Social Structure and Human Capital Dynamics

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Motivation and Questions

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- What is the importance of local effects for human capital dynamics and therefore inequality and long term economic growth?

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- Neighborhood/local effects: the social environment affects people's behavior
- What is the importance of local effects for human capital dynamics and therefore inequality and long term economic growth?
- How and why can different social structures in otherwise identical societies generate significant differences in long run human capital, output, growth?

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  - **Local externalities**: peer effects, neighborhood externalities
- Social structures affect the economy through the local externalities

What we do not do (yet?)

- Endogenous/dynamic network formation

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Overview of results

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3. Higher growth rates associated with high inequality in the long run.
4. During transition,
(a) inequality is lower when society is cohesive, and
(b) growth rates can be high even when inequality is relatively low (star network).

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- Related work

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- Environment

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- Future plans

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Households and firm

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- Budget constraint

$$c_{it} + e_{it} p_t = h_{it}.$$

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- Parameter restrictions: $\theta > 0,\ \eta > 0,\ \beta_1 \geq 0,\ \beta_2 \geq 0,\ \beta_1 + \beta_2 < 1$

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$$p_t = \bar{h}_t.$$

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- Relative human capital: $x_{it} = h_{it} / \bar{h}_t$

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- Human capital accumulation

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Local externality

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- Social structure = network

Networks: some basic concepts

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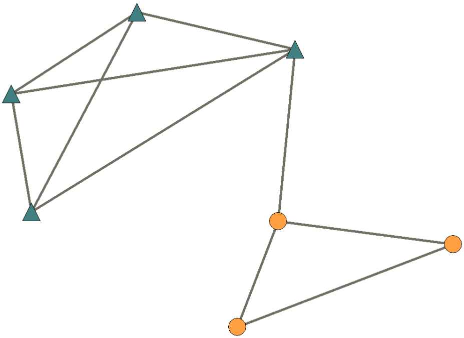
- The **adjacency matrix** A has typical element a_{ij} (symmetric matrix of zeros and ones)
- Let $G = A + I_n$

An example: Bridge network



$$G = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

Social structure, local externality and human capital

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- I.e. $\bar{h}_{it}$ is the average human capital of household i's neighborhood

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- Let $\gamma_{it} = h_{it+1} / h_{it}$ be the growth rate for a household i
- Equilibrium fully characterized by

$$\begin{aligned} x_{it+1} &= \frac{\gamma_{it}}{\gamma_t} x_{it} = \frac{x_{it}\gamma_{it}}{\frac{1}{n}\sum_{k=1}^{n} x_{kt}\gamma_{kt}} \\ \gamma_{it} &= \left(\theta + \max\left\{0, \frac{\psi\eta x_{it} - \theta}{1+\psi\eta}\right\}\right)^{\eta} x_{it}^{-\beta_1-\beta_2} \left(\frac{\sum_{j=1}^{n} g_{ij} x_{jt}}{\sum_{j=1}^{n} g_{ij}}\right)^{\beta_2} \end{aligned}$$

for all $i = 1, ..., n$

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- Also, it must be that

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- Define auxiliary parameters

$$\begin{aligned}\delta &= \frac{\eta}{1+\theta} \\ \phi &= \left(\frac{\psi\eta}{1+\psi\eta}\right)^{\eta}\end{aligned}$$

Long run equality

- Growth rate

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- Growth rate

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- And

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- Trivially a solution to dynamic system
- The growth rate at the BGP does not depend on the externalities or the network structure

Long run equality

Proposition 1: **Homogeneity**

When all households are identical and have equal initial capital h_0 then the local externality is irrelevant and the economy is always at the balanced growth path with equality for all $i \in N$ and $t = 1, 2, \ldots$.

Long run equality

Proposition 2: **Heterogeneity**

Suppose that the population is heterogeneous, i.e. that households may have different initial human capital. A necessary and sufficient condition for local existence, uniqueness and stability of the BGP with equality is that

$$\beta_1 + \kappa\beta_2 > \delta$$

Long run equality

Proposition 2: **Heterogeneity**

Suppose that the population is heterogeneous, i.e. that households may have different initial human capital. A necessary and sufficient condition for stability of the BGP with equality is that

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What is δ?
Summarizes importance of investment in education
for human capital accumulation

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What is κ?

