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### **A Model of Equilibrium Institutions**

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# A model of equilibrium institutions\*

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Preliminary

## Abstract

In an environment populated by ex-ante identical agents, institutions (the “rules of the game”) are set to maximize payoffs of those individuals in power. They are constrained by the threat of rebellion, where the rebels would be similarly constrained by further threats. Equilibrium institutions are the fixed point of this constrained maximization problem. With the possibility of private investment and the threat of government expropriation, power is shared among a larger group of people. Endogenously, this enables the group in power to act as government committed to protection of property rights, which would otherwise be time inconsistent. But since sharing power implies sharing rents, capital taxation is inefficiently high.

JEL CLASSIFICATIONS: D7; H1; H2; O1; P4.

KEYWORDS: government; political economy; public goods; capital taxation; time inconsistency.

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And Samuel told all the words of the LORD unto the people that asked of him a king.

And he said, This will be the manner of the king that shall reign over you: He will take your sons, and appoint *them* for himself, for his chariots, and *to be* his horsemen; and *some* shall run before his chariots. . . .

And he will take your daughters *to be* confectionaries, and *to be* cooks, and *to be* bakers. . . .

He will take the tenth of your sheep: and ye shall be his servants.

And ye shall cry out in that day because of your king which ye shall have chosen you; and the LORD will not hear you in that day.

Nevertheless the people refused to obey the voice of Samuel; and they said, Nay; but we will have a king over us;

That we also may be like all the nations; and that our king may judge us, and go out before us, and fight our battles.

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1 SAMUEL 8:10–20

## 1 Introduction

Institutions are defined by [North \(1990\)](#) as the “rules of the game”, or the “humanly devised constraints that shape human interaction”. Differences in institutions go a long way in explaining the huge disparities in income across the globe as they affect incentives for agents to invest, produce and exchange.<sup>1</sup> Even though institutions may serve the interests of elites, a fundamental question is why this gives rise to huge economic inefficiencies, that is, why is there no “political” equivalent of the Coase Theorem?

This paper builds a model to address that question. The model starts from a world of ex-ante identical individuals. The elite in power and the rules laying down the allocation of resources will both be endogenously determined in equilibrium, with the goal of maximizing the payoffs of those who will be in the elite. The elite and the rules comprise the equilibrium institutions.<sup>2</sup>

The only constraint on the choice of institutions is the threat of “rebellions”, which destroy the prevailing institutions. There are no other “technological” constraints on which institutions

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<sup>1</sup>For example, see [North and Weingast \(1989\)](#), [Engerman and Sokoloff \(1997\)](#), [Hall and Jones \(1999\)](#) and [Acemoglu, Johnson and Robinson \(2005\)](#).

<sup>2</sup>Recently, some models have been developed aiming at understanding institutions. [Greif \(2006\)](#) combines a rich historical analysis of trade and institutions in medieval times with economic modelling, part of which focuses on the form of government and political institutions that emerged in Genoa. [Acemoglu and Robinson \(2006, 2008\)](#) analyse conditions leading to democracy or dictatorship in an environment where an elite is trying to maintain its power, while citizens prefer a more egalitarian state. In [Besley and Persson \(2009a,b, 2010\)](#), society comprises two groups of agents that alternate in power, and make investments in two technologies that respectively allow the state to tax people and to enforce contracts. The exogenous parameters are the extent of political turnover and institutional (or demographic) features that determine how much one group cares about the other. They obtain predictions consistent with the data on state capacity and fiscal capacity, civil wars, and different forms of taxation.

are feasible, thus the issue will be the incentives for creating and destroying institutions. In the absence of rebellion, the rules prescribed by the chosen institutions are followed. This means that an elite can establish institutions laying down credible rules as long as it avoids rebellions. So although institutions will be chosen in the interests of an elite, this does not preclude the emergence of a “political” Coase Theorem. But crucially, once institutions have been destroyed, the new institutions that will arise cannot be constrained by the past rules or any other deals made earlier. This reflects the commitment problems analysed in [Acemoglu \(2003\)](#) and [Acemoglu, Johnson and Robinson \(2005\)](#).

Rebellions are costly, reflecting the fact that conflict is costly. Importantly, anyone can participate in a rebellion, including members of the current elite. Moreover, there is no limit to the number of rebellions: after any group takes power, there is always another opportunity for rebellion. The cost of conflict for rebels is proportional to the number of individuals in the elite who do not take part in the rebellion.

Since individuals are ex-ante identical and self-interested, the notion of equilibrium institutions is independent of the competence, benevolence, or factional affiliation of the individuals comprising the ruling group. Hence, once a rebellion has succeeded in destroying the current institutions, the rebels will have the same objectives and face the same constraints as the elite formerly in power. As in George Orwell’s *Animal Farm*, there is no intrinsic difference between the “men” and the “pigs”, but in equilibrium, some individuals will be “more equal” than others.

In the model, it is not conflict per se, but rather the threat of conflict that shapes institutions by constraining the actions of those in the elite. No conflict occurs in equilibrium owing to the absence of uncertainty in the conflict technology and the actions of individuals.

The incentives to rebel depend on what members of a new elite can extract once in power, which is also true of rebellions against institutions set up by the rebels, and so on. Given that all individuals are ex-ante identical, there is no fundamental reason why different institutions would be chosen following a rebellion, so the paper focuses on Markovian equilibria. The equilibrium institutions are thus the fixed point of the constrained maximization problem of the elite in power subject to the threat of rebellion, where the rebels would be similarly constrained by further threats of rebellion.

There is no restriction on the composition of a group launching a rebellion. Since the most disgruntled individuals have the most to gain from a rebellion, discouraging the “most profitable” rebellion a fortiori discourages all possible rebellions. It follows that the equilibrium institutions assign payoffs to producers according to a maximin rule, and hence, payoffs of those outside the elite (the “producers”) are equalized. Furthermore, as the survival of institutions depends on those in the elite not defecting and joining a rebellion, payoffs of those inside the elite must also be equalized. In other words, sharing power implies sharing rents.

In an endowment economy where there is a basic trade-off that characterizes the equilibrium institutions. The larger the elite (and hence the greater the number of people with a stake in defending the institutions), the greater the amount of taxes that can be levied, but the proceeds need to be divided among more people. The problem can be represented as a choice of the size of the elite and taxes to maximize the payoff of a member of the elite, subject to the constraint of avoiding a rebellion by producers. This “no-rebellion” constraint acts as a participation constraint for the producers.

Do the equilibrium institutions lead to the elite acting as a “government” in any meaningful sense of the term? That is, are institutions that are designed to maximize the payoffs of those in power ever congruent with the interests of the people? In some cases, the answer turns out to be yes. For example, suppose there is a technology that transforms the output of the producers into a public good that benefit everyone, and which has no impact on any other aspect of the environment. Such a public good will be optimally provided in equilibrium, as if it had been chosen by a benevolent government.

This outcome is consistent with a world characterized by a political Coase Theorem. The ability of the elite to set down rules is analogous to the possibility of contracting in the “regular” Coase Theorem. The notion of the elite constrained by the threat of rebellion means that there is a “price” attached to policy actions affecting those outside the elite, analogous to the existence of markets in the Coase Theorem.

A natural application of the model is to the taxation of investment proceeds. The model is extended so that once institutions are formed and after no rebellion has occurred, producers have access to an investment technology. However, the fruits of this investment are realized only after a lag, during which time there is another round of opportunities for rebellion in which both producers and members of the elite can participate. Since the cost of investing is sunk, it does not affect individuals’ incentives to rebel, so by the maximin principle for payoffs in equilibrium explained earlier, the group in power would have incentives to expropriate fully individuals’ investments.

Although institutions could in principle prescribe any level of capital taxation, whatever is chosen must be “rebellion-proof” after investment decisions have been made. The problem is that the capital arising from investment increases incentives for attacks on the current institutions that lead to new rules permitting full capital expropriation. Thus it is necessary to raise the cost of rebelling. This can only be done by expanding the size of the elite. The problem cannot be solved through taxes and transfers as such instruments can only redistribute disgruntlement, decreasing rebellion incentives for some, but raising them for others.

In equilibrium, to ensure there will be no incentives to rebel against the existing institutions — leading inevitably to full expropriation — the elite has to be large so that if a rebellion were to take place after investment decisions had been made, the equilibrium size of the

subsequent elite would be smaller than the current one. This is the only way to prevent members of the elite simply launching a costless rebellion from within that maintains intact the elite's composition (a rebellion is costless if the entire elite is willing to defect). Following a rebellion, there would then be insufficient places for all the current elite in the subsequent one. Thus, some members of the elite will oppose changing the institutions, and so conflict with them makes it costly to expropriate capital.

Adding the possibility of investment to the model thus gives rise to a larger elite. Sharing power among a wider group of individuals allows the elite to act as a government committed to a certain set of policies that would otherwise be time inconsistent. The model highlights the importance of sharing power as a way to guarantee stability of institutions and thus incentives for investment. This resonates with Montesquieu's doctrine of the separation of powers, which is now accepted and followed in all well-functioning systems of government. It is important to note that in the model, power is not shared among those individuals who are actually investing. The extra individuals in the elite in no sense represent or care about those who invest — but they do care about their own rents under the status quo. Thus, this group of self-interested individuals acts a government that commits to some protection of property rights.

Although it is possible to sustain protection against expropriation in equilibrium, capital taxation is not set efficiently, so the political Coase Theorem breaks down. While in general it would be optimal from the point of view of society to have a larger group in power in order to guarantee that the fruits of investment would not be expropriated, the equilibrium elite chooses taxes on investment that are too high and too little sharing of power.<sup>3</sup>

There are two reasons for the inefficiently high level of capital taxation. The first, and more interesting, follows from the distributional effects of protecting against expropriation. Since members of the elite can bring down the institutions more easily if they take part in a rebellion, they must receive rents. As lower capital taxes require more power sharing, they also imply more rent sharing. This goes against the interests of each individual in the elite. The second is because the elite cannot extract all surplus from investors as the effort required to invest is private information.

