Short and Long Interest Rate Targets

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Policy Question

- At the zero bound for the short term nominal interest rate, are there additional policy instruments?
- Can monetary policy be conducted with both short and long interest rates?
- Conventional wisdom: Short and long interest rates are not independent policy instruments (Woodford, 2005).
  - Under a Taylor rule for the short rate (with the Taylor principle) there is a locally determinate equilibrium.
  - At the determinate solution, the long rates are obtained by arbitrage from the short rates.
  - It is not possible to use both short and long rates if the locally determinate equilibrium is the single equilibrium. But it is not.

- Even if there is a locally determinate equilibrium, in general, there are multiple equilibria, globally.
- Short and long rates are, in general, independent instruments,
- and they should both be used as a way of solving the classic problem of multiplicity of equilibria when only short rates are used.
- The zero bound is an exception.

Literature

- Sargent and Wallace (1975). There are multiple equilibria with interest rate policy.
- McCallum (1981). Interest rate feedback rule: locally determinate equilibria at the expense of multiple explosive solutions.
- Attempts at solving the multiplicity problem
  - Fiscal theory of the price level.
  - Escape clauses. Atkeson, Chari and Kehoe (2009), Christiano and Rostagno (2002), Nicolini (1996), Obstfeld and Rogoff (1983).
  - There are interest rate rules that deliver global uniqueness. Loisel (2009) and Adao, Correia and Teles (2009).

- Nakajima and Polemarchakis (2003). Measure the degree of multiplicity when the instruments are the money supply or the nominal interest rate.
  - With an infinite horizon, cannot count.
  - Restrictions on the structure so that the infinite horizon is the limit of a finite horizon economy.
  - The rules in Loisel (2009) and Adao, Correia and Teles (2009) are a counterexample.
- Angeletos (2002) and Buera and Nicolini (2004). It is possible to replicate state contingent public debt with debt of different maturities.

Outline

1. Multiplicity of equilibria with interest rate rules in a simple endowment economy.
2. If monetary policy targets state-contingent interest rates (as well as the initial money supply), there is a unique equilibrium globally. Flexible price economy.
3. Instead of targeting the state-contingent interest rates, can target the prices of nominal assets of different maturities.
4. This does not work at the zero bound.
5. Extension of the results to a sticky price model with prices set (one period) in advance.

Multiple equilibria with interest rate rules

- Endowment economy
- Euler equation for the representative household with $C_t = Y_t$

$$\frac{u_c(Y_t)}{P_t} = R_t E_t \frac{\beta u_c(Y_{t+1})}{P_{t+1}}$$

- In log deviations from a deterministic steady state with constant inflation π^*:

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{P}_{t+1} - \widehat{P}_t,$$

where $r_t = \frac{u_c(Y_t)}{\beta E_t u_c(Y_{t+1})}$

- Interest rate target

$$\widehat{R}_t = \widehat{R}_t^*$$

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{\pi}_{t+1}$$

  - Unique path for the conditional expectation of inflation $E_t \widehat{\pi}_{t+1}$,
  - but not for the initial price level, nor the distribution of realized inflation across states.

- Current feedback rule:

$$\widehat{R}_t = \widehat{r}_t + \tau\widehat{\pi}_t$$

$$\widehat{R}_t = \widehat{r}_t + E_t\widehat{\pi}_{t+1}$$

- Equilibria:

$$\tau\widehat{\pi}_t - E_t\left(\widehat{\pi}_{t+1}\right) = 0.$$

  – Equilibrium with $\widehat{\pi}_t = 0$

  – Multiple other solutions
    * If $\tau > 1$ (Taylor principle): Continuum of divergent solutions
      The equilibrium with $\widehat{\pi}_t = 0$ is locally unique
    * If $\tau < 1$: Continuum of solutions converging to $\widehat{\pi}_t = 0$

- In general, interest rate or money supply rules do not solve multiplicity.
- Exception: a particular rule (ACT (2009), Loisel (2008)).