It is a measure of network cohesion

A measure of network cohesion

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$$\bar{r}_l = \frac{1}{n} \sum_i r_{il}$$

- Let

$$f_{il} = r_{il} - \bar{r}_l$$

The larger $f_{il} - s$ are, the more variability there is in the importance of social interactions in this economy

A measure of network cohesion

- Given a network described by G, we define

$$\kappa = 1 - \rho(F)$$

where $\rho\left(F\right)$ is the spectral radius of the matrix F, with elements f_{il}

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- Important property: $0 < \kappa < 1$ for any network.

A measure of network cohesion

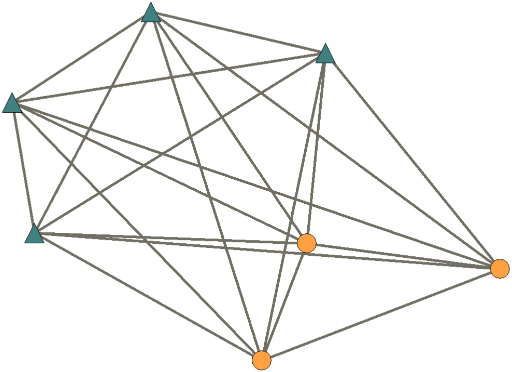
- Given a network described by G, we define

$$\kappa = 1 - \rho(F)$$

where $\rho(F)$ is the spectral radius of the matrix F, with elements f_{il}

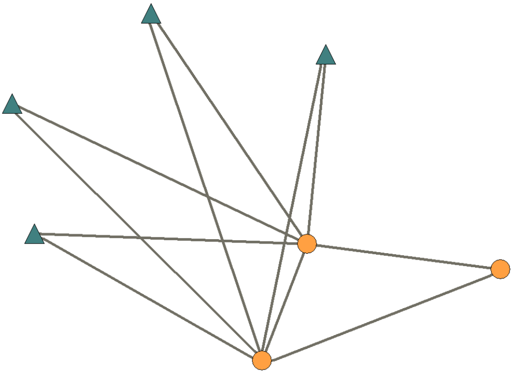
- Important property: $0 < \kappa < 1$ for any network.
- The smaller $\rho(F)$ or larger κ is, the more uniform (and less fragmented) society is.

Example 1: Complete network, F = 0



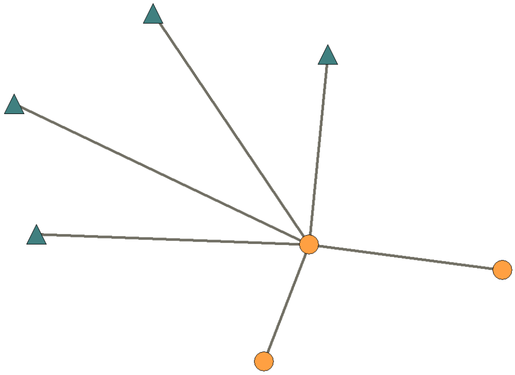
- Highest cohesion possible, $\kappa = 1$

Example 2: Double star



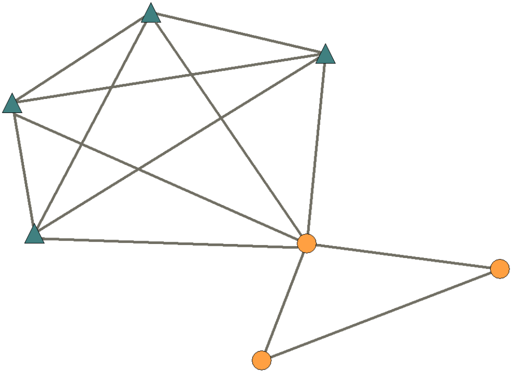
- High cohesion, $1/2 < \kappa < 1$, here $\kappa = 0.6190$