These reasons allow us to understand why a political Coase Theorem breaks down in this case. The first reason mentioned above is essentially the inseparability of power and rents. In other words, it is not possible in equilibrium to add individuals to the elite and grant them a payoff lower than their peers. There is thus an endogenous limitation (which is binding) on the set of possible transfers. The second reason for the failure of the Coase Theorem is the unobservability of investors' surpluses.

While the model is quite abstract, it is congruent with a number of historical examples,

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<sup>3</sup>This is in accordance with the empirical literature that highlights the importance of institutions guaranteeing protection against government expropriation (for example [Acemoglu and Johnson, 2005](#)).

some of which are discussed later in the paper: the disappearance of private corporations (the *societas publicanorum*) when power was concentrated under the Roman emperors; the need for a militarily strong leader (*podestà*) to guarantee stability in a society (medieval Genoa) where other strong groups could seize power; the tenacious resistance of the Stuart Kings of England to sharing power with Parliament.

Section 2 presents the model, with the extension to the case of private investment in section 3. Section 4 draws some conclusions.

## 1.1 Related literature

Since Downs (1957) emphasized the importance of studying governments composed of self-interested agents, a vast literature on political economy has developed (see, for example, Persson and Tabellini, 2000). Most of this literature focuses on democracies, so institutions are not themselves explained in terms of the decisions of self-interested agents. But in much of the developing world and during most of human history, political regimes have differed greatly from democracies.

This paper shares important similarities with the literature on coalition formation, as analysed by Ray (2007).<sup>4</sup> As in that literature, the process of establishing rules is non-cooperative, but it is assumed that such rules are followed. Moreover, the modelling of rebellions here is related to the idea of blocking in coalitions (Part III of Ray, 2007) in the sense that there is no explicit game-form, as in cooperative game theory. The distinguishing feature of this model is the “rebellion technology” that must be used in order to replace existing institutions by new ones.

The model assumes that the institutions established by the incumbent army determine the allocation of resources once production has taken place. But how would those institutions manage to affect the allocation of goods ex post? As pointed out by Basu (2000) and Mailath, Morris and Postlewaite (2001), laws and institutions do not change the physical nature of the game, all they can do is affect how agents coordinate on some pattern of behaviour. But in reality, laws and institutions are seen to have a strong impact on behaviour, and this feature must be present in any model of institutions.

The view of this paper is similar to the application of Schelling’s (1960) notion of focal points in the organization of society, as put forward by Myerson (2009). The “rules of the game” are self-enforcing as long as society coordinates on punishing whomever deviates from the rules — and whomever deviates from punishing the deviators. For example, if the laws specify how much an individual must pay or receive from another, and if both expect to be

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<sup>4</sup> Baron and Ferejohn (1989) analyse bargaining in legislatures using this approach. Levy (2004) studies political parties as coalitions. Recent contributions include Acemoglu, Egorov and Sonin (2008) Piccione and Razin (2009).

harshly punished if they fail to comply (along with any “higher-order” deviators) then the laws will be self enforcing.

Following this, theorizing about institutions is theorizing about (i) how rules (or focal points) are chosen, and (ii) how rules can change. For example, Myerson (2004) explores the idea of justice as a focal point influencing the allocation of resources in society. This paper takes a more cynical view towards our fellow human beings. Here, the individuals in power choose the laws and institutions to maximize their own payoffs, and those institutions can only be destroyed by a rebellion — wiping out the old institutions, and making way for new ones. There is no modelling of the post-production game.

This paper is also related to the literature on social conflict and predation, surveyed by Garfinkel and Skaperdas (2007).<sup>5</sup> It is easy to envisage how conflict could be important in a state of nature: individuals could devote their time to fighting and stealing from others. However, when there are fights, there are deadweight losses. Thus, people would be better off if they could agree on transfers to avoid conflict. This paper presupposes such deals are possible: producers pay taxes to the group in power, which allocates resources according to some predetermined rules. Here, differently from the literature on conflict, individuals fight to be part of the group that sets the rules, not over what has been produced. Moreover, they fight in groups, not as isolated individuals.

There are theoretical models focused on political issues that lead to inefficiencies in protection of property rights. Examples include Glaeser, Scheinkman and Shleifer (2003), Acemoglu (2008), Guriev and Sonin (2009) and Myerson (2010). Here, the possibility of capital expropriation and the consequent need to protect property rights is just a natural consequence of the possibility of investment and the “rebellion technology” that allows institutions to be destroyed and replaced.

Lastly, it is possible to draw an analogy between this paper and models of democracy (Persson and Tabellini, 2000) in the sense that the “election technology” there is replaced by a “rebellion technology” here.

## 2 The model

### 2.1 Environment

There is an area containing a measure-one continuum of ex-ante identical individuals. An individual will either be a member of the *elite* or a *producer*. The *institutions* (the “rules of the game”) determine the composition of the elite and the allocation of resources. Once they are created, those institutions cannot be modified, unless a *rebellion* occurs. A rebellion

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<sup>5</sup>For instance, see Grossman and Kim (1995), Hirshleifer (1995), and also Hafer (2006), Dal Bó and Dal Bó (2010) for some recent contributions.

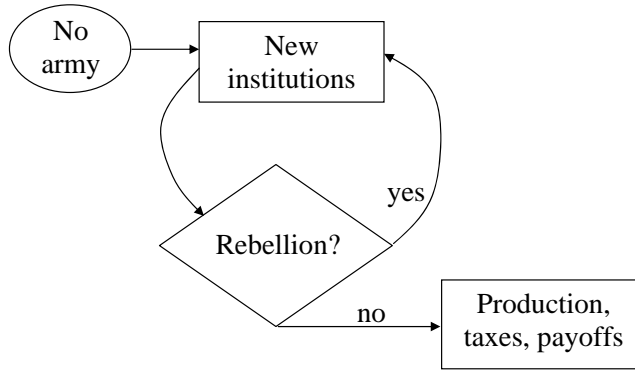


destroys existing institutions, allowing for the creation of new ones. After institutions are formed, there are opportunities for rebellions, and everyone has the opportunity to rebel.

Producers have access to a production technology that yields an exogenous endowment of  $q$  units of a homogeneous good. Members of the elite cannot produce, but are able to establish and defend institutions. The status of the elite relies on defending their institutions against rebellions, so they make up the *incumbent army*.

The sequence of events is as shown in Figure 1. A group takes power and establishes institutions, becoming the elite. There are then opportunities for rebellion. If a rebellion occurs, new institutions are established, potentially changing the elite, with these institutions also being subject to the threat of rebellion. When no rebellions occur, all individuals outside the elite become producers, production takes place, rules are followed according to the prevailing institutions, and payoffs are received.

**Figure 1:** *Sequence of events*



### 2.1.1 Establishing institutions

Institutions determine who belongs to the elite, and how resources in society are distributed.

It is assumed that the size of the elite  $a$  is such that it maximizes the average utility of its members.<sup>6</sup> Thus the elite cannot have incentives to expel or recruit new members. That captures the idea that the distribution of power reflects the interests of the elite, not the welfare of society as a whole. Given the size  $a$ , the composition of the elite depends on a predetermined ordering set at the time of the previous rebellion.<sup>7</sup>

The elite maximizes over the distribution of taxes  $\tau(\cdot)$  levied on producers, and the distribution of consumption of its members  $C_a(\cdot)$ . No exogenous restrictions are imposed on

<sup>6</sup>Moving away from the assumption that the elite maximizes its average payoff would require modelling its hierarchy, which is beyond the scope of this paper. For a model of this kind, see Myerson (2008).

<sup>7</sup>Starting from no pre-existing institutions, a random ordering is used.

taxation, beyond the natural bound of what producers possess.<sup>8</sup>

The choices of  $a$  and the distribution of resources are constrained by the threat of rebellion, as described below. Once institutions are established, each producer gets to know the tax  $\tau(\iota)$  he faces and each member of the elite gets to know his consumption  $C_a(\iota)$ . Further rebellions are then considered.

### 2.1.2 A rebellion

Understanding incentives for rebellion requires knowing individuals' beliefs about what institutions would be created, and what payoffs would be received, following a rebellion. Let  $a^e$  and  $U_a^e$  be beliefs about the size of the elite and the average utility of its members, respectively, that will characterize the next elite if the current institutions are destroyed. Let  $U(\iota)$  denote the utility of individual  $\iota$  if no rebellion occurs. The difference  $U_a^e - U(\iota)$  corresponds to the gain individual  $\iota$  obtains if the current institutions are destroyed and he belongs to the subsequent elite. So the maximum fighting cost (in utility units) that an individual  $\iota$  who expects to have a place in the subsequent elite is willing to make is

$$F(\iota) = U_a^e - U(\iota). \quad [2.1]$$

A rebellion is an ordering of the whole population. The corresponding *rebel army*  $\mathcal{R}$  is the subset of size  $a^e$  of individuals ordered first. This might include members of the elite. Denote by  $d$  the measure of individuals in the rebel army that also belong to the elite. This rebellion is profitable if:

$$\int_{\mathcal{R}} F(\iota) d\iota > \delta(a - d). \quad [2.2]$$

The right-hand side is the total fighting effort required to destroy the current institutions. The rebels have to fight against the incumbent army, which comprises those members of the elite who do not belong to the rebel army. The fighting effort is proportional to the measure of the incumbent army  $a - d$ . The parameter  $\delta$  measures the power of the incumbent army. It is assumed that individuals' utility is linear in the amount of fighting effort exerted. Furthermore, total fighting effort is the integral of the effort put in by all individuals in the rebel army  $\mathcal{R}$ , which is the left-hand side of the inequality above.

Thus for a rebellion to be viable the total fighting effort required to displace the current institutions must be smaller than the total effort those in the rebel army would be willing to make. This quantity of effort depends on beliefs about the subsequent institutions, and these beliefs must be consistent with the institutions that will emerge if the rebellion succeeds. Note

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<sup>8</sup>Taxes can be contingent on any observable variables, but this does not play any role in the basic version of the model. Taxes cannot be contingent on agents' identities. This would imply identities would become state variables in the model, making the analysis much more complicated. In equilibrium, though, the elite has no incentives to use identity-contingent taxes.

that if all members of the elite defect ( $d = a$ ), there is no cost whatsoever of destroying the current institutions.