$$\widehat{R}_t = \widehat{r}_t + E_t\widehat{P}_{t+1}$$

$$\widehat{R}_t = \widehat{r}_t + E_t\widehat{P}_{t+1} - \widehat{P}_t$$

implying

$$\widehat{P}_t = 0$$

An interest rate peg

- In a deterministic world: Peg noncontingent nominal returns
- In an uncertain world: Peg state-contingent nominal returns
- Or different maturities

A model with flexible prices

- Representative household, competitive firms, and a government.
- Preferences over consumption and leisure.
- The production uses labor only with a linear technology.
- There are shocks to productivity $A\left(s^t\right)$ and government consumption $G\left(s^t\right)$.
  - Discrete distribution. In each time period $t = 1$, $2...$, one of finitely many events $s_t \in S_t$ occurs. The history of events up to period t, $(s_0, s_1, ..., s_t)$ is $s^t \in S^t$ and the initial realization s_0 is given.
  - The variables are indexed by the history s^t:
    - $C\left(s^t\right)$, $L\left(s^t\right)$, $M\left(s^t\right)$, $B\left(s^t\right)$, $Z\left(s^{t+1}/s^t\right)$, $Q\left(s^{t+1}/s^t\right)$, $R\left(s^t\right)$, $P\left(s^t\right)$, $W\left(s^t\right)$, $T\left(s^t\right)$
    - To simplify the notation: C_t, L_t, M_t, B_t, $Z_{t,t+1}$, $Q_{t,t+1}$, R_t, P_t, W_t, T_t
- Cash-in-advance constraint on the households' transactions with the timing structure as in Lucas (1980).

Households

- Preferences

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(C_t, L_t\right) \right\}$$

- Budget constraints

$$M_t + B_t + E_t Q_{t,t+1} Z_{t,t+1} \leq \mathbb{W}_t$$

$$\mathbb{W}_{t+1} = M_t - P_t C_t + R_t B_t + Z_{t,t+1} + W_t N_t - T_t$$

- Cash-in-advance constraint

$$P_t C_t \leq M_t$$

- Marginal conditions

$$\frac{u_C\left(s^t\right)}{u_L\left(s^t\right)} = \frac{P_t R_t}{W_t}$$

$$\frac{u_C\left(s^t\right)}{P_t} = R_t E_t \left[\frac{\beta u_C\left(s^{t+1}\right)}{P_{t+1}}\right]$$

$$Q_{t,t+1} = \beta \frac{u_C\left(s^{t+1}\right)}{u_C\left(s^t\right)} \frac{P_t}{P_{t+1}}$$

Firms

- The firms are competitive and prices are flexible
- Production function of the representative firm is linear

$$Y_t = A_t N_t$$

- The equilibrium real wage is

$$\frac{W_t}{P_t} = A_t$$

Government

- Government budget constraints

$$\sum_{s=0}^{\infty} E_t Q_{t,t+s+1} \left[M_{t+s} \left(R_{t+s} - 1 \right) + T_{t+s} - P_{t+s} G_{t+s} \right] = \mathbb{W}_t$$

Equilibria

$$C_t + G_t = A_t(1 - L_t)$$

$$\frac{u_C\left(s^t\right)}{u_L\left(s^t\right)} = \frac{R_t}{A_t}$$

From these get $C_t = C(R_t, .)$, $L_t = L(R_t, .)$

$$P_t C_t \leq M_t$$

$$\frac{u_C\left(s^t\right)}{P_t} = R_t E_t \left[\frac{\beta u_C\left(s^{t+1}\right)}{P_{t+1}}\right]$$

$$Q_{t,t+1} = \beta \frac{u_C\left(s^{t+1}\right)}{u_C\left(s^t\right)} \frac{P_t}{P_{t+1}}$$

$$\sum_{s=0}^{\infty} E_t Q_{t,t+s+1} \left[M_{t+s}\left(R_{t+s} - 1\right) + T_{t+s} - P_{t+s} G_{t+s}\right] = \mathbb{W}_t$$

