Centre for Economic Policy Research

BANCO DE ESPAÑA
Eurosistema

EUROPEAN SUMMER SYMPOSIUM IN INTERNATIONAL MACROECONOMICS (ESSIM) 2010

Hosted by
Banco de España

Tarragona, Spain; 25-28 May 2010

How Inefficient is Worker Reallocation?

Alejandro Justiniano, David López-Salido and Claudio Michelacci

How Inefficient is Worker Reallocation?*

Alejandro Justiniano
Federal Reserve Bank of Chicago

David Lopez-Salido
Federal Reserve Board and CEPR

Claudio Michelacci
CEMFI and CEPR

May 2010

Very preliminary and Incomplete

Abstract

We estimate a real-business-cycle-search model that features three welfare ine¢ ciencies: (i) a failure of the Hosios' condition, (ii) appropriability problems in the job creation costs, and (iii) real wage rigidity. We evaluate the contribution of these ine¢ ciencies in replicating the cyclical volatility of vacancies, unemployment, hours worked, output, labor productivity, wages, consumption, and investment. We evaluate the ine¢ ciencies in the process of reallocation of workers by comparing, in terms of welfare, the equilibrium allocations of the estimated model to the social planner ones. While ine¢ ciencies are small in steady state, they are potentially large at business cycle frequencies. This provides a new dimension to evaluate the costs of business cycles.

JEL classi…cation: E00, J60, O33.
Key words: Search frictions, technological progress, creative destruction.

*We would like to thank Robert Hall for useful suggestions.

1 Introduction

At least since Hosios (1990), several papers have argued that the combination of ex-post wage bargaining with labor market search frictions generates inefficiencies in the process of reallocation of workers across productive units and there is much debate on the role played by these inefficiencies in replicating the observed cyclical fluctuations of unemployment. There is also much debate on whether standard labor market models can replicate the cyclical volatility of labor market variables and on the welfare costs of the resulting fluctuations. Shimer (2005) has argued that the standard search model (as described in Pissarides (2000)) can not replicate the high degree of volatility of key labor market variables observed in the data. Hagedorn and Manovskii (2008) have argued that a specification of the model where unemployment has almost no cost can replicate several features of the data. This conclusion has been challenged by Shimer (2004), Hall (2005), Hall and Milgrom (2008), and more recently by Gertler and Trigari (2009) who have instead argued that modelling wage rigidities is needed to account for the cyclical volatility of vacancies and unemployment. Caballero Hammour (1994, 1996, 1998) and more recently Pissarides (2009) and Silva and Toledo (2009) have emphasized the role of adjustment costs. These three mechanisms (low cost of unemployment, wage rigidity and adjustment costs) could have different implications for the cyclical properties of wages, aggregate hours worked, consumption, investment, labor productivity and output. They also imply different welfare costs of the process of worker reallocation that arises over business cycle and a different role for stabilization policies. The relative merit of these different mechanisms and their welfare consequences have never been analyzed jointly in a fully specified business cycle model.

We embed these mechanisms in a conventional business cycle model that we extend to incorporate many ingredients regarded as important in the business cycle literature. We consider a model with several possible shocks that different strands of the literature have regraded as important for the analysis of business cycle fluctuations.[1]

[1] Technology shocks are typically regarded as an important source of business cycle fluctuations as first emphasized in the pioneering contributions by Kydland and Prescott (1982) and Prescott (1986). Galí (1999) raise doubts about their relative importance. Greenwood, Hercowitz, and Krusell (2000), Fisher (2006) and Michelacci and Lopez Salido (2007) and Justiniano, Primiceri, and Tambalotti (2008) emphasize the role of investment specific technology shocks. There is also substantial debate of the importance of aggregate relative to reallocative shocks in explaining business cycle fluctuations. This debate dates back at least to the original contribution of Lilien (1982). More recently Davis and Haltiwanger (1999) have argued that shocks leading to an increase in job reallocation are important in explaining business cycles. Also preference shocks as in Smets and Wouters (2007) or shocks to

We allow for non linear preferences and for adjustment costs to capital. Under some model specifications the decentralized equilibrium is efficient, but under others there are three possible sources of inefficiencies: 1) failure of the Hosios (1990) condition; 2) appropriability problems in creation costs as in Caballero and Hammour (1996, 1998) and Caballero (2007); and 3) wage rigidity as in Shimer (2004) and Hall (2005). To model the importance of wage rigidity we assume that the wage setting mechanism is such that in a fraction of jobs wages are rigid as in Hall (2005), while in the remaining fraction wages are set through Nash bargaining as in the conventional formulation of the search model described in Pissarides (2000).

In the model we allow for two key features that Hall (2009a) argue to be important to fit the cyclical co-movements in consumption, employment, labor productivity, and weekly hours. In particular we consider non separable preferences in consumption and hours worked and we allow for the possibility that consumptions and hours worked are complements as in Hall and Milgrom (2008). Our modelling specification borrows from Shimer (2010) and Trabandt and Uhlig (2009). Intuitively, Hall (2009a) justifies the possibility of consumption-hours complementarity by arguing that "unemployed workers, having more time to produce at home, choose lower levels of consumption of market goods than they would if employed, at the same level of well-being." Shimer (2009) argues that consumption-hours complementarity is consistent with Beckerian model of time allocation (see Becker 1965) We test for the relevance of this assumption. Hall (2009a) also claims that to explain the data we need to get rid of the assumption of Nash bargaining wages. This is controversial given the conclusions by Hagedorn and Manovskii (2008) and Pissarides (2009) that the standard search model can replicate the observed volatility of key labor market variables, even in the absence of wage rigidity. As previously discussed we consider a wage setting formulation that nests wage rigidity and Nash bargaining as special cases. The data are used to test for the importance of wage rigidities. We estimate the model and we evaluate the contribution of different transmission mechanisms in replicating the volatility of key cyclical variables and their implications for the cyclical properties of aggregate hours worked, output, labor productivity, wages, consumption, and investment. Then we evaluate the inefficiencies that arise in the cyclical process of worker reallocation and we address the following questions:

1. How much wage rigidity? Our estimates imply that data are consistent with a

discount factor as in Primiceri, Schaumburg, and Tambalotti (2006) have recently been emphasized as an important source of cyclical fluctuations.

model speci…cation where approximately half of the jobs have rigid wages close to marginal productivity as in Hall (2005) and half of the jobs are set through Nash bargaining with workers having little (almost no) bargaining power. This is implies that steady unemployment is almost identical to the one that would arise in a model speci…cation where all wages are set through Nash bargaining and workers and …rm have identical bargaining power. But over the business cycle, wage rigidity is pervasive. Since wages ‡uctuate little relative to labour productivity wage rigidity is an important mechanism to explain why the labor share is countercyclical.

2. What are the source of business cycle ‡uctuations? We …nd that just two shocks explain around 80 per cent of the cyclical ‡uctuations of unemployment, vacancies, the job …nding rate and the job separation rates. Stationary shocks to the neutral technology are the most important driving force of cyclical ‡uctuations: they explain around 60 per cent of the cyclical ‡uctuation of the labor market and they operate pretty much as in a standard real business cycle model proposed by Prescott (1986). The second most important shocks are exogenous shocks to the separation rate, that explain around twenty percent of the cyclical ‡uctuations of vacancies and unemployment. Overall these two shocks alone account well for the magnitude of the cyclical ‡uctuations of the labor market.

3. How costly is involuntary unemployment? In the literature there is substantial controversy on the costs of unemployment. In Hagedorn and Manovskii (2006) the cost of unemployment is basically nil since workers are almost indi¤erent between being employed or unemployed, in Shimer (2005) and in Hall and Milgrom (2008) the cost is substantial. According yo our estimates, this costs is sizeable. Overall workers's welfare increase on average by 7 percent upon becoming employed.

4. What is the importance of the intensive margin relative to the extensive margin in explaining aggregate labor market ‡uctuations? Do their contribution di¤er in di¤erent business cycle episodes? Aggregate hours worked can ‡uctuate due to either ‡uctuations in hours per employee (the intensive margin of labor market adjustment) or in the number of employed workers (the extensive margin).

are pretty irrelevant for business cycle analysis. As in Hall (2009a) we also find that modeling the intensive margin is important to replicate the business cycle volatility of key labor market variables.

5. What are the welfare costs of search inefficiencies? There is a relevant literature that argues that the combination of labor market search frictions with ex post bargaining cause inefficiencies in the process of reallocation of workers across productive unit, see for example Hosios (1990) and Caballero and Hammour (1996, 1998). But are these search inefficiencies on average large or small? [2] We find that inefficiencies due to search, wage rigidity and adjustment costs are pretty irrelevant for the determination of steady state unemployment, but they imply important costs over the business cycle. We also study whether search inefficiencies have been particularly harmful in some business episode or in response to some specific shocks.

Estimating a general version of the standard search model as described in Pissarides (2000) represents a formidable task, see Yashiv (2002) for an estimation of the model using Israeli data. Yashiv (2002) does not however analyze the cost induced by search inefficiencies and the source of aggregate fluctuations. We later further discuss the relation of our contribution to his.

Shimer (2005) argues that the search model can not account for key labor market variables. Hagedorn and Manovskii (2008) say that it can. The two calibrations differ in two key elements (value of bargaining power and income during unemployment). Both papers assume that there is just one productivity shock that affects labor productivity and under this strategy it is not entirely obvious how one can discriminate between the two calibrations. We have a model where we consider several shocks and we analyze the effects on several additional variables such the labor share, consumption, investment and labor productivity. This provides other margins to evaluate the relative merits of the two calibrations. By considering shocks that mainly affects the dynamics of the labor market through the outside options of workers, we also indirectly address the criticism of the Hagedorn and Manovskii specification put forward by Costain and Reiter (2008) who argue that shocks that affect the outside options of workers have counterfactually too large effects under the Hagedorn and Manovskii's parametrization.

[2] There is substantial debate on this. For example the directed search literature tends to argue that search inefficiencies should vanish in equilibrium, see for example Acemoglu and Shimer (1999). Michelacci and Suarez (2006) show instead that directed search is not enough to eliminate search inefficiencies in the presence of adverse selection problems.

Section 2 characterizes the economy while Section 3 presents the equilibrium conditions of the decentralized economy. Section 4 deals with the social planner problem and it discusses the relation between the social planner allocation and the decentralized allocation. It analyzes the welfare properties of the economy under di¤erent speci…cations for the job creation costs and the wage determination process. Section 5 discusses the relation with Hall (2009a) two factors model. Section 6 discusses the data used to estimate the model and some identi…cation issues. Appendix A derives some properties of preferences. Appendix B solves the social planner problem while Appendix C discuss how we solve for the allocation chosen by the social planner. Appendix E contains computational details about the model estimation.

2 Description of the decentralized equilibrium

We now describe the assumptions that characterize the decentralized equilibrium.

Job output and technologies There is one consumption good, the numeraire. Output is produced according to

$$Y = F_t(K, N) = A_t Z_t K^{\alpha} N^{1-\alpha}, \tag{1}$$

where K is the capital stock, N the amount of labor intensive intermediate goods used in production, and $0 < \alpha < 1$. A_t is the standard source of temporary cyclical ‡uctuations considered for example in general equilibrium versions of the standard search model.[3] We assume that

$$a_t \equiv \ln A_t = \rho_a a_{t-1} + \varepsilon_{at}. \tag{2}$$

We also introducee Z_t such that $z \equiv \ln Z$ evolves as:

$$z_t = \mu_z + z_{t-1} + \varepsilon_{zt}, \tag{3}$$

where $\mu_z > 0$ and ε_{zt} is iid normal with standard deviation σ_z. This exogenous process captures one of the two sources of long run productivity growth.

Labor intensive intermediate goods are produced in jobs which consist of …rm-worker pairs. A worker can be employed in at most one job. A job produces an amount of intermediate goods equal to h^{ϕ}, where h denotes the amount of hours worked by the worker in the job, and $\phi \in [0, 1]$.

[3] See, for example, Andolfatto (1996), Merz (1995), and Den Haan, Ramey, and Watson (2000).

Capital Accumulation Households accumulate capital and rent it out to the …rms. Following Christiano, Eichenbaum and Evans (2005), we allow for the presence of investment adjustment cost. Thus, the law of accumulation of the capital in the hand of the representative household (see below) can be described as follows:

$$K_{t+1} = (1-\delta)K_t + e^{q_t+\varphi_t}\left[1 - T\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{4}$$

where the function T satis…es that $T = T' = 0$ in deterministic steady state and $T'' > 0$. Out of the deterministic steady state, T, T' and T'' are all strictly positive. In the above expression I_t is the amount of investment expenditures measured in …nal output. The variables $q_t + \varphi_t$ represent the sum of the two components that jointly characterize the investment speci…c technology.[4] The q_t component thus corresponds to the other source of long-run productivity growth and it evolves according to:

$$q_t = \mu_q + q_{t-1} + \varepsilon_{qt}, \tag{5}$$

where $\mu_q > 0$ and ε_{qt} is iid normal with standard deviation σ_q. The second component, φ_{t-1}, is a temporary shock that evolves according to the following AR(1) process:

$$\varphi_t = \rho_\varphi \varphi_{t-1} + \varepsilon_{\varphi t}. \tag{6}$$

As in Solow (1960) and Greenwood, Hercowitz, and Krusell (1997) the stochastic trend in the economy, X_t, will be given by:

$$X_t \equiv e^{\frac{z_t}{1-\alpha}}\ e^{\frac{\alpha q_t}{1-\alpha}}. \tag{7}$$

Search frictions The labor market for workers is subject to search frictions. The matching process within a period takes place at the same time as production for that period. Workers and …rms whose matches are severed can enter their respective matching pools and be re-matched within the same period. All separated workers are assumed to reenter the unemployment pool (i.e. we abstract from workers' labor force participation decisions). Workers and …rms that are matched in period t begin active relationships at the start of period $t+1$, while unmatched workers remain in the unemployment pool. Following Pissarides (2000), we model the ‡ow of viable matches using a matching function

$$n_t(u, v) = (M_t u)^\eta\, v^{1-\eta}, \tag{8}$$

[4] As in Solow (1960) and Greenwood, Hercowitz, and Krusell (1997)

whose arguments denote the current masses of unemployed workers (u) and vacancies (v), respectively. This function is homogeneous of degree one, increasing in each of its arguments, concave, and continuously differentiable.[5] Under these assumptions the probability that a vacancy gets filled is given by:

$$q\left(f\right) = \frac{n_t(u,v)}{v} = \left(\frac{n}{u}\right)^{-\frac{\eta}{1-\eta}} = f^{\,-\frac{\eta}{1-\eta}},$$

which is decreasing in the rate at which an unemployed worker finds a job given by $f = n/u$. We assume that

$$m_t \equiv \ln(M_t) = (1-\rho_m)m + \rho_m m_{t-1} + \varepsilon_{mt}, \tag{9}$$

which characterizes a shock to the matching technology, i.e. a skill mismatch shock. These shocks do not affect the productivity of a job, but have a direct effect on the outside options of workers.[6]

Job creation Free entry by firms determines the size of the vacancy pools. Processing the applications for a vacancy requires some recruiting services that are exchanged in a perfectly competitive market. Recruiting services are produced by combining labour intensive intermediate goods and some other output services whose unitary cost of production is X_t units of final output. The amount of recruiting services obtained by using l units of labour intensive intermediate goods and o units of output services is given by the Cobb-Douglas production function $\left(\frac{l}{\kappa}\right)^{\kappa}\left(\frac{o}{1-\kappa}\right)^{1-\kappa}$. Cost minimization implies that the unitary cost of producing a recruiting service is

$$r_t = p_t^{\kappa} X_t^{1-\kappa}, \tag{10}$$

where $0 < \kappa < 1$, and p_t denotes the equilibrium price of a labor intensive intermediate good. This is achieved by using

$$l_t = \kappa \left(\frac{X_t}{p_t}\right)^{1-\kappa} \tag{11}$$

units of labour intensive intermediate goods and by spending

$$o_t X_t = (1-\kappa)\, r_t \tag{12}$$

[5] We will check that, over the relevant range, it always satisfies $n_t(u,v) \le \min(u,v)$.

[6] These shocks can induce positive comovements in the finding and separation rate. Shocks with this property are usually dubbed reallocative, see Blanchard and Diamond (1989, 1990), Davis and Haltiwanger (1999) and Balakrishnan and Michelacci (2001) for evidence about the relevance of these shocks. Cheremukhin and Echavarria (2009) also argue that these shocks are important to explain the cyclical volatility of labor market variables and of the labor wedge.

units of final output.

The expression for the cost of recruiting services in (10) allows for differences in the factor content of recruiting costs which Shimer (2009) has shown to matter for the response of the economy to shocks. When recruiting services are obtained by just using labor –which in our formulation corresponds to the case $\kappa = 1$– Shimer (2009) derives a neutrality proposition whereby shocks to labor productivity have no effects on unemployment.[7] In standard formulations of the search model with technology growth (see for example Aghion and Howitt (1994), Mortensen and Pissarides (1998), Hornstein, Krusell, and Violante (2005, 2007), and Pissarides and Vallanti (2007)) the existence of a steady state is typically guaranteed by assuming that recruiting costs exogenously increase with the aggregate trend in the economy which, in our formulation, would correspond to the case $\kappa = 0$.