This approach to modelling the threat of conflict allows for a simple representation of the constraints faced by those choosing institutions, without accounting explicitly for the punches and sword thrusts. By assumption, all the fighting is between those in the incumbent army (drawn only from the elite) and those in the rebel army, which comprises only those individuals who expect a place in the subsequent elite. This means that those who will be producers regardless of the success of a particular rebellion would not fight. The rationale for this assumption is that armies face a problem of incentivizing their members to fight. This incentive problem is likely to be much more severe for those who will not have a place in the elite once the fighting is over. The basic problem is that those designing the new institutions after a rebellion have no interest in honouring past inducements to fight to the extent that these would reduce their own payoffs. Promises to pay producers for fighting (the carrot) are costly by definition, while punishments (the stick) would also be costly because they create disgruntlement with new institutions and increase incentives for another rebellion. But denying a place in the incumbent army to those who do not exert enough effort does not need to be costly owing to the presence of a pool of identical replacements that would like to join.<sup>9</sup>

### 2.1.3 Production, taxation and payoffs

Once there is no profitable rebellion against the existing institutions, production takes place, taxes are collected, and payoffs are received.

Let the set of individuals in the elite be  $\mathcal{A}$  and the set of producers be  $\mathcal{P}$ . Producer  $i$  subject to institutions imposing a tax  $\tau(i)$  on him receives consumption

$$C_p(\tau(i)) = q - \tau(i). \quad [2.3]$$

Total consumption of all members of the elite must be equal to the amount collected in taxes:

$$\int_{\mathcal{A}} C_a(i) di = \int_{\mathcal{P}} \tau(i) di. \quad [2.4]$$

An individual who receives consumption  $C$  obtains utility  $U = u(C)$ , net of any past fighting costs that have been incurred, and where  $u(C)$  is a strictly increasing and weakly concave function.

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<sup>9</sup>The formulation necessarily abstracts from coordination issues, and it might be thought that these would be a more pressing problem for the rebels than the incumbent. If this were the case, the power parameter  $\delta$  could be adjusted to account for such differences.

## 2.2 Equilibrium

Equilibrium institutions are those that maximize the average utility of members of the elite subject to there being no profitable rebellion. The choice variables are the size of the elite  $a$ , the payoffs of each member of the elite, and a tax distribution  $\tau(\cdot)$  specifying a lump-sum tax for each producer  $\iota \in \mathcal{P}$ . The equilibrium institutions are the result of the following constrained maximization problem:

$$\max_{a, \tau(\cdot)} \frac{1}{a} \int_{\mathcal{A}} U_a(\iota) d\iota \quad \text{s.t.} \quad \int_{\mathcal{R} \cap \mathcal{P}} (U_a^{te} - U_p(\tau(\iota))) d\iota + \int_{\mathcal{R} \cap \mathcal{A}} (U_a^{te} - U_a(\iota)) d\iota \leq \delta(a - d), \quad [2.5]$$

for all sets  $\mathcal{R}$  such that  $\mathbb{P}(\mathcal{R}) = a^{te}$ , where  $a^{te}$  is the belief about the size of the elite if a rebellion succeeds,  $U_a^{te}$  is the expected utility of those in the elite under the institutions that would be established in case of a successful rebellion, and  $d = \mathbb{P}(\mathcal{R} \cap \mathcal{A})$  is the measure of defectors from the current elite. The solution to [2.5] depends on  $a^{te}$  and  $U_a^{te}$ .

Once the current institutions are destroyed by a rebellion, new ones are formed to maximize the average utility of those who will be in the new elite. The choice of  $a'$  is not constrained by the size of the rebel army  $\mathbb{P}(\mathcal{R})$ , the assumption being that the extension of power to an additional individual must be in the interests of all other members of the elite. The order in which individuals join the elite is set down by the predetermined ordering that characterizes the rebellion. The new elite also maximizes over a tax distribution  $\tau'(\cdot)$ . In equilibrium, the earlier beliefs  $a^{te}$  and  $U_a^{te}$  must be equal to the actual values of  $a'$  and  $U_a'$  given the absence of uncertainty.

The maximization problem characterizing  $a'$  and  $\tau'(\cdot)$  is of an identical form to that in [2.5] for  $a$  and  $\tau(\cdot)$  owing to the irrelevance of history: the cost of conflict is sunk and additive; and the choice of new institutions is not constrained by the size of the rebel army. Thus, there are no state variables, and hence no fundamental reason why different institutions would be chosen at each point. It is therefore natural to focus on Markovian equilibria.

A Markovian equilibrium is a solution to the maximization problem [2.5] with  $a = a' = a^{te}$ ; an identical distribution of taxes over producers:  $\mathbb{P}(\tau(\iota) \leq \tau) = \mathbb{P}(\tau'(\iota) \leq \tau)$  for all  $\tau$ ; an identical distribution of consumption over members of the elite:  $\mathbb{P}(C_a(\iota) \leq C) = \mathbb{P}(C_a'(\iota) \leq C)$  for all  $C$ . The following result demonstrates some features of any Markovian equilibrium.

**Proposition 1** *Any Markovian equilibrium must have the following properties:*

- (i) *Equalization of producers' payoffs:  $U_p(\iota) = U_p$  for all  $\iota$  (with measure one)*
- (ii) *Sharing power implies sharing rents:  $U_a(\iota) = U_a$  for all  $\iota$  (with measure one)*
- (iii) *The set of constraints in [2.5] is equivalent to a single "no-rebellion" constraint:*

$$U_p(\tau) \geq U_{a'} - \delta \frac{a}{a'}. \quad [2.6]$$

(iv) *Power determines rents:*  $U_a - U_p = \delta$

PROOF See [appendix A.1](#). ■

These results do not rely on risk aversion: they also hold for a linear utility function. Since all producers receive the same endowment here, payoff equalization is equivalent to tax equalization. The intuition for the equalization of producer payoffs is that as only a subset of individuals takes part in a rebellion, the elite wants to maximize the utility of the subset with the minimum utility. This is achieved by equalizing producers' utility. Introducing inequalities in payoffs reduces the average utility of a subset of size  $a'$  with minimum utility, so it necessarily makes it harder to avoid a rebellion.<sup>10</sup>

An analogous argument implies that heterogeneity in elite payoffs is undesirable because it makes it harder to avoid rebellions from within the elite at the same time as ensure an overall high payoff for them. Because the cost of rebelling is proportional to the measure of members of the elite who do not defect, any inequality will lead the least satisfied individuals in the elite to participate in a rebellion.

The single “no-rebellion” constraint in the third part of the proposition is the effective constraint faced by the elite when the rebellion includes no defectors and payoffs of producers are equalized. Once this is satisfied, all other constraints are redundant. An immediate consequence of this is that the power parameter  $\delta$  determines the size of the rents received by members of the elite in a Markovian equilibrium.

Therefore, the maximization problem characterizing the equilibrium institutions has the following recursive form

$$\max_{a, \tau} U_a(a, \tau) \quad \text{s.t.} \quad U_a(a', \tau') - \delta \frac{a}{a'} \leq U_p(\tau), \quad [2.7]$$

where  $a'$  and  $\tau'$  solve an identical problem taking  $a''$  and  $\tau''$  as given, and so on.

As the tax revenue  $(1 - a)\tau$  is equally shared among the elite, utility is

$$U_a(a, \tau) = u \left( \frac{1 - a}{a} \tau \right).$$

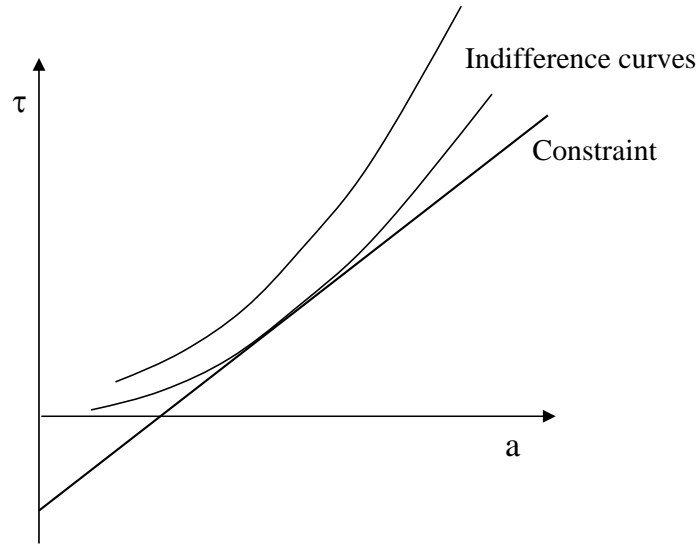
The payoff of those in the elite is increasing in the tax  $\tau$  and decreasing in the size of the elite  $a$ . The tradeoff between the two is represented by the convex indifference curves in [Figure 2](#). The elite has two margins to ensure that it avoids rebellions. It can reduce taxes (the “carrot”), or increase its size, as this means that rebels must fight a larger incumbent army (the “stick”). This corresponds to the upward-sloping no-rebellion constraint. The maximum

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<sup>10</sup>This result is different from those found in some models of electoral competition such as [Myerson \(1993\)](#). In the equilibrium of that model, politicians offer different payoffs to different agents. But there is a similarity with the model here because in neither case will agents' payoffs depend on their initial endowments.

is at the tangency point. With linear preferences, the constraint is a straight line, as depicted in Figure 2. With risk aversion, the constraint implies  $\tau$  is a concave function of  $a$ .

**Figure 2:** Trade-off between army size and taxation



## 2.3 Examples

There are two exogenous parameters in the model: the power parameter  $\delta$ , and the endowment  $q$  of a producer. The following examples illustrate the workings of the model for some particular utility functions.