The equilibrium conditions can be summarized by

$$\frac{u_C(R_t)}{P_t} = \beta R_t E_t \left[\frac{u_C(R_{t+1})}{P_{t+1}}\right], t \geq 0$$

$$Q_{t,t+1} = \beta \frac{u_C(R_{t+1})}{u_C(R_t)} \frac{P_t}{P_{t+1}}, t \geq 0$$

- A target for the (noncontingent) nominal interest rate

$$\frac{u_C(R_t)}{P_t} = \beta R_t E_t \left[\frac{u_C(R_{t+1})}{P_{t+1}} \right], \; t \geq 0$$

- Can money supply policy solve the multiplicity?

$$\frac{u_C(R_t)}{P_t} = \beta R_t E_t \left[\frac{u_C(R_{t+1})}{P_{t+1}} \right], t \geq 0$$

and

$$P_t C(R_t) = M_t$$

so that

$$\frac{u_C(R_t)}{\frac{M_t}{C(R_t)}} = \beta R_t E_t \left[\frac{u_C(R_{t+1})}{\frac{M_{t+1}}{C(R_{t+1})}} \right], t \geq 0$$

Policy with state contingent interest rates

$$Q_{t,t+1} = \beta \frac{u_C(R_{t+1})}{u_C(R_t)} \frac{P_t}{P_{t+1}}, \, t \geq 0$$

$$E_t Q_{t,t+1} = \frac{1}{R_t}, \, t \geq 0$$

Proposition 1 If the state contingent interest rates are set exogenously for every date and state, there is a unique equilibrium for the allocations and prices for a given initial price level P_0.

State-contingent debt in zero net supply

- The budget constraints are

$$\sum_{s=0}^{\infty} E_t Q_{t,t+s+1} \left[M_{t+s} \left(R_{t+s} - 1 \right) + T_{t+s} - P_{t+s} G_{t+s} \right] = \mathbb{W}_t$$

where, with $Z_t = 0$, $\mathbb{W}_t = M_{t-1} + R_{t-1} B_{t-1} + P_{t-1} G_{t-1} - T_{t-1}$ is not state-contingent.

- T_{t+s} satisfies the constraints. Ricardian policies.

Term Structure

- Equivalence between pegging state-contingent prices and noncontingent interest rates of different maturities.
- Suppose there is noncontingent debt of maturity $j = 1, ...n$, with gross (compound) interest rate $R_{n,t}$

$$\frac{u_C(R_t^1)}{P_t} = \beta^n R_t^n E_t \left[\frac{u_C(R_{t+n}^1)}{P_{t+n}} \right]$$

- Maturities $n = 2$

$$\frac{u_C(R_t^1)}{P_t} = \beta^2 R_t^2 E_t \left[\frac{u_C(R_{t+2}^1)}{P_{t+2}} \right]$$

$$\frac{u_C(R_t^1)}{P_t} = \beta R_t^1 E_t \left[\frac{u_C(R_{t+1}^1)}{P_{t+1}} \right]$$

$$\frac{u_C(R_{t+1}^1)}{P_{t+1}} = \beta R_{t+1}^1 E_{t+1} \left[\frac{u_C(R_{t+2}^1)}{P_{t+2}} \right]$$

$$\frac{u_C(R_t^1)}{P_t} = \beta^2 R_t^1 E_t \left[R_{t+1}^1 E_{t+1} \left[\frac{u_C(R_{t+2}^1)}{P_{t+2}} \right] \right]$$

$$R_t^2 E_t\left[\frac{u_C(R_{t+2}^1)}{P_{t+2}}\right] = R_t^1 E_t\left[R_{t+1}^1 E_{t+1}\left[\frac{u_C(R_{t+2}^1)}{P_{t+2}}\right]\right]$$

- If $Cov = 0$, then

$$R_t^2 = R_t^1 E_t\left[R_{t+1}^1\right]$$

  The long rate is given by the sequence of short rates.

