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# Is There a Fiscal Free Lunch in a Liquidity Trap?

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#### Is There a Fiscal Free Lunch in a Liquidity Trap?\*

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#### Abstract

This paper uses a DSGE model to examine the effects of an expansion in government spending in a liquidity trap. If the liquidity trap is very prolonged, the spending multiplier can be much larger than in normal circumstances, and the budgetary costs minimal. But given this "fiscal free lunch," it is unclear why policymakers would want to limit the size of fiscal expansion. Our paper addresses this question in a model environment in which the duration of the liquidity trap is determined endogenously, and depends on the size of the fiscal stimulus. We show that even if the multiplier is high for small increases in government spending, it may decrease substantially at higher spending levels; thus, it is crucial to distinguish between the average and marginal multiplier.

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#### 1. Introduction

During the past two decades, a voluminous empirical literature has attempted to gauge the effects of fiscal policy shocks. This literature has been instrumental in identifying the channels through which fiscal policy affects the economy, and, in principle, would seem a natural guidepost for policymakers seeking to assess how alternative fiscal policy actions could mitigate the current global recession.

However, it is unclear whether estimates of the effects of fiscal policy from this empirical literature – which focuses almost exclusively on the postwar period – should be regarded as applicable under conditions of a recession-induced liquidity trap.<sup>1</sup> Keynes (1933, 1936) argued in support of aggressive fiscal expansion during the Great Depression exactly on the grounds that the fiscal multiplier was likely to be much larger during a severe economic downturn than in normal times, and the burden of financing it correspondingly lighter.

In this paper, we use a New-Keynesian DSGE modeling framework to examine the implications of an increase in government spending for output and the government budget when monetary policy faces a liquidity trap. A key advantage of the DSGE framework is that it allows explicit consideration of how the conduct of monetary policy – and, in particular, the zero bound constraint on nominal interest rates – affects the multiplier.

We begin by showing that the government spending multiplier can be amplified substantially in the presence of a prolonged liquidity trap. This corroborates previous analysis by Eggertson (2008) and Davig and Leeper (2009), which shows that government spending can have outsized effects when monetary policy reacts passively by allowing real interest rates to fall, and recent work by Christiano, Eichenbaum and Rebelo (2009) in a model with endogenous capital accumulation. While our workhorse model is a variant of the Christiano, Eichenbaum and Evans (2005) and Smets-Wouters (2007) models, we show that the spending multiplier is even larger in versions that embed hand-to-mouth agents (as in Galí, López-Salido, and Vallés 2007) and financial frictions (as in Bernanke, Gertler, and Gilchrist 1999, and Christiano, Motto, and Rostagno 2007). Moreover,

¹ The bulk of research suggests a government spending multiplier in the range of 0.5 to slightly above unity. One strand of the literature – originating with Barro (1981, 1990) – has estimated the multiplier by examining the response of output to changes in military spending. This approach tends to yield multipliers in the range of 0.5-0.8, including in more recent work by Ramey (2009) and Hall (2009); however, as emphasized by Hall, the estimates hinge critically on the relationship between output and spending during WWII and the Korean War, and may be somewhat downward-biased due to the "command-economy" features prevalent in WWII, and because taxes were raised markedly during the Korean War. An alternative approach involves identifying the government spending multiplier using a structural VAR – as in Blanchard and Perotti (2002), and Gali, Lopez-Salido, and Valles (2007). These studies report a government spending multiplier of unity or somewhat higher (after 1-2 years), though the cross-county evidence of Perotti (2007) and Mountford and Uhlig (2008) is suggestive of a lower multiplier.

an increase in government spending against the backdrop of a deep liquidity trap puts less upward pressure on public debt than under normal circumstances, reflecting that the larger output response translates into much higher tax revenues.

At first blush, these results seem highly supportive of Keynes' argument for fiscal expansion in response to a recession-induced liquidity trap – the benefits are extremely high, and the budgetary expense to achieve it very low. But this raises the important question of why policymakers would want to limit the magnitude of fiscal expansion, and thus pass up on what appears to be a "fiscal free lunch."

Our paper addresses this question by showing that the spending multiplier in a liquidity trap decreases with the level of government spending. The novel feature of our approach is to allow the economy's exit from a liquidity trap – and return to conventional monetary policy – to be determined endogenously, with the consequence that the multiplier depends on the size of the fiscal response. Quite intuitively, a large fiscal response pushes the economy out of a liquidity trap more quickly. Because the multiplier is smaller upon exiting the liquidity trap – reflecting that monetary policy reacts by raising real interest rates – the marginal impact of a given-sized increase in government spending on output decreases with the magnitude of the spending hike. This dependence of the government spending multiplier on the scale of fiscal expansion evidently contrasts with a standard linear framework in which the multiplier is invariant to the size of the spending shock.

The qualitative implication that the multiplier declines in the level of spending suggests an important rationale for limiting the size of fiscal spending packages in a liquidity trap: in particular, even if the multiplier is high for small increments to government spending, it may be relatively low at higher spending levels. Therefore, it is crucial to characterize the behavior of the marginal spending multiplier to make informed judgements about the appropriate scale of fiscal intervention in a liquidity trap. Accordingly, a major focus of our paper consists of a quantitative characterization of how the government spending multiplier varies with the level of spending in an array of nested DSGE models.

Section 2 analyzes the effects of government spending shocks in a simple three equation New Keynesian model in which policy rates are constrained by the zero lower bound. Similar to previous research (e.g., Eggertson 2008), the liquidity trap is generated by an adverse taste shock that sharply depresses the potential real interest rate. A key result of our analysis is that the marginal government spending multiplier – measured as the contemporaneous impact on output

of a very small increment in government spending – is a step function in the level of government spending. If the level of spending is sufficiently small, higher government spending does not affect the economy's exit date from the liquidity trap, and the multiplier is constant at a value that is higher than in a normal situation in which monetary policy would raise real interest rates. However, as spending rises to higher levels, the economy emerges from the liquidity trap more quickly, and the marginal multiplier drops. The multiplier continues to drop discretely as government spending rises further – reflecting a progressive shortening of the liquidity trap – until spending is high enough to keep the economy from falling into a liquidity trap. Beyond this level of spending, the marginal multiplier levels out at a value equal to that under normal conditions in which policy rates are unconstrained.

The simple New Keynesian model is a very convenient tool for illustrating the salient role of inflation expectations in determining the marginal multiplier. If prices are fairly responsive to marginal cost—as implied by relatively short-lived price contracts—the multiplier is extremely high for small increments to government spending, but drops quickly at higher spending levels. Thus, the large multipliers that apply to small fiscal expansion should not be inferred to carry over to much larger fiscal expansions, and it is crucial to take account of the endogeneity of the multiplier precisely under those conditions in which the marginal multiplier is very high. By contrast, the multiplier function is much flatter under a flatter Phillips Curve slope, and even at low spending levels it isn't dramatically different than in normal times.

The simple model is also convenient tool for assessing other empirically relevant factors that may affect the multiplier, including implementation lags in spending. Implementation lags may dampen the multiplier significantly, and even cause the multiplier to be negative if the lags are sufficiently long. Thus, echoing Friedman (1953), the efficacy of fiscal policy in macroeconomic stabilization – even in a liquidity trap – can be hampered by "long and variable lags."

These considerations garnered from the stylized model prove useful in interpreting the behavior of the government spending multiplier in more empirically-realistic models. In Section 3, we investigate a model that is very similar to the estimated models of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). Given the relevance of initial conditions that determine the duration and depth of the liquidity trap for the spending multiplier, we analyze the multiplier against the backdrop of a "severe recession scenario" that attempts to capture some of the features of the recession-induced liquidity trap experienced by the United States and other countries during the recent financial crisis. This scenario is constructed by a sequence of adverse

consumption demand shocks that depress output by about 8 percent relative to steady state, and generates a liquidity trap lasting two years.

We begin by considering impulse response functions to a 1 percent of GDP increase in government spending. The impact on output is about 0.8 after four quarters, almost twice as large as under normal conditions in which monetary policy would raise real interest rates immediately. The larger multiplier translates into a smaller rise in government debt. Against the backdrop of an even deeper recession with an 11 quarter liquidity trap, the government spending multiplier exceeds 2, and government debt declines.

However, impulse response functions only indicate the average effects of a given-sized fiscal impetus, and may be a poor gauge of the marginal impact. Under our benchmark calibration, the marginal multiplier falls substantially below the average multiplier even for fairly modest increases in government spending. For example, the marginal multiplier is nearly 3 when initial conditions imply an 11 quarter liquidity trap for spending increases less than 0.5 percent of GDP, but drops to 1.1 for spending increases exceeding 1.5 percent of GDP. Correspondingly, the marginal impact of higher government spending on government debt is considerably greater than the average response.

Although the peak multiplier can be much higher under calibrations in which expected inflation is more responsive to shocks, the marginal multiplier declines more abruptly with the size of the fiscal stimulus. Our benchmark calibration implies a fairly sluggish price and wage adjustment, even though the slope of both the price- and wage-setting schedules are in line with recent empirical estimates: for example, price contracts have an effective duration of ten quarters.<sup>2</sup> If both prices and wages were more responsive,<sup>3</sup> the deflationary pressure caused by the recession would be larger, but expansionary fiscal policy would be more effective in reversing it – implying a higher multiplier. But as the fiscal stimulus shortens the duration of the trap, this deflationary pressure quickly recedes, and the marginal benefit of additional stimulus falls sharply. Thus, with four quarter price and wage contracts, the marginal multiplier is about 10 for a very small increase in government spending, but drops to unity when government spending is boosted more than 1 percent of baseline GDP. As discussed below, the resilence of short-run expected inflation during the past

<sup>&</sup>lt;sup>2</sup> In this paper, we find it convenient to simply map a given slope of the Phillips curve into an effective contract duration under the assumption that marginal costs are identical across firms. However, empirical work by e.g. Galí, Gertler, and López-Salido (2001) and Altig, Christiano, Eichenbaum and Lindé (2010) has shown that if capital is firm-specific, a given slope of the Phillips curve can be consistent with a considerably lower degree of price stickiness under plausible assumptions about capital utilization costs and the elasticity of firm demand. Consequently, even the relatively long contract durations we consider in this paper are consistent with empirical estimates of the slope of the Phillips Curve.

<sup>&</sup>lt;sup>3</sup> As we show below, even short-lived price contracts may be consistent with a relatively muted response of inflation if wages are sufficiently sticky.

recession makes us skeptical of calibrations that imply such extreme variations in the multiplier across spending levels.

Section 4 analyzes an augmented model that incorporates both financial frictions, and Keynesian "hand-to-mouth" households. The inclusion of Keynesian households is appealing insofar as Galí, López-Salido, and Vallés (2007) have shown that it can account for the positive response of private consumption to a government spending shock documented in structural VAR studies by e.g., Blanchard and Perotti (2002) and Perotti (2007).<sup>4</sup> These additional features amplify the effect of government spending on the potential real interest rate, and hence boost the multiplier. If households that consume their after-tax income comprise half the population – which seems at the upper end of the plausible range – the peak government spending multiplier is about 1.5 for an eight quarter liquidity trap; even so, the marginal multiplier drops to around unity for spending increases beyond 1 percent of GDP.<sup>5</sup>

Taken together, our results suggest a somewhat nuanced view of the role of fiscal policy in a liquidity trap. For an economy facing a protracted recession and for which monetary policy seems likely to be constrained by the zero bound for a very prolonged period – exceeding 2 years – there is a strong argument for increasing government spending on a temporary basis. Consistent with the views originally espoused by Keynes, this temporary boost can have much larger effects than under usual conditions, and comes at a low cost to the Treasury. For a shorter-lived liquidity trap of less than two years, the multiplier is somewhat larger than under normal conditions, provided that the spending is modest in scale, and can be implemented quite rapidly to coincide with the liquidity trap; but larger spending programs are likely to suffer from sharply diminishing returns, and boost government debt at the margin.

#### 2. A stylized New Keynesian model

As in Eggertsson and Woodford (2003), we use a standard log-linearized version of the New Keynesian model that imposes a zero bound constraint on interest rates. Our framework allows exit from the liquidity trap to be determined endogenously, rather than fixed arbitrarily, an innovation

<sup>&</sup>lt;sup>4</sup> As discussed by the recent paper by Leeper, Walker and Yang (2009) and in Ramey (2009), identified VARs can produce misleading results if some of the fiscal expansion is anticipated. Accordingly, Fisher and Peters (2009) identify government spending shocks with statistical innovations to the accumulated excess returns of large U.S. military contractors, and find that positive spending shocks are associated with an output multiplier above unity and increases in hours and consumption.

<sup>&</sup>lt;sup>5</sup> Uhlig (2009) emphasizes that highly persistent (or permanent) increases in the level of government spending tend depress output significantly at horizons beyond a couple of years if the higher spending must be financed by a hike in the labor income tax rate. The short-run multiplier is also damped significantly if the labor income tax rate responds rapidly to government debt.

that is crucial in showing how the multiplier varies with the level of fiscal spending.

