Illiquidity, over-diversification, and macroprudential regulation*

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Abstract

We examine optimal macroprudential regulation of the liquidity and diversification choices of financial intermediaries in a model of systemic bank runs. Individually rational diversification results in systemic fragility, exposing each bank to runs on others because of an endogenous liquidation cost externality. Therefore, a bank's privately optimal portfolio features inefficient illiquidity and over-diversification. Optimal macroprudential regulation attains constrained efficiency by setting a systemic liquidity buffer, which also endogenously reduces diversification. The optimal liquidity buffer is large when liquidation costs are high or when expected investment returns are low, such as during asset-price booms or 'search-for-yield' episodes.

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1 Introduction

There has been a secular decline in liquid assets as a share of bank balance sheets over the last three decades. Figure 1 depicts the liquidity ratios of the banking systems in the United States and the United Kingdom during this period. While the average liquidity ratio for US banks was roughly constant at a level of 5–7% during the 1980s and early 1990s, it dropped to below 1% before the outbreak of the financial crisis in 2007. A similar picture arises for the UK, where the liquidity ratio was steady at a level of about 3% during the 1980s and early 1990s, dropping to a level of 1% and below in the 2000s.

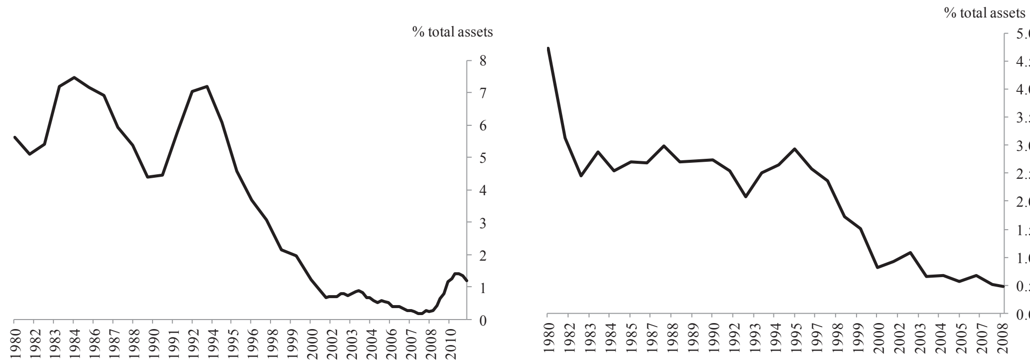


Figure 1: Liquid assets as a share of banks' balance sheet in percentage points for the US (left panel, 1980–2010) and the UK (right panel, 1980–2008). Note: The chart for the US shows obligations of the US Treasury held by FDIC-insured commercial banks as a proportion of total FDIC-insured commercial bank assets. Source: `www2.fdic.gov/hsob`, Commercial Bank reports. The chart for the UK shows the 'broad' liquidity ratio of UK banks reported in Bank of England Financial Stability Report (October, 2008), which shows cash, central bank balances, money at call, eligible bills and UK gilts held by the UK banking sector as a proportion of total UK banking sector assets. Source: `www.bankofengland.co.uk.`

Liquidity regulation plays a major role in recent proposals for financial reform. These proposals include the introduction of rules governing the composition of banks' balance sheets envisaged under the Basel Committee on Banking Supervision's proposed Liquidity Coverage Ratio (LCR) or Net Stable Funding Ratio (BCBS (2010)). Both regulatory tools seek to impose limits on the degree of liquidity mismatch on a bank's balance sheet by, for example, imposing a lower bound on banks' liquidity ratios. Liquidity regulation is also being considered for use as part of the macroprudential toolkit in the United Kingdom (Bank of England (2011)).

This paper studies optimal macroprudential regulation in an interconnected financial system subject to system-wide bank runs. To what extent would individual institutions self-insure against bank runs by holding liquidity buffers over and above those needed to meet expected withdrawals or diversify their portfolio holdings across regions? And would they do so to a socially optimal degree? If not, which type of macroprudential regulation attains constrained efficiency?

To address these questions, we construct a model in which regionally distinct banks make portfolio choices over their holdings of a liquid asset and two risky claims – a domestic asset and a foreign asset. Costly liquidation of the domestic risky asset generates strategic complementarity between the creditors of a given bank, as in a standard bank run setting (Diamond and Dybvig (1983)). But costly liquidation also means that inter-regional diversification results in strategic complementarity between creditors of distinct but interconnected banks, as the value of foreign claims depends on the liquidation decisions of foreign banks. As such, a systemic bank run can be triggered by a rational belief that creditors of other institutions will run.

Runs can be discouraged if banks allocate more of their portfolios to liquid assets, thereby reducing the required liquidation volume when facing many withdrawals. Holding additional liquidity is costly, however, as the higher expected return on risky assets is foregone. But holding claims on foreign assets can also discourage runs. In particular, holding the foreign asset has diversification benefits because asset returns are imperfectly correlated across regions. As such, holding foreign claims reduces a bank's exposure to strategic complementarities between domestic creditors' withdrawal decisions, but increases exposure to strategic complementarities between foreign creditors' withdrawal decisions. Since portfolio choices are endogenous, the extent of the exposure to strategic complementarity within and between regions is endogenously determined.

Banks' privately optimal liquidity holdings and diversification trade off these two sources of strategic risk. We show that these portfolio allocations are socially inefficient, however. While financial institutions' private choices take into account the effect of another bank's liquidation on its creditors, it does not internalise the effect of its costly liquidation on the returns of foreign creditors, and therefore foreign institutions' likelihood of experiencing a run. Therefore, private liquidity and diversification choices are socially constrained inefficient.

The constrained-efficient allocation entails banks allocating more of their portfolios to liquid assets and less to inter-regional claims, thereby raising welfare relative to laissez-faire. The larger liquidity holding under the constrained-efficient allocation offers a partial substitute for inter-regional diversification. This substitution away from inter-regional diversification has the beneficial effect of reducing the size of the channel through which financial contagion occurs.

We show that a macroprudential liquidity buffer implements the constrained-efficient allocation. The larger liquidity holding forces banks to internalise the social cost of their liquidation decisions. And banks also optimally choose lower inter-regional exposures in the presence of macroprudential liquidity regulation. In the absence of further distortions, no additional macroprudential regulation of diversification is required once banks hold the socially optimal amount of liquid assets. Further, a macroprudential cap on diversification alone would not attain the constrained-efficient allocation.

How does the optimal liquidity buffer vary with the parameters of the economy? We show that larger liquidation costs result in a larger macro-prudential liquidity buffer. Intuitively, larger liquidation costs exacerbate the strategic complementarities between creditors, and therefore amplify the channel through which the liquidation cost externality operates. We also show that there are two reasons for the optimal liquidity buffer to be high when investment returns are low, such as in *search-for-yield* episodes - akin to the compression of spreads seen in the run up to the recent crisis. First, a systemic bank run is more likely when the expected returns on banks' investments are low. Second, the opportunity cost of higher liquidity holdings is low when alternative investments yield low payoffs. This establishes a *leaning against the wind* property of optimal macroprudential liquidity regulation, as the efficient liquidity buffer is high in low-return conditions, which typically accompany cyclical upswings (Fama and French (1989), Campbell and Diebold (2009)).

Variation in the optimal liquidity buffer entails variation in the optimal degree of inter-regional diversification. For example, as liquidity requirements tighten, the extent of inter-regional diversification rises. This mitigates the cost of tighter liquidity standards, despite the fact that optimal regulation always imposes a lower level of interconnectedness relative to laissez-faire.

Our model builds on the literature that uses global games techniques to study bank runs (Goldstein and Pauzner (2005), Morris and Shin (2001)). As in their models, there is coordination failure among bank creditors that leads to inefficient bank runs, and these runs are initiated by adverse news about the bank's profitability. In contrast to these papers, we also study the bank's optimal portfolio choice, its welfare implications, and optimal regulation of these portfolios. While Wagner (2010) and Wagner (2011) also consider the systemic implications of portfolio choices of banks, we explicitly analyse the coordination problem of bank creditors. In contrast to these papers, we derive the implications for optimal regulation in this context.

Our setup nests the Goldstein (2005) who studies twin crises in a model of reinforcing strategic complementarities between bank creditors and currency speculators. In contrast to that model, the strategic complementarities within and between groups are endogenous in our model: the incidence and exposure to strategic complementarity arises from banks' portfolio choices. As such, banks optimally trade off the strategic complementarities between their own creditors with strategic compelemntarities between the creditors of other banks. Our analysis shows how the privately optimal trade-off differs from the socially efficient one, and studies optimal regulation designed to attain the constrained efficient trade-off between these forms of strategic complementarity through its influence on bank portfolios.

Contagion occurs in Goldstein and Pauzner (2004) because of a wealth effect on investors which is assumed to be identical across regions. By contrast, we analyse the strategic interaction of bank creditors and focus on the efficient regulation of intermediaries' portfolio choices. As our agents are risk neutral and are allowed to differ across regions, our paper provides a complementary explanation for the potential undesirability of full diversification. Similarly, Dasgupta (2004) features strategic complementarities within groups of bank creditors of banks that share liquidity risk via an interbank market. By contrast, we allow for strategic interaction between creditors of different banks, focussing on asset risk diversification and optimal regulation.

Our policy implications complement a small but growing literature on macroprudential interventions. Pecuniary externalities due to fire sales in the presence of incomplete markets opens the door for a social planner to make a welfare-improving intervention by imposing capital requirements (Lorenzoni (2008)), taxing debt (Bianchi (2011)) or taxing risk taking (Korinek (2011)). The mechanism at work in our study is complementary but distinct from that operating in these papers, and the intervention we study is on the asset-side (liquidity) rather than the liability side (bank capital). In Bianchi (2011), for example, agents subject to borrowing constraints do not internalise the effect their leverage decisions have on others' ability to maintain leverage in the event of adverse shocks that trigger fire sales. This can motivate leverage restrictions *ex-ante*. On liquidity requirements, Morris and Shin (2008) argue that these may be a necessary complement to proposals for reform focussed on capital, or leverage, requirements.[1] For a comprehensive survey of these and other macroprudential interventions, see Bank of England (2011).

This remainder of this paper is as follows. Section 2 describes the economy and section 3 characterises its unique equilibrium. Section 4 and 5 analyse the constrained planner's problem, establish the optimality of a macro-prudential liquidity buffer, and study its comparative statics. Finally, section 6 discusses our results and section 7 concludes. Most derivations and all proofs are relegated to the Appendix in section 8.

2 The Model

The economy extends over three dates $t = 0, 1, 2$ and consists of two regions $k = A, B$. Our notion of a region is generic, comprising a particular sector, a country within an economic union, or a bank in a banking system. There is a bank and a continuum of creditors in each region. Our notion of creditors is also broad: it is not limited to the traditional case of retail creditors and commercial banks but incorporates, for instance, money market funds (creditors) and investment banks (banks).[2] A single good is used for consumption and investment.

2.1 Investment opportunities

Table 1 summarises the two types of investment opportunity available in each region at the initial date. First, storage yields a unit safe return. Second, a long term regional investment project, such as loans to a productive sector, matures after two periods and yields a stochastic return $\mathbf{r}_k$ with mean $\bar{r}$. Similar to Morris and Shin (2001), premature liquidation $l_k \in [0, 1]$ of

[1] Castiglionesi et al. (2010) study the effects of financial integration, in the form of interbank borrowing, on banks' incentives to hold liquid reserves. If integration reduces self-insurance, aggregate liquidity shocks can have more severe effects, since the total level of liquid resources in the banking system is lower. The authors note the possibility of welfare-improving regulation in their model.

[2] This model's creditors and banks may also be re-interpreted as local and global banks in the spirit of Uhlig (2010). Then, a prematurely withdrawing creditor represents a run of one (local) bank on another (global) bank, an arguably reasonable feature of the recent financial crises.

Asset	$t = 0$	$t = 1$	$t = 2$
Storage ($0 \to 1$)	-1	1	
Storage ($1 \to 2$)		-1	1
Project ($0 \to 2$)	-1	l_k	$r_k - (1+\chi)l_k$

Table 1: Returns from investment opportunities

the invested resources at par at the interim date results in the reduction of the final-date return to $r_k - (1+\chi)l_k$. The cost of premature liquidation $\chi > 0$ is the source of strategic interaction between late creditors within a given region. To avoid strict dominance of either investment opportunity, we assume $1 < \bar{r} < 1 + (1+\chi)$. Our assumptions about the mean return and the liquidation cost can be summarized:

Assumption 1.

$$-\chi < 0 < \bar{r} - 1 < 1 + \chi \tag{1}$$

Costly liquidation is a key but well-motivated feature of our model. As discussed in James (1991) and Mullins and Pyle (1994), in practice these costs comprise both direct liquidation expenses together with reductions in the 'going concern' value of bank assets under distress. The empirical literature typically finds these liquidation costs to be large: of the order of 30% of bank assets on average.[3]

2.2 Information structure

Building on the global games framework pioneered by Carlsson and van Damme (1993), we extend the notion of uncertainty about investment returns to a multi-regional setup.[4] All distributions are common knowledge. Bold variables denote random variables. The stochastic economy-wide investment return $\boldsymbol{r}$, such as a key macroeconomic variable common to both regions, is distributed according to:

$$\boldsymbol{r} \sim \mathcal{N}\left(\bar{r}, \frac{1}{\alpha}\right) \tag{2}$$

[3] Mullins and Pyle (1994) and Brown and Epstein (1992) estimates of direct liquidation expenses of around 10%, varying between 17% for assets relating to owned real estate, down to 0% for liquid securities for assets in receivership at the FDIC. Adding to direct expenses losses associated with forced liquidation, James (1991) gives an average total cost of 30% of a failed bank's assets. Similar orders of magnitude are reported in Bennett and Unal (2011) whose sample runs for much longer, covering 1986-2007.

[4] See Morris and Shin (2001) and Goldstein and Pauzner (2005) for setups with one region.

where $\bar{r}$ is the mean of the economy-wide return, $\alpha \in (0, \infty)$ its precision, and r its realisation. The returns to the regional investment project consist of the economy-wide investment return as a common component and a regional disturbance $\boldsymbol{\eta}_k$:

$$\boldsymbol{r}_k \equiv \boldsymbol{r} + \boldsymbol{\eta}_k, \tag{3}$$

$$\boldsymbol{\eta}_k \sim \mathcal{N}\left(0, \frac{1}{\beta}\right) \tag{4}$$

where the regional shock $\boldsymbol{\eta}_k$ has precision $\beta \in (0, \infty)$, is identically and independently distributed across regions and independent of the economy-wide investment return. Regional noise adds a second layer of uncertainty relative to the setting of Morris and Shin (2001). Creditors use of the information structure to infer that regional returns are distributed according to:

$$\boldsymbol{r}_k \sim \mathcal{N}\left(\bar{r}, \frac{1}{\delta}\right) \tag{5}$$

where $\delta \equiv \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^{-1}$ is the precision of the public signal of the regional return. The correlation between both investment project returns is $corr(\boldsymbol{r}_A, \boldsymbol{r}_B) = \frac{1}{1+\alpha/\beta} \in (0, 1)$. Given the imperfect positive correlation, there is a diversification opportunity for banks.

