing bankers (i.e. old bankers who do not convert back into workers at the end of a period) have the opportunity to reinvest the past returns of their wealth as bank capital for at least one more period. Let $v_{t+1}$ denote the (stochastic) marginal value of one unit of an old banker’s wealth at the time of receiving the returns from his past investment (right before learning whether he will remain active at $t+1$). If $R_{jt+1}$ is the (stochastic) return paid by some security $j$ at $t+1$, then an active banker’s valuation of the security at date $t$

\(^{26}\)A systemic bank is indeed detected if the systemic shock realizes, but at that point its capital is depleted and, under limited liability, there is no further punishment that can be imposed to its owners. An observable characteristic that might help differentiate the systemic bank from the non-systemic bank, if there were just one bank of each type, would be size. But the underlying constant returns to scale technologies are compatible with having many banks within each class and arbitrary sizes for each bank.

\(^{27}\)The appropriability of the excess return from risk-shifting by bank managers might justify why the investment in bank equity is in the first place limited to the especial class of agents that we call bankers (who might be interpreted as agents with the ability to prevent being expropriated by bank managers). This is consistent with the view in Diamond and Rajan (2000).
will be $\beta E(v_{t+1}R_{jt+1})$, where $\beta v_{t+1}$ plays the role of a stochastic discount factor.\(^{28}\)

To find a recursive expression for $v_t$, notice that when a banker converts into a worker, which happens with probability $\psi$, his only alternatives are either to save the wealth as a bank deposit (earning a gross return $1+r$ at $t+1$) or to consume it (in which case one unit of wealth is worth just $1$ at $t$). Given the agent’s impatience and the small probability of ever becoming a banker or an entrepreneur again, we will assume that consuming is the optimal decision and, thus, the value of one unit of wealth for the old banker who converts into a worker is just $1$.\(^{29}\) Considering the optimization over the possible uses of one unit of wealth for the banker who remains active at $t+1$, the Bellman equation for $v_t$ can be expressed as:

$$v_t = \psi + (1 - \psi) \max\{1, \beta \max\{(1 + r)E_t(v_{t+1}), E_t(v_{t+1}R_{0t+1}), E_t(v_{t+1}R_{1t+1})\}\}.$$  \(^{(5)}\)

The terms within the max operator reflect, in this order, the following possibilities: (i) consuming the wealth, (ii) investing in deposits, (iii) investing in equity of the non-systemic bank (which yields a gross return $R_{0t+1}$), and (iv) investing in equity of the systemic bank (which yields a gross return $R_{1t+1}$).

Equation (5) implies a number of properties for $v_t$ and the various possible equilibrium allocations of bankers wealth. Continuing bankers’ possibility of consuming their wealth at $t$ implies $v_t \geq 1$. But they may decide to keep part of their wealth aside as bank deposits (rather than consuming it) if $(1 + r)E_t(v_{t+1}) \geq 1$ and the returns on bank equity ($R_{0t+1}$ or $R_{1t+1}$) are small enough, i.e. $(1 + r)E_t(v_{t+1}) \geq \max\{E_t(v_{t+1}R_{0t+1}), E_t(v_{t+1}R_{1t+1})\}$. However, in equilibrium, the last condition will never hold with strict inequality because in that case no banker would invest in bank capital and banks would not be able to give loans, which is incompatible with equilibrium under the technology described in (1).\(^{30}\)

For brevity, the equilibrium conditions presented in the main text focus on the case with $\beta \max\{E_t(v_{t+1}R_{0t+1}), E_t(v_{t+1}R_{1t+1})\} > \max\{1, \beta(1 + r)E_t(v_{t+1})\}$. Then active bankers’

\(^{28}\)This reflects that bankers’ valuation of a unit of wealth may be different in different states of nature (e.g. depending on the scarcity of bankers’ aggregate wealth). At an individual level, however, an old banker’s wealth exhibits constant returns to scale, i.e. $e_t$ units of wealth are worth $v_{t+1}$.\(^{29}\)We check the validity of this assumption in all the parameterizations explored in the numerical part.\(^{30}\)The Cobb-Douglas production technology and the Walrasian determination of equilibrium wages tends to make the supply of loans infinitely profitable when the amount of supplied loans tends to zero.
optimal portfolio decisions can be described as follows:

- Invest all wealth in equity of the non-systemic bank if \( E_t(v_{t+1}R_{0t+1}) > E_t(v_{t+1}R_{1t+1}) \).
- Invest all wealth in equity of the systemic bank if \( E_t(v_{t+1}R_{1t+1}) > E_t(v_{t+1}R_{0t+1}) \).
- Invest in equity of any of the banks if \( E_t(v_{t+1}R_{0t+1}) = E_t(v_{t+1}R_{1t+1}) \).

We will refer to \( Q_t \equiv \max\{E_t(v_{t+1}R_{0t+1}), E_t(v_{t+1}R_{1t+1})\} \) as bankers’ required value-weighted return on wealth. Moreover, to avoid problems interpreting the pooling equilibrium in which the systemic bank mimics the non-systemic bank in all dimensions (except in setting \( \xi_{it} = 1 \) for all its funded firms), we will focus on parameterizations under which the equity of the non-systemic bank is always sufficiently attractive to bankers in equilibrium, in which case \( Q_t = E_t(v_{t+1}R_{0t+1}) \) for all \( t \).

4.2 Firm-bank lending contracts

This subsection describes how the non-systemic bank sets the terms of the lending relationship with each of its funded firms. The systemic bank will just mimic all these terms, except the unobservable systemic orientation of its funded firms. The contract signed between the non-systemic bank and each of its funded entrepreneurs will set \((\xi_{it}, k_{it}, n_{it}, l_{it}, B_{it}) = (0, k_t, n_t, l_t, B_t)\), where \( k_t, n_t, l_t, \) and \( B_t \) solve the following problem:

\[
\max_{(k_t, n_t, l_t, B_t, d_t, e_t)} \left(1 - p_0\right)[AF(k_t, n_t) + (1 - \delta)k_t - B_t] \\
\text{s.t.} \quad E\{v_{t+1}[\left(1 - p_0\right)B_t + p_0(1 - \lambda)k_t - (1 + r)d_t]\} \geq Q_te_t, \\
l_t = k_t + wtn_t, \quad l_t = d_t + e_t, \quad e_t \geq \gamma_t l_t.
\]

\(^{31}\) It is possible to analytically show that having a small measure of active bankers (\( \phi \to 0 \)) or low risk-shifting incentives (\( p_1 \to (p_0 - \varepsilon)/(1 - \varepsilon) \)) is sufficient to rule out equilibria with all bankers’ wealth is invested in equity of the systemic bank (\( x_t = 1 \)).

\(^{32}\) It can be shown that entrepreneurs are eventually indifferent between undertaking production in the systemic or the non-systemic mode since in both cases the equilibrium lending contract leaves them with a net payoff of zero in all states. As in neoclassical production theory, this is a result of perfect competition under constant returns to scale.

\(^{33}\) Notice that constant returns to scale will make the optimal size of individual firms (and, hence, of individual loans) undetermined in equilibrium, so it is useful to drop the firm subscripts \( i \) and to think of \((0, k_t, n_t, l_t, B_t)\) as the terms of a representative (linearly scalable) non-systemic loan.
This problem maximizes the expected payoff of any of the funded entrepreneurs at the end of period $t$, subject to the constraints faced by the bank. The entrepreneur obtains the difference between the gross output $AF(k_t, n_t) + (1 - \delta)k_t$ and the loan repayment $B_t$ when his firm does not fail.

The first constraint in (6) reflects bankers’ participation constraint. The bank knows that an arbitrary stochastic payoff $P_{t+1}$ offered in exchange for one unit of equity capital is acceptable to bankers if and only if $E(v_{t+1}P_{t+1}) \geq Q_t$, where $v_{t+1}$ and $Q_t$ are taken as given. The payoffs that bankers receive at $t + 1$ from the non-systemic bank (which will definitely be positive in equilibrium) are the gross repayments from the performing loans, $(1 - p_0)B_t$, plus the payment coming from the recovery of depreciated physical capital in failed firms, $p_0(1 - \lambda)k_t$, minus the payments due to depositors, $(1 + r)d_t$.

The last three constraints in problem (6) reflect (i) the use of loans to pay firms’ capital and labor in advance, (ii) the balance sheet identity $l_t = d_t + e_t$, and (iii) the regulatory capital requirement $e_t \geq \gamma_t l_t$.