Example 3: Star



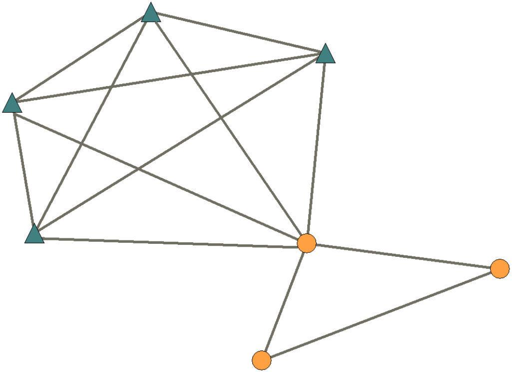
- Medium cohesion, $\kappa = 1/2$

Example 4: Many links



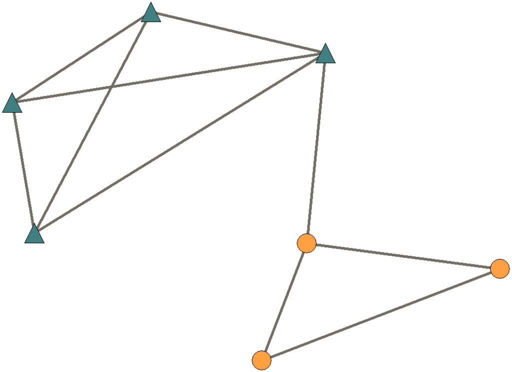
- Low cohesion, $0 < \kappa < 1/2$, here $\kappa = 0.2841$

Example 4: Many links



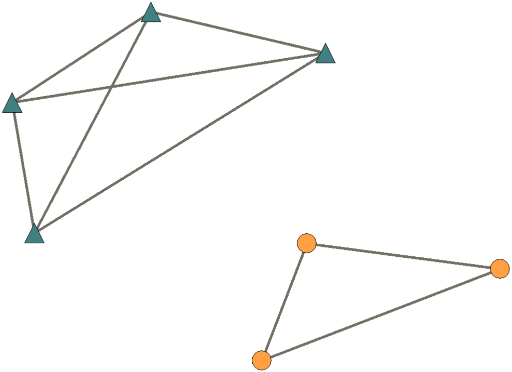
- Low cohesion, $0 < \kappa < 1/2$, here $\kappa = 0.2841$
- Note: # links $= 13 > 6 =$ # links of star

Example 5: Bridge



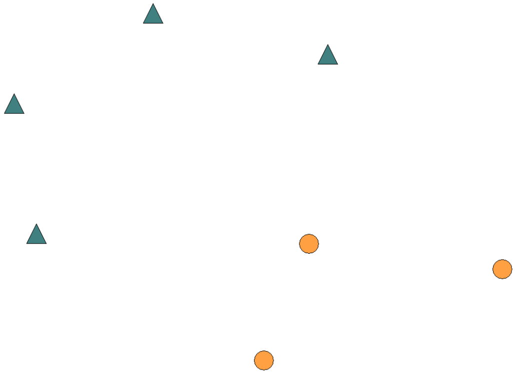
- Very low cohesion, $0 < \kappa << 1/2$, here $\kappa = 0.1134$

Example 6: Two components



- No cohesion, $\kappa = 0$

Example 7: Empty network



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Long run equality

- Condition for long run equality

$$\underbrace{\beta_1}_{\text{global externality}} + \underbrace{\kappa\beta_2}_{\substack{\text{local externality} \\ \times \text{ network cohesion}}} > \underbrace{\delta}_{\text{investment}}$$

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- The stronger the externalities are and the more network cohesion there is, the easier it is to obtain long run equality

Long run inequality

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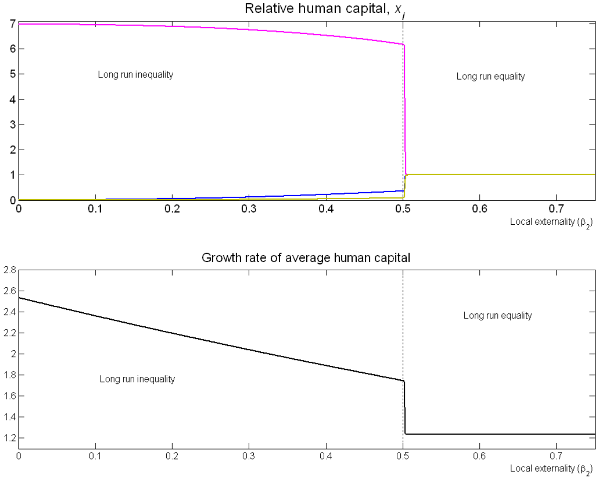
  - If $\beta_1 + \kappa_g \beta_2 << \delta$, inequality solutions become stable, depending on network