- But what if the covariance is not zero, which is the general case?

- Two states in each period $t \geq 1$, $\{h, l\}$. $\pi(l, h/s^t)$ is the probability of occurrence of state (s^t, h, l) conditional on s^t.
- One and two period noncontingent bonds.
- Arbitrage conditions

$$\frac{u_C(R^1(s^t))}{P(s^t)} = \beta R^1\left(s^t\right)\left[\pi\left(h/s^t\right)\frac{u_C\left(R^1(s^t,h)\right)}{P(s^t,h)} + \pi\left(l/s^t\right)\frac{u_C(R^1(s^t,l))}{P(s^t,l)}\right]$$

$$\frac{u_C\left(R^1(s^t)\right)}{P(s^t)} = \beta R^2\left(s^t\right)\left[\pi\left(h/s^t\right)\frac{u_C\left(R^1(s^t,h)\right)}{R^1(s^t,h)\,P(s^t,h)} + \pi\left(l/s^t\right)\frac{u_C\left(R^1(s^t,l)\right)}{R^1(s^t,l)\,P(s^t,l)}\right]$$

- Given $P(s^t)$, these conditions determine $P(s^t, h)$ and $P(s^t, l)$, provided $R^1(s^t, l) \neq R^1(s^t, h)$.
- If $R^1(s^t, l) = R^1(s^t, h)$, then $R^2(s^t) = R^1(s^t)\, R^1\left(s^{t+1}\right)$ and the price levels are not pinned down.

$$\frac{u_C(R^1(s^t))}{P(s^t)} = \beta R^1(s^t)\left[\pi(h/s^t)\frac{u_C(R^1(s^t,h))}{P(s^t,h)} + \pi(l/s^t)\frac{u_C(R^1(s^t,l))}{P(s^t,l)}\right]$$

$$\frac{u_C(R^1(s^t))}{P(s^t)} = \beta R^2(s^t)\left[\begin{array}{c}\pi(h,h/s^t)\frac{u_C(R^1(s^t,h,h))}{P(s^t,h,h)} + \pi(l,h/s^t)\frac{u_C(R^1(s^t,h,l))}{P(s^t,h,l)} \\ +\pi(h,l/s^t)\frac{u_C(R^1(s^t,l,h))}{P(s^t,l,h)} + \pi(l,l/s^t)\frac{u_C(R^1(s^t,l,l))}{P(s^t,l,l)}\end{array}\right]$$

$$\frac{u_C(R^1(s^t,h))}{P(s^t,h)} = \beta R^1(s^t,h)\left[\pi(h/s^t,h)\frac{u_C(R^1(s^t,h,h))}{P(s^t,h,h)} + \pi(l/s^t,h)\frac{u_C(R^1(s^t,h,l))}{P(s^t,h,l)}\right]$$

$$\frac{u_C(R^1(s^t,l))}{P(s^t,l)} = \beta R^1(s^t,l)\left[\pi(h/s^t,l)\frac{u_C(R^1(s^t,l,h))}{P(s^t,l,h)} + \pi(l/s^t,l)\frac{u_C(R^1(s^t,l,l))}{P(s^t,l,l)}\right]$$

- Generally, for $S_t = \{s_1, s_2, ..., s_n\}$

$$\frac{u_C(R_t^1)}{P_t} = \beta R_t^1 E_t \left[\frac{u_C\left(R_{t+1}^1\right)}{P_{t+1}} \right]$$

$$\frac{u_C(R_t^1)}{P_t} = \beta R_t^2 E_t \left[\frac{u_C\left(R_{t+1}^1\right)}{R_{t+1}^1 P_{t+1}} \right]$$

...

$$\frac{u_C(R_t^1)}{P_t} = \beta R_t^n E_t \left[\frac{u_C\left(R_{t+1}^1\right)}{R_{t+1}^{n-1} P_{t+1}} \right]$$

Proposition 2 Let $S_t = \{s_1, s_2, ..., s_n\}$ and suppose there are nominal noncontingent assets of maturity $j = 1, ..., n$. If the returns on these assets are set exogenously, then, in general, there is a unique equilibrium for the allocations and prices (if the money supply is set exogenously in the initial period).

