Precautionary price stickiness

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Motivation

- An old question: does money matter for real allocations?
- How responsive is the *aggregate* price level to shocks?
- Far-reaching implications for how the economy behaves and for the design of suitable policies
Motivation

- Evidence: economic conditions fluctuate continuously, yet at the firm level price adjustment is intermittent
- Some discussion still about the typical (or relevant) frequency of price changes (a lot of heterogeneity)
- But clearly adjustment is not continuous
Motivation

- For any given frequency of micro adjustment, aggregate price stickiness depends on one’s preferred model of pricing
  - e.g. Calvo (1983) vs. fixed menu cost (Golosov-Lucas, 2007)
- The most popular pricing models have unappealing features:
  - Calvo: Lucas critique, cannot study optimal rate of inflation
  - Fixed menu cost: relies on a large cost of changing price tags; ignores cognitive costs
- Both Calvo and FMC have serious difficulties matching the micro evidence on price changes
Motivation

- The search goes on for a “good” model of price stickiness
  - microfounded: not subject to the Lucas critique
  - tractable: can be simulated in general equilibrium
  - matches at least the most salient features of micro evidence
  - hopefully produces plausible IRFs to shocks
Overview of popular pricing models

Elements shared by most existing price-setting models:

- a state variable affecting period profits: \( \frac{P_i}{P_A} \)
- a control variable: \( P_i \)
- factors that shift the state away from the optimum absent control: \( A_i, P \)
- a friction that prevents continuous adjustment of the control:
  - fixed menu cost (Mankiw, Barro, Golosov-Lucas)
  - stochastic menu cost (Dotsey-King-Wolman)
    - switching between 0 and \( \infty \) (Calvo)
  - “generalized (S,s)” (Caballero-Engel, Costain-Nakov)
  - time-dependent models (Taylor)
Overview of popular pricing models

- Useful to think of price-setting as two problems
  - a “timing” problem: is it time to change prices?
  - a “size” problem: by how much to change prices?

- In all of the above models there is a friction in the *timing* of price changes

- The decision on *size* itself is frictionless: the new price is chosen optimally given the friction in timing

- This seems arbitrary. Why consider a friction only in the timing but none in the size choice?
Idea of this paper

- Study a DSGE model in which there is a friction (only) in the size of price changes
- Price stickiness arises endogenously, as a precaution against errors in pricing
- Idea: when firms change prices, they are liable to make a mistake
- Firms are aware of this risk
  - if the current price is close to optimal, potential errors are costly, and firms choose to stick with their current price
  - if the current price is far from optimal, the expected gain from adjustment is positive, and firms change their price
Idea of this paper

- Need discipline when talking about errors: we model mistakes as firms playing quantal (logit) strategies
- Quantal response equilibrium (McKelvey and Palfrey, 1995): a statistical generalization of Nash equilibrium
- Offers a disciplined deviation from REE, controlled by a single “precision” parameter
  - With infinite “precision”, the model becomes the frictionless neoclassical paradigm
  - With bounded precision, prices are sticky
- Choose the precision parameter so as to replicate the median duration of regular prices in the data (10 months)
Idea of this paper

- Evaluate model’s ability to match the evidence on:
  - distribution of price changes by size
    - Data: large and small price changes coexist even within narrowly defined product categories
    - Calvo: too many small changes; FMC: either all large, or all small
  - adjustment hazard rate over time
    - Data: first downward sloping, then flat
    - Calvo: flat; FMC: upward sloping
  - size of price changes as a function of time elapsed since last adjustment
    - Data: flat
    - Calvo & FMC: upward sloping
- Study macro behavior of the model economy: IRFs to (money) shocks
Preview of the results

Size distribution of price changes

Density of price changes

- AC Nielsen data
- PPS model
Preview of the results
Size distribution of price changes

Distribution of non-zero price changes

Size of (log) price changes

Density of price changes

AC Nielsen
Calvo
Preview of the results
Size distribution of price changes

Distribution of non-zero price changes
Size of (log) price changes
Density of price changes

AC Nielsen
Fixed menu cost
Preview of the results

Probability of price change over time

Hazard rate: \( h(k) = \frac{f(k)}{s(k-1)} \)
Preview of the results

Size of price changes over time

Mean absolute price change as a function of time since adjustment

Months elapsed since last price adjustment
Related literature
On state-dependent pricing

► In partial equilibrium

► In general equilibrium
  ► Dotsey-King-Wolman (1999) (stochastic menu costs; aggr. shock only)
  ► Golosov-Lucas (2007) (fixed menu cost; aggr. + idiosync. shocks)
  ► Gertler-Leahy (2008) (simplify for analytic results)
  ► Costain-Nakov (2008) (generalized (S,s); aggr. + idiosync.)
Related literature