### 2.3.1 Linear utility

When the utility function  $u(C) = C$ , the maximization problem of the elite is

$$\max_{a, \tau} \frac{1-a}{a} \tau \quad \text{s.t.} \quad C'_a - \delta \frac{a}{a'} \leq q - \tau. \quad [2.8]$$

Substituting tax  $\tau$  from the constraint into the objective function yields:

$$C_a = \frac{1-a}{a} \left( q - C'_a + \delta \frac{a}{a'} \right). \quad [2.9]$$

The following first-order condition with respect to  $a$  is obtained:

$$\frac{C_a}{1-a} = (1-a)\frac{\delta}{a'}. \quad [2.10]$$

Imposing Markovian equilibrium ( $a = a'$  and  $C'_a = C_a$ ) in [2.9] yields:

$$C_a = (1-a)(q + \delta).$$

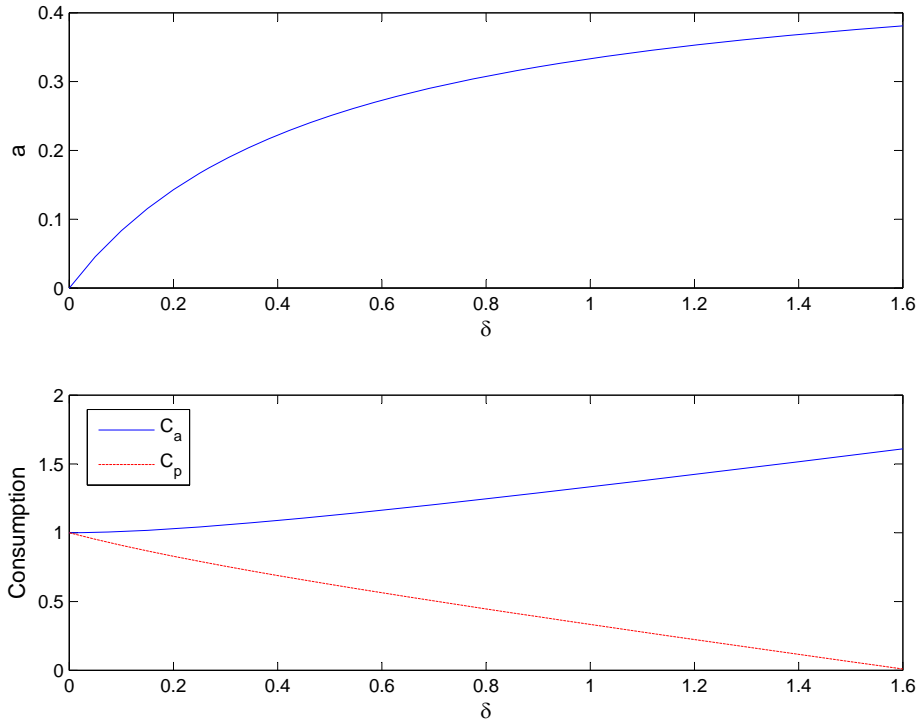
Combining this equation with [2.10] implies:

$$a = \frac{\delta}{q + 2\delta}, \quad C_a = \frac{(q + \delta)^2}{q + 2\delta}, \quad \text{and} \quad C_p = \frac{(q + \delta)^2}{q + 2\delta} - \delta.$$

With a linear utility function, the size of the elite is a function of  $q/\delta$ .

The relationship between the power parameter  $\delta$  and the key endogenous variables of the model is shown in Figure 3 for  $q = 1$ .

**Figure 3:** *The case of linear utility*



If  $\delta/q \geq (1 + \sqrt{5})/2$ , consumption of producers is zero. To make the problem interesting, it is sensible to restrict the parameters to ensure producers obtain positive consumption, which requires  $\delta/q < (1 + \sqrt{5})/2$ .

The power parameter  $\delta$  affects the equilibrium in three ways. First, an increase in  $\delta$  makes

the incumbent army stronger because the rebels have to bear a higher cost to defeat it. This leads to an increase in  $\tau$  and a decrease in  $a$ . Second, the payoff that the rebels will receive once in power increases as their position would also be stronger once they have supplanted the current elite, making rebellion more attractive. This effect makes the position of the elite weaker, leading it to decrease  $\tau$ , and increase  $a$ . Third, an increase in  $\delta$  raises the effectiveness of the marginal fighter in the incumbent army, leading the elite to increase its size in order to extract higher taxes. As long as  $C_p > 0$ , the last effect dominates and the size of the elite is increasing in  $\delta$ .

### 2.3.2 Log utility

When the utility function  $u(C) = \log C$ , the maximization problem of the elite is

$$\max_{a,\tau} \log\left(\frac{1-a}{a}\tau\right) \quad \text{s.t.} \quad \log(C'_a) - \delta\frac{a}{a'} \leq \log(q - \tau). \quad [2.11]$$

Substituting  $\tau$  from the constraint into the objective function, the following first-order condition is obtained:

$$\frac{1}{1-a} + \frac{1}{a} = \frac{\delta C'_a \exp\{-\delta a/a'\}}{a'\tau}.$$

Imposing Markovian equilibrium ( $a = a'$  and  $C'_a = (1-a)\tau/a$ ) in the above yields:

$$a = (1-a)^2 \delta \exp\{-\delta\},$$

which implies that:

$$a = \frac{2\delta \exp\{-\delta\}}{1 + 2\delta \exp\{-\delta\} + \sqrt{1 + 4\delta \exp\{-\delta\}}}.$$

The size of the elite is independent of a producer's endowment  $q$ . As consumption of a producer increases, so does the amount of goods he is willing to surrender in order to avoid conflict.

It is also instructive to note how the output of the economy is distributed among individuals. The following condition is obtained by imposing equilibrium and using the no-rebellion constraint:

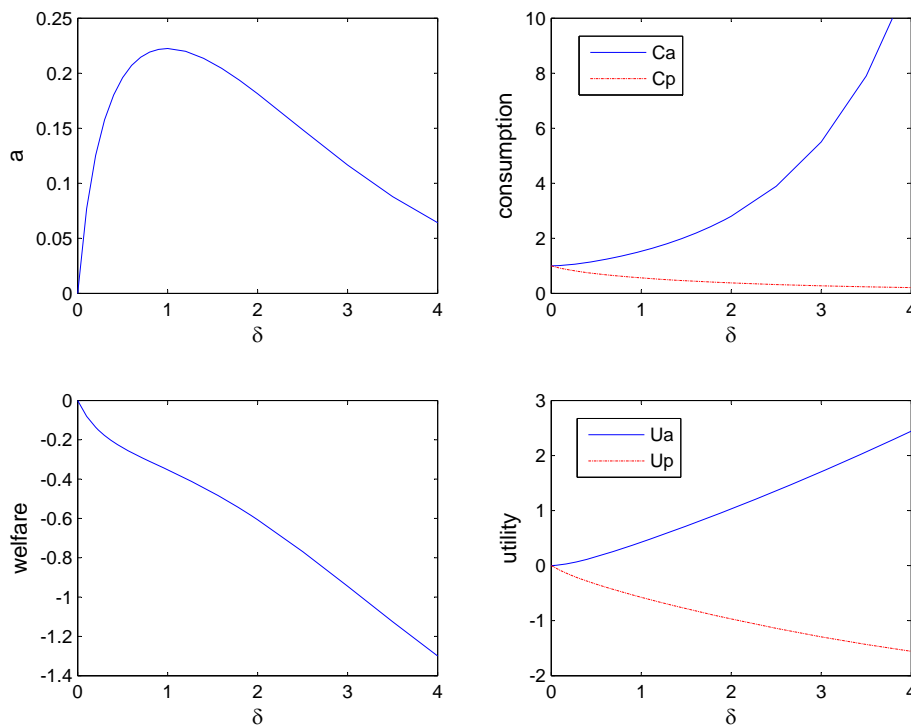
$$C_a = \frac{(1-a)q}{a + (1-a) \exp\{-\delta\}}, \quad \text{and} \quad C_p = \frac{\exp\{-\delta\}(1-a)q}{a + (1-a) \exp\{-\delta\}}. \quad [2.12]$$

The consumption of members of the elite and producers is proportional to  $q$ . The output of the economy  $(1-a)q$  is divided among all individuals in the economy, with producers getting a fraction  $\exp\{-\delta\}$  of what someone in the elite receives. The value of  $q$  (affecting the size of the pie) has no influence on the shares of each individual owing to log utility in consumption.



The relationship between the power parameter  $\delta$  and the key endogenous variables of the model is shown in Figure 4.

**Figure 4:** *The case of log utility*



The size of the elite  $a$  is positively related to  $\delta$  for  $\delta < 1$  and decreasing in  $\delta$  otherwise. An increase in  $\delta$  raises the effectiveness of the marginal fighter in the incumbent army. When  $\delta$  is relatively small, this leads to an increase in the size of the elite and higher taxes. But as  $\delta$  becomes larger, reducing the consumption of producers leads to greater incentives for them to rebel, leading the elite to choose a smaller  $a$  and a smaller  $\tau$ .

Output in the economy is negatively related to  $a$ . So it reaches its lowest level at  $\delta = 1$  when  $a$  reaches its maximum. However, welfare is everywhere decreasing in  $\delta$  owing to the negative distributional effects of increases in  $\delta$ .

## 2.4 The provision of public goods

In the previous section there no scope for the elite to do what governments are customarily thought to do, such as the provision of public goods. This section introduces a technology that allows for production of public goods. It is then natural to ask whether those would be provided, since unlikely atomistic individuals, the elite can set up institutions that determine taxes and spending on the provision of public goods. The question is of whether a political Coase Theorem will arise in this setting.

The new technology converts units of output into public goods. If  $g$  units of goods per capita are converted using the technology then everyone receives  $f(g)$  units of the consumption good. The utility function of an individual is thus  $u(C + f(g))$ .

Per-capita consumption is

$$(1 - a)q - g + f(g).$$

A benevolent social planner would choose  $g$  such that

$$f'(g) = 1, \tag{2.13}$$

to maximize the final amount of goods available for consumption. Note that the choice of  $g$  is independent of  $a$ .

The assumptions of [section 2](#) are now modified so that the institutions include the provision of public goods  $g$ . All individuals observe the choice of  $g$  and take it into account when determining how much fighting effort they are willing to make in a rebellion.