The amount of recruiting services required for training n workers and processing applications for v vacancies is given by:

$$\bar{R}(n, v) = \bar{r} n^{\gamma_1} v^{\gamma_2}, \ \gamma_1 \geq 0, \gamma_2 > 0, \tag{13}$$

and $\gamma_1 + \gamma_2 = 1$ to guarantee that the sector producing recruiting services is perfectly competitive. Notice that after using (8) to substitute for $v = (M_t u)^{-\frac{\eta}{1-\eta}} n^{\frac{1}{1-\eta}}$, the amount of recruiting services can be written as

$$R_t(u, n) = \bar{r} (M_t u)^{-\eta_0} n^{\eta_1}, \tag{14}$$

where $\eta_0 = \frac{\eta \gamma_2}{1-\eta}$ and $\eta_1 = \gamma_1 + \frac{\gamma_2}{1-\eta}$, which highlights the role of unemployment in reducing creation costs. We will see below that, as first stressed by Caballero and Hammour (1996), the parameter γ_1 plays an important role in determining the costs of search inefficiencies. This parameter measures the gains from smoothing job creation in response to shocks and the incentive to synchronize the level of unemployment and vacancies over the business cycle.[8] In the standard formulation of the search model (see for example Pissarides, 2000) creation costs are linear in vacancies which corresponds to the case $\gamma_1 = 0$ and $\gamma_2 = 1$. Pissarides (2009) and Rotemberg (2006) emphasize the case where the average cost of posting a vacancy falls in the number of posted vacancies which corresponds to the case $\gamma_2 < 1$.

[7] As will be discussed below, in addition to $\kappa = 1$, the analysis in Shimer (2010) regarding the neutrality proposition would require that capital is absent $\alpha = 0$ and that $\gamma_1 = 0$ and $\gamma_2 = 1$ so that $\bar{R}(n, v) = \bar{r} v$.

[8] Pissarides (2009) and Silva and Toledo (2009b) have also emphasized the role of creation cost in matching the cyclical volatility of key cyclical variables.

Wages If a firm and a worker who have met separated, both would loose the opportunity of producing and each would have to go through a time-consuming process of search before meeting a new suitable partner. Hence, there is a surplus from a job.

We allow for different ways of splitting such surplus. The wage setting mechanism is determined at the time when the match is formed and it will be in effect until the match is destroyed. We assume that with probability $1-\theta$ the wage determination process in a job is governed by Nash bargaining (Pissarides (2000)). In this case, at each point in time, the worker and the firm split the net surplus of a job by using a generalized Nash bargaining solution in which the bargaining powers of the worker and the firm are β and $1-\beta$, respectively. Division of the surplus is accomplished via wage payments. With probability θ the job is instead characterized by rigid wages as in Shimer (2004) and Hall (2005). If these wages are inside the bargaining set, then the firm pays to the worker $\boldsymbol{\omega} X_t$.[9] If the wage is outside the bargaining set the outside option of either the worker or the firm binds.

To address Barro's criticism that two parties in direct contact with one another can always arrange the terms of their relationship so to achieve private bilateral efficiency, we follow Hall (2009a) in assuming that the worker at the start of the relationship buys out the firm by paying to the firm the full value of the job. This guarantees that the decision on hours in the job are privately efficient and equal to those that would arise as under Nash bargaining. We also assume that the rigid wage, $\boldsymbol{\omega}$, is randomly draw from a given distribution G.[10] To simplify notation we will also assume that $\boldsymbol{\omega}$ is iid over time, but this will be without loss of generality given the assumption that the worker buys out the firm at the start of the employment relationship.

In the presence of a (non stochastic) unique rigid wage, the wage would jump discontinuously in response to a shock that makes the outside option of one party binding. This makes solution methods based on linearizing the equilibrium conditions of the model inappropriate. A possible solution would be to assume—as for example Hall (2005), Hall and Milgrom (2008) and Gertler and Trigari (2009) do—that wages are always strictly within the bargaining set so that rigid wages never jump discontinuously. Although this assumption is reasonable, this approach is unfeasible when it comes to estimating the model since we also have to evaluate the relative likelihood of parametrizations where the outside options of workers or firms could become binding with non negligible probability. By allowing for stochastic wages, we instead have that

[9] Notice that X_t is the stochastic trend of the economy defined in (7)

[10] This technical assumption is convenient for estimating the model using linear methods.

outside options always bind for at least some workers and some …rms and shocks a¤ect the fraction of workers and …rms for which outside options are binding. Since fractions move continuously in response to shocks, solution methods based on linearization are appropriate. This permits estimating the model with standard methods.

Representative household The economy is populated by a continuum of identical in…nitely-lived households of measure one. Each household is thought of as a large extended family which contains a continuum of workers. For simplicity, the population of workers in the economy is assumed to be constant and normalized to one. We follow, among others, Andolfatto (1996) and den Haan et al. (2000) in assuming that workers pool their income at the end of the period and choose consumption and e¤ort costs to maximize the sum of the expected utility of the household's members; thus a representative household exists. The utility obtained by the representative household in a period is given by

$$u_t U^u(C_t) + (1-u_t)U_t^e(C_t^e, h_t) - \frac{1}{1-\chi}, \tag{15}$$

where C_t is consumption of an unemployed workers, C_t^e is consumption of an employed worker, and h_t is his hours worked. As discussed in the Appendix, the last term in expression (15) is introduced to obtain logarithmic preferences as a particular case. In this formulation the utility of an unemployed worker U^u is a standard constant relative risk aversion utility function given by:

$$U^u(C_t) = \frac{C_t^{1-\chi}}{1-\chi}, \quad \chi > 0, \tag{16}$$

where $\chi^{-1} > 0$ determines the intertemporal substitution for consumption. Following Shimer (2009) and Trabandt and Uhlig (2009), the utility of an employed worker is given by:

$$U_t^e(C_t^e, h_t) = \frac{(C_t^e)^{1-\chi}\left[S_t(h_t)\right]^{\chi}}{1-\chi}, \quad \chi > 0, \tag{17}$$

with

$$S_t(h_t) = 1 + (\chi - 1)\left(\Psi_0 + \Psi_{1,t}\frac{h_t^{1+\nu}}{1+\nu}\right),$$

represents the e¤ort cost of working, which is equal to the the sum of a …xed cost of going to work Ψ_0 and a term which is increasing in the number hours worked in the job h. The parameter $\nu > 0$ is equal to the inverse of the Frisch labor supply elasticity (see Appendix A for further details). The parameter $\chi > 0$ determines both the intertemporal substitution for consumption and the complementarity between consumption and hours. When $\chi = 1$, after taking into account the constant in (15), the preferences in (17) yield the standard separable log-preferences as a particular case—i.e. $U_t^e(C_t^e, h_t)$ becomes equal to $\log C_t^e - \Psi_0 - \Psi_{1,t}\frac{h_t^{1+\nu}}{1+\nu}$. When $\chi < 1$ we have that consumption and leisure are complements, while $\chi > 1$ makes the marginal utility of consumption higher when households work which implies that consumption and hours worked are complements.[11] Shimer (2009), among others, argues that consumption hours complementarity is consistent with standard models of time allocation while Sbordone (2006) and Hall (2009a) provide evidence that consumption hours complementarity is important to explain key feature of aggregate fluctuations in the labor market. Hall (2009b) argues that it is also important to explain the effects of shocks to government expenditures. We also assume that

$$\psi_t \equiv \ln \Psi_{1t} = (1 - \rho_\psi)\psi + \rho_\psi \psi_{t-1} + \varepsilon_{\psi t}, \tag{18}$$

which characterizes a shock to the intensive margin, as in Smets and Wouters (2007) and Hall (2008).

The representative household maximizes the expected present value of its instantaneous utility (15). The household's discount factor is B_t where

$$b_t \equiv \ln B_t = (1 - \rho_b)b + \rho_b b_{t-1} + \varepsilon_{bt}, \tag{19}$$

so that as in Primiceri, Schaumburg, and Tambalotti (2008) and Justiniano, Primiceri, and Tambalotti (2008) we allow for shocks to the discount factor. We assume that the claims on the profit streams of firms are traded. In equilibrium the household owns a diversified portfolio of all such claims, implying that the discount factor used by firms to discount future profits from time $t + j$ to t is consistent with the household's intertemporal decisions and so they share the same discount factor.

Aggregate resource constraint The aggregate resource constraint is: $Y_t = I_t + C_t + D_t + L_t$, where D_t is an aggregate demand exogenous component (say due to government expenditures or net trade balances) that we assume evolves as

$$D_t \equiv \widetilde{D}_t X_t,$$

[11] In the literature there are alternative representation of preferences that allows for consumption-hours complementarity (see Basu and Kimball (2002), Malin (2008), and Hall and Milgrom (2008)). But these other formulation impose less transparent restrictions on the set of parameters that guarantees that the utility function is well behaved. These restrictions are particularly problematic when it comes toe estimating the model.

where

$$d_t \equiv \ln \widetilde{D}_t = (1-\rho_d)d + \rho_d d_{t-1} + \varepsilon_{dt} \tag{20}$$

which guarantees the existence of a balanced growth path.[12] Finally, after combining (12) and (13), we have that

$$L_t = (1-\kappa)\, r_t \bar{R}(n_t, v_t) \tag{21}$$

represent the total amount of output units spent for job creation purposes. The quantity

$$L_t^l = \kappa r_t \bar{R}(n_t, v_t), \tag{22}$$

denotes instead the recruiting costs due to the purchase of labour intensive intermediate goods.

Job destruction At every point in time jobs are exogenously destroyed with probability $\Lambda_t \equiv \frac{1}{1+e^{\lambda_t}}$, where λ_t evolves as follows:

$$\lambda_t = (1-\rho_\lambda)\lambda + \rho_\lambda \lambda_{t-1} + \xi_t \tag{23}$$

The disturbance ξ_t is equal to

$$\xi_t = \sum_{i \in \{z,a,q,\phi,\psi,m,d,b,\lambda\}} \pi_i \varepsilon_{it},$$

where the impact coe¢ cients π_i $i \in \{z, a, q, \phi, \psi, m, d, b, \lambda\}$ are left unrestricted with the normalization $\pi_\lambda \equiv 1$. This simply means that all shocks can a¤ect the separation rate on impact. The shock $\varepsilon_{\lambda t}$ is instead a speci…c shock to the job separation rate.

Timing We adopt the following convention about the timing of events within a period t:

i. Aggregate shocks ε_{zt}, ε_{qt}, ε_{at}, $\varepsilon_{\varphi t}$, ε_{mt}, $\varepsilon_{\lambda t}$, $\varepsilon_{\psi t}$, ε_{bt}, and $\varepsilon_{dt}$are realized;

ii. Old jobs realize whether they are destroyed (which occurs with probability Λ_t). New jobs (resulting from matches at time $t-1$) start producing.

iii. Decisions about job creation, consumption and investment are taken;

iv. Output is produced, income pooled, invested and consumed. Next period begins.

[12] In principle we could model D_t as equal to a varying fraction of total output. This would however introduce an additional source of ine¢ ciencies orthogonal to those stemming from institutional features of the labor market.

Parameters
α : elasticity of output to capital
β : workers' bargaining power
μ_i, $i = z, q$: average rate of growth of z and q
ρ_i, $i = a, \varphi, \psi, \lambda, b, d, m$: serial correlation of stationary shocks
σ_i, $i = z, q, a, \varphi, \psi, \lambda, b, d, m$: sd of innovation to shocks
Ψ_0,Ψ_1 : e¤ort cost parameter
ϕ : sensitivity of output to hours
δ : depreciation rate of capital
κ : weight of labor goods in creation costs
γ_1, γ_2 : parameters of recruiting costs function
$\bar{r}$: constant in recruiting costs function
η : elasticity of matching function wrt u
η_0, η_1 : parameters in social planner creation cost function
ω : (detrended) wage in sticky wage jobs (random variable)
χ : consumption-hours complementarity parameter
ν: inverse of labor supply Frisch elasticity
θ : importance of wage stickiness

Shocks
a (A) : stationary component in neutral technology
z (Z) : trend in neutral technology
q (Q) : trend in investment-speci…c technology
ϕ (Φ) : stationary component in investment speci…c technology
ψ (Ψ) : shock to preferences
λ (Λ) : job destruction shock
b (B) : shock to discount factor
d (D) : shock to aggregate demand
m (M) : shock to matching technology

Other functions
F : production function
R, $\bar{R}$: recruiting services for job creation
g (G): density (CDF) of realization of rigid wage
U^u, U^e : utility of an unemployed and an employed worker
S : e¤ort cost of working
n : matching function
q : probability of …lling a vacancy
E : expected value
T : physical capital adjustment cost

Table 1: Legend

Values
V (V^*) ($\widetilde{V}$): net private (social) surplus of a job (detrended)
H ($\widetilde{H}$): value of unemployment (detrended)
W^b ($\widetilde{W^b}$) : net value to the worker of a bargained wage job (detrended)
W^r ($\widetilde{W^r}$) : expected net value to the worker of a sticky wage job (detrended)
$W^r(\omega)$: net value to the worker of a job with sticky wage ω
$\overline{W}(\omega)$: value of a job with sticky wage ω when options do not bind
P^b ($\widetilde{P^b}$) : value to the …rm of a bargained wage job (detrended)
P^r ($\widetilde{P^r}$) : expected value to the …rm of a sticky wage job (detrended)
Ω_t ($\widetilde{\Omega}_t$): marginal value of capital (detrended)

Variables
X_t : stochastic trend
Y_t ($\widetilde{Y}_t$) : output (detrended)
K_t ($\widetilde{K}_t$) : capital (detrended)
N_t : amount of labor intensive intermediate goods used to produce …nal output
I_t ($\widetilde{I}_t$) : investment expenditures (detrended)
C_t ($\widetilde{C}_t$) : consumption of unemployed (detrended)
C_t^e ($\widetilde{C}_t^e$) : consumption of employed (detrended)
D_t ($\widetilde{D}_t$) : exogenous aggregate demand component (detrended)
L_t ($\widetilde{L}_t$): total output units cost of job creation (detrended)
n_t : new jobs created
u_t : unemployment rate
s_t : separation rate
f_t : …nding rate
h_t : hours per worker
l_t : requirement of labour intensive goods to produce one recruting service
o_t : requirement of output services to produce one recruting service
p_t ($\widetilde{p}_t$): marginal value of one labor intensive intermediate good
w_t^b : wages paid to worker in a bargained wage job
w_t^r : expected wage paid to a worker in rigid wage job
r_t ($\widetilde{r}_t$): cost of one unit of recruiting services
r_{vt} ($\widetilde{r}_{vt}$): cost of processing applications for one vacancy (detrended)
r_{vt} ($\widetilde{r}_{nt}$) : cost of training one worker (detrended)
π_t ($\widetilde{\pi}_t$): marginal value of wealth (detrended)

Table 2: Legend (continued)

3 Equilibrium conditions

We now characterize the equilibrium conditions of the decentralized economy

Consumption At every point in time the marginal value of wealth π_t (i.e. the Lagrange multiplier of the representative household's budget constraint) satisfies

$$\pi_t = \frac{1}{C_t^{\chi}} = \left[\frac{S_t(h_t)}{C_t^e}\right]^{\chi} \tag{24}$$

which means the the representative households equates the marginal utility of consumption of unemployed and employed workers. The Euler condition for the optimal choice of investment is

$$\pi_t = e^{q_t+\varphi_t}\left[1\text{-}T\left(\frac{I_t}{I_{t\text{-}1}}\right)\text{-}T'\left(\frac{I_t}{I_{t\text{-}1}}\right)\frac{I_t}{I_{t\text{-}1}}\right]\Omega_t + B_t E_t\left[e^{q_{t+1}+\varphi_{t+1}}T'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\Omega_{t+1}\right], \tag{25}$$

which establishes a marginal indifference condition between increasing C_t or increasing I_t. In the expression Ω_t is the time-t expected shadow value of capital at time $t+1$, which satisfies the following arbitrage condition:

$$\Omega_t = B_t E_t\left[(1-\delta)\,\Omega_{t+1} + \pi_{t+1}\alpha A_{t+1} Z_{t+1}\left(\frac{N_{t+1}}{K_{t+1}}\right)^{1-\alpha}\right]. \tag{26}$$

Value of a job The value of being unemployed H_t, measured in utils, solves the asset type equation

$$H_t = U^u(C_t) - \pi_t C_t + B_t E_t\left\{H_{t+1} + f_t\left[(1-\theta)W_{t+1}^b + \theta W_{t+1}^r\right]\right\} \tag{27}$$

where H_{t+1} is next period value, $1-\theta$ is the probability that the job found is governed by bargaining, while W_{t+1}^b and W_{t+1}^r denote the expected net increase in value from finding a job with a bargained wage or with a rigid wage, respectively. The first two terms in expression (27) represent the instantaneous utility of being unemployed which is equal to the per period utility from consumption minus the cost to the representative household of providing that utility to the worker. The third term is just the discounted expected future value of unemployment.[13]