#### 2.1. The Model

The key equations of the model are:

$$x_t = x_{t+1|t} - \hat{\sigma}(i_t - \pi_{t+1|t} - r_t^{pot}), \tag{1}$$

$$\pi_t = \beta \pi_{t+1|t} + \kappa_p x_t, \tag{2}$$

$$i_t = \max\left(-i, \gamma_\pi \pi_t + \gamma_x x_t\right),\tag{3}$$

$$r_t^{pot} = \frac{1}{\hat{\sigma}} \left( 1 - \frac{1}{\phi_{mc} \hat{\sigma}} \right) \left[ g_y (g_t - g_{t+1|t}) + (1 - g_y) \nu_c (\nu_t - \nu_{t+1|t}) \right]$$
 (4)

where  $\hat{\sigma}$ ,  $\kappa_p$ , and  $\phi_{mc}$  are composite parameters defined as:

$$\hat{\sigma} = \sigma(1 - g_y)(1 - \nu_c) \tag{5}$$

$$\kappa_p = \frac{(1 - \xi_p)(1 - \beta \xi_p)}{\xi_p} \phi_{mc} \tag{6}$$

$$\phi_{mc} = \frac{\chi}{1 - \alpha} + \frac{1}{\hat{\sigma}} + \frac{\alpha}{1 - \alpha} \tag{7}$$

and where  $x_t$  is the output gap,  $\pi_t$  is the inflation rate,  $i_t$  is the short-term nominal interest rate, and  $r_t^{pot}$  is the potential (or "natural") real interest rate. All variables are measured as percent or percentage point deviations from their steady state level.<sup>6</sup>

Equation (1) expresses the "New Keynesian" IS curve in terms of the output and real interest rate gaps. Thus, the output gap  $x_t$  depends inversely on the deviation of the real interest rate  $(i_t - \pi_{t+1|t})$  from its potential rate  $r_t^{pot}$ , as well as directly on the expected output gap in the following period. The parameter  $\hat{\sigma}$  determines the sensitivity of the output gap to the real interest rate; as indicated by (5), it depends on the household's intertemporal elasticity of substitution in consumption  $\sigma$ , the steady state government spending share of output  $g_y$ , and a (small) adjustment factor  $\nu_c$  which scales the consumption taste shock  $\nu_t$ . The price-setting equation (2) specifies current inflation to depend on expected inflation and the output gap, where the sensitivity to

<sup>&</sup>lt;sup>6</sup> We use the notation  $y_{t+j|t}$  to denote the conditional expectation of a variable y at period t+j based on information available at t, i.e.,  $y_{t+j|t} = E_t y_{t+j}$ . The superscript 'pot' denotes the level of a variable that would prevail under completely flexible prices, e.g.,  $y_t^{pot}$  is potential output.

the latter is determined by the composite parameter  $\kappa_p$ . Given the Calvo-Yun contract structure, equation (6) implies that  $\kappa_p$  varies directly with the sensitivity of marginal cost to the output gap  $\phi_{mc}$ , and inversely with the mean contract duration  $(\frac{1}{1-\xi_p})$ . The marginal cost sensitivity equals the sum of the absolute value of the slopes of the labor supply and labor demand schedules that would prevail under flexible prices: accordingly, as seen in equation (7),  $\phi_{mc}$  varies inversely with the Frisch elasticity of labor supply  $\frac{1}{\chi}$ , the composite parameter  $\hat{\sigma}$  determining the interest-sensitivity of aggregate demand, and the labor share in production  $(1-\alpha)$ .

Equation (4) indicates that the potential real interest rate is driven by two exogenous shocks, including a consumption taste shock  $\nu_t$  and government spending shock  $g_t$ . Each shock, if positive, raises the marginal utility of consumption associated with any given output level, which puts upward pressure on the real interest rate if the shock is front-loaded.<sup>7</sup> The consumption taste shock and government spending shock are assumed to follow an AR(1) process with the same persistence parameter  $(1 - \rho_{\nu})$ , e.g., the taste shock follows:

$$\nu_t = (1 - \rho_{\nu})\nu_{t-1} + \varepsilon_{\nu,t},\tag{9}$$

Given the same stochastic structure for the shocks, it is evident from equation (4) that these shocks affect the potential real interest rate in an identical way.

The log-linearized equation for the stock of government debt is given by:

$$b_t = (1+r)b_{t-1} + g_y g_t - \tau_L(l_t + \zeta_t) - \tau_t, \tag{10}$$

where  $l_t$  is labor hours,  $\zeta_t$  is the real wage,  $b_t$  is end-of-period government debt, and  $\tau_t$  is a lumpsum tax (with both  $b_t$  and  $\tau_t$  expressed as a share of steady state GDP). The government derives tax revenue from a fixed tax on labor income  $\tau_L$ , and from the time-varying lump-sum tax  $\tau_t$ . The tax rate  $\tau_L$  is set so that government spending is financed exclusively by the distortionary labor tax in the steady state (with the government debt stock zero in steady state). Lump-sum taxes adjust according to the reaction function:

$$\tau_t = \varphi_\tau \tau_{t-1} + \varphi_b b_{t-1},\tag{11}$$

Given that agents are Ricardian and that only lump-sum taxes adjust dynamically, the fiscal rule only affects the evolution of the stock of debt and lump-sum taxes, with no effect on other macro

$$\lambda_{c,t} = -\frac{1}{\hat{\sigma}}c_t + \frac{\nu_c(1 - g_y)}{\hat{\sigma}}\nu_t = \frac{1}{\hat{\sigma}}\left[\frac{(g_y g_t - y_t)}{1 - g_y} + \nu_c(1 - g_y)\nu_t\right]$$
(8)

where  $c_t$  is consumption,  $y_t$  output, and  $g_t$  government spending.

<sup>&</sup>lt;sup>7</sup> The effect of each shock on the marginal utility of consumption  $\lambda_{ct}$  can be expressed:

variables. In Sections 3, we will consider the implications of rules in which distortionary taxes adjust dynamically.

Our benchmark calibration is fairly standard. The model is calibrated at a quarterly frequency. We set the discount factor  $\beta=0.995$ , and the steady state net inflation  $\pi=.005$ ; this implies a steady state interest rate of i=.01 (one percent at a quarterly rate, or four percent at an annualized rate). We set the intertemporal substitution elasticity  $\sigma=1$  (i.e. assume logarithmic period utility), the capital share parameter  $\alpha=0.35$ , the Frisch elasticity of labor supply  $\frac{1}{\chi}=0.4$ , the government share of steady state output  $g_y=0.2$ , and the scale parameter on the consumption taste shock  $\nu_c=0.01$ . We examine a range of values of the price contract duration parameter  $\xi_p$  to highlight the senstivity of the fiscal multiplier to the Phillips Curve slope  $\kappa_p$ . It is convenient to assume that monetary policy would completely stabilize output and inflation in the absence of a zero bound constraint, which can be regarded as a limiting case in which the coefficient on inflation  $\gamma_{\pi}$  in the interest rate reaction function becomes arbitrarily large. Finally, the preference and government spending shocks are assumed to follow an AR(1) process with persistence of 0.9, so that  $\rho_{\nu}=0.1$  in equation (9).

#### 2.2. Impulse Responses to a Front-Loaded Rise in Government Spending

The effects of fiscal policy in a liquidity trap depend on agents' perceptions about how long the liquidity trap would last in the absence of fiscal stimulus, as well as the severity of the associated recession.<sup>8</sup> Thus, we must first specify a shock(s) that pushes the economy into a liquidity trap, and then consider how fiscal policy operates against the backdrop of these initial conditions. Clearly, a number of different shocks could cause the "notional" interest rate – the level of the interest rate if monetary policy were unconstrained – to fall persistently below zero, and consequently generate a liquidity trap.

For simplicity, we follow the recent literature – including Eggertson and Woodford (2003), and Eggertson (2009), and Adam and Billi (2008) – by assuming that the liquidity trap is caused by an adverse taste shock  $\nu_t$  that sharply depresses the potential real interest rate  $r_t^{pot}$ . The response of  $r_t^{pot}$  to the adverse taste shock is depicted by the solid line in Figure 1a. Given the assumption that monetary policy would fully stabilize inflation and the output gap if feasible, the nominal interest rate  $i_t$  simply tracks  $r_t^{pot}$  provided that the implied nominal rate is non-negative (i.e.,  $i_t$ 

<sup>&</sup>lt;sup>8</sup> In the more empirically realistic models considered in Sections 3 and 4, the underlying shock process is chosen to roughly match the decline in U.S. GDP during the recent recession. However, in this section we simply scale the taste shock to generate a liquidity trap of the same duration (8 quarters) as in the models considered subsequently.

 $=r_t^{pot}$ , recalling that both variables are measured as percentage point deviations from baseline). The concurrence of the nominal and potential real interest rate is apparent in Figure 1a beginning in period  $T_n$ , which is the first period in which  $r_t^{pot}$  exceeds -i = -1 percent (the figure shows the annualized interest rate, so -4 percent). However, because  $r_t^{pot} < -i$  prior to  $T_n$ , equations (1)-(3) imply that the nominal interest rate must equal its lower bound of -i. The taste shock is scaled so that the liquidity trap lasts for  $T_n = 8$  quarters.

The solid lines in Figure 2 shows the effects of the taste shock on the output gap, inflation, the real interest rate, and government debt (relative to baseline GDP). To highlight the role of expected inflation in amplifying the effects of the shock, it is useful to begin by illustrating the effects in a limiting case in which inflation (and hence expected inflation) is constant. This is achieved by assuming that the average duration of price contracts is arbitrarily long, so that  $\kappa_p$  (in equation 2) is close to zero.<sup>9</sup> The left column of Figure 2 shows this limiting case. The real interest rate declines in lockstep with the nominal interest rate (i.e., by i=4 percent). However, because the potential interest rate declines by more, and remains persistently below -i, output falls persistently below potential. The government debt/GDP ratio increases substantially, mainly because revenue from the labor income tax declines in response to lower labor demand and falling real wages.

We next consider the effect of a one percentage point of (steady state) GDP rise in government spending against this backdrop. As seen in Figure 1a, the higher government spending simply offsets some of the decline in  $r_t^{pot}$  induced by the negative taste shock, so that the path of  $r_t^{pot}$  shifts upward in a proportional manner. Because the government spending hike is too small to affect the duration of the liquidity trap, monetary policy continues to hold the nominal interest rate unchanged for  $T_n = 8$  quarters. As seen in the left column of Figure 2, this invariance of the nominal rate implies that the higher government spending has no effect on the path of the real interest rate (dashed lines) over this period. Accordingly, the output gap is less negative in response to the combined shocks, since the real interest rate remains unchanged even though the potential real interest rate path is higher. The (partial) effect of the government spending rise – the difference between the response to the combined shocks and taste shock alone – is depicted by the dash-dotted line(s) in the figure. Recalling that government spending has no effect on the output gap outside of the liquidity trap, and only affects potential output, the spending multiplier

<sup>&</sup>lt;sup>9</sup> The parameter  $\xi_p$  is set equal to .9995, implying a mean duration of price contracts of 2000 quarters, and a value of  $\kappa_p$  of .0000028.

is clearly larger in a liquidity trap. The bigger effect on output translates into higher tax revenue, so that the higher government spending reduces government debt slightly for a couple of quarters.

The government spending multiplier  $\frac{1}{g_y}\frac{dy_t}{dg_t}$ , i.e., the percentage increase in output in response to a one percent of baseline GDP rise in government spending, is about 0.7 in this case. The multiplier is computed as the sum of the output gap response of 0.4 shown in the figure, plus the effect on potential output (not shown) of 0.3. The multiplier is amplified substantially more when expected inflation responds to shocks, as illustrated in the right column of Figure 2 for a calibration implying a mean duration of price contracts of 5 quarters ( $\xi_p = 0.8$ ). In this case, the negative output gap due to the taste shock causes inflation to fall persistently (solid lines). With expected inflation falling more than the nominal interest rate, the real interest rate rises, which reinforces the contraction in output. Thus, the same-sized fall in  $r_t^{pot}$  has much larger adverse effects on output when expected inflation reacts. Conversely, in addition to boosting  $r_t^{pot}$ , higher government spending causes expected inflation to rise, and hence exerts a more stimulative effect on output than when expected inflation remains constant. The peak output gap response of 1.8 seen in the figure implies a spending multiplier of 2.1, and translates into a substantial and persistent reduction in government debt.

#### 2.3. The Multiplier and the Size of Fiscal Spending

In the log-linearized model that ignores the zero bound constraint, the government spending multiplier is invariant to the size of the change in spending, so that the effects of an X percent of GDP spending hike can be inferred by multiplying the responses in Figure 2 by X. By contrast – as we next proceed to show – the multiplier in a liquidity trap declines in the level of government spending. Intuitively, this behavior reflects that the multiplier varies positively with the duration of the liquidity trap, and that the duration shortens as the level of spending rises.