The fact that equity returns at the non-systemic bank are deterministic allows us to divide both sides of the first constraint in (6) by $E(v_{t+1})$ and obtain

$$(1 - p_0)B_t + p_0(1 - \lambda)k_t - (1 + r)d_t \geq R_{0t+1}e_t,$$

where $R_{0t+1}$ is to be thought of the market-determined “required” return on equity at the non-systemic bank (taken as given by banks). Relying on this substitution, the following lemma uses the optimization conditions emanating from (6), together with labor market clearing, to establish a number of relationships between the key endogenous variables of the model. The proof is in Appendix A.

**Lemma 1** For a given expected return on equity at the non-systemic bank, $R_{0t+1}$, optimal firm-bank lending contracts and labor market clearing imply that, in a pooling equilibrium:

(a) firms’ aggregate demand for physical capital $k_t$ satisfies

$$(1 - p_0)[AF_k(k_t, 1) + (1 - \delta)] + p_0(1 - \lambda) = (1 - \gamma_t)(1 + r) + \gamma_t R_{0t+1},$$

15
(b) the market clearing wage rate \( w_t \) satisfies

\[
(1 - p_0)AF_n(k_t, 1) = [(1 - \gamma_t)(1 + r) + \gamma_t R_{0t+1}]w_t,
\]

(c) the aggregate demand for equity capital \( e_t \) satisfies

\[
e_t = \gamma_t(k_t + w_t), \text{ and}
\]

(d) the loan rate \( r_{Lt} \) satisfies

\[
1 + r_{Lt} = \frac{1}{1 - p_0} \left\{ [(1 - \gamma_t)(1 + r) + \gamma_t R_{0t+1}] - p_0(1 - \lambda) \frac{k_t}{k_t + w_t} \right\}.
\]

Equations (7) and (8) reflect how the production problem solved by banks and firms in our economy extends the canonical problem of perfectly-competitive firms in static production theory. First, the production process is intertemporal and subject to failure risk. Second, expected gross output at \( t + 1 \) is partly net output and partly depreciated capital. Third, the factors \( k_t \) and \( n_t \) are pre-paid at \( t \) using bank loans and, hence, their effective cost is affected by the funding bank’s weighted average cost of funds, \( (1 - \gamma_t)(1 + r) + \gamma_t R_0 \).

Bank frictions affect the real sector through the cost of the loans that firms use to finance their factors of production. For given capital requirement \( \gamma_t \), increasing the required rate of return on bank capital \( R_{0t+1} \) increases the competitive bank loan rate, pushing firms to reduce their scale, which, after taking labor market clearing into account, implies that both \( k_t \) by (7) and, recursively, \( w_t \) by (8) fall.\(^{34}\) Hence, the demand for bank capital described in (9) is decreasing in \( R_{0t+1} \). With these ingredients, determining the equilibrium path for \( R_{0t+1} \) will be conceptually as simple as adding the supply side of the market for bank capital and making sure that such market clears at all dates.

4.3 The supply of bank capital

For the purposes of this subsection, let us think of \( e_{t+1} \) as the aggregate supply of bank capital at date \( t + 1 \). Along a full reinvestment path, \( e_{t+1} \) coincides with the total wealth of active bankers at the beginning of period \( t + 1 \), which is made up of two components: (i)\(^{34}\) The same effects follow from an increase in \( \gamma_t \), for given \( R_{0t+1} > 1 + r \).
the capitalized value $\phi(1 + r)\omega_t$ of the labor income earned by currently active bankers in the prior date (which they are assumed to save), and (ii) the gross returns on the wealth $(1 - \psi)\epsilon_t$ that continuing bankers invested as bank capital at date $t$.\(^{35}\) This results in the following law of motion for $e_{t+1}$:

$$e_{t+1} = \phi(1 + r)\omega_t + (1 - \psi)[(1 - x_t)R_{0t+1} + x_tR_{1t+1}]\epsilon_t,$$

where, as previously defined, $x_t \in [0, 1]$ is the fraction of total bank capital invested in the systemic bank at date $t$.

From the point of view of date $t$, $R_{0t+1}$ is deterministic while $R_{1t+1}$ is a random variable that solely depends on the realization of $\epsilon_{t+1}$. Under most reasonable parameterizations, if the systemic shock does not realize, one unit of capital of the systemic bank yields some gross return $R_{1t+1}^{1 - \epsilon} > R_{0t+1}$, while if it realizes, the bank becomes insolvent and, by limited liability, the bankers recover just $R_{1t+1}^\epsilon = 0 < R_{0t+1}$ per unit of equity.\(^{36}\) Using the fact that the systemic bank mimics the non-systemic bank in every decision except its systemic orientation, and taking into account the different default rates, $p_0$ and $p_1$, experienced by each type of bank when $\epsilon_{t+1} = 0$, it is possible to write

$$R_{1t+1}^{1 - \epsilon} = \frac{1 - p_1}{1 - p_0}R_{0t+1} + \frac{1}{\gamma_t} \frac{p_0 - p_1}{1 - p_0}[(1 - \gamma_t)(1 + r) - (1 - \lambda)\frac{k_t}{k_t + \omega_t}].$$

This expression makes clear that, when the capital requirement $\gamma_t$ is lower than the rate of depreciation of physical capital in failed projects $\lambda$, having $p_1 < p_0$ (by A1) implies $R_{1t+1}^{1 - \epsilon} > R_{0t+1}$.\(^{37}\)

From date $t$ perspective, the aggregate bank capital available in the next date, $e_{t+1}$, is a dichotomous random variable whose law of motion can be expressed as:

$$e_{t+1} = \begin{cases} \phi(1 + r)\omega_t + (1 - \psi)[(1 - x_t)R_{0t+1} + x_tR_{1t+1}^{1 - \epsilon}]\epsilon_t \equiv e_{t+1}^{1 - \epsilon}, & \text{if } \epsilon_{t+1} = 0, \\ \phi(1 + r)\omega_t + (1 - \psi)(1 - x_t)R_{0t+1}\epsilon_t \equiv e_{t+1}^\epsilon, & \text{if } \epsilon_{t+1} = 1, \end{cases}$$

\(^{35}\) Appendix C states equilibrium conditions for the general case in which active bankers may find it optimal to consume part of their wealth or to keep part of it inverted as bank deposits. For simplicity, the assumption that bankers save all their labor income will not be relaxed, but we will check that such behavior is always optimal under our parameterization of the model.

\(^{36}\) With a slight abuse of notation, the superscripts $\epsilon$ and $1 - \epsilon$ will identify, respectively, the realizations of the random variables contingent on whether the systemic shock realizes or not.

\(^{37}\) The condition $\gamma_t < \lambda$ holds in all the quantitative analysis below, even when $\gamma_t$ is set at its social welfare maximizing value.
which clearly shows its dependence of the aggregate shock $\varepsilon_{t+1}$.

Looking back at (5) and using (13), it is immediate to write down the conditions for the compatibility of particular values of $x_t$ with bankers’ optimal portfolio decisions.

**Lemma 2** In equilibria with $x_t \in [0, 1)$ we must have:

\[
[(1 - \varepsilon)v(e_{t+1}^{1-\varepsilon}) + \varepsilon v(e_{t+1}^\varepsilon)]R_{0t+1} \geq (1 - \varepsilon)v(e_{t+1}^{1-\varepsilon})R_{1t+1}^{1-\varepsilon}. \quad (14)
\]

Moreover, if (14) holds with equality, then any $x_t \in [0, 1)$ is compatible with bankers’ optimization, otherwise only the corner solution $x_t = 0$ is compatible.

The corner solution without systemic risk-taking ($x_t = 0$) that emerges when (14) holds with equality can be formally captured by imposing:

\[
\{[(1 - \varepsilon)v(e_{t+1}^{1-\varepsilon}) + \varepsilon v(e_{t+1}^\varepsilon)]R_{0t+1} - (1 - \varepsilon)v(e_{t+1}^{1-\varepsilon})R_{1t+1}^{1-\varepsilon}\}x_t = 0, \quad (15)
\]

which can be interpreted as a complementary slackness condition.