To analyze the job creation decision, it is useful to focus on the time-t (private) net value in utils of a job which is equal to

$$V_t = \max_h \pi_t p_t h^{\phi} + U_t^e(C_t^e, h_t) - \pi_t C_t^e - H_t + B_t E_t\,(H_{t+1}) + B_t E_t\left[(1-\Lambda_{t+1})\,V_{t+1}\right] \tag{28}$$

[13] Notice that, for simplicity, we have dropped the constant $1/(1-\chi)$ from the expression of the utility function.

where

$$p_t = (1-\alpha) A_t Z_t \left(\frac{K_t}{N_t}\right)^{\alpha}, \tag{29}$$

is the price of one unit of labor intensive intermediate goods which is equal to its marginal productivity— since final goods are produced in competitive markets according to the production function in (1). To understand the expression, notice that the first five terms compute the instantaneous net value of the job by subtracting the user cost of labor $H_t - B_t E_t (H_{t+1})$ from the sum of the value of job output (measured in utils) and the net utility from working in the period (equal to the utility obtained by the worker minus the cost to the representative household of providing that utility). The last term is just the expected present value of the future job net surplus. After using (17) to write the first order condition with respect to hours worked in expression (28) and after some rearranging we obtain that the choice of hours satisfies

$$h_t = \left(\frac{\phi \pi_t p_t}{\chi \Psi_{1,t}}\right)^{\frac{1}{1+\nu-\phi}} \left[\frac{S_t(h_t)}{C_t^e}\right]^{\frac{1-\chi}{1+\nu-\phi}}. \tag{30}$$

Splitting of the job net surplus Let P_t^i, $i = b, r$ denote the expected value to the firm of a job where wages are set through Nash bargaining $i = b$, or where wages are rigid $i = r$. Recall that W_t^i, $i = b, r$ denotes the analogous value to the worker. Notice that for any i we have $V_t = P_t^i + W_t^i$. Nash bargaining implies that $W_t^b = \beta V_t$ and $P_t^b = (1-\beta) V_t$ while the expected value to the worker of a job with rigid wages satisfies

$$W_t^r \equiv \int_R W_t^r(\omega)\, dG(\omega)$$

where $W^r(\omega)$ is the net value to the worker of a job with sticky wage ω, which solves

$$W_t^r(\omega) = \min\left[\max\left(0,\ \overline{W}_t^r(\omega)\right), V_t\right]. \tag{31}$$

The $\max$ and $\min$ operator in expression (31) guarantees that when ωX_t is out of the bargaining set the outside options of either the worker or the firm bind, and thus, in equilibrium, inefficient separations never arise. The term $\overline{W}_t^r(\omega)$ denotes the expected value to the worker (net of his outside options) of the job when the rigid wage ωX_t is strictly within the bargaining set. Its value satisfies the following asset equation:

$$\overline{W}_t^r(\omega) = \pi_t \omega X_t + U_t^e(C_t^e, h_t) - \pi_t C_t^e - H_t + B_t E_t (H_{t+1}) + B_t E_t \left[(1-\Lambda_{t+1}) W_{t+1}^r\right] \tag{32}$$

where hours are still given by expression (30). Notice that the expression incorporates the convention that shocks to ω are iid over time. Notice that (31) implies that realized wages can never be smaller than $\underline{\omega}_t X_t$, where $\underline{\omega}_t$ is the value of ω that makes $\overline{W}_t^r(\omega)$ equal to zero. This means that $\underline{\omega}_t$ solves

$$\pi_t \underline{\omega}_t X_t + U_t^e(C_t^e, h_t) - \pi_t C_t^e - H_t + B_t E_t (H_{t+1}) + B_t E_t \left[(1 - \Lambda_{t+1}) W_{t+1}^r\right] = 0 \quad (33)$$

Wages can also never be greater than $\overline{\omega}_t X_t$ where $\overline{\omega}_t$ is the value of ω that makes $\overline{W}_t^r(\omega)$ equal to V_t. After using (32) and (33) we have that $\overline{\omega}_t$ solves

$$\pi_t (\overline{\omega}_t - \underline{\omega}_t) X_t = V_t. \quad (34)$$

Then integrating (31) over all possible value of ω, we obtain that the expected value to the worker of a job with rigid wages is equal to

$$W_t^r = V_t + \pi_t \left[\int_{\underline{\omega}_t}^{\overline{\omega}_t} \omega dG(\omega) + G(\underline{\omega}_t) \underline{\omega}_t - G(\overline{\omega}_t) \overline{\omega}_t\right] X_t \quad (35)$$

The expected value to the …rm of a rigid wage job satis…es

$$P_t^r = V_t - W_t^r.$$

Free entry Since vacancies are posted till the exhaustion of any rents, in equilibrium their value would be equal to zero so the following free entry condition will hold in equilibrium

$$\pi_t r_{vt} + q(f_t) \pi_t r_{nt} = q(f_t) B_t E_t \left[(1-\theta) P_{t+1}^b + \theta P_{t+1}^r\right] \quad (36)$$

where r_{vt} and r_{nt} denotes the cost of processing the applications for a vacancy and training a worker, respectively. This expression equates the expected cost of …lling the vacancy (equal to the sum of the cost of processing application for the vacancies plus the cost for the worker training) to the expected net capital gains (the term in the right hand side). In equilibrium the cost of processing the applications is equal to its marginal cost, that given the functional form for $\tilde{R}$ is equal to

$$r_{vt} = r_t \frac{\partial \tilde{R}(n_t, v_t)}{\partial v_t} = \gamma_2 r_t \overline{r} n_t^{\gamma_1} v_t^{\gamma_2 - 1},$$

which after using the fact that

$$v_t = (M_t u_t)^{-\frac{\eta}{1-\eta}} n_t^{\frac{1}{1-\eta}} \quad (37)$$

can be expressed as

$$r_{vt} = \gamma_2 r_t \overline{r} (M_t u_t)^{-\eta_0} n_t^{\eta_1 - 1} q(f_t) = \gamma_2 q(f_t) \frac{r_t R(u_t, n_t)}{n_t} \quad (38)$$

while the marginal cost of training a worker is given by

$$r_{nt} = r_t \frac{\partial \tilde{R}(n_t, v_t)}{\partial n_t} = \gamma_1 r_t \overline{r} n_t^{\gamma_1 - 1} v_t^{\gamma_2},$$

which, again after using (37), can be expressed as

$$r_{nt} = \gamma_1 r_t \overline{r} (M_t u_t)^{-\eta_0} n_t^{\eta_1 - 1} = \gamma_1 \frac{r_t R(u_t, n_t)}{n_t} \tag{39}$$

We can now use (38) and (39) to substitute for r_{vt} and r_{nt} into (36). After some rearranging and after remembering that $\gamma_1 + \gamma_2 = 1$ we obtain the following free entry condition:

$$\frac{\pi_t r_t R(u_t, n_t)}{n_t} = B_t E_t \left[(1-\theta) P_{t+1}^b + \theta P_{t+1}^r \right], \tag{40}$$

which will be convenient to analyze the welfare properties of the equilibrium. This conditions simply says that jobs are created up to the point when the average cost of creating a new job is equal to its expected net value. Notice that the elasticity of $R(u_t, n_t)/n_t$ to n_t is smaller when γ_1 is larger which clarifies why a greater γ_1 amplifies the magnitude of the fluctuation in the labor market, which is the point made by Pissarides (2009) and Silva and Toledo (2009).

Unemployment dynamics The separation rate is $s_t = \Lambda_t$ while the job finding probability for workers searching between time $t-1$ and time t is $f_t = \frac{n_{t-1}}{u_{t-1}}$ so that unemployment evolves as

$$u_t = u_{t-1} + s_t (1 - u_{t-1}) - f_t u_{t-1}. \tag{41}$$

Notice that in the data we do not observe s_t and f_t but only their average monthly rate in the quarter, that we denote as f_t^m and s_t^m, respectively. Their values can be recovered by using the relations $1 - f_t = (1 - f_t^m)^3$ and $1 - s_t = (1 - s_t^m)^3$, respectively.

Average hours per worker are simply given by h_t while the total amount of labor intensive intermediate goods produced is equal to $(1 - u_t) h_t^{\phi}$. Given (11) and (13), the amount of labor intensive intermediate goods used for producing final output is then given by

$$N_t = (1 - u_t) h_t^{\phi} - \kappa \left(\frac{X_t}{p_t} \right)^{1-\kappa} \bar{R}(n_t, v_t) \tag{42}$$

Calculating workers' wage and labor share To calculate wage payments notice that the value to the firm of a bargained wage job satisfies

$$P_t^b = \left(p_t {h_t}^{\phi} - w_t^b \right) \pi_t + B_t E_t \left[(1 - \Lambda_{t+1}) (1 - \beta) V_{t+1} \right] \tag{43}$$

where w_t^b denotes the total wage payments (measured in …nal output) received by the worker for the hours he worked in the job and the last term uses Nash bargaining which implies that $P_t^b = (1-\beta)V_t$. After rearranging we obtain that

$$w_t^b = p_t h_t{}^{\phi} + \frac{1}{\pi_t}(1-\beta)\left\{B_t E_t\left[(1-\Lambda_{t+1})V_{t+1}\right] - V_t\right\}.$$

The expected wage paid in a rigid wage job is instead equal to

$$w_t^r = \left[G\left(\underline{\omega}_t\right)\underline{\omega}_t + \int_{\underline{\omega}_t}^{\overline{\omega}_t} \omega dG(\omega) + \left(1 - G\left(\overline{\omega}_t\right)\right)\overline{\omega}_t\right] X_t.$$

The labor share can then be expressed as equal to

$$y_{lt} = \frac{(1-u_t)\left[(1-\theta)w_t^b + \theta w_t^r\right]}{Y_t - L_t}. \tag{44}$$

Notice that $Y_t - L_t$ corresponds to the standard de…nition of GDP, that can be allocated to either consumption, investment or to satisfy the exogenous demand component D_t.

Decentralized equilibrium definition One can check that the economy evolves around the stochastic trend given by

$$X_t \equiv e^{\frac{z_t}{1-\alpha}}\ e^{\frac{\alpha q_t}{1-\alpha}}$$

To make the environment stationary we de…ned the following scaled quantities:

$$\begin{aligned}
\widetilde{K}_t &\equiv \frac{K_t}{e^{q_t}X_t},\ \widetilde{C}_t^e \equiv \frac{C_t^e}{X_t},\ \widetilde{C}_t \equiv \frac{C_t}{X_t},\ \widetilde{I}_t \equiv \frac{I_t}{X_t},\ \widetilde{L}_t \equiv \frac{L}{X_t},\ \widetilde{p}_t \equiv \frac{p_t}{X_t},\ \widetilde{r}_t \equiv \frac{r_t}{X_t},\\
\widetilde{\pi}_t &\equiv X_t^{\chi}\pi_t,\ \widetilde{V}_t \equiv X_t^{\chi-1}V_t,\ \widetilde{H}_t \equiv X_t^{\chi-1}H_t,\ \widetilde{P}_t^i \equiv X_t^{\chi-1}P_t^i,\ \widetilde{W}_t^i \equiv X_t^{\chi-1}W_t^i,
\end{aligned}$$

and $\widetilde{\Omega}_t \equiv X_t^{\chi}e^{q_t}\Omega_t$. We can show that an equilibrium consists of a stationary tuple

$$(\widetilde{K}_{t+1}, \widetilde{C}_t^e, \widetilde{C}_t, \widetilde{I}_t, \widetilde{L}_t, \widetilde{\pi}_t, \widetilde{\Omega}_t, \widetilde{p}_t, \widetilde{r}_t, u_t, h_t, n_t, N_t, \widetilde{V}_t, \widetilde{H}_t, \widetilde{P}_t^b, \widetilde{P}_t^r, \widetilde{W}_t^b, \widetilde{W}_t^r, \underline{\omega}_t, \overline{\omega}_t), \tag{45}$$

which satis…es the following conditions— which I produce here just for convenience even if they would more naturally …t to an Appendix— :

1. The stationary version of the law of motion of capital in (4) :

$$\widetilde{K}_{t+1} e^{\frac{\mu_z+\mu_q+\varepsilon_{z,t+1}+\varepsilon_{q,t+1}}{1-\alpha}} = \left\{(1-\delta)\widetilde{K}_t + e^{\varphi_t}\left[1 - T\left(\frac{\widetilde{I}_t e^{\frac{\mu_z+\alpha\mu_q+\varepsilon_{z,t}+\alpha\varepsilon_{q,t}}{1-\alpha}}}{\widetilde{I}_{t-1}}\right)\right]\widetilde{I}_t\right\} \tag{46}$$

2. The optimal consumption level for the unemployed in (24):

$$\widetilde{\pi}_t = \frac{1}{\widetilde{C}_t^{\chi}} \tag{47}$$

3. The optimal consumption level for the employed in (24):

$$\widetilde{\pi}_t = \left[\frac{S_t(h_t)}{\widetilde{C}_t^e}\right]^{\chi} \tag{48}$$

where

$$S_t(h_t) = 1 + (\chi - 1)\left(\Psi_0 + \Psi_{1,t}\frac{h_t^{1+\nu}}{1+\nu}\right), \quad \nu, \chi > 0.$$

4. The total cost of job creation

$$\widetilde{L}_t = (1-\kappa)\,\widetilde{r}_t\overline{r}\,(M_t u_t)^{-\eta_0}\,n_t^{\eta_1} \tag{49}$$

5. The aggregate resource constraint:

$$A_t\left(\widetilde{K}_t\right)^{\alpha}(N_t)^{1-\alpha} = u_t\widetilde{C}_t + (1-u_t)\,\widetilde{C}_t^e + \widetilde{I}_t + \widetilde{D}_t + \widetilde{L}_t \tag{50}$$

6. The marginal productivity of a labor intensive intermediate good in (29):

$$\widetilde{p}_t = (1-\alpha)\,A_t\left(\frac{\widetilde{K}_t}{N_t}\right)^{\alpha} \tag{51}$$

7. The cost of one recruiting service in (10):

$$\widetilde{r}_t = \widetilde{p}_t^{\kappa} \tag{52}$$

8. The evolution for the marginal value of capital in (26):

$$\widetilde{\Omega}_t = B_t E_t\left\{\left[\widetilde{\pi}_{t+1}\alpha A_{t+1}\left(\frac{N_{t+1}}{\widetilde{K}_{t+1}}\right)^{1-\alpha} + (1\text{-}\delta)\,\widetilde{\Omega}_{t+1}\right]e^{-\frac{\chi\left(\mu_z+\varepsilon_{z,t+1}\right)+(\chi\alpha+1-\alpha)\left(\mu_q+\varepsilon_{q,t+1}\right)}{1-\alpha}}\right\}, \tag{53}$$

9. The stationary version of the Euler equation for investment (25):

$$\begin{aligned}\widetilde{\pi}_t = e^{\varphi_t}&\left[1 - T\left(\frac{\widetilde{I}_t e^{\frac{\mu_z+\alpha\mu_q+\varepsilon_{z,t}+\alpha\varepsilon_{q,t}}{1-\alpha}}}{\widetilde{I}_{t-1}}\right) - T'\left(\frac{\widetilde{I}_t e^{\frac{\mu_z+\alpha\mu_q+\varepsilon_{z,t}+\alpha\varepsilon_{q,t}}{1-\alpha}}}{\widetilde{I}_{t-1}}\right)\frac{\widetilde{I}_t e^{\frac{\mu_z+\alpha\mu_q+\varepsilon_{z,t}+\alpha\varepsilon_{q,t}}{1-\alpha}}}{\widetilde{I}_{t-1}}\right]\widetilde{\Omega}_t \\ &+ B_t E_t\left[e^{\frac{\left(\mu_z+\alpha\mu_q+\varepsilon_{z,t+1}+\alpha\varepsilon_{q,t+1}\right)(2-\chi)}{1-\alpha}+\varphi_{t+1}}T'\left(\frac{\widetilde{I}_{t+1}e^{\frac{\mu_z+\alpha\mu_q+\varepsilon_{z,t+1}+\alpha\varepsilon_{q,t+1}}{1-\alpha}}}{\widetilde{I}_t}\right)\left(\frac{\widetilde{I}_{t+1}}{\widetilde{I}_t}\right)^2\widetilde{\Omega}_{t+1}\right]\end{aligned} \tag{54}$$

