Discussion of Bacchetta et al: Rational Risk Panics

Paul Klein
IIES, Stockholms universitet
($\rightarrow$ University of Southampton)
pklein2@gmail.com

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Multiplicity of equilibria

- If the current price of an asset depends on its future variance, the price process is not uniquely pinned down.
- A low volatility and a high volatility equilibrium
- This contrasts with the case where the current price depends on its future level. Then stability considerations pin down the equilibrium.
- $p_t = \mathrm{Var}_t[p_{t+1}]$

Multiplicity of equilibria

- Let s_t be a sunspot process
- $s_{t+1} = \rho s_t + \varepsilon_{t+1}$
- $\mathsf{E}_t[\varepsilon_{t+1}] = 0$
- $\mathsf{E}_t[\varepsilon_{t+1}^2] = \sigma^2$
- $\mathsf{E}_t[\varepsilon_{t+1}^3] = 0$
- $\mathsf{E}_t[\varepsilon_{t+1}^4] = 3\sigma^4$
- $\mathsf{Var}_t[\varepsilon_{t+1}^2] = 2\sigma^4$

Multiplicity of equilibria

- If p_t is linear (affine) in s_t, $p_t = 0$ is the unique equilibrium.
- If p_t is quadratic in s_t, there are two distinct equilibria.

Multiplicity of equilibria

- $p_t = a_0 + a_1 s_t + a_2 s_t^2$
- $\mathrm{Var}_t[p_{t+1}] = a_1^2\sigma^2 + a_2^2(2\sigma^4 + 4\rho^2\sigma^2 s_t^2)$
- $p_t = \mathrm{Var}_t[p_{t+1}]$
- Equating coefficients:
  - $a_0 = a_1^2\sigma^2 + 2a_2^2\sigma^4$
  - $a_1 = 0$
  - $a_2 = 4a_2^2\rho^2\sigma^2$

Multiplicity of equilibria

- $a_1 = 0$ for sure
- One solution has $a_2 = 0$ which implies $a_0 = 0$
- If $a_2 \neq 0$ then
  - $1 = 4a_2\rho^2\sigma^2$
  - $p_t = \frac{1}{8\rho^4} + \frac{1}{4\rho^2\sigma^2} s_t^2$
- Notice that an i.i.d. sunspot does not work
- If we are free to vary ρ, there is a continuum of equilibria

Switching equilibria

- Now introduce a second sunspot that (in a manner of speaking) shifts between the two equilibria by coordinating beliefs
- It takes two possible values, corresponding to high and low risk
- If the second sunspot is perfectly persistent, then the low-risk regime is like the zero-risk equilibrium and vice versa
- If not, things get a little bit more complicated...
- Given enough persistence, there is an equilibrium with two distinct regimes (low risk, high risk)

Comments and questions

- How new is the result? Compare with McCafferty & Driskill (1980)
- How general is it? Does it apply to a Lucas (1978) or Mehra & Prescott (1985) environment?
- Connect to the data
  - Is there evidence of regime shifts in stock market prices?
  - Can the parameter values be chosen to match the long-term evidence (or just a single episode)?