### 4.4 Equilibrium

Along a full-reinvestment equilibrium, the state of the economy at any date $t$ (right before investment decisions are made for one more period) can be summarized by a single state variable: the total wealth available to the active bankers $e_t$. As described in (13), the stochastic evolution of $e_t$ is driven by the realization or not of the systemic shock at the end of every period. The equilibrium value of other variables of the model can be thought of as functions of $e_t$ that satisfy the individual optimization and market clearing conditions established in previous sections. More formally:

**Definition 1** A full-reinvestment equilibrium is (i) a stationary law of motion for the state variable $e$ on a bounded support $[\underline{e}, \overline{e}]$ and (ii) a tuple $(v(e), x(e); k(e), w(e), R_0(e), R_1^{1-\varepsilon}(e))$ describing the key endogenous variables as functions of $e \in [\underline{e}, \overline{e}]$, such that all the sequences\{\{e_t\}_t=0,1,... \text{ and } \{v_t, x_t, k_t, w_t, R_{0t+1}, R_{1t+1}^{1-\varepsilon}\}_{t=0,1,...}\text{ that they generate satisfy:}

1. Optimization by price-taking workers, bankers, entrepreneurs, banks, and firms.
2. The clearing of all markets.

3. The investment as bank capital of all the wealth available to active bankers.

Thus, the equilibrium values of the marginal value of bank capital \( v_t \), the fraction of bank capital allocated to the systemic bank \( x_t \), the physical capital used by firms \( k_t \), the wage rate \( w_t \), the return on equity at the non-systemic bank \( R_{0t+1} \), and the return on equity at the systemic bank when the systemic shock does not occur \( R_{1t+1}^{1-\varepsilon} \) that arise when aggregate bank capital is \( e_t = e \) will be found by evaluating the various components of the tuple \( (v(e), x(e); k(e), w(e), R_{0}(e), R_{1}^{1-\varepsilon}(e)) \).

Appendix C relaxes requirement 3. in the definition of equilibrium, so as to allow for solutions in which bankers find it optimal to consume part of their wealth or to invest part of it in bank deposits in some states. Appendix C also describes the numerical solution method that we use to compute the equilibrium under the parameterizations explored in the quantitative part.

4.5 Key equilibrium forces

This subsection is going to intuitively explain some of the key mechanism at work in the determination of equilibrium in our economy. Given the fixed supply of labor and the underlying constant-returns-to-scale technology, the aggregate returns to bank lending are marginally decreasing. This makes one unit of bankers’ wealth (which can be used to expand banks’ lending capacity) more valuable when bankers’ aggregate wealth is more scarce. Specifically, we have \( v'(e) < 0 \) if bankers invest all their wealth in bank equity and \( v'(e) = 0 \) if, at some point, they devote it, in the margin, to alternative uses.

Intuitively, when bankers invest all their wealth in bank equity, increasing \( e \) expands banks’ lending capacity, loans become cheaper, and firms expand their activity, which in equilibrium, after wages adjust, implies devoting more physical capital to production. Like in the neoclassical growth model, the fixed supply of labor makes the aggregate equilibrium returns to physical capital marginally decreasing, implying that the marginal value of bank
lending and, hence, the scarcity rents appropriated by bank capital are marginally decreasing in $e$.$^{38}$

For the dynamics described in (13), the aggregate marginally decreasing returns to $e$ imply that, after sufficiently many periods without suffering a systemic shock, this economy will converge to what we denote as its pseudo-steady state (PSS): a state in which all aggregate variables remain constant insofar as the systemic shock does not realize (but agents are fully aware of the possible occurrence of such shock at any date). If the shock realizes, all the $e$ invested as equity of the systemic bank is lost and the process of accumulation of bankers’ wealth and the convergence to the PSS (possibly disrupted by the arrival of another systemic shock) starts over again.

To understand the intuition driving bankers’ systemic risk-taking decisions (represented by the aggregate fraction of bank equity $x$ invested in the systemic bank), the crucial equations are (14) and (15). As previously mentioned, (12) implies $R_{1t+1}^{1−ε} > R_{0t+1}$ so satisfying (14) requires a sufficiently large value of $εv(ε_{t+1})$ or, given that $ε$ is small and $v(e)$ is decreasing, having a sufficiently low value of $ε_{t+1}$ (or a very steep $v(e)$). Intuitively, for bankers to abstain from systemic risk taking (i.e. giving up the gains from risk-shifting), they must be compensated by the expectation of obtaining a large revaluation of their wealth if it survives the systemic shock.

By (13), a larger $x_t$ implies, other things equal, a larger aggregate loss of bank capital when the shock occurs, and hence a lower $ε_{t+1}$ and a larger $v(ε_{t+1})$. This establishes a self-equilibrating mechanism for the determination of $x_t$. Equation (15) embeds the no-arbitrage (or indifference) condition required for producing an interior $x_t ∈ (0, 1)$ which is possible thanks to the just commented equilibrating force.

### 4.6 Social welfare

A natural measure of social welfare $W_t$ in this economy is the expected present value of the aggregate net consumption flows that it provides to the various agents. This measure can be obtained and decomposed in various forms, depending on the dimension along which

---

38 This result also arises, with identical intuition, in e.g. Gertler and Kiyotaki (2010).
the relevant overall aggregation is performed. One can infer the net consumption flow that
the economy generates for each class of agents in each date \( t \) and aggregate across agents.
Alternatively, one can just look at the differences between the aggregate quantity of the
consumption good the economy produces at the end of a typical period and the quantity
which is reutilized as a factor of production (physical capital) in the next period. Appendix
B provides an explicit expression for \( W_t \) and an associated flow measure of welfare \( \omega_t \), which
are explained there using two intuitive decompositions.

Following convention, we will describe below the gains and losses in expected welfare,
\( E(W_t) \), as percentage differences in the certainty-equivalent permanent aggregate net con-
sumption flow that would give rise to such welfare, which can be calculated as \((1-\beta)E(W_t)\).\(^{39}\)

5 Numerical results

Our baseline quantitative results are obtained under a time-invariant capital requirement,
i.e. with \( \gamma_t = \gamma \) for all \( t \). For illustration purposes, we will compare the results obtained with
a reference capital requirement of 7% (\( \gamma=0.07 \)) with those obtained with the requirement
that, under the parameterization presented in Table 1 below, maximizes the unconditional
expected value of our social welfare measure \( W_t \). By “unconditional” we mean that the
starting points of the simulated paths over which \( E(W_t) \) is computed are extracted from
the ergodic distribution of \( e_t \) in proportion to the relevant probabilities.\(^{40}\) We analyze some
cases of time-varying or state contingent capital requirements in Section 6. In particular, we
assess the implications of moving from a regime with \( \gamma=0.07 \) to a regime with higher capital
requirements in a gradual way. We also assess potential gains from giving a pro-cyclical or
counter-cyclical profile to \( \gamma_t \).

\(^{39}\)The use of impatient agents’ discount factor \( \beta \) in the discounting of the relevant consumption flows is
justified in Appendix B.

\(^{40}\)The support of the ergodic distribution is made of the values reached with strictly positive probability
by \( e_t \) along sufficiently long histories of the economy. Our baseline numerical results are based on simulating
one path of 10,000 periods.
5.1 Calibration

Our quantitative results are based on assuming that one period in the model corresponds to one year in calendar time. Table 1 contains the parameters chosen for our tentative calibration of the model. We have tried numerous other parameter configurations to check the performance of the solution method and the generality of the qualitative results, in both cases with positive results. Some of these robustness checks will be discussed in Section 6.

Table 1

Baseline parameter values
(One period is one year; all rates are yearly rates)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patient agents’ discount rate</td>
<td>$r$ 0.02</td>
</tr>
<tr>
<td>Impatient agents’ discount factor</td>
<td>$\beta$ 0.96</td>
</tr>
<tr>
<td>Total factor productivity</td>
<td>$A$ 2</td>
</tr>
<tr>
<td>Physical capital elasticity</td>
<td>$\alpha$ 0.3</td>
</tr>
<tr>
<td>Depreciation rate in successful firms</td>
<td>$\delta$ 0.05</td>
</tr>
<tr>
<td>Depreciation rate in failed firms</td>
<td>$\lambda$ 0.35</td>
</tr>
<tr>
<td>Idiosyncratic default rate of non-systemic firms</td>
<td>$p_0$ 0.03</td>
</tr>
<tr>
<td>Idiosyncratic default rate of systemic firms</td>
<td>$p_1$ 0.018</td>
</tr>
<tr>
<td>Probability of a systemic shock</td>
<td>$\varepsilon$ 0.03</td>
</tr>
<tr>
<td>Bankers’ exit rate</td>
<td>$\psi$ 0.20</td>
</tr>
<tr>
<td>Fraction of wages devoted to forming new bank capital</td>
<td>$\phi$ 0.05</td>
</tr>
</tbody>
</table>

The model is quite parsimonious: it has the 11 parameters listed in Table 1 (plus the capital requirement $\gamma$, if taken as given) and a single binary iid aggregate shock (the systemic shock), whose probability of occurring $\varepsilon$ is one of the parameters. The discount rate of the patient agents $r$ is chosen equal to 2% to capture a situation with low real interest rates such as the one observed in developed economies in the years leading to 2007 financial crisis. The discount factor of the impatient agents embeds a discount rate which is approximately twice as large as $r$. In the literature on external financing frictions it is standard to assign values of this order of magnitude, or even larger, to their discount rate.\footnote{For instance, in Iacoviello (2005) the spread between the discount rate of the borrowing entrepreneurs and that of the patient households that finance them is 4%. In Carlstrom and Fuerst (1997) and Gomes, Yaron, and Zhang (2003), the spread is 5.6%.} The value for the total factor productivity parameter $A$ is inconsequential, except for the scale of the variables in
levels. With $A = 2$, most macroeconomic aggregates take two-digit values in levels, making them just easier to report.