When $T = T\prime = 0$ this becomes:

$$\frac{1}{\left(\widetilde{C}_t\right)^{\chi}} = B_t e^{\varphi_t} E \left\{ \frac{\left[\alpha A_{t+1}\left(\frac{N_{t+1}}{\widetilde{K}_{t+1}}\right)^{1-\alpha} + (1-\delta)e^{-\varphi_{t+1}}\right]}{\left(\widetilde{C}_{t+1}\right)^{\chi}\ e^{\frac{\chi\left(\mu_z+\varepsilon_{z,t+1}\right)+(\chi\alpha+1-\alpha)\left(\mu_q+\varepsilon_{q,t+1}\right)}{1-\alpha}}} \right\}$$

10. The definition of unemployment in (41):

$$u_t = 1 - (1-\Lambda_t)(1-u_{t-1}) - n_{t-1} \tag{55}$$

11. The optimal choice of hours worked in (30):

$$h_t = \left(\frac{\phi\widetilde{\pi}_t\widetilde{p}_t}{\chi\Psi_{1,t}}\right)^{\frac{1}{1+\nu-\phi}} \left[\frac{S_t(h_t)}{\widetilde{C}_t^e}\right]^{\frac{1-\chi}{1+\nu-\phi}} \tag{56}$$

12. The definition of the amount of labor intensive intermediate goods used in production in (42):

$$N_t = (1-u_t)\, h_t^{\phi} - \kappa\left(\frac{1}{\widetilde{p}_t}\right)^{1-\kappa} \overline{r}\,(M_t u_t)^{-\eta_0}\, n_t^{\eta_1} \tag{57}$$

13. The expression for the value of unemployment in (27)

$$\widetilde{H}_t = \frac{\widetilde{C}_t^{1\text{-}\chi}}{1\text{-}\chi} - \widetilde{\pi}_t\widetilde{C}_t + B_t E_t\left\{\left(\widetilde{H}_{t+1} + \frac{n_t}{u_t}\left[(1\text{-}\theta)\widetilde{W}_{t+1}^b + \theta\widetilde{W}_{t+1}^r\right]\right) e^{\frac{\left(\mu_z+\alpha\mu_q+\varepsilon_{z,t+1}+\alpha\varepsilon_{q,t+1}\right)(1-\chi)}{1-\alpha}}\right\} \tag{58}$$

14. The job net surplus in (28):

$$\widetilde{V}_t = \widetilde{\pi}_t\widetilde{p}_t h_t^{\phi} + \frac{\left(\widetilde{C}_t^e\right)^{1\text{-}\chi}[S_t(h_t)]^{\chi}}{1\text{-}\chi} - \widetilde{\pi}_t\widetilde{C}_t^e - \widetilde{H}_t + B_t E_t\left[\widetilde{H}_{t+1} e^{\frac{\left(\mu_z+\alpha\mu_q+\varepsilon_{z,t+1}+\alpha\varepsilon_{q,t+1}\right)(1-\chi)}{1-\alpha}}\right]$$
$$+ B_t E_t\left[(1-\Lambda_{t+1})\,\widetilde{V}_{t+1} e^{\frac{\left(\mu_z+\alpha\mu_q+\varepsilon_{z,t+1}+\alpha\varepsilon_{q,t+1}\right)(1-\chi)}{1-\alpha}}\right] \tag{59}$$

15. The free entry condition in (40):

$$(\gamma_1+\gamma_2)\,\widetilde{\pi}_t\widetilde{r}_t\overline{r}\,(M_t u_t)^{-\eta_0}\, n_t^{\eta_1-1} = B_t E_t\left\{\left[(1\text{-}\theta)\widetilde{P}_{t+1}^b + \theta\widetilde{P}_{t+1}^r\right] e^{\frac{\left(\mu_z+\alpha\mu_q+\varepsilon_{z,t+1}+\alpha\varepsilon_{q,t+1}\right)(1-\chi)}{1-\alpha}}\right\} \tag{60}$$

16. The value to the firm of a bargained wage job:

$$\widetilde{P}_t^b = (1-\beta)\,\widetilde{V}_t \tag{61}$$

17. The value to the …rm of a rigid wage job:

$$\widetilde{P}_t^r = \widetilde{V}_t - \widetilde{W}_t^r. \tag{62}$$

18. The value to a worker of a bargained wage job:

$$\widetilde{W}_t^b = \beta \widetilde{V}_t, \tag{63}$$

19. The lowest wage in the bargaining set in (33):

$$\begin{aligned} & \widetilde{\pi}_t \underline{\omega}_t + \frac{\left(\widetilde{C}_t^e\right)^{1-\chi} [S_t(h_t)]^{\chi}}{1-\chi} - \widetilde{\pi}_t \widetilde{C}_t^e - \widetilde{H}_t \\ & + B_t E_t \left\{ \left[\widetilde{H}_{t+1} + (1 - \Lambda_{t+1}) \widetilde{W}_{t+1}^r \right] e^{\frac{\left(\mu_z + \alpha \mu_q + \varepsilon_{z,t+1} + \alpha \varepsilon_{q,t+1}\right)(1-\chi)}{1-\alpha}} \right\} = 0 \end{aligned} \tag{64}$$

20. The highest wage in the bargaining set in (34):

$$\widetilde{\pi}_t \left(\overline{\omega}_t - \underline{\omega}_t \right) = \widetilde{V}_t \tag{65}$$

21. The expected value to a worker of a rigid wage job in (35):

$$\widetilde{W}_t^r = V_t + \widetilde{\pi}_t \left[\int_{\underline{\omega}_t}^{\overline{\omega}_t} \omega dG(\omega) + G\left(\underline{\omega}_t\right) \underline{\omega}_t - G\left(\overline{\omega}_t\right) \overline{\omega}_t \right] \tag{66}$$

22. The laws of motion of the two nonstationary technology shocks z_t and q_t are given in (3) in (5) while the laws of motion of the seven stationary aggregate shocks a_t, λ_t, φ_t, d_t, m_t, b_t and ψ_t are given in (2), (23), (6), (20), (9), (19), and (18), respectively.

4 Social planner problem

In characterizing the social planner we denote with a "$*$" the corresponding quantities in the decentralized equilibrium.

Social planner allocation We show in the Appendix B that, after adding $*$'s to the corresponding quantities, saving and consumption decisions are still governed by (24), (25), and (26). A key di¤erence relative to the decentralized equilibrium is the expression for the net value of a job that according to the social planner is given by

$$\begin{aligned} V_t^* &= \max_{h_t^*} \pi_t^* p_t^* h_t^{*\phi} + U_t^e \left(C_t^{e*}, h_t^*\right) - U^u \left(C_t^*\right) - \pi_t^* \left(C_t^{e*} - C_t^*\right) + \pi_t^* r_t^* \frac{\partial R_t(u_t, n_t)}{\partial u_t} \\ & + B_t E_t \left[(1 - \Lambda_{t+1}) V_{t+1}^* \right] \end{aligned} \tag{67}$$

where π_t^* is the social shadow value of wealth. To understand the expression, notice that the …rst four terms in the …rst row are the sum of the value of output and the utility obtained by an employed worker minus the utility of an employed. The last term in the …rst row simply subtracts from the …rst the value of unemployment. Here unemployment has value because it reduces the cost of creating new jobs, R. Given (14, this value is simply equal to minus the partial derivative of R with respect to u. The last term represents instead the expected present value of the future job net surplus The …rst order condition with respect to hours worked in (67) reads as (30) after adding *'s to the corresponding quantities. The optimal number of newly created jobs solves

$$\pi_t^* r_t^* \frac{\partial R_t(u_t, n_t)}{\partial n_t} = B_t E_t \left(V_{t+1}^* \right) \tag{68}$$

which, given (14) equates the social marginal cost of creating a new job (equal to the partial derivative of R with respect to n) to its expected future net value. For convenience the equilibrium conditions that characterizes the social planner allocation are fully summarized in Appendix C.

Inefficiencies in the decentralized equilibrium We can compare the equilibrium conditions of the social planner problem described in the previous Section with those that characterize the decentralized equilibrium described in Section 3. The two allocations coincide if at every point in time the number of newly created job is identical. In that case the evolution of unemployment, output, consumption and investment would also coincide. Clearly n_t and n_t^* are equal if the private net value of a job V_t is equal to its social value V_t^* and if the free entry condition of the decentralized equilibrium in (40) coincides with condition (68) that characterizes the optimal level of job creation chosen by the social planner.

By comparing the expression for the net private value of a job V_t in (28) and the social net value V_t^* in (67), we have that the two values coincide if and only if

$$H_t - B_t E_t \left(H_{t+1} \right) = \frac{C_t^{*1-\chi}}{1-\chi} - \pi_t^* C_t^* - \pi_t^* r_t^* \frac{\partial R_t(u_t, n_t)}{\partial u_t}. \tag{69}$$

This means that the user cost of labor in the decentralized equilibrium is equal to its social marginal value.

By comparing the free entry condition (40) to the e¢ cient job creation condition (68), and after remembering the properties of Nash bargaining, we have that the two conditions coincide if there are no wage rigidities, $\theta = 0$, $V_t = V_t^*$ and if

$$\eta_1 = \frac{(\gamma_2 + \gamma_1)}{(1-\beta)}.$$

After using the de…nition of η_1, this condition can also be expressed as

$$(1-\beta) = \frac{\gamma_1 + \gamma_2}{\gamma_1 (1-\eta) + \gamma_2} \cdot (1-\eta)\,. \tag{70}$$

Also notice that this last result can be used to simplify the expression in (69). We can …rst use (27) and the fact that Nash bargaining implies that $W_{t+1}^b = \beta V_{t+1}$ to replace $H_t - B_t E_t \{H_{t+1}\}$ in (69). Then, after using (40) to manipulate the resulting expression, and after assuming that there are no wage rigidities $\theta = 0$, we obtain that condition (69) simpli…es to

$$\frac{1-\beta}{\beta} = \frac{\gamma_1 + \gamma_2}{\gamma_2} \cdot \frac{1-\eta}{\eta}. \tag{71}$$

Conditions (70) and (71) represent two su¢ cient conditions that must both hold for the decentralized equilibrium to be socially e¢ cient. If $\gamma_1 > 0$, (70) and (71) represent two linearly independent conditions that workers bargaining power β should satisfy for the decentralized equilibrium to be e¢ cient. So provided that $\gamma_1 > 0$, it never exists a value of β that makes the decentralized equilibrium e¢ cient. If instead γ_1 is equal to 0, the two conditions coincide and the decentralized equilibrium is socially e¢ cient if and only if $\beta = \eta$. This is the well known conditions …rst derived by Hosios (1990). This highlights how the parameter γ_1 that determines the gains from smoothing job creation over time plays a key role in determining the costs of search ine¢ ciencies. This is one of the main point made by Caballero and Hammour (1996, 1998) and it is further discussed in Caballero (2007). This suggests that $\gamma_1 > 0$ is likely to increase the welfare of the search ine¢ ciencies in worker reallocation. Overall we have proved that:

Proposition 1 The decentralized equilibrium is socially e¢ cient if the Hosios condition holds $(\beta = \eta)$, there are no appropriability problems $(\gamma_1 = 0)$, and wages are ‡exible $(\theta = 0)$.

The Proposition highlights that there are three possible sources of ine¢ ciencies in the model. All them stem from frictions in the labour and they have already been emphasized before. Ine¢ ciencies arise because of either failure of the Hosios (1990) condition (β is di¤erent from η); or because of appropriability problems in creation costs as in Caballero and Hammour (1996, 1998) (γ_1 is different from zero); or because of wage rigidity as in Shimer (2004) and Hall (2005) (θ is different from zero). Notice that these frictions could have different welfare costs depending of the type of shocks and their sign.

The Hosios' benchmark To clarify the operation of the different inefficiency margins let's assume that the parameters of the model are such that at every point in time (1-θ)$W_t^b + \theta W_t^r = \eta V_t$ and $(1-\theta)P_t^b + \theta P_t^r = (1-\eta)V_t$. These are the net surplus splitting surplus rule that correspond to the Hosios (1990) condition.

When this condition holds we have that

$$\frac{R(u_t, n_t)}{(1-\eta)n_t} \geq \frac{\partial R(u_t, n_t)}{\partial n_t} \tag{72}$$

since $1/(1-\eta) \geq \gamma_1 + \gamma_2/(1-\eta)$ with strict inequality if $\gamma_1 > 0$.[14] This means that the average cost of job creation is greater than the marginal cost.

Moreover we have that

$$\begin{aligned} H_t - B_t E_t(H_{t+1}) &= \frac{C_t^{*1-\chi}}{1-\chi} - \pi_t^* C_t^* + \frac{n_t}{u_t} B_t E_t\left[(1-\theta)W_{t+1}^b + \theta W_{t+1}^r\right] \\ &\geq \frac{C_t^{*1-\chi}}{1-\chi} - \pi_t^* C_t^* - \pi_t^* r_t^* \frac{\partial R_t(u_t, n_t)}{\partial u_t}. \end{aligned} \tag{73}$$

To see this notice that in the parametrization that implements the Hosios's condition we have that $B_t E_t\left[(1-\theta)W_{t+1}^b + \theta W_{t+1}^r\right] = \frac{\eta}{1-\eta} \cdot \frac{\pi_t r_t R(u_t, n_t)}{n_t}$ which makes use of the free entry condition (40). But since $-\frac{\partial R_t(u_t,n_t)}{\partial u_t} \leq \frac{\eta}{1-\eta} \cdot \frac{R_t(u_t,n_t)}{u_t}$ (with strict inequality when $\gamma_1 > 0$), it follows immediately that the inequality in (73) is satisfied. But with a greater user cost of labor we immediately have that the net surplus of the decentralized equilibrium is such that

$$V_t < V_t^* \tag{74}$$

Equations (72) and (74) imply that unemployment is inefficiently high at a net surplus splitting rule that satisfies the Hosios conditions. Equation (74) implies that the private net surplus of a job is too low relative to its social value while equation (72) implies that the average cost of production is greater than marginal cost. Both forces make job creation too low and as result the unemployment rate becomes inefficiently high.

[14] To see that the inequality is satisfied multiply the left and right hand side by $(1-\eta)$ and the rearrange to obtain that the inequality is equivalent to

$$\gamma_1 \geq \gamma_1 (1-\eta).$$

5 Relation with Hall's two factors model

It is easy to see that our model embeds the framework considered by Hall (2009a) as a particular case. Hall (2009a) follows Jackman, Layard, and Pissarides (1989) and Shimer (2005) in assuming that actual unemployment is well characterized by the following fictional unemployment rate

$$\widehat{u}_t = \frac{s_t}{s_t + f_t} \tag{75}$$

Actual unemployment is well approximated by $\widehat{u}_t$ since, for realistic values of the job finding rate, unemployment tend to converge very quickly to the steady state value that would arise if the finding and the separation rate were to remain forever unchanged at their current value. Under this approximation, Hall (2009a) considers a model where only two factors $\varkappa_{1t}$ and $\varkappa_{2t}$ drive the dynamics of detrended consumption, productivity, hours per worker and unemployment. The first factor is labour productivity

$$\varkappa_{1t} = A_t \left(\frac{\widetilde{K}_t}{N_t} \right)^{\alpha} h_t^{\phi - 1},$$

the second is the marginal value of wealth

$$\varkappa_{2t} = \widetilde{\pi}_t.$$

It is easy to see that in our model, after approximating u_t with $\widehat{u}_t$, there are four factors in addition to $\varkappa_{1t}$ and $\varkappa_{2t}$ that drive the dynamics of consumption, productivity, hours per worker and unemployment. These are the i) separation rate Λ_t, ii) the matching efficiency technology M_t, iii) the discount factor B_t, and iv) the marginal value for leisure Ψ_{1t}. We can analyze the contribution of these four additional factors in explaining the fluctuations of the four cyclical variables (consumption, productivity, hours per worker and unemployment) considered by Hall (2009a). Notice that the other five shocks in the model (the four technology shocks z_t, a_t, q_t, and φ_t and the aggregate demand shock d_t) are simply the source of the exogenous variation in the two factors emphasized by Hall (2009a). We can compare our results to his and emphasize the role of the four additional factors in explaining specific business cycle episodes.

6 Data and prior distribution of the parameters

In this section we discuss the data used to estimate the model and the priors we impose on the model parameters.

Data The …rst four variables used are the same as in Hall (2009a). They are non-durable and services consumption, unemployment, weekly hours per worker, and labor productivity. In addition we will use the …nding and the separation rates. Given that we would like to identify the role of wage stickiness in the model, it seems appropriate to use information on compensation so our variable number seven is the labor share.[15] These seven variables leave us with seven shocks. In the current version of the model we have nine. We would like to add an extra variable, vacancies. We think that adding the variable will help in identifying the shocks to the matching technology and ine¢ ciencies in reallocation, which is a key point of Caballero and Hammour (1996). But adding this variable might add extra issues. In particular, it is well recognized in the literature that the proxy for vacancies based on the Help-Wanted Index is subject to severe low frequency measurement error. We can detrend the series and then allows for classical measurement error. Our last variable, number nine, is investment as in Justiniano and Primiceri (2008)

So we have nine variable and nine shocks plus measurement error in vacancies. It will also be natural to add measurement error to …nding rates and separation rates since the model is estimated with quarterly data while in the data we only observe the average monthly …nding and separation rate in the quarter, that we denote as f_t^m and s_t^m, respectively. The value of the …nding and separation rate in the model can be recovered by using the relations $1-f_t = (1-f_t^m)^3$ and $1-s_t = (1-s_t^m)^3$, respectively. Since these equalities will only hold as an approximation we add measurement error to the two series. To recap, the variables used for the estimation are consumption, unemployment, hours per worker, labor productivity, …nding rate, separation rate, vacancies, labor share, and investment. Variables 1-4 are as in Hall (2009a); 5-6 and 8 seem important to characterize the labor market and to identify shocks to the matching technology; 7 seems key to identify the importance of wage rigidity; and 9 follows from the analysis in Justiniano and Primiceri (2008).