Because government spending and taste shocks have the same linear effects on  $r_t^{pot}$ , and only  $r_t^{pot}$  matters for the output gap and inflation response, it is convenient to simply analyze how  $r_t^{pot}$  affects those variables in a liquidity trap. Solving the IS curve forward yields:

$$x_{t} = -\hat{\sigma} \sum_{j=0}^{T_{n}-1} (-i - r_{t+j|t}^{pot}) + \hat{\sigma}iT_{n} + \hat{\sigma} \sum_{j=1}^{T_{n}} \pi_{t+j|t} + x_{T_{t}|t}$$
(12)

The output gap  $x_t$  in the current period depends on four terms. First, it depends on the *cumulative* gap between the nominal interest rate -i and the potential real interest rate over the interval in

which the economy remains in a liquidity trap. This cumulative interest rate gap  $\sum_{j=0}^{T_n-1} (-i-r_{t+j|t}^{pot})$  can be interpreted as indicating how shocks to the potential real interest rate would affect the output gap if expected inflation remained constant. Second, the output gap depends on cumulative expected inflation over the liquidity trap (or equivalently, the log change in the price level  $log(P_{Tn}) - log(P_t)$ ); as indicated above, the effects of shocks to the potential real rate on the output gap can be amplified through changes in expected inflation. Third, the current output gap also depends on the expected output gap  $x_{T_n|t}$  when the economy exits the liquidity trap, though both the terminal output gap and inflation terms drop under the assumption that monetary policy completely stabilizes the economy  $(x_{T_n|t} = \pi_{Tn|t} = 0)$ . Finally, the exit date  $T_n$  is determined endogenously as the first period in which the expected potential real interest rate exceeds -i. Thus:

$$T_n = \min_j(r_{t+j|t}^{pot} > -i) \tag{13}$$

In general, this exit date depends both on the size and persistence of the shock to  $r_t^{pot}$ . The relation between the exit date and  $r_t^{pot}$  under our baseline calibration with  $(1-\rho_{\nu})=0.9$  is shown in Figure 1b. Because the exit date is only affected as  $r_t^{pot}$  exceeds certain threshold values, it is a step function in the level of  $r_t^{pot}$  (rising as  $r_t^{pot}$  assumes more negative values). Thus, a slightly larger adverse taste shock that caused  $r_t^{pot}$  to drop more than shown in Figure 1a would leave the duration of the liquidity trap unchanged at 8 quarters; but a large enough adverse shock would extend the duration of the trap, and a sufficiently smaller shock would shorten it.

In the limiting case in which expected inflation remains constant, we can derive a simple closed form solution for the multiplier. In particular, given the AR(1) specification of the taste and government spending shocks,  $r_t^{pot}$  also follows an AR(1) with persistence parameter 1- $\rho_v$ , equation (12) implies that the output gap  $x_t$  equals:

$$x_t = -\hat{\sigma} \sum_{i=0}^{T_n - 1} (-i - (1 - \rho_v)^j r_t^{pot}) = \hat{\sigma} i T_n + \hat{\sigma} r_t^{pot} \frac{1 - (1 - \rho_v)^{T_n}}{\rho_v} < 0$$
 (14)

For changes in government spending that are small enough to keep the liquidity trap duration unchanged at  $T_n$  periods, the multiplier  $\frac{1}{g_y} \frac{dy_t}{dg_t}$  is derived by differentiating equation (14) with respect to  $g_t$ , and adding the effect on potential output:  $\left(\frac{dy_t^{pot}}{dg_t}\right)$ 

$$\frac{1}{g_y}\frac{dy_t}{dg_t} = \frac{1}{g_y}\frac{d(y_t - y_t^{pot} + y_t^{pot})}{dg_t} = \frac{1}{g_y}(\frac{dx_t}{dg_t} + \frac{dy_t^{pot}}{dg_t}) = \hat{\sigma}\frac{1 - (1 - \rho_v)^{T_n}}{\rho_v}\frac{1}{g_y}\frac{dr_t^{pot}}{dg_t} + \frac{1}{g_y}\frac{dy_t^{pot}}{dg_t}$$
(15)

The first term – the output gap component – is positive. It varies directly with the duration of the underlying liquidity trap  $T_n$  (induced by the taste shock), reflecting that fiscal policy can only affect the output gap over the period in which the economy remains in the trap. The second term  $\frac{1}{g_y} \frac{dy_t^{pot}}{dg_t}$  is equal to the spending multiplier in the flexible price equilibrium, as well as during normal times given our assumption that monetary policy, if unconstrained, keeps output at potential. The latter may be expressed as  $\frac{1}{g_y} \frac{dy_t^{pot}}{dg_t} = \frac{1}{\phi_{mc}\hat{\sigma}} < 1$  (since  $\phi_{mc}\hat{\sigma} = 1 + \frac{(\alpha + \chi)\hat{\sigma}}{1-\alpha} > 1$ ).

Substituting  $\frac{1}{g_y} \frac{dr_t^{pot}}{dg_t} = \frac{1}{\hat{\sigma}} (1 - \frac{1}{\phi_{mc}\hat{\sigma}}) \rho_v$  into equation (15), the marginal multiplier can be expressed in the simple form:

$$\frac{1}{g_y}\frac{dy_t}{dg_t} = 1 - (1 - \frac{1}{\phi_{mc}\hat{\sigma}})(1 - \rho_v)^{T_n}$$
(16)

The upper panel of Figure 3 plots how the marginal multiplier varies with the duration of the liquidity trap, where the latter is indicated by the tick marks along the upper axis. Thus, the multiplier associated with a tiny increment to government spending in an 8 quarter liquidity trap is about 0.7, but rises to about 0.8 against the backdrop of an 11 quarter liquidity trap (caused by a larger contractionary taste shock than in Figure 1a). The marginal multiplier increases monotonically with the duration of the trap, but in a concave manner; and importantly, the multiplier remains below unity provided that the liquidity trap is of finite duration, however long.

These results are useful in illuminating how the marginal multiplier varies with the size of the increase in government spending. While the previous exercise examined the effects of tiny increments to government spending against the backdrop of different initial conditions (i.e., associated with liquidity traps of varying length), we now take "initial conditions" – summarized by a given-sized taste shock – as fixed, and assess how increases in government spending affect the multiplier by reducing the duration of the liquidity trap. For a liquidity trap of duration  $T_n$  induced by the taste shock, the government spending multiplier remains constant at the value implied by equation (15) until government spending exceeds a threshold level  $g_t(0)$  that boosts the potential real interest rate just enough to shorten the liquidity trap by one period (with this threshold determined by equation 13). The multiplier then jumps down to the level implied by a  $T_{n-1}$  period trap, where it remains for sufficiently small additional increments to spending.

In this vein, the upper left panel of Figure 3 can be reinterpeted as showing how the marginal multiplier varies with alternative levels of government spending. For concreteness, we assume that the liquidity trap is generated by the same adverse taste shock shown in Figure 1a, so that the liquidity trap would last 8 quarters in the absence of any government spending response. Thus, for

a government spending hike of less less than 1.2 percent of GDP, the duration of the liquidity trap remains unchanged at  $T_n = 8$  quarters, and the multiplier of 0.7 is exactly equal to the impact multiplier shown in Figure 2. If government spending increases more than 1.2 percent, but less than 3.1 percent, the liquidity trap is shortened by one period, and the marginal multiplier falls discontinuously to 0.66 (i.e., the value implied by equation (15) with  $T_n = 7$ ). The marginal multiplier continues to decline in a step-wise fashion – with equation (13) implicitly determining the threshold levels of spending at which the multiplier drops discontinuously – until levelling off at a constant value of  $\frac{1}{g_y} \frac{dy_t^{pot}}{dg_t}$  (equal to about 0.3) corresponding to a spending level high enough to keep the economy from entering a liquidity trap. Conversely, reductions in government spending exert a progressively more negative marginal impact as they become large enough to extend the duration of the liquidity trap. The figure can also be used to assess how the multiplier varies with spending for alternative initial conditions after shifting the horizontal axis appropriately; for example, if the initial conditions implied an 11 quarter trap, then the 0 government spending level would be shifted left toward the origin.

The relationship between the marginal spending multiplier and level of spending can be given an alternative graphical interpretation using Figure 1a. Recall that absent any fiscal response, the adverse taste shock would depress the path of the potential real interest rate as shown by the solid line (labeled "taste shock only"). The effect on the output gap is proportional to  $\sum_{j=0}^{T_n-1}(-i-r_{t+j|t}^{pot})$ , which is simply the sum of the bold vertical line segments between -i and the path of  $r_{t+i}^{pot}$  implied by the taste shock through period  $T_{n-1}$  (the factor of proportionality is  $-\hat{\sigma}$ ). A small enough increase in government spending – including the 1 percent of GDP rise shown by the dashed line – has no effect on the duration of the liquidity trap, so that the higher government spending narrows the gap between between -i and  $r_{t+i}^{pot}$  (the "interest rate gaps") over  $T_n = 8$  periods. The quantitative effect on the output gap of incremental spending is equal to  $\hat{\sigma} \frac{1-(1-\rho_v)^8}{\rho_v} \frac{dr_t^{pot}}{dg_t} dg_t > 0$ . But as government spending rises above the threshold of 1.2 percent of GDP, the potential interest rate at  $T_{n-1}$  rises above -i, and the liquidity trap duration shortens to 7 quarters. Thus, increments to spending in the range of 2 percent of GDP (the dash-dotted line) have no effect on the interest rate gap at  $T_{n-1}$ , as the increase in the potential real rate due to the spending increment is completely offset by monetary policy. Accordingly, fiscal policy only affects the interest rate gaps for 7 periods, and the incremental effect on the output gap falls to  $\hat{\sigma} \frac{1 - (1 - \rho_v)^7}{\rho_v} \frac{dr_t^{pot}}{dg_t} dg_t.$ 

The "outsized" multiplier in a liquidity trap implies that small increases in government spending

actually reduce the government deficit. This is shown in the upper right panel, which plots the government deficit (relative to baseline GDP) as a function of the level of government spending. However, for spending levels high enough to shorten the duration of the liquidity trap by just two quarters, marginal increments to spending boost the deficit, albeit to a smaller degree than in normal times.

The variation in the marginal multiplier with the level of spending is even more pronounced in the more plausible case in which the Phillips Curve is upward-sloping. When expected inflation responds, movements in the potential real interest rate  $r_t^{pot}$  have larger effects on the output gap than implied by equation (14), so that the same taste shock has a larger contractionary effect, and higher government spending has a more stimulative effect. To see how the effects of variation in  $r_t^{pot}$  are magnified, equations (1) and (2) can be solved forward (imposing the zero bound constraint that  $i_t = -i$ ) to express inflation in terms of current and future interest rate gaps:

$$\pi_t = -\hat{\sigma}\kappa_p \sum_{j=0}^{T_n - 1} \psi(j) (-i - r_{t+j|t}^{pot}), \tag{17}$$

where the weighting function  $\psi(j)$  is given by

$$\psi(j) = \lambda_1 \psi(j-1) + \lambda_2^j, \tag{18}$$

with the initial condition  $\psi(0) = 1$ , and where  $\lambda_1$  and  $\lambda_2$  are determined as:

$$\lambda_1 + \lambda_2 = 1 + \beta + \hat{\sigma}\kappa_p,\tag{19}$$

$$\lambda_1 \lambda_2 = \beta. \tag{20}$$

Given that  $\kappa_p > 0$ , the coefficients  $\psi(j)$  premultiplying the interest rate gap grow exponentially with the duration of the liquidity trap  $T_n$ . Moreover, the contour is extremely sensitive to  $\kappa_p$ , as illustrated in Figure 4a for several values of  $\kappa_p$  associated with price contraction durations ranging from four to ten quarters.

The convex pattern of weights reflects that deflationary pressure associated with any given-sized interest rate gap ( $-i-r_{t+j|t}^{pot}$ ) is compounded as liquidity trap lengthens by the interaction between the response of the output gap and expected inflation. A small interest rate gap of  $\varepsilon$  at  $T_{n-1}$  would reduce the output gap  $x_{T_{n-1}}$  by  $-\hat{\sigma}\varepsilon$ , as inflation would be expected to return to baseline at  $T_n$ . But because the lower output gap at  $T_{n-1}$  also reduces inflation by  $\kappa_p \hat{\sigma}\varepsilon$ , the same-sized interest rate gap of  $\varepsilon$  at  $T_{n-2}$  would imply a fall in  $x_{T_{n-2}}$  of  $2\hat{\sigma}\varepsilon + \beta\kappa_p\hat{\sigma}\varepsilon$ , with  $\beta\kappa_p\hat{\sigma}\varepsilon$  reflecting the contribution

from lower expected inflation. This nonlinear effect on the output gap arising from the expected inflation channel helps account for the sharply nonlinear response of inflation in Figure 4a.