The elasticity of physical capital in the production function $\alpha$ is fixed according to standard macro practice, so as to produce a share of labor income in GDP of about 70%. The depreciation rates of physical capital in successful and failing firms, $\delta$ and $\lambda$, are chosen so as to match an aggregate physical capital to GDP ratio in the range of 3 to 4 as well as a loss-given-default (LGD) for bank loans of about 45%, which is the LGD fixed for unrated corporate exposures in the standardized approach of Basel II.42

The values of $p_0$, $p_1$, and $\varepsilon$ are set so as to have sufficient potential room for risk shifting and for significant aggregate losses due to it. The current choices are compatible with the conditions $p_1 < p_0 < (1 - \varepsilon)p_1 + \varepsilon$ established in assumptions A1 and A2. They imply unconditional expected default rates in the range from 3% (if all firms were non-systemic) to 4.7% (if all firms were systemic). The probability of a systemic shock (situation in which all systemic firms fail simultaneously) is set at 3%, so that this shock occurs on average once in every 33 years. These parameter choices help us extract our quantitative results in a context in which the efficiency losses due to risk-shifting are significant, while bankers’ inclination towards risk-shifting is potentially large.43

The choices for the values of the bank capital dynamics parameters, $\psi$ and $\phi$, are quite tentative. The rate $\psi$ at which bankers convert back into workers and stop providing their accumulated wealth as bank capital implies that they have an average active life of 5 years. The value of $\phi$ implies that bankers’ represent 5% of the efficiency units of labor in the economy, so that their yearly labor income provides additions to the aggregate wealth available to active bankers equivalent to 5% of total labor income per period.44

42To explain why $\lambda = 0.35$ produces a LGD of 45%, notice that loans in this model also finance firms’ wages (what empirically might correspond to funding their working capital) and the recoveries obtained by banks on that part of the loans is zero when a firm fails.

43The robustness of our key qualitative results to changes in the parameters is analyzed in Section 6.

44In the pseudo-steady state, the increase in active bankers’ wealth due to $\phi$ exactly compensates the wealth taken out by the fraction $\psi$ of non-continuing bankers, making the effective supply of bank capital constant until a systemic shock occurs.
5.2 Graphical presentation of the results

The numerical method used for solving the model relies on value function iteration in order to obtain \( v(e) \). As described in Appendix C, we use the relevant equilibrium conditions in their full, potentially non-linear form.\(^{45}\) All the results correspond to the equilibrium conditions described there, which consistently accommodate the possibilities of bankers wanting to consume or save part or all of their wealth as bank deposits.\(^{46}\) No problem of multiplicity of equilibria was detected.

![Graph](image)

**Figure 1** Social welfare \( W \) as a function of the capital requirement \( \gamma \)

Figure 1 is generated by solving the model for a grid of values of \( \gamma \) and by computing the unconditional expected value of social welfare, \( E(W_t) \), under each of them. The figure describes welfare as the certainty-equivalent consumption flow \((1-\beta)E(W_t)\) which, if received

---

\(^{45}\) Linearization is only used locally, for interpolation purposes, when the value function has to be evaluated at values of \( e \) not included in the initial grid.

\(^{46}\) Situations in which bankers do not want to fully reinvest their wealth as bank equity are typically confined to values of \( e \) out of the support of the ergodic distribution.
as a perpetuity, would have a present discounted value of $E(W_t)$.

For illustration purposes, we will henceforth compare the results obtained with a capital requirement of 7% ($\gamma = 0.07$) with those obtained with a requirement of 14% ($\gamma = 0.14$), which is the one that maximizes social welfare under our baseline parameterization. For these two requirements, Figure 2 depicts the function $v(e)$ that describes the marginal value of one unit of bank capital, and the function $x(e)$ that describes the fraction of bank capital invested in the systemic bank (i.e. devoted to make systemic loans). Both functions are depicted for the union of the ranges of $e$ which are relevant in the economies with $\gamma = 0.07$ (low capital requirement) and $\gamma = 0.14$ (optimal capital requirement). As shown below, in the economy with a low $\gamma$, total bank capital $e$ fluctuates over a wider but lower range of values than in the economy with the optimal $\gamma$.

The results tell, quite intuitively, that the greater scarcity of bank capital induced by a higher capital requirement implies a higher marginal value of capital $v(e)$ at every level of capital $e$. More importantly, systemic risk-taking, which is non-decreasing in $e$ (and strictly increasing in $e$ up to the point in which bankers stop to fully reinvest their wealth as bank equity), is lower at every $e$ with $\gamma = 0.14$ than with $\gamma = 0.07$. The monotonicity of the effect of $\gamma$ on the schedule $x(e)$ holds for all the range of capital requirements that we have tried. The intuition for this result is that the impact of $\gamma$ on $v(e)$ (i.e. on its position and slope) affects the incentives of bankers to guarantee that (part of) their capital survives a systemic shock. This effect is driven by the impact of $\gamma$ on the effective scarcity of bank capital in equilibrium and its endogenous return. The key logic is consistent with the traditional charter value effect stressed in the microeconomic literature, but the general equilibrium channel for the impact of capital requirements on risk-taking incentives that we identify is, to the best of our knowledge, a novel contribution of our paper.47

---

47The possible negative impact of capital requirements on risk-taking identified by, for example, Hellmman et al. (2000) is due to the negative impact of the requirements on banks’ profitability. However, such a negative effect is established in a partial equilibrium setup with a perfectly elastic supply of bank capital.
Figure 2 $v(e)$ and $x(e)$ under low and optimal capital requirements (CR)
The interaction between bankers’ systemic risk taking, as reflected in $x(e)$, and the endogenous dynamics of bank capital can be further explained by looking at Figure 3. The panels on the left correspond to the economy with $\gamma = 0.07$ and those on the right to the economy with $\gamma = 0.14$. The solid lines in the top panels represent the phase diagram mapping the amount of bank capital in one period, $e_t$, onto its amount in the next period if the systemic shock does not occur “at the end” of period $t$, $e_{t+1}^{1-\varepsilon}$ (see (13)). This schedule is strictly increasing except when $e_t$ is large enough for continuing bankers to consume part of it as a voluntary dividend (an option considered in the extended definition of equilibrium described in Appendix C), in which case the schedule becomes flat.

The dashed downward sloping curve in the top panels represent the mapping from $e_t$ to the capital available to bankers in the next date if the systemic shock occurs, $e_{t+1}^\varepsilon$. The vertical distances between the solid and the dashed curves measure the loss of bank capital when the economy is hit by the systemic shock. The loss is larger not only in absolute but also in relative terms for higher values of $e$ because the fraction of equity invested in the systemic bank increases with $e$ (see Figure 2). Indeed, for sufficiently low values of $e$, we have $x(e) = 0$, in which case the two curves merge ($e_{t+1}^{1-\varepsilon} = e_{t+1}^\varepsilon$).

The points where the solid no-shock phase diagrams intersect the 45-degree line in each of the top panels identify the corresponding pseudo-steady state. Interestingly, the PSS value of $e$ is the highest point in the ergodic support of $e$ and, thus, is associated with the highest level of systemic risk-taking (since we have $x'(e) > 0$). This is also the point where the realization of the systemic shock has the most devastating consequences: the largest loss of bank capital and the largest subsequent contraction of credit. The arrows on each panel identify the path of crisis and recovery for the (most frequent) situation in which the economy fully returns to its PSS without suffering a second systemic shock. With $\gamma = 0.07$ ($\gamma = 0.14$) our economy fully recovers in a minimum of 5 (8) years.\textsuperscript{48,49}

\textsuperscript{48}Recall that in our calibration systemic shocks are iid and occur with a probability of just 3% per period.
\textsuperscript{49}The differences between the two economies in this respect are hard to see in the figures since in the last periods prior to returning to PSS, the economy is very close to it.
Figure 3 Equilibrium dynamics with low (CR=7%) and optimal (CR=14%) capital requirements
The bottom panels in Figure 3 depict the relative frequencies with which different values of $e$ are visited along sufficiently long histories of the economy. Consistently with the aggregate shock being so little frequent, our economy spends most of the time (about 80% or more) in the pseudo-steady state. However, when the systemic shock realizes the consequences are very sizeable. Other points with positive frequencies correspond to the various recovery paths (or paths with some more crises and subsequent recoveries) that may occur.