The consumption of a producer is now

$$C_p(\tau, g) = q - \tau + f(g), \tag{2.14}$$

and the consumption of a member of the elite is:

$$C_a(a, \tau, g) = \frac{\tau(1 - a) - g}{a} + f(g). \tag{2.15}$$

The equilibrium institutions are the solution of the following maximization problem:

$$\begin{aligned} \max_{a, \tau, g} u \left( \frac{\tau(1 - a) - g}{a} + f(g) \right) \quad \text{s.t.} \quad U'_a - \delta \frac{a}{a'} \leq U(q + f(g) - \tau), \\ \text{with } a = a', \quad \tau = \tau' \quad \text{and } g = g'. \end{aligned} \tag{2.16}$$

Setting up the Lagrangian and taking the first-order conditions with respect to  $\tau$  and  $g$  yields:

$$\begin{aligned} u'(C_a) \left( -\frac{1}{a} + f'(g) \right) &= \lambda U'(C_p) f'(g), \\ u'(C_a) \left( \frac{1 - a}{a} \right) &= -\lambda U'(C_p), \end{aligned}$$

and combining both leads to:

$$f'(g) = 1.$$

This is the same equation as [\[2.13\]](#) for the case of the benevolent social planner, so the public good is optimally provided.

Who benefits from public good provision? The distribution of output among the individuals depends on the particular utility function. For example, with log utility, the following payoffs analogous to [2.12] are obtained:

$$C_a = \frac{(1-a)q + f(g) - g}{a + (1-a)\exp\{-\delta\}}, \quad \text{and} \quad C_p = \frac{\exp\{-\delta\}((1-a)q + f(g) - g)}{a + (1-a)\exp\{-\delta\}}.$$

Thus, even though the elite is extracting rents from producers, this does not preclude it from acting as if it were benevolent in other contexts. An implication is that the welfare of producers could be larger or smaller compared to a world in which no-one can compell others to act against their will. This reflects the ambivalence effects of having a ruling elite (or a king) on ordinary people.<sup>11</sup>

The “no-rebellion” constraint implies that the elite cannot disregard the interests of the producers. Provision of public goods slackens the “no-rebellion” constraint, while the taxes raised to finance them tighten the constraint. By optimally trading off the benefits of the public good against the cost of provision, the elite effectively maximizes the size of the pie, making use of transfers to ensure everyone is indifferent between rebelling or not. By not rebelling, those outside the elite essentially acquiesce to the “offer” made by the elite, analogous to the contracting that underlies the regular Coase Theorem.

The result is far from surprising and can be obtained in several environments. This is discussed by [Persson and Tabellini \(2000\)](#) in the context of voting and elections. Here the result provides a benchmark where the political Coase Theorem holds.

### 3 Investment

This section adds the possibility of investment to the model. Producers can exert effort to obtain a larger amount of goods in the future, but there is a time lag between the effort being made and the fruits of that investment being received. During this span of time, there are opportunities for rebellion against the prevailing institutions. The model is otherwise identical.

There are no changes here to the mechanism through which institutions are created and destroyed. However, the presence of investment proceeds changes incentives for rebellion, and thus the incentives of the elite over the choice of institutions. The first question is what the equilibrium institutions are. The second is whether those institutions are consistent with a political Coase Theorem.

#### 3.1 Environment

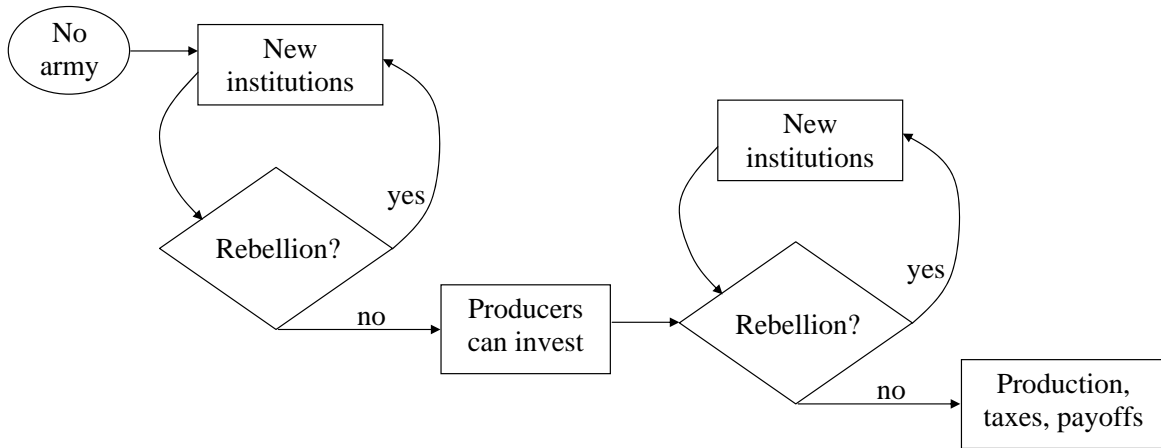
[Figure 5](#) shows the sequence of events in the model. The first part of the sequence resembles

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<sup>11</sup>As stressed by the prophet Samuel.

the basic model (Figure 1), but now after institutions triggering no rebellion are chosen, each producer receives an investment opportunity and decides whether to invest. Once all these decisions are made, there is another round of opportunities for rebellion, with new institutions established if a rebellion occurs. When the prevailing institutions do not trigger a rebellion, production takes place, taxes are paid, and payoffs are received. In equilibrium, the elite at the pre-investment stage will choose institutions that survive rebellion at all points (as in the basic model).

**Figure 5:** *Sequence of events*



The investment opportunity is as follows. Each producer gets to know an idiosyncratic effort cost  $\epsilon$  (in utility terms), an independent draw from a uniform distribution  $U[\epsilon_L, \epsilon_H]$ . Any producer choosing to incur this cost receives an extra  $\kappa$  units of goods, referred to as *capital*, at the production and payoffs stage of Figure 5. It is assumed that the distribution of effort costs is such that a sufficiently large fraction of producers will not invest in equilibrium.  $k \in \{0, \kappa\}$  denotes an individual producer's holdings of capital at the post-investment stage.

Producers' effort costs  $\epsilon$  are private information, while their capital  $k$  is publicly observable. This means that taxes can be contingent on capital, but not on effort. As before, lump-sum taxes are available.

For analytical tractability, agents' preferences are linear in consumption, so  $u(C) = C$ . This allows for relatively simple closed-form solutions.

### 3.2 Equilibrium

Characterizing the equilibrium requires working backwards from the post-investment stage, determining the equilibrium institutions if a rebellion were to occur, and then analysing what institutions will be chosen at the pre-investment stage.

### 3.2.1 Post-investment stage after a rebellion

Suppose that a rebellion occurs at some point during the post-investment stage. An argument similar to [Proposition 1](#) shows that the institutions chosen at this point would equalize consumption of all producers, so consumption cannot be contingent on how much capital one producer has relative to what others have. The producers' effort costs at the pre-investment stage are sunk, so they do not affect the willingness of individuals to participate in rebellions.

Let  $\hat{U}_p$  denote the utility common to all producers after a rebellion has occurred at the post-investment stage, net of any effort costs (for fighting or investment) incurred earlier. Similarly,  $\hat{U}_a$  is the utility of those in the elite after a post-investment-stage rebellion, and  $\hat{a}$  is the size of such an elite. Let  $K$  denote the total amount of capital resulting from producers' earlier investment decisions. The economy's resource constraint is

$$\hat{a}\hat{U}_a + (1 - \hat{a})\hat{U}_p = K + (1 - \hat{a})q,$$

and thus the payoff of those in the elite is

$$\hat{U}_a = \frac{K + (1 - \hat{a})(q - \hat{U}_p)}{\hat{a}}.$$

The no-rebellion constraint faced by the elite is

$$\hat{U}_p \leq \hat{U}'_a - \delta \frac{\hat{a}}{\hat{a}'}$$

Maximizing the elite payoff subject to the no-rebellion constraint and imposing Markovian equilibrium yields:

$$\hat{a} = \frac{\delta}{q + 2\delta}, \quad \hat{U}_a = \frac{(q + \delta)^2}{q + 2\delta} + K, \quad \text{and} \quad \hat{U}_p = \frac{(q + \delta)^2}{q + 2\delta} - \delta + K, \quad [3.1]$$

assuming that  $\delta/q$  is smaller than  $(1 + \sqrt{5})/2$  (otherwise  $\hat{U}_p$  would be zero).

The size of the elite  $\hat{a}$  is the same as that found in [section 2.3.1](#). It is independent of  $K$ , which is analytically convenient. Thus, were a rebellion to occur at the post-investment stage, the total capital stock  $K$  would be expropriated and equally distributed among the whole population.

### 3.2.2 Equilibrium institutions

The now familiar argument of [Proposition 1](#) implies that the elite has incentives to avoid creating dispersion in producers' payoffs orthogonal to capital holdings. This means that the elite's maximization problem reduces to the choice of its size  $a$ , a level of lump-sum taxation

$\tau_q$  levied on all producers, and a tax  $\tau_\kappa$  paid by all those producers who hold  $\kappa$  units of capital.

The choice of capital taxation  $\tau_\kappa$  determines producers' effort threshold for investment. A producer will invest if his idiosyncratic effort cost  $\epsilon$  is smaller than  $\epsilon^*$ , where

$$\epsilon^* = \kappa - \tau_\kappa.$$

This determines the fraction  $s$  of producers who will choose to invest:

$$s = \frac{\epsilon^* - \epsilon_L}{\epsilon_H - \epsilon_L}, \quad [3.2]$$

and the capital stock at the second stage,  $K = s(1 - a)\kappa$ .

