How does the Bond Market Perceive FOMC Interventions?

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Abstract

The last decades have shown that the FOMC intervenes in the business cycle. This paper analyzes how the bond market perceives such discretionary interventions. The paper sets-up an endowment economy, in which the representative investor is exposed to Knightian uncertainty about the underlying FOMC intervention model. The equilibrium endogenizes a priced and time-varying FOMC intervention premium. The estimation reveals that the FOMC is perceived as a benevolent institution that intervenes in the business cycle to boost output growth, i.e. "Greenspan put". The equilibrium slope of the real and nominal yield curve is positive and a predictor for positive bond premiums.

Keywords:

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[1] I thank my colleagues and seminar participants at Columbia Business school for helpful comments and discussions. Any remaining errors are my responsibility.

March 10, 2011

1. Introduction

At a 2010 conference, former FOMC Chairman Alan Greenspan responds to the question of whether he thinks there was a "Greenspan put" in the market during his Chairman tenure:[2]

We are responding to the economy, not to the markets, not to interest rates. We are responding essentially to what our job is, namely to stabilize the system. Now, if in effect the Greenspan put is a notion which says you are stabilizing the system, I say well I hope so. [3]

This paper analysis how the bond market perceives such FOMC interventions. Whether the FOMC intervenes in the business cycle and by which measures and strength is at its full discretion. Its dual mandate allows it to intervene into the real economy to promote growth in production/employment, while preserving stable prices.[4] While the notion of a "Greenspan put" indicates that the stock market likes the prospect of the FOMC intervening in the business cycle, because it raises expected output growth and as a result stock valuations, it is not clear how the bond market perceives these discretionary interventions.

To formally analyze the question of how the bond market perceives the

[2] The term "Greenspan put" describes the stock market perception that the FOMC, under former Chairman Greenspan, used its discretionary but mandate consistent interventions to "bail out" the stock market. Compare minutes 44 to 50 of the recorded panel discussion: *http://www.frbatlanta.org/news/conferences/10jekyll_webcast.cfm.*

[3] Compare minute 47 of the above mentioned recorded panel discussion.

[4] Compare the monetary objectives within the Federal Reserve Act: http://www.federalreserve.gov/aboutthefed/section2a.htm.

possibility of discretionary FOMC interventions into the business cycle, I utilize an equilibrium set-up with one representative agent and a business cycle component in consumption growth that is potentially systematically affected by FOMC interventions. The agent knows that the FOMC can discretionally raise or lower the business cycle component. This exposes the representative agent to Knight (1921) uncertainty about the model that best describes FOMC interventions in the business cycle.[5] In addition, I explicitly control for other sources of macro ambiguity, such as Knightian uncertainty about inflation and consumption growth.[6]

So far, there is little agreement on how FOMC interventions are perceived by the bond market. Following the standard interpretation of the literature, one might want to argue that the agent is averse towards FOMC intervention ambiguity.[7] Taking into account the dual mandate of the FOMC and the notion of a "Greenspan put", one might argue that the representative agent loves FOMC intervention ambiguity. The implications for the bond market depend dramatically on how the agent perceives potential FOMC interventions, because they determine whether the investor's belief about growth in

[5]Model uncertainty in the spirit of Knight (1921) and Ellsberg (1961) has been successful in addressing several asset pricing puzzles. For methodological contributions compare Hansen and Sargent (2008), Chen and Epstein (2002), Ghirardato and Marinacci (2002), Klibanoff et al. (2005), Epstein and Schneider (2003) and Anderson et al. (2003).

[6]Ellison and Sargent (2010) assume the FOMC itself is averse against Knightian uncertainty towards inflation and unemployment. Ulrich (2010b) studies the option market in an economy where the investor is averse against inflation and consumption ambiguity.

[7]Recent applications of ambiguity assume the representative investor is averse against ambiguity. Applications for equity markets are Epstein and Miao (2003), Uppal and Wang (2003), Maenhout (2004), Maenhout (2006), Sbuelz and Trojani (2002a), Sbuelz and Trojani (2008), Leippold et al. (2008), Trojani and Vanini (2004), Liu et al. (2005), Drechsler (2009). Applications for interest rates are Gagliardini et al. (2009b), Ulrich (2010a), Kleshecheslski and Vincent (2009).

the economy is lower or higher, compared to an economy without FOMC interventions.[8]

The equilibrium is solved in closed-form and it accounts explicitly for both perceptions about FOMC interventions. The market prices of risk and uncertainty in the economy have three components: a consumption risk premium, a premium for macro ambiguity, and a premium for FOMC intervention ambiguity. The sign of the premium for FOMC interventions depends on how the bond market perceives these discretionary interventions. If the investor is averse against these ambiguous interventions, he will lower his expected growth rate for the economy, which increases the attractiveness of bond investments. On the other hand, if the investor loves the prospect of discretionary FOMC interventions, he will price assets with a higher expected growth rate for the economy, which lowers the valuation of bond investments.

The latter scenario induces a positive term premium into the yield curve of real and nominal bonds, making them both slope upwards. This feature of the model adds a new perspective to recent general equilibrium explanations of the term structure, which can explain why the nominal yield curve slopes upwards but have problems to explain why the U.S. real term structure is upward sloping.[9]

I perform a Maximum-Likelihood estimation of the model with a panel

[8]From a methodological point of view, Ghirardato and Marinacci (2002) and Klibanoff et al. (2005) show that it is straightforward to implement ambiguity aversion and ambiguity love.

[9]The average yield curve on Treasury Inflation Protected Securities (TIPS) is upward sloping. Long-run risk models explain the upward sloping nominal yield curve but not the upward sloping real yield curve, compare Bansal and Shaliastovich (2010), Piazzesi and Schneider (2010). The habit formation model of Wachter (2006) is an alternative explanation for an upward sloping nominal and real yield curve.

of macro and bond yield data. The estimation results reveal that the investor perceives FOMC interventions as "benevolent" actions. This means that the representative agent beliefs the FOMC employs within its discretionary power the most favorable (in terms of high expected consumption growth) intervention model. Such a belief is consistent with the notion of a "Greenspan put". At the same time, this implies a positive term premium and positive excess holding period returns for real and nominal bonds. The intuition is that the agent's love for FOMC intervention ambiguity makes the investor expect a higher growth path for the economy, which lowers the value of real and nominal bonds, because these assets pay out well in periods of low growth (recession hedge).

The estimation reveals the following additional results. First, due to model ambiguity the real and nominal yield curve slope upwards. Real bonds are a good hedge against macro ambiguity, which leads to a negative and downward sloping term premium in real bonds. The term premium for FOMC intervention ambiguity is positive, upward sloping, and it dominates the downward sloping term premium that is induced by macro ambiguity.

Second, the FOMC intervention ambiguity premium was highest during the FOMC Chairmanship of Paul Volcker and lowest during the first term of FOMC Chairman Ben Bernanke. Third, a variance decomposition of model implied yields implies that changes in the amount of FOMC intervention ambiguity explains 11% (65%) of variations in the 10-year nominal yield (real yield).

Fourth, an impulse response analysis which measures how a one percent increase in FOMC intervention ambiguity affects the yield curve and bond excess returns concludes the following. Real and nominal yields are affected the same way. The level of the yield curve goes up by 60 basis points, the yield curve steepens by 30 basis points and the expected excess return of a 10-year bond increases by 5.8%. It takes five years for the one percent increase in FOMC intervention ambiguity to die out.

The rest of the paper is organized as follows. Section 2 sets up the model for the real economy, solves the Knight (1921) uncertainty problem, and derives the equilibrium results for the real economy. Section 3 derives the equilibrium results for the nominal economy. Section 4 performs the estimation of the model and summarizes empirical findings. Section 5 checks the robustness of the identification strategy of ambiguity premia. Chapter 6 concludes the paper. Technical details, proofs of all propositions and the likelihood specification, are collected in the appendix.

2. The Model for the Real Economy

The complete filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, Q^0)$ describes the reference belief in the economy and expectations under that belief are denoted as $E[.]$. All Brownian motions in the economy are pairwise orthogonal.

2.1. Reference Belief

The representative logarithmic utility investor faces model ambiguity with regard to three economic shocks that affect the business cycle component z of expected consumption growth $E_t[d \ln c_t]$: shocks to expected inflation W^w, FOMC interventions W^f, shocks to real economic growth (W^r)

$$E_t\left[d\ln c_t\right] := (c_0 + z_t)dt \tag{1}$$

$$dz_t := \kappa_z z_t dt + \sigma_z \cdot dW_t^z \tag{2}$$

$$\sigma_z \cdot dW_t^z := \sigma_{1z} dW_t^r + \sigma_{2z} dW_t^w + \sigma_{3z} dW_t^f \tag{3}$$

with $c_0 \in \mathcal{R}^+$, $\kappa_z \in \mathcal{R}^-$, $\sigma_{1z} \in \mathcal{R}^+$, $\sigma_{2z} \in \mathcal{R}^-$, $\sigma_{3z} \in \mathcal{R}^-$.[10]

Shock $\sigma_{2z} dW^w$ captures in "reduced-form" short-term consumption non-neutrality of expected inflation. Volatility component $\sigma_{2z} < 0$ implies that an unexpected increase in expected inflation leads to an unexpected fall in z (Piazzesi and Schneider (2006), Ulrich (2010a)). Similarly, shock $\sigma_{3z} dW^f$ is a "reduced-form" description of short-term real effects of extra liquidity that the FOMC provides to the economy, for example along the business cycle or immediately after financial market crashes. Volatility component $\sigma_{3z} < 0$ models that an expansionary Fed policy, defined as $W^f < 0$, leads to a short-lived increase in z.[11]

The conditional volatility of consumption growth is assumed to be homoscedastic

$$d\ln c_t - E_t\left[d\ln c_t\right] = \sigma_c dW^c \tag{4}$$

with $\sigma_c \in \mathcal{R}^+$. Shock W^w is assumed to drive time-variations in expected

[10]The logarithmic utility set-up is advantageous for emphasizing the main implications, while allowing a high degree of tractability.

[11]My analysis focuses on how the bond market perceives these interventions and I therefore do not model the FOMC decision of why it intervenes or through which channels it achieves the intervention. Instead I take this intervention as given.

inflation

$$E_t\left[d\ln p_t\right] := (p_0 + w_t)dt \tag{5}$$

$$dw_t := \kappa_w w_t dt + \sigma_w dW_t^w \tag{6}$$

where p_t is the aggregate price level, $p_0 \in \mathcal{R}^+$, $\sigma_w \in \mathcal{R}^+$, $\kappa_w \in \mathcal{R}^-$.

2.2. Model Misspecification

Similar to the modeling of risk, working with model ambiguity requires assumptions about the investor's "ambiguity attitude" and the amount of "revealed ambiguity" (Ghirardato et al. (2004), Klibanoff et al. (2005) and Klibanoff et al. (2009)).[12]

"Revealed ambiguity" is characterized by an observed (revealed) non-singleton set of probability measures that could all be the probability measure of the economy. The "ambiguity attitude" of an investor determines which probability model from the set of models describes the investor's belief. It can be the worst-case probability measure (ambiguity aversion), the best-case probability measure (ambiguity loving) or any convex combination of both.[13]

2.2.1. Amount of "Revealed Ambiguity"

I characterize the amount of revealed ambiguity by a time-varying class of probability models or perturbations, Q^h, that are absolutely continuous with respect to Q^0 ((Chen and Epstein (2002)). Time-varying perturbations of the reference belief coincide with stochastic instantaneous drift distortions to W^z. This means that $W_t^{z,h} = W_t^z + \int_0^t h_s ds$ defines a Brownian motion

[12]The analog for modeling risk are risk attitude of the investor and riskiness of an event.
[13]Ghirardato et al. (2004) call the latter α-maxmin expected utility with multiple priors.

process under the distorted belief Q^h. As I will explain in the next section, the investor's attitude towards ambiguity (ambiguity aversion or ambiguity love) makes him lower or increase his belief about the instantaneous expected growth grate of $d \ln c_t$ by $\sigma_z \cdot h_t dt$, compared to the reference belief. Proposition 1 will show that h_t is an endogenous outcome of the equilibrium.

The investor quantifies in every period the amount of revealed ambiguity through a likelihood ratio test. Such a test measures whether a distorted belief or the reference belief generated the realized observation of z. I denote such a likelihood ratio with a_T where T symbolizes the number of observations that are used to compare models

$$a_T := \frac{dQ_T^h}{dQ_T^0} = \exp\left(-\frac{1}{2}\int_0^T h_t' h_t dt + \int_0^T h_t \cdot dW_t^z\right) \tag{7}$$

with $h_t = (h_t^r, h_t^w, h_f^f)'$ being instantaneous distortions to the business cycle shocks (W^r, W^w, W^f). This likelihood ratio is also called relative entropy between a distorted model Q^h and the reference model Q^0 (Chen and Epstein (2002), Anderson et al. (2003)).