Figures 2 and 3 reflect some of the considerations relevant for the welfare comparison between the economies with $\gamma = 0.07$ and $\gamma = 0.14$. The amount of systemic risk taking $x(e)$ over the range of values of $e$ in the support of the ergodic distribution (and, particularly, at the PSS where the economy spends most of the time) tells about the importance of the unresolved risk-shifting problem. Risk shifting produces “static” as well as “dynamic” losses in this economy. The “static” losses are due to the inefficiency of operating the production technology in the systemic mode, which implies a larger unconditional failure rate, with the subsequent waste of resources. The “dynamic” losses are due the loss of bank equity (and subsequent loss in banks’ lending capacity) that follows the realization of a systemic shock, which implies the amplification and propagation over time of the macroeconomic implications of the shock.

Since previous figures suggest than an economy with larger capital requirements would involve lower systemic risk-taking, it might be tempting to conclude that its performance in welfare terms should always be superior. But the story is not so simple. Capital requirements reduce the extent to which a given amount of bank equity gets translated into banks’ lending capacity and, despite $\gamma = 0.14$ is twice as high as $\gamma = 0.07$, one can see in Figure 3 that the PSS level of bank equity with $\gamma = 0.14$ is not twice as large as with $\gamma = 0.07$. So increasing the capital requirements in our economy reduces aggregate bank credit (and thus the level of economic activity), at least in the PSS, producing a major counterbalancing negative effect to their beneficial welfare effects along the risk-taking dimension. This counterbalancing cost of high capital requirements will become apparent when looking at the tables included below.
5.3 Quantitative details

Table 2 reports the unconditional expected values of the main endogenous variables of the model under each level of the capital requirement, as well as the percentage difference in each statistic when comparing the economy with the low requirement $\gamma = 0.07$ and the economy with the optimal requirement $\gamma = 0.14$.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Main endogenous variables</th>
<th>(Unconditional expected values and percentage difference across regimes)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma = 7%$</td>
<td>$\gamma = 14%$</td>
</tr>
<tr>
<td>Welfare* (=equivalent perpetual consumption flow)</td>
<td>2.973</td>
<td>3.000</td>
</tr>
<tr>
<td>GDP*</td>
<td>4.404</td>
<td>4.116</td>
</tr>
<tr>
<td>Bank credit ($l$)</td>
<td>19.243</td>
<td>15.254</td>
</tr>
<tr>
<td>Bank equity ($e$)</td>
<td>1.347</td>
<td>2.136</td>
</tr>
<tr>
<td>Loan rate ($r_L$) (in %)**</td>
<td>4.1</td>
<td>5.6</td>
</tr>
<tr>
<td>Deposit insurance costs*</td>
<td>0.158</td>
<td>0.037</td>
</tr>
<tr>
<td>Value of one unit of bank capital ($v$)</td>
<td>1.107</td>
<td>1.786</td>
</tr>
<tr>
<td>Fraction of capital invested in systemic banks ($x$)</td>
<td>0.706</td>
<td>0.244</td>
</tr>
</tbody>
</table>

*See Appendix B for exact definitions of these variables. **Difference reported in percentage points.

The first variable in the list is social welfare, reported as $(1 - \beta)E(W_t)$, like in Figure 1. We find that moving from the low to the optimal capital requirement increases social welfare in an amount equivalent to a perpetual increase of 0.9% in aggregate net consumption. The main reason behind the larger welfare attained in the economy with higher capital requirements is the lower average fraction of bank capital invested in systemic banks (and hence the fraction of “systemic loans”): roughly 24% rather than 71%.

Table 2 also reports the average levels of several macroeconomic and banking variables. Importantly, imposing capital requirements of 14% rather than 7% implies making bank equity relatively more scarce and, consequently, bank loans more expensive (the unconditional of the mean loan rate shifts from 4.1% to 5.4%). This implies obtaining significantly lower values in macroeconomic aggregates such as GDP and bank credit, whose unconditional means are, respectively, 6.5% and 20.7% lower with $\gamma = 0.14$ than with $\gamma = 0.07$. The
analysis evidences that, in the presence of systemic risk, the unconditional means of these variables (and even more their values in the PSS) are pretty bad proxies of social welfare. How cheap is credit or how large is GDP, in particular, do not properly reflect the relatively low social net present value of the marginal investments undertaken when systemic risk-taking is large and, in particular, the very significant losses (e.g. in physical capital not recovered from failed projects) suffered when systemic risk materializes.

When the systemic shock occurs, the loss in bank capital reduces credit and investment for a number of periods, until the economy returns to its pseudo-steady state. Table 3 reports the fall in the variables listed in Table 2 in the period that follows a systemic shock: we compare their value one year after the shock if the economy gets hit at the PSS with the value in the PSS if no shock occurs. The shock wipes out a fraction of bank capital, affecting the supply of credit and the cost of bank loans. The increase in loan rates following the shock is of 11.6 percentage points with capital requirements of 7% and of only 2.5 percentage points with capital requirements of 14%. Aggregate net consumption, GDP, and bank credit fall by 17%, 32%, and 65% with capital requirements of 7% and by only 5%, 10%, and 24% with capital requirements of 14%.

Finally, Table 4 describes the values of some relevant macroeconomic and financial ra-
tios across the economies with $\gamma = 0.07$ and $\gamma = 0.14$, respectively. Some ratios show that our calibration is consistent with some standard macroeconomic calibration targets. Other ratios are suggestive of a relationship between weak capital regulation (low $\gamma$) and financial exuberance. Specifically, the ratio of credit to GDP is positively correlated with the underlying endogenous level of systemic risk-taking ($x$) and, thus, higher in the economy with low capital requirements. This correlation, very much in line with the perceptions of the early proponents of a macroprudential approach to bank regulation (e.g. Borio, 2003), is endogenously generated by the model.

### Table 4

Other macroeconomic and financial ratios  
(Unconditional expected values)

<table>
<thead>
<tr>
<th></th>
<th>$\gamma = 7%$</th>
<th>$\gamma = 14%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor income/GDP*</td>
<td>0.69</td>
<td>0.68</td>
</tr>
<tr>
<td>Physical capital/GDP</td>
<td>3.68</td>
<td>3.03</td>
</tr>
<tr>
<td>Bank credit/GDP</td>
<td>4.37</td>
<td>3.71</td>
</tr>
<tr>
<td>Deposit insurance costs/GDP</td>
<td>0.036</td>
<td>0.009</td>
</tr>
<tr>
<td>ROE at non-systemic banks</td>
<td>1.104</td>
<td>1.168</td>
</tr>
<tr>
<td>ROE at systemic banks if systemic shock does not realize</td>
<td>1.189</td>
<td>1.213</td>
</tr>
<tr>
<td>Banks’ payout/Bank capital</td>
<td>0.213</td>
<td>0.224</td>
</tr>
</tbody>
</table>

*See Appendix B for an exact definition of GDP.

All in all the results summarized in prior tables suggest that the quantitative implications of capital requirements can be quite sizeable. They also point to the fact that the design of optimal capital requirements should respond to an economic risk-management logic: caring about inefficient systemic risk-taking and its normally-invisible threat to macroeconomic stability. Standard macroeconomic variables (such as GDP or credit), evaluated at the unconditional means or in the PSS (or along paths in which the systemic shock does not realize), give wrong indications about the desirability of setting capital requirements at one level or another. As we have seen, the optimal capital requirement of 14% seems to have a “large cost” relative to the suboptimal requirement of 7% when evaluated in terms of these variables.
6 Extensions and discussion

This section is structured in four parts. First, we analyze the transition from a regime with low capital requirements to one with a higher target level of capital requirements, assessing the value of gradualism in the imposition of the new target. Second, we consider a flexible class of state-dependent capital requirements and assess the effects of giving them a higher or lower pro- or counter-cyclical profile. Third, we discuss the effects of bailout policies (e.g. transfers of wealth to bankers aimed at reducing the contraction of credit supply after a systemic shock). Finally, we look at the comparative statics of the model (and the robustness of our baseline results) by analyzing the effects of changing the value of some of its key parameters.

6.1 Transitional dynamics and the value of gradualism

Prior sections have focused on economies in which the capital requirement $\gamma_t$ remains constant over time, but the model can be extended to analyze the transition from a regime with some initial level of capital requirements, say $\gamma^0$, to a target level to be reached after $T$ periods, $\gamma^*$. To limit computational costs, we consider linear adjustment paths $\{\gamma_t\}$ with

$$\gamma_t = \gamma^0 + \frac{(\gamma^* - 0.07)}{T} t, \text{ for } t = 1,...,T - 1.$$ 

We set $\gamma^0 = 0.07$ so as to start from the “low capital requirements” regime of prior sections.