The payoff of a member of the elite is

$$U_a = \frac{(1 - a)(\tau_q + s\tau_\kappa)}{a}.$$

At the point where the pre-investment institutions are determined, individual producers do not know their idiosyncratic effort costs  $\epsilon$ . The expected utility of a producer net of effort costs is

$$\mathbb{E}U_p = q - \tau_q + s\mathbb{E}[\kappa - \tau_\kappa - \epsilon | \epsilon \leq \epsilon^*] = q - \tau_q + s\frac{\epsilon^* - \epsilon_L}{2}.$$

Therefore, similar to what is obtained in [section 2](#), the first stage no-rebellion constraint is:

$$q - \tau_q + s\frac{\epsilon^* - \epsilon_L}{2} \geq U'_a - \delta\frac{a}{a'} \quad [3.3]$$

The consumption of those producers who choose not to invest is  $q - \tau_q$ , whereas those who did invest are able to consume  $q - \tau_q + \kappa - \tau_\kappa$ , which is larger than the former (because  $\kappa - \tau_\kappa > 0$ ). Therefore, producers that have invested are not willing to make as much fighting effort to destroy the current institutions. Consequently, their no-rebellion constraint is not binding, creating incentives for the elite to expropriate their capital.

Using the results for the post-investment stage after a rebellion and the assumption that there are more producers choosing not to invest than the post-rebellion elite size  $\hat{a}$ , the no-rebellion constraint [\[2.2\]](#) can be written as

$$\hat{a}\hat{U}_a - (\hat{a} - d)(q - \tau_q) - dU_a \leq \delta(a - d).$$

This constraint must hold for all values of  $d$ . Owing to the linearity of this constraint in  $d$ , it is equivalent to the following pair of constraints (one corresponding to  $d = 0$ , the other to

$d = \hat{a}$ ):

$$q - \tau_q \geq \frac{(q + \delta)^2}{q + 2\delta} + (1 - a)s\kappa - \delta \frac{a}{\hat{a}}, \quad \text{and} \quad [3.4]$$

$$U_a \geq \frac{(q + \delta)^2}{q + 2\delta} + (1 - a)s\kappa - \delta \frac{a}{\hat{a}} + \delta. \quad [3.5]$$

**Proposition 2** Any Markovian equilibrium with  $s > 0$  must have the following features:

(i) Both no-rebellion constraints [3.4] and [3.5] for members of the elite and producers are binding at the post-investment stage, while the producers' pre-investment-stage constraint [3.3] is slack.

(ii) The equilibrium size of the elite is

$$a = \frac{\delta \hat{a} + s\epsilon^*}{\delta + s\epsilon^*}. \quad [3.6]$$

(iii) The payoff of a member of the elite is

$$U_a = \frac{\delta(q + \delta)}{q + 2\delta} \left( \frac{q + \delta + s(\kappa - \epsilon^*)}{\delta + s\epsilon^*} \right). \quad [3.7]$$

PROOF See [appendix A.2](#). ■

If some producers are to invest then the size of the elite  $a$  must be larger than that which would prevail if a rebellion were to occur after investment decisions had been made. To sustain investment, the elite must convince producers that there will be no rebellions leading to expropriation of capital. The proposition shows that this is equivalent to removing incentives for rebellion from members of the elite and those producers with no capital. The presence of capital increases incentives for rebellion. On the one hand, discouraging rebellion by producers requires either a larger elite or *lower* taxes, on the other hand, discouraging rebellion by members of the elite requires either a larger elite or *higher* taxes. A larger elite is thus needed to satisfy both constraints simultaneously.

If the size of the elite is greater than the size associated with the equilibrium institutions following a post-investment rebellion then the post-rebellion elite would not include all members of the existing elite. Since some of them will lose their status, there can be no rebellion which commands the unanimous support of the elite. In the language of the model, for any possible rebellion, even if the rebel army were exclusively drawn from the current elite, others in the elite would constitute the incumbent army that would defend the current institutions against the rebellion, as destruction of their institutions would lead to them becoming producers. This incumbent army would impose costs on those wanting to destroy the current

institutions. This cost however would be zero for the elite if they could all be guaranteed a place in the post-rebellion elite.

Creating incentives for investment thus requires power sharing: the elite is larger than what would otherwise prevail without the possibility of investment. But importantly, in the model, power is not shared with those who are actually investing. The extra members of the elite have no special function and have no access to any technology directly protecting property rights. Their role is to oppose changes to the status quo, with them fighting against rebellions from producers, and especially from other members of the elite. They do this merely because they would lose from rebellions that dislodge them from the ruling elite.

This analysis relies on two key assumptions. First, institutions survive and cannot be modified without a rebellion. This assumption is necessary for there to be any chance of elites establishing an environment where rules are followed. It captures the idea that elites can in principle create institutions that tie their hands when they would want to expropriate capital or deviate from commitments to provide public goods. Second, were a rebellion to occur, there can be no enforcement of deals made prior to the rebellion. This reflects the commitment problem highlighted by [Acemoglu \(2003\)](#). The underlying idea is that institutions are needed to enforce deals, but there are no “meta-institutions” to enforce deals concerning the choice of institutions themselves.

Because the destruction of institutions leads to unconstrained optimization over all dimensions of the new institutions, the current elite might struggle to agree to launch a rebellion, even if they are only participants in the rebellion. If it were possible for optimization following a rebellion to be restricted to certain areas, this paradoxically might make credible commitments impossible. For example, if the composition of the elite is defined by page one of the constitution, but limitations on expropriation are on page two, then being able to rebel against page two but at the same time committing not to touch page one annihilates the credibility of page two.

Finding the equilibrium institutions implies maximizing the elite payoff in equation [3.7] with respect to the threshold effort for investment  $\epsilon^*$  (noting that the elite size  $a$  and all taxes have been eliminated by substituting the binding constraints, as shown by [Proposition 2](#), and the fraction  $s$  of producers who choose to invest is given by [3.2]):

$$(\epsilon_H - \epsilon_L)\kappa s^2 + 2(q + 2\delta)(\epsilon_H - \epsilon_L)s + [\epsilon_L(q + 2\delta) - \delta\kappa] = 0.$$

There is a maximum of one economically meaningful root of this quadratic equation. When there is an interior solution, it is given by:

$$s^* = \sqrt{\left(\frac{q + 2\delta}{k}\right)^2 + \frac{1}{\epsilon_H - \epsilon_L} \left(\delta - (q + 2\delta) \frac{\epsilon_L}{k}\right)} - \frac{q + 2\delta}{k} \quad [3.8]$$



This is positive if and only if

$$\delta\kappa - \epsilon_L(q + 2\delta) > 0 \Rightarrow \frac{\kappa}{\epsilon_L} - 1 > 1 + \frac{q}{\delta}. \quad [3.9]$$

As discussed above, the no-rebellion constraint for producers with low effort costs is not binding. Thus, the distribution of income does not correspond exactly to the distribution of power.

This result shows that under some circumstances, the elite chooses to establish institutions with capital taxes that are low enough to incentivize some investment. But is the choice of capital tax efficient, that is, is it consistent with a political Coase Theorem?

### 3.3 The efficient choice of capital taxes

Since lump-sum taxes are available, if there were no need to consider the feasibility of different tax systems (in the sense of not triggering rebellions), a zero capital tax would be efficient. However, this benchmark is not informative because it assumes the institutions would be chosen by a benevolent social planner who does not face the threat of rebellion. Instead, the appropriate efficient benchmark is what level of capital taxes would be imposed by a benevolent planner knowing that all other institutional features would be chosen in the interests of an elite subject to the threat of rebellion. Since investment has been seen to require a larger elite, the opportunity cost of the larger elite must be considered by the planner.

Formally, the efficient benchmark is determined as follows. Taxes on capital  $\tau_\kappa$  are chosen by a benevolent agent, knowing all other institutional variables (the elite size and all other transfers) will be determined in equilibrium.

In the analysis of [section 2.4](#), having public-good provision  $g$  chosen by such a benevolent agent would not improve upon the equilibrium outcome when  $g$  is chosen by the elite. Thus, the political Coase Theorem holds in that case. Does this conclusion apply more broadly?

Taking the benevolent agent's choice of  $\tau_\kappa$  as given, the no-rebellion constraints determine  $a$  and  $\tau_q$ . The capital tax  $\tau_\kappa$  determines the fraction of investors  $s$ , which pins down the elite size,  $a$ , as any smaller size would lead to a rebellion at the post-investment stage. Lump-sum taxes are then chosen to make the no-rebellion constraint [\[3.4\]](#) bind.

Welfare (average utility) of all agents is:

$$W = aU_a + (1 - a)(q - \tau_q) + (1 - a)s \frac{\epsilon^* - \epsilon_L}{2},$$

and since  $U_a = q - \tau_q + \delta$  in equilibrium:

$$W = U_a - (1 - a)\delta + (1 - a)s \frac{\epsilon^* - \epsilon_L}{2}.$$

There are two differences between the expression for welfare  $W$  and the elite payoff  $U_a$ . The second term  $(1-a)\delta$  is related to the distribution of resources in the economy. The third term is the surplus of investors.

By substituting the equilibrium elite size from equation [3.7] in the expression for welfare:

$$W = \frac{\delta(q + \delta)}{q + 2\delta} \left( \frac{q + s(\kappa - \epsilon)}{\delta + s\epsilon^*} + \frac{s^2(\epsilon_H - \epsilon_L)}{2(\delta + s\epsilon^*)} \right).$$

The second term inside the brackets corresponds to the investors' surplus. To analyse the planner's choice of  $s$  it is helpful to consider the two terms in the expression above separately. Define  $W_0$  to be welfare net of investors' surplus:

$$W_0 \equiv \frac{\delta(q + \delta)}{q + 2\delta} \left( \frac{q + s(\kappa - \epsilon)}{\delta + s\epsilon^*} \right),$$

with investors' surplus as the difference between  $W$  and  $W_0$ :

$$W - W_0 = \frac{\delta(q + \delta)}{q + 2\delta} \left( \frac{s^2(\epsilon_H - \epsilon_L)}{2(\delta + s\epsilon^*)} \right).$$

Taking the derivative of this difference with respect to  $s$  (recalling that the threshold  $\epsilon^*$  is a function of  $s$ ) implies:

$$\frac{\partial W}{\partial s} > \frac{\partial W_0}{\partial s}.$$

The constrained welfare-maximizing choice of  $s$  leads to more investment than when the elite chooses  $s$  (through its choice of capital taxes) because the planner takes into account the investor's surplus. This constitutes one distortion that will lead to a breakdown of the political Coase Theorem.