The investor does not fully trust his reference model for z. He acknowledges that the true data generating process of z could be any distorted model Q^h within the set of potential models. Before seeing a new realization of z in $t + dt$, the investor expects in t to experience at worst an instantaneous change in the log-likelihood ratio (under the distorted belief) of

$$\frac{1}{2}(h_t^r)^2 dt + \frac{1}{2}(h_t^w)^2 dt + \frac{1}{2}(h_t^f)^2 dt, \tag{8}$$

where I assume that

$$\frac{1}{2}(h_t^i)^2 dt \leq A_i \left(\eta_t^i\right)^2 dt, \quad \forall i \in \{r, w, f\} \; \forall t \geq 0 \tag{9}$$

with $A_i > 0$ and the process $(\eta_t^i)^2$ being observed. The last assumption has several interesting implications. First, it postulates separate relative entropy bounds for each ambiguous business cycle shock.[14] Second, each bound is time-varying which allows for a time-varying set of models.[15] Third, time-variation in the amount of revealed ambiguity is driven by $(\eta_t^i)^2$, while A_i is a positive scaling parameter. Periods of higher model ambiguity coincide with rising $(\eta_t^i)^2$. Periods in which $(\eta_t^i)^2 \to 0$ coincide with the rational expectations set-up of a single model. Fourth, such a modeling of revealed ambiguity is consistent with a rectangular set of priors and supports a dynamically consistent preference ordering (Chen and Epstein (2002), Epstein and Schneider (2003)).

The process $(\eta^i)^2$, $\forall i \in \{r, w, f\}$, is observed and positive for each time period and state of the world. For analytical convenience I model η to follow an Ornstein-Uhlenbeck process

$$d\eta_t^i = (a_{\eta^i} + \kappa_{\eta^i}\eta_t^i)dt + \sigma_{\eta^i} dW^{\eta^i}, \quad \forall i \in \{r, w, f\} \tag{10}$$

with $a_{\eta^i} \in \mathcal{R}^+$, $\kappa_{\eta^i} \in \mathcal{R}^-$, $\sigma_{\eta^i} \in \mathcal{R}^+$.

2.2.2. Ambiguity Attitude

The sources of ambiguity are grouped into *macro ambiguity* and *FOMC intervention ambiguity*. The former denotes ambiguity towards (W^r, W^w), while the latter refers to ambiguity towards W^f.

[14] Separate relative entropy bounds allow for greater analytical flexibility compared to one joint relative entropy bound as in Gagliardini et al. (2009b) and Gagliardini et al. (2009a). Separate bounds were introduced in Chen and Epstein (2002) and applied by Ulrich (2010b).

[15] Time-varying bounds for continuous-time models were introduced by Chen and Epstein (2002) and applied by Sbuelz and Trojani (2002b), Sbuelz and Trojani (2008), Ulrich (2010a), Ulrich (2010b), Drechsler (2009).

I assume the investor has a constant ambiguity attitude for each group of ambiguity. I follow the literature and assume the investor's attitude is to be averse towards *macro ambiguity*. I implement this by assuming that the representative investor is concerned about worst-case instantaneous distortions to (W^r, W^w).[16] Moreover, I assume the investor's attitude towards ambiguity about discretionary *FOMC interventions* can be either aversion or love. An ambiguity love attitude characterizes an investor who hopes the best-case (in terms of high consumption growth) distortion to W^f is actually the correct model of discretionary FOMC interventions.

The dual mandate of the FOMC could be a natural motivation for the investor's ambiguity loving attitude towards discretionary *FOMC interventions*. The 1977 amendment to the Federal Reserve Act assigns two goals to the FOMC: to promote growth in production/employment and stable prices.[17] This dual mandate is different to other central bank mandates like the one the European Central Bank or the Bank of England have, which focus on the single (or hierarchial) mandate of "stable prices".[18] The out-

[16]Ellison and Sargent (2010) assume the FOMC itself is confronted and averse against macro ambiguity, which corresponds to inflation and unemployment in their set-up. Ulrich (2010b) assumes the representative investor is averse against macro ambiguity and studies implications for the option market.

[17]The Federal Reserve Act specifies the following monetary policy objectives: "The Board of Governors of the Federal Reserve System and the Federal Open Market Committee shall maintain long run growth of the monetary and credit aggregates commensurate with the economy's long run potential to increase production, so as to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates.". Compare http://www.federalreserve.gov/aboutthefed/section2a.htm.

[18]Article 2 of the Statute of the ECB specifies: "... the primary objective of the ESCB shall be to maintain price stability." Compare $http://www.ecb.int/ecb/legal/pdf/en\_statute_2.pdf$.
The Bank of England Act 1998 specifies the objectives of the Bank of England as follows: "In relation to monetary policy, the objectives of the Bank of England shall be (a)

put stabilization mandate of the FOMC might make investors hope that discretionary FOMC interventions in the business cycle follow the best possible model, compared to the reference belief of no systematic impact, i.e. $E[W^f] = 0$.

An ambiguity loving attitude towards discretionary FOMC interventions implies that the representative investor beliefs that from the class of discretionary FOMC intervention models, it is the most favorable model (in terms of high expected consumption growth) that the FOMC follows. Thus, love for *FOMC intervention ambiguity* incorporates the belief that discretionary and mandate consistent FOMC interventions support the business cycle. He therefore, subjectively expects a higher consumption growth path, compared to the reference belief, which such an investor could interpret as a "Greenspan put".

To summarize, ambiguity aversion and ambiguity love are modeled, respectively, by assuming that the representative investor solves (i) a max-min consumption optimization with respect to the set of models for ambiguous macro shocks (W_t^r, W_t^w) and (ii) a max-min or max-max consumption optimization with respect to the family of ambiguous FOMC intervention models for W^f. The set of models is constrained by time-varying and positive relative entropy bounds $A_i(\eta_t^i)^2$, $i \in \{r, w, f\}, t \geq 0$. Ultimately, this implies that investors optimize consumption with respect to a (i) lower and (ii) lower or higher drift distortion, generated by a constant attitude towards ambiguity

to maintain price stability and (b) subject to that, support the economic policy of Her Majesty's Government, including its objectives for growth and employment.". Compare $http://www.bankofengland.co.uk/about/legislation/1998act.pdf$.

on (i) (W^r, W^w) and (ii) W^f, respectively.

2.3. Min-Max Expected Utility

If the representative investor was ambiguity averse with regard to *macro ambiguity* and *FOMC intervention ambiguity* he would formally solve

$$\min_{\{h_t^r, h_t^w, h_t^f\}} E^H \left[\int_0^\infty e^{-\rho t} \ln c_t dt \right] \tag{11}$$

$$s.t. H := \{H \in \mathcal{H} : \frac{1}{2}(h_t^i)^2 \leq A_i(\eta_t^i)^2 \ \ (\forall i \in \{r, w, f\}, \forall t \geq 0)\} \tag{12}$$

with $\mathcal{H}$ being a well defined set of probability measures and $\rho \in \mathcal{R}^+$ being the subjective time discount factor.

On the other hand, if the investor is averse with respect to *macro ambiguity* but loves *FOMC intervention ambiguity* he will solve the following mathematical problem

$$\min_{\{h_t^r, h_t^w\}} \max_{\{h_t^f\}} E^H \left[\int_0^\infty e^{-\rho t} \ln c_t dt \right] \tag{13}$$

$$s.t. H := \{H \in \mathcal{H} : \frac{1}{2}(h_t^i)^2 \leq A_i(\eta_t^i)^2 \ \ (\forall i \in \{r, w, f\}, \forall t \geq 0)\}. \tag{14}$$

The following proposition summarizes the solution to these optimization problems.[19]

Proposition 1. *The optimal amount of instantaneous perturbations in the ambiguous business cycle shocks* (W^r, W^w) *are*

$$h_t^r dt = m_r \eta_t^r dt, \quad m_r \equiv -\sqrt{2A_r} \tag{15}$$

$$h_t^w dt = m_w \eta_t^w dt, \quad m_w \equiv \sqrt{2A_w}. \tag{16}$$

[19]The solution methodology follows other research, such as Ulrich (2010b), Chen and Epstein (2002), Sbuelz and Trojani (2002a), Sbuelz and Trojani (2008) and others.

The optimal amount of instantaneous perturbations in the ambiguous FOMC intervention W^f is

$$h_t^f dt = m_f \eta_t^f dt \tag{17}$$

with $m_f \equiv \sqrt{2A_f} > 0$ *if the investor is averse, while* $m_f \equiv -\sqrt{2A_f} < 0$ *if the investor loves* FOMC intervention ambiguity.[20]

The intuition of the implications of the proposition are as follows. First, compared to the reference belief, aversion against *macro ambiguity* makes the investor expect lower instantaneous consumption growth, i.e. $\sigma_{1z} m_r \eta_t^r dt < 0$ and $\sigma_{2z} m_w \eta_t^w dt < 0$. Second, if the investor is averse against *FOMC intervention ambiguity*, his expected instantaneous growth rate for consumption would be lower than under the reference belief, i.e. $\sigma_{3z} m_f \eta_t^f dt < 0$. Third, if the investor loves *FOMC intervention ambiguity* his distorted instantaneous expected growth rate for consumption is higher than under the reference belief, i.e. $\sigma_{3z} m_f \eta_t^f dt > 0$. The last distortion depends multiplicatively on (i) the amount of revealed FOMC ambiguity ($\eta_t^f > 0$), (ii) the investor's ambiguity loving attitude ($m_f < 0$) and (iii) the magnitude with which FOMC interventions affect the business cycle ($\sigma_{3z} < 0$).

2.4. Real Yield Curve

The price of a real bond with maturity in $\tau \in \mathcal{R}^+$ periods is denoted as $B_t(\tau)$.

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[20]The assumed Gaussian dynamic for η implies that in a mathematically more rigorous formulation one could write the results in Proposition 1 in terms of $|\eta|$. Given that I use an observed and strictly positive process for η in the empirical section, I simplify notation and use η instead of $|\eta|$.

Proposition 2. *The equilibrium price of a real bond is*

$$B_t(\tau) = e^{A(\tau)+B_z(\tau)z_t+\sum_{i\in\{r,w,f\}} B_{\eta^i}(\tau)\eta_t^i} \tag{18}$$

with $B_z(\tau)$, $B_{\eta^i}(\tau)$, $\forall i \in \{r, w, f\}$ and $A(\tau)$ being deterministic functions of the economy and fully characterized in the appendix.

I denote the FOMC premium in the real yield curve with $FP_t(\tau)$.

Proposition 3. *The term premium for ambiguous FOMC interventions equals*

$$FP_t(\tau) = a_{PF}(\tau) - \frac{1}{\tau} B_{\eta^f}(\tau)\eta_t^f \tag{19}$$

where $a_{PF}(\tau)$ is a deterministic function of time to maturity and fully specified in the appendix.

The FOMC premium affects the entire term structure of real bonds. Its time-variation is driven by variations in η_t^f. Abstracting from the small impact of precautionary savings in $a_{PF}(\tau)$, $FP_t(\tau)$ is positive if the investor perceives FOMC interventions as benevolent and $FP_t(\tau)$ is negative if the investor perceives FOMC interventions as malevolent.[21]

Proposition 3 implies that if the representative agent perceives FOMC interventions as ambiguous and benevolent, he requires a positive term premium for bonds. This will make the real yield curve slope upwards. This result is intuitive for the following reason. The FOMC mandate to increase production/employment makes the investor belief that consumption might

[21] A "benevolent" player is someone who chooses from a set of multiple models the model that maximizes the expected life-time utility of the investor. On the opposite, a "malevolent" player is someone who chooses from a set of multiple models the model that is most harmful for the investor's expected life-time utility.

be higher than the reference model predicts. This is advantageous for stocks because they tend to benefit from strong growth in the economy. Investors typically shift money from bonds to stocks during periods of strong growth, which leads to falling bond prices.[22] In general equilibrium, the bond market must stay in zero net supply which forces bonds to offer an additional positive premium for these "benevolent" and growth generating ambiguous FOMC interventions. The longer the duration of the bond the higher the positive FOMC intervention premium because different models differ more substantially in their growth forecast the longer the forecast horizon.

If the agent perceives business cycle interventions of the FOMC as benevolent, the slope of the real yield curve will go up in periods of higher η^f. This happens because the gap between the best-case belief and the reference belief increases if η^f increases. The difference in expected growth prospects under both beliefs widens the further out the prediction. If the best-case belief was indeed true, long-term real bonds would return significantly less than under the reference belief. As a result, long-term bond yields increase more than short-term bond yields, leading to a steepening of the real yield curve.