Because even small interest rate gaps in the future can have large effects on expected inflation when  $\kappa_p$  is high enough, fiscal policy actions which reduce the real interest rate gap can potentially exert much larger real effects than when inflation is fixed. But a corrollary is that, to the extent that fiscal stimulus shortens the duration of the liquidity trap, deflationary pressure arising from the intial contractionary shock should recede, reducing the impact of additional stimulus (even more than the case in which expected inflation is fixed). To illustrate both these observations, the lower left panel of Figure 3 plots the impact government spending multiplier as a function of the level of government spending under alternative specifications of the price contract duration parameter (i.e. the slope of the Phillips curve). Focusing first on the multiplier associated with an increase in government spending of 1 percent of GDP, it is clear that the multiplier is extremely sensitive to the price contract duration parameter. The multiplier in the case of 10 quarter price contracts is 1.4, or roughly double that in the limiting case in which price inflation is unresponsive (shown in the upper panel); but the multiplier rises to 2.1 with 5 quarter contracts, and to about 4 with 4 quarter contracts. This high sensitivity to the price contract duration – and hence to the expected inflation response – confirms our previous discussion of the impulse response functions in Figure But even more striking is the wide divergence between the step functions across the different The marginal multiplier for the case of 10 quarter price contracts is relatively flat, decreasing only gradually in the level of spending. By contrast, the marginal multiplier for 4 quarter contracts drops precipitously as spending rises.

Thus, structural parameters influencing the expected inflation response play a salient role in affecting the multiplier. If expected inflation is responsive enough, the multiplier increases in a convex manner with the duration of the liquidity trap (as the lower panel of Figure 3 for 4 quarter contracts), in contrast to the concave relation when expected inflation is less responsive. As seen in the lower right panel, small increments to government spending can reduce the fiscal deficit substantially when prices are relatively flexible and the liquidity trap sufficiently persistent. But the policy response itself can greatly reduce the benefit of additional stimulus, and cause the marginal impact on government debt to quickly turn positive. Accordingly, given the sharp decline in the multiplier in an environment with highly responsive inflation expectations, it is vital to distinguish the marginal impacts of fiscal actions on output and the government budget from the average effects captured by impulse response functions.

There is admittedly some uncertainty about the "appropriate" calibration of the Phillips Curve slope  $\kappa_p$ , which in our formulation is simply pinned down by contract duration parameter. But calibrations with shorter price contract durations imply an enormous reaction of expected inflation to the adverse taste shock which generates the underlying liquidity trap; as seen in Figure 2, the 9 percent fall in output below baseline is accompanied by a 20 percentage point fall in inflation under the assumption of 5 quarter price contracts. To the extent that such implications seem at odds with historical experience, including during the recent recession, the calibration with the flatter Phillips Curve (corresponding to 10 quarter contracts) – and relatively flat marginal multiplier schedule – seems more plausible.

#### 2.4. Effects of Implementation Lags

We conclude this section by examining the implications of lags between the announcement of higher fiscal spending and its implementation. In particular, we assume that the government announces a new stimulus plan immediately in response to the adverse preference shock, but that it takes some time for spending to peak. To capture such delays, we assume that government spending follows an AR(2) as in Uhlig (2009):

$$g_t - g_{t-1} = \rho_{q_1}(g_{t-1} - g_{t-2}) - \rho_{q_2}g_{t-1} + \varepsilon_{g,t}, \tag{21}$$

This representation makes clear that there is some persistence in the growth rate of government spending, even though the level is stationary due to the "error correction" term  $\rho_{g2}$ .

The solid lines in Figure 4b show the effects of a rise in government spending that peaks after eight quarters (achieved by setting  $\rho_{g1} = .90$  and  $\rho_{g2} = 0.025$ ) against the backdrop of the same adverse preference shock considered previously (again depicted by the dashed lines).<sup>10</sup> Given the implementation lag, the higher spending depresses  $r_t^{pot}$  over the entire period in which the economy is in the liquidity trap, while leaving the duration of the trap unchanged at 8 quarters. As seen by equation (4), the expectation that government spending will grow in the future depresses the potential real interest rate  $r_t^{pot}$  by encouraging saving. Interestingly, the response of output is significantly negative, reflecting that aggregate demand is weaker over the entire period in which the economy is in the liquidity trap. The negative output response induces a larger deterioration of the fiscal balance, and consequent boost in the government debt/GDP ratio.

The rather dramatic consequences for the multiplier shown in the figure are dependent on the

<sup>&</sup>lt;sup>10</sup> The mean duration of price contracts is set to 5 quarters, as in Figure 2.

monetary policy specification; it can be shown that the output response is in fact uniformly positive under a less aggressive monetary policy rule. Even so, implementation lags may have substantial implications for the multiplier, with the spending multiplier shrinking considerably if the bulk of the higher spending occurs after monetary policy is no longer constrained by the zero bound.

#### 3. An Empirically-Validated New Keynesian Model with Capital

In this section, we present a fully-fledged model with endogenous capital accumulation. Our objectives are to assess whether the factors identified as playing a major role in influencing the multiplier in the simple New Keynesian model continue to be important in a more empirically realistic framework, as well as to provide a more reasonable quantitative assessment of the multiplier.

Our model can be regarded as a slightly simplified version of the model developed and estimated by Christiano, Eichenbaum and Evans (2005), and Smets and Wouters (2003, 2007). Christiano, Eichenbaum and Evans (2005) show that their model can account well for the dynamic effects of a monetary policy innovation during the post-war period. Smets and Wouters (2003, 2007) consider a much broader set of shocks, and estimate their model using Bayesian methods. They argue that it is able to fit many key features of U.S-. and euro area-business cycles.

#### 3.1. The Model

As outlined below, our benchmark model incorporates nominal rigidities by assuming that labor and product markets exhibit monopolistic competition, and that wages and prices are determined by staggered nominal contracts of random duration (following Calvo (1983) and Yun (1996)). The model includes an array of real rigidities, including habit persistence in consumption, and costs of changing the rate of investment. Monetary policy follows a Taylor rule, and fiscal policy specifies that taxes respond to government debt.

#### 3.1.1. Firms and Price Setting

Final Goods Production As in Chari, Kehoe, and McGrattan (2000), we assume that there is a single final output good  $Y_t$  that is produced using a continuum of differentiated intermediate goods  $Y_t(f)$ . The technology for transforming these intermediate goods into the final output good is constant returns to scale, and is of the Dixit-Stiglitz form:

$$Y_{t} = \left[ \int_{0}^{1} Y_{t} \left( f \right)^{\frac{1}{1+\theta_{p}}} df \right]^{1+\theta_{p}} \tag{22}$$

where  $\theta_p > 0$ .

Firms that produce the final output good are perfectly competitive in both product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index  $Y_t$ , taking as given the price  $P_t(f)$  of each intermediate good  $Y_t(f)$ . Moreover, final goods producers sell units of the final output good at a price  $P_t$  that is equal to the marginal cost of production:

$$P_{t} = \left[ \int_{0}^{1} P_{t}\left(f\right)^{\frac{-1}{\theta_{p}}} df \right]^{-\theta_{p}} \tag{23}$$

It is natural to interpret  $P_t$  as the aggregate price index.

Intermediate Goods Production A continuum of intermediate goods  $Y_t(f)$  for  $f \in [0,1]$  is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces a demand function for its output good that varies inversely with its output price  $P_t(f)$ , and directly with aggregate demand  $Y_t$ :

$$Y_{t}\left(f\right) = \left[\frac{P_{t}\left(f\right)}{P_{t}}\right]^{\frac{-\left(1+\theta_{p}\right)}{\theta_{p}}}Y_{t} \tag{24}$$

Each intermediate goods producer utilizes capital services  $K_t(f)$  and a labor index  $L_t(f)$  (defined below) to produce its respective output good. The form of the production function is Cobb-Douglas:

$$Y_t(f) = K_t(f)^{\alpha} L_t(f)^{1-\alpha}$$
(25)

Firms face perfectly competitive factor markets for hiring capital and the labor index. Thus, each firm chooses  $K_t(f)$  and  $L_t(f)$ , taking as given both the rental price of capital  $R_{Kt}$  and the aggregate wage index  $W_t$  (defined below). Firms can costlessly adjust either factor of production. Thus, the standard static first-order conditions for cost minimization imply that all firms have identical marginal cost per unit of output.

The prices of the intermediate goods are determined by Calvo-Yun style staggered nominal contracts. In each period, each firm f faces a constant probability,  $1-\xi_p$ , of being able to reoptimize its price  $P_t(f)$ . The probability that any firm receives a signal to reset its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, we follow Christiano, Eichenbaum and Evans (2005) and assume that it simply adjusts its price by a weighted combination of the lagged and steady state rate of inflation (i.e.,  $P_t(f) = \pi_{t-1}^{\iota_p} \pi^{1-\iota_p} P_{t-1}(f)$  for the non-optimizing firms). When  $\iota_p$  is set close to unity, this formulation introduces structural inertia into the inflation process.

#### 3.1.2. Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector; that is, goods-producing firms regard each household's labor services  $N_t(h)$ ,  $h \in [0,1]$ , as an imperfect substitute for the labor services of other households. It is convenient to assume that a representative labor aggregator (or "employment agency") combines households' labor hours in the same proportions as firms would choose. Thus, the aggregator's demand for each household's labor is equal to the sum of firms' demands. The labor index  $L_t$  has the Dixit-Stiglitz form:

$$L_{t} = \left[ \int_{0}^{1} N_{t} \left( h \right)^{\frac{1}{1+\theta_{w}}} dh \right]^{1+\theta_{w}} \tag{26}$$

where  $\theta_w > 0$ . The aggregator minimizes the cost of producing a given amount of the aggregate labor index, taking each household's wage rate  $W_t(h)$  as given, and then sells units of the labor index to the production sector at their unit cost  $W_t$ :

$$W_{t} = \left[ \int_{0}^{1} W_{t} \left( h \right)^{\frac{-1}{\theta_{w}}} dh \right]^{-\theta_{w}} \tag{27}$$

It is natural to interpret  $W_t$  as the aggregate wage index. The aggregator's demand for the labor hours of household h – or equivalently, the total demand for this household's labor by all goodsproducing firms – is given by

$$N_t(h) = \left\lceil \frac{W_t(h)}{W_t} \right\rceil^{-\frac{1+\theta_w}{\theta_w}} L_t \tag{28}$$

The utility functional of a typical member of household h is

$$\mathbb{E}_{t} \sum_{i=0}^{\infty} \beta^{j} \left\{ \frac{1}{1-\sigma} C_{t+j}(h) - \varkappa C_{t+j-1} - \nu_{c} \nu_{t} \right\}^{1-\sigma} - \frac{\chi_{0}}{1+\chi} N_{t+j}(h)^{1+\chi} \right\}$$
 (29)

where the discount factor  $\beta$  satisfies  $0 < \beta < 1$ . The period utility function depends on household h's current consumption  $C_t(h)$ , as well as aggregate per capita consumption in the previous period  $C_{t-1}$ . This formulation allows the possibility of external habit persistence in consumption spending as in Smets and Wouters (2003). As in the simple model considered in the previous section, a positive taste shock  $\nu_t$  raises the marginal utility of consumption associated with any given consumption level. The period utility function also depends on current leisure  $1 - N_t(h)$ .

Household h's budget constraint in period t states that its expenditure on goods and net purchases of financial assets must equal its disposable income:

$$P_{t}C_{t}(h) + P_{t}I_{t}(h) + \frac{1}{2}\psi_{I}P_{t}\frac{(I_{t}(h) - I_{t-1}(h))^{2}}{I_{t-1}(h)} + P_{B,t}B_{G,t+1} - B_{G,t} + \int_{s} \xi_{t,t+1}B_{D,t+1}(h) - B_{D,t}(h)$$

$$= (1 - \tau_{N,t})W_{t}(h)N_{t}(h) + (1 - \tau_{K,t})R_{K,t}K_{t}(h) + \delta\tau_{K,t}P_{t}K_{t}(h) + \Gamma_{t}(h) - T_{t}(h)$$
(30)

Thus, the household purchases the final output good (at a price of  $P_t$ ), which it chooses either to consume  $C_t(h)$  or invest  $I_t(h)$  in physical capital. The total cost of investment to each household h is assumed to depend on how rapidly the household changes its rate of investment (as well as on the purchase price). Our specification of such investment adjustment costs as depending on the square of the change in the household's gross investment rate follows Christiano, Eichenbaum, and Evans (2005). Investment in physical capital augments the household's (end-of-period) capital stock  $K_{t+1}(h)$  according to a linear transition law of the form:

$$K_{t+1}(h) = (1 - \delta)K_t(h) + I_t(h)$$
(31)

In addition to accumulating physical capital, households may augment their financial assets through increasing their government bond holdings  $(P_{B,t}B_{G,t+1} - B_{G,t})$ , and through the net acquisition of state-contingent bonds. We assume that agents can engage in frictionless trading of a complete set of contingent claims. The term  $\int_s \xi_{t,t+1}B_{D,t+1}(h) - B_{D,t}(h)$  represents net purchases of state-contingent domestic bonds, with  $\xi_{t,t+1}$  denoting the state price, and  $B_{D,t+1}(h)$  the quantity of such claims purchased at time t. Each member of household h earns after tax labor income  $(1 - \tau_{N,t})W_t(h)N_t(h)$ , after-tax capital rental income of  $(1 - \tau_{K,t})R_{K,t}K_t(h)$ , and a depreciation allowance of  $\delta\tau_{K,t}P_tK_t(h)$ . Each member also receives an aliquot share  $\Gamma_t(h)$  of the profits of all firms, and pays a lump-sum tax of  $T_t(h)$  (this may be regarded as taxes net of any transfers).