At the interim date the investment project returns are realized but not publicly observed. Each creditor receives a private signal x_{ik} about his regional return:

$$\boldsymbol{x}_{ik} \equiv \boldsymbol{r}_k + \boldsymbol{\epsilon}_{ik}, \tag{6}$$

$$\boldsymbol{\epsilon}_{ik} \sim \mathcal{N}\left(0, \frac{1}{\gamma}\right), \tag{7}$$

where the idiosyncratic noise $\boldsymbol{\epsilon}_{ik}$ has precision $\gamma \in (0, \infty)$ and is independently and identically distributed across creditors, and independent of the regional noise and the economy-wide return. Idiosyncratic noise can be thought of as slightly different interpretations of some public information about the regional investment profitability.

2.3 Creditors

Each region has ex-ante identical creditors of mass $1 + \lambda$ that face idiosyncratic liquidity risk (Diamond and Dybvig (1983)). A creditor wishes to consume either at the interim date ('early')

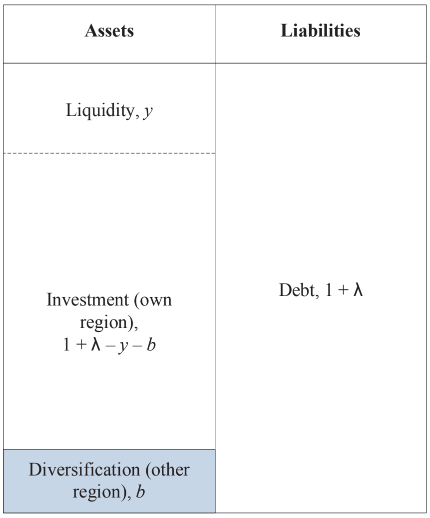


Figure 2: A bank's balance sheet at the initial date

or at the final date ('late'). The ex-ante probability of being early is $\lambda/(1+\lambda) \in (0,1)$ and is identical across creditors. By the law of large numbers, this is also the share of early creditors in each region. Creditors are uncertain about their consumption needs at the initial date, but learn their preference privately at the beginning of the interim date. A creditor's utility function is:

$$U(c_1, c_2) = \begin{cases} c_1 & \text{w.p. } \frac{\lambda}{1+\lambda} \\ c_2 & \text{w.p. } \frac{1}{1+\lambda} \end{cases}, \tag{8}$$

where c_t is consumption at date t. Creditors are endowed with one unit at the initial date only, implying an aggregate endowment of $1+\lambda$, to be stored or deposited.

2.4 Banks

Banks specify their investment plans at the initial date. As depicted in figure 2, part of the raised deposits is stored $y_k \in [0, 1+\lambda]$, which we refer to as *liquidity*, and the remainder $I_k \equiv 1 + \lambda - y_k$ is invested in long term projects. Banks also *diversify* their portfolios by holding some of the long-term investment in the other region's project $b_k \in [0, I_k]$, which could represent either syndicated loans or outright purchases.[5] Diversification is beneficial as it reduces the volatility of a bank's portfolio. This mitigates the coordination problem between a bank's creditors, thereby lowering the probability of a bank run. The portfolio choices of banks are publicly observed.

[5]See Sufi (2007) for an empirical work on syndicated loans.

There is free entry to the banking sector such that a bank maximizes creditors' expected utility. If it did not, another bank would emerge and offer a better investment plan, attracting all deposits. At the final date a bank shares the proceeds from its investment equally among the remaining creditors. Given the alignment of interest between the bank and its creditors as well as the bank's enhanced access to projects, all creditors deposit in full.[6]

Throughout our analysis, we focus on demand deposit contracts that allow regional creditors to withdraw at par at the interim date.[7] While we acknowledge that this demand deposit contract is suboptimal in our setup, there are three reasons for maintaining this assumption. First, we could generalise our results to risk averse creditors such that the considered contract can be optimal. However, this would not substantively change our main conclusions, while complicating the analysis substantially. Second, the empirical evidence strongly supports this contract as an empirical regularity. Third, Diamond and Rajan (2001) show that this contract is second-best optimal in a setup with limited commitment. Our welfare analysis and prescriptions for macroprudential regulation are conditional on the specified demand deposit contract and the bank run risk this creates.

As in Allen and Gale (2000), the bank has an optimal pecking order when serving withdrawals at the interim date, using liquidity first. Let $w_k \in [0,1]$ denote the proportion of late creditors in region k that withdraw at the interim date. If withdrawals from late creditors are sufficiently high ($w_k > y_k - \lambda$), the bank partially liquidates its investment projects. It starts with its own region's project, which is more easily or faster liquidated.[8] A generalisation of of this pecking order is discussed in section 6.

Liquidity is useful in preventing costly liquidation (precautionary motive). It drives a wedge between the proportion of late creditors that withdraw w_k and the proportion of the investment project that is liquidated l_k. Therefore, a bank may hold excess liquidity ($y_k > \lambda$), that is more cash than needed to service withdrawals from early creditors.

The following time line summarizes the model description:

[6] While Diamond and Dybvig (1983) demonstrate a role for the bank as provider of liquidity insurance in case of risk-averse creditors, banking arises in our model from the bank's enhanced access to investment projects. This common assumption can be motivated as an equilibrium outcome in a perturbed setting with moral hazard and monitoring. Delegation is optimal as long as banks are better at monitoring or can obtain monitoring technology more cheaply.

[7] See also Dasgupta (2004) and a similar assumption in Goldstein (2005).

[8] For syndicated loans, the lead bank may decide on liquidation such that the liquidation decision is forfeited. See also Sufi (2007).

Initial date $t = 0$

- Creditors receive endowment.
- Banks offer an investment plan to regional creditors.
- Creditors deposit or store their endowment.
- Banks make their portfolio choices by storing an amount y_k and investing in long term projects I_k, part of which in the other region b_k. Portfolio choices are publicly observed.

Interim date $t = 1$

- Creditors privately observe their consumption preferences (early or late).
- Each creditor receives a private signal x_{ik} about the regional investment profitability.
- Creditors may withdraw.
- Banks service withdrawals, potentially by liquidating investment projects.
- Early creditors consume. Late creditors store their withdrawal.

Final date $t = 2$

- The investment project matures.
- Banks service withdrawals from remaining creditors.
- Late creditors consume.

2.5 Payoffs

We are now ready to determine a creditor's payoff. An early creditor always withdraws at the interim date and receives unity. An early creditor's payoff is unaffected by other creditors' withdrawal decisions. This follows from viability of banks at the interim date, guaranteed by the fact that the promised payment does not exceed the unit liquidation value. A late creditor who withdraws at the interim date receives the same payoff since the bank does not observe

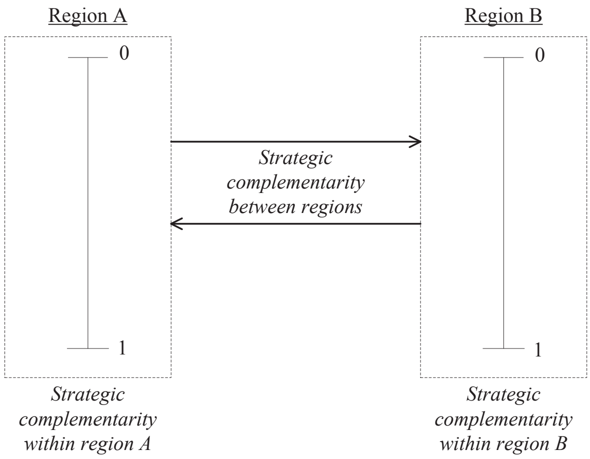


Figure 3: Interaction of two forms of strategic complementarities

creditors' types. A late creditor who keeps the funds at the bank receives an equal share of the bank's final-date assets A_{2k}:

$$c_{2k} = \frac{\overbrace{\max\{0, y_k - \lambda - w_k\}}^{\text{remaining liquidity}} + \overbrace{(I_k - b_k)[r_k - (1+\chi)l_k]}^{\text{investment proceeds}} + \overbrace{b_k[r_{-k} - (1+\chi)l_{-k}]}^{\text{proceeds from diversification}}}{1 - w_k} \tag{9}$$

where $l_k \equiv \max\{0, w_k + \lambda - y_k\}$ is the total amount of liquidation by bank k, and where we use subscript $-k$ to denote the other region ('not k') throughout. It decreases in the amount of liquidity y_k chosen by that region's bank at the initial date and increases in the proportion of prematurely withdrawing late creditors w_k.

There are two dimensions to the strategic behaviour of a late creditor as depicted in figure (3). The first dimension is strategic interaction between the withdrawal decisions of late creditors in a given region. More withdrawals by other late creditors in the same region have two effects. First, the bank draws down its excess liquidity and then liquidates a larger share of its investment in the long term projects. This effect is favourable for those late creditors who withdraw early. Second, there are fewer late creditors to share the remaining resources with at the final date. This effect is detrimental to those late creditors who withdraw early. It is straightforward to derive the partial derivative of the final-date payoff for a late creditor with respect to the share of prematurely withdrawing late creditors in his region, $\frac{\partial d_{2k}}{\partial w_k}$. Under the condition that the liquidation cost is sufficiently high ($\chi > \underline{\chi}$), there is strategic complementarity between depositors of a given region, where $(1 + \underline{\chi})(I_k - b_k) \equiv 1$ (see Appendix 8.3).

This condition is consistent with assumption 1 and states that the cost of liquidating the long term investment project exceeds unity.

The second dimension is a strategic complementarity between the withdrawal decisions of late creditors across regions that arises from diversification ($b_k > 0$). The more late creditors in region $-k$ withdraw, the more of region $-k$'s investment project is liquidated (once excess liquidity is exhausted), reducing the final-date return to region $-k$'s investment project. Because of diversification and the liquidation cost externality, the bank in region k is also affected and its final-date asset value is reduced. This increases the incentive for a late creditor in region k to withdraw as well. See Appendix (8.4) for a proof.

The combination of regional cross-holdings and liquidation costs generate the possibility of contagion across regions in our model. The liquidation cost externality captures the idea that the liquidation of local projects by local banks has an adverse effect on the value of others' claims on nearby projects. Imagine each regional bank manages a local orchard, and that banks share cross-holdings on the produce from each other's harvest. If a local bank has to meet interim withdrawals, it does so by harvesting fruit early, damaging the trees in its plot. The fruit that remains to be harvested at the final date, of which some is pledged to foreigners, is damaged in the process, so final-date returns are impaired.

A more concrete example of this phenomenon in practice is given in Campbell et al. (2011). Suppose the assets held by banks are loans backed by claims on real estate. In the context of the housing market, Campbell *et al* show, first, that forced sales following bank foreclosures take place at a discount of up to 27 per cent, a measure of the liquidation cost parameter χ in our model.[9] Second, the authors show that, at a local level, these foreclosures also impose negative valuation spillovers on the prices of surrounding houses. This means the foreclosure decision of one bank imposes a negative externality on the holders of other claims on the surrounding housing stock, which has a direct parallel with our simple model. Campbell *et al* argue that the negative spillovers associated with liquidation reflect physical damage to neighbourhoods in the housing context.

A final interpretation of our liquidation cost externality relates to the concept of market liquidity, in the context in which projects can be used as collateral. The haircuts associated with this collateral depend on market conditions, and larger haircuts are applied in times of

[9] This is of the order of the estimate of James (1991), cited earlier.

stress (eg Brunnermeier and Pedersen (2009)). In our case, a run on a domestic bank in the interim period triggers a reduction in the pledgeable returns due to final-date creditors. This is the flip side of there being a rise in haircuts following a run. This is detrimental to foreign banks because the rise in haircuts cuts into the returns that can be pledged to them at the final date.

As such, there is a system-wide effect stemming from the negative externality of one bank's liquidation decision on the other bank's portfolio value. This externality motivates policy intervention in our setup. Indeed, one can think of the (constrained) planner as a *macro*-prudential authority, as the necessary policy intervention is based on an economy-wide or across-the-system externality.

3 Equilibrium

We define and solve for the equilibrium in our two-stage game. At the interim date, there is an imperfect-information sub-game between creditors. Upon receiving a private signal, each creditor updates his beliefs about the investment returns and the proportion of withdrawing late creditors in both regions. The posterior distributions are derived in Appendix (8.1) and denoted by capital letters:

$$\boldsymbol{R^k_{i,k}} \equiv r_k|x_{ik} \tag{10}$$

$$\boldsymbol{R^k_{i,-k}} \equiv r_{-k}|x_{ik} \tag{11}$$

$$W^k_{i,k} \equiv w_k|x_{ik} \tag{12}$$

$$W^k_{i,-k} \equiv w_{-k}|x_{ik} \tag{13}$$

where the posterior distribution in the subscripted region is viewed by a creditor in the superscripted region.

We determine the Bayesian Nash equilibrium of the sub-game at the interim date. Each creditor decides whether or not to withdraw his funds from the bank based on his posterior belief about returns. A *strategy* is a plan of action of late creditor i in region k for each private signal x_{ik}. A *profile of strategies* is a Bayesian Nash equilibrium in the sub-game if the actions described by creditor i's strategy maximize his expected utility conditional on the information available and given the strategies followed by all other creditors. We will prove the uniqueness of a Bayesian Nash equilibrium in *threshold strategies*. Late creditor i in region k withdraws if and only if his signal falls short of a regional threshold ($x_{ik} < x_k^*$).

At the initial date, there is a perfect-information game in which banks choose their portfolios, taking the choice of the other bank as given (Nash equilibrium). Solving backwards, a bank takes into account the effect of its portfolio choice at the initial date on the withdrawal threshold at the interim date. Because of free entry, the bank chooses the Bayesian Nash equilibrium in the sub-game that is best for its creditors.[10]

To obtain intuition, we first consider exogenous amounts of diversification ($0 < b_k < I_k$), while studying endogenous diversification in section (5).

3.1 Withdrawal subgame at the interim date

We now study the equilibrium withdrawal behaviour of creditors in the sub-game. Early creditors always withdraw. Strategic complementarity in withdrawal decisions, both within and between regions, arises from the strategic withdrawal decision of late creditors. The threshold posterior mean R_k^*, equivalent to the threshold signal x_k^*, is defined as the posterior mean in region k that makes a late creditor indifferent between withdrawal and continuation and implicitly given by:

$$1 = c_{2k}(R_k^*, R_{-k}^*) \tag{14}$$

where the left-hand side is the payoff from withdrawing and the right-hand side is the expected payoff from not withdrawing conditional on receiving the threshold signal. Taking the other region's withdrawal threshold R_{-k}^* as given, equation 14 implicitly defines the best response function $R_k^*(R_{-k}^*)$. Each region's withdrawal threshold depends on the other's, owing to the effect of expected withdrawals in the other region on final period payoffs. The equilibrium of the sub-game is obtained as the intersection of the best-response functions and is summarised by the following lemma that states its existence, uniqueness, and symmetry:

Lemma 1. *Suppose that the idiosyncratic noise is sufficiently small and the regional noise is sufficiently large ($\gamma > \underline{\gamma} < \infty$, $\delta < \overline{\delta} \in (0, \infty)$). Then there exists a unique and symmetric*

[10]This is similar to Gale (2010), in which the bank's optimal behaviour under free entry and subject to the creditors participation constraint can be expressed as the solution to a contracting problem in which the welfare of creditors is maximised subject to the zero-profit constraint of the bank.

equilibrium threshold $R_k^* = R^*(y_k, y_{-k})$ *in the sub-game, solving:*

$$\begin{aligned} R_k^* &= 1 + \frac{(1+\chi)(I_k - b_k) - 1}{I_k - (1-\kappa)b_k}[W_k^k(R_k^*) + \lambda - y_k] \\ &\quad - \frac{(1-\kappa)(\bar{r}-1) - (1+\chi)[W_{-k}^k(R_k^*, R_{-k}^*) + \lambda - y_{-k}]}{I_k - (1-\kappa)b_k} b_k \end{aligned} \tag{15}$$

where κ *is a constant that collects precision parameters, defined in the Appendix.*

Proof. See Appendix (8.5). □

Note that a sufficiently precise private signal relative to the public signal is a standard condition for a unique (global game) equilibrium (eg Morris and Shin (2001), Goldstein and Pauzner (2005)). A low precision of the regional investment return can be attained by a low precision of the global investment return and of the regional noise.