I denote the macro ambiguity premium in the real yield curve with $MP_t(\tau)$.

Proposition 4. *The term premium in the real yield curve for macro ambiguity equals*

$$MP_t(\tau) = a_{MP}(\tau) - \frac{1}{\tau}\left(B_{\eta^w}(\tau)\eta_t^w + B_{\eta^r}(\tau)\eta_t^r\right) \tag{20}$$

[22]A standard consumption-based Euler equation confirms that the price of real and nominal bonds falls when growth in the economy picks up.

where $a_{MP}(\tau)$ is a deterministic function of time to maturity and fully specified in the appendix.

The macro ambiguity term premium affects the entire yield curve and varies over time with the time-varying amount of revealed *macro ambiguity*.

Proposition 4 implies that an increase in the observed amount of *macro ambiguity* leads to a lowering of real yields, while long-term yields fall particularly strongly. The term premium for macro ambiguity is negative and downward sloping for the real yield curve (Ulrich (2010b)). The reason for this behavior is that an investor who is averse against *macro ambiguity* lowers his expected growth prospects, compared to his reference belief. Real bonds would pay off better, compared to the reference belief, if the worst-case model for (W^r, W^w) was indeed the correct model. In general equilibrium, the real yield must fall, compared to the reference belief, so that the market for real bonds remains in zero net supply.

If the investor perceives business cycle interventions of the FOMC as benevolent, the overall term premium in the real yield curve can be positive and negative, depending on whether $FP_t(\tau)$ or $MP_t(\tau)$ dominates. If the investor perceived business cycle interventions of the FOMC as malevolent, the real yield curve would be downward sloping.

2.5. Market Price of Risk and Market Price of Model Ambiguity

The real stochastic discount factor (SDF), M^r, shows that shocks to consumption and shocks to aggregate model ambiguity are priced in the economy

$$M^r_{t,t+\Delta} = e^{-\rho\Delta}\left(\frac{c_{t+\Delta}}{c_t}\right)^{-1}\frac{a_{t+\Delta}}{a_t}. \tag{21}$$

The market price of risk, $\sigma_c \in \mathcal{R}^+$, compensates for unpredictable instantaneous variations in aggregate consumption. The market price for model ambiguity, $-h_t = (-h_t^r, -h_t^w, -h_t^f)$, rewards for ambiguous shocks to z, i.e. (W^r, W^w, W^f).

The market price for model ambiguity is time-varying and increasing in periods where the set of potential models increases. In periods where the set of potential models converges to a single model, i.e. $(\eta_t^i)^2 \to 0$, $\forall i \in \{r, w, f\}$, it is the market price of risk that dominates because the market price of model ambiguity converges to zero. In states of high model uncertainty it is the market price of model ambiguity that dominates the market price of risk.

The unconditional expected value of the market price of ambiguity is non-zero, i.e. $-m_i \frac{a_{\eta^i}}{\kappa_{\eta^i}} \neq 0$, $\forall i \in \{r, w, f\}$. This makes the ambiguity model observationally distinct from models with time-varying biases in beliefs that arise in heterogeneous economies with a single model. [23]

2.6. *Risk Premium and Model Ambiguity Premium in Excess Returns of Real Bonds*

Standard results reveal that real bonds carry a bond premium if the bond return correlates with M^r (Duffie (2001)). The instantaneous risk premium for a τ maturity real bond is defined as

$$xHPR_t(\tau) = - < \frac{dM_t^r}{M_t^r}, \frac{dB_t(\tau)}{B_t(\tau)} > \quad =< \frac{dc_t}{c_t}, \frac{dB_t(\tau)}{B_t(\tau)} > - < \frac{da_t}{a_t}, \frac{dB_t(\tau)}{B_t(\tau)} > \tag{22}$$

[23] Basak (2005), Buraschi and Jiltsov (2006), Dumas et al. (2009) are examples where the expected value of the market price for differences in opinions (sentiment) converges to zero in steady state. Xiong and Yan (2010) show that under special assumptions on the overconfidence of investors and without a learning algorithm differences in belief might not converge to zero in steady state.

where $< dx, dy >$ denotes the instantaneous covariation between x and y.

The risk premium in excess returns is zero, i.e. $< \frac{dc_t}{c_t}, \frac{dB_t(\tau)}{B_t(\tau)} >= 0$, because the state variables are orthogonal to W^c. Excess returns of real bonds contain a model ambiguity premium because the return on bonds correlates with changes in the perceived amount of model ambiguity. The ambiguity premium in bond returns equals

$$- < \frac{da_t}{a_t}, \frac{dB_t(\tau)}{B_t(\tau)} > = -B_z(\tau)\left(\sigma_{1z}h_t^r + \sigma_{2z}h_t^w + \sigma_{3z}h_t^f\right)dt. \qquad (23)$$

where $-B_z(\tau) \in \mathcal{R}^+$, $\sigma_{1z}h_t^r \in \mathcal{R}^-$, $\sigma_{2z}h_t^w \in \mathcal{R}^-$.[24] The term $\sigma_{3z}h_t^f \in \mathcal{R}^-$ if the investor is ambiguity averse towards discretionary FOMC interventions and $\sigma_{3z}h_t^f \in \mathcal{R}^+$ if the investor loves ambiguity about discretionary FOMC interventions.

Equation (23) confirms that macro ambiguity induces a negative bond premium, while the sign of the *FOMC intervention ambiguity* premium can be positive and negative, depending on how the investor perceives FOMC interventions. The latter premium is positive if the investor loves *FOMC intervention ambiguity*, which coincides with an investor who perceives discretionary FOMC interventions as benevolent actions (Ghirardato and Marinacci (2002), Klibanoff et al. (2005)). On the other hand, if the investor believes the FOMC uses its output mandate in a "malevolent" fashion, he would be averse against ambiguous FOMC interventions and the resulting ambiguity premium in real bonds would be negative.

The reason that aversion against *macro ambiguity* and *FOMC interven-*

[24]The structure of the bond premium is consistent with Gagliardini et al. (2009a, 2009b), who show that factors that are uncorrelated with aggregate consumption can be the main driver for the bond premium.

tion ambiguity generates a negative bond premium is that real bonds pay out unexpectedly well in periods where the investor mistrusts his reference model more than anticipated. In the model, it is only if the investor loves *FOMC intervention ambiguity* that the bond premium can switch sign over the business cycle, depending on whether it dominates the negative macro ambiguity premium.

3. The Model for the Nominal Economy

I complete the structure of the nominal economy by assuming a constant volatility for inflation

$$d\ln p_t - E_t[d\ln p_t] = \bar{\sigma}_p dW^c + \sigma_p dW^p \tag{24}$$

with $\bar{\sigma}_p \in \mathcal{R}$ and $\sigma_p \in \mathcal{R}^+$.

3.1. Nominal Yield Curve

The price of a nominal bond with maturity in $\tau \in \mathcal{R}^+$ periods is denoted as $N_t(\tau)$.

Proposition 5. *The equilibrium price of a nominal bond is*

$$N_t(\tau) = e^{A^\$(\tau)+B_z(\tau)z_t+B^\$_w(\tau)w_t+\sum_{j\in\{r,f\}}B_{\eta^j}(\tau)\eta^j_t+B^\$_{\eta^w}(\tau)\eta^w_t} \tag{25}$$

with $B_z(\tau), B_{\eta^r}(\tau), B_{\eta^f(\tau)}$ *loadings being the same as for real bonds. All nominal bond loadings are derived in the appendix.*

The nominal yield curve inherits the FOMC premium from the real yield curve. The following proposition summarizes this result.

Proposition 6. *The real and nominal yield curve share the same FOMC term premium*

$$FP_t(\tau) = a_{PF}(\tau) - \frac{1}{\tau} B_{\eta^f}(\tau)\eta_t^f \tag{26}$$

where $a_{PF}(\tau)$ is a deterministic function of time to maturity and fully specified in the appendix.

3.2. Inflation Premium

Ulrich (2010a) shows that the inflation premium in a τ-period nominal bond price coincides with

$$cov_t\left(M^r_{t,t+\tau}, \frac{p_t}{p_{t+\tau}}\right) = e^{-\rho\tau} cov_t\left(\frac{c_t}{c_{t+\tau}} \frac{a_{t+\tau}}{a_t}, \frac{p_t}{p_{t+\tau}}\right). \tag{27}$$

The inflation premium has two components: (i) an inflation risk premium and (ii) an inflation ambiguity premium.

The inflation risk premium arises in equilibrium because aggregate consumption and the aggregate price index are correlated. In the model the inflation risk premium in nominal bond yields, $IRP_t(\tau)$, is constant,

$$IRP_t(\tau) = -\sigma_c \bar{\sigma}_p. \tag{28}$$

The inflation ambiguity premium emerges in equilibrium because periods of increased model mistrust about the dynamics of expected inflation (W^w) and ultimately about z, coincide with periods in which the real value of nominal bonds falls. Being long a nominal bond is a bad hedge against an increase in inflation ambiguity. Similar to Ulrich (2010a), the inflation ambiguity premium, $IAP_t(\tau)$, depends on the perceived amount of inflation ambiguity. The following proposition summarizes the result:

Proposition 7. *The inflation ambiguity premium in nominal bond yields, $IAP_t(\tau)$, is affine in the perceived amount of inflation ambiguity*

$$IAP_t(\tau) = a_{IAP}(\tau) - \frac{1}{\tau} B^{\$}_{\eta^w}(\tau)\eta^w_t \tag{29}$$

where $a_{IAP}(\tau)$ is a deterministic function of time to maturity and fully specified in the appendix.

The inflation ambiguity premium increases in periods where the set of potential inflation models increases. It converges to zero if $(\eta^w_t)^2 \to 0$, which coincides with the rational expectations world where the investor knows the unique model that drives W^w. FOMC interventions W^f do not affect the inflation premium because W^f shocks are orthogonal to shocks to inflation (W^p and W^w).

3.3. Market Price of Risk and Market Price of Model Ambiguity

The nominal stochastic discount factor $M^{\$}$ is given by

$$-\frac{dM^{\$}_t}{M^{\$}_t} = R_t dt + (\sigma_c + \bar{\sigma}_p)dW^c + \sigma_p dW^p - h_t \cdot dW^z_t \tag{30}$$

$$R_t = c_0 + p_0 - \rho - \frac{1}{2}(\sigma_c^2 + \bar{\sigma}_p^2 + \sigma_p^2) - \bar{\sigma}_p\sigma_c + z_t + w_t \tag{31}$$

where R is the nominal short rate. The market price for consumption risk is $\sigma_c + \bar{\sigma}_p \in \mathcal{R}$ and the market price for inflation risk is $\sigma_p \in \mathcal{R}^+$. The market price for model ambiguity, $-h_t = (-h^r_t, -h^w_t, -h^f_t)$, is the same for nominal and real assets.

Time-variations in the market prices of model ambiguity are entirely driven by the time-varying amount of revealed model ambiguity. The comments for the market price of ambiguity from the real economy hold also for the nominal economy.

3.4. Risk and Model Ambiguity Premium in Excess Returns of Nominal Bonds

The instantaneous bond premium that investors earn when holding a nominal bond coincides with the bond premium of real bonds

$$xHPR_t(\tau) = - < \frac{dM_t^\$}{M_t^\$}, \frac{dN_t(\tau)}{N_t(\tau)} >= -B_z(\tau) \left(\sigma_{1z} h_t^r + \sigma_{2z} h_t^w + \sigma_{3z} h_t^f \right) dt, \tag{32}$$

where $-B_z(\tau) \in \mathcal{R}^+$, $\sigma_{1z} h_t^r \in \mathcal{R}^-$, $\sigma_{2z} h_t^w \in \mathcal{R}^-$. The term $\sigma_{3z} h_t^f \in \mathcal{R}^-$ if the investor is averse against *FOMC intervention ambiguity* and $\sigma_{3z} h_t^f \in \mathcal{R}^+$ if the investor loves *FOMC intervention ambiguity.*

Excess returns of nominal bonds are time-varying although consumption and inflation shocks are constant. The expected excess return on nominal bonds carries a positive FOMC intervention premium if ambiguous FOMC interventions W^f are perceived to be benevolent actions of the Fed.

The slope of the nominal yield curve steepens if the amount of ambiguity about expected inflation increases and/or if the dynamics of FOMC interventions are perceived to be more ambiguous, assuming the representative investor regards these interventions as benevolent. The latter case leads also to a steepening of the real yield curve and an increase in the expected bond excess returns for real and nominal bonds.