On quantal response equilibrium

- McKelvey and Palfrey (1995)
  - A generalization of Nash equilibrium that allows for noisy optimizing behavior while maintaining the internal consistency of rational expectations
  - Very successful in matching observed behavior in experiments
    - Fey, McKelvey, and Palfrey (1996): centepede game
Related literature
On quantal response equilibrium

- Camerer, Ho, Chong (2004):
  - “Quantal response equilibrium, a statistical generalization of Nash, almost always explains the direction of deviations from Nash, and should replace Nash as the static benchmark to which other models are routinely compared”

- Haile, Horacsu, Kosenok (AER, 2008):
  - Need additional maintained assumptions to be able to test the model empirically, e.g. logit choice
Outline of the talk

1. Introduction ✓
2. Model
3. Calibration
4. Results
5. Conclusions
Model: main features

- Firm output: \( Y = A_i N \)
- Profits: \( U = PY - WN \)
- Firm value: \( V(P, A, ...) = U + E(QV(P', A', ...)) \)
- Optimal price choice (neoclassical):
  \( P^*(A) = \arg \max_P V(P, A) \)
- Instead, we assume noisy optimization:
  - firms’ price is drawn from a (logit) distribution over possible prices, with probabilities proportional to the payoff associated with each price, adjusted by precision parameter \( \xi \in [0, \infty) \):

\[
\pi(P|A) = \frac{\exp(\xi V(P, A))}{\sum_P \exp(\xi V(P, A))}
\]
Model: main features

\[ \pi(P|A) = \frac{\exp(\xi V(P, A))}{\sum_P \exp(\xi V(P, A))} \]

- Parameter \( \xi \in [0, \infty) \) controls the “degree of rationality”:
  - When precision is infinite (\( \xi = \infty \)), firms choose the optimal price \( P^* \) with probability \( \pi(P^*|A) = 1 \) (neoclassical)
  - When precision is zero (\( \xi = 0 \)), firms choose a uniform distribution over possible prices (myopic firms)
  - When precision is positive but bounded (\( 0 < \xi < \infty \)), the probability of choosing the optimal price \( 0 < \pi(P^*|A) < 1 \)
  - The optimal price has the highest probability of being chosen
  - And the probability of choosing a “good” price is (much) higher than choosing a “bad” price
Model: main features

- Firm’s expected value if it decides to change its price

\[ E(V(A)) = \sum_P \pi(P|A)V(P, A) \]

Weighted average over all possible prices, including some which are worse than the current price, so

- Expected gain from adjustment,
  \[ G = E(V(A)) - V(P, A) \geq 0 \]

- Adjustment (timing) decision:
  - change price if \( G > 0 \)
  - stay with old price if \( G < 0 \)
Model: main features

- Changing the price itself is costless (zero “menu” cost), the friction comes from the possibility of errors in pricing
- Errors occur only in the size of price changes, not in the timing (later we relax this)
Model: main features

- Whether $G \geq 0$ depends on the proximity of the current price to the optimum “(S,s)-type model”
  - If the current price is far from the optimal (there was a big fundamental shock), then $G > 0$
    - the firm resets its price
  - If the current price is close to optimal (there was a small shock), then $G < 0$
    - the firm chooses to stick with its old price
- Price stickiness is “precautionary”
Model: the rest is standard

- Household utility: $\frac{C^{1-\gamma}}{1-\gamma} - \chi N + \nu \log(M/P)$ with discount factor $\beta$

- Period budget constraint:
  \[ P_t C_t + M_t + R_t^{-1} B_t = W_t N_t + M_{t-1} + T_t + B_{t-1} + \Pi_t \]

- Consumption bundle:
  \[ C_t = \left[ \int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} \, di \right]^{\frac{\epsilon}{\epsilon-1}} \] with price index $P_t \equiv \left[ \int_0^1 P_{it}^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}}$

- Money supply: $M_t = \mu_t M_{t-1}$ where $\mu_t = \mu \exp(z_t)$, and $z_t = \phi_z z_{t-1} + \epsilon_t^z$
Solution method

- Challenge: firms are ex-ante identical, but not ex-post, so the entire distribution of firms on \((P, A)\) is a state variable

- Solution: Reiter’s (*JEDC*, 2009) method of “projection and perturbation”

- Typically idiosyncratic shocks are much bigger than aggregate shocks

- Combines
  - Nonlinear solution of the aggregate steady-state on a grid (projection)
  - Linearization around the steady-state wrt aggregate shocks (perturbation)
Solution method: steady state (projection)