Long run inequality: example



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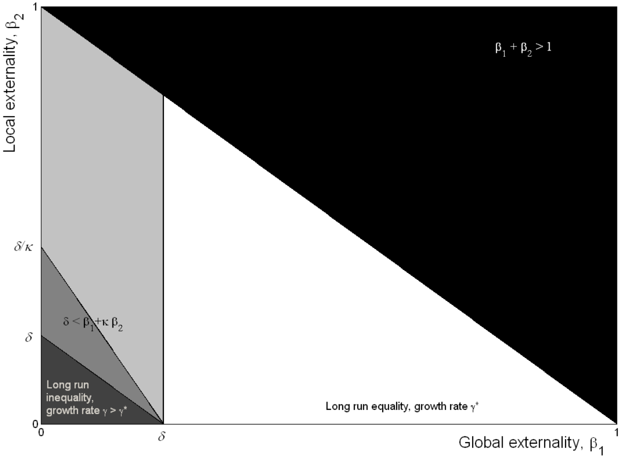
- High long run inequality generally associated with high growth rates

Summary of long run properties



Transition

$$U_{it} = \ln c_{it} + \psi \ln h_{it+1}, \quad h_{it+1} = (\theta + e_{it})^{\eta} h_{it}^{1-\beta_1-\beta_2} \bar{h}_{it}^{\beta_2} \bar{h}_t^{\beta_1}$$

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  - Education expenditures to final expenditure is 10.64%
  - Assuming long run equality, the annual growth rate of output per capita is 2% (model period is 35 years)

The questions we ask

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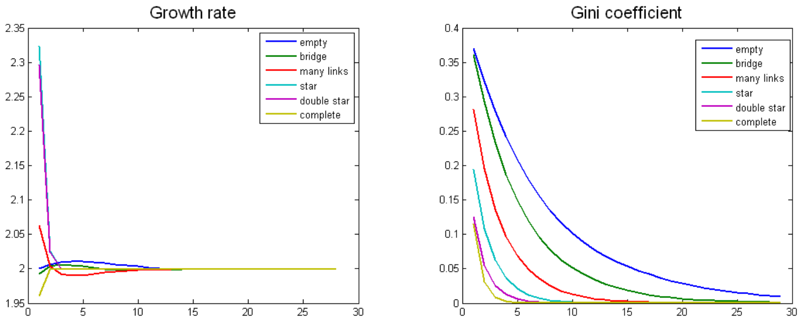
  - Transition when we switch from one social structure to another

Transition to long run equality



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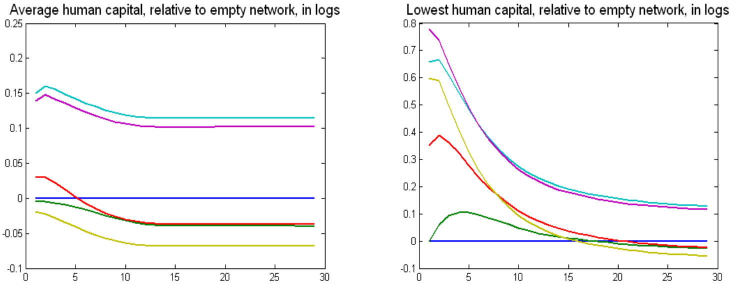
What is the relation of network cohesion and inequality/growth in the short run?