We now turn to the efficiency properties of the equilibrium threshold R_k^* in the sub-game, defined in equation 15. The threshold can be grouped into three terms. First, the efficient level for the withdrawal threshold is unity, or $R^{eff} = 1$. Second, the coefficient on the posterior liquidation share $L_k^* \equiv W_k^k(R_k^*) + \lambda - y_k$ is positive if and only if there is strategic complementarity within a region's creditors, or $\chi > \underline{\chi}$ (see also Appendix 8.3). Third, there is a term associated with diversification. Thus, there are efficient bank runs in the absence of diversification and strategic interaction ($b_k = 0, \chi = \underline{\chi}$). Absent any diversification ($b_k = 0$), strategic complementarities in the withdrawal decision of late creditors ($\chi > \underline{\chi}$) result in co-ordination failure between creditors and a withdrawal threshold above the efficient level. In other words, a late creditor withdraws even though his posterior over mean returns exceeds unity, such that there is an inefficient bank run, similar to Morris and Shin (2001).

Finally, we establish an upper bound and lower bound on the equilibrium withdrawal threshold. Each bound can be derived in two steps. First, equation 15 defining R_k^* implies that neither $R_k^* = \bar{r}$ nor $R_k^* = 1$ are equilibria under the conditions required for uniqueness in the sub-game. The left-hand side is too high and too low, respectively, in these cases. Second, the conditions for equilibrium uniqueness imply that the slope of the left-hand side of 15 exceeds the slope of the right-hand side of 15, such that the equilibrium level R_k^* must lie above unity and below $\bar{r}$. Further, given the conditions sufficient for a unique equilibrium, the upper bound also falls short of $1 + \chi$. This establishes the presence of inefficient bank runs, summarised in the following corollary:

Corollary 1. *The equilibrium withdrawal threshold always exceeds unity, but never reaches either the total cost of liquidation or the expected investment return, so* $1 < R_k^* < \min\{\bar{r}, 1+\chi\}$.

3.2 Comparative statics on the withdrawal threshold R_k^*

Next we consider how changes in the parameters of the economy affect the likelihood of a bank run. Of primary interest are changes in expected investment returns $\bar{r}$ and liquidation costs χ. But since, in the sequel, banks take into account the effect of their portfolio choices on the withdrawal threshold, and therefore the likelihood of a run, we also state the effects of changes in the levels of liquidity (both y_k and y_{-k}) and the amount of diversification b_k on the withdrawal threshold. We show:

Lemma 2. *The withdrawal threshold* R_k^* *varies according to*

(a) $\frac{\partial R_k^*}{\partial \chi} > 0$*: the withdrawal threshold is high when liquidation costs are high.*

(b) $\frac{\partial R_k^*}{\partial \bar{r}} < 0$*: the withdrawal threshold is low when the expected investment return is high.*

(c) $\frac{\partial R_k^*}{\partial y_k} < 0$*: the withdrawal threshold is low when own-region liquidity holding is large.*

(d) $\frac{\partial R_k^*}{\partial y_{-k}} < 0$*: the withdrawal threshold is low when the other-region liquidity holding is large.*

(e) $\frac{\partial R_k^*}{\partial b_k} < 0$*: the withdrawal threshold is low when the own-region bank is more diversified.*

Proof. See Appendix (8.6). □

The intuition for these results is as follows. High liquidation costs imply a low value of a late creditor's funds at the final date when many other late creditors withdraw early. Thus, a late creditor finds it optimal to withdraw his funds for a larger range of posterior means. Higher liquidation costs are therefore associated with intensified co-ordination failure between late creditors. By contrast, high expected returns on the investment project makes it worthwhile abandoning the investment project only if private signals are sufficiently bad. Hence, early withdrawal happens for a smaller range of posterior means.

The composition of a bank's portfolio will also affect the withdrawal threshold. A high level of liquidity has two implications. First, more own-region liquidity makes liquidation of the risky project less likely, tending to make bank runs less likely. Second, with higher liquidity, high returns are foregone as otherwise high-yielding assets are overlooked in favour of low-yielding liquidity (the opportunity cost is R_k^* in equilibrium). At low levels of liquidity, such as around the unique equilibrium with positive liquidation, the first effect dominates, such that higher liquidity reduces the incidence of bank runs.

Similarly, a larger degree of inter-region diversification reduces the variance of an individual bank's portfolio and thus the individual probability of a bank run. While exposing late creditors to strategic complementarity with the other region, the strategic complementarity within the region is alleviated by more, as implied by the lower precision of the information about the other region relative to the own region. Premature withdrawals then occur for a smaller range of posterior means. As such, inter-regional diversification is similar to *co-insurance* as in Brusco and Castiglionesi (2007), and reduces the individual probability of bank failure.

Finally, a high level of liquidity held by the foreign bank prevents costly liquidation in that region for a larger range of posterior means. This benefits not only creditors in the foreign region, but also creditors in the domestic region who hold a stake in the other region's investment project through inter-region diversification. This constitutes a positive externality associated with holding liquidity: greater liquidity holdings in one region reduce the withdrawal threshold in the other region, motivating macro-prudential policy. The externality is the stronger the larger is the amount of inter-regional diversification and the larger are liquidation costs.

3.3 Optimal portfolio choice

We turn next to the characterisation of banks' portfolio decisions. Each bank's objective function is the expected utility of its creditors, derived in Appendix (8.7), which can be written as:

$$EU_k = y_k + I_k \bar{r} - (1+\chi) b_k \int_{-\infty}^{\infty} \Phi_{-k}(\cdot) dF(r) - (I_k - b_k) \int_{-\infty}^{\infty} (r-1)\Phi_k(\cdot) - \frac{\phi_k(\cdot)}{\beta} dF(r) \quad (16)$$

where $F(r)$ is the cumulative density of the economy-wide investment return r, and $\Phi_k(\cdot) \equiv \Phi\left(\sqrt{\beta}[R_k^* - r]\right)$ is the probability of a bank run in region k for a given investment return r. Note that the probability of a bank run decreases in the investment return r ($\partial \Phi_k(\cdot)/\partial r < 0$).

The expression for expected utility consists of three terms. The first term is the expected return in the absence of bank runs: liquidity yields unity and the investment project yields $\bar{r}$ in expectation. The second term reflects the negative externality arising from a bank run in the other region, which happens with probabilty $\Phi_{-k}(\cdot)$. The third term measures the loss from a bank run in the creditor's region. The measure of exposed assets is $I_k - b_k$, a run occurs with probability $\Phi_k(\cdot)$, and the unit loss given default is $[1 + \frac{\phi_k(\cdot)}{\beta\Phi_k(\cdot)} - r]$ for any given investment return r.

We can simplify the expression for expected utility by using the conditions sufficient for delivering uniqueness in the co-ordination game. In particular, uniqueness relies on a sufficiently small value of $\delta = \alpha\beta/(\alpha + \beta)$, such that there are two degrees of freedom (α, β) in ensuring uniqueness. For simplicity, we normalise $\beta \equiv 1$ and pick α such that the sufficient condition on the precision of the regional signal δ holds. Expected utility in region k then becomes:

$$EU_k = y_k + I_k\bar{r} - (I_k - b_k)\int_{-\infty}^{\infty}(r-1)\Phi(R_k^* - r) - \phi(R_k^* - r)dF(r) - b_k(1+\chi)\int_{-\infty}^{\infty}\Phi(R_{-k}^* - r)dF(r) \tag{17}$$

A higher withdrawal threshold implies a larger area of inefficient runs, and therefore lowers the expected utility, $\partial EU_k/\partial R_k^* < 0$ (see Appendix 8.7). Therefore, the positive externality from holding liquidity, or the equivalent negative externality from liquidation, shows up in expected utility, since $\frac{dEU_k}{dy_{-k}^*} = \frac{\partial EU_k}{\partial R_k^*}\frac{\partial R_k^*}{\partial y_{-k}} > 0$. This externality is stronger the larger are liquidation costs and the more diversified the bank is. Set against this is the negative direct effect of holding liquidity, which arises because the opportunity cost of liquidity, measured by the expected return on the investment project, exceeds the return to liquidity. As such, $\partial EU_k/\partial y_k < 0$.

The following then characterises the bank's privately optimal portfolio choice:

Definition 1. *The privately optimal equilibrium level of liquidity y_k^* solves the bank's portfolio choice problem at the initial date, taking creditors' responses at the interim date into account, and taking the other bank's level of liquidity y_{-k} as given:*

$$y_k^* \equiv \arg\max_{y_k} EU_k \;\; s.t. \;\; R_k^* = R^*(y_k, y_{-k}) \tag{18}$$

The competitive bank's first-order condition yields:

$$0 = \frac{\partial EU_k}{\partial y_k} + \frac{\partial EU_k}{\partial R_k^*}\frac{\partial R_k^*}{\partial y_k} \tag{19}$$

This expression balances the (direct) private cost of holding liquidity in terms of foregone investment return ($\partial EU_k/\partial y_k < 0$) with the (indirect) private benefits from holding liquidity in terms of a lower withdrawal threshold ($\partial R_k^*/\partial y_k < 0$). The equilibrium liquidity holding is symmetric ($y_k^* = y^*$) and is the unique global maximizer of the bank's objective function. Uniqueness here arises from the global concavity of the objective function, which stems from the effect of liquidity holdings on the withdrawal threshold, as established in Appendix (8.8).

4 Welfare

This section compares the bank's optimal portfolio choice with that of a social planner. Given the system-wide externality, the planner can be thought of as the macro-prudential authority. As in Lorenzoni (2008), we adopt the notion of constrained efficiency: the (constrained) planner takes the demand-deposit contract and the withdrawal decision of depositors at the interim date as given. In contrast to a bank, the planner internalizes the beneficial effects of holding liquidity on the creditors of the *other* bank.

Definition 2. *The socially efficient levels of liquidity (y_A^{SP}, y_B^{SP}) solve the planner's portfolio choice problem at the initial date, taking creditors' responses at the interim date into account.*

$$(y_A^{SP}, y_B^{SP}) \equiv \arg\max_{y_A, y_B} EU_A + EU_B \ \ s.t. \ \ R_k^* = R^*(y_k, y_{-k})$$

The first-order conditions for the social planner's problem are ($k \in \{A, B\}$):

$$0 = \frac{\partial EU_k}{\partial y_k} + \frac{\partial EU_k}{\partial R_k^*}\frac{\partial R_k^*}{\partial y_k} + \frac{\partial EU_{-k}}{\partial R_{-k}^*}\frac{\partial R_{-k}^*}{\partial y_k} \tag{20}$$

The planner balances the (direct) social cost of holding liquidity in terms of foregone investment return ($\partial EU_k/\partial y_k < 0$) with the (indirect) social benefits from holding liquidity in terms of a lower withdrawal threshold. There are now two components to the social benefits from holding liquidity. Along with the beneficial effect of holding liquidity on the 'domestic' creditor ($\partial R_k^*/\partial y_k < 0$), the macroprudential planner accounts for the beneficial effect of holding liquidity on the other region's creditors ($\partial R_{-k}^*/\partial y_k < 0$). While the private and social costs of holding liquidity coincide, the social benefit from holding liquidity exceeds the private benefit.

The global concavity of the social welfare function $EU_A + EU_B$ is established in Appendix (8.8) such that the planner's allocation is the unique global maximiser of the social welfare function. Symmetry is again obtained ($y_k^{SP} = y^{SP}$). Comparing the optimal and the efficient levels of liquidity yields our first main result:

Proposition 1. *The constrained social planner holds more liquidity than the bank. Thus, the planner imposes a macro-prudential liquidity buffer $y^{SP} - y^* > 0$.*

See Appendix 8.9 for a proof. As the constrained planner takes into account all economy-wide effects, it imposes a macro-prudential liquidity requirement in order to internalise the social costs of liquidation that arise in an interlinked financial system.

We now study the equilibrium allocation and the planner's allocation vary with the exogenous parameters of the model, summarized by the following proposition:

Proposition 2. *The privately optimal and socially efficient levels of liquidity vary according to*

(a) $\partial y^{SP}/\partial \chi > \partial y^*/\partial \chi > 0$, such that higher liquidation costs, which generate more costly distress, raise the liquidity levels as chosen privately and socially;

(b) $\partial y^{SP}/\partial b_k < \partial y^*/\partial b_k < 0$, such that more diversification by banks requires fewer liquidity holdings as chosen privately and socially;

(c) $\partial y^{SP}/\partial \bar{r} < \partial y^*/\partial \bar{r} < 0$, such that higher investment returns (low asset prices) require lower private and social levels of liquidity.

See Appendix (8.10) for a proof. Corollary 2 follows directly:

Corollary 2. *The macro-prudential liquidity buffer $y^{SP} - y^*$ rises as*

(a) there are higher liquidation costs, χ;

(b) there is a lower amount of diversification, b_k;

(c) there is a lower expected investment return, $\bar{r}$.

The mechanisms underlying these results are as follows. First, the strength of the externality is captured by the liquidation cost parameter χ. Thus, when the costs of financial distress are higher, both the bank and the social planner hold more liquidity since individual and systemic runs are more likely (Lemma 2). As the planner takes the effect on all creditors into account, it raises it liquidity level by more than the bank, leading to an increase in the buffer.

Second, when banks' portfolios are more diversified, a run in a given region is less likely (Lemma 2). This diversification provides a natural hedge against region-specific return shocks, reducing the need for liquidity buffers. This highlights the substitutability between liquidity (self-insurance) and diversification (co-insurance).

Third, when expected asset returns are low (or asset prices high), the social planner requires a high liquidity buffer. This result is the consequence of two forces going in the same direction. On the one hand, low asset returns mean a low opportunity cost of holding liquidity from an ex-ante perspective, thus making holding liquidity relatively inexpensive. On the other hand, a bank run is more probable from an ex-ante perspective as the economy-wide investment return is low (Lemma 2). In sum, both effects imply that the macro-prudential buffer is small when investment returns are high.