The no-rebellion constraint for investors is slack. This is because individual effort  $\epsilon$  is not observable to the elite, so it is impossible for them to extract rents up to the point where the constraint would bind. Consequently the elite disregards those non-marginal rebels when determining institutions. Capital taxes will therefore be too high. The political Coase theorem does not hold here because of the non-observability of effort, which makes it impossible to write rules contingent on it.

This is not the only distortion, nor the most fundamental. Note that  $W_0$  is different from  $U_a$  in [3.7] in that there is an additional  $\delta$  term in the numerator of the latter. The value of  $s$  maximizing  $W_0$  (assuming an interior solution) is:

$$s_0 = \sqrt{\left(\frac{q + \delta}{\kappa}\right)^2 + \frac{\delta\kappa - \epsilon_L(q + \delta)}{\kappa(\epsilon_H - \epsilon_L)}} - \frac{q + \delta}{\kappa}.$$

This is positive if

$$\delta\kappa - \epsilon_L(q + \delta) > 0 \Rightarrow \frac{\kappa}{\epsilon_L} - 1 > \frac{q}{\delta},$$

which is a weaker condition than [3.9] in the case of the elite choosing  $s$ . As long as either  $s$  or  $s_0$  is strictly between 0 and 1, it follows that  $s < s_0$ .

This distortion is related to sharing power. Protection of property rights requires a larger elite, thus it requires sharing power. But sharing power requires sharing rents, because a rebellion that includes members of the elite involves less fighting effort. Thus even if protection leads to higher output, it also implies that a lower fraction of output going to a member of the elite. Hence, the elite chooses higher taxes on capital even though it leads to lower output, and hence the political Coase Theorem does not hold.

The cause of the breakdown of the political Coase Theorem in this case is the inability of the elite to make transfers to and from agents irrespective of the distribution of power. In particular, the elite cannot expand its size and claw back the rents received by the new members because this would trigger a rebellion including the new members.

Because sharing power requires sharing rents, public goods or services that have to be provided by people with power will be underprovided. Hence, the threat of expropriation from the government turns out to be a serious issue in the model, reflected in capital taxes that are too high from the society's point of view. That is in accordance with the empirical evidence in [Acemoglu and Johnson \(2005\)](#).

Last, a note on the effect of the power parameter,  $\delta$ . In [section 2.4](#), welfare of producers in an economy with institutions and a self-interested elite might be larger than the welfare of producers in an economy where everyone gets  $q$  and no transfers can be enforced. That is because the possibility of establishing rules allows for the provision of public goods. However, in that case,  $\delta \rightarrow 0$  is ideal. Intuitively, it is important to have power to collect and enforce taxes, but power in itself leads to larger elite size  $a$  and a worse income distribution.

However, this section shows that in an economy with a very small  $\delta$ , there cannot be investment in equilibrium. Any capital would be fully expropriated. If  $\delta$  is low enough, it is not even efficient to create institutions that avoid expropriation — it would require too many people in the elite. A larger  $\delta$  allows the elite to hold on to power more easily, which while being used for their own good, might also induce them to choose institutions protecting private property — something that would not be feasible with a lower  $\delta$ .

### 3.4 Analogies with historical examples

The results show the importance of sharing power to stimulate investment and prevent changes in the rules of the game that lead to expropriation. They also show that rulers will not share power as much as would be efficient. Although the model is too abstract to match historical

examples precisely, this idea squares with our reading of history.

Malmendier (2009) studies what he calls the earliest predecessor of modern business corporation, the Roman *societas publicanorum*. Their demise occurred with the transition from the Roman Republic to the Roman Empire. One possible reason for their demise was that “the Roman Republic was a system of checks and balances. But the emperors centralized power and could, in principle, bend law and enforcement in their favor”. In other words, while power was decentralized, it was possible to have rules that guarantee the property rights of the *societas publicanorum*, presumably because changing the rules would constitute an attack on the interests of some powerful people, which would be costly. But once power was centralized, protection against expropriation was not possible any longer.

Greif (2006) analyses in detail the building of a state in Genoa, in Medieval times. As in many cities in medieval Italy, Genoa employed a *podestà*, someone coming from another city that would govern Genoa for a year. Greif analyses the importance of the *podesteria* system for interclan cooperation. The *podestà* was generously paid, but it was worth having him there because it allowed for cooperation and investment. Interestingly, Greif argues that the *podestà* had to be sufficiently strong because otherwise if one clan defeated the other, it could easily defeat the *podestà* as well. Translating this into the language of the model, the strength of the *podestà* ( $a - \hat{a}$ ) had to be large enough to prevent a clan from expelling him and changing the rules.<sup>12</sup>

The Glorious Revolution in England established power sharing between the king and parliament. By accepting the Declaration of Rights and the Bill of Rights, King William accepted that power is shared between him and the parliament. A result of this was greater security of property rights and the elimination of confiscatory government. One issue in this case is that members of parliament were either directly interested or very related to those who were interested in the protection of their own wealth against the Crown. But North and Weingast (1989) show that after the Glorious Revolution, England could borrow much more, at substantially lower rates. That was certainly in the interest of the king, but was not possible without sharing power between king and parliament, as otherwise it would be too easy for the king to rewrite the rules.

Although this change is advantageous to the king in terms of financing government debt, many kings strongly resisted earlier attempts by parliament to force them to share power. Presumably the existence of a powerful parliament implied that rents would have to be shared between king and the parliament, so even if the pie was larger, the share of the king could have been smaller. This is related to the result of the model that the elite will not share power as much as is needed to achieve the efficient level of investment.

Broadly speaking, the extra people in the elite ( $a - \hat{a}$ ) might perhaps be interpreted as a

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<sup>12</sup>Greif (2006), pp. 239–240.

“parliament” or any other group of people with power to deter changes in government — from outsiders and, especially, from insiders. Parliaments usually are thought of as representing the people that elected their members, but then so are democratically elected presidents, and parliaments are not only useful to defend minorities. Power sharing makes institutions more stable because it makes costly for some members in power to alter the institutions that establish how power is distributed and limit how much can be taken from producers.

## 4 Conclusions

Governments and institutions play a key role in development by making it possible to direct resources to the provision of public goods and by enforcing collective choices. However, what is called a government in some of the poorest countries bears little resemblance to its counterpart in the most developed nations. As noted by the bible passage in the book of Samuel, those with power to choose the “rules of the game”, guided by their own interests, might choose policies that are not necessarily good from the point of view of society. Indeed, in far too many cases, the warnings of Samuel remain as relevant as ever.<sup>13</sup>

This paper provides a model where institutions are set to maximise the payoffs of the group in power. That assumption and the threat of rebellions are the main elements of the model. When private investment is considered, the risk of expropriation emerges. In order to protect investment, sharing power is needed. That is the only way to prevent those in power to tear up the old rules and expropriate capital. However, sharing power requires sharing rents, because those with power can use it against the other members of the ruling group. Hence, there is not enough sharing power and too high capital taxes — too little protection of investment.

This framework could be used in a number of ways. First, it can be explored to understand which factors allow for more or less power sharing — and thus investment. It would also be interesting to analyse other sources of inefficiencies. Also, agents are homogeneous ex ante, with heterogeneity as an equilibrium outcome, but ex-ante heterogeneity of various forms could be incorporated into the model (for example, ethnic differences, as in [Caselli and Coleman \(2006\)](#)). There are no rebellions in equilibrium, but they could arise if the model had some stochastic elements.

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<sup>13</sup>For a model on the trade-off that having a government (or a “king”) entails, see [Grossman \(2002\)](#).

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## A Technical appendix

### A.1 Proof of Proposition 1

Start by fixing a particular choice of institutions, that is, a choice of army size  $a$  and the distribution of consumption across agents. These choices also determine the composition of  $\mathcal{A}$  and  $\mathcal{P}$ . This particular choice of institutions does not lead to any rebellion if and only if

$$\max_{\mathcal{R}} \left\{ \int_{\mathcal{R} \cap \mathcal{P}} (U'_a - U_p(i)) di + \int_{\mathcal{R} \cap \mathcal{A}} (U'_a - U_a(i)) di + \delta d \right\} \leq \delta a \quad \text{s.t.} \quad \mathbb{P}(\mathcal{R}) = a',$$

where the size of the subsequent incumbent army  $a'$ , as well as  $U'_a$ , is taken as given. For a given  $\mathcal{R}$ ,  $d$  is the measure of the set  $\mathcal{R} \cap \mathcal{A}$ . Now define disjoint sets  $\mathcal{R}_p \equiv \mathcal{R} \cap \mathcal{P}$  and  $\mathcal{R}_a \equiv \mathcal{R} \cap \mathcal{A}$ , and note that the maximization problem above is equivalent to

$$\max_{\mathcal{R}_p, \mathcal{R}_a} \left\{ (a' - d)U'_a - \int_{\mathcal{R}_p} U_p(i) di + d(U'_a + \delta) - \int_{\mathcal{R}_a} U_a(i) di \right\} \leq \delta a \quad \text{s.t.} \quad \mathbb{P}(\mathcal{R}_p \cup \mathcal{R}_a) = a'. \quad [\text{A.1.1}]$$



Now make the following definitions:

$$\bar{U}_p \equiv \frac{1}{1-a} \int_{\mathcal{P}} U_p(\iota) d\iota, \quad \bar{U}_a \equiv \frac{1}{a} \int_{\mathcal{A}} U_a(\iota) d\iota, \quad \text{and} \quad [\text{A.1.2a}]$$

$$\underline{U}_p(r_p) = \min_{\mathcal{R}_p \text{ s.t. } \mathbb{P}(\mathcal{R}_p)=r_p} \left\{ \frac{1}{r_p} \int_{\mathcal{R}_p} U_p(\iota) d\iota \right\}, \quad \underline{U}_a(r_a) = \min_{\mathcal{R}_a \text{ s.t. } \mathbb{P}(\mathcal{R}_a)=r_a} \left\{ \frac{1}{r_a} \int_{\mathcal{R}_a} U_a(\iota) d\iota \right\}. \quad [\text{A.1.2b}]$$

These definitions imply

$$\underline{U}_p(r_p) \leq \bar{U}_p, \quad [\text{A.1.3a}]$$

with equality if and only if (i)  $C_p(\iota) = C_p$  for all  $\iota$  (with measure one), or (ii)  $u''(\cdot) = 0$  and  $r_p = 1 - a$ . Analogously,

$$\underline{U}_a(r_a) \leq \bar{U}_a, \quad [\text{A.1.3b}]$$

with equality if and only if (i)  $C_a(\iota) = C_a$  for all  $\iota$  (with measure one), or (ii)  $u''(\cdot) = 0$  and  $r_a = a$ .