Figure 4 displays the social welfare (in permanent certainty-equivalent net consumption terms) associated with different target requirements $\gamma^*$ when the economy starts in the pseudo-steady state that corresponds to $\gamma^0 = 0.07$. Each curve corresponds to a value of $\gamma^*$ ranging from 8% to 15% and the horizontal axis represents the number of transition periods $T$. The shortest transition, $T = 1$, implies announcing at $t = 0$ (when the capital requirement is 7%) that the new requirement $\gamma^*$ will come into effect at $t = 1$. With $T = 2$, the announcement is that half of the total increase in the requirement will take place at $t = 1$. 

33
and the remaining half at $t = 2$. And so on.

**Figure 4** Social welfare for different target capital requirements as a function of the length of the transition (starting from the PSS with $\gamma = 0.07$)

To obtain the results, we first consider the economies in which capital requirements remain constant at the relevant target $\gamma^*$ and use the obtained value functions, valid for $t \geq T$, together with backward induction on (5), to solve for the value functions relevant at each of the transitional periods $t = 1, 2, \ldots T - 1$. After obtaining the transitional value functions, we simulate 1000 equilibrium paths of 1000 years each starting from the PSS of the economy with $\gamma = 0.07$ and compute the welfare associated with each path.

Under our baseline calibration, the maximum welfare is obtained with a target capital requirement of 13% and 10 years of transition—although welfare is very similar with a target requirement of 12% and 7 years of transition. Interestingly, if capital requirements were to be increased only up to a (suboptimal) level of 10%, then the (conditional) optimal transition length would just be $T = 1$, i.e. there would be no net gains from increasing the requirements
in a gradual way. For higher target requirements, however, maximizing (conditional) welfare requires transitional periods whose lengths increase with the target.

Figure 4 evidences the decreasing marginal social gains from increasing capital requirements in our economy—even when allowing for their gradual introduction. While there are significant gains from increasing capital requirements from 7% to up to 10% (even without gradualism), realizing significant gains from raising the target to, e.g., 11% (rather than 10%) requires gradualism (4 years). And the target of 12% (13%) only improves over the target of 11% (12%) if the transition is extended to 7 (10) years.

6.2 Cyclically-adjusted capital requirements

The model can also be extended to analyze the potential value of cyclically-adjusted capital requirements, i.e. potentially time-variant capital requirements of the form $\gamma_t = g(e_t)$, where $g$ is an arbitrary function taking values in the interval $[0, 1]$. Recent debates among bank regulators suggest that having $g' > 0$ might have virtuous counter-cyclical effects: it would contract (expand) the amount of credit that can be extended per unit of available bank capital when bank capital $e_t$ is more abundant (scarcer). Symmetrically, current wisdom attributes undesirable procyclical effects to $g' < 0$, i.e. to making capital requirements tighter when bank capital is scarcer.

To check the effects of cyclically-adjusted capital requirements in a quantitatively easy manner, we consider the following flexible functional form for $g$:

$$g(e_t) = \min\{\max\{g_0 + g_1 (\log(e_t) - \log(\bar{e})), 0\}, 1\},$$

where $g_0$ and $g_1$ are constant parameters, and $\bar{e}$ is the amount of bank capital that the economy would accumulate in its PSS under a time-invariant capital requirement $\gamma_t = g_0$. Accordingly, the capital requirement increases by about $g_1$ percentage points for each 1% difference between $e_t$ and the reference value $\bar{e}$.

To find out which cyclical adjustments would make more sense under our baseline parameterization of the model, we set $g_0 = 0.14$ (since a requirement of 14% maximizes social welfare among the class of time-invariant requirements) and compute the welfare attained
under alternative values of $g_1$ around zero. To our surprise, we find that welfare increases smoothly as $g_1$ is set further and further below zero, up to a value of about $g_1 = -0.8$ (where the welfare gains relative to the time-invariant benchmark are of about 0.75%). In contrast, welfare falls quite sharply if $g_1$ is increased (with a welfare loss of about 2.5% relative to the benchmark for $g_1 = 0.2$). This implies that trying to produce countercyclical effects by relaxing capital requirements after a crisis is actually counterproductive in our model. If anything, there would be social welfare gains from making capital effectively scarcer for low values of $e$.

These results come from the endogeneity of systemic risk taking in the model. Bankers face a dynamic trade-off when choosing between systemic and non-systemic lending. One motivation for bankers to stay away from systemic risk taking is the expectation of earning scarcity rents on their surviving capital if the systemic shock realizes and other bankers lose their capital. Having $g_1 > 0$ would reduce the after-shock gains of the owners of surviving capital, pushing more bankers into inefficient systemic risk taking and increasing the severity of any systemic crisis. In other words, although $g_1 > 0$ reduces the credit crunch that follows any given loss of aggregate bank capital $e_t$, it also encourages ex ante risk taking by banks, implying that the endogenous size of bank capital losses is higher with $g_1 > 0$ than with $g_1 \leq 0$.

### 6.3 Bailout policies

What does our model say about bailout policies? Are they at all desirable? Do they have undesirable side effects? Due to space constraints, a detailed quantitative assessment of the many forms that these policies might take is beyond the scope of the current paper, but it is worth making a few remarks directly emanated from the logic of the model. In a spirit similar to the countercyclical capital requirements that we have just discussed, the rationale for bailout policies in the context of the model would be to try to avoid the sharp contraction in credit that follows the realization of a systemic shock (especially if it occurs at or near the PSS). The tool to expand credit supply after the shock in this case would be transferring wealth to bankers (so as to increase the effective supply of bank capital) rather than touching
the capital requirement.

Instrumenting the bailouts in the form of transfers of wealth to bankers is consistent with our maintained assumption that bank ownership requires possessing bank management talent, which is exclusive to bankers. It is useful to distinguish between three classes of possible recipients of the bail-out transfers: (i) failed bankers, (ii) solvent bankers, and (iii) novel bankers. Among the first two classes, one may further distinguish between retiring and non-retiring bankers. In terms of the implications for the capital effectively available to banks after the bailout (and, hence, the resulting relief of the credit crunch), transfers to retiring bankers are clearly a waste, while all other transfers are perfect substitutes, since they contribute equally to reducing the ex post credit crunch. But in addition to the direct effects on credit supply there are two important incentive effects to consider.

First, the wealth transferred to the non-retiring bankers will reduce the equilibrium value of each unit of bank capital after the crisis. In terms of Figure 2, the $v(e)$ schedule will become less steep, producing a negative impact on incentives (i.e. increasing systemic risk taking as represented by $x(e)$). Second, there will also be direct incentives effects associated with the candidate recipients’ prospects of receiving wealth transfers after the systemic shock. These are the effects that informal discussions about bank bailouts refer to as “moral hazard” effects. Transfers to failed bankers only would constitute a reward to systemic risk taking, those to solvent bankers only would be a reward to staying away from systemic risk, and those to novel bankers would have no direct effects on incentives.

Combining the three aspects of this discussion suggests that the potentially most welfare enhancing manner of implementing a bailout would be by directing the wealth transfers exclusively to (non-retiring) bankers with wealth invested in banks that remain solvent after the systemic shock. At the cost of redistributing wealth from tax payers to bankers, it might well be the case that the relief of the credit crunch achieved in this manner, together with the direct incentive effects, offset the negative welfare implications of the indirect incentive effects due to the reduction in the unit value of bank capital after a systemic shock.
Figure 5 Dependence of the optimal $\gamma$ (left axis) and systemic risk taking $E(x)$ (right axis) on some of the parameters.
6.4 Comparative statics

Figure 5 summarizes the comparative statics of the model relative to four of its key parameters. Each of its panels shows the effects of changing, one at a time, one of the parameters. We consider several alternative values around the baseline values reported in Table 1.

Each panel shows, as a different curve, the effects on the optimal time-invariant capital requirement ($\gamma^*$), the associated unconditional expected fraction of systemic loans, $E(x(e))$, and what that number would be if $\gamma$ were held constant at the value of 14% which is optimal in our baseline parameterization. From left to right, from top to bottom, the four panels show the effects of moving the discount rate of the patient agents ($r$), the frequency or yearly unconditional probability of the systemic shock ($\varepsilon$), the rate at which previously active bankers retire ($\psi$), and the (constant) size of the population of active bankers ($\phi$).

7 Concluding remarks

The paper describes a dynamic general equilibrium model of endogenous systemic risk-taking. The model incorporates a meaningful definition of systemic risk and allows a formal assessment of the macroprudential role of bank capital requirements. Importantly, our normative analysis is performed using an internally consistent welfare measure, based on adding up the net consumption flows of the various agents in the model.