A literature following the seminal paper by Fama and French (1989) links cyclical variation in the macro-economy to expected investment returns. Expected returns are high during recessions and low during booms, such that there exists a counter-cyclicality in asset returns (see also Campbell and Diebold (2009)). In light of this, our result is consistent with a macro-prudential authority requiring a high macro-prudential liquidity buffer during booms – when expected returns are low. For example, the period prior to the recent financial crisis was characterised by low expected returns and a 'search for yield, suggesting that, were macro-prudential regulation operative at the time, the liquidity buffer would have optimally been high.

5 Endogenous diversification

5.1 Optimal portfolio choices

In this section we relax the assumption of exogenous inter-regional diversification and allow banks to choose the amounts of both liquidity y_k and diversification b_k they hold in their portfolios. Enriching the analysis with this extra choice dimension, we find that the socially efficient allocation features more liquidity and less diversification relative to laissez-faire. As in the case of exogenous diversification, optimal regulation takes the form of a macro-prudential liquidity buffer. Further, a macroprudential cap on the amount of inter-regional diversification alone does not attain the socially efficient allocation.

Allowing for inter-regional diversification to be a choice variable generates an additional first-order condition that complements the one for liquidity. However, a given bank's first-order condition for diversification is identical to that of the planner since there are no externalities associated with diversification per se. The relevant additional first-order condition is:

$$0 = \frac{\partial EU_k}{\partial b_k} + \frac{\partial EU_k}{\partial R_k^*}\frac{\partial R_k^*}{\partial b_k} \tag{21}$$

which again yields a symmetric solution. Appendix (8.8) verifies the validity of the second-order conditions, which now involve a Hessian, and demonstrates the uniqueness of the allocation.

To gain intuition, the Figure 4 depicts the first-order conditions of both the bank and the planner in diversification - liquidity space (b_k, y_k). Let the superscripts y refer to the first-order condition for liquidity and b for diversification. Likewise, we label the competitive equilibrium and the social planner allocations by $*$ and SP, respectively. Consider the bank's portfolio choice first. The first-order condition for diversification is steeper than the first-order condition for liquidity (proven in Appendix (8.8)):

$$\left(\frac{dy_k}{db_k}\right)_*^b < \left(\frac{dy_k}{db_k}\right)_*^y < 0 \tag{22}$$

Next, we determine how the planner's allocation will differ from the competitive equilibrium. Solving for the slope of the planner's first-order condition for liquidity in diversification-liquidity space (see Appendix (8.8)), we learn that the planner's first-order condition for liquidity is flatter than the bank's one:

$$\left(\frac{dy_k}{db_k}\right)_{SP}^y < \left(\frac{dy_k}{db_k}\right)_*^y \tag{23}$$

This result is intuitive: the planner is less willing to trade off liquidity for diversification as it considers the full social cost of liquidation, internalising the negative externality of liquidation on the other bank's creditors. Note also that the planner's first-order condition for liquidity lies strictly above the one for the competitive bank.[11] This is one of our key results, stated in graphical form: the planner holds a higher level of liquidity for any level of diversification because the social benefits from holding liquidity exceed the private benefits.

[11] To see this, fix diversification. Then, note that the additional term in the planner's first-order condition for liquidity, relative to the bank's first-order condition, is positive. Hence, liquidity needs to change to restore equality, which is brought by a rise in liquidity levels as $d^2EU_k/dy_k^2 < 0$.

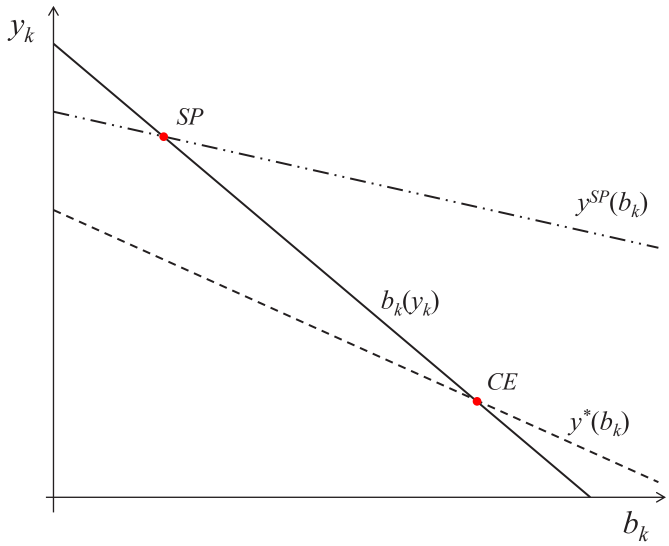


Figure 4: Allocations with endogenous diversification and liquidity. Qualitative illustration only. Note: Liquidity is on the vertical axis and intra-financial linkages or diversification on the horizontal axis. The social planner allocation (SP) is at the top left, while the private competitive equilibrium (CE) is at the bottom right, implying that the planner chooses a higher level of liquidity and a lower level of diversification.

Taking these three observations together, we obtain an extension of our key result to the case of the endogenous diversification:

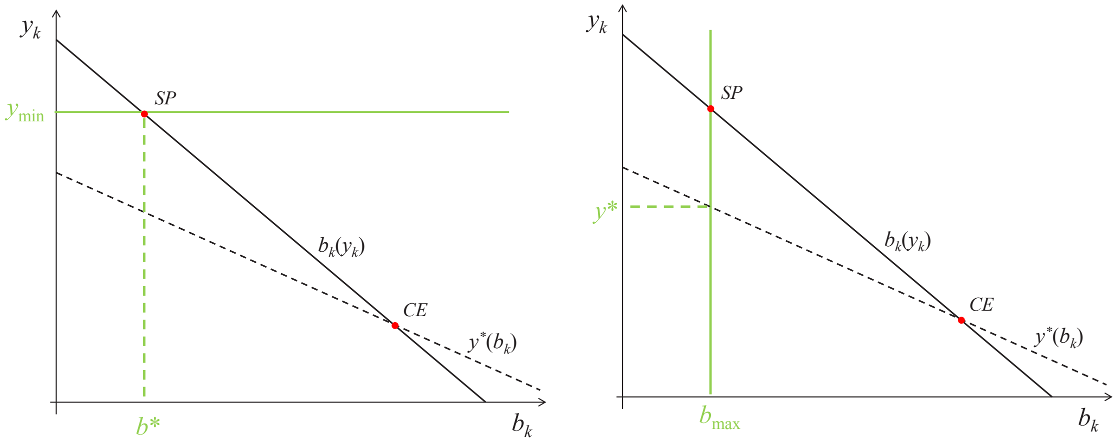
Proposition 3. *If both liquidity and diversification are endogenous, the planner chooses a higher amount of liquidity and a lower amount of diversification than laissez-faire,* $y_k^{SP} > y_k^*$ *and* $b_k^{SP} < b_k^*$.

What is the intuition for the planner to hold more liquidity and diversify less? First, the systemic consequences of liquidation costs lead the planner to require higher liquidity holdings, as in our baseline setting. This diminishes the need for inter-region diversification in avoiding bank runs at individual banks. So when inter-region exposure is endogenous, its level is adjusted optimally downwards to reflect the diversification-return trade-off. At the same time, lower inter-regional diversification contracts the channel of contagion through which liquidation in one region triggers distress in another.

5.2 On implementation of the social optimum

We have seen that the macroprudential authority imposes a liquidity buffer $y_k^{SP} > y_k^*$ and lower inter-regional holdings $b_k^{SP} < b_k^*$ relative to laissez-faire. How is this implemented? There are

Figure 5: Implementing the constrained social optimum. Note: Charts show liquidity holdings (y-axis) against inter-regional diversification (x-axis). The optimal portfolio conditions under laissez-faire $b_k(y_k)$ and $y^*(b_k)$ are shown. The competitive equilibrium allocation occurs at CE, while the social planner chooses allocation SP. In the left-hand panel, the planner imposes a regulatory minimum liquidity requirement y_{min}. In the right-hand panel, the planner imposes a maximum diversification limit b_{max}.



three possibilities, under which (a) the social planner imposes a minimum liquidity requirement, (b) the planner imposes a maximum diversification requirement, or (c) both. Here we show that a minimum liquidity requirement is sufficient to achieve the constrained social optimum.[12]

Figure 5 shows the private optimality conditions of the bank, analogous to Figure 4. As there, $b_k(y_k)$ gives the bank's first-order condition with respect to diversification, while $y^*(b_k)$ gives the bank's first-order condition with respect to liquidity. We omit the social planner's first-order condition, but note that the planner's optimal allocation, at point SP in the diagrams, is the same as that shown in Figure 4, and lies to the north-west of the competitive equilibrium, CE.

In the left-hand panel of Figure 5, the planner imposes a minimum liquidity requirement at y_{min}, which coincides with the constrained efficient level of liquidity. Given this liquidity level, the bank chooses its inter-regional diversification which, reading off from the first-order condition for diversification, yields the level b^*. This coincides exactly with the planner's optimum, SP.

By contrast, the right-hand panel of Figure 5 shows the case in which the planner imposes a maximum diversification requirement that coincides with the constrained efficient level of

[12]See Perotti and Suarez (2011) for a discussion of the relative benefits of quantity versus price regulation in various setups, including unobserved heterogeneity and moral hazard.

diversification at SP. Now the bank chooses liquidity freely, and chooses a level y^*. This allocation clearly does not achieve the constrained-efficient portfolio allocation, as liquidity is insufficiently high.

We conclude that implementing the constrained efficient allocation in this economy entails imposing a minimum liquidity requirement. Note that, conditional on achieving y_{min}, banks then choose the socially efficient level of diversification (which is lower than laissez-faire). Thus, option (c) above is not required. The planner can maximise welfare by imposing the liquidity requirement alone. Intuitively, this achieves the efficient allocation as this tool directly tackles the liquidation cost externality at the heart of the model. Once the level of self insurance has been corrected relative to laissez-faire, the level of co-insurance takes its optimal level as well.

5.3 Comparative statics

Now we consider how the equilibrium allocation and the planner's allocation vary with the exogenous parameters of the model when both liquidity and diversification are endogenous. The comparative statics on the macroprudential liquidity buffer found in our baseline setting carry over to this extended setting. In addition, we derive how optimal and constrained efficient levels of diversification adjust as the liquidation cost and the expected investment return rise. In particular:

Proposition 4. *The privately optimal and socially efficient levels of liquidity and diversification vary according to*

(a) $\partial y^{SP}/\partial\chi > \partial y^*/\partial\chi > 0$, *such that higher liquidation costs, which generate more costly distress, raise the liquidity levels as chosen privately and socially.*

(b) $\partial y^{SP}/\partial\bar{r} < \partial y^*/\partial\bar{r} < 0$, *such that higher investment returns (low asset prices) require lower private and social levels of liquidity.*

(c) $\partial b^{SP}/\partial\chi > \partial b^*/\partial\chi > 0$, *such that higher liquidation costs raises the diversification levels as chosen privately and socially.*

(d) $\partial b^{SP}/\partial\bar{r} < \partial b^*/\partial\bar{r} < 0$, *such that higher investment returns require lower private and social levels of diversification.*

Proof. See Appendix (8.10). □

Corollary 3 follows directly:

Corollary 3. *The macro-prudential liquidity buffer $y^{SP} - y^* > 0$ increases in magnitude and the macro-prudential diversification buffer $b^{SP} - b^* < 0$ decreases in magnitude as*

(a,c) there is a higher liquidation cost χ;

(b,d) there is a lower expected return of the investment project $\bar{r}$.

The results on the level of liquidity are as before. To interpret the results on diversification, it is instructive to recall its two roles in our model. First, it substitutes the strategic complementarity in creditors' withdrawal decisions within a region for strategic complementarity between regions. Given the information structure, particularly the lower precision about the other region's fundamentals, this substitution beneficially reduces the probability of a bank run. Second, diversification is also the source of the liquidation cost externality, thereby being detrimental to creditors.

Now consider a rise in liquidation costs, illustrated in Figure (5.3). Two effects take place: the liquidation cost externality is more severe and the strategic complementarity within a region is stronger, both of which generate inefficient bank runs of solvent but illiquid banks for a larger range fundamentals. With higher liquidation costs, a larger macro-prudential liquidity buffer is required to mitigate the rise in the bank-run threshold due to the first effect of a more severe liquidation cost externality. This is shown by shifts to the right of the $y(b)$ schedules in the figure, whereby the planner's schedule shifts more strongly as it internalises the negative effect of its liquidation decision in the other region's creditors. If nothing else changed (ie if the $b(y)$ schedules did not move), inter-region diversification would fall, such that diversification benefits would diminish. The second force, more strategic complementarity within a region's creditors, induces more diversification that substitutes strategic complementarity within a region with strategic complementarity across regions, reducing the bank-run threshold. This effect is shown by a shift to the right of the $b(y)$ schedule. In sum, both liquidity and diversification increase overall at the expense of reduced exposure to banks' own regions. While the macro-prudential diversification buffer shrinks, the socially optimal level of inter-region exposure always remains below the laissez-faire level.

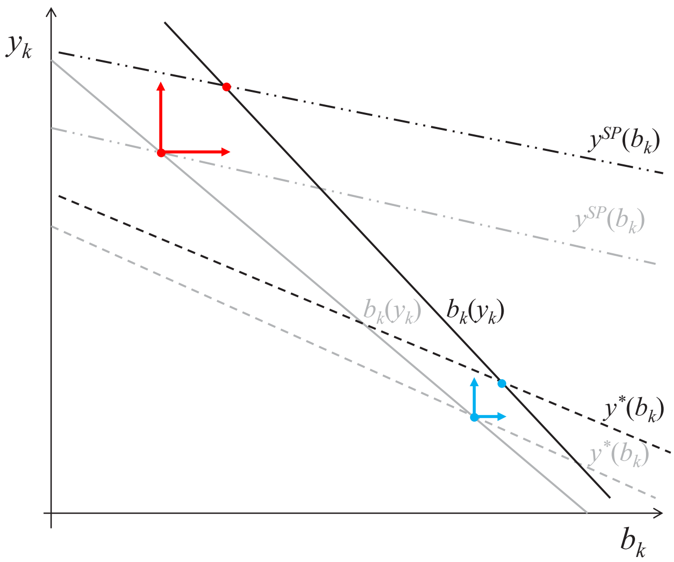


Figure 6: Comparative static on allocations with endogenous diversification and liquidity for a rise in liquidation costs. Qualitative illustration only. Note: Liquidity is on the vertical axis and intra-financial linkages or diversification on the horizontal axis. The social planner allocation (SP) is at the top left, while the private competitive equilibrium (CE) is at the bottom right. The rise in liquidation costs widens the macro-prudential liquidity buffer but narrows the difference between inter-regional diversification allocations.

6 Discussion

We discuss further interpretation of our model as well as limitations that could provide fruitful avenues for subsequent research.

First, liquidity serves a precautionary motive in our paper by preventing costly liquidation. Other motives for holding liquidity are possible. Gale and Yorulmazer (2011) consider a speculative motive for liquidity: liquidity is hoarded to speculate on a bargain in a fire-sale. Calomiris et al. study how easier verifiability of liquidity and its invariance to unobservable action by a bank manager (moral hazard) affect the mix of macro-prudential capital and liquidity requirements. Both papers abstract from the creditor co-ordination problem central to our analysis.