In every period t, each member of household h maximizes the utility functional (29) with respect to its consumption, investment, (end-of-period) capital stock, bond holdings, and holdings of contingent claims, subject to its labor demand function (28), budget constraint (30), and transition equation for capital (31). Households also set nominal wages in Calvo-style staggered contracts that are generally similar to the price contracts described above. Thus, the probability that a household receives a signal to reoptimize its wage contract in a given period is denoted by  $1 - \xi_w$ ; as in the case of price contracts, this probability is independent of the date at which the household last reset its wage. In addition, we specify a dynamic indexation scheme for the adjustment of the wages of those households that do not get a signal to reoptimize, i.e.,  $W_t(h) = \omega_{t-1}^{\iota_w} \omega^{1-\iota_w} W_{t-1}(h)$ ,

where  $\omega_{t-1}$  is the gross nominal wage inflation in period t-1 and  $\omega=\pi g_z$  is the steady state rate of change in the nominal wage (gross price inflation times steady state gross productivity growth). Dynamic indexation of this form introduces some element of structural persistence into the wage-setting process.

#### 3.1.3. Fiscal and Monetary Policy and the Aggregate Resource Constraint

Government purchases  $G_t$  are assumed to follow an exogenous stochastic process given by eq. (21). These purchases are assumed neither to affect the utility of households in a multiplicative fashion (i.e. we retain the assumption of separability, in which case purchases in equation (29) can be disregared for positive analysis purposes), nor to serve as an input into goods production. Government expenditures are assumed to be financed by a combination of labor taxes, taxes on capital income, and lump sum taxes. However, the government does not need to balance its budget each period, and issues nominal debt to finance budget deficits according to

$$P_{B,t}B_{G,t+1} - B_{G,t} = P_tG_t - T_t - \tau_{N,t}W_tL_t - \tau_{K,t}(R_{K,t} - \delta P_t)K_t. \tag{32}$$

In eq. (32), all quantity variables are aggregated across households, so that  $B_{G,t}$  is the aggregate stock of government bonds,  $K_t$  is the aggregate capital stock, and  $T_t = (\int_0^1 T_t(h) dh)$  are aggregate lump-sum taxes. Throughout the analysis, we will assume that capital taxes  $\tau_{Kt}$  are given by an exogenous stochastic process with mean  $\tau_K$ . However lump-sum taxes adjust endogenously in our benchmark specification. The tax rate reaction has the same basic form as in Section 2, but also allows taxes to respond to the gross budget deficit (i.e. the first difference of the debt as share of trend output):

$$\tau_t - \tau = \varphi_\tau (\tau_t - \tau) + \varphi_b (b_{G,t} - b_G) + \varphi_d (b_{G,t} - b_{G,t-1}), \tag{33}$$

where  $b_{G,t} = B_{G,t}/\bar{Y}$ . For sensitivity analysis, we also consider a case in which the distortionary tax rate on labor income adjusts according to eq. (33), in which case  $\tau_{N,t}$  replaces  $\tau_t$ . Some simple econometric analysis suggest that these specifications fit the US post-1980 evidence quite well if  $\varphi_b$  and  $\varphi_d$  are set to small values.

Monetary policy is assumed to be given by a policy rule similar to eq. (3) except allowing for a smoothing coefficient  $\gamma_i$ :

$$i_t = \{ \max(-i, (1 - \gamma_i) (\gamma_\pi \pi_t + \gamma_x x_t) + \gamma_i i_{t-1}) \}$$
 (34)

We set  $\gamma_i = 0.7$ ,  $\gamma_{\pi} = 3$  and  $\gamma_x = 0.25$  based on the estimation results reported by Erceg, Guerrieri and Gust (2006) for the 1983:1-2003:4 period.<sup>11</sup>

Finally, total output of the service sector is subject to the resource constraint:

$$Y_t = C_t + I_t + G_t + \psi_{I,t} \tag{35}$$

where  $\psi_{I,t}$  is the adjustment cost on investment aggregated across all households (from eq. 30,  $\psi_{I,t} \equiv \frac{1}{2} \psi_I \frac{(I_t(h) - I_{t-1}(h))^2}{I_{t-1}(h)}$ ).

#### 3.1.4. Solution and Calibration

To analyze the behavior of the model, we log-linearize the model's equations around the non-stochastic steady state. Nominal variables, such as the contract price and wage, are rendered stationary by suitable transformations. To solve the unconstrained version of the model, we compute the reduced-form solution of the model for a given set of parameters using the numerical algorithm of Anderson and Moore (1985), which provides an efficient implementation of the solution method proposed by Blanchard and Kahn (1980). When we solve the model subject to the non-linear monetary policy rule (34), we use the techniques described in Hebden, Lindé and Svensson (2009). An important feature of the Hebden, Lindé and Svensson algorithm is that the duration of the liquidity trap is endogenous, and is affected by the size of the fiscal impetus.

As in Section 2, we set the discount factor  $\beta=0.995$ , and steady state (net) inflation  $\pi=.005$ , implying a steady state nominal interest rate of i=.01 at a quarterly rate. The subutility function over consumption is logarithmic, so that  $\sigma=1$ , and the parameter determining the degree of habit persistence in consumption  $\varkappa$  is set at 0.6 (similar to the empirical estimate of Smets and Wouters 2003). The Frisch elasticity of labor supply  $\frac{1}{\chi}$  of 0.4 is well within the range of most estimates from the empirical labor supply literature (see e.g. Domeij and Flodén, 2006).

The capital share parameter  $\alpha$  is set to 0.35. The quarterly depreciation rate of the capital stock  $\delta = 0.025$ , implying an annual depreciation rate of about 10 percent. We set the cost of adjusting investment parameter  $\psi_I = 3$ , which is somewhat smaller than the value estimated by Christiano, Eichenbaum, and Evans (2005) using a limited information approach; however, the analysis of Erceg, Guerrieri, and Gust (2006) suggests that a lower value may be better able to capture the unconditional volatility of investment.

<sup>&</sup>lt;sup>11</sup> Some simple regression analysis for the sample period 1993:4-2008:3 supports the estimation results in Erceg, Guerrieri and Gust (2006) and suggest that our benchmark parameterization is in line with historical correlations. Our own analysis suggest that the federal funds rate has become somewhat more responsive to movements in the output gap and inflation in the more recent years.

We maintain the assumption of a relatively flat Phillips curve by setting the price contract duration parameter  $\xi_p = 0.9$ . As in Christiano, Eichenbaum and Evans (2005), we also allow for a fair amount of intrinsic persistence by setting the price indexation parameter  $\iota_p = 0.9$ . It bears emphasizing that our choice of  $\xi_p$  does not necessarily imply an average price contract duration of 10 quarters. Altig et al. (2010) show that even a model with a low slope of the Phillips curve can be consistent with frequent price reoptimization. The structure of our model is essentially identical to theirs, and for a moderate markup of  $\theta_p = .10$  (i.e., a 10 percent markup) our choice of  $\xi_p$  (and implied slope of the Phillips curve of about 0.012) is consistent with about 4-5 quarters between reoptimization of price contracts under the assumption that capital is firm-specific. Hence, our choice of  $\xi_p$  accords well with empirical evidence on the Phillips curve slope, including the work of Adolfson et al. (2005), Altig et al. (2010), Gali and Gertler (1999), Gali, Gertler, and Lopez-Salido (2001), Lindé (2005), and Smets and Wouters (2003, 2007).

Given strategic complementarities in wage-setting across households, the wage markup influences the slope of the wage Phillips curve. Our choices of a wage markup of  $\theta_W = 1/3$  and a wage contract duration parameter of  $\xi_w = 0.85-$  along with a wage indexation parameter of  $\iota_w = 0.9$ - imply that wage inflation is about as responsive to the wage markup as price inflation is to the price markup.

The parameters pertaining to fiscal policy are set as follows. The share of government spending of total expenditure is set equal to 20 percent. The steady state capital income tax rate,  $\tau_K$ , is set to 0.2 while the lump sum tax revenue to GDP ratio is set to 0.02. The government debt to GDP ratio is 0.5, close to the total estimated federal government debt to output ratio at end-2009. The government's intertemporal budget constraint implies that labor income tax rate  $\tau_N$  equals 0.27 in steady state.<sup>12</sup> The parameters in the fiscal policy rule in eq. (33) are set to  $\varphi_{\tau} = 1$ ,  $\varphi_b = 0.05$  and  $\varphi_d = 0.10$ . These coefficients are in line with the historical correlations between total tax revenues, government debt, and the deficit,<sup>13</sup> and imply that the tax rule is not very aggressive. The choice of the tax rule parameters only matters for equilibrium allocations in variants of the model with distortionary taxes or Keynesian households.

<sup>12</sup> The results are not much affected if we consider a steady state in which the government debt to output ratio is set to zero.

<sup>&</sup>lt;sup>13</sup> We collected data on total nominal tax revenues as share of trend nominal GDP, and estimated (33) with OLS. Imposing these coefficients only reduced the  $R^2$  from 0.97 to 0.95 relative to the best- fitting OLS estimates.

#### 3.1.5. Initial Economic Conditions

As emphasized in Section 2, the effects of fiscal policy depend on the perceived depth and duration of the underlying liquidity trap. Accordingly, we begin by using our workhorse model to generate initial macroeconomic conditions that capture some key features of the recent U.S. recession, including a sharp and persistent fall in output, some decline in inflation, and a protracted period of near-zero policy rates.

The solid lines in Figure 5a depict this "severe U.S. recession" scenario under the benchmark calibration of our model. The scenario is generated by a sequence of three unanticipated negative taste shocks  $\nu_t$  that begin in 2008:Q3 and continue through 2009:Q1. Each shock  $\nu_t$  follows an AR(2) to allow for some persistence in the growth rate.<sup>14</sup> The shock innovations are scaled to induce a maximum output contraction of about 8 percent relative to steady state. Although the initial shock is too small to push the economy into a liquidity trap, the subsequent shocks deepen the recession and generate a liquidity trap. Upon the arrival of the final shock in 2009:Q1, agents expect that the short-term nominal interest rate will remain at its lower bound of zero for 8 quarters through 2010:Q4 (to highlight the zero bound constraint, the short-term nominal interest rate and inflation rates are shown in levels). Inflation falls from its steady state level of 2 percent to a trough of slightly below zero, and remains close to zero for about a year.

The peak output contraction in this scenario comes close to matching the maximum decline in U.S. output relative to trend that occurred following the intensification of the financial crisis in 2008:Q3 (the detrended U.S. output series is depicted by cross-hatches). The implication of a prolonged liquidity trap seems consistent with historical experience thus far, as actual policy rates have remained at zero since late 2008 (the cross-hatches show realized values of the federal funds rate). Moreover, given that the *perceived* duration of the liquidity trap plays a crucial role in determining the effects of fiscal stimulus, we also compare the expected duration of the liquidity trap based on our model simulation with an empirical proxy for the expected path of the policy rate based on overnight index swap rates. These projections are available 1-24 months ahead, and 36 months ahead. As seen in the lower panel, the "projected" path of the federal funds rate in January 2009 – shortly after the current federal funds rate target was reduced to nearly zero – is below 1 percent for a horizon extending out two years. Although there are difficulties with interpreting this path as measuring the expected policy rate due to e.g., time-varying risk premia,

Paralleling the case in which government spending requires implementation lags, we assume that  $\nu_t$  follows  $\nu_t - \nu_{t-1} = \rho_{\nu_1} (\nu_{t-1} - \nu_{t-2}) - \rho_{\nu_2} \nu_{t-1} + \varepsilon_{\nu,t}$ , where  $\rho_{\nu_1} = 0.2$  and  $\rho_{\nu_2} = 0.05$ .

this evidence suggests that the implications of our baseline model are not unreasonable; we also investigate the sensitivity of our results to the duration of the liquidity trap.