Unemployment in the data evolves as

$$u_t = u_{t-1} + (1-\Lambda_t)(1-u_{t-1}) - n_{t-1}$$

where u_t is unemployment in the model while $\overline{d}$ is an extra parameter to be estimated. In principle the mapping between unemployment in the data and unemployment in the model is imperfect because it is not clear whether unemployment in the data is mea-

[15] Ríos-Rull and Santaeulalia-Llopis (2009) also argue in favor of matching the dynamics of the labor share to test macroeconomic models.

sured before or after the search process in the period has been completed. Measurement error in unemployment can take care of this data issue.

Priors Tables ?? reports the prior used in the estimation of the model.

Preferences The parameter χ that governs consumption-hours complementarity parameter is set around 1.75, inside the range (between 1 and 5) of values considered by Shimer (2009). We also impose a moment condition on the ratio between consumption of an unemployed worker and consumption of a n employed worker $\frac{C_t}{C_t^e}$ which we centered around 0.85, which is consistent with Hall (2009, page 301) and Shimer (2009, page 21) who argues in favor of value of 0.80.[16] The parameters ν measures the inverse of labor supply Frisch elasticity which according to Hall (2009) is equal to $\simeq \frac{1}{0.7}$. This number is justi…ed using Pistaferri(2003). Most micro people (Blundell and others) will feel more comfortable with a value of ν equal to 2 (which corresponds to a Frisch elasticity of 0.5). These priors implicitly set a prior on χ, ψ_0 and ν.

Technologies The elasticity of output to capital α is centered around 0.4. At quarterly frequency, the rate of growth of investments speci…c technology μ_q has a prior centered at $\frac{0.035}{4}$. The depreciation rate of capital δ is set at 10 percent per year, 2.5 percent per quarter which is similar to Greenwood, Hercowitz and Krusell (1997). The elasticity of matching function with respect to unemployment, η, is centered around 0.6. The value of 0.6 accords well with the summary of the evidence reported in Petrongolo and Pissarides (2001) who argue that "A plausible range for the empirical elasticity of unemployment is 0.5 to 0.7". Shimer (2005) argues in favor of a value of $\eta = 0.72$. The parameters ϕ, that governs the sensitivity of output to hours, is left pretty unrestricted. The same applies to adjustment costs to capital parameter T'' and the parameter κ that measures the weight of labor goods in creation costs.[17] The parameter $\bar{r}$ that characterizes the constant in recruiting costs function is again left unrestricted, yet its value is implicitly restricted by the prior on the …nding rate.

Source of inefficiencies We consider pretty loose priors for all the parameters that drive the possible sources of ine¢ ciency: (i) the importance of wage stickiness

[16] These numbers are based on the evidence by Browning and Crossley (2001) and Low, Meghir, Pistaferri (2008). They report a fall in consumption upon unemployment in the range 0.15−0.2.

[17] There is \scant evidence about its value. Shimer (2009) is …rst in arguing that the parameter κ might matter for cyclical ‡uctuations.

θ, the workers' bargaining power β, and the job creation cost γ_1. There is a hot debate in the literature about the importance of wage stickiness θ. Hall (2005) argue in favor of $\theta = 1$, while Hagedorn and Manovskii (2009) are inclined toward $\theta = 0$. Similar considerations apply for the value of β that characterize workers' bargaining power. There is indeed substantial debate on the value of β in the literature. For the recruiting costs function we impose the constant-return-to scale (CRS) assumption on the production function of recruiting services, $\gamma_1 + \gamma_2 = 1$. This will simplify the estimation problem substantially since now parameters values are bounded and we could have a uniform prior on $U[0, 1]$ for $\gamma_1 = \gamma$. The parameter γ_2 will then be equal to $1 - \gamma$. We then impose a uniform prior on $U[0, 1]$ for $\gamma_1 = \gamma$. We could later test for the CRS assumption. The prior for the mean wage in sticky wage jobs μ_ω is set equal to 0.96 of marginal productivity. This is taken from Hall (2005, Table).[18] The standard deviation in sticky wage jobs σ_ω is left pretty unrestricted.

Shocks and their dynamics All shocks are normalized to zero (in logs, that means to one in levels) except Λ (separation rate), ψ_1 (labor cost parameter), B (discount factor), and D (exogenous demand component). Some parameters will be identi…ed just by using dynamics. These will be ρ_i, $i = a, \varphi, \psi, \lambda, b, d, m,$which denotes the serial correlation of stationary shocks, and the standard innovation to shocks σ_i, $i = z, q, a, \varphi, \psi, \lambda, b, d, m$. The prior on the value of Λ was given above. The prior on ψ_1 comes from the normalization that employed workers work 40 per cent of their leisure time when employed, $h_{ss} = 0.4$. The prior for the discount factor,B, is driven by having a reasonable steady state real rate of return of 0.7 per cent per quarter (i.e. 2.8 percent per year given the trend in productivity growth which will be approximately equal to 2 per cent on an annual basis.) The prior on the exogenous demand component comes from imposing that 18 per cent of observable output Y^* is allocated for purposes other than private consumption and investment. Finally, the value of the impact coe¢ cients π's on the separation rate are left pretty unrestricted both in terms of sign and magnitude.

Priors on steady state quantities We impose some priors on some relevant steady state quantities. We start specifying the model at the quarterly frequency, so we convert the available average monthly …nding and separation rate that we denote as f_t^m and s_t^m, respectively. The quarterly …nding rate f_t and the quarterly separation rate

[18] Recall that the expression of the marginal productivity is: $(1-\alpha)(\frac{K}{N})^\alpha h^\phi$.

Parameters	Distri.	Mean	SD	Min	Max
χ	G	1.75	0.25	0.99	30
$\frac{C}{C^e}$	B	0.85	0.05	0.5	0.99
ν	G	1.6	0.2	0.01	20
f_{ss}	G	0.2	0.1	0.001	2
α	N	0.4	0.05	0.01	1
δ	C	0.025	0	0.02	0.03
ϕ	U	0.5	0.28	0	1
T''	U	2.5	1	0.001	100
μ_z (%)	N	0.3	0.025	0.01	1
μ_q (%)	C	0.01	0	0.01	1
g_{ss}	C	0.18	0	0.18	0.22
λ	B	0.045	0.01	0.005	0.5
h_{ss}	B	0.37	0.02	0.05	1
η	U	0.6	0.1	0.001	1
$\overline{r}$	U	3	2.8	0.001	30
κ	U	0.5	0.28	0.001	0.998
γ_1	B	0.5	0.28	0.001	0.998
β	U	0.5	0.28	0.001	0.998
θ	U	0.5	0.28	0.001	0.998
τ	N	0.97	0.04	0.1	5
ς	G	1.5	1	0.001	20
dd	B	0.5	0.28	0	1
$case\_correl$	C	2	0	0.99	2
$\pi_i,\ i \in \{z, a, q, \phi, \psi, m, d, b, \lambda$					

s_t can be recovered by using the relations $1 - f_t = (1 - f_t^m)^3$ and $1 - s_t = (1 - s_t^m)^3$, respectively. If $f_t^m = 0.45$ and $s_t^m = 0.03$ we would have $f_t = 1 - (1 - f_t^m)^3 = 0.8336$ and we set a tight prior at the steady state state unemployment rate at 5.8 per cent. We also set a tight prior on the labor share. We are also interested in setting prior on the total cost of job creation over observed output de...ned as $Y_t^* \equiv Y_t - L_t$:

$$L\text{to}Y = \frac{L_t^l + L_t}{Y_t - L_t} = \frac{L_t^l + L_t}{Y_t^*}$$

This ratio depends on the frequency at which the model is calibrated. Silva and Toledo (2008) calculate that hiring a worker requires 4.3 percent of the quarterly wage of a newly hired worker. Hiring also requires some training costs that amounts to 55 percent of quarterly wage. These are the recruiting costs intensive in labor. We can think that they should correspond to L_t^l. The value of 0.045, as a measure of recruiting costs, is in line with both Shimer (2009) and Hagedorn and Manovskii (2009).[19] The existence of recruiting costs is much more controversial (especially given their magnitude). At the quarterly frequency we can calculate the ratio $\frac{L^l}{Y^*}$ as equal to

$$\begin{aligned}\frac{L^l}{Y^*} &= \frac{y_{lt}}{(1-u)}(0.043 + 0.55) \cdot u \cdot f \\ &= \frac{2}{3(1 - 0.055)}(0.043 + 0.55) \cdot 0.055 \cdot 0.75 = 0.017\end{aligned}$$

where y_{lt} denotes the labor share. Notice that $\frac{y_{lt}}{(1-u)}$ measure the ratio of quarterly wage to output Y^*. So $\frac{y_{lt}}{(1-u)}(0.043 + 0.55)$ measures recruiting costs for the purchase of labor intensive intermediate goods per each worker recruited. The product $u \cdot f$ denotes the number of newly recruited workers hired in a steady state. We evaluate this quantity when unemployment is $u = 0.055$, the ...nding rate is $f = 0.75$, the labor share is equal to $2/3$. This implies an $\frac{L^l}{Y^*}$ ratio of 1.7 percent. If we think that κ is never smaller than 0.5 we have that $\frac{L_t^l + L_t}{Y_t - L_t}$ can never be greater than 3.4 percent. With $\kappa = 1/3$ this ratio rise to a value just above 5 per cent. Our prior for the $\frac{L_t^l + L_t}{Y_t^*}$ is centered around small values. This is intended to penalize parametrization with very large job creation costs.

7 Estimation Results

We next discuss the resulting parameters estimate, the contribution of di¤erent shock to the cyclical ‡uctuations of labor market variable, the model ...t and the welfare implications of our estimates.

[19] Hagedorn and Manovskii (2009) also add some costs from keeping capital idle.

7.1 Parameters estimates

The parameters estimates are reported in Table 4, other steady state quantities are reported in Table 5.

Our estimates imply that data are consistent with a model speci…cation where approximately half of the jobs have rigid wages ($\theta = 0.43$) close to marginal productivity as in Hall (2005) ($\tau = 1$) and half of the jobs are set through Nash bargaining with workers having little (almost no) bargaining power ($\beta = 0.06$). This is implies that steady unemployment is almost identical to the one that would arise in a model speci…cation where all wages are set through Nash bargaining and workers and …rm have almost identical bargaining power— close to a speci…cation where bargaining power is equal to $0.43 + (1 - 0.43) \cdot 0.06 = 0.464$. But over the business cycle, wage rigidity is pervasive. Since wages ‡uctuate little relative to labour productivity wage rigidity is an important mechanism to explain why the labor share is countercyclical.

In our estimates $\kappa = 0.28$ recruiting costs comes from one third in labor units. $\gamma = 0.6$ so that recruiting costs are an important component of job creation costs. $\chi = 4$ which is a bit high but still under a reasonable range. The parameter $\eta = 0.71$ which is close the preferred value proposed by Shimer (2005).

The impact coe¢ cient on the separation rate have the expected sign. A shock to the matching technology leads to an increase in the separation rate, so that the shock is reallocating. A positive stationary neutral technology shocks makes the separation rate falls. A shock to the long run level of technology make the separation rate increase as in the Schumpeterian creative destruction story by Michelacci and Lopez-Salido (2007). A rise in the discount factor or an increase in the e¤ort costs of working make the separation rate falls. Similarly an improvement in the investment speci…c technology make the separation rate fall although the e¤ects are small. An increase in the exogenous demand component make the separation rate increases. Adjustment costs to capital are positive but in the plausible range. The short run marginal productivity of hours decrease very fast ($\phi = 0.16$).

Table 5 reports the value of relevant steady state quantities. The unemployment rate, the …nding rate and the labor share have all very plausible value. The ratio of recruiting costs to output is around 4 per cent. The $\frac{C}{C^e}$ ratio is about 0.93 is a bit higher than the value proposed buy Shimer (2009). The $\frac{C}{Y}$ and $\frac{K}{Y}$ ratios are plausible.

χ	4.04	τ	1.00	ρ_m	0.81
$\frac{c}{c^e}$	0.93	ς	0.85	$bound_{me}$	4.00
ν	2.10	dd	0.00	$bound_f$	4.00
fss	0.41	$case\_correl$	2.00	$bound_s$	4.00
α	0.37	π_a	5.48	$bound_v$	4.00
δ	0.03	π_z	-3.68	σ_a (%)	0.60
ϕ	0.16	π_q	0.00	σ_z (%)	0.39
T''	0.03	π_{var}	-0.01	σ_q (%)	0.01
μ_z (%)	0.29	π_ψ	-0.19	σ_φ (%)	32.81
μ_q (%)	0.01	π_b	9.45	σ_ψ (%)	1.11
g_{ss}	0.18	π_d	-0.01	σ_λ (%)	1.93
λ	0.04	π_m	-0.12	σ_b (%)	0.09
h_{ss}	0.38	ρ_a	0.95	σ_d (%)	1.93
η	0.71	ρ_φ	0.68	σ_m (%)	1.65
$\overline{r}$	2.39	ρ_ψ	0.99	σ_u^{me} (%)	0.26
κ	0.28	ρ_λ	0.89	σ_f^{me} (%)	0.61
γ_1	0.61	ρ_b	0.99	σ_s^{me}	0.92
β	0.06	ρ_d	0.97	σ_v^{me}	0.14
θ	0.43				
logPost	-1469.44				
logLikel	-1348.50				

Table 4: Parameters' estimates

The cost of involuntary unemployment can be measured by the ratio

$$\frac{(1-\theta)W^b + \theta W^r}{|H|}$$

which is close to seven per cent. This cost is sizeable.

C	1.37	W^r	0.99	$\frac{C}{C^e}$	0.93
f	0.88	$\overline{\omega}$	1.76	$\frac{L}{Y}$	0.04
$\underline{\omega}$	-1.74	μ_ω	1.74	L^*	0.69
V	0.99	$G(\underline{\omega})$	0.00	$\frac{K}{Y}$	7.86
u	0.05	$G(\overline{\omega})$	0.94	ν	2.10
n	0.04	P^b	0.93	Ψ_0	0.01
N	0.80	P^r	0.01	Ψ_1	1.06
$S(h)$	1.07	H	-6.65	w^b	1.55
Y	2.57	Ω	0.28	w^r	1.74
C^e	1.47	π	0.28	y_l	0.62
I	0.58	p	2.04	u (%)	4.69
L	0.08	r	1.22	$\frac{L^e+L}{Y}$	4.25
D	0.45	$\frac{C}{Y}$	0.59	$\frac{L}{Y}$	0.03
K	19.59	$\frac{I}{Y}$	0.23	$\frac{L^e}{Y}$	0.01
W^b	0.06				

Table 5: Relevant steady state quantities

7.2 Variance covariance decompositions

We will next discuss shocks and their identi…cation.

1. Neutral technology shocks: identi…ed by their long run e¤ects on labour productivity.

2. Investment speci…c shocks: identi…ed by the long run e¤ects on the relative price of investment. In the absence of the relative price of investment their identi…cation may be more di¢ cult, but Alejandro is an expert on this.

3. Stationary shock to job productivity. This will operate as a standard RBC shock. Consumption, output, job …nding rates increase while the separation rate falls. This shock would operate through the intertemporal substitution of labour. So hour per worker are particularly sensitive to this shock.

4. Stationary shock to the investment speci…c technology.

5. Shocks to matching technology. These will tend to generate a positive comovement in the separation rate and in the job …nding rate. The literature on SVAR has usually refereed to these shocks as Reallocative shocks. Davis and Haltiwanger have an AER paper that argues that these shocks are important using sign restrictions in SVAR. Cheremukhin and Echavarria (2009) also argue that they are important to reproduce the cyclical properties of the labor wedge. It is typically di¢ cult to …nd a shock that tend to make both increase (or decrease) the job …nding rate and the job separation rate. This is a nice margin of identi…cation. The recent conventional wisdom is that they are not too important. They are also an important margin to address the criticism by Costain and Reiter (2008) on the Hagedorn and Manovskii (2008) calibration.

6. Shocks to the separation rate. These shocks would have very similar e¤ects of stationary shocks to job productivity (shock 3) but without having a direct e¤ect on productivity. These shocks a¤ect the outside option of workers and they are another important source of variation. They are also an important margin to address the criticism by Costain and Reiter (2008) on the Hagedorn and Manovskii (2008) calibration. The return to working hours in the job is also unchanged so maybe hours per worker would respond less than in shock 3.