Our results suggest that capital requirements have qualitatively and quantitatively important effects on systemic risk-taking, standard macroeconomic variables, banking indicators, and welfare. Under our calibration of the model, socially optimal capital requirements are quite high, have a sizeable negative impact on GDP, and should be gradually introduced. We also find that counter-cyclical adjustments to these requirements would be welfare decreasing and argue that, in the context of the model, bank bailouts may have negative effects on systemic risk taking and are most likely to be welfare enhancing if instrumented as transfers of wealth to the owners of the banks that remain solvent after the systemic shock.

We think of our contribution in this paper as essentially qualitative, with the current calibrations showing the suitability of the model for quantitative analysis, as well as the
potential quantitative importance of the effects. Enhancing the quantitative reliability of the implications might require integrating the key elements of the current construction (its representation of banks, bank capital dynamics, and systemic risk taking) into a fully fledged DSGE model, as well as fine tuning the calibration of the parameters in such a context.
Appendix

A Proof of Lemma 1

Bankers net payoffs from the portfolio of loans made by the non-systemic bank are, as described in problem (6), \((1 - p_0)B_t + p_0(1 - \lambda)k_t - (1 + r)d_t\). Conditional on the information available at \(t\), this payoff is deterministic since the default rate on non-systemic loans does not depend on the realization of the systemic shock (and idiosyncratic default risk is diversified away). Hence this whole expression can be taken out of the expectations operator in the first constraint in problem (6), leaving bankers’ participation constraint as follows:

\[ E(v_{t+1})[(1 - p_0)B_t + p_0(1 - \lambda)k_t - (1 + r)d_t] \geq Q_te_t. \]  

(16)

Additionally, when investing in the non-systemic bank is optimal for bankers we have \(Q_t = E(v_{t+1}R_{0t+1}) = E(v_{t+1})R_{0t+1}\) since, for what we just said, the return on equity at the non-systemic bank is not random. But then (16) can be simplified to:

\[ (1 - p_0)B_t + p_0(1 - \lambda)k_t - (1 + r)d_t \geq R_{0t+1}e_t. \]  

(17)

Under this writing, the required rate of return on equity at the non-systemic bank, \(R_{0t+1}\), and the wage rate, \(w_t\), are the sole channels through which the firm-bank decisions \((k_t, n_t, l_t, B_t, d_t, e_t)\) are affected by the state of the economy at date \(t\).\(^{50}\)

Back to the optimization problem stated in (6), it should the obvious that both (17) and the constraint associated with the minimum capital requirement will be binding. So eventually the optimization problem involves six variables and four binding constraints. These constraints can be conveniently used to make substitutions that reduce the problem to one of unconstrained optimization with just two variables: \(k_t\) and \(n_t\). Variables \(l_t\), \(d_t\), and \(e_t\) can be found recursively using the binding constraints \(l_t = k_t + w_tn_t\), \(d_t = (1 - \gamma_t)(k_t + w_tn_t)\), and \(e_t = \gamma_t(k_t + w_tn_t)\).

As for the loan repayment \(B_t\), (17) and some further substitutions yield:

\[ B_t = \frac{1}{1 - p_0}\{[(1 - \gamma_t)(1 + r) + \gamma_tR_{0t+1}](k_t + w_tn_t) - p_0(1 - \lambda)k_t\}. \]  

(18)

Intuitively, the loan repayment at \(t + 1\) must compensate the bank in expected terms for the weighted average cost \((1 - \gamma_t)(1 + r) + \gamma_tR_{0t+1}\) of the funds \(k_t + w_tn_t\) lent to the firm at date \(t\).\(^{50}\)

\(^{50}\)Having a deterministic \(R_{0t+1}\) makes the system of equations that characterize equilibrium recursive by blocks. With a non-deterministic \(R_{0t+1}\) (say, if non-systemic firms also had some exposure to systemic shocks), the model would still be solvable but at a larger computational cost.
The term \( p_0(1 - \lambda)k_t \) credits for the depreciated capital that the bank recovers if the firm fails.

Now, using (18) to substitute for \( B_t \) in the objective function of (6) gives rise to

\[
\max_{(k_t, n_t)} (1 - p_0)[AF(k_t, n_t) + (1 - \delta)k_t] + p_0(1 - \lambda)k_t - [(1 - \gamma_t)(1 + r) + \gamma_t R_0](k_t + w_t n_t),
\]

which is the reduced unconstrained maximization problem. The objective function of this problem is homogeneous of degree one in \((k, n)\) so, like in neoclassical models with this feature, obtaining finite non-zero solutions requires the value of the objective function to be zero at the optimum. The FOCs of the unconstrained problem (19) when evaluated at \( n_t = 1 \) (which is the aggregate supply of labor) uniquely determine, for each value of \( R_{0t+1} \), an equilibrium wage rate \( w_t \) and a physical capital to labor ratio \( k_t \) consistent with firm-bank optimization and labor-market clearing.\(^{51}\) Specifically, we obtain (7), which determines a \( k_t \) for each \( R_{0t+1} \), and (8), which recursively determines a \( w_t \) for each \( R_{0t+1} \) and \( k_t \).

The demand for bank capital that emanates from the above discussion is \( e_t = \gamma_t l_t \) or, equivalently, \( e_t = \gamma_t(k_t + w_t) \), as given by (9). Finally, we can obtain the expression for the loan rate that appears in (10) by using the definition \( 1 + r_{Lt} = B_i/l_t \), where \( B_i \) is found by evaluating (18) at \( n_t = 1 \) and \( l_t = k_t + w_t \).

## B Social welfare

Importantly, the patient agents who provide (insured) deposit funding to the banks at rate \( r \) just break-even in terms of their own net present value in all periods. Thus their net consumption flows as depositors make a zero net addition to social welfare \( W_t \) and we can safely leave them out in the derivations that follow. All other agents have a discount factor \( \beta < 1/(1 + r) \), which is the one that we will use to discount the remaining consumption flows, including the negative flows associated with the taxes needed to cover the costs of deposit insurance when the systemic shock realizes (and the systemic bank goes bankrupt).\(^{52}\) Focusing on the case in which bankers always reinvest 100% of their accumulated wealth as

---

\(^{51}\)The presence of firms operated in the systemic mode does not alter the aggregation implicit in this argument since they mimic the \((k, n)\) decisions associated with the lending contract of the non-systemic bank.

\(^{52}\)By assigning the cost of deposit insurance to the impatient agents, we prevent the possibility of increasing our utilitarian measure of social welfare by using deposit insurance as an indirect means of redistributing wealth from the patient agents to the impatient agents.
bank capital, social welfare at any period $t$, $W_t$, can be expressed as follows:

$$W_t = E_t \left( \sum_{s=0}^{\infty} \beta^s \omega_{t+s} \right),$$

(20)

where

$$\omega_t = -e_t + [1 - \phi(1 + \psi)]w_t + \beta\{y_{t+1} - (1 + r)[d_t - \phi(1 + \psi)w_t]\},$$

(21)

$$y_{t+1} = gdp_{t+1} + (1 - \Delta_{t+1})k_t,$$

(22)

$$gdp_{t+1} = [(1 - x_t)(1 - p_0) + x_t(1 - \varepsilon_{t+1})(1 - p_1)]AF(k_t, 1),$$

$$\Delta_{t+1} = \delta + \{(1 - x_t)p_0 + x_t[(1 - \varepsilon_{t+1})p_1 + \varepsilon_{t+1}]\}(\lambda - \delta),$$

(23)

and $\varepsilon_{t+1} \in \{0, 1\}$ indicates whether the systemic shock realizes ($\varepsilon_{t+1} = 1$) or not ($\varepsilon_{t+1} = 0$) at the end of period $t$. In this decomposition, $\omega_t$ is the present value of the net consumption flows that the impatient agents derive from the activity of the economy in the production period between dates $t$ and $t + 1$. Such activity initially absorbs bank capital $e_t$ from the bankers and pre-pays wages $w_t$ to all agents. However, some of these wages are not immediately consumed. Specifically, wages paid to the active bankers, $\phi w_t$, and to the workers who will be bankers at $t + 1$, $\phi \psi w_t$, are saved in the form of bank deposits. Finally, banks also advance the funds needed for firms to prepay physical capital at date $t$ but, since those funds are invested at date $t$, they bring about zero net consumption at date $t$.

At date $t + 1$ (which explains the discount factor $\beta$ in (21)), the impatient agents in the economy (including the taxpayers who pay, if needed, for the net costs associated with deposit insurance) appropriate (if positive) or contribute (if negative) the difference between gross output $y_{t+1}$ and the gross repayments $(1 + r)[d_t - \phi(1 + \psi)w_t]$ to the patient agents who hold bank deposits. Gross output is the sum of GDP as conventionally defined, $gdp_{t+1}$, and depreciated physical capital, $(1 - \Delta_{t+1})k_t$. The expressions for these two components, (22) and (23), respectively, make clear that the GDP and the depreciation experienced by physical capital at the end of a given period are affected both by the endogenous systemic risk-taking variable $x_t$ and the realization of the systemic shock $\varepsilon_{t+1}$.