Second, we consider the role of prudential, that is ex-ante, policy with a focus on crisis prevention, abstracting from ex-post interventions aimed at crisis management. See, for instance, Diamond and Rajan (2011) and Uhlig (2010) for discussion of crisis management options and Farhi and Tirole (2012) for an analysis of the ex-ante effects, such as moral hazard, of ex-post policy intervention. A commonly discussed ex-ante policy is standard (unconditional) deposit insurance. This would be undesirable in our set-up as liquidation is efficient for sufficiently detrimental shocks to solvency.

Third, we consider asset-side diversification in our model, in that banks' portfolio choices are limited to choosing between a menu of assets. In reality, banks also face alterative sources of funding, such that liability-side diversification is both a consideration for banks and regulators. For example, the proposed Net Stable Funding Ratio of Basel III seeks to limit maturity mismatch across both sides of the balance sheet, accounting for temporal mismatches between assets and liabilities. While our paper captures the idea that banks 'fund locally' but 'invest globally', an extension to a funding choice could be interesting.

Forth, for tractability, we focus on the case of risk-neutral creditors. The existence of banks in our model is motivated by their privileged access to investment projects, for instance due to better monitoring capabilities, rather than an insurance motive per se. Goldstein and Pauzner (2005) examine the latter, by allowing for risk aversion and the derived motive for liquidity insurance. Incorporating this into our model would substantially complicate the analysis while obscuring the main message of the paper: that private liquidity and diversification choices are unlikely to be socially optimal in the presence of strategic complementarities, inter-linkages, and liquidation costs.

Fifth, we allow for liquidation decisions over assets held in banks' own regions, while liquidation over other-region assets are forfeit. However, our main result would obtain whenever the liquidation of one bank imposes a negative externality on the creditors of another. This feature would prevail under a more general model in which banks choose both the region they invest in and the amount of inter-region assets to liquidate at the interim. Our simpler environment abstracts from this complication.

Sixth, while we interpret the inter-regional claims as direct diversification, alternatives are possible. With liquidation decisions over other-region assets foregone, the inter-regional claims in our model resemble the real-world structure of, for example, syndicated loans, where it is the lead bank that plays a key role in the liquidation decision (eg Sufi (2007)).

7 Conclusion

We examine the optimal design of macroprudential regulation in an interconnected financial system subject to systemic crises. We explore an economy in which regionally distinct banks make portfolio choices over their holdings of a liquid asset and two risky claims – a domestic asset and a foreign asset. Liquidation costs and diversification lead to strategic complementarities both between creditors of a given bank, and between creditors of distinct but interconnected banks. Individually rational diversification results in systemic fragility: each bank is exposed to runs on others, generating a positive externality from holding liquid assets.

Such spillovers in an interconnected financial system generate a role for macroprudential regulation. A constrained planner that internalises the costs of liquidation imposed on all creditors in the financial system requires banks to hold more liquidity and fewer cross-regional claims relative to laissez-faire. We show that this constained-efficient allocation can be implemented via a macroprudential liquidity buffer, but not via a cap on interregional diversification alone. The optimal liquidity buffer is large when the cost of liquidation is high or when the return on banks' investments is low, such as during asset-price boom 'search for yield' episodes.

A corollary of the macroprudential liquidity buffer is that banks optimally attain a lower level of interconnectedness than under laissez-faire. The reason is that liquidity and inter-regional claims are substitutable in avoiding runs by domestic creditors. However, inter-regional claims allow a run in one region to be transmitted to another. As such, a macroprudential authority maximising aggregate welfare requires banks to rely less on 'co-insurance' (inter-regional diversification), and more on self-insurance (liquidity holdings), in trading-off returns against the risk of creditor runs. In the absence of further distortions, a liquidity requirement alone is sufficient for the authority to achieve this welfare-maximising allocation.

Our framework provides a natural laboratory for studying macroprudential policies in a general micro-founded setting. While preserving the co-ordination failure aspect of bank creditors, the presence of spillovers between regions puts the macroprudential aspect of regulation at the center of our analysis in an intuitive way. Consequently, we have abstracted from micro-prudential regulation motives, such as moral hazard. We also abstracted from other regulatory tools like capital requirements or taxes on interbank exposures in our current analysis, but plan to approach these in subsequent research.

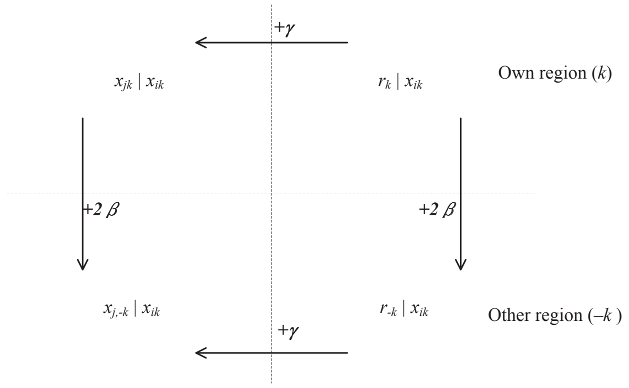


Figure 7: Updating: additional noise

8 Appendix

8.1 Posterior distributions

Figure 8.1 summarizes the updating. Additional noise arises from another creditor's private signal (γ) and from regional investment return shocks (2β):

Investment return in region k The mean of the posterior distribution is a weighted average of the mean of the prior distribution and the private signal, in which the relative weights are given by the respective precisions. The precision of the posterior distribution is the sum of the prior's and signal's precisions. Normality is preserved, such that:

$$\boldsymbol{R}^k_{ik} - \bar{r} \sim \mathcal{N}\left(R^k_{i,k} - \bar{r}, \frac{1}{\gamma + \delta}\right) \tag{24}$$

$$R^k_{i,k} - \bar{r} \equiv \frac{x_{ik} - \bar{r}}{1 + \frac{\delta}{\gamma}} \tag{25}$$

where $\delta \equiv \frac{\alpha\beta}{\alpha+\beta}$ is the precision of the public signal about the regional return. Three remarks are in order. First, the ratio of the prior's precision to the private signal, $\frac{\delta}{\gamma}$, determines the extent to which the posterior relies on the private signal. The more precise the private signal relative to the prior, the more the posterior is determined by the private signal. In the limit ($\frac{\delta}{\gamma} \to 0$), the posterior mean converges to the private signal. Second, the one-region result of Morris and Shin (2001) is obtained for vanishing regional noise ($\beta \to \infty$). Third, it is convenient to rewrite the threshold strategy in terms of a threshold of the posterior mean R^*_k: A late creditor i in region k withdraws if and only if $R^k_{i,k} < R^*_k$.

Investment return in region $-k$ Because of the common component of regional returns, a creditor's private signal is also informative about the other region. The other region's posterior return is distributed according to:

$$\boldsymbol{r}_{-k} \sim \mathcal{N}\left(x_{ik}, \frac{1}{\mu}\right) \tag{26}$$

where $\mu \equiv \left(\frac{1}{\gamma} + \frac{1}{\beta} + \frac{1}{\beta}\right)^{-1} < \gamma$ is the precision of the private signal of a creditor i in k about region $-k$. The fact that $\mu < \gamma$ implies that the private signal is more precise about region k (the creditor's own region) than about region $-k$ (the other region). The precision of regional noise β determines the usefulness of the private signal for predicting the other region's return. If regional noise is large, knowledge about the common component is less valuable, and the private signal becomes less useful for predicting the other region's return. The private signal ceases to be included into the updating as regional noise becomes large ($\beta \to 0$) — even if idiosyncratic noise vanishes ($\gamma \to \infty$).

The posterior return of creditor i in region k over region $-k$'s return is:

$$\boldsymbol{R}^k_{i,-k} - \bar{r} \sim \mathcal{N}\left(R^k_{i,-k} - \bar{r}, \frac{1}{\mu + \delta}\right) \tag{27}$$

$$R^k_{i,-k} - \bar{r} \equiv = \kappa[R^k_{i,k} - \bar{r}] \tag{28}$$

$$\kappa \equiv \frac{1 + \frac{\delta}{\gamma}}{1 + \frac{\delta}{\mu}} \in (0, 1) \tag{29}$$

where $\kappa < 1$ as $\mu < \gamma$. Relative to the posterior mean of his own region's investment return, creditor i in k relies less on his private signal and more on the prior ($\kappa < 1$) in forming his posterior mean over region $-k$'s return due to the additional regional noise. Thus, $R^k_{i,-k} = \kappa R^k_{ik} + (1 - \kappa)\bar{r} > R^k_{i,k}$ for any signal below its mean, $x_{ik} < \bar{r}$. The difference between the posterior means vanishes ($\kappa \to 1$) as regional noise becomes small ($\beta \to \infty$).

Figure 8 illustrates the posterior distributions of regional returns. It also contains the posterior distribution for creditor i over another creditor j's signal. Creditor i uses this to form the posterior of the proportion of withdrawing late creditors. When inferring another creditor's possible signal $x_{jk}|x_{ik}$, additional idiosyncratic noise γ leads to an unchanged posterior mean but a more dispersed posterior distribution.

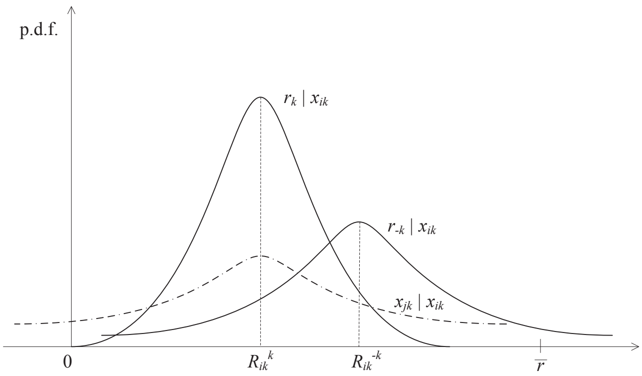


Figure 8: Different posteriors (for a given $x_{ik} < \bar{r}$)

Proportion of withdrawing late creditors in region *k* This posterior proportion $W_{i,k}$ can be written as the conditional probability of the withdrawal of an arbitrary late creditor:[13]

$$
\begin{aligned}
W_{ik}^{k} &= \boldsymbol{w}_k | x_{ik} && (30)\\
&= \int_{j\in[0,1]} \mathbf{1}\{\text{agent j in region k withdraws}|x_{ik}\} dj && (31)\\
&= \Phi\left(\sqrt{\delta_1}\left[R_k^* - \bar{r}\right] + \sqrt{\delta_1}\gamma\delta\left[R_k^* - R_{ik}^k\right]\right) && (32)\\
\delta_1 &\equiv \frac{\frac{\delta^2}{\gamma^2}}{\frac{1}{\delta+\gamma}+\frac{1}{\gamma}} = \frac{\delta^2}{\gamma}\frac{\delta+\gamma}{\delta+2\gamma} && (33)
\end{aligned}
$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution and δ_1 summarizes precision parameters. A late creditor that receives the threshold signal x_k^* thus forms the following posterior mean:

$$
W_k^k(R_k^*) \equiv W_{ik}^k\Big|_{R_{ik}^k = R_k^*} = \Phi\left(\sqrt{\delta_1}\left[R_k^* - \bar{r}\right]\right) \qquad (34)
$$

Note that $\delta_1 \to 0$ as the private signal becomes very precise ($\gamma \to \infty$) such that $W_k^* \to \frac{1}{2}$. As regional noise vanishes ($\beta \to \infty$), the multi-region model collapses to the one-region model of Morris and Shin (2001), with the coefficient being equivalent to equation (2.6) in Morris and Shin (2001).

[13]The second line uses the definition of the proportion $\boldsymbol{w}_k$, the third line the the definition of threshold strategy, the fourth line the posterior distribution about the investment return by late household $j \neq i$, the fifth line j's private signal, and the sixth line the independence of idiosyncratic noise from both the economy-wide investment return and the return to region's investment project return.

Proportion of withdrawing late creditors in region $-k$ The posterior proportion $W^k_{i,-k}$ is again the probability of the event that an arbitrary late creditor in the other region withdraws:

$$W^k_{i,-k} = \Pr\{\boldsymbol{x}_{j,-k} \leq x^*_{-k}|x_{ik}\} \tag{35}$$

where the conditional probability takes the same form as before but differs in the distributions $\boldsymbol{x}_{j,-k}|x_{ik}$:

$$\begin{aligned}
(x_{j,-k} - \bar{r})|x_{ik} &= (\boldsymbol{r}_{-k} - \bar{r})|x_{ik} + \boldsymbol{\epsilon}_{j,-k} && (36)\\
&= \mathcal{N}\left(R^k_{i,-k} - \bar{r}, \left[\frac{\gamma\left[\delta + \gamma\mu\right]}{\delta + 2\gamma\mu}\right]^{-1}\right) && (37)\\
W^k_{i,-k} &= \Phi\left(\sqrt{\delta_2}\left[R^*_{-k} - \bar{r}\right] + \sqrt{\frac{\gamma(\delta+\mu)}{\delta+\gamma+\mu}}\left[R^*_{-k} - R^k_{i,-k}\right]\right) && (38)\\
&= \Phi\left((1 + \frac{\delta}{\gamma})\sqrt{\delta_2}\left[R^*_{-k} - \bar{r}\right] - \kappa\sqrt{\delta_2}\left[R_{ik} - \bar{r}\right]\right) && (39)\\
\delta_2 &\equiv \frac{\gamma[\beta + (2 + \frac{\beta}{\gamma})\delta]}{2(\beta+\gamma) + (2 + \frac{\beta}{\gamma})\delta} && (40)
\end{aligned}$$

A late creditor who receives the threshold signal x^*_k will thus form the following posterior proportion of withdrawing late creditors in its neighbouring region:

$$\begin{aligned}
W^k_{-k}(R^*_k, R^*_{-k}) &\equiv W^k_{i,-k}\Big|_{R^k_{ik}=R^*_k} = \Phi\left(\sqrt{\rho_1}\left(R^*_{-k} - \bar{r}\right) - \sqrt{\rho_2}\left(R^*_k - \bar{r}\right)\right) && (41)\\
\rho_1 &\equiv \left(\frac{\delta+\gamma}{\gamma}\right)^2 \frac{1}{\frac{1}{\delta+\mu} + \frac{1}{\gamma}} && (42)\\
\rho_2 &\equiv \kappa^2 \frac{1}{\frac{1}{\delta+\mu} + \frac{1}{\gamma}} && (43)
\end{aligned}$$

where $\rho_1 > \rho_2$.

The remaining analysis is from the perspective of a creditor in region k, which is without loss of generality. Therefore, we drop the superscript in the subsequent analysis.

8.2 Positive expected liquidation share in equilibrium

Lemma (3) ensures positive equilibrium liquidation shares:

Lemma 3. *Let private signals be sufficiently precise. Then, there is always a positive amount* *of liquidation of the investment project $L_k^* > 0$ in equilibrium.*

A proof follows below. The intuition is as follows: given the low level of liquidation, the final-date consumption level of a late depositor always exceeds unity if the private signal is sufficiently precise. Thus, it is strictly dominant to wait, contradicting the supposition of equilibrium.