The upward sloping yield curve of U.S. Treasury Inflation-Protected Securities (TIPS) favors a positive premium for *FOMC intervention ambiguity* over the positive inflation ambiguity premium, because the latter induces a negative slope on the real yield curve, while the premium for *FOMC intervention ambiguity* induces a positive slope on real and nominal yields.

4. Empirical Analysis

For the empirical analysis, I discretize the Gaussian model (Euler-Marujama method) to estimate it with quarterly data. It is estimated with maximum-likelihood and a panel of quarterly macro variables (1981.III - 2009.II), nominal bond yields of ten different maturities (1981.III - 2009.II) and real bond yields of six different maturities (2003.I - 2009.II). The next section explains the data in more detail, the identification of the state variables, the estimation methodology and empirical results. The section finishes with a robustness section which discusses and contrasts in more detail the proposed empirical methodology to alternative specifications.

4.1. Macro and Bond Data

Because of better data availability I focus on GDP data instead of consumption data. Realized GDP growth and realized inflation are matched with the quarterly continuously compounded GDP growth rate and the quarterly continuously compounded GDP implicit price deflator growth rate, respectively. The nominal short rate R is matched with the fed funds rate. All three series are published by the FRED data base and are from 1981.III to 2009.II.

State variables z and w are identified with the demeaned one quarter ahead median forecast of GDP growth and inflation, respectively. The forecasts are published by the Survey of Professional Forecasters (SPF).[25] I use c_0 and p_0 to demean the respective median forecast, where c_0 is matched with

[25] Compare: $http://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/$.

the sample mean of quarterly GDP growth and p_0 is set to coincide with the sample mean of quarterly inflation.

The state variables that capture the observed amount of model ambiguity, η_t^i, $\forall i \in \{r, w, f\}$, are identified with the dispersion in the corresponding one quarter ahead SPF forecasts. In detail, $(\eta_t^r)^2$ and $(\eta_t^w)^2$ coincide with the cross-sectional variance in time t of one quarter ahead GDP growth and inflation SPF forecasts. This follows Anderson et al. (2009) and Ulrich (2010b).[26] Forecasts for ambiguity towards FOMC interventions are not directly accessible. The next paragraph explains the methodology that I apply to indirectly infer $(\eta_t^f)^2$ from SPF data.

For each quarter, I use the SPF forecast data on the 3-month T-bill, one-quarter ahead output growth and one-quarter ahead inflation from each investor to estimate a cross-sectional Taylor rule. Based on work of Woodford (2003), Ang et al. (2008), Taylor (1993), Clarida and Gertler (1997), Clarida et al. (2000), researchers and investors usually interpret deviations of the nominal short rate from such an interest rate rule as an FOMC intervention. I apply this insight and use for each quarter the cross-sectional variance of the Taylor rule error to match $(\eta_t^f)^2$. Periods of high η_t^f coincide with periods in which the Taylor rule implies very different FOMC intervention forecasts. Periods where $(\eta_t^f)^2$ is close to zero coincide with periods in which the Taylor rule implies a very small set of potential FOMC intervention dynamics and therefore a very small amount of *FOMC intervention ambiguity.*

[26] The robustness section provides details on results of Patton and Timmerman (2010) who find that model ambiguity is a likely explanation for why macroeconomic forecasters disagree.

The theoretical implications for the FOMC premium and the interpretation of how the bond market perceives FOMC interventions are robust with regard to how the time-series of *FOMC intervention ambiguity* is approximated. A different approximation, or even a latent factor approach, changes only the time-series of the FOMC intervention ambiguity premium, but none of the main equilibrium interpretations and conclusions. The advantage of my approximation is that it treats η^f as observable and it extracts that series from a benchmark tool in monetary policy (Taylor rule) together with high quality forecast data that is provided by the SPF.

The quarterly bond yield data consists of continuously compounded yields on nominal U.S. Treasury bonds for maturity 1,2,3,4,5,6,7,8,9,10 years and continuously compounded yields on CPI-indexed U.S. Treasury bonds of maturity 5,6,7,8,9,10 years. The latter are used as a proxy for real government bond yields. The yield data is published by the Board of Governors of the Federal Reserve System.

4.2. Econometric Methodology

The model is estimated with Maximum-Likelihood (ML). The model has five Gaussian state equations ($w, z, \eta^i, \ \forall i \in \{r, w, f\}$) and nineteen measurement equations (GDP growth, inflation, fed funds rate, nominal yields, real yields). The measurement errors for the yields and the fed funds rate are assumed to be Gaussian and orthogonal to each other and to the other processes in the economy.[27]

[27] An implication of Trojani et al. (2010) is that this standard assumption implies that the finite sample distribution of estimated ambiguity and risk premia is independent of whether ML or robust ML is applied. Trojani et al. (2010) provide additional examples that show that such an assumption might not be innocuous.

The panel of yields dominates the likelihood. I use the following procedure to ensure that the macro dynamics are consistent with the data counterpart. First, I estimate the state equations to obtain confidence intervals for the state parameters. During the ML estimation, I constrain these parameters to lie within the 99% confidence interval. Second, I constrain the norm $||\sigma_z||$ to be smaller than 0.003^2 which prevents an amplification of the model ambiguity premium through an unrealistically high volatility of expected GDP growth. Third, the Detection Error Probability (DEP) is constrained to be at least 15%. This constraint puts an upper bound on the scaling parameters $m_i, i \in \{r, w, f\}$ and ensures that the reference belief and the distorted belief are sufficiently close to each other such that a likelihood ratio test between both models induces a model detection error of at least 15 percent.[28]

The bond model itself does also constrain the amount of model ambiguity. The amount of perceived model ambiguity enters the yield curve as state variables. If the time variation of η does not help to explain time variations in the real and nominal yield curve, the numerical optimizer would set $m = 0$ and indicate that ambiguity does not help to explain bond yields. On the other hand, if η helps to explain variations in the yield curve, above and beyond expected GDP growth and expected inflation, the numerical optimizer would like to set m as high as necessary. The last constraint on the DEP prohibits this.

[28] Hansen and Sargent (2008) suggest to use a DEP of at least 10%.

4.3. Empirical Findings

All yields and premiums are presented in quarterly units.

Tables (2) compares model implied nominal and real yields with the data counterpart. The model captures the level and positive slope of the real- and nominal yield curve. The estimation strategy ensures a perfect fit to the mean growth rate of GDP and the GDP implicit price index. The estimated detection error probability is 15%, implying that a likelihood ratio test between the reference and distorted model leaves the econometrician with a model detection error of 15%. The appendix derives the detection error probability.

Litterman and Scheinkman (1991) find that three latent factors explain nearly all time-variations in the nominal yield curve. Figure (1) shows that in my model expected inflation is the persistent level factor, expected GDP growth is the slope factor and the perceived amount of *FOMC intervention ambiguity* is the curvature factor. *Macro ambiguity* corresponds to an additional slope factor but its impact coincides with the 4th and 5th principal component only. From a practical point of view, *macro ambiguity* is an unspanned macro factor which affects the bond premium.[29]

Panel A of Table (3) presents the model implied real yield curve and its ambiguity premiums for the period 1981.III to 2009.II. Without model ambiguity the real yield curve would be downward sloping. Since TIPS hedge *macro ambiguity*, the corresponding *macro ambiguity* premium is negative

[29] This is consistent with findings in Ludvigson and Ng (2009) and Ulrich (2010b). The former find that part of the bond excess returns are driven by macro variables that do not explain variations in the yield curve. The latter builds a structural model with unspanned macro ambiguity driving bond excess returns and implied volatilities of bond options.

and downward sloping. The premium for *FOMC intervention ambiguity* is positive and upward sloping. It is 0.05% for a 4-quarter TIPS yield and 0.87% for a 40-quarter TIPS yield.

This indicates that bond investors require a positive premium for ambiguity about potential FOMC interventions. The estimation reveals that the representative investor loves *FOMC intervention ambiguity.* This means he hopes the FOMC uses its discretion to intervene in the business cycle such that his expected life-time utility is maximized. While this is advantageous for GDP growth, it is disadvantageous for bond investors, hence the positive *FOMC intervention ambiguity* premium.

Table (4) presents the model implied bond premium for ambiguity about FOMC interventions for the current and the previous two Fed Chairmen. The FOMC premium was highest during the Volcker period, 0.18% for a 4-quarter TIPS yield and 1.02% for a 40-quarter TIPS yield. During the Chairmanship of Greenspan, the FOMC premium fell to 0.01% for a 4-quarter TIPS yield and 0.84% for a 40-quarter TIPS yield. The FOMC premium fell further during the first term of Chairman Bernanke, even becoming slightly negative for the 4-quarter TIPS yield due to an increase in the pre-cautionary savings motive.[30] Overall, Table (4) points out that the ambiguity premium for discretionary FOMC interventions has fallen over the last decades.

Panel B of Table (3) decomposes the term structure of nominal yields into: real yields, inflation expectations, inflation risk premia and inflation ambiguity premia. The model implied real yield curve for the period 1981.III to

[30] The term $a_{FP}(\tau)$ in Proposition 3 contains a pre-cautionary savings term for ambiguity about FOMC interventions. The increase in the savings motive reduces the interest rate.

2009.II is upward sloping, 0.33% for a 4-quarter TIPS yield and 0.53% for a 40-quarter TIPS yield. The real yield curve is upward sloping because the love attitude for *FOMC intervention ambiguity* dominates the aversion attitude against *macro ambiguity.* The term structure of inflation expectations is weakly upward sloping, from 0.93% for a 4-quarter horizon to 0.99% for a 40-quarter horizon. The inflation risk premium is constant at 0.03%. The inflation ambiguity premium is positive and non-monotone.

Compared to Ulrich (2010a), the non-monotonicity arises in this set-up because the love attitude towards *FOMC intervention ambiguity* endogenizes an upward sloping real and nominal term premium, which dominates inflation ambiguity which itself endogenizes an upward sloping nominal term premium, but at the cost of a downward sloping real term premium. Moreover, the perceived amount of *FOMC intervention ambiguity* is more persistent than the amount of inflation ambiguity, making *FOMC intervention ambiguity* have a relatively bigger impact on long-duration bonds.

Panel A of Table (5) presents that variations in w are the most important driver for nominal yields, explaining 82.1% of variations in the 40-quarter nominal yield. Variations in z explain 23% of variations in the 4-quarter nominal yield and 5.8% of variations in the 40-quarter nominal yield, while η^f explains 11.4% of variations in the 40-quarter nominal yield. Although (η^r, η^w) affect the nominal yield curve, their small contribution to the variance of the nominal yield curve, makes these factors appear to be unspanned by the yield curve (Ulrich (2010b)).

Panel B of Table (5) reveals that variations in z explain most of variations in short-term TIPS yields, while variations in η^f explain most of variations in long-term TIPS yields. Variations in η^f explain 65.2% of variations in the 40-quarter TIPS yield, while variations in z explain the remaining part. Ambiguity about FOMC interventions affects long-term TIPS yields stronger, because of their higher duration which allows this ambiguity premium to accumulate to a sizeable spread.

Table (6) shows that the instantaneous expected excess return on real and nominal bonds is 0.16% for a 4-quarter bond and 0.68% for a 40-quarter bond. As explained in the model section, the expected excess return has two components, one coming from *macro ambiguity* and the other one arising from *FOMC intervention ambiguity*. Bonds are a hedge against macro ambiguity and bond investors are willing to pay a premium for this hedge.

On the other hand, an investor who has a loving attitude for *FOMC intervention ambiguity* requires a positive premium for holding real and nominal bonds because they pay out less, compared to the reference belief, in case the FOMC intervenes according to the best-case model (in terms of high consumption growth). The positive ambiguity premium for discretionary FOMC interventions dominates the negative macro ambiguity premium across all maturities.

Figure (2) presents impulse responses for the yield curve and the bond excess return for a one percent increase in η^f. A one percent increase in η^f increases the 4-quarter yield by 0.6%, the 20-quarter yield by 0.9% and the 40-quarter yield by 0.7%. Yields with a medium-term maturity are affected strongest because η^f is the curvature factor. The slope of the yield curve (20-quarter yield minus 4-quarter yield) increases by 0.3%. While the slope increases, the expected excess return for bonds increases as well. The excess return for a 20-quarter bond increases by 5% and it increases by 5.8% for a 40-quarter bond. It takes 20-quarters for the one percent increase in η^f to die out. In summary, the impulse responses show that a rise in perceived *FOMC intervention ambiguity* has a significant impact on the bond market.

5. Robustness of Empirical Findings and Identification of Ambiguity Premia

This section discusses the robustness of the empirical findings and the empirical identification strategy of the ambiguity premium. First, I discuss evidence for why dispersion in forecasts is a good proxy for model ambiguity. Second, I explain advantages of treating η^f as observable, compared to treating it as latent. Third, I further challenge the identification of η^f by testing the hypothesis: "There is no anticipated FOMC intervention in forecast data".