As seen in Figure 5a, the decline price inflation implied by our benchmark calibration is somewhat larger than in the corresponding data (the price inflation measure is the core CPI inflation rate, and is depicted by cross-hatches). In fact, a striking feature of the recession is that both actual inflation and inflation forecasts responded little to large and persistent output declines. The right panel of Figure 5b plots the median forecast *path* of expected inflation over the next six quarters from the Survey of Professional Forecasters, beginning in 2008-Q3 (the solid line) and continuing through 2009-Q2. Clearly even short-term inflation expectations remained quite stable as the recession deepened.

Even so, it bears emphasizing that our benchmark calibration implies much less movement in inflation (and expected inflation) than other commonly-adopted calibrations, reflecting that our chosen values of both the contract duration parameters and the coefficient on inflation in the monetary rule are towards the higher side of empirical estimates. To highlight this, Figure 5a also reports results for two alternative calibrations. In the case labelled "more flexible p and w," the mean duration of price and wage contracts is reduced to four quarters, while another alternative labelled "loose rule" adopts the standard Taylor rule coefficients in the monetary policy rule (i.e., the parameters  $\gamma_{\pi}$  and  $\gamma_{x}$  in equation (34) are set to 1.5 and 0.125, respectively, compared with 3 and 0.25 under our benchmark). Under each of these alternatives, the taste shock is rescaled (reduced modestly) to account for roughly the same-sized output contraction as in the benchmark, and to imply an eight quarter liquidity trap. Inflation declines by considerably more under either of these alternative calibrations. Overall, we take these results as providing support for our benchmark calibration relative to these alternatives, while acknowledging the possibility suggested by fitting the recent recession that even our benchmark may perhaps overstate the response of inflation to highly persistent economic shocks.

#### 3.2. Dynamic Effects of Government Spending

The solid lines in Figure 6 (labelled "benchmark ZLB 8 qtr") show the effects of a front-loaded increase in government expenditures equal to 1 percent of steady state output against the backdrop of the negative taste shocks described above above. The government spending shock follows an AR(1) with a persistence of 0.9. The impulse response functions shown are computed as the difference between this scenario which includes both the consumption taste shocks and government

spending shock, and the previous scenario (i.e., the benchmark in Figure 5) with only the taste shocks. While the government spending shock occurs in 2009:Q1 – at which point agents expect the liquidity trap would last 8 quarters in the absence of stimulus– this corresponds to "period 0" in the figure.<sup>15</sup> The fiscal expansion is assumed to be financed by lump-sum taxes that respond endogenously to government debt and the budget deficit.

As in the stylized model analyzed in Section 2, the fiscal policy expansion implies larger effects on output relative to a normal situation in which policy is unconstrained (the dotted line). The amplified multiplier reflects that higher government spending boosts the potential real interest rate, while the (ex ante) real interest rate falls as nominal interest rates do not respond and expected inflation rises. The fall in real interest rates "crowds in" investment spending, and pushes the multiplier above unity at impact. The amplified multiplier significantly dampens the response of government debt/GDP relative to a normal situation, though the debt to output ratio increases above zero after only a couple of quarters.<sup>16</sup>

The same factors identified as having a major influence on the multiplier in Section 2 – including the duration of the liquidity trap and structural parameters determining the response of expected inflation – also have a major influence on the multiplier in this more empirically-realistic model. Figure 6 shows impulse response functions to the same government spending shock against the backdrop of a longer-lived liquidity trap of 11 quarters. The multiplier is more than twice as large in this case, reflecting the more persistent rise in inflation and larger implied fall in the real interest rate. The bigger multiplier contributes to a highly persistent fall in the government debt/GDP ratio, which is tantamount to a "fiscal free lunch." Similarly, the multiplier is much larger under a more accommodative monetary policy rule (not shown), again due to a larger expected inflation response.

Conversely, the multiplier shows much less variation with the duration of the liquidity trap under conditions that give rise to a small inflation response. As in Section 2, the multiplier depends on the gap that the higher fiscal spending induces between the potential real interest rate and actual real interest rate over the period in which the economy remains in a liquidity trap. In the limiting case in which inflation is constant, the effect of a increasing the liquidity trap duration on the multiplier simply hinges on how government spending affects the potential real interest rate at relatively

<sup>&</sup>lt;sup>15</sup> The American Reinvestment and Recovery Act was passed in 2009:Q1, the first quarter in which the federal funds rate averaged less than 1/4 percentage point.

The tax-rule (33) responds to government debt as share of annualized trend nominal output ratio,  $b_{G,t} = \frac{B_{Gt}}{4P_tY}$ . The figure reports government debt relative to actual output  $\tilde{b}_{G,t} \equiv \frac{B_{Gt}}{4P_tY_t}$ .

distant horizons; and since these effects are small, the multiplier shows relatively little increase with the duration of the trap. To illustrate this, Figure 6 shows the multiplier under a calibration labelled 'ZLB long duration' which imposes extremely long-lived price and wage contracts with a mean duration of 40 quarters, and against the backdrop of a Great Depression-sized output decline of almost 30 percent (the initial conditions are depicted in Figure 5a with the same label). Despite a 16 quarter liquidity trap, the multiplier only grazes unity during the impact period before quickly declining.

#### 3.2.1. Marginal vs. Average Responses

In Section 2, we used the simple New Keynesian model to illustrate that the marginal effects of government spending shocks can diverge substantially from the average effects captured by impulse response functions. We next provide a parallel analysis in the more empirically-realistic model, with the upper panel of Figure 7 depicting marginal and average multipliers as a function of government spending under our benchmark calibration. Because it is useful to first show how the multiplier behaves in a long-lived liquidity trap, the negative taste shocks which generate the initial conditions are scaled so that the liquidity trap would last 11 quarters in the absence of fiscal stimulus (as in the "ZLB benchmark 11 quarter" case in Figure 6).

As in the stylized model in Section 2, the marginal multiplier follows a step function. The multiplier is constant until government spending reaches a threshold value that is large enough to shorten the duration of the liquidity trap by one period, and then drops discretely as spending surpasses this threshold. Quantitatively, the multiplier is nearly 3 for small additions to spending less than 0.4 percent of GDP. However, as seen from the tick marks on the upper axis, the marginal multiplier drops abruptly as the duration of the liquidity trap shortens just a couple of periods, and is close to unity for a liquidity trap duration in the range of 8-9 quarters (corresponding to a spending level around 1.5 percent of GDP).<sup>17</sup> Clearly, the marginal multiplier is well below the average even for relatively modest increases in spending.

The right upper panel plots the marginal and average responses of the government deficit/GDP ratio, inclusive of interest payments, to government spending. Given the long duration of the underlying liquidity trap, the marginal response is negative for spending increments less than 3 percent of GDP. As additional spending shrinks the duration of the liquidity trap below 8 quarters

<sup>&</sup>lt;sup>17</sup> The spending multipliers are computed as the average increase in output (relative to trend) for the first 4 quarters divided by the increase in government spending to trend output in the first period. We consider a four quarter average of output in order to account for possible hump-shaped dynamics of output to fiscal stimulus.

(as seen by the upper tick marks), the marginal effect on the government deficit turns positive, even though the average response remains negative. Thus, the gap between the average and marginal spending multiplier translates into a similar gap between the marginal and average response of the government deficit, underlying how marginal increments to fiscal spending may put significant upward pressure on budget deficits even when the average effects appear small.

The lower panel of Figure 7 shows the marginal multiplier under our benchmark calibration (solid lines) and several alternatives; in each case, the initial conditions are chosen to imply a shorter-lived liquidity trap of 8 quarters. The marginal multiplier under our benchmark calibration is below unity at all spending levels (as can be surmised from the upper panel as well). The multiplier can be much higher under calibrations that imply a larger response of expected inflation. The amplification of the multiplier is particularly dramatic under the case of "more flexible prices" and wages," which specifies that both price and wage contracts last only four quarters. The multiplier exceeds 10 for low spending levels below 0.2 percent of GDP. The high potency of fiscal policy reflects that it offsets the strong deflationary pressure that would otherwise occur in response to the contractionary (taste) shocks. However, the multiplier drops precipitously as spending increases and the liquidity trap duration (indicated by the upper tick markets for this calibration) shortens. For a spending increment a bit over 1 percent of GDP, the liquidity trap lasts only 6 quarters, and the marginal multiplier is only about unity. A second alternative examines a case in which prices are less sticky than under our baseline (i.e.  $\xi_p$  is lowered from 0.90 to 0.75), but wage-setting remains unaltered. The multiplier function in this case (the dotted line) is only slightly higher than under our benchmark, reflecting that the sluggish behavior of wages keeps price inflation from moving as much as under the previous alternatives. This calibration underscores that the dramatically higher multiplier under the "more flexible price and wage" calibration hinges on both prices and wages being considerably more flexible than under our benchmark. The third alternative ("loose rule") assumes that monetary policy is less aggressive in responding to inflation and the output gap than under our benchmark ( $\gamma_{\pi}=1.5$  and  $\gamma_{x}=0.125$ ). The multiplier is boosted noticeably, though only for fairly low spending levels.

Overall, these results provide a strong rationale for expanding fiscal spending in a very deep liquidity trap lasting roughly 3 years, with the caveats that it is important to take account of how the multiplier declines with higher spending, and that the high multipliers assume no implementation lags. For a shorter-lived liquidity trap of two years of less, the multiplier is only modestly higher than normal under plausible calibrations. Although the workhorse model can generate very high

multipliers if prices and wages adjust fairly quickly, we are skeptical of this implication. Such calibrations imply a higher sensitivity of the price-setting and wage setting equations to their forcing variables than typically estimated, and also imply that the adverse taste shocks that generate the underlying liquidity trap would induce an extremely large decline in inflation: recalling Figure 5a, inflation falls 7 percentage points below baseline under the "more flexible price and wage" calibration in response to the shocks that depress output about 8 percent.

#### 3.2.2. Effects of Implementation Lags

Figure 8 examines the sensitivity of the results to alternative assumptions about how the fiscal stimulus is financed and how quickly it can be implemented. First, we consider the implications of gradual rise in government spending of 1 percent of GDP that peaks after eight quarters, rather than immediately, while continuing to assume lump-sum tax adjustment. The response of output (the dotted line, labelled "8 quarter implementation lag lump-sum") is only about half as large as under our benchmark with immediate implementation (the solid line), and government debt rises immediately. The smaller output response reflects that the higher spending reduces the potential real interest rate in the short-run, as the promise to increase future public spending encourages households to save. Second, we replace the benchmark assumption of lump-sum tax adjustment with an alternative in which labor taxes adjust dynamically in response to higher government debt. The output response (dashed line) is reduced modestly relative to the benchmark, reflecting that the higher labor taxes damp labor supply. But it is worth emphasizing that the parameters in the tax rate reaction function are fairly small, and that output would be affected more if the parameters of the tax rate reaction function implied a larger tax rate response. Finally, we also consider a third variant that merges these alternatives by embedding both implementation lags, and the assumption of labor tax adjustment. The output response (dash-dotted line) under this alternative is only slightly positive. Clearly, implementation lags and the requirement that government purchases be financed by distortionary taxes can significantly reduce the multiplier, though the mode of financing doesn't seem to have especially large quantitative effects provided that the government spending hike is temporary, and taxes adjust gradually.<sup>18</sup>

We have also examined a case in which capital income taxes adjust dynamically to debt. The alternative implies an even lower output response than in Figure 8 (negative if capital taxes adjust quickly enough).

### 4. Robustness analysis: The Empirical Model Augmented with Financial Frictions and Keynesian Households

The workhorse model in Section 3 has been criticized by Galí, López-Salido and Vallés (2007), among others, for its inability to account for VAR-based empirical evidence indicating that private consumption rises in response to higher government spending. As shown by GLV, the inclusion of non-Ricardian households helps account for this empirical evidence, and allows their model to generate a somewhat higher government spending multiplier even in normal times when monetary policy raises interest rates. In addition, the workhorse model of Section 3 omits potentially important financial channels, such as movements in private credit spreads due to balance sheet effects. Given these potential shortcomings of the workhorse model, we investigate the sensitivity of our results to the inclusion of non-Ricardian households and financial frictions.

#### 4.1. Key Model Equations and Calibration

To incorporate these features, we modify the model in Section 3, along two dimensions. First, we assume that a fraction  $s_{kh}$  of the population consists of "Keynesian" households whose members consume their current after-tax income each period, and set their wage equal to the average wage of the optimizing households. Because all households face the same labor demand schedule, each Keynesian household works the same number of hours as the average optimizing household. Thus, the consumption of Keynesian households  $C_t^K(h)$  is simply determined as

$$P_{t}C_{t}^{K}\left(h\right)=\left(1-\tau_{Nt}\right)W_{t}\left(h\right)N_{t}\left(h\right)-T_{t}.$$

Second, we incorporate a financial accelerator following the basic approach of Bernanke, Gertler and Gilchrist (1999). Thus, entrepreneurs acquire capital to supply to homogeneous factor markets, but must pay an external finance premium on the funds they borrow from households due to an agency problem. We follow Christiano, Motto and Rostagno (2007) by assuming that the debt contract between the entrepreneurs and lenders (households) is written in nominal terms (rather than real terms as in BGG 1999). At an aggregate level, the corporate finance premium varies with the degree of leverage of the economy.