- Monotonic ranking of short-run inequality and network cohesion
- Non-monotonic relation of κ with growth
- High growth and relatively low inequality for star network (assumes high human capital at the centre)

Levels



Average human capital, relative to empty network, in logs
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
0
5
10
15
20
25
30
Lowest human capital, relative to empty network, in logs
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
5
10
15
20
25
30

Observations

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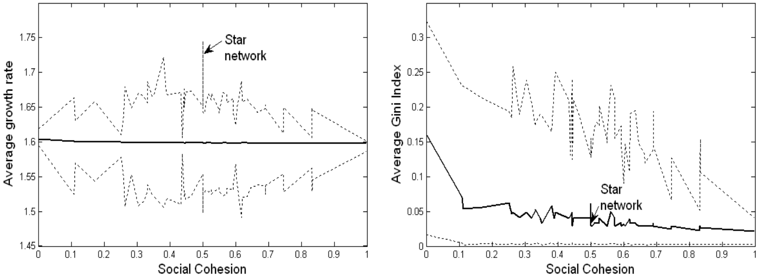
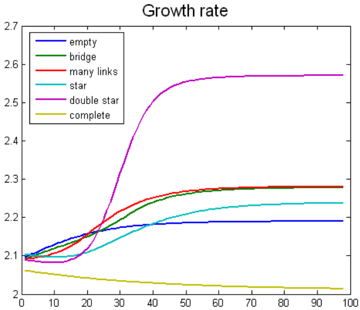
- Long run levels of human capital (and output) depend on:
  - initial human capital of each group
  - size of each group
  - relative importance of local and global externality
  - social structure
- In general there seems to be a **trade-off** between long run equality and long run levels of output

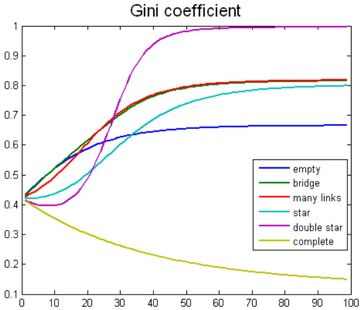
Simulations



Sample means, min and max of average growth rates, and gini coefficients, for 100, 000 replications.

Transition to long run inequality





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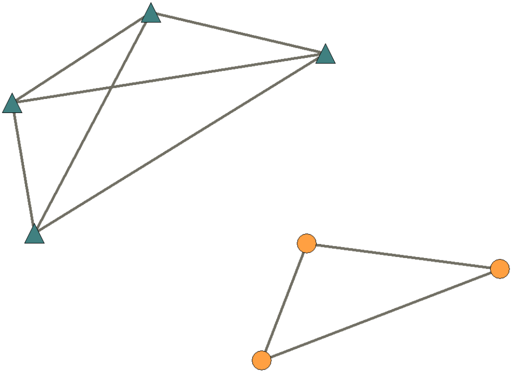
- Neighbors/externalities very important

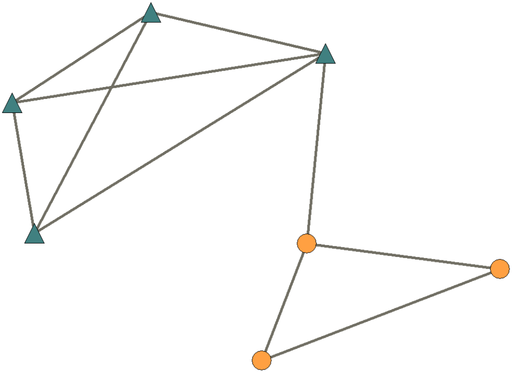
Adding one link to bring equality

Two complete components



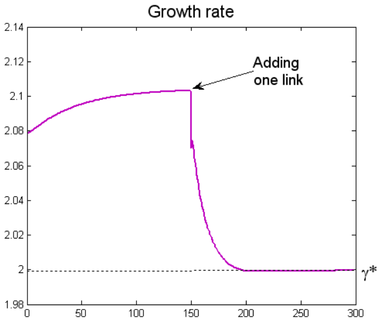
Adding one link to bring equality

Bridge network



Dynamics

Adding one link to bring equality



Two (small) extensions/variations

1. Three period model with savings and capital in production function:

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   - Networks matter only for dynamics and transition

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- The "golden" social structure seems to be the **star** network, which implies low inequality but also high levels of long run human capital and output
- US economy resembles a scenario of inequality and high growth

Further work

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