7. Shocks to the discount factor. Primiceri, Schaumburg, and Tambalotti (2006) argue they are important. I believe you. Moreover we do not have monetary policy and sticky prices, so those shocks may also capture "interest rate shocks" due to changes in monetary policy. In a reduced form they may also capture "new shocks" since they characterize the relevance of the future relative to the present.

8. Shocks to the marginal disutility of working: In the model we have a …xed cost of working and a parameter that multiplies the marginal disutility of working. We are not sure whether it is better adding the shock to the …xed cost or the marginal disutility of working. Given Hall we should go for latter, since this will amplify the response of hours per worker.

9. Aggregate demand shock. These will characterize trade balances shocks or changes in net export and oil prices or shocks to government expenditures.. They are nice shocks, since most of the transmission will occur through consumption. It seems one natural source of variation of one of the factors emphasized by Hall (2009a).

Table 6 presents the variance covariance decomposition of the different shocks. We find that just two shocks explain around 80 per cent of the cyclical fluctuations of unemployment, vacancies, the job finding rate and the job separation rates. Stationary shocks to the neutral technology are the most important driving force of cyclical fluctuations: they explain around 60 per cent of the cyclical fluctuation of the labor market and they operate pretty much as in a standard real business cycle model proposed by Prescott (1986). The second most important shocks are exogenous shocks to the separation rate, that explain around twenty percent of the cyclical fluctuations of vacancies and unemployment. Overall these two shocks alone account well for the magnitude of the cyclical fluctuations of the labor market.

	a	dz	dq	ϕ	ψ	λ	b	d	m	me-u	me-f	me-s	me-v
y	84.2	12.0	0	0.30	0.3	1.9	0.9	0.1	0.2	0	0	0	0
c	28.9	14.9	0	10.6	0.3	0.3	33.1	12.0	0.0	0	0	0	0
i	63.9	0.1	0	03.9	0.1	1.9	23.7	6.2	0.2	0	0	0	0
Δy	84.2	12.1	0	0.2	0.3	2.0	0.9	0.1	0.2	0	0	0	0
Δc	27.5	14.4	0	13.9	0.2	0.3	31.9	11.5	0.0	0	0	0	0
Δi	62.6	0.1	0	5.7	0.1	1.9	23.2	6.1	0.3	0	0	0	0
u	70.7	12.2	0	1.6	0.3	9.7	3.6	0.1	1.4	0.3	0	0	0
f	65.1	9.4	0	2.2	0.3	0.1	3.5	0.4	10.9	0	8.1	0	0
s	61.5	11.9	0	1.2	0.2	20.9	4.0	0	0.2	0	0	0	0
v	54.3	5.9	0	6.6	0.4	17.4	4.7	1.4	9.0	0	0	0	0
h	1.1	0.7	0	5.1	72.4	0.4	15.0	5.3	0.0	0	0	0	0
ls	02.7	0.0	0	84.7	0.0	0.0	0.6	0.7	11.2	0	0	0	0
L	12.3	4.1	0	8.1	0.1	67.1	4.2	0.9	3.2	0	0	0	0
P	84.6	11.5	0	1.6	0.2	0.4	1.2	0.3	0.1	0	0	0	0
n	57.2	11.3	0	2.2	0.2	24.5	3.4	0.0	1.2	0	0	0	0
N	63.7	9.6	0	0.6	5.6	12.1	6.7	0.2	1.4	0	0	0	0

Table 6: Variance Covariance Decomposition: Percentage of variance explined by the shock in colun at periodicity 6 to 32 quarters. me-i stands for measurement error of variable i.

7.3 Fits

See mapping on cross correlation we do a pretty good job.

8 Welfare inefficiencies

There is a relevant literature that argues that the combination of labor market search frictions with ex post bargaining cause inefficiencies in the process of reallocation of workers across productive unit, see for example Hosios (1990) and Caballero and Hammour (1996, 1998). But are these search ine¢ ciencies on average large or small? [20] We …nd that ine¢ ciencies due to search, wage rigidity and adjustment costs have small e¤ects on the determination of steady state unemployment, but they imply important costs over the business cycle. We also study whether search ine¢ ciencies have been particularly harmful in some business episode or in response to some speci…c shocks.

Statistic	Decentralized	E¢ cient
	(%)	(%)
u	4.69	5.85
f	87.6	69.3
Λ	4.00	4.00
L^{*}/Y	4.25	3.37
C/Y	58.7	58.8
I/Y	23.3	23.2
C	136.6	136.7
C^{e}	146.7	146.8

Table 7: Welfare comparison: steady state (percentage)

Statistic	Decentralized	E¢ cient	SD(E)/SD(D)
	(%)	(%)	
u-rate	15.3	8.50	0.38
Finding	11.3	3.60	0.20
Separation	8.90	8.90	1.00
Vacancies	24.6	6.90	0.18
Hours per worker	1.56	1.54	0.99
Consumption	0.87	0.87	1.00
Investment	2.76	2.97	1.08

Table 8: Median Standard deviation

E¢ cient allocation: unemployment and vacancies are positively correlated separation and vacancies are positively correlated

This table indicates that the responses to neutral shocks is mitigated (but consumption and investment respond more). Matching shocks become more relevant

[20] There is substantial debate on this. For example the directed search literature tends to argue that search ine¢ ciencies should vanish in equilibrium, see for example Acemoglu and Shimer (1999). Michelacci and Suarez (2006) show instead that directed search is not enough to eliminate search ine¢ ciencies in the presence of adverse selection problems.

	a	dz	ϕ	ψ	λ	b	d	m
u	59	12	5	0	18	3	0	3
f	7	2	0	15	0	0	0	44
v	39	6	14	0	23	5	0	12
y	86	11	0	0	0	1	0	0
c	34	14	10	0	0	29	13	0
i	65	1	4	0	0	22	6	0

Table 9: Contribution of shocks: e¢ cient vs decentralized: Percentage of variance explained by each shock: periodicity 6-32 quarters

9 Conclusions

We estimated a "real model". Nominal rigidities are missing. Monetary policy is not analyzed. It would also be interesting to endogenize the separation rate along the lines of Mortensen and Pissarides (1994, 1998). We leave these as extensions for further research.

There are potentially other sources of ine¢ ciencies stemming for example from …nancial markets. Some have argued that these are paramount to the analysis of business cycles. Incorporating them into the model would be interesting. But any of these additional ine¢ ciencies would propagate in the system and would generate welfare costs also due to the ine¢ ciencies emphasized in this paper.

References

[1] Acemoglu, D. and Shimer, R. (1999): "Holdups and E¢ ciency with Search Frictions," International Economic Review, 40, 827-851.

[2] Aghion, P. and Howitt, P. (1994): "Growth and Unemployment," Review of Economic Studies, 61, 477-494.

[3] Andolfatto, D. (1996): "Business Cycles and Labour-Market Search", American Economic Review, 86, 112-132.

[4] Balakrishnan, R. and Michelacci, C. (2001): "Unemployment Dynamics across OECD Countries", European Economic Review, 45, 135-165.

[5] Barro, Robert J. (1977): "Long-Term Contracting, Sticky Prices, and Monetary Policy," Journal of Monetary Economics 3, 305–16.

[6] Basu, S. and Kimball, M., (2002): "Long-Run Labor Supply and the Elasticity of Intertemporal Substitution for Consumption," mimeo University of Michigan.

[7] Becker, G. (1965): "A Theory of the Allocation of Time," Economic Journal 75, 493-517.

[8] Blanchard, Olivier J. and Diamond, Peter, (1989): "The Beveridge Curve," Brookings Papers on Economic Activity, 1989, (1), pp. 1-60.

[9] Blanchard, Olivier J. and Diamond, Peter, (1990): "The Cyclical Behavior of Gross Flows of Workers in the United States," Brookings Papers on Economic Activity 1990, (2), . 85-155.

[10] Caballero, R. and Hammour, M. (1994): "The Cleansing E¤ect of Recessions," American Economic Review, 84, 1350-1368.

[11] Caballero, R. and Hammour, M. (1996): "On the Timing and E¢ ciency of Creative Destruction," Quarterly Journal of Economics, 111, 805-852.

[12] Caballero, R. and Hammour, M. (1998): "The Macroeconomics of Speci…city," Journal of Political Economy 106-4: 724-767.

[13] Caballero, R. (2007): Speci…city and the Macroeconomics of Restructuring, MIT Press.

[14] Cheremukhin, A. and Echavarria, P. (2009): "The Labor Wedge as a Matching Friction", Mimeo UCLA University

[15] Costain, J. S. and Reiter, M. (2008): "Business Cycles, Unemployment Insurance, and the Calibration of Matching Models," Journal of Economic Dynamics and Control 32, 1120-1155.

[16] Davis, S. and Haltiwanger, J. (1999): "On the Driving Forces behind Cyclical Movements in Employment and Job Reallocation," American Economic Review, 89, 1234-1258.

[17] Den Haan, W. , Ramey, G. and Watson, J. (2000): "Job Destruction and Propagation of Shocks," American Economic Review, 90, 482-498.

[18] Fisher, J. (2006): "The Dynamic E¤ects of Neutral and Investment-Speci…c Technology Shocks," Journal of Political Economy, 114, 413-451.

[19] Galí, J. (1999): "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?," American Economic Review, 89, 249-271.

[20] Gertler, M. and Trigari, A. (2009): "Unemployment Fluctuations with Staggered Nash Wage Bargaining," Journal of Political Economy, 117(1): 38-86.

[21] Greenwood, J. Hercowitz, Z. and Krusell, P. (1997): "Long-Run Implications of Investment-Speci…c Technological Change", American Economic Review, 87, 342-362.

[22] Greenwood, J., Hercowitz, Z. and Krusell, P. (2000): "The role of Investment-Speci…c technological change in the Business Cycle," European Economic Review 44, 91-115.

[23] Haefke, C., M. Sonntag, and Van Rens, T. (2007): "Wage Rigidity and Job Creation," Institute for the Study of Labor (IZA): Bonn, Discussion Paper 3714.

[24] Hagedorn, M. and I. Manovskii (2008): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," American Economic Review, 98, 1692-1706.

[25] Hall, R. (2005): "Job Loss, Job Finding, and Unemployment in the US Economy over the past Fifty Years," NBER Macroeconomic Annual, Vol. 20, Mark Gertler and Kenneth Rogo¤ (editors).

[26] Hall, R. (2005): "Employment Fluctuations with Equilibrium Wage Stickiness," American Economic Review, 95, 50-65.

[27] Hall, R. (2009a): "Reconciling Cyclical Movements in the Marginal Value of Time and the Marginal Product of Labor," Journal of Political Economy, 117, 281-323.

[28] Hall, R. (2009b): "By How Much Does GDP Rise If the Government Buys More Output? ", forthcoming Brookings Paper on Economic Activity, September.

[29] Hall, R. and Milgrom, P. (2008): "The Limited In‡uence of Unemployment on the Wage Bargain," American Economic Review 98:4, 1653–1674

[30] Hornstein, A. Krusell, P. and Violante, G. (2005): "The Replacement Problem in Frictional Economies: A Near Equivalence Result", Journal of the European Economic Association, 3, 1007-1057.

[31] Hornstein, A., P. Krusell, and G. L. Violante. (2005). "Unemployment and Vacancy Fluctuations in the Matching Model: Inspecting the Mechanisms."Federal Reserve Bank of Richmond Economic Quarterly, 91, 19-51.

[32] Hornstein, A. Krusell, P. and Violante, G. (2007): "Technology-Policy Interaction in Frictional Labor Markets," Review of Economic Studies 74(4), 1089-1124.

[33] Hosios, A. (1990). "On the E¢ ciency of Matching and Related Models of Search and Unemployment," Review of Economic Studies, 57, 279-298.

[34] Jackman, R., Layard, R., and Pissarides, C (1989), "On Vacancies", Oxford Bulletin of Economics and Statistics, 51, 377-394.

[35] Jovanovic, B. and Lach, S. (1989): "Entry, Exit, and Di¤usion with Learning by Doing," American Economic Review, 79, 690-699.

[36] Justiniano, A., and Primiceri, G. E. (2008): "The Time Varying Volatility of Macroeconomic Fluctuations," American Economic Review 98(3), 604-641.

[37] Justiniano, A. Primiceri, G. and Tambalotti, A. (2008): "Investment Shocks and the Business Cycle", Mimeo, Northwestern University.

[38] Kydland, Finn E and Prescott, Edward C. (1982): "Time to Build and Aggregate Fluctuations," Econometrica 50, 1345-70.

[39] Lilien, David (1982): "Sectoral Shifts and Cyclical Unemployment," Journal of Political Economy 90, 777-93.

[40] López-Salido, David and Rabanal Pau (2006): "Government Spending and Consumption-Hours Preferences," La Caixa Working Paper Series No. 02/2006.

[41] Malin, B. (2008): "Lower-Frequency Macroeconomic Fluctuations: Living Standards and Leisure," Board of Governors of the Federal Reserve System, mimeo.

[42] Merz, M. (1995): "Search in the Labour Market and the Real Business Cycle", Journal of Monetary Economics, 36, 269-300.

[43] Michelacci, C. and Lopez-Salido, D. (2007): "Technology Shocks and Job Flows," Review of Economic Studies 74, 1195–1227

[44] Michelacci, C. and Suarez, J. (2006): "Incomplete Wage Posting," Journal of Political Economy, 114, 1098-1123.

[45] Mortensen, D. and Pissarides, C. (1994): "Job Creation and Job Destruction in the Theory of Unemployment," Review of Economic Studies, 61, 397-415.

[46] Mortensen, D. and Pissarides, C. (1998): "Technological Progress, Job Creation, and Job Destruction," Review of Economic Dynamics, 1, 733-753.

[47] Pissarides, C. (2000): Equilibrium Unemployment Theory, 2nd Edition, MIT Press.

[48] Pissarides, C. (2009): "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?," forthcoming Econometrica.

[49] Pissarides, C. and Vallanti, G. (2007): "The Impact of TFP Growth on Steady-State Unemployment," International Economic Review 48, 607-639.

[50] Prescott, E. (1986): "Theory Ahead of Business Cycle Measurement," Federal Reserve Bank of Minneapolis Quarterly Review, 10, 9-22.

[51] Primiceri, G., Schaumburg, E. and Tambalotti, A. (2006): "Intertemporal Disturbances," NBER Working Paper 12243.

[52] Ríos-Rull, J. and Santaeulalia-Llopis, R. (2009): "Redistributive Shocks and Productivity Shocks," Mimeo Washington University in St. Louis.

[53] Rogerson, R. and Shimer R. , (2010): "Search in Macroeconomic Models o fthe Labor Market," NBER Working Paper 15901.

[54] Rotemberg, J. (2006): "CyclicalWages in a Search-and-Bargaining Model with Large Firms," Centre for Economic Policy Research, London, Discussion Paper no. 5791.

[55] Sbordone, Argia (2006): “An Optimizing Model of U.S. Wage and Price Dynamics,” Journal of Money, Credit and Banking, forthcoming.

[56] Shimer, R. (2004): “The Consequences of Rigid Wages in Search Models,” Journal of the European Economic Association (Papers and Proceedings): 2, 469-479.

[57] Shimer, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” American Economic Review, 95(1): 25-49.

[58] Shimer, R. (2005): “Reassessing the Ins and Outs of Unemployment,” Mimeo, University of Chicago.

[59] Shimer, R. (2010): Labor Markets and Business Cycles, Princeton University Press

[60] Smets F. and Wouters, R. (2007): “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” American Economic Review, 97, 586-606.

[61] Solow, R. (1960): “Investment and Technical Progress”, in Mathematical Methods in the Social Sciences, 1959, edited by Arrow, K., Karlin, S. and Suppes, P., Stanford University Press, Stanford California.

[62] Trabandt, M. and Uhlig, H. (2009) “How Far Are We From the Slippery Slope? The La¤er Curve Revisited,” NBER Working Paper 15343.

[63] Toledo, M. and Silva, J. (2009a) “Investment Shocks and Cyclical Fluctuations in the Labor Market,” Mimeo, Universidad Carlos III Madrid.

[64] Toledo, M. and Silva, J. (2009b) “The Unemployment Volatility Puzzle: The Role of Matching Cost Revisited,” Mimeo, Universidad Carlos III Madrid.

[65] Violante, G. (2002): “Technological Acceleration, Skill Transferability and the Rise of Residual Inequality,” Quarterly Journal of Economics, 117, 297-338.

[66] Yashiv, E., 2000. “The Determinants of Equilibrium Unemployment,” American Economic Review 90 (5): 1297–1322.

[67] Yashiv, E. (2007): Labor Search and Matching in Macroeconomics,” European Economic Review, 51, 1859-1895.