5.1. Dispersion as a Measure for Model Ambiguity

Economies with model ambiguity and economies with learning and heterogenous agents with differences in beliefs use dispersion data to quantify the amount of disagreement.[31] Patton and Timmermann (2010) try to distinguish between both mechanisms by analyzing term structure data of dispersion in macro forecasts. They find, first, forecasts of macro variables become more disperse the longer the forecast horizon. This finding is consistent with my model ambiguity set-up because the steady-state value of the set of potential models does not converge to zero, i.e. $-m_i \frac{a_{\eta^i}}{\kappa_{\eta^i}} \neq 0$. Disagreement

[31] An example for ambiguity is Anderson et al. (2009) and Ulrich (2010b), while Buraschi and Jiltsov (2006) is an example for differences in belief under a common model.

on the models persists and becomes particularly pronounced for long-term forecasts.

In contrast, economies where heterogeneous agents have differences in beliefs imply that expected disagreement converges to zero because different investors agree on the underlying model which means that long-term forecasts converge to the same steady state value.[32] It requires special assumptions on the model, as in Xiong and Yan (2010), such that differences in beliefs do not converge to zero, in the absence of model uncertainty.

The second finding of Patton and Timmermann (2010) is that dispersion persists over time which indicates that investors do either practically not learn or the economic reason for observing disperse forecasts is that forecasters use different time-series models to forecast macro variables. This finding is also consistent with model ambiguity because this set-up implies that investors do not expect to learn the true model over time. In contrast, Bayesian learning in heterogeneous economies implies that investors eventually learn about the true model.[33]

5.2. Treating η^f as Observable or as Latent

One could treat η^f as a latent factor and regress it on several macro variables to learn what it correlates with. This approach has several disadvantages. First, η^f as a latent factor will be treated as a measurement error that is set to minimize pricing errors across all bond yields. Second, η^f would increase in periods where the model fails otherwise to produce a good yield

[32] Examples include Basak (2005), Buraschi and Jiltsov (2006), Dumas et al. (2009), among others.

[33] Compare Buraschi and Jiltsov (2006).

curve fit. Both features separately would imply a disconnect between the latent η^f and its model interpretation of an uncertainty factor for discretionary FOMC interventions. A whole research agenda in finance, which started with Ang and Piazzesi (2003) focuses on using observable macro variables to explain financial variables, because this is essentially what theoretical asset pricing models urge the econometrician to do. I therefore regard the option of using η^f as a latent factor as a viable but less attractive alternative.

5.3. Hypothesis: "There is no anticipated FOMC intervention in forecast data"

A valid question is whether the identified $(\eta^f)^2$ can be described by a linear combination of the cross-sectional variance of GDP growth forecasts and the cross-sectional variance of inflation forecasts. This could be motivated in a very special set-up where different forecasters are assumed to use the same Taylor rule but different loadings on inflation expectations and GDP growth expectations.

In more detail, one can write such a special Taylor rule with investor specific loadings as

$$R_t^i = c^i + a^i z_t^i + b^i w_t^i \tag{33}$$

where index i stands for a particular forecaster and R coincides with the forecast of next quarter 3-month T-Bill yield. Defining $\Delta a^i := a^i - \bar{a}$, $\Delta b^i := b^i - \bar{b}$ and $\Delta c^i := c^i - \bar{c}$ allows to rewrite this Taylor rule as

$$R_t^i = \bar{c} + \bar{a} z_t^i + \bar{b} w_t^i + f_t^i \tag{34}$$

$$f_t^i := \Delta c^i + \Delta a^i z_t^i + \Delta b^i w_t^i. \tag{35}$$

The cross-sectional Taylor rule that I estimate to identify $(\eta^f)^2$ is similar to equation (34), where $\bar{a}, \bar{b}$ coincide with the Taylor weights for a particular quarter that all SPF forecasters use. The term f_t coincides with the monetary policy shock (FOMC intervention). Equation (35) implies that such a special Taylor rule induces the cross-sectional variance of the monetary policy shock to be a linear function in the cross-sectional variance of z and w. A test of this hypothesis is to regress the cross-sectional variance of the FOMC intervention on the cross-sectional variance of z and w and check whether the R^2 is close to 100% and whether the loadings are significant.

I run such a regression and find first, the loading on $(\eta^r)^2$ is not significant. Second, the R^2 is only 39%, which suggests that 61% of variations in $(\eta^f)^2$ cannot be explained by such a special Taylor rule model.

To summarize, variations in the empirical proxy for FOMC interventions cannot be explained by inflation and GDP dispersion alone. Moreover, the main conclusions of the paper do not rely on approximation errors that might be present in the proxy of $(\eta^f)^2$. Only the time-variation and time-series characteristics of the ambiguity premium for FOMC intervention might depend on this. The sign of the FOMC premium and the finding that investors have an ambiguity loving attitude towards discretionary FOMC interventions, which results in a positive premium in the bond market, are not affected by potential approximation errors in $(\eta^f)^2$.

6. Conclusion

This paper finds that investors like discretionary FOMC interventions in the business cycle. The results indicate that the investor understands that the output mandate of the FOMC urges the FOMC to intervene into the business cycle in a favorable (in terms of high GDP growth) way. While these interventions are at full discretion of the FOMC, the market prices financial assets with the anticipation that FOMC interventions lift expected GDP growth higher.

Anticpated FOMC interventions reduce the likelihood and severity of recessions. Real and nominal bonds become less attractive, because they are essentially recession hedges. As a result, the paper shows that real and nominal bonds will contain a positive FOMC intervention premium. This premium explains why real and nominal bonds slope upwards in the U.S. and why bond excess returns go up in periods where the yield curve steepens.

7. Appendix

7.1. Proof Proposition 1

Rewrite the constrained minimization as a relative entropy bound constrained HJB. J denotes the value function. It depends on $J = J(\ln c, \eta^w, \eta^r, z_t)$. The time varying Lagrange multipliers for the relative entropy bounds are θ_t^r and θ_t^w.

$$\rho J(\ln c_t, \eta^w, \eta^r, z_t) = \min_{h_t^r, h_t^w} \ln c_t + \theta_t^r \left(\frac{(h_t^r)^2}{2} - A_r (\eta_t^r)^2 \right) + \theta_t^w \left(\frac{(h_t^w)^2}{2} - A_w (\eta_t^w)^2 \right) + + \mathcal{A}^h J(\ln c_t, \eta^w, \eta^r, z_t), \tag{36}$$

where $\mathcal{A}^h$ is the second order differential operator (under the macro ambiguity adjusted measure) applied to the value function J. Guess the value function is linear in the states, i.e. $J = \delta_0 + \rho^{-1} \ln c_t + \delta_z z_t + \delta_{\eta^w} \eta_t^w + \delta_{\eta^r} \eta_t^r$. The second order differential operator applied to the value function is

$$\mathcal{A}^h J = \rho^{-1}(c_0 + z_t) + \delta_z(\kappa_z z_t + \sigma_{1z} h_t^r + \sigma_{2z} h_t^w) + \delta_{\eta^r}(a_{\eta^r} + \kappa_{\eta^r} \eta_t^r) + \delta_{\eta^w}(a_{\eta^w} + \kappa_{\eta^w} \eta_t^w) \tag{37}$$

First-order conditions with regard to h_t^r and θ_t^r reveal

$$\theta_t^r = \frac{-\sigma_{1z}\delta_z}{\pm\sqrt{2A_r}\eta_t^r} \tag{38}$$

$$h_t^r = \pm\sqrt{2A_r}\eta_t^r. \tag{39}$$

Note ($\delta_z > 0, \sigma_{1z} > 0$), the robust HJB is minimized at

$$h_t^r = -\sqrt{2A_r}\eta_t^r \equiv m_r \eta_t^r, \quad m_r \in \mathcal{R}^- \tag{40}$$

$$\theta_t^r = \frac{-\sigma_{1z}\delta_z}{-\sqrt{2A_r}\eta_t^r} \equiv \frac{b_0}{\eta_t^r}, \quad b_0 \in \mathcal{R}^+, \tag{41}$$

where I defined $m_r \equiv -\sqrt{2A_r} < 0$ and $b_0 \equiv \frac{-\sigma_{1z}\delta_z}{-\sqrt{2A_r}} > 0$.

First-order conditions with regard to h_t^w and θ_t^w reveal

$$\theta_t^w = \frac{-\sigma_{2z}\delta_z}{\pm\sqrt{2A_w}\eta_t^w} \tag{42}$$

$$h_t^w = \pm\sqrt{2A_w}\eta_t^w. \tag{43}$$

Note ($\delta_z > 0, \sigma_{2z} < 0$), the robust HJB is minimized at

$$h_t^w = \sqrt{2A_w}\eta_t^w \equiv m_w \eta_t^w, \quad m_w \in \mathcal{R}^+ \tag{44}$$

$$\theta_t^w = -\frac{\sigma_{2z}\delta_z}{\sqrt{2A_w}\eta_t^w} \equiv \frac{b_1}{\eta_t^w}, \quad b_1 \in \mathcal{R}^+, \tag{45}$$

where we defined $m_w \equiv \sqrt{2A_w} < 0$ and $b_1 \equiv \frac{-\sigma_{2z}\delta_z}{\sqrt{2A_w}} > 0$.

Now I analyze the ambiguity problem with regard to the FOMC intervention. I first pretend the investor loves the FOMC ambiguity, which means he maximizes expected life-time utility across all potentially correct FOMC models. The value function J depends on $J = J(\ln c, \eta^f, z_t)$. The time varying Lagrange multiplier for the relative entropy bound is θ_t^f.

$$\rho J(\ln c_t, \eta^f, z_t) = \max_{h_t^f} \ln c_t + \theta_t^f \left(\frac{(h_t^f)^2}{2} - A_f (\eta_t^f)^2 \right) + \mathcal{A}^h J(\ln c_t, \eta^f, z_t), \tag{46}$$

where $\mathcal{A}^h$ is the second order differential operator (under the FOMC ambiguity adjusted measure) applied to the value function J. Guess the value function is linear in the states, i.e. $J = \delta_0 + \rho^{-1} \ln c_t + \delta_z z_t + \delta_{\eta^f} \eta_t^f$. The second order differential operator applied to the value function is

$$\mathcal{A}^h J = \rho^{-1}(c_0 + z_t) + \delta_z(\kappa_z z_t + \sigma_{3z} h_t^f) + \delta_{\eta^f}(a_{\eta^f} + \kappa_{\eta^f} \eta_t^f) \tag{47}$$

First-order conditions with regard to h_t^f and θ_t^f reveal

$$\theta_t^f = \frac{-\sigma_{3z}\delta_z}{\pm\sqrt{2A_f}\eta_t^f} \tag{48}$$

$$h_t^f = \pm\sqrt{2A_f}\eta_t^f. \tag{49}$$

Note ($\delta_z > 0, \sigma_{3z} < 0$), the robust HJB is maximized at

$$h_t^f = -\sqrt{2A_f}\eta_t^f \equiv m_f \eta_t^f, \quad m_f \in \mathcal{R}^- \tag{50}$$

$$\theta_t^f = \frac{-\sigma_{3z}\delta_z}{-\sqrt{2A_f}\eta_t^f} \equiv \frac{b_2}{\eta_t^f}, \quad b_2 \in \mathcal{R}^-, \tag{51}$$

where we defined $m_f \equiv -\sqrt{2A_f} < 0$ and $b_2 \equiv \frac{-\sigma_{3z}\delta_z}{-\sqrt{2A_f}} < 0$.

Assume the investor regards FOMC interventions as malevolent. He minimizes expected life-time utility with regard to all potentially correct FOMC intervention models. The first-order condition is the same as (49). The difference is that the minimum is achieved in

$$h_t^f = \sqrt{2A_f}\eta_t^f \equiv m_f \eta_t^f, \quad m_f \in \mathcal{R}^+ \tag{52}$$

$$\theta_t^f = \frac{-\sigma_{3z}\delta_z}{\sqrt{2A_f}\eta_t^f} \equiv \frac{b_2}{\eta_t^f}, \quad b_2 \in \mathcal{R}^+. \tag{53}$$

I now verify the solution to the HJB is correct. I plug the optimal distortions back into the HJB

$$\rho[\delta_0 + \delta_z z_t + \sum_{i\in\{r,w,f\}} \delta_{\eta^i}\eta_t^i] + \ln c_t \overbrace{=}^{!} \ln c_t + \rho^{-1}(c_0 + z_t) + \delta_0 + \delta_z(\kappa_z z_t + \sigma_{1z} m_r \eta_t^r + \sigma_{2z} m_w \eta_t^w + \sigma_{3z} m_f \eta_t^f)$$
$$+ \sum_{i\in\{r,w,f\}} \delta_{\eta^i}(a_{\eta^i} + \kappa_{\eta^i}\eta_t^i). \tag{54}$$

Matching coefficients reveals:

$$\delta_z = \frac{\rho^{-1}}{\rho - \kappa_z} \tag{55}$$

$$\delta_{\eta^i} = \frac{\rho^{-1}\sigma_{iz} m_i}{(\rho - \kappa_{\eta^i})(\rho - \kappa_z)}, \quad \forall i \in \{r, w, f\} \tag{56}$$

$$\delta_0 = \frac{\rho^{-1}c_0 + \sum_{i\in\{r,w,f\}} \delta_{\eta^i} a_{\eta^i}}{\rho - 1} \tag{57}$$

$$\tag{58}$$

with $\sigma_{rz} \equiv \sigma_{1z}$, $\sigma_{wz} \equiv \sigma_{2z}$ and $\sigma_{fz} \equiv \sigma_{3z}$.