We set the population share of the Keynesian households to optimizing households,  $s_{kh}$ , to 1/2, which implies that the Keynesian households' share of total consumption is about 1/3. This calibration perhaps overstates the role of non-Ricardian households in affecting consumption behavior, but seems useful to help put plausible bounds on how the multiplier may vary with the degree of

non-Ricardian behavior in consumption (recognizing that the workhorse model is a special case in which  $s_{kh} = 0$ ). Our calibration of the parameters affecting the financial accelerator follow BGG (1999).<sup>19</sup>

#### 4.2. Dynamic Effects of Fiscal Policy Expansions

We analyze the effects of a government spending shock against the backdrop of initial economic conditions that are generated in the same manner as described in Section 3.1.5. In particular, we calibrate the same AR(2) process for the consumption taste shock, and again feed in a sequence of three innovations beginning in 2008:Q3 to roughly match the peak decline in U.S. output relative to trend that occurred during the financial crisis. The short-term interest rate falls to zero in 2009:Q1, and remains at this level for 8 quarters, while inflation falls about 2.5 percent below its steady state value of 2 percent. Given the stronger propagation mechanisms in this model, the size of the consumption innovations are considerably smaller than those considered in the previous section.

Figure 9 shows the effects of a front-loaded increase in government expenditures of 1 percent to trend GDP (as in Figure 6, the impulse responses show the "partial effect" of the rise in government spending). We report results both for the benchmark parameterization of the model where monetary policy is constrained by the zero lower bound (labeled 'ZLB Full Model'), and for the case in which policy is unconstrained (labeled 'Normal Full Model'). The multiplier is about 1.5 in a liquidity trap, considerably larger than its value of unity in a normal situation in which policy raises real interest rates.

The figure also shows the spending multiplier for the benchmark calibration of the model analyzed in the previous section for both the constrained ('ZLB CEE/SW) and unconstrained ('Normal CEE/SW) settings of the policy rule. The government spending multiplier in the liquidity trap case is substantially enhanced by the inclusion of Keynesian households and financial frictions. This is mainly due to a much larger rise in the potential real interest rate relative to the CEE/SW model: because government spending boosts wage income and hence the consumption of Keynesian households, real interest rates must rise more to keep output at potential.<sup>20</sup> In the normal case when monetary policy is unconstrained, the larger increase in the potential real rate in the full

The monitoring cost,  $\mu$ , expressed as a proportion of entrepreneurs' total gross revenue, is 0.12. The default rate of entrepeneurs is 3 percent per year, and the variance of the idiosyncratic productivity to entrepreneurs is 0.28. Conditioning on the duration of the liquidity trap, the bulk of the difference in the spending multiplier between the workhorse CEE/SW model and the full model is driven by the inclusion of Keynesian households (and the assumption that they account for half of all households).

model has less impact on the multiplier, since monetary policy simply boosts real interest rates by more than in the workhorse CEE/SW model. But in a liquidity trap, the larger increase in the potential real rate in the full model means that higher government spending brings about a greater reduction in the gap between the real interest rate and potential real interest rate, ande thereby a bigger multiplier.

The upper left panel of Figure 10 reports the marginal and average government spending multipliers as a function of the size of the increment to government spending for the full model under our benchmark calibration. The initial conditions imply a liquidity trap lasting 10 quarters to emphasize how the multiplier behaves in a very long-lived trap. The marginal multiplier is almost 3.5 for increments to spending below 1/2 percent of GDP, and remains above unity even for relatively large spending increases above 3 percent of GDP. Moreover, even the marginal impact on the government budget balance is negative for spending increments in this range. These results provide an even stronger rationale for increasing fiscal spending in a very deep liquidity trap than the workhorse model.

The lower left panel shows the marginal multiplier when initial conditions imply a liquidity trap lasting only 8 quarters. Under our benchmark calibration, the multiplier remains above unity for spending levels ranging up to roughly 2 percent of GDP, and the government deficit actually declines. Thus, the full model suggests that the multiplier is considerably larger than under normal conditions even for fairly substantial increases in government spending, though increases in spending much beyond 2 percent of GDP have a considerably lower multiplier, and boost the deficit. The multiplier is somewhat higher under the standard Taylor rule ("loose policy rule"), but only for small increments to spending. As in the workhorse model, the marginal multiplier is dramatically higher under the calibration with four quarter price and wage contracts ("more flexible prices and wages"), exceeding 8 for spending below 0.2 percent of GDP; however, the multiplier declines sharply as spending increases.

#### 5. Conclusions

Taken together, our results suggest a somewhat nuanced view of the role of fiscal policy in a liquidity trap. For an economy facing a protracted recession and for which monetary policy seems likely to be constrained by the zero bound for a very prolonged period – roughly 2 years or more – there is a strong argument for increasing government spending on a temporary basis. The fiscal multiplier can be enhanced even more if monetary policy is accommodative and allows expected inflation

to rise substantially in the short- and medium-term, and if the fiscal expansion is well-timed to coincide with the period in which the economy remains in the trap. For shorter-lived liquidity traps of less than two years, the multiplier is larger than under 'normal conditions' for small increases in spending, but drops relatively quickly at higher spending levels. Thus, larger spending programs may suffer from sharply diminishing returns, and may boost government debt significantly.

We have throughout the paper assumed that no alternative measures are available to the central bank in a liquidity trap. In practice, the Federal Reserve and other central banks have deployed a number of policy tools after policy rates declined to nearly zero in the wake of the financial crisis. One such tool is forward guidance, i.e. communicating that economic conditions are likely to warrant the policy rate to be kept at zero for an extended period. To the extent that such statements extends the horizon for which households and firms expect the policy rate to remain at zero, this will stimulate aggregate demand. Apart from providing forward guidance, many central banks have used the asset side of their balance sheet to support credit markets by providing liquidity and purchase of long-term securities. Although the models considered in this paper does not allow for an assessment of the effectiveness of such actions, our analysis highlights the importance of analyzing the effects of such actions jointly with the fiscal stimulus packages in order to properly assess their marginal impact.

There are also are number of interesting issues that we leave for future research. For instance, we have intentionally focused on a positive analysis of government expansions, and not studied normative issues such as assessing the conditions under which a government spending hike is welfare-enhancing. Christiano, Eichenbaum and Rebelo (2009) and Nakata (2009) argue that an increase in government consumption might be welfare-enhancing in a liquidity trap. It would be interesting to explore the welfare implications of fiscal stimulus when allowing for endogenous exit. Moreover, this paper has focused on government consumption spending exclusively as the tool of fiscal policy; clearly, it would be interesting to extend our analysis by considering alternative fiscal measures such as tax cuts and targeted transfers. Finally, as the multiplier may well be affected through cross country linkages, it would interesting to extend our analysis to an open economy setting. <sup>21</sup>

<sup>&</sup>lt;sup>21</sup> Eggertsson (2009) argues that tax cuts aimed at stimulating aggregate demand rather than aggregate supply are preferable in a liquidity trap (e.g. sales taxes and implementing an investment tax credit).

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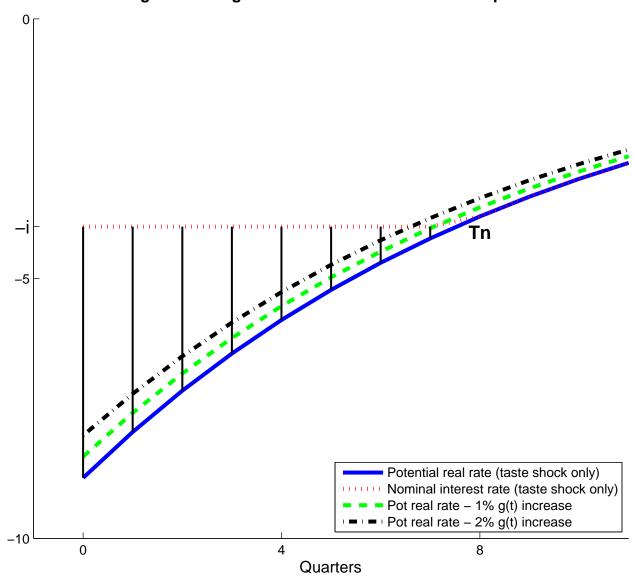
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Figure 1a: Negative Taste Shock and Fiscal Response



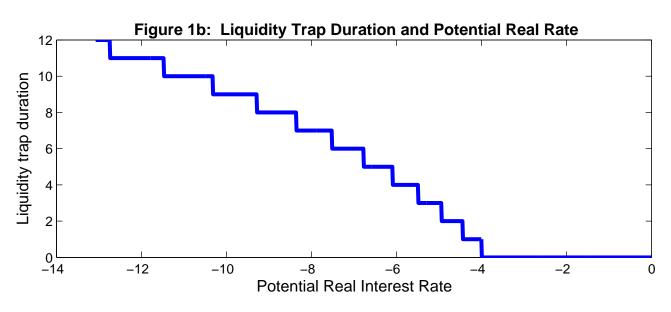


Figure 2: Immediate Rise in Government Spending

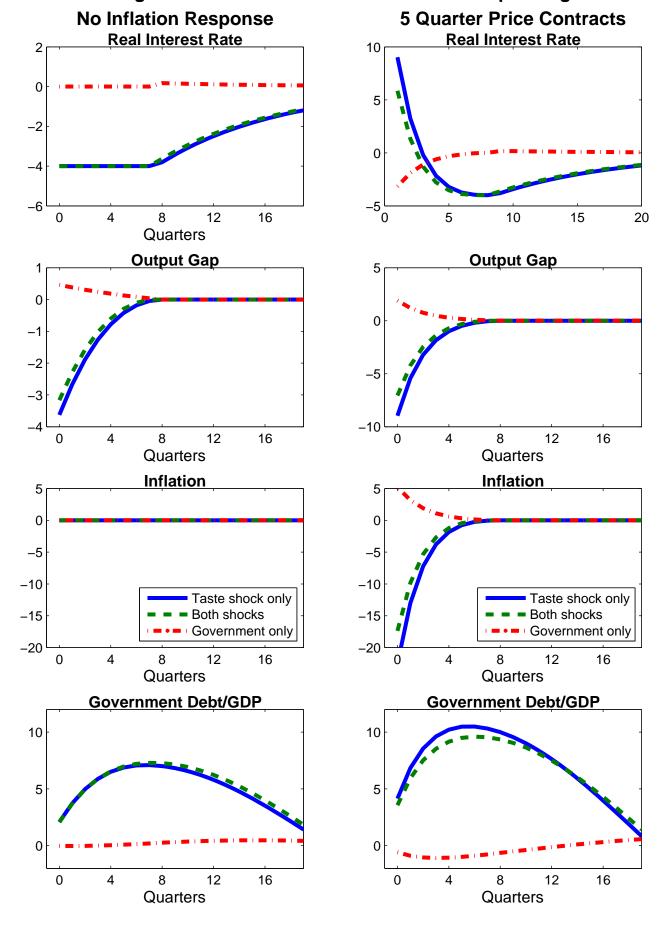


Figure 3: Marginal Output and Government Deficit Spending Multipliers in the Simple New-Keynesian Model