A Consumption-Hour complementarity

Shimer (2009) proposes a nice simple preferences speci…cation which satis…es balanced growth while allowing for the possibility that consumption and hours could be complement. In particular he assumes that

$$U\left(c,h\right)=\frac{c^{1-\chi}\left[S(h)\right]^{\chi}-1}{1-\chi},\quad \chi>0. \tag{76}$$

where

$$S(h)=1+(\chi-1)\left(\Psi_0+\Psi_1\frac{h^{1+\nu}}{1+\nu}\right),\quad \nu>0.$$

represents the disutility of working with Ψ_0 and $\Psi_1>0$ parametrizing the disutility of work and $\nu>0$ equal to the inverse of the Frisch labor supply elasticity (see below). The parameter $\chi>0$ determines both the intertemporal substitution for consumption and the complementarity between consumption and hours. When $\chi=1$, these preferences nest the standard separable case with log-preferences since they converge to $\log c-\left(\Psi_0+\Psi_1\frac{h^{1+\nu}}{1+\nu}\right)$. The case $\chi>1$ is of particular interest, since this implies the marginal utility of consumption is higher when households work which Shimer (2009) among others argue to be consistent with standard Beckerian model of time allocation. Similar but less elegant formulations which impose less transparent restrictions on parameters were proposed by Basu and Kimball (2002), Malin (2008), and Hall and Milgrom (2008).

It is easy to verify that:

$$\begin{aligned}
U_c &= \left(\frac{S(h)}{c}\right)^{\chi}>0\\
U_h &= -\chi\left(\frac{c}{S(h)}\right)^{1-\chi}\Psi_1 h^{\nu}<0\\
U_{cc} &= -\frac{\chi U_c}{c}<0\\
U_{hh} &= -\chi\left(1-\chi\right)^2\left(\frac{c}{S(h)}\right)^{1-\chi}\frac{1}{S(h)}\left(\Psi_1 h^{\nu}\right)^2-\chi\nu\left(\frac{c}{S(h)}\right)^{1-\chi}\Psi_1 h^{\nu-1}\\
&= -\chi\left[\frac{\left(1-\chi\right)^2\Psi_1 h^{\nu}}{S(h)}+\frac{\nu}{h}\right]\left(\frac{c}{S(h)}\right)^{1-\chi}\Psi_1 h^{\nu}<0
\end{aligned}$$

while

$$U_{ch}=\chi(\chi-1)\left(\frac{S(h)}{c}\right)^{\chi}\frac{\Psi_1 h^{\nu}}{S(h)}$$

which is positive if only if $\chi>1$. From the previous expressions it follows that these preferences impose a constant elasticity of intertemporal substitution, χ, and that $\chi>1$ will guarantee consumption and hours worked complementarity $U_{ch}>0$. These preferences also imply that the optimal condition for the supply if labor $-U_h = U_c w$ reads as follows:

$$\chi \Psi_1 h^{\nu} = \frac{S(h)w}{c} \tag{77}$$

The Frisch elasticity of labor supply is de…ned as the elasticity of hours worked to the wage rate, given a constant marginal utility of consumption which is generally di¤erent from $\frac{U_h}{hU_{hh}}$. In particular under our perference the Frisch elasticity is constant and equal to $\frac{1}{\nu}$, see Rogerson and Shimer (2010) page 22. According to Shimer (2010) and Trabandt and Uhlig (2009) our utility function is the only one that 1) it is consistent with a balanced growth and 2) it has a constant Frisch elasticity of labor supply. To see that the Frisch elasticity is indeed constant, notice that the marginal utility of cosumption is equal to $u_c = \left(\frac{S(h)}{c}\right)^{\chi}$ so a constant u_c implies that $\frac{S(h)}{c}$ is constant. Then (77) immediately implies that the elasticity of hours worked to the wage rate, keeping constant the marginal utility of consumption, is equal to $\frac{1}{\nu}$.

B Derivation of the social planner allocation

Let K_t, I_{t-1}, e_t, Σ_t denote the current capital stock, previous period investment, employment, and the nine dimensional vector containing value of the exogenous driving forces, $\Sigma_t = (z_t, q_t, a_t, \varphi_t, d_t, \lambda_t, m_t, b_t, \psi_t)$, respectively. The social planner problem of our economy can then be written as follows

$$\begin{aligned} W(K_t, I_{t-1}, e_t, \Sigma_t) &= \max_{C_t, C_t^e, I_t, n_t, h_t,} e_t \left\{ \frac{(C_t^{e*})^{1-\chi} [S_t(h_t)]^{\chi}}{1-\chi} \right\} + (1-e_t) \left(\frac{C_t^{*1-\chi}}{1-\chi} \right) - \frac{1}{1-\chi} \\ &\quad + B_t E_t [W(K_{t+1}, I_t, e_{t+1}, \Sigma_{t+1})] \end{aligned} \tag{78}$$

which is subject to the following set of transition equations

$$\begin{aligned} K_{t+1} &= (1-\delta) K_t + e^{q_t + \varphi_t} \left[1 - T\left(\frac{I_t}{I_{t-1}} \right) \right] I_t, \\ e_{t+1} &= (1 - \Lambda_{t+1}) e_t + n_t \end{aligned}$$

the law of motion of the nine exogenous driving forces and to the three following identities:

$$N_t = e_t h_t^{\phi} - \kappa \left(\frac{X_t}{p_t} \right)^{1-\kappa} \overline{r} (M_t u_t)^{-\eta_0} n_t^{\eta_1} \tag{79}$$

$$u_t = 1 - e_t \tag{80}$$

$$A_t Z_t K^{\alpha} N_t^{1-\alpha} = e_t C_t^e + (1-e_t) C_t + I_t + D_t + (1-\kappa) r_t \overline{r} (M_t u_t)^{-\eta_0} n_t^{\eta_1} \tag{81}$$

$$p_t = (1-\alpha) A_t Z_t \left(\frac{K_t}{N_t} \right)^{\alpha} \tag{82}$$

$$r_t = p_t^{\kappa} X_t^{1-\kappa} \tag{83}$$

B.1 Savings decisions

By deriving with respect to the consumption level of employed and unemployed we obtain that the allocation chosen by the social planner should satisfy:

$$\pi_t^* = \frac{1}{C_t^{*\chi}} = \left(\frac{S_t(h_t^*)}{C_t^{e*}} \right)^{\chi}$$

where π_t^* is the Lagrange multiplier of the budget constraint (81). After adding *'s this is equation (24) in the text. By deriving with respect to the level of investment expenditures in (78) and after taking into account (79) and (81) we obtain:

$$\pi_t^* = e^{q_t + \varphi_t} \left[1 - T\left(\frac{I_t}{I_{t-1}} \right) - T'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] \Omega_t^* + B E_t \left(\frac{\partial W_{t+1}}{\partial I_t} \right). \tag{84}$$

where $\Omega_t^* \equiv B_t E_t (\partial W_{t+1} / \partial K_{t+1})$ is the current period expected shadow value of capital at time $t+1$, (here $\partial W_{t+1} / \partial K_{t+1}$ denotes the partial derivative of the value function of next period with respect to capital) while $\partial W_{t+1} / \partial I_t$ is the next period value of the partial derivative of the value function with respect to I_t. The envelope condition with respect to capital reads as:

$$\frac{\partial W_t}{\partial K_t} = \left[B_t E_t \left(\frac{\partial W_{t+1}}{\partial K_{t+1}} \right) (1-\delta) + \pi_t^* e^{q_t + \varphi_t} \alpha A_t \left(\frac{N_t}{K_t} \right)^{1-\alpha} \right].$$

After evaluating this expression in the next period we obtain

$$\frac{\partial W_{t+1}}{\partial K_{t+1}} = \left[(1-\delta)\Omega_{t+1}^* + \pi_{t+1}^* e^{q_{t+1} + \varphi_{t+1}} \alpha A_{t+1} \left(\frac{N_{t+1}}{K_{t+1}} \right)^{1-\alpha} \right] \tag{85}$$

which implies that

$$\Omega_t^* \equiv B_t E_t \left(\frac{\partial W_{t+1}}{\partial K_{t+1}} \right) = B_t E_t \left[(1-\delta)\Omega_{t+1}^* + \pi_{t+1}^* e^{q_{t+1} + \varphi_{t+1}} \alpha A_{t+1} \left(\frac{N_{t+1}}{K_{t+1}} \right)^{1-\alpha} \right]$$

which, after adding *'s, is simply equation (26) in the text. The envelope condition with respect to previous period investment yields

$$\frac{\partial W_t}{\partial I_{t-1}} = e^{q_t + \varphi_t} T' \left(\frac{I_t}{I_{t-1}} \right) \left(\frac{I_t}{I_{t-1}} \right)^2 \Omega_t^*$$

which substituted into (84) yields

$$\pi_t^* = e^{q_t + \varphi_t} \left[1 - T\left(\frac{I_t}{I_{t-1}} \right) - T'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] \Omega_t^* + B_t E_t \left[e^{q_{t+1} + \varphi_{t+1}} T' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \Omega_{t+1}^* \right].$$

After adding *'s, this is equation (25) in the paper

B.2 Choice of hours

By deriving with respect to hours in (78), after taking into account (79) and (81) we immediately obtain (30) amended by adding the corresponding*'s.

B.3 Creation

The …rst order condition with respect to n_t reads as follows:

$$\pi_t^* r_t \bar{r} \eta_1 \left(e^{m_t} u_t \right)^{-\eta_0} n_t^{\eta_1 - 1} = B_t E_t \left(V_{t+1}^* \right) \tag{86}$$

where $V_{t+1}^* \equiv \partial W_{t+1} / \partial e_{t+1}$ is the next period value of a job that produces with technological gap zero (i.e. a newly created job). This is equal to the partial derivative of the value function of next period with respect to the number of jobs. The envelope condition allows to write

$$\begin{aligned} V_t^* &= \pi_t^* (1-\alpha) A_t Z_t \left(\frac{K_t}{N_t} \right)^{\alpha} (h_t^*)^{\phi} + \frac{(C_t^{e*})^{1-\chi} [S_t(h_t)]^{\chi}}{1-\chi} - \pi_t^* C_t^{e*} - \frac{C_t^{*1-\chi}}{1-\chi} + \pi_t^* C_t^* \\ &\quad - \pi_t^* r_t \bar{r} \eta_0 (M_t)^{-\eta_0} (u_t^*)^{-\eta_0 - 1} (n_t^*)^{\eta_1} + B_t E_t \left[(1 - \Lambda_{t+1}) V_{t+1}^* \right] \end{aligned}$$

which is simply the value of the function V_t^* in (67). Thus (86) is equivalent to (68) in the paper.

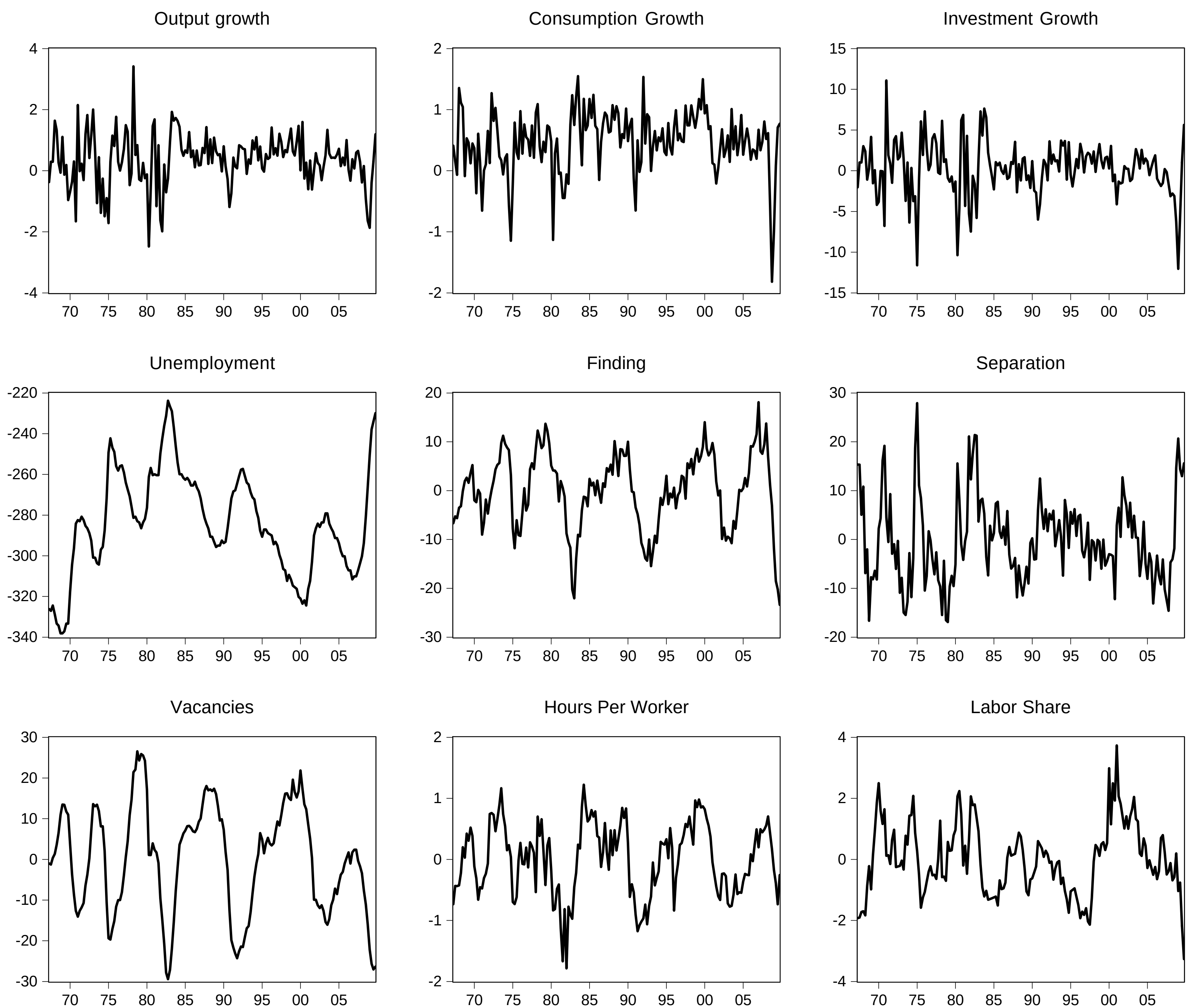
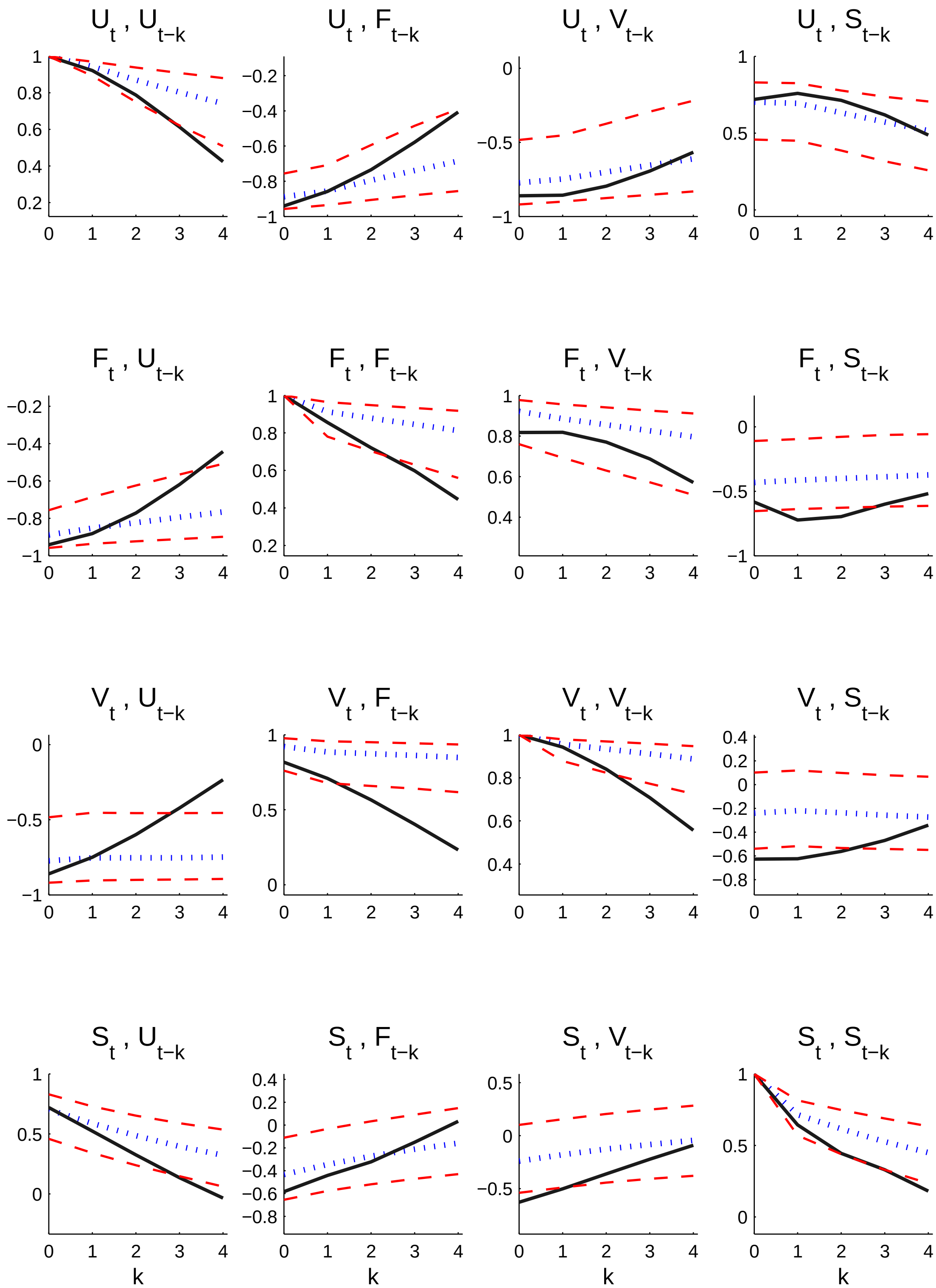