- At the zero bound:
- $R^1(s^t, h) = R^1(s^t, l) = 1$, then $R^2(s^t) = 1$.
  For a given initial price level there are multiple equilibria.
- Robustness. If the rates are arbitrarily close to the zero bound.

Sticky prices

- Need price setters:
  - There is a continuum of goods, indexed by $i \in [0,1]$.
    Each good i is produced by a different firm.
  - The composite private consumption is

$$C_t = \left[\int_0^1 c_{it}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \theta > 1,$$

    and public consumption is

$$G_t = \left[\int_0^1 g_{it}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}.$$

  - Monopolistic competitive firms set prices in advance.
- Households equilibrium conditions are the same as under flexible prices.

- Firms that decide the price for period t with the information up to period $t-1$:

$$p_t = \frac{\theta}{(\theta-1)} E_{t-1}\left[\eta_t \frac{W_t}{A_t}\right]$$

where

$$\eta_t = \frac{Q_{t-1,t} P_t^{\theta} Y_t}{E_{t-1}\left[Q_{t-1,t} P_t^{\theta} Y_t\right]}$$

- If there are only these firms, market clearing implies

$$C_t + G_t = A_t(1 - L_t).$$

- Can use the firms and households conditions to write

$$E_{t-1}\left[\frac{u_C\left(s^t\right)}{R_t^1}A_t\left(1-L_t\right)-\frac{\theta}{\left(\theta-1\right)}u_L\left(s^t\right)\left(1-L_t\right)\right]=0,\ t\geq 1$$

  which imposes Φ_{t-1} (intratemporal) constraints on the allocations, where Φ_t is the number of states at t.

- There are less restrictions, but there are also less variables to determine: Φ_{t-1} prices in every period t, instead of Φ_t.

- Allocations are not pinned down by the nominal interest rates.

- Equilibrium conditions:

$$E_{t-1}\left[\frac{u_C\left(s^t\right)}{R_t^1}A_t\left(1-L_t\right)-\frac{\theta}{(\theta-1)}u_L\left(s^t\right)\left(1-L_t\right)\right]=0,\ t\geq 1$$

$$C_t+G_t=A_t(1-L_t),\ t\geq 1$$

$$\frac{u_C\left(s^t\right)}{P_t}=\beta R_t^1 E_t\left[\frac{u_C\left(s^{t+1}\right)}{P_{t+1}}\right],\ t\geq 0$$

$$\frac{u_C\left(s^t\right)}{P_t}=\beta R_t^2 E_t\left[\frac{u_C\left(s^{t+1}\right)}{R_{t+1}^1 P_{t+1}}\right],\ t\geq 0$$

...

$$\frac{u_C\left(s^t\right)}{P_t}=\beta R_t^n E_t\left[\frac{u_C\left(s^{t+1}\right)}{R_{t+1}^{n-1} P_{t+1}}\right],\ t\geq 0$$

- At any $t\geq 1$, given P_{t-1}, C_{t-1}, and L_{t-1}, there are Φ_{t-1} intratemporal conditions, Φ_t resource constraints, and Φ_t intertemporal conditions.

- These determine Φ_t consumption, C_t, and Φ_t labor allocations, $1 - L_t\left(s^t\right)$, and Φ_{t-1} price levels, P_t.
- For $t = 0$, there is one condition, the resource constraint, to determine two variables: C_0 and L_0. P_0 is exogenous.

Conclusion

- Under certainty the nominal interest rate would be the right instrument.
- Under uncertainty need to target state contingent returns (plus initial money supply).
- Long and short term rates can be used to implement a unique equilibrium.
- The returns on assets with different maturities are independent monetary instruments,
- but not at the zero bound.
- Robust to price or wage stickiness.