- Guess: $w$
- Labor FOC: $C = (\chi / w)^{1/\gamma}$
- Payoffs in different $(p, a)$ grid points:
  $$u_{ij} = (p_i - w / a_j) C(p_i)^{-\epsilon}$$
- Solve Bellman equation: $V = U + \beta R' (V + G^\xi) S$
- Calculate distributions:
  - $\tilde{\Psi} = R\Psi S'$
  - $\Psi = (1 - \Lambda) \ast \tilde{\Psi} + \Pi \ast (1 \ast (\Lambda \ast \tilde{\Psi}))$
- Check if $1 = \left[ \sum_{ij} \psi_{ij} (p_j)^{1-\epsilon} \right]^{1/(1-\epsilon)}$ and adjust $w$ until it is satisfied
Solution method: dynamics (perturbation)

- Dynamic Bellman equation:
  \[ V_t = U_t + \beta E_t \left[ \frac{u'(C_{t+1})}{u'(C_t)} R'_{t+1} \left( V_{t+1} + G_{t+1}^{\xi} \right) S \right] \]

- Distribution dynamics:
  \[ \tilde{\Psi}_t = R_t \Psi_{t-1} S' \]
  \[ \Psi_t = (1 - \Lambda_t) \cdot \tilde{\Psi}_t + \Pi_t \cdot (1 \ast (\Lambda_t \ast \tilde{\Psi}_t)) \]

- Collect variables in vector:
  \[ X_t = (\text{vec}(V_t)', C_t, p_t, \text{vec}(\Psi_{t-1}')) \]

- Model: \[ E_t F (X_{t+1}, X_t, z_{t+1}, z_t) = 0 \]

- Linearization:
  \[ E_t A \Delta X_{t+1} + B \Delta X_t + E_t C z_{t+1} + D z_t = 0 \]

- Solve with Klein’s QZ method for linear models
Calibration

Discount factor \( \beta = 1.04^{-1/12} \)  
Golosov-Lucas (2007)

CRRA \( \gamma = 2 \)  
Ibid.

Labor supply \( \chi = 6 \)  
Ibid.

MIUF coeff. \( \nu = 1 \)  
Ibid.

Elast. subst. \( \epsilon = 7 \)  
Ibid.

Money growth \( \mu = 1 \)  
AC Nielsen dataset: zero inflation

Persistence \( \rho = 0.95 \)  
Blundell-Bond (2000)

Std. dev. prod. \( \sigma = 0.06 \)  
Eichenbaum et. al. (2009)

Precision \( \xi = 16.67 \)  
Nakamura-Steinsson (2008): 10m

- Note: noise = 1/precision = 0.06; even less noise than is typically estimated in applied GT experiments
Some steady-state objects

Expected adjustment gain (% of median firm value)

Price distribution probabilities

(S,s) adjustment bands

Density of firms after adjustment
Responses to a correlated money growth shock
### Phillips curves

<table>
<thead>
<tr>
<th></th>
<th>Calvo</th>
<th>FMC</th>
<th>PPS</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly frequency (%)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Std. dev. money shock (%)</td>
<td>0.33</td>
<td>0.12</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>Std. dev. quart. inflation (%)</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Std. dev. quart. output (%)</td>
<td>1.08</td>
<td>0.19</td>
<td>0.40</td>
<td>0.51</td>
</tr>
<tr>
<td>Slope $\beta_2$ of Phillips curve</td>
<td>1.10</td>
<td>0.15</td>
<td>0.37</td>
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</tr>
</tbody>
</table>

$c_t = \beta_1 + \beta_2 \hat{\pi}_t + \varepsilon_t$, where $\hat{\pi}_t$ is instrumented by the money supply
Sensitivity to noise

- Shock process
- Inflation
- Nominal interest rate
- Consumption
- Labor
- Price dispersion
- Real interest rate
- Real wage
- Real money holdings
Summary

- We propose a disciplined approach for studying near-rational price setting in DSGE models based on logit equilibrium.
- Logit choice generates price stickiness even if firms are free to change their price costlessly in each period.
- When prices are close to optimal, firms prefer to leave them unchanged.
- The model matches several “puzzles” from micro data on pricing which existing models are unable to match.
- Money shocks have real effects inbetween the fixed menu cost and the Calvo model.
Extensions

- The model can easily accommodate errors in both the size and timing of price changes. “Nested logit”:
  - decide whether to adjust
  - decide by how much to adjust

- The solution method can easily be applied to a larger scale (Smets-Wouters type) DSGE
  - Straightforward to model monetary policy with a Taylor rule, to study other aggregate shocks, add frictions etc.

- The framework can be applied to other situations where action is intermittent