This implies

$$b_0 = -\frac{\sigma_{1z}\delta_z}{-\sqrt{2A_r}} = \frac{\sigma_{1z}}{\rho\sqrt{2A_r}(\rho - \kappa_z)} \tag{59}$$

$$b_1 = -\frac{\sigma_{2z}\delta_z}{\sqrt{2A_w}} = -\frac{\sigma_{2z}}{\rho(\rho - \kappa_z)\sqrt{2A_w}} \tag{60}$$

$$\tag{61}$$

and for ambiguity aversion about FOMC interventions

$$b_2 = -\frac{\sigma_{3z}\delta_z}{\sqrt{2A_f}} = -\frac{\sigma_{3z}}{\rho(\rho - \kappa_z)\sqrt{2A_f}} \tag{62}$$

and for ambiguity love about FOMC interventions

$$b_2 = -\frac{\sigma_{3z}\delta_z}{-\sqrt{2A_f}} = \frac{\sigma_{3z}}{\rho(\rho - \kappa_z)\sqrt{2A_f}} \tag{63}$$

7.2. Proof Proposition 2

Price of a τ-maturity real bond is $F = F_t(\tau) = e^{A(\tau)+B'(\tau)X_t}$ with $X_t = (z_t, \eta_t^i \forall i \in \{r, w, f\})'$. F solves

$$r \cdot F = \mathcal{A}^H F - F_\tau \tag{64}$$

$$r \equiv c_0 - \rho - \frac{1}{2}\sigma_c^2 + z_t \tag{65}$$

$$F_\tau \equiv F[\dot{A}(\tau) + \dot{B}(\tau)X_t] \tag{66}$$

$$\mathcal{A}^H F \equiv F[B_z(\tau) \cdot (\kappa_z z_t + \sigma_{1z} m_r \eta_t^r + \sigma_{2z} m_w \eta_t^w + \sigma_{3z} m_f \eta_t^f)] \tag{67}$$

$$= F[\frac{1}{2}B_z^2(\tau)(\sigma_{1z}^2 + \sigma_{2z}^2 + \sigma_{3z}^2)] + F[\sum_{i\in\{r,w,f\}} B_{\eta^i}(\tau) \cdot (a_{\eta^i} + \kappa_{\eta^i}\eta_t^i)] \tag{68}$$

$$+ F[\frac{1}{2}\sum_{i\in\{r,w,f\}} B_{\eta^i}^2(\tau)\sigma_{\eta^i}^2] \tag{69}$$

Matching coefficients of fundamental pde and solving the resulting one dimensional ode's gives

$$z_t : B_z(\tau) = \frac{1 - e^{\kappa_z \tau}}{\kappa_z} \tag{70}$$

$$\eta^i : B_{\eta^i}(\tau) = -\frac{\sigma_{iz} m_i}{\kappa_z}\left(\frac{1 - e^{\kappa_{\eta^i}\tau}}{\kappa_{\eta^i}} + \frac{e^{\kappa_z \tau} - e^{\kappa_{\eta^i}\tau}}{\kappa_z - \kappa_{\eta^i}}\right),\ \forall i \in \{r, w, f\} \tag{71}$$

with $\sigma_{rz} \equiv \sigma_{1z}$, $\sigma_{wz} \equiv \sigma_{2z}$, $\sigma_{fz} \equiv \sigma_{3z}$.

$$A(\tau) = (-c_0 + \rho + \frac{1}{2}\sigma_c^2)\tau + \sum_{i\in\{r,w,f\}} a_{\eta^i}\int_0^\tau B_{\eta^i}(k)dk + \frac{(\sigma_{1z}^2 + \sigma_{2z}^2 + \sigma_{3z}^2)}{2}\int_0^\tau B_z^2(k)dk \tag{72}$$

$$+ \frac{1}{2}\sum_{i\in\{r,w,f\}} \sigma_{\eta^i}^2 \int_0^\tau B^2(\eta^i)(k)dk. \tag{73}$$

Define $a(\tau) := -\frac{A(\tau)}{\tau}$ and $b(\tau) := -\frac{B(\tau)}{\tau}$ which gives: $y_t(\tau) := -\frac{\ln F_t(\tau)}{\tau} = a(\tau) + b'(\tau)X_t$.

7.3. Proof Proposition 3 and Proposition 6

The FOMC premium $FP_t(\tau)$ in real and nominal bond yields coincides with

$$y_t(\tau) - y_t(\tau)|_{m_f \equiv 0} = y_t^{\$}(\tau) - y_t^{\$}(\tau)|_{m_f \equiv 0}. \tag{74}$$

The affine yield curve allows to solve for this difference in closed-form:

$$FP_t(\tau) = -\frac{1}{\tau}B_{\eta_f}(\tau)\eta_t^f - \frac{1}{\tau}\left[a_{\eta_f}\int_0^\tau B_{\eta_f}(k)dk + \frac{\sigma_{\eta_f}^2}{2}\int_0^\tau B_{\eta_f}^2(k)dk\right]. \tag{75}$$

Solving for the integrals reveals

$$\begin{aligned} FP_t(\tau) = &-\frac{1}{\tau}B_{\eta^f}(\tau)\eta_t^f - \frac{a_{\eta^f}\sigma_{3z}m_f}{\tau\kappa_z}\left[\frac{-\tau}{\kappa_{\eta^f}} + \frac{e^{\kappa_{\eta^f}\tau} - 1}{\kappa_{\eta^f}^2} - \frac{e^{\kappa_z\tau} - 1}{\kappa_z(\kappa_z - \kappa_{\eta^f})} + \frac{e^{\kappa_{\eta^f}\tau} - 1}{\kappa_{\eta^f}(\kappa_z - \kappa_{\eta^f})}\right] \\ &- \frac{\sigma_{\eta^f}^2\sigma_{3z}^2 m_f^2}{2\kappa_z^2\tau}\left[\frac{\tau}{\kappa_{\eta^f}^2} - 2\frac{e^{\kappa_{\eta^f}\tau} - 1}{\kappa_{\eta^f}^2} + \frac{e^{2\kappa_{\eta^f}\tau} - 1}{2\kappa_{\eta^f}^3} + \frac{e^{2\kappa_z\tau} - 1}{2\kappa_z(\kappa_z - \kappa_{\eta^f})^2} - 2\frac{e^{(\kappa_z + \kappa_{\eta^f})\tau} - 1}{(\kappa_z + \kappa_{\eta^f})(\kappa_z - \kappa_{\eta^f})^2} + \frac{e^{2\kappa_{\eta^f}\tau} - 1}{2\kappa_{\eta^f}(\kappa_z - \kappa_{\eta^f})^2}\right] \\ &- \frac{\sigma_{\eta^f}^2\sigma_{3z}^2 m_f^2}{2\kappa_z^2\tau}\left[2\frac{e^{\kappa_z\tau} - 1}{\kappa_z\kappa_{\eta^f}(\kappa_z - \kappa_{\eta^f})} - 2\frac{e^{\kappa_{\eta^f}\tau} - 1}{\kappa_{\eta^f}^2(\kappa_z - \kappa_{\eta^f})} - 2\frac{e^{(\kappa_{\eta^f} + \kappa_z)\tau} - 1}{\kappa_{\eta^f}(\kappa_z - \kappa_{\eta^f})(\kappa_z + \kappa_{\eta^f})} + \frac{e^{2\kappa_{\eta^f}\tau} - 1}{\kappa_{\eta^f}^2(\kappa_z - \kappa_{\eta^f})}\right] \end{aligned} \tag{76}$$

which I define as

$$FP_t(\tau) \equiv -\frac{1}{\tau}B_{\eta^f}(\tau)\eta_t^f + a_{FP}(\tau). \tag{77}$$

7.4. Proof Proposition 4

The macro ambiguity premium $MP_t(\tau)$ in real bond yields coincides with

$$y_t(\tau) - \left(y_t(\tau)|_{m_w \equiv 0} + y_t(\tau)|_{m_r \equiv 0}\right). \tag{78}$$

The affine yield curve allows to solve for this difference in closed-form:

$$MP_t(\tau) = -\frac{1}{\tau}\left(\sum_{j\in\{r,w\}} B_{\eta_j}(\tau)\eta_t^j\right) - \frac{1}{\tau}\left[\sum_{j\in\{r,w\}} a_{\eta_j}\int_0^\tau B_{\eta_j}(k)dk + \frac{\sigma_{\eta_j}^2}{2}\int_0^\tau B_{\eta_j}^2(k)dk\right]. \tag{79}$$

Direct integration along the lines of the equation (76) establishes the result

$$MP_t(\tau) = a_{MP}(\tau) - \frac{1}{\tau}\left(\sum_{j\in\{r,w\}} B_{\eta^j}(\tau)\eta_t^j\right). \tag{80}$$

7.5. Proof Proposition 5

Price of a τ-maturity real bond is $F = F_t(\tau) = e^{A^\$(\tau)+B^{\$\prime}(\tau)X_t}$ with $X_t = (w_t, z_t, \eta_t^i \forall i \in \{r,w,f\})'$. F solves

$$R\cdot F = \mathcal{A}^H F - F_\tau \tag{81}$$

$$R \equiv c_0 - \rho - \frac{1}{2}\sigma_c^2 + p_0 - \frac{1}{2}(\bar\sigma_p^2 + \sigma_p^2) - \bar\sigma_p\sigma_c + w_t + z_t \tag{82}$$

$$F_\tau \equiv F[\dot A(\tau) + \dot(B)(\tau)X_t] \tag{83}$$

$$\mathcal{A}^H F \equiv F[B_z^\$(\tau)\cdot(\kappa_z z_t + \sigma_{1z}m_r\eta_t^r + \sigma_{2z}m_w\eta_t^w + \sigma_{3z}m_f\eta_t^f)] \tag{84}$$

$$= F[\frac{1}{2}(B_z^\$(\tau))^2(\sigma_{1z}^2+\sigma_{2z}^2+\sigma_{3z}^2)] + F[\sum_{i\in\{r,w,f\}} B_{\eta^i}^\$(\tau)\cdot(a_{\eta^i} + \kappa_{\eta^i}\eta_t^i)] \tag{85}$$

$$+ F[\frac{1}{2}\sum_{i\in\{r,w,f\}}(B_{\eta^i}^\$(\tau))^2\sigma_{\eta^i}^2] \tag{86}$$

$$+ F[B_w^\$(\tau)\cdot(\kappa_w w_t + \sigma_w m_w\eta_t^w) + \frac{1}{2}(B_w^\$(\tau))^2\sigma_w^2] \tag{87}$$

Matching coefficients of fundamental pde and solving the resulting one dimensional ode's gives

$$z_t : B_z^\$(\tau) = B_z(\tau) \tag{88}$$

$$w_t : B_w^\$(\tau) = \frac{1-e^{\kappa_w\tau}}{\kappa_w} \tag{89}$$

$$\eta^j : B_{\eta^j}^\$(\tau) = B_{\eta^j}(\tau),\ \forall j\in\{r,f\} \tag{90}$$

$$\eta^w : B_{\eta^w}^\$(\tau) = B_{\eta^w}(\tau) - \frac{\sigma_w m_w}{\kappa_w}\left(\frac{1-e^{\kappa_{\eta^w}\tau}}{\kappa_{\eta^w}} + \frac{e^{\kappa_w\tau}-e^{\kappa_{\eta^w}\tau}}{\kappa_w - \kappa_{\eta^w}}\right) \tag{91}$$

with $\sigma_{rz}\equiv\sigma_{1z}$, $\sigma_{wz}\equiv\sigma_{2z}$, $\sigma_{fz}\equiv\sigma_{3z}$.

$$A^\$(\tau) = (-c_0 + \rho + \frac{1}{2}\sigma_c^2 - p_0 + \frac{1}{2}(\bar\sigma_p^2+\sigma_p^2) + \bar\sigma_p\sigma_c)\tau + \sum_{i\in\{r,w,f\}} a_{\eta^i}\int_0^\tau B_{\eta^i}^\$(k)dk + \frac{(\sigma_{1z}^2+\sigma_{2z}^2+\sigma_{3z}^2)}{2}\int_0^\tau (B_z^\$(k))^2 dk \tag{92}$$

$$+ \frac{1}{2}\sum_{i\in\{r,w,f\}}\sigma_{\eta^i}^2\int_0^\tau (B^\$(\eta^i)(k))^2dk + \frac{\sigma^w}{2}\int_0^\tau (B_w^\$(k))^2dk. \tag{93}$$

Define $a^\$(\tau) := -\frac{A^\$(\tau)}{\tau}$ and $b^\$(\tau) := -\frac{B^\$(\tau)}{\tau}$. Factor loadings on z, η^f, η^r coincide with the corresponding factor loading for real bond yield yields. Exploiting this, I rewrite nominal yields as:
$y_t^\$(\tau) := -\frac{\ln F_t(\tau)}{\tau} = a^\$(\tau) + b_z(\tau)z_t + b_{\eta^f}(\tau)\eta_t^f + b_{\eta^r}(\tau)\eta_t^r + b_w^\$(\tau)w_t + b_{\eta^w}^\$(\tau)\eta_t^w$.