This proof is by contradiction: suppose that creditors expect no liquidation in equilibrium, $L_k^* = 0$. The idea of the proof is to show that this implies $c_{2k} \neq 1$ for a sufficiently large precision of the private signal. For the following steps, we employ the posterior distributions derived in Appendix (8.1).

Step 1: Fix any amount of liquidity $\lambda \leq y_k < \frac{1}{2} + \lambda$. We focus on parameters that make it never optimal to hold a larger amount of liquidity in equilibrium, that is a sufficiently high mean return $\bar{r}$. Then, we have:

$$W_k^* \leq y_k - \lambda \tag{44}$$

Step 2: Using the posterior mean of the proportion of withdrawing late creditors in region k, we find the implied bound on the equilibrium mean investment return R_k^*:

$$R_k^* \leq \bar{r} + \frac{1}{\sqrt{\delta_1}} \left[\Phi^{-1}(y_k - \lambda)\right] \tag{45}$$

where $\Phi^{-1}(\cdot)$ denotes the inverse of cumulative distribution function of a standard normal random variable. Note that $R_k^* \to -\infty$ as $\gamma \to \infty$.

Step 3: The implied final-date payoff to late despitors is:

$$c_{2k}^* = \frac{I_k R_k^* + (1-\kappa) b_k (\bar{r} - R_k^*) - (1+\chi) b_k (W_{-k}^* + \lambda - y_{-k})}{1 - W_k^*} \tag{46}$$

where $L_{-k}^* = W_{-k}^* + \lambda - y_{-k}$ is the equilibrium liquidation share in the other region.

Step 4: Given $y_k < \lambda + \frac{1}{2}$, there exists a boundary γ_0 such that $c_{2k}^* \neq 1$ for any $\gamma > \gamma_0$. This contradicts the supposition of R_k^* being an equilibrium in the withdrawal game. (If $y_k = \lambda + \frac{1}{2}$, then there is exists only one b_k that satisfies the equation $c_{2k}^* = 1$. However, the withdrawal subgame equilibrium threshold R_k^* ought to be determined for a range of portfolio choices. The portfolio choices in turn are determined at the initial stage, implying that this is not an equilibrium either.)

In sum, we have shown there is always liquidation in equilibrium for a sufficiently precise private signal and $y_k < \frac{1}{2} + \lambda$.

8.3 Proof: Strategic complementarities within region

The relevant case around equilibrium is a positive liquidation by the regional bank (see 8.2). Thus, the final-date payoff of a late creditor who keeps his funds at the bank is:

$$c_{2k} = \frac{(I_k - b_k)[r_k - (1+\chi)(w_k + \lambda - y_k)] + b_k[r_{-k} - (1+\chi)l_{-k}]}{1 - w_k} \tag{47}$$

Partially differentiating with respect to the porportion of withdrawing late creditors:

$$\frac{\partial c_{2k}}{\partial w_k} = \frac{(I_k - b_k)[r_k - I_k(1+\chi)] + b_k[r_{-k} - (1+\chi)l_{-k}]}{(1 - w_k)^2} \tag{48}$$

Locally around the equilibrium threshold R_k^*, we have:

$$\frac{\partial c_{2k}}{\partial w_k} = \frac{1 - (1+\chi)(I_k - b_k)}{(1 - w_k)} \tag{49}$$

such that $1 + \chi > 1 + \underline{\chi} \equiv \frac{1}{I_k - b_k}$ ensures the presence of *strategic complementarities* within a region.

8.4 Proof: Strategic complementarities between region

We show that the partial derivative of the final-date payoff to a late creditor with respect to the share of withdrawing late creditors in the *other* region is (strictly) negative if and only if the amount of interbank diversification is (strictly) positive and the excess liquidity exhausted:

$$\frac{\partial c_{2k}}{\partial w_{-k}} = -\frac{(1+\chi)b_k}{(1 - w_k)} < 0 \tag{50}$$

8.5 Proof: uniqueness of threshold equilibrium

Equation 14 implicitly defines the best response function $R_k^*(R_{-k}^*)$, taking the other region's withdrawal threshold R_{-k}^* as given. Each region's withdrawal threshold depends on the other's, owing to the effect of expected withdrawals in the other region on final period payoffs. By virtue of Lemma 3, the best response function of creditors in region k reduces to:

$$\begin{aligned} R_k^* \quad = \quad & 1 + \frac{(1+\chi)(I_k - b_k) - 1}{I_k - (1-\kappa)b_k}[W_k(R_k^*) + \lambda - y_k] \\ & - \frac{(1-\kappa)(\bar{r} - 1) - (1+\chi)[W_{-k}(R_k^*, R_{-k}^*) + \lambda - y_{-k}]}{I_k - (1-\kappa)b_k} b_k \end{aligned} \tag{51}$$

where κ is a constant defined in the Appendix that collects precision parameters.

The existence and uniqueness proof of the threshold equilibrium has two steps. First, we show that the best response function is bounded and that its lies strictly within zero and one, $\frac{dR_k^*}{dR_{-k}^*} \in (0, 1)$. This establishes that there exists an threshold equilibrium and that it is unique. Second, an individual creditor must not find it profitable to deviate from the threshold R_k^* given that all other creditors in both regions play a threshold strategy with threshold R_k^*. Using the iterated deletion of strictly dominated strategies, this establishes that any equilibrium strategy in threshold strategies features the threshold R_k^*. Hence, the subgame has a unique Bayesian Nash equilibrium in threshold strategies. We discuss both steps in turn.

8.5.1 Slope of best-response function

Consider the bounds of the best-response functions first. If the threshold in the other region becomes smaller ($R_{-k} \to -\infty$), creditors in the other region never withdraw ($W_{-k} \to 0$). Let $\underline{R_k}$ denote the best response of a creditor in region k to that strategy. It is given by the solution to (15). We have $-\infty < \underline{R_k} < \bar{r}$. Likewise, we determine $\overline{R_k}$ for $R_{-k} \to +\infty$, then $W_{-k} \to 1$, that is all late creditors always withdraw in the other region. We have $\underline{R_k} < \overline{R_k} < \infty$.

Existence and uniqueness of equilibrium requires the slope of the best-response function to lie strictly within zero and one, $\frac{dR^*_k}{dR^*_{-k}} \in (0,1)$. The partial derivative is given by:

$$\frac{dR^*_k}{dR^*_{-k}} = \frac{(1+\chi)\sqrt{\rho_1}\phi(z_2)b_k}{D_k} \tag{52}$$

$$D_k \equiv I_k - (1-\kappa)b_k + (1+\chi)\sqrt{\rho_2}\phi(z_2)b_k - \sqrt{\delta_1}\phi(z_1)\left[(1+\chi)(I_k - b_k) - 1\right] \tag{53}$$

$$z_1 \equiv \sqrt{\delta_1}\left[R^*_k - \bar{r}\right] \tag{54}$$

$$z_2 \equiv \sqrt{\rho_1}\left[R^*_{-k} - \bar{r}\right] - \sqrt{\rho_2}\left[R^*_k - \bar{r}\right] \tag{55}$$

Both conditions are ensured when $D_k > (1+\chi)\sqrt{\rho_1}\phi(z_2)b_k > 0$. The second inequality never binds. Letting idiosyncratic noise vanish ($\gamma \to \infty$) and with the upper bound on the pdf ($\phi(z) \leq \frac{1}{\sqrt{2\pi}}$), the first inequality yields:

$$\frac{b_k}{I_k}\left[(1+\chi)\sqrt{\frac{\beta}{4\pi}}\left(\frac{2\frac{\alpha}{\beta}}{\sqrt{3(\frac{\alpha}{\beta})^2 + 4\frac{\alpha}{\beta} + 1}}\right)\frac{2\frac{\alpha}{\beta}}{1+3\frac{\alpha}{\beta}}\right] < 1 \tag{56}$$

As $b_k \leq I_k$, a sufficiently low precision of the regional noise for a given correlation between the investment projects ρ suffices to ensure this condition. In other words, for any portfolio choice (b_k, y_k) there exists ($\underline{\gamma}$, $\overline{\delta}$) such that for any ($\beta < \overline{\delta}$, $\gamma > \underline{\gamma}$) that ensures a the existence of a unique (and symmetric) equilibrium.

8.5.2 No profitable deviation

We are left to demonstrate that there is no profitable deviation for an individual creditor if all other creditors play the threshold stategy with threshold R^*_k. We employ a standard argument of iterated deletion of strictly dominated strategies (see Morris and Shin (2001)) to show that it is indeed optimal for late creditor i from region k to follow a threshold strategy with threshold R^*_k.

First, we determine the payoff $v(\cdot)$ of the deviating creditor. If all late players in region k and $-k$ play a threshold strategy characterised by R_k and R_{-k}, late creditor i's uses its private signal to update his beliefs about regional returns and the shares of withdrawing late creditors. His payoff from keeping its funds in the bank is given by:

$$v\left(R_{ik}, R_k, R_{-k}\right) \equiv \frac{(I_k - b_k)[R_{ik} - (1+\chi)(W_{i,k} + \lambda - y_k)]}{1 - W_{ik}} + \tag{57}$$
$$+ b_k \frac{[\kappa R_{ik} + (1-\kappa)\bar{r}(1+\chi)]W_{i,-k} + \lambda - y_{-k}}{1 - W_{ik}}$$

$$W_{i,k} = \Phi(z_{1i}) \tag{58}$$

$$W_{i,-k} = \Phi(z_{2i}) \tag{59}$$

$$z_{1i} \equiv (\sqrt{\delta_1}[R_k^* - \bar{r}] + \delta\gamma\sqrt{\delta_1})[R_k^* - R_{ik}] \tag{60}$$

$$z_{2i} \equiv (\sqrt{\rho_1}[R_{-k}^* - \bar{r}] - \sqrt{\rho_2}[R_{i,k} - \bar{r}] \tag{61}$$

$$\tag{62}$$

It is useful for the following argument to define the partial derivatives of the payoff function:

$$v_1 \equiv \frac{\partial v}{\partial R_{ik}} = (1 - W_{ik})^{-1}\left[I_k - (1-\kappa)b_k + \gamma\delta\sqrt{\delta_1}\phi(z_1 i)[(1+\chi)(I - k - b_k) - d_{2ik}]\right] > 0 \tag{63}$$

$$v_2 \equiv \frac{\partial v}{\partial R_k^*} = -\frac{\sqrt{\delta_1}(1+\delta\gamma)\phi(z_{1i})}{1 - W_{ik}}[A_{2ik} - (1+\chi)(I_k - b_k)] < 0 \tag{64}$$

$$v_3 \equiv \frac{\partial v}{\partial R_{-k}^*} = -\frac{(1+\chi)\sqrt{\rho_1}\phi(z_{2i})}{1 - W_{ik}} b_k < 0 \tag{65}$$

Note that the strong monotonicity of the payoff function in the first argument arises for the sufficiency conditions of a unique equilibrium and implies the optimality of a threshold strategy. Likewise, the strong monotonicity of the second partial derivative is due to the sufficiency conditions. Take the first step of the iteration argument.

Step 1: If the realised regional return r_k is sufficiently bad, agent i's private signal and posterior mean R_{ik} will be so bad that withdrawing is a dominant strategy, i.e. other players' actions do not matter. Formally, there exists a $\underline{R_k^{(1)}}$ such that

$$v(R_{ik}, R_k, R_{-k}) < 1 \ \ \forall (R_k, R_{-k}) \tag{66}$$

for *any* $R_{ik} < \underline{R_k^{(1)}}$. Note that this lower bound applies to any creditor in both regions by symmetry. The lower bound is implicity defined by

$$\lim_{R_k \to -\infty} \lim_{R_{-k} \to -\infty} \left(v(\underline{R_k^{(1)}}, R_k, R_{-k}) \right) = 1 \tag{67}$$

Hence, $R_{ik} \geq \underline{R_k^{(1)}}$. The same analysis is done by other late creditors in region k: $R_k \geq \underline{R_k^{(1)}}$. By symmetry, this lower bound holds for the other region's threshold as well.

Step 2: Given that other late creditors ruled out all thresholds below the lower bound $\underline{R_k^{(1)}}$, there is a second-stage lower bound $\underline{R_k^{(2)}}$ for late creditor i defined by:

$$1 = v\left(\underline{R_k^{(2)}}, \underline{R_k^{(1)}}, \underline{R_k^{(1)}}\right) \tag{68}$$

Given the partial derivatives of the payoff function v, we have $\underline{R_k^{(2)}} > \underline{R_k^{(1)}}$, where any strategy that keeps funds in the bank for a posterior lower than $\underline{R_k^{(2)}}$ is strictly dominated in the second round.

Step 3: Iterating on these steps, we obtain a strictly increasing sequence $\underline{R_k^{(1)}} < \underline{R_k^{(2)}} < \cdots < \underline{R_k^{(l)}} < ...$, where the initial value $\underline{R_k^{(1)}}$ is given above and $\underline{R_k^{(l+1)}}$ is recursively defined by

$$1 = v\left(\underline{R_k^{(l+1)}}, \underline{R_k^{(l)}}, \underline{R_k^{(l)}}\right) \quad l = 1, 2, ... \tag{69}$$

Any posterior mean $R < \underline{R_k^{(m)}}$ does not survive the m^{th} round of deletion of strictly dominated stratgies. Let $\underline{R_k}$ be the limit of this sequence defined by

$$1 = v\left(\underline{R_k}, \underline{R_k}, \underline{R_k}\right) \tag{70}$$

An identical argument holds for a strictly decreasing sequence $\overline{R^{(1)}} > \overline{R^{(2)}} > \cdots > \underline{R_k^{(l)}} > ...$ that converges to $\overline{R} : 1 = v\left(\overline{R}, \overline{R}, \overline{R}\right)$.

Step 4: We proved that there is a unique R^* that solves $1 = v\left(R^*, R^*, R^*\right)$. Thus, both sequences converge to $\underline{R_k} = R^* = \overline{R}$. This establishes that there is a unique Bayesian Nash equilibrium in the subgame at the interim date. It prescribes that every creditor in either region will use a threshold strategy with threshold R^*, withdrawing his funds if and only if his posterior mean falls short of this threshold. This completes the proof for the existence and uniqueness of the equilibrium in threshold strategies. ■

8.6 Proofs: changes of threshold R_k^*

In analogy to the previous short hands (z_1,z_2), we define

$$z_1' \equiv \sqrt{\delta_1}\,[R_k^* - \bar{r}] \tag{71}$$

$$z_2' \equiv (\sqrt{\rho_1} - \sqrt{\rho_2})\,[R_k^* - \bar{r}] \tag{72}$$

All partial derivatives share a common denominator $\tilde{D}_k = D_k - (1+\chi)\sqrt{\rho_1}\phi(z_2')b_k > 0$. The difference to the previous denominator arises as a regional bank takes into account the effect of its portfolio choice decision on both best response functions, that is on R_{-k} as well, while a creditor takes R_{-k}^* as given. (The bank still takes the other bank's portfolio choice as given.) Mathematically, this yields the above an additional negative term. The positive sign is assured by the sufficiency conditions of the unique equilibrium. Recall $\rho_1 > \rho_2$ and that $\delta_1 \to 0$ as $\gamma \to \infty$, such that $\sqrt{\delta_1} < (\sqrt{\rho_1} - \sqrt{\rho_2})$ as private noise vanishes. Consequently, $W_k(R_k^*) > W_{-k}(R_k^*, R_{-k}^*)$ as $R_k^* < \bar{r}$. The smaller precision about the other regions makes a depositor belief that the equilibrium proportion of withdrawals is higher in his region than in the other region.