The definitions in [A.1.2a] imply that the constraint [A.1.1] is equivalent to

$$\max_{d \in \mathcal{D}(a, a')} \left\{ (a' - d)U'_a - (a' - d)\underline{U}_p(a' - d) + d(U'_a + \delta) - d\underline{U}_a(d) \right\} \leq \delta a, \quad [\text{A.1.4}]$$

where  $\mathcal{D}(a, a') \equiv [\max\{0, a + a' - 1\}, \min\{a, a'\}]$ . Rearranging terms yields

$$a'(U'_a - \bar{U}_p) + \max_{d \in \mathcal{D}(a, a')} \left\{ d(\bar{U}_p - \bar{U}_a + \delta) + (a' - d)(\bar{U}_p - \underline{U}_p(a' - d)) + d(\bar{U}_a - \underline{U}_a(d)) \right\} \leq \delta a, \quad [\text{A.1.5}]$$

making use of the definitions in [A.1.2a] again.

Now consider a Markovian equilibrium with  $a = a'$  and  $U'_a = \bar{U}_a$ . Let

$$D \equiv \max_{d \in [\max\{0, 2a-1\}, a]} \left\{ d(\bar{U}_p - \bar{U}_a + \delta) + (a - d)(\bar{U}_p - \underline{U}_p(a - d)) + d(\bar{U}_a - \underline{U}_a(d)) \right\}, \quad [\text{A.1.6}]$$

where  $d^*$  is the smallest value (without loss of generality) of  $d$  that maximizes the above expression. The single constraint [A.1.4] to which the equilibrium institutions are subject must be binding. Thus, by substituting the solution of the problem in [A.1.6] into [A.1.4], and after some rearrangement:

$$a\bar{U}_a - d^*\underline{U}_a(d^*) = (a - d^*)(\delta + \underline{U}_p(a - d^*)). \quad [\text{A.1.7}]$$

First, note that  $d^* < a$ , otherwise higher taxes on producers would be feasible. Substituting the inequalities in [A.1.3a] and [A.1.3b] into [A.1.7] yields

$$\bar{U}_p - \bar{U}_a + \delta \geq 0,$$

with equality if payoffs are equalized among army members and among producers. Thus, if payoffs are equalized, then the value of  $D$  in [A.1.6] is necessarily zero.

Now suppose that payoffs among the producers are not equalized. Then  $(\bar{U}_p - \underline{U}_p(a - d))$  is strictly positive for  $d > 2a - 1$  since this ensures  $a - d < 1 - a$ , the total measure of producers. Moreover, for  $d < a$ , the coefficient multiplying this term in [A.1.6] is positive as well. As the other terms in [A.1.6] would be non-negative, and since such a choice of  $d$  is feasible (as  $a < 1$ ), the value of  $D$  would be positive. As has been seen,  $D = 0$  is possible if institutions equalize payoffs within groups. This can be achieved without affecting the other terms in the constraint [A.1.5], and without decreasing the objective function. Hence, in a Markovian equilibrium, payoffs of producers must be equalized.

Analogously, if payoffs among army members are not equalized, then  $(\bar{U}_a - \underline{U}_a(d))$  is strictly

positive for  $d < a$ . For  $d > 0$ , this term has a positive coefficient in [A.1.6]. Such a choice is feasible ( $a < 1$ ), and the other terms appearing in [A.1.6] would be non-negative. Again,  $D$  would hence be positive. Since  $D = 0$  is feasible, a similar argument to that above shows that payoffs of army members must be equalized.

## A.2 Proof of Proposition 2

0. If  $s > 0$ ,  $a^* > \hat{a}$ .

If  $s > 0$ , consumption at the second stage is not the same for all producers: those who invested have to get more than the others (to compensate for effort). So, the no-rebellion constraint of those producers who invested cannot be binding. Now, that implies that the army would benefit from expropriating their capital, so that their consumption would be exactly the same as the consumption of the other producers. But if  $a^* \leq \hat{a}$ , a rebellion in the second stage is costless for the members of the incumbent army: they can expropriate capital from those who invested at no cost. But that cannot happen in equilibrium.

1. If  $s > 0$ , [3.3] and [3.5] cannot both bind.

Suppose they do, then imposing equilibrium, they imply:

$$q - \tau_q + s \frac{\epsilon^* - \epsilon_L}{2} = U_a - \delta$$

$$U_a = \frac{(q + \delta)^2}{q + 2\delta} + (1 - a)s\kappa - \delta \frac{a}{\hat{a}} + \delta$$

Plugging the value of  $U_a$  in the first one,

$$q - \tau_q + s \frac{\epsilon^* - \epsilon_L}{2} = \frac{(q + \delta)^2}{q + 2\delta} + (1 - a)s\kappa - \delta \frac{a}{\hat{a}}$$

which contradicts [3.4].

2. [3.5] cannot be the only one that binds.

Trivial.

3. [3.3] cannot be the only one that binds.

Suppose is the only one that binds. Getting  $\tau_q$  from equation [3.3] and substituting it at the objective function, we get:

$$U_a = \frac{1 - a}{a} \left( q - U'_a + \delta \frac{a}{a'} + s \frac{2\kappa - \epsilon^* - \epsilon_L}{2} \right)$$

The first order condition with respect to  $a$  yields

$$a^* = \frac{\delta}{q + 2\delta + s \frac{2\kappa - \epsilon^* - \epsilon_L}{2}} < \hat{a}$$

But  $a^* < \hat{a}$  is a contradiction.

4. So [3.4] binds. Using the constraint to get an expression for  $\tau$  and substituting it in the objective function, we obtain

$$U_a = \frac{1 - a}{a} \left( as\kappa + \delta \frac{a}{\hat{a}} - s\epsilon^* - \frac{\delta^2}{q + 2\delta} \right) \quad [\text{A.2.1}]$$

5. Substituting the expression for  $U_a$  from [A.2.1] into [3.5] and doing algebra, we get:

$$a^* \geq \frac{\delta \hat{a} + s\epsilon^*}{\delta + s\epsilon^*} \quad [\text{A.2.2}]$$

6. Now need to show that [3.5] is binding. To show that, (a) we show a contradiction when assuming that only [3.4] is binding and (b) we show a contradiction when assuming that [3.3] and [3.4] are binding.

6a. If [3.4] is the only constraint binding, the solution is given by the unconstrained maximization of [A.2.1]. Taking the first order condition with respect to  $a$  and rearranging yields

$$a^* = \frac{\delta \hat{a} + s\epsilon^*}{a^*(s\kappa + q + 2\delta)}$$

Condition [A.2.2] implies

$$\frac{\delta \hat{a} + s\epsilon^*}{a^*(s\kappa + q + 2\delta)} \geq \frac{\delta \hat{a} + s\epsilon^*}{\delta + s\epsilon^*}$$

Thus

$$a^*(s\kappa + q + 2\delta) \leq \delta + s\epsilon^* \Rightarrow a^* \leq \frac{\delta + s\epsilon^*}{s\kappa + q + 2\delta}$$

Using again condition [A.2.2],

$$\frac{\delta + s\epsilon^*}{s\kappa + q + 2\delta} \geq \frac{\delta \hat{a} + s\epsilon^*}{\delta + s\epsilon^*}$$

Rearranging shows a contradiction.

6b. The Lagrangian of the problem of maximizing  $U_a$  subject to [3.3] and [3.4] is:

$$\mathcal{L} = \frac{1-a}{a} (\tau_q + s(\kappa - \epsilon^*)) + \aleph_1 \left( q - \tau_q + s \frac{\epsilon^* - \epsilon_L}{2} - U'_a + \delta \frac{a}{a'} \right) + \aleph_2 \left( -\tau_q - \frac{\delta^2}{q + 2\delta} - (1-a)s\kappa + \delta \frac{a}{\hat{a}} \right)$$

The first order condition with respect to  $\tau_q$  implies

$$\aleph_2 = \frac{1-a}{a} - \aleph_1$$

The first order condition with respect to  $a$  implies:

$$-\frac{1}{a^2} (\tau_q + s(\kappa - \epsilon^*)) + \aleph_1 \frac{\delta}{a'} + \aleph_2 \left( s\kappa + \frac{\delta}{\hat{a}} \right) = 0$$

Substituting the value of  $\tau_q$  implied by the binding constraint [3.4] and the value of  $\aleph_2$  from the first order with respect to  $\tau_q$  and rearranging yields

$$\frac{1}{a^2} (\delta \hat{a} + s\epsilon^*) = s\kappa + q + 2\delta + \aleph_1 \left( s\kappa + \frac{\delta}{\hat{a}} - \frac{\delta}{a} \right)$$

As in equilibrium  $a \geq \hat{a}$  and  $\aleph_1 \geq 0$ , the last term is positive. Hence

$$\frac{1}{a^2} (\delta \hat{a} + s\epsilon^*) > s\kappa + q + 2\delta \Rightarrow \frac{\delta \hat{a} + s\epsilon^*}{a^*(s\kappa + q + 2\delta)} > a^*$$

Now using the same steps as 6a, we get the proof.

As [A.2.2] is a translation of [3.5] using the objective function and the binding constraint [3.4], it is binding and yields the equilibrium value of  $a^*$ .

Rearranging [A.2.1] yields

$$U_a = (1 - a)(s\kappa + q + 2\delta) - \frac{1 - a}{a} \left( \frac{\delta^2}{q + 2\delta} + s\epsilon^* \right),$$

and substituting the value of  $a$  implies equation [3.7].