7.6. Proof Proposition 7

The inflation ambiguity premium $IAP_t(\tau)$ in nominal bond yields coincides with

$$y_t^{\$}(\tau) - y_t^{\$}(\tau)|_{m_w \equiv 0}. \tag{94}$$

The affine yield curve allows to solve for this difference in closed-form:

$$MP_t(\tau) = -\frac{1}{\tau} B_{\eta_w}^{\$}(\tau)\eta_t^w - \frac{1}{\tau} a_{\eta_w} \int_0^\tau B_{\eta_w}^{\$}(k)dk + \frac{\sigma_{\eta_w}^2}{2} \int_0^\tau (B_{\eta_w}^{\$})^2(k)dk. \tag{95}$$

Direct integration along the lines of the equation (76) establishes the result

$$IAP_t(\tau) = a_{IAP}(\tau) - \frac{1}{\tau} B_{\eta^w}^{\$}(\tau)\eta_t^w. \tag{96}$$

7.7. Derivation of Detection Error Probability

The derivation of the detection-error probabilities $p_T(m_f, m_r, m_w)$ follows directly from Maenhout (2006):

$$p_T(m_f, m_r, m_w) = \frac{1}{2}\left(Pr\left(\ln \frac{dQ_T^h}{dQ_T^0} > 0|dQ^0, \mathcal{F}_0\right) + Pr\left(\ln \frac{dQ_T^0}{dQ_T^h} > 0|dQ^h, \mathcal{F}_0\right)\right) \tag{97}$$

$$\begin{aligned} &= \frac{1}{2}\left(Pr\left(-\frac{1}{2}\int_0^T h_m' h_m dm + \int_0^T h_m \cdot dW_m^z > 0|dQ^0, \mathcal{F}_0\right)\right) \\ &+ \frac{1}{2}\left(Pr\left(-\frac{1}{2}\int_0^T h_m' h_m dm - \int_0^T h_m \cdot dW_m^{z,h} > 0|dQ^h, \mathcal{F}_0\right)\right) \end{aligned} \tag{98}$$

where $h_t = (m_f \eta_t^f \;\; m_w \eta_t^w \;\; m_r \eta_t^r)'$ is the endogenous distortion to expected consumption growth. The last equation coincides with

$$p_T(m_f, m_r, m_w) = \frac{1}{2} - \frac{1}{2\pi}\int_0^\infty \left(Re\left(\frac{\phi^h(k,0,T)}{ik}\right) - Re\left(\frac{\phi(k,0,T)}{ik}\right)\right) dk \tag{99}$$

where $i = \sqrt{-1}$, $\phi(.)$ is defined as $\phi(k,0,T) := E\left[e^{i\cdot k\cdot \xi_{1,T}}|\mathcal{F}_0\right]$ and $\phi^h(.)$ is defined as $\phi^h(k,0,T) := E^h\left[e^{i\cdot k\cdot \xi_{1,T}}|\mathcal{F}_0\right]$ and $\xi_{1,T} = \ln \frac{dQ_T^h}{dQ_T^0}$.

Applying Feynman-Kac theorem to ϕ^h and ϕ reveals that they are an exponentially quadratic function in the amount of inflation distortion h_t:

$$\phi^h(k,t,T) = z_t^{ik+1} e^{G(\tau,k) + \sum_{j\in\{f,w,r\}} E_j(\tau,k)h_j(t) + \sum_{j\in\{f,w,r\}} \frac{F_j(\tau,k)}{2} h_j^2(t)} \tag{100}$$

$$\phi(k,t,T) = z_t^{ik} e^{\hat{G}(\tau,k) + \sum_{j\in\{f,w,r\}} \hat{E}_j(\tau,k)h_j(t) + \sum_{j\in\{f,w,r\}} \frac{\hat{F}_j(\tau,k)}{2} h_j^2(t)} \tag{101}$$

$$z_T := e^{\xi_{1,T}}, \tag{102}$$

where $G(\tau,k)$, $E_j(\tau,k)$, $F_j(\tau,k)$, $\hat{G}(\tau,k)$, $\hat{E}_j(\tau,k)$, $\hat{F}_j(\tau,k)$ are deterministic solutions to standard complex valued Riccati equations. We provide some details on the derivation of the Riccati equations for ϕ^h. The derivation of ϕ is analogous. $\phi^h(k,t,T)$ solves $\phi_\tau^h = \mathcal{A}\phi^h$ where $\tau = T - t$ and ϕ_τ^h stands for $\frac{\partial \phi^h}{\partial \tau}$.

$$\phi_\tau^h = \phi^h\left(\dot{G}(\tau,k) + \sum_{j\in\{f,w,r\}} \dot{E}_j(\tau,k)h_j(t) + \frac{1}{2}\sum_{j\in\{f,w,r\}} \dot{F}_j(\tau,k)h_j^2(t)\right) \tag{103}$$

$$\begin{aligned} \frac{\mathcal{A}\phi^h}{\phi^h} &= \sum_{j\in\{f,w,r\}} \left[\left(E_j(\tau,k) + F_j(\tau,k)h_j(t)\right)\left(a_{\eta^j} + \kappa_{\eta^j} h_j(t)\right)\right] + 0.5ik(k+1)\left(h_f^2(t) + h_w^2(t) + h_r^2(t)\right) \\ &+ \sum_{j\in\{f,w,r\}} \left(E_j^2(\tau,k) + F_j^2(\tau,k)h_j^2(t) + 2E_j(\tau,k)F_j(\tau,k)h_j(t)\right) m_j \sigma_{\eta^j} \end{aligned} \tag{104}$$

Set $\phi^h_\tau = \mathcal{A}\phi^h$ and match coefficients:

$$F_j(\tau,k) = F^r_j(\tau,k) + F^c_j(\tau,k) \tag{105}$$

$$F^c_j(\tau,k) = k \cdot \tau \tag{106}$$

$$F^r_j(\tau,k) = \frac{(a_j + d_j)(1 - e^{d_j \tau})}{2b^r_{2j}(1 - g_j e^{d_j \tau})} \tag{107}$$

where F^r is the real part of F and F^c is the complex part and

$$a_j = -b^r_{1j}; \quad d_j = \sqrt{a_j^2 - 4b^r_{0j}b^r_{2j}}; \quad g_j = \frac{a_j + d_j}{a_j - d_j}; \quad b^r_{0j} = -k^2 \tag{108}$$

$$b^r_{1j} = 2\kappa_{\eta^j}; \quad b^r_{2j} = m_j^2 \sigma^2_{\eta^j} \tag{109}$$

where $j \in \{f, w, r\}$. The stable steady state solution of F is

$$F_j(\infty,k) = -\frac{b^r_{1j} + d_j}{2b^r_{2j}}. \tag{110}$$

The loadings $E_j(\tau,k), j \in \{f, w, r\}$ solve the following ode

$$\dot{E}_j(\tau,k) = \kappa_{\eta^j} E_j(\tau,k) + m_j a_{\eta^j} F_j(\tau,k) + E_j(\tau,k)F_j(\tau,k)m_j^2\sigma^2_{\eta^j}. \tag{111}$$

We obtain an analytical approximation by approximating $F_j(\tau,k)$ around its steady state value $F_j(\infty,k)$.

$$E_j(\tau,k) = -\frac{\hat{a}_j}{\hat{b}_j}(1 - e^{\hat{b}_j \tau}) \tag{112}$$

$$\hat{a}_j = F_j(\infty,k)m_j a_{\eta^j} \tag{113}$$

$$\hat{b}_j = F_j(\infty,k)m_j^2\sigma^2_{\eta^j} + \kappa_{\eta^j}. \tag{114}$$

The loading $G(\tau,k)$ is obtained through straightforward integration

$$G(\tau,k) = \sum_{j\in\{f,w,r\}} \left(m_j a_{\eta^j} \int_0^\tau E_j(u,k)du \right) + \frac{1}{2} \sum_{j\in\{f,w,r\}} m_j^2 \sigma^2_{\eta^j} \int_0^\tau E_j^2(u,k)du. \tag{115}$$

The required expression $\phi^h(k,0,T)$ is therefore

$$\phi^h(k,0,T) = e^{G(T,k) + \sum_{j\in\{f,w,r\}} E_j(T,k)h_j(\infty) + \frac{1}{2}\sum_{j\in\{f,w,r\}} F_j(T,k)h_j^2(\infty)}, \tag{116}$$

where we assumed that $h_j(0)$ started in its steady state $h_j(\infty) = \frac{m_j a_{\eta^j}}{-\kappa_{\eta^j}}$.

7.8. Derivation Likelihood

Model is cast into a ML model. The frequency is quarterly. The macroeconomic measurement equations are:

$$m_t := \begin{pmatrix} \ln \frac{c_{t+1}}{c_t} \\ \ln \frac{p_{t+1}}{p_t} \end{pmatrix} \sim N\left(\mu_m(t), \Sigma_m \Sigma_m^{'}\right), \tag{117}$$

where $\mu_m(t)$ and $\Sigma_m\Sigma_m^{'}$ follow from the parameters of the corresponding discretized stochastic differential equation.

The macroeconomic state equations are observed and follow the following dynamic:

$$X_{t+1} := \begin{pmatrix} w_{t+1} \\ z_{t+1} \\ \eta^f_{t+1} \\ \eta^w_{t+1} \\ \eta^r_{t+1} \end{pmatrix} \sim N\left(\mu_X(t), \Sigma_X \Sigma_X^{'}\right), \tag{118}$$

where $\mu_X(t)$ and $\Sigma_X\Sigma_X^{'}$ follow from the corresponding discretized stochastic differential equation.

There are eleven measurement equations for nominal bond yields. We denote $u_{t,\tau,\$}$ to be the measurement error at time t for the τ-maturity nominal bond yield, and $y_t^{\$}(\tau)$ to be the corresponding observed yield:

$$u_{t,\tau,\$} := y_t^{\$}(\tau) + \frac{1}{\tau}\left(A^{\$}(\tau) + B^{\$'}(\tau)X_t\right), \quad u_{t,\tau,\$} \sim N(0, \sigma^2_{\tau,\$}), \tag{119}$$

where $X_t = (w_t, z_t, \eta_t^f, \eta_t^w, \eta_t^r)$. We incorporate the Federal funds rate as a nominal yield.

There are six measurement equations for real yields. We denote $u_{t,\tau,r}$ to be the measurement error at time t for the τ-maturity real bond yield and $y_t^r(\tau)$ to be the corresponding observed yield:

$$u_{t,\tau,r} := y_t^r(\tau) + \frac{1}{\tau}\left(A^r(\tau) + B^{r'}(\tau)S_t\right), \quad u_{t,\tau,r} \sim N(0, \sigma^2_{\tau,r}), \tag{120}$$

where $S_t = (z_t, \eta_t^f, \eta_t^w, \eta_t^r)$.