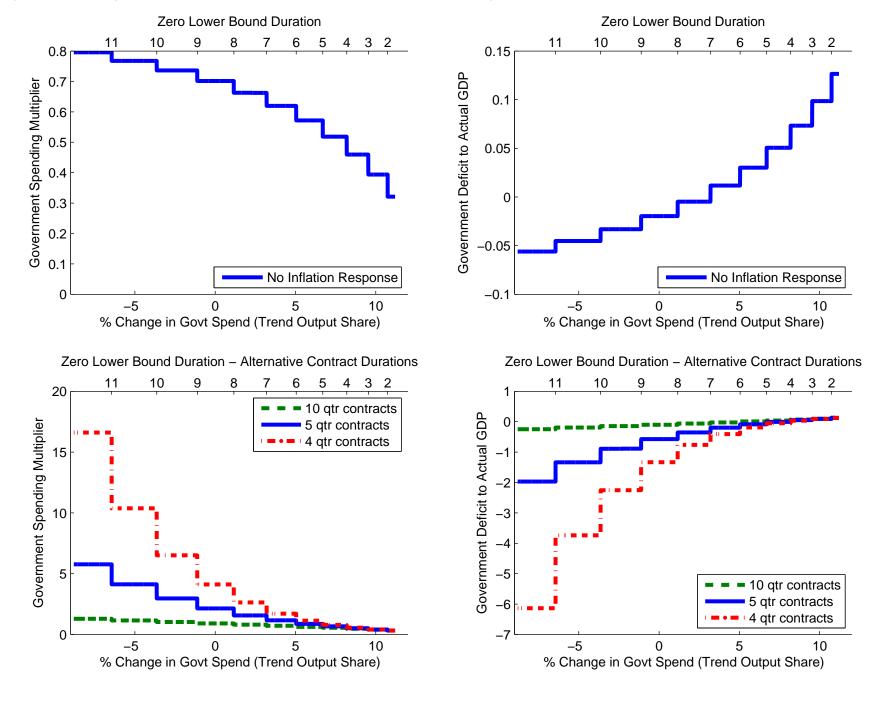


Figure 4a: Weights on Leads of the Interest Rate Gap in Inflation Equation

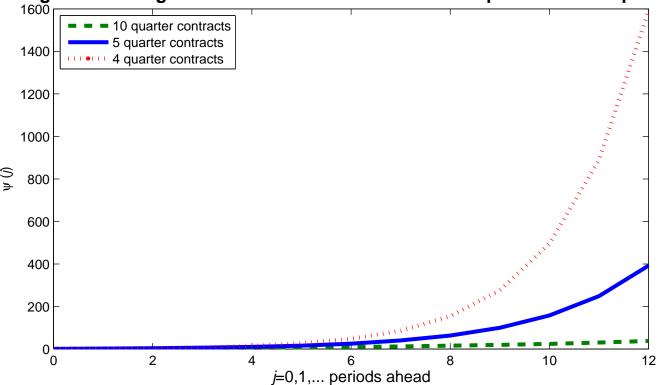


Figure 4b: Government Spending Peaks after Eight Quarters

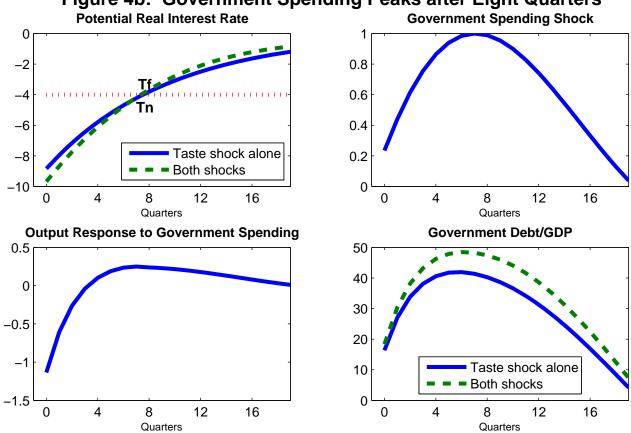


Figure 5a: Simulated and Actual Paths for Key Macroeconomic Variables in CEE/SW Model

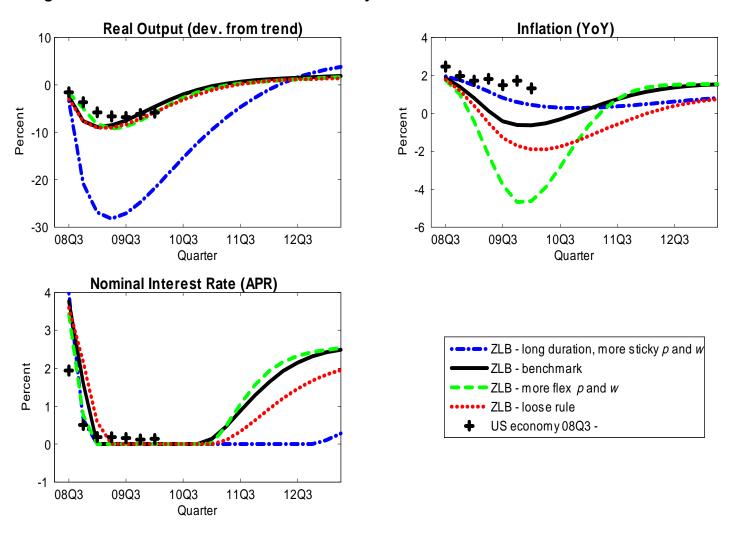
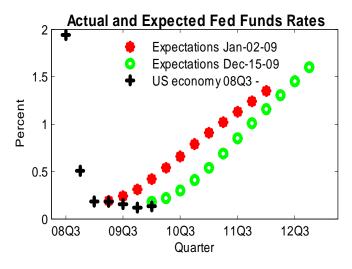


Figure 5b: Actual and Expected FFR and Core Inflation Rates



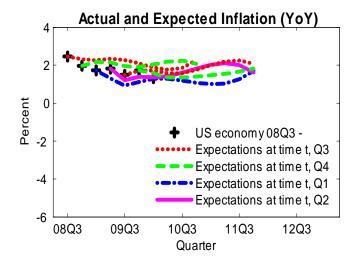
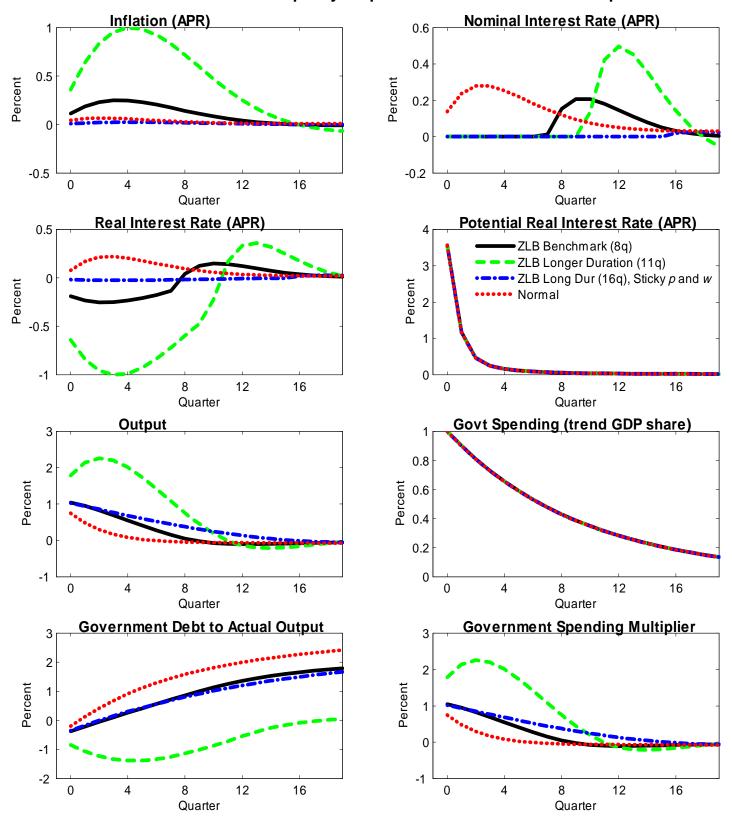


Figure 6: Responses to a Front-Loaded Increase in Government Spending in Normal Times and in a Liquidity Trap in the CEE-SW Model with Capital



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Figure 7: Average and Marginal Multipliers in the CEE-SW Model With Capital and Their Sensitivity to Alternative Parameterizations

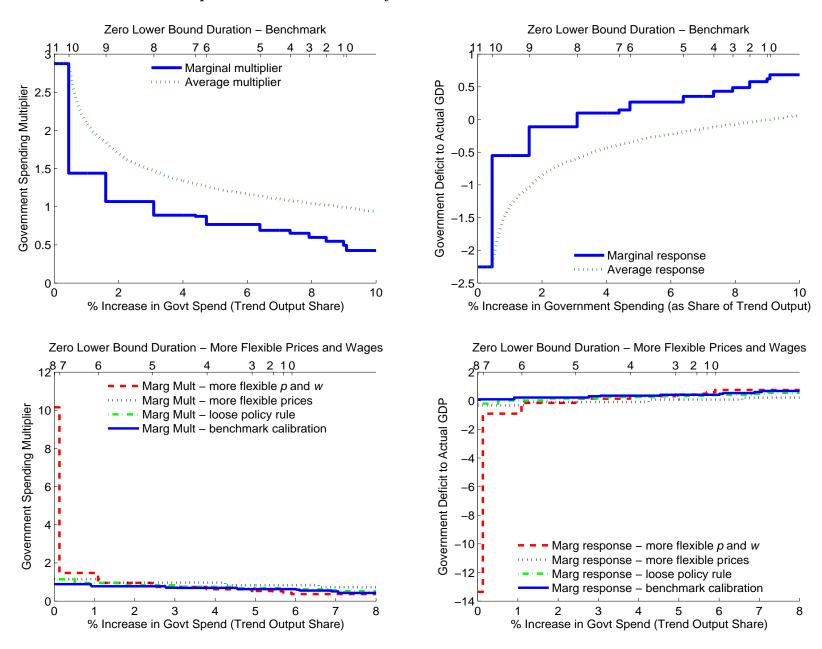
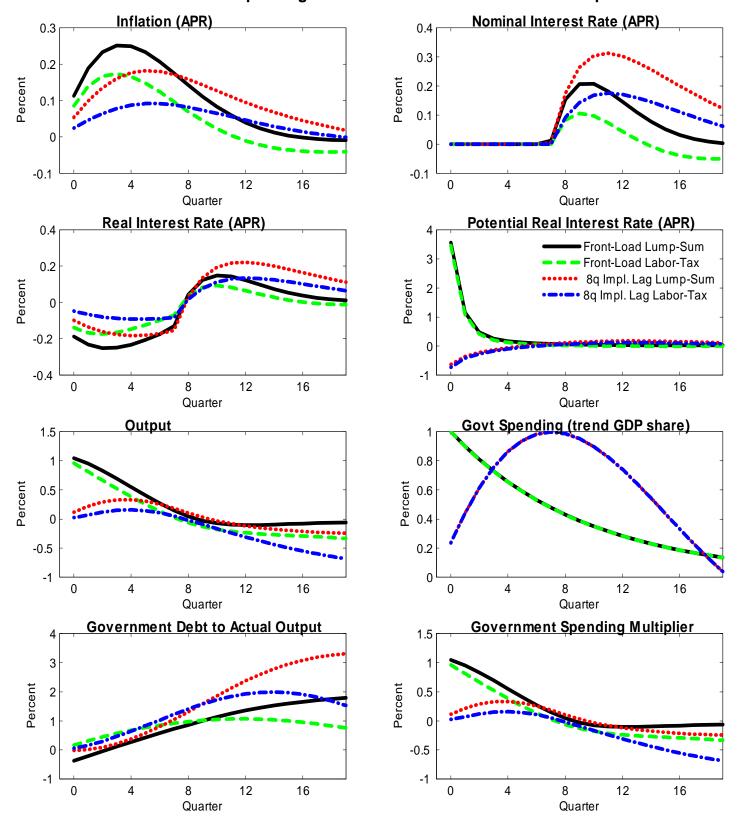


Figure 8: Responses to Alternative Implementation and Financing of the Government Spending Increase in the CEE-SW Model with Capital



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Figure 9: Responses to a Front-Loaded Government Spending hike in Normal Times and a Liquidity Trap in Model With Keynesian Agents and Financial Frictions and Benchmark CEE-SW Model

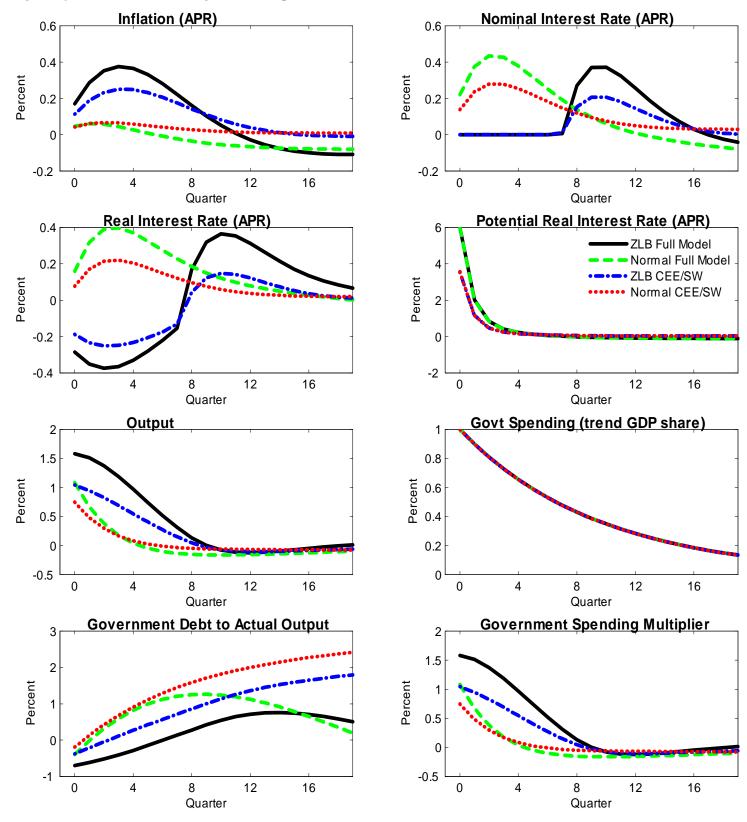


Figure 10: Average and Marginal Multipliers in the CEE-SW Model Extended With Financial Frictions and Keynesian Agents and Their Sensitivity to Alternative Parameterizations