To further understand the sources of welfare in the expression above, notice that the per-period welfare flow $\omega_t$ might be alternatively decomposed as the sum of net present value of the payoffs associated with the stakes held by each class of agents during the corresponding period:

1. Impatient agents other than active bankers and next-period bankers, who receive wages at $t$ and consume: $+[1 - \phi(1 + \psi)]w_t$. 43
2. Patient agents who act as depositors, who break even in NPV terms: +0.

3. Entrepreneurs, who in such task break even state-by-state: +0.

4. Tax payers, who pay deposit insurance costs at $t+1$: $-\beta[(1+r)d_t - (1-\lambda)k_t]x_t \varepsilon_{t+1}$.

5. Bankers who, as bank capital suppliers, contribute $e_t$ at $t$ and receive bank equity returns at $t+1$:
   
   $$-e_t + \beta[(1-x_t)R_{0t+1} + x_tR_{1t+1}]e_t.$$

6. Active and next-period bankers who, as suppliers of labor, receive at $t+1$ the proceeds from having invested the wages earned at date $t$ in bank deposits: $+\beta(1+r)\phi(1+\psi)w_t$.

Notice that this decomposition is not explicit about bankers’ net consumption. Along a full-reinvestment path bankers only consume when they convert into workers again. Their consumption flow is implicit in the above components. Specifically, the total income inflow assigned to old and new bankers at $t+1$ in the expressions above are $[(1-x_t)R_{0t+1} + x_tR_{1t+1}]e_t + (1+r)\phi(1+\psi)w_t$, while the only income outflow assigned to them at that date is the equity capital $e_{t+1}$ contributed to banks for their next period of activity. Using (11), we obtain a net income flow of $\psi\{[(1-x_t)R_{0t+1} + x_tR_{1t+1}]e_t + (1+r)\phi w_t\} > 0$ which indeed corresponds to the gross returns of the accumulated equity and the (deposited) last-period wages of the old bankers who convert into workers at date $t+1$ (which they entirely consume at that date).

All these expressions can be easily extended to the case in which, in certain periods, bankers voluntarily consume part of their wealth or keep part of it invested in deposits. All welfare computations in the quantitative part are based on the extended expressions, whose details we skip for brevity.

C Solution method

The numerical solution procedure used in order to compute the equilibrium of the model can be described as follows:

1. Create a grid $\{e_i\}$, with $i = 1, 2, ..., N$ (with some large $N$), over a range of values that includes the conjectured relevant range $[\underline{e}, \bar{e}]$ of the state variable. Parameterize a possible state-dependent capital requirement as $\gamma_i = \Gamma(e_i)$, where $\Gamma(\cdot)$ is a given function. In the baseline calibrations, we have a constant requirement $\gamma_i = \gamma$ for all $i$.  

44
2. For each point in the grid, define \((k(e_i), w(e_i), R_0(e_i))\) as the (unique) non-negative triple \((k_i, w_i, R_{0i})\) that, for the given \(e_i\), solves the following version of equilibrium conditions (7)-(9):

\[
(1 - p_0)[AF_k(k_i, 1) + (1 - \delta)] + p_0(1 - \lambda) - [(1 - \gamma_i)(1 + r) + \gamma_i R_{0i}] = 0, \\
(1 - p_0) AF_n(k_i, 1) - w_i[(1 - \gamma_i)(1 + r) + \gamma_i R_{0i}] = 0, \\
\gamma_i(k_i + w_i) - e_i = 0.
\]

3. Identify, if it exists, the point in the grid \(j\) for which \(R_{0j} \geq 1 + r\) but \(R_{0j+1} < 1 + r\).

(a) For \(i < j + 1\) set \(\hat{e}_i = e_i\).

(b) For \(i \geq j + 1\) set \(\hat{e}_i = e_j\).

[In this formulation \(e_i - \hat{e}_i\) stands for the candidate amount of bankers’ wealth invested in deposits.]

4. Set \((k_i, w_i, R_{0i}) = (k(\hat{e}_i), w(\hat{e}_i), R_0(\hat{e}_i))\) for each point \(i\) in the grid.

5. Consider the candidate \(\{v_i\} = \{v(e_i)\}\). As an initial guess for \(\{v(e_i)\}\), take some positive, non-increasing function.

6. Identify, if it exists, the point in the grid \(m\) for which \(v(e_m) \geq 1\) but \(v(e_{m+1}) < 1\).

(a) For \(i < m + 1\) set \(c_i = 0\).

(b) For \(i \geq m + 1\) set \(c_i = e_i - e_m\), and reset \((k_i, w_i, R_{0i}) = (k_m, w_m, R_{0m})\) and \(\hat{e}_i = \hat{e}_m\).

[In this formulation \(c_i\) stands for the candidate amount of bankers’ wealth that active bankers consume. This procedure takes care of having \(v(e_i) \geq 1\) for all \(e_i\).]

7. Use the following version of (12) to uniquely determine \(R_{1i}^{1-\varepsilon}\) for each \(i\):

\[
(1 - p_0)R_{1i}^{1-\varepsilon} - (1 - p_1)R_{0i} - \frac{1}{\gamma_i}(p_0 - p_1)[(1 - \gamma_i)(1 + r) - (1 - \lambda)\frac{k_i}{k_i + w_i}] = 0.
\]

8. Use the following extended version of (13) to find \(e_i^{1-\varepsilon}\) and \(e_i^\varepsilon\) for each \(i\):

\[
e_i^{1-\varepsilon} = \phi(1 + r)w_i + (1 - \psi)[(1 - x_i)R_{0i+1} + x_iR_{1i+1}]\hat{e}_i + (1 + r)(e_i - c_i - \hat{e}_i)), \\
e_i^\varepsilon = \phi(1 + r)w_i + (1 - \psi)[(1 - x_i)R_{0i+1}\hat{e}_i + (1 + r)(e_i - c_i - \hat{e}_i)].
\]
9. Use the following version of (14) and (15) to check for a solution \( x_i \in [0, 1) \) for each \( i \):

\[
[(1 - \epsilon)v(e_i^{1-\epsilon}) + \epsilon v(e_i^\epsilon)]R_{0i} - (1 - \epsilon)v(e_i^{1-\epsilon})R_{1i}^{1-\epsilon} \geq 0,
\]

\[
\{[(1 - \epsilon)v(e_i^{1-\epsilon}) + \epsilon v(e_i^\epsilon)]R_{0i} - (1 - \epsilon)v(e_i^{1-\epsilon})R_{1i}^{1-\epsilon}\}x_i = 0.
\]

If no such solution exists, set \( x_i = 1 \), reset \((k_i, w_i, R_{0i}) = (k(e_i - c_i), w(e_i - c_i), R_0(e_i - c_i))\) and check whether bankers could be interested in investing as bank capital some alternative \( \widehat{e}_i \leq e_i - c_i \), keeping \( e_i - c_i - \widehat{e}_i \) in a safe account.

[This extends the notion of equilibrium in the main text to deal with equilibria in which all loans are systemic and active bankers may not necessarily reinvest all their wealth as bank capital.]

10. Use the following version of (5) to update the value function:

\[
v(e_i) = \begin{cases} 
\psi + (1 - \psi)\beta[(1 - \epsilon)v(e_i^{1-\epsilon}) + \epsilon v(e_{i+1}^\epsilon)]R_{0i+1}, & \text{if } x_i \in [0, 1), \\
\psi + (1 - \psi)\beta(1 - \epsilon)v(e_{i+1}^{1-\epsilon})R_{1i+1}^{1-\epsilon}, & \text{if } x_i = 1.
\end{cases}
\]

11. Check convergence, i.e. the proximity between the previous \( \{v_i\} \) and the new \( \{v(e_i)\} \).

In case of convergence, save and report the solution, and finish. Otherwise, go to Step 5 and iterate again.
References


Darracq Pariès, Matthieu, Christoffer Kok Sørensen, and Diego Rodriguez Palenzuela (2010), “Macroeconomic propagation under different regulatory regimes: evidence from an estimated DSGE model for the Euro Area,” ECB WP No. 1251.

Dewatripont, Mathias, and Jean Tirole (1994), The Prudential Regulation of Banks, MIT Press.


Freixas, Xavier, and Jean-Charles Rochet (2008), Microeconomics of Banking, 2nd edition, MIT Press.