We denote the Gaussian density functions of the macroeconomic state variables X_t, the macroeconomic measurement equation m_t, the nominal bond yield error $u_{t,\tau,\$}$ and the real bond yield error $u_{t,\tau,r}$ as f_X, f_m, $f_{u_\$}$, f_{u_r} respectively. The joint log-likelihood $\log(\mathcal{L}(\theta))$, for a given parameter vector θ, is given by (we drop the θ dependence) $\frac{1}{T}\log L(\theta)$, where

$$\begin{aligned}
\log(\mathcal{L}(\theta)) &= \sum_{t=2}^{T} \log f_X(X_t|X_{t-1}) + \log f_m(m_{t-1})|X_{t-1}) \\
&\quad + \log f_{u_\$}(u_{t-1,\tau,\$}|X_{t-1}) + \log f_{u_r}(u_{t-1,\tau,r}|S_{t-1}) && (121)\\
\log f_X(X_t|X_{t-1}) &= -\frac{5}{2}\log(2\pi) - \frac{1}{2}\log(det(\Sigma_X\Sigma_X^{'})) \\
&\quad - \frac{1}{2}\left(X_t - \mu_X(t-1)\right)'\left(\Sigma_X\Sigma_X^{'}\right)^{-1}\left(X_t - \mu_X(t-1)\right) && (122)\\
\log f_m(m_{t-1}|X_{t-1}) &= -\log(2\pi) - \frac{1}{2}\log\left(det(\Sigma_m\Sigma_m^{'})\right) \\
&\quad - \frac{1}{2}\left(m_{t-1} - \mu_m(t-1)\right)'\left(\Sigma_X\Sigma_X^{'}\right)^{-1}\left(m_{t-1} - \mu_m(t-1)\right) && (123)\\
f_{u_\$}(u_{t-1,\tau,\$}|X_{t-1}) &= -\frac{11}{2}\log(2\pi) - \frac{1}{2}\log\left(\sum_{l=1}^{11}\sigma^2_{l,\$}\right) \\
&\quad - \frac{1}{2}\sum_{l=1}^{11}\frac{u^2_{t-1,l,\$}}{\sigma^2_{l,\$}} && (124)\\
\log f_{u_r}(u_{t-1,\tau,r}|S_{t-1}) &= -\frac{6}{2}\log(2\pi) - \frac{1}{2}\log\left(\sum_{l=1}^{6}\sigma^2_{l,r}\right) \\
&\quad - \frac{1}{2}\sum_{l=1}^{6}\frac{u^2_{t-1,l,r}}{\sigma^2_{l,r}}. && (125)
\end{aligned}$$

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Table 1: PARAMETER ESTIMATES (Standard Errors)

Panel A: State Variables

	Drift, Volatility				
	κ	σ			a
w	0.0047 (0.2e-6)	0.0026 (0.1e-6)			0 (fixed)
z	-0.06 (0.8e-5)	0.0017 (0.1e-6)	-0.0014 (0.3e-6)	-0.002 (0.1e-6)	0 (fixed)
η^f	-0.25 (0.3e-4)	0.001 (0.2e-6)			0.00036 (0.3e-7)
η^w	-0.6 (0.4e-4)	0.001 (0.4e-7)			0.0018 (0.2e-6)
η^z	-1.0 (0.9e-4)	0.003 (0.5e-5)			0.003(0.3e-6)

Panel B: Growth and Inflation

c_0	0.0068 (fixed)
p_0	0.0066 (fixed)
σ_c	0.0548 (0.3e-6)
σ_p	0.0029 (0.6e-6)
$\bar{\sigma}_p$	-0.055 (0.7e-3)
ρ	0.001 (fixed)
m^f	-290.8 (0.02)
m^w	69.2 (0.4e-2)
m^r	-99.7 (0.8e-2)

Note: The table presents ML parameter estimates and their standard error (in parenthesis). The asymptotic standard errors are determined based on the score of the log likelihood. The ML estimation uses bond yield and macro data from 1981.III to 2009.II.

Table 2: Yield Curve, in %, per quarter

Panel A: Nominal Yields

$y^{\$}$

maturity	R	4	8	12	16	20	24	28	32	36	40
data	1.41	1.4345	1.5096	1.5655	1.6098	1.648	1.6822	1.7115	1.7373	1.7593	1.7789
model	1.28	1.3935	1.5997	1.6686	1.6923	1.6978	1.6950	1.6881	1.6794	1.67	1.6603

Panel B: Real Yields

y^r

maturity	20	24	28	32	36	40
data	0.3917	0.422	0.4488	0.4712	0.4902	0.5061
model	0.4904	0.4944	0.4968	0.4984	0.4994	0.5

Note: Panel A compares model implied nominal bond yields with the data counterpart. R stands for the nominal short rate (federal funds rate), while the other maturities refer to quarters. The yields are in quarterly percentage units. Panel B compares model implied real bond yields with the data counterpart. Maturity is in quarterly units, and interest rates are in quarterly percentage units. The ML estimation uses bond yields and macro data from 1981.III to 2009.II.

Table 3: Yield Curve Decomposition, in %, per quarter

Panel A: Real Yield Curve

maturity	y^r	MP	FP	$y^r - (MP + FP)$
4	0.3264	-0.3539	0.0465	0.6338
8	0.4502	-0.3869	0.2577	0.5794
12	0.4983	-0.4600	0.4085	0.5499
16	0.5197	-0.5364	0.5221	0.5340
20	0.5295	-0.6075	0.6113	0.5257
24	0.5339	-0.6714	0.6834	0.5219
28	0.5353	-0.7283	0.7430	0.5207
32	0.5353	-0.7787	0.7931	0.5209
36	0.5344	-0.8233	0.8358	0.5219
40	0.5332	-0.8627	0.8724	0.5235

Panel B: Nominal Yield Curve

maturity	y^r	$E[\pi]$	IRP	IAP
4	0.3264	0.9324	0.0300	0.1047
8	0.4502	0.8824	0.0300	0.2371
12	0.4983	0.8844	0.0300	0.2559
16	0.5197	0.8993	0.0300	0.2434
20	0.5295	0.9172	0.0300	0.2211
24	0.5339	0.9349	0.0300	0.1962
28	0.5353	0.9515	0.0300	0.1713
32	0.5353	0.9665	0.0300	0.1476
36	0.5344	0.9801	0.0300	0.1254
40	0.5332	0.9921	0.0300	0.1050

Note: The upper panel of this table decomposes the real yield curve, (y^r), into the hypothetical real yield curve in a world without model ambiguity, $(y^r - (MP + FP))$, and the two premiums for model ambiguity (macro ambiguity (MP) and FOMC intervention ambiguity (FP)). The lower panel of this table decomposes the nominal yield curve into the real yield curve (y^r), expected inflation $(E[\pi])$ and inflation premium. The inflation premium consists of an inflation risk premium (IRP) and an inflation ambiguity premium (IAP). The ML estimation uses bond yield and macro data from 1981.III to 2009.II. The yields are in quarterly units.

Table 4: Premium for FOMC Intervention Ambiguity

Real Yield Curve

y^r

maturity	*Volcker*	*Greenspan*	*Bernanke*
4	0.1836	0.0127	-0.0221
8	0.4508	0.2101	0.1611
12	0.6199	0.3564	0.3027
16	0.7342	0.4698	0.4160
20	0.8161	0.5608	0.5088
24	0.8775	0.6356	0.5863
28	0.9253	0.6981	0.6518
32	0.9636	0.7511	0.7078
36	0.9951	0.7965	0.7561
40	1.0213	0.8358	0.7980

Note: This table presents the premium for FOMC intervention ambiguity inherent in the yield curve of TIPS. The model is estimated over the entire sample 1981.III to 2009.II, and sample averages are used for the Chairman Volcker period (2nd column), Chairman Greenspan period (3rd column) and Chairman Bernanke (4th column) period. The premium is in quarterly units and in %.

Table 5: Variance Decomposition

Panel A: Nominal Yield Curve

$y^{\$}$

maturity	w	z	η^f	η^w	η^r
4	0.6953	0.2302	0.0692	0.0006	0.0046
8	0.6759	0.1820	0.1360	0.0012	0.0050
12	0.6776	0.1498	0.1665	0.0017	0.0045
16	0.6926	0.1268	0.1745	0.0021	0.0040
20	0.7138	0.1093	0.1709	0.0025	0.0035
24	0.7372	0.0953	0.1615	0.0029	0.0031
28	0.7605	0.0836	0.1498	0.0033	0.0027
32	0.7827	0.0738	0.1373	0.0037	0.0024
36	0.8029	0.0655	0.1253	0.0041	0.0022
40	0.8213	0.0584	0.1139	0.0044	0.0020

Panel B: Real Yield Curve

y^r

maturity	z	η^f	η^w	η^r
4	0.7553	0.2272	0.0023	0.0152
8	0.5619	0.4199	0.0029	0.0153
12	0.4656	0.5177	0.0028	0.0139
16	0.4143	0.5700	0.0027	0.0129
20	0.3843	0.6007	0.0026	0.0123
24	0.3656	0.6199	0.0026	0.0119
28	0.3532	0.6327	0.0025	0.0116
32	0.3448	0.6413	0.0025	0.0114
36	0.3387	0.6476	0.0025	0.0113
40	0.3344	0.6520	0.0025	0.0112

Note: This table presents a variance decomposition of the nominal (upper panel) and real (lower panel) yield curve. The ML estimation uses bond yield and macro data from 1981.III to 2009.II. The state variables are extracted from the Survey of Professional Forecasters.

Table 6: Decomposition

Bond Excess Return

$y^{\$}$ and y^r

maturity	$xHPR$	(h^r, h^w)	h^f
4	0.1592	-0.3034	0.4626
8	0.2845	-0.5420	0.8265
12	0.3830	-0.7297	1.1128
16	0.4605	-0.8774	1.3379
20	0.5215	-0.9936	1.5151
24	0.5694	-1.0850	1.6544
28	0.6072	-1.1568	1.7640
32	0.6369	-1.2134	1.8502
36	0.6602	-1.2579	1.9181
40	0.6786	-1.2928	1.9714

Note: This table presents the instantaneous bond excess return (2nd column) and its two components, i.e. premium for macro ambiguity (third column) and premium for FOMC intervention ambiguity (fourth column). The ML estimation uses bond yield and macro data from 1981.III to 2009.II. The data is in quarterly units.

Figure 1: **Nominal Yield Curve: Factor Loadings**

This figure presents the estimated factor loadings for the nominal yield curve model. The state variables are expected inflation w, trend GDP growth z, cross-sectional dispersion in FOMC intervention forecast η^f, cross-sectional dispersion in inflation forecast η^w and cross-sectional dispersion in GDP growth forecast η^r. The ML estimation uses bond yield and macro data from 1981.III to 2009.II.

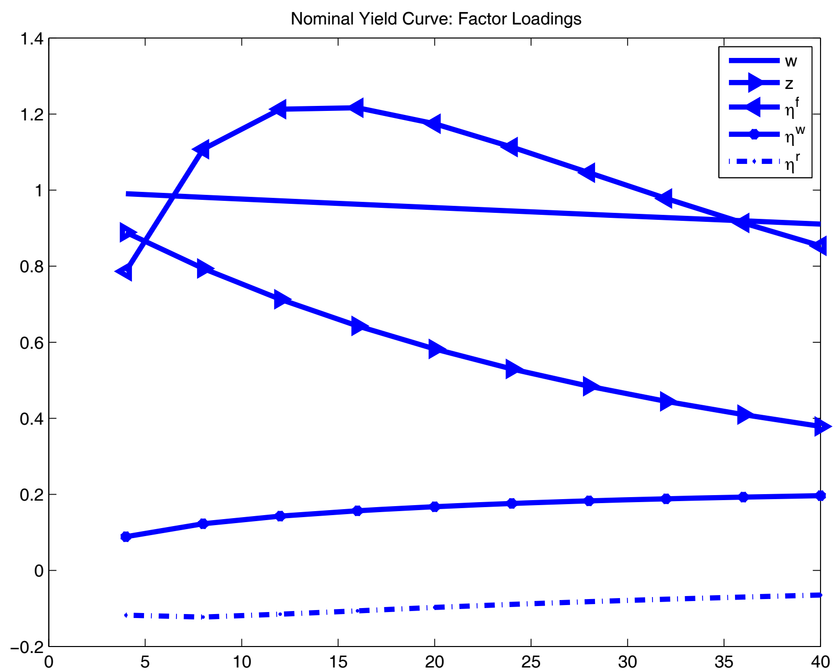

Figure 2: **Impulse Response**

This figure presents impulse response functions for a one percent increase in the amount of FOMC intervention ambiguity. The latter coincides with the cross-sectional dispersion of one quarter ahead FOMC interventions, relative to a forward looking Taylor rule. Ambiguity about FOMC interventions affect real and nominal yields in the same way, i.e. y^4 stands for a 4-quarter yield, y^{20} for a 20-quarter yield, y^{40} for a 40-quarter yield, $y^{20} - y^4$ for the yield curve slope, RP^{20} for the expected excess return of a 20-quarter bond and RP^{40} for the 40-quarter analog.