The partial derivatives with respect to parameters are:

$$\begin{aligned}
\tilde{D}_k \frac{\partial R_k^*}{\partial \chi} &= [I_k - b_k](W_k^* + \lambda - y_k) + b_k(W_{-k}^* + \lambda - y_{-k}) > 0 \\
\tilde{D}_k \frac{\partial R_k^*}{\partial \bar{r}} &= -(1-\kappa)b_k - [(1+\chi)(I_k - b_k) - 1]\sqrt{\delta_1}\phi(z_1') - (1+\chi)b_k(\sqrt{\rho_1} - \sqrt{\rho_1})\phi(z_2') < 0
\end{aligned}$$

The positive sign of the first bracket is ensured by the sufficient condition for strategic complementarity within a region ($\chi > \underline{\chi}$). Moving to the partial derivatives with respect to a bank's portfolio choice variables:

$$\tilde{D}_k \frac{\partial R_k^*}{\partial y_{-k}} = -(1+\chi)b_k < 0 \tag{73}$$

$$\tilde{D}_k \frac{\partial R_k^*}{\partial y_k} = R_k^* - (1+\chi)[1 + W_k^* - b_k] < 0 \tag{74}$$

$$\tilde{D}_k \frac{\partial R_k^*}{\partial b_k} = -[(1-\kappa)(\bar{r} - R_k^*) + (1+\chi)(W_k^* - y_k - W_{-k}^* + y_{-k})] \tag{75}$$

The first partial derivative illustrates the key channel for a beneficial policy intervention in our model. This beneficial effect of reducing the withdrawal threshold is only internalised by the planner. The partial derivatives for the portfolio choice variables of a bank b_k and y_k are both negative due to the sufficiency conditions of equilibrium uniqueness. Note that $\tilde{D}_k \frac{\partial R_k^*}{\partial b_k} < 0$ arises in any symmetric equilibrium $y_k^* = y^*$, as implied by our model.

8.6.1 Second-order derivatives

To varify the second-order conditions and to compute comparative statics on the privately optimal and socially efficient portfolio allocation, we also obtain the second-order effects on the withdrawal threshold. Note that the partial derivatives of $\tilde{D}_k$ are:

$$
\begin{aligned}
\frac{\partial \tilde{D}_k}{\partial y_k} &= -1 + (1+\chi)\phi(z_1')\sqrt{\delta_1} < 0 \\
\frac{\partial \tilde{D}_k}{\partial b_k} &= -(1-\kappa) + \sqrt{\delta_1}(1+\chi)\phi(z_1') - (1+\chi)\phi(z_2')(\sqrt{\rho_1} - \sqrt{\rho_2}) < 0 \\
\frac{\partial \tilde{D}_k}{\partial y_{-k}} &= 0 \\
\frac{\partial \tilde{D}_k}{\partial R_k^*} &= -(\bar{r} - R_k^*)[(1+\chi)(\sqrt{\rho_1} - \sqrt{\rho_2})^2\phi(z_2') - [(1+\chi)(I_k - b_k) - 1](\sqrt{\delta_1})^2\phi(z_1')] < 0
\end{aligned}
$$

where the signs arise as γ becomes large. This implies the following second-order partial derivatives:

$$\frac{\partial^2 R_k^*}{\partial y_k^2} = \underbrace{-\frac{\partial \tilde{D}_k}{\partial y_k}}_{+} \frac{1}{\tilde{D}_k} \frac{\partial R_k^*}{\partial y_k} < 0 \tag{76}$$

$$\frac{\partial^2 R_k^*}{\partial b_k^2} = \underbrace{-\frac{\partial \tilde{D}_k}{\partial b_k}}_{+} \frac{1}{\tilde{D}_k} \frac{\partial R_k^*}{\partial b_k} < 0 \tag{77}$$

$$\frac{\partial^2 R_k^*}{\partial y_k \partial b_k} = \frac{1}{\tilde{D}_k}\left[(1+\chi) - \frac{\partial \tilde{D}_k}{\partial b_k}\frac{\partial R_k^*}{\partial y_k}\right] > 0 \tag{78}$$

$$\frac{\partial^2 R_k^*}{\partial b_k \partial y_k} = \frac{1}{\tilde{D}_k}\left[(1+\chi) - \frac{\partial \tilde{D}_k}{\partial y_k}\frac{\partial R_k^*}{\partial b_k}\right] > 0 \tag{79}$$

$$\frac{\partial^2 R_k^*}{\partial b_k \partial y_{-k}} = \frac{\partial^2 R_k^*}{\partial y_{-k} \partial b_k} = -\frac{(1+\chi)}{\tilde{D}_k} < 0 \tag{80}$$

Thus $\frac{\partial^2 R_k^*}{\partial y_k^2}$ inherits the sign of $\frac{\partial R_k^*}{\partial y_k}$ and $\frac{\partial^2 R_k^*}{\partial b_k^2}$ inherits the sign of $\frac{\partial R_k^*}{\partial b_k}$. The sign of the cross second partial derivatives is unambiguous under the sufficient conditions.

8.7 Derivation of expected utility EU_k

To determine the expected utility in region k, EU_k, we first obtain the payoffs to creditors in four cases. As regional noise vanishes, there are two possible cases: full runs (partial bank runs exist only with non-vanmishing noise) or no run at all. Combining these cases for each region $k = \{A, B\}$, we have four cases. A run takes place if the realization of the regional return is below the threshold R_k^*. Note that the private signal converges to the regional return as the individual noise vanishes. In the limit, $x_{ik} \to r_k$ as $\gamma \to \infty$. Then, $R_{ik} < R_k^* \Leftrightarrow r_k < R_k^*$.

First, case (a) occurs when investment returns in both regions are sufficiently good to prevent runs, so $r_k \geq R_k^*$ and $r_{-k} \geq R_{-k}^*$. In this case no liquidation takes place in either region. Early creditors of mass λ receive their promised payment and late creditors of unit mass receive the remainder. Let π_k^a be the total payoff in region k in case (a), then:

$$\pi_k^a = [1 \times \lambda] + [(y_k - \lambda) + (I_k - b_k)r_k + b_k r_{-k}] \times 1 \tag{81}$$

$$= y_k + (I_k - b_k)r_k + b_k r_{-k} \tag{82}$$

Second, case (b) occurs when investment returns in k are sufficient to prevent a run in k, but bad investment returns in $-k$ generate a run there, so $r_{-k} < R_{-k}^*$ and $R_k^* \leq r_k$. Then, no liquidation takes place region k but full liquidation takes place in $-k$. This imposes liquidation costs on the bank in region k. Early creditors of mass λ receive their promised payment and late creditors of unit mass receive the remainder, giving:

$$\pi_k^b = [1 \times \lambda] + [(y_k - \lambda) + (I_k - b_k)r_k + b_k (r_{-k} - (1 + \chi))] \times 1 \tag{83}$$

$$= \pi_k^a - b_k(1 + \chi). \tag{84}$$

Next, case (c) occurs when bad investment returns in region k trigger a run there, but good investment returns in the other region prevent contagion, so $r_k < R_k^*$ and $R_{-k}^* \leq r_{-k}$. In this case liquidation takes place in region k only. The bank in region k liquidates all of its assets and all agents receive an equal share of the funds (pro rata). Part of the liquidation proceeds of the bank is a share of the interbank claim. Ex-post trade ensures that these claims are held by late creditors without any welfare loss. Thus, the competitive bank and the planner are concerned with the total value of the assets only. Again, liquidation costs are imposed on the other bank in the other region $-k$:

$$\pi_k^c = [1 \times \lambda] + [(y_k - \lambda) + (I_k - b_k) + b_k r_{-k}] \times 1 \tag{85}$$

$$= 1 + \lambda - b_k + b_k r_{-k}. \tag{86}$$

Finally, case (d) occurs when bad investment returns in both regions mean $r_k < R_k^*$ and $r_{-k} < R_{-k}^*$. Liquidation takes place in both regions. Banks liquidate their assets and mutually impose liquidation costs on each other. Note that the amount creditors obtain from withdrawing at the interim date falls short of unity in this case. This is consistent with the withdrawal threshold derived earlier as withdrawing is a strictly dominant strategy in this case. All agents receive an equal share of the remaining funds (pro rata), giving

$$\pi_k^d = [1 \times \lambda] + [(y_k - \lambda) + (I_k - b_k) + b_k (r_{-k} - (1 + \chi))] \times 1 \tag{87}$$

$$= \pi_k^c - b_k(1 + \chi) \tag{88}$$

We can identify the benefits of a asset diversification. In case (c), diversification is beneficial. Also, some strategic complementarity between creditors within a given region is substituted with that between regions. This arises in the form of a negative externality imposed by the liquidation decision of one bank the other. This is reflected in the payoffs: $\pi_k^a > \pi_k^b$ and $\pi_k^c > \pi_k^d$.

Also note that the payoff π_k is weakly increasing in the investment return r_k with a discrete jump at R_k^*, reflecting liquidation costs. Thus, a marginal decrease in the threshold R_k^* – as implied by larger liquidity holdings in the other region – strictly increases welfare.

We are now ready to determine the expected utilty by integrating over the four cases. Let $f(r) = \phi(\sqrt{\alpha}(r - \bar{r}))$ and $g(\eta_k) = \phi(\sqrt{\beta}\eta_k)$ denote the probability distribution functions of $\boldsymbol{r}$ and $\boldsymbol{\eta}_k$, respectively, where $\phi(\cdot)$ is the probability distribution function of the standard normal distribution. Note that the event $r_k < R_k^*$ can be rewritten as $\eta_k < \bar{\eta}(r) \equiv R_k^* - r$ for a given economy-wide investment return r. Using these distribution functions and the payoffs derived above, total welfare in region k is given by

$$EU_k \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\bar{\eta}(r)} \left[\int_{-\infty}^{\bar{\eta}(r)} \pi_k^d g(\eta_{-k}) d\eta_{-k} + \int_{\bar{\eta}(r)}^{\infty} \pi_k^c g(\eta_{-k}) d\eta_{-k} \right] g(\eta_k) d\eta_k + \\ + \int_{\bar{\eta}(r)}^{\infty} \left[\int_{-\infty}^{\bar{\eta}(r)} \pi_k^b g(\eta_{-k}) d\eta_{-k} + \int_{\bar{\eta}(r)}^{\infty} \pi_k^a g(\eta_{-k}) d\eta_{-k} \right] g(\eta_k) d\eta_k f(r) dr.$$

Integrating over η_{-k}, we have:

$$\begin{aligned}
\int_{\bar{\eta}(r)}^{\infty} \pi_k^c g(\eta_{-k}) d\eta_{-k} &= [1+\lambda+b_k(r-1)][1-\Phi(\sqrt{\beta}\bar{\eta})] \\
&\quad +b_k \mathbb{E}\left[\eta_{-k}, \eta_{-k} \geq \bar{\eta}\right] \\
\int_{-\infty}^{\bar{\eta}(r)} \pi_k^d g(\eta_{-k}) d\eta_{-k} &= \Phi(\sqrt{\beta}\bar{\eta})[1+\lambda+b_k(r-1)-b_k(1+\chi)] \\
&\quad +b_k \mathbb{E}\left[\eta_{-k}, \eta_{-k} \leq \bar{\eta}\right] \\
\int_{\bar{\eta}(r)}^{\infty} \pi_k^a g(\eta_{-k}) d\eta_{-k} &= [y_k+(1+\lambda-y_k)r+(1+\lambda-b_k-y_k)\eta_k][1-\Phi(\sqrt{\beta}\bar{\eta})] \\
&\quad +b_k \mathbb{E}\left[\eta_{-k}, \eta_{-k} \geq \bar{\eta}\right] \\
\int_{-\infty}^{\bar{\eta}(r)} \pi_k^b g(\eta_{-k}) d\eta_{-k} &= [y_k+(1+\lambda-y_k)r+(1+\lambda-b_k-y_k)\eta_k-b_k(1+\chi)]\Phi(\sqrt{\beta}\bar{\eta}) \\
&\quad +b_k \mathbb{E}\left[\eta_{-k}, \eta_{-k} \leq \bar{\eta}\right]
\end{aligned}$$

The conditional expectations on η_{-k} vanish when adding up. We find a closed-form expression for the conditional expectation on η_k by substitution:

$$\mathbb{E}\left[\eta_k, \eta_k \geq \bar{\eta}\right] = \frac{\phi\left(\sqrt{\beta}[R_k^* - r]\right)}{\beta} \tag{89}$$

Expected utility can thus be written as:

$$\begin{aligned}
EU_k &= y_k + I_k \bar{r} - (1+\chi) b_k \int_{-\infty}^{\infty} \Phi\left(\sqrt{\beta}[R_k^* - r]\right) dF(r) \\
&\quad -(I_k - b_k) \int_{-\infty}^{\infty} (r-1)\Phi\left(\sqrt{\beta}[R_k^* - r]\right) - \frac{\phi\left(\sqrt{\beta}[R_k^* - r]\right)}{\beta} dF(r)
\end{aligned} \tag{90}$$

Note that the level of your region's liquidity y_k enters directly via payoffs and indirectly via the withdrawal threshold R_k^*, while the level of the other region's lquidity y_{-k} enters indirectly through the withdrawal threshold only. We consider the partial derivatives with respect to both liquidity and diversification as well as the withdrawal threshold in turn:

$$
\begin{aligned}
\frac{\partial EU_k}{\partial y_k} &= (1-\bar{r}) + \int_{-\infty}^{\infty} (r-1)\Phi(\sqrt{\beta}[R_k^* - r]) - \frac{\phi(\sqrt{\beta}[R_k^* - r])}{\beta} dF(r) && (91)\\
&< \int_{-\infty}^{\infty} -\frac{\phi(\sqrt{\beta}[R_k^* - r])}{\beta} dF(r) < 0 && (92)\\
\frac{\partial EU_k}{\partial b_k} &< \int_{-\infty}^{\infty} -\frac{\phi(\sqrt{\beta}[R_k^* - r])}{\beta} + [(r-1) - (1+\chi)]\Phi(\sqrt{\beta}[R_{-k}^* - r]) dF(r) < 0 && (93)\\
\frac{\partial EU_k}{\partial R_k^*} &= \int_{-\infty}^{\infty} -(1+\chi)\sqrt{\beta} b_k - (I_k - b_k)\left[(r-1)\sqrt{\beta} + (R_k^* - r)\right] dF(r) && (94)
\end{aligned}
$$

The inequality of the partial derivative w.r.t. liquidity ($\frac{\partial EU_k}{\partial y_k}$) arises from the larger weight put on low realisation of the global return r by the cumulative distribution function $\Phi(\cdot)$. The inequality of the partial derivative w.r.t. diversification ($\frac{\partial EU_k}{\partial b_k}$) arises from assumption 1 and the information structure that implies $W_k(R_k^*) > W_{-k}(R_k^*, R_{-k}^*)$. Consequently, $\frac{dEU_k}{dy_k}$ and $\frac{dEU_k}{db_k}$ have ambiguous sign. The sign of the partial derivative w.r.t. the withdrawal threshold ($\frac{\partial EU_k}{\partial R^*}$) is unambigouosly negative if $\beta = 1$. The precision of the global return α can be chosen accordingly to satisfy the sufficiency constraint on equilibrium uniqueness.

Higher-order derivatives evaluated at $\beta = 1$ are:

$$
\begin{aligned}
\frac{\partial^2 EU_k}{\partial y_k^2} &= 0 = \frac{\partial^2 EU_k}{\partial b_k^2} = \frac{\partial^2 EU_k}{\partial y_k \partial b_k} && (95)\\
\frac{\partial^2 EU_k}{\partial y_k \partial R_k^*} &= (R_k^* - 1)\int_{-\infty}^{+\infty} \phi(r - R_k^*) f(r) dr > 0 && (96)\\
\frac{\partial^2 EU_k}{\partial b_k \partial R_k^*} &= (R_k^* - 1 - (1+\chi))\int_{-\infty}^{+\infty} \phi(r - R_k^*) f(r) dr < 0 && (97)\\
\frac{\partial^2 EU_k}{\partial (R_k^*)^2} &= -(I_k - b_k)\int_{-\infty}^{+\infty} \phi(r - R_k^*) dF(r) + && (98)\\
&\quad -[(R_k^* - 1)(I_k - b_k) - (1+\chi)b_k]\int_{-\infty}^{+\infty} (r - R_k^*)\phi(\cdot) dF(r) < 0
\end{aligned}
$$

where the first line states that the concavity of the objective function EU_k comes from the indirect effect on the withdrawal threshold R_k^* only, while the second and third line state second-order partial derivatives with respect to a choice variable and the withdrawal threshold, which will be used later. Some remarks for the fourth line are in order. Note that the second integral is positive. This term contains two probability distributions. First, $\phi(\cdot)$ is centered around R_k^* and would on its own imply a zero term. However, the pdf $f(r)$ that is centered around $\bar{r} > R_k^*$ pushes the joint distribution up.

8.8 Second-order conditions

First, consider the case of the competitive bank. We find the second-derivative of the objective function with respect to liquidity:

$$\frac{d^2 EU_k}{dy_k^2} = \frac{\partial^2 EU_k}{\partial y_k^2} + \frac{\partial^2 EU_k}{\partial y_k \partial R_k^*}\frac{\partial R_k^*}{\partial y_k} + \left[\frac{\partial^2 EU_k}{\partial y_k \partial R_k^*} + \frac{\partial^2 EU_k}{\partial (R_k^*)^2}\frac{\partial R_k^*}{\partial y_k}\right]\frac{\partial R_k^*}{\partial y_k} + \frac{\partial EU_k}{\partial R_k^*}\frac{\partial^2 R_k^*}{\partial y_k^2} \tag{99}$$

$$= \frac{\partial^2 EU_k}{\partial (R_k^*)^2}\left(\frac{\partial R_k^*}{\partial y_k}\right)^2 + \frac{\partial^2 EU_k}{\partial y_k \partial R_k^*}\frac{\partial R_k^*}{\partial y_k} + \frac{\partial}{\partial y_k}\underbrace{\left[-\frac{\partial EU_k}{\partial y_k} + \frac{\partial EU_k}{\partial R_k^*}\frac{\partial R_k^*}{\partial y_k}\right]}_{=0} \tag{100}$$

$$= \underbrace{\frac{\partial R_k^*}{\partial y_k}}_{-}\left[\underbrace{\frac{\partial^2 EU_k}{\partial y_k \partial R_k^*}}_{+} + \underbrace{\frac{\partial^2 EU_k}{\partial (R_k^*)^2}}_{-}\frac{\partial R_k^*}{\partial y_k}\right] < 0 \tag{101}$$

The signs of the second derivatives of the objective function are derived in Appendix (8.7).

Second, an identical argument applies for the social planner's objective function:

$$\frac{d^2(EU_k + EU_{-k})}{dy_k^2} = \frac{\partial^2 EU_k}{\partial y_k^2} + \frac{\partial^2 EU_k}{\partial y_k \partial R_k^*}\left(\frac{\partial R_k^*}{\partial y_k} + \frac{\partial R_k^*}{\partial y_{-k}}\right) + \left[\frac{\partial^2 EU_k}{\partial y_k \partial R_k^*} + \frac{\partial^2 EU_k}{\partial (R_k^*)^2}\frac{\partial R_k^*}{\partial y_k}\right]\frac{\partial R_k^*}{\partial y_k} + \cdots$$

$$\cdots + \frac{\partial EU_k}{\partial R_k^*}\left(\frac{\partial^2 R_k^*}{\partial y_k^2} + \frac{\partial^2 R_k^*}{\partial y_{-k}\partial y_k}\right)$$

$$= \frac{\partial^2 EU_k}{\partial (R_k^*)^2}\left(\frac{\partial R_k^*}{\partial y_k}\right)\left(\frac{\partial R_k^*}{\partial y_k} + \frac{\partial R_k^*}{\partial y_{-k}}\right) + \frac{\partial^2 EU_k}{\partial y_k \partial R_k^*}\left(\frac{\partial R_k^*}{\partial y_k} + \frac{\partial R_k^*}{\partial y_{-k}}\right) + \cdots$$

$$\cdots + \frac{\partial}{\partial y_k}\underbrace{\left[-\frac{\partial EU_k}{\partial y_k} + \frac{\partial EU_k}{\partial R_k^*}\left(\frac{\partial R_k^*}{\partial y_k} + \frac{\partial R_k^*}{\partial y_{-k}}\right)\right]}_{=0} < 0$$

Third, consider the case of endogenous diversification. We have:

$$\frac{d^2 EU_k}{db_k^2} = \frac{\partial R_k^*}{\partial b_k}\left[\frac{\partial^2 EU_k}{\partial b_k \partial R_k^*} + \frac{\partial^2 EU_k}{\partial (R_k^*)^2}\frac{\partial R_k^*}{\partial b_k}\right] < 0 \tag{102}$$

$$\frac{d^2 EU_k}{db_k dy_k} = \frac{\partial R_k^*}{\partial b_k}\left[\frac{\partial^2 EU_k}{\partial y_k \partial R_k^*} + \frac{\partial^2 EU_k}{\partial (R_k^*)^2}\frac{\partial R_k^*}{\partial y_k}\right] < 0 \tag{103}$$

$$\Rightarrow \frac{d^2 EU_k}{dy_k^2}\frac{d^2 EU_k}{db_k^2} - \left(\frac{d^2 EU_k}{db_k dy_k}\right)^2 > 0 \tag{104}$$

Thus,

$$\left(\frac{dy_k}{db_k}\right)_*^y = -\frac{\frac{d^2EU_k}{dy_k db_k}}{\frac{d^2EU_k}{dy_k^2}} < 0 \tag{105}$$

$$\left(\frac{dy_k}{db_k}\right)_*^{b_k} = -\frac{\frac{d^2EU_k}{db_k^2}}{\frac{d^2EU_k}{db_k dy_k}} < \left(\frac{dy_k}{db_k}\right)_*^y \tag{106}$$

and

$$\frac{d^2(EU_k + \mathrm{E}[U_{-k}])}{dy_k^2} = \left[\frac{\partial R_k^*}{\partial y_k} + \frac{\partial R_k^*}{\partial y_{-k}}\right]\left(\frac{\partial^2 EU_k}{\partial (R_k^*)^2}\left[\frac{\partial R_k^*}{\partial y_k} + \frac{\partial R_k^*}{\partial y_{-k}}\right] + \frac{\partial^2 EU_k}{\partial y_k \partial R_k^*}\right) < 0 \tag{107}$$

$$\frac{d^2(EU_k + \mathrm{E}[U_{-k}])}{dy_k db_k} = \frac{\partial R_k^*}{\partial b_k}\left(\frac{\partial^2 EU_k}{\partial (R_k^*)^2}\left[\frac{\partial R_k^*}{\partial y_k} + \frac{\partial R_k^*}{\partial y_{-k}}\right] + \frac{\partial^2 EU_k}{\partial y_k \partial R_k^*}\right) < 0 \tag{108}$$

8.9 Proof of macro-prudential liquidity buffer

Observe that the right-hand side of 20 has an additional positive term, the positive externality of holding liquidity on the other region. Thus, the social benefits from holding liquidity exceed the social cost of liquidity when evaluated at the competitive level y^*. In other words, $\left.\frac{d\mathrm{E}[U_k]+\mathrm{E}[U_{-k}]}{dy_k}\right|_{y_k=y_k^*} > 0$. Given the strict global concavity of the objective function, we obtain $y^{SP} > y^*$.

8.10 Comparative Statics

This section examines the dependence of the competitve equilibrium allocation and the (constrained) social planner's allocation on the exogenous parameters of the model.

Consider the case of exogenous diversification first. We proceed by totally differentiating the equilibrium conditions (19) and (20). Using an envelope-theorem argument, the partial derivative of the liquidity level with respect to any variable in $a \in A = \{\chi, b_k, \bar{r}\}$, can be written as:

$$\frac{\partial y^*}{\partial a} = \Gamma^* \frac{\partial R_k^*}{\partial a} \tag{109}$$

$$\frac{\partial y^{SP}}{\partial a} = \Gamma^{SP} \frac{\partial R_k^*}{\partial a}\ , \tag{110}$$

where Γ^* and Γ^{SP} are positive parameters with $\Gamma^{SP} > \Gamma^* > 0$ given by:

$$\Gamma^* = -\frac{\frac{\partial^2 EU_k}{\partial y_k \partial R_k^*} + \frac{\partial^2 EU_k}{\partial (R_k^*)^2}\frac{\partial R_k^*}{\partial y_k}}{\frac{\partial^2 EU_k}{\partial (R_k^*)^2}(\frac{\partial R_k^*}{\partial y_k})^2} \tag{111}$$

$$\Gamma^{SP} = -\frac{\frac{\partial^2 EU_k}{\partial y_k \partial R_k^*} + \frac{\partial^2 EU_k}{\partial (R_k^*)^2}[\frac{\partial R_k^*}{\partial y_k} + \frac{\partial R_k^*}{\partial y_{-k}}]}{\frac{\partial^2 EU_k}{\partial (R_k^*)^2}(\frac{\partial R_k^*}{\partial y_k} + \frac{\partial R_k^*}{\partial y_{-k}})^2} \tag{112}$$

The ranking $\Gamma^* < \Gamma^{SP}$ arises from direct differentiation with respect to the new term $\frac{\partial R_k^*}{\partial y_{-k}}$.

Consider the case of endogenous diversification, where a similar argument holds. Totally differentiating the equilibrium conditions of the private bank and the social planner and using an envelope-theorem argument, the partial derivative of the levels of liquidity and diversification with respect to the liquidation cost χ and the expected investment return $\bar{r}$, where $a' \in \{\chi, \bar{r}\}$, are:

$$\frac{\partial y^*}{\partial a} = \Gamma_y^* \frac{\partial R_k^*}{\partial a} \tag{113}$$

$$\frac{\partial y^{SP}}{\partial a} = \Gamma_y^{SP} \frac{\partial R_k^*}{\partial a} \tag{114}$$

$$\frac{\partial b^*}{\partial a} = \Gamma_b^* \frac{\partial R_k^*}{\partial a} \tag{115}$$

$$\frac{\partial b^{SP}}{\partial a} = \Gamma_b^{SP} \frac{\partial R_k^*}{\partial a} \tag{116}$$

where Γ_y^*, Γ_b^*, Γ_y^{SP}, and Γ_b^{SP} are positive parameters as insured by the respective sufficient conditions of optimality. Let $W \equiv EU_A + EU_B$ denote the total social welfare.

$$\Gamma_y^* = -\frac{\frac{d^2 EU_k}{db_k^2}[\frac{\partial^2 EU_k}{\partial y_k \partial R_k^*} + \frac{\partial^2 EU_k}{\partial (R_k^*)^2}\frac{\partial R_k^*}{\partial y_k}]}{\frac{d^2 EU_k}{dy_k^2}\frac{d^2 EU_k}{db_k^2} - (\frac{d^2 EU_k}{db_k dy_k})^2} > 0 \tag{117}$$

$$\Gamma_b^* = -\frac{\frac{d^2 EU_k}{dy_k^2}[\frac{\partial^2 EU_k}{\partial b_k \partial R_k^*} + \frac{\partial^2 EU_k}{\partial (R_k^*)^2}\frac{\partial R_k^*}{\partial b_k}]}{\frac{d^2 EU_k}{dy_k^2}\frac{d^2 EU_k}{db_k^2} - (\frac{d^2 EU_k}{db_k dy_k})^2} > 0 \tag{118}$$

$$\Gamma_y^{SP} = -\frac{\frac{d^2 W}{db_k^2}[\frac{\partial^2 EU_k}{\partial y_k \partial R_k^*} + \frac{\partial^2 EU_k}{\partial (R_k^*)^2}(\frac{\partial R_k^*}{\partial y_k} + \frac{\partial R_k^*}{\partial y_{-k}})]}{\frac{d^2 W}{dy_k^2}\frac{d^2 W}{db_k^2} - (\frac{d^2 W}{db_k dy_k})^2} > 0 \tag{119}$$

$$\Gamma_b^{SP} = -\frac{\frac{d^2 W}{dy_k^2}[\frac{\partial^2 EU_k}{\partial b_k \partial R_k^*} + \frac{\partial^2 EU_k}{\partial (R_k^*)^2}\frac{\partial R_k^*}{\partial b_k}]}{\frac{d^2 W}{dy_k^2}\frac{d^2 W}{db_k^2} - (\frac{d^2 W}{db_k dy_k})^2} > 0 \tag{120}$$

Note that:

$$\frac{d^2W}{dy_k^2} = \frac{d^2EU_k}{dy_k^2} + \frac{\partial R_k^*}{\partial y_{-k}}\left[\frac{\partial^2 EU_k}{\partial y_k \partial R_k^*} + \frac{\partial^2 EU_k}{\partial (R_k^*)^2}(\frac{\partial R_k^*}{\partial y_k} + \frac{\partial R_k^*}{\partial y_{-k}})\cdot 2\right] < \frac{d^2EU_k}{dy_k^2} \quad (121)$$

$$\frac{d^2W}{db_k^2} = \frac{d^2EU_k}{db_k^2} \quad (122)$$

$$\frac{d^2W}{dy_k db_k} = \frac{d^2EU_k}{dy_k db_k} + \frac{\partial R_k^*}{\partial y_{-k}}\left[\frac{\partial^2 EU_k}{\partial b_k \partial R_k^*} + \frac{\partial^2 EU_k}{\partial (R_k^*)^2}\frac{\partial R_k^*}{\partial b_k}\right] + \frac{\partial EU_k}{\partial R_k^*}\frac{\partial^2 R_k^*}{\partial y_k \partial b_k} > \frac{d^2EU_k}{dy_k db_k} \quad (123)$$

(124)

Hence, $\Gamma_y^{SP} > \Gamma_y^*$ and $\Gamma_b^{SP} > \Gamma_b^*$.

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