Output growth
Consumption Growth
Investment Growth
Unemployment
Finding
Separation
Vacancies
Hours Per Worker
Labor Share
70
75
80
85
90
95
00
05

Figure 1: Cross−correlations for labor market variables



Note: Data (solid), model median (dotted) and [5,95] posterior bands (dashed)

Figure 2: IRF to Neutral stationary

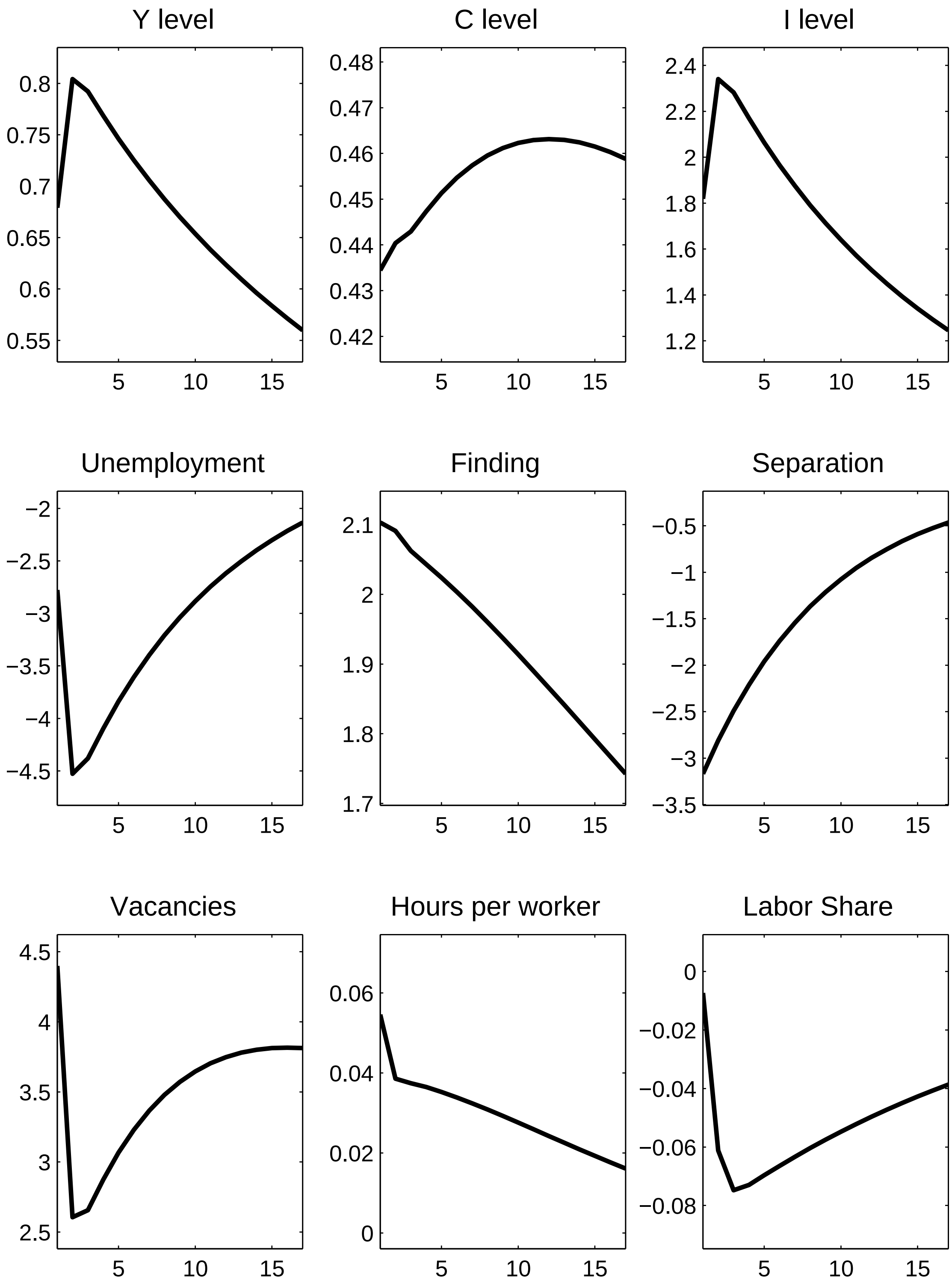


Figure 3: IRF to Neutral stochastic trend

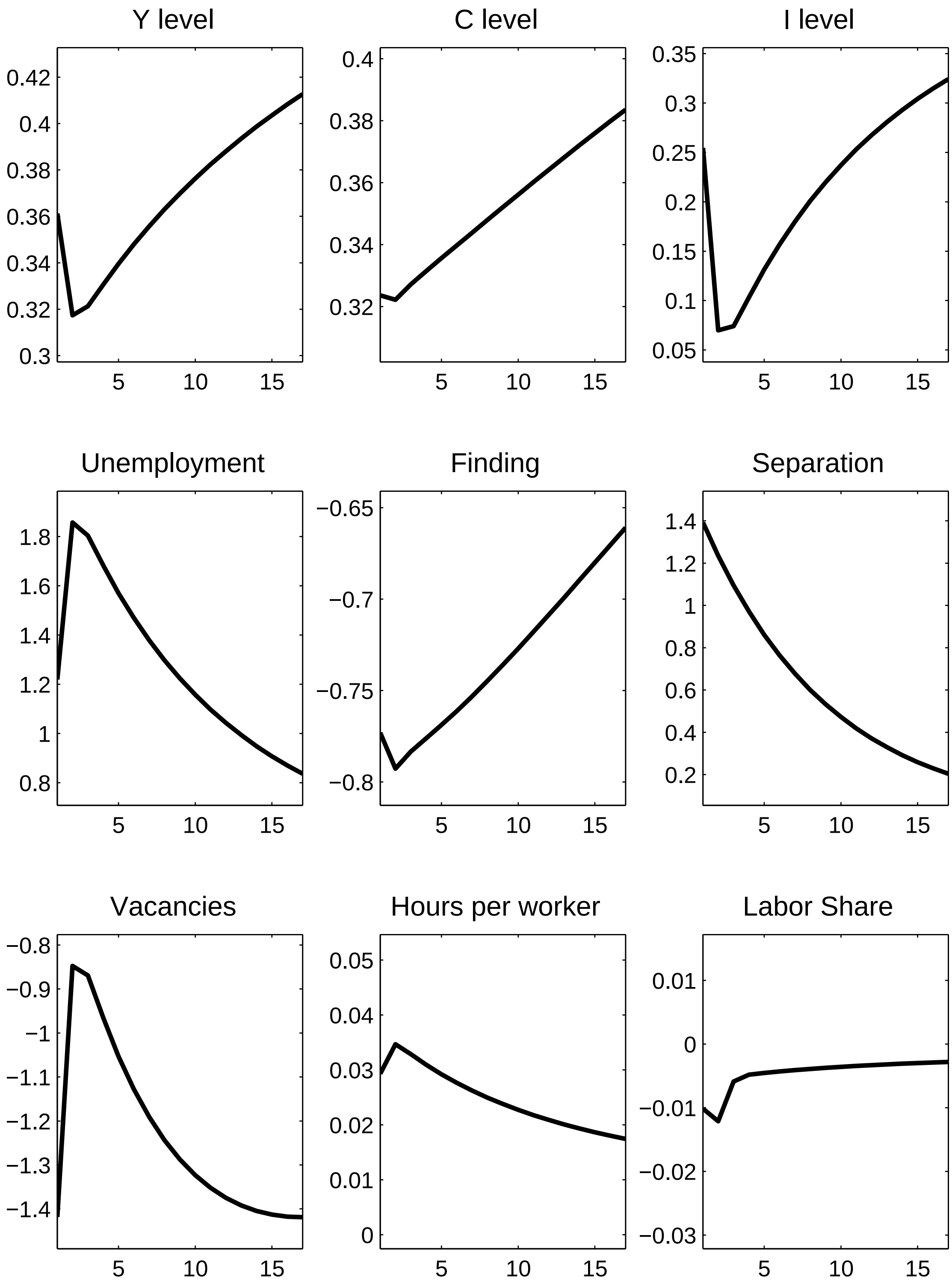


Figure 4: IRF to ME of Investment

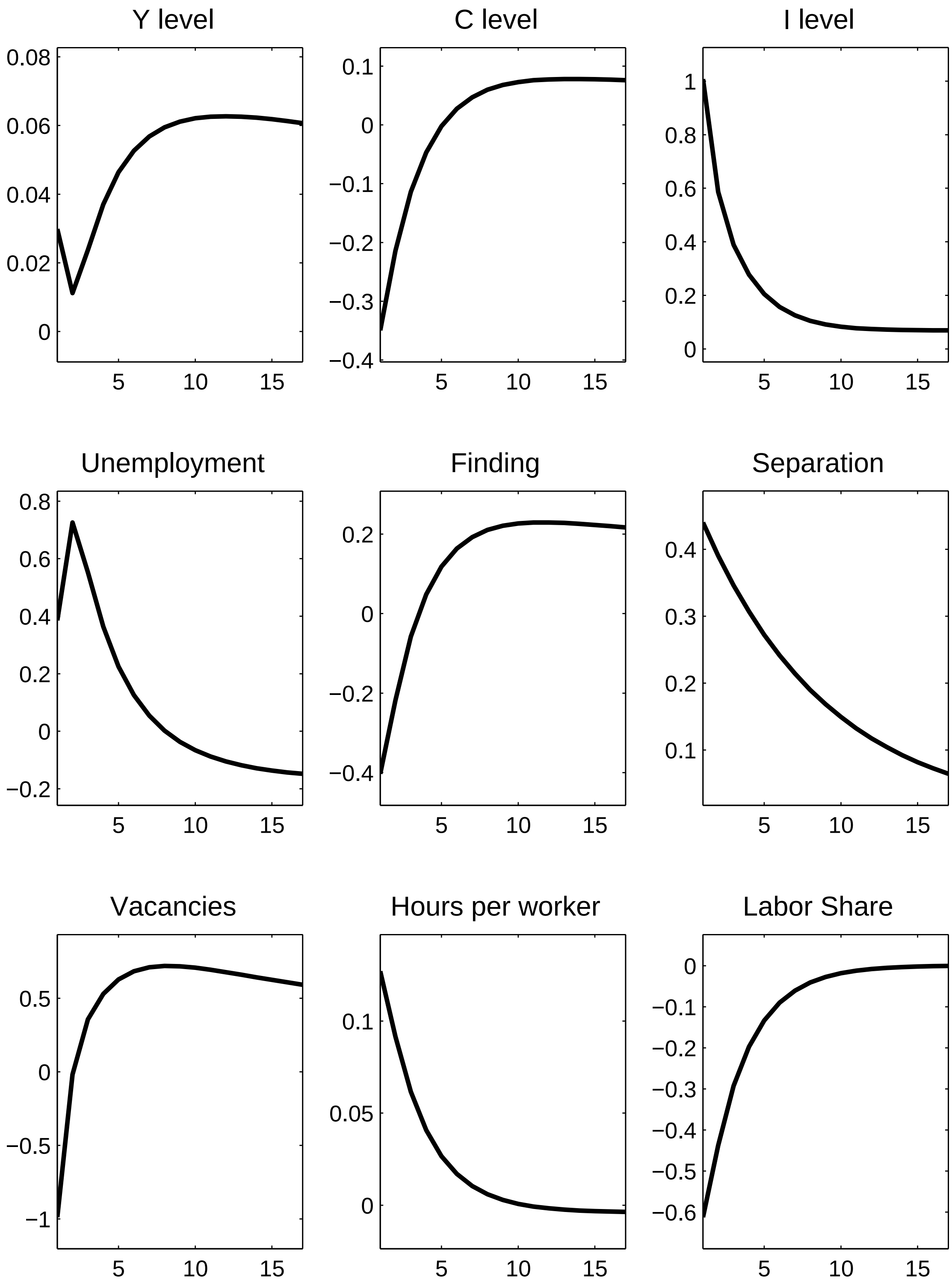


Figure 5: IRF to Labor Preference

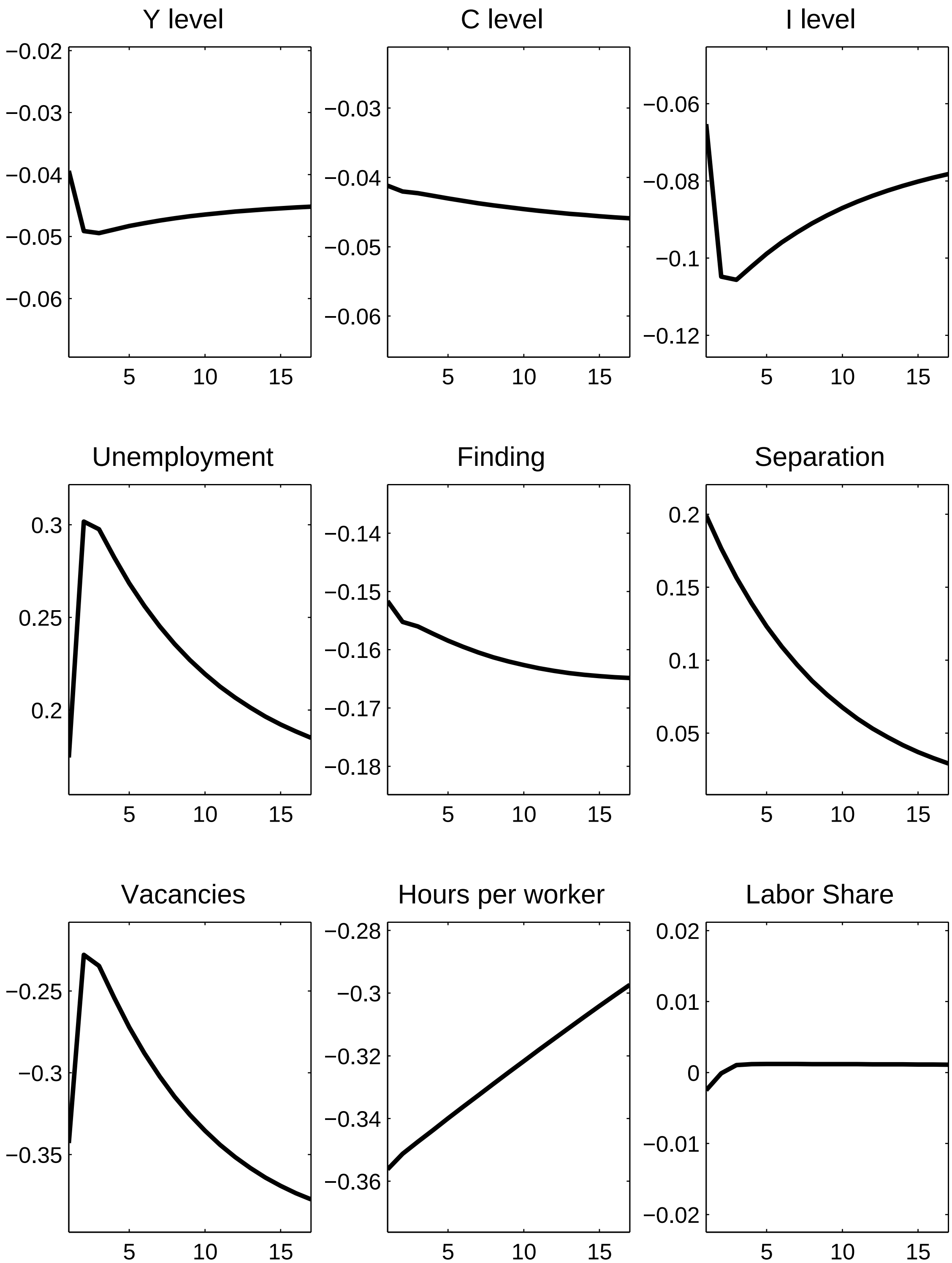


Figure 6: IRF to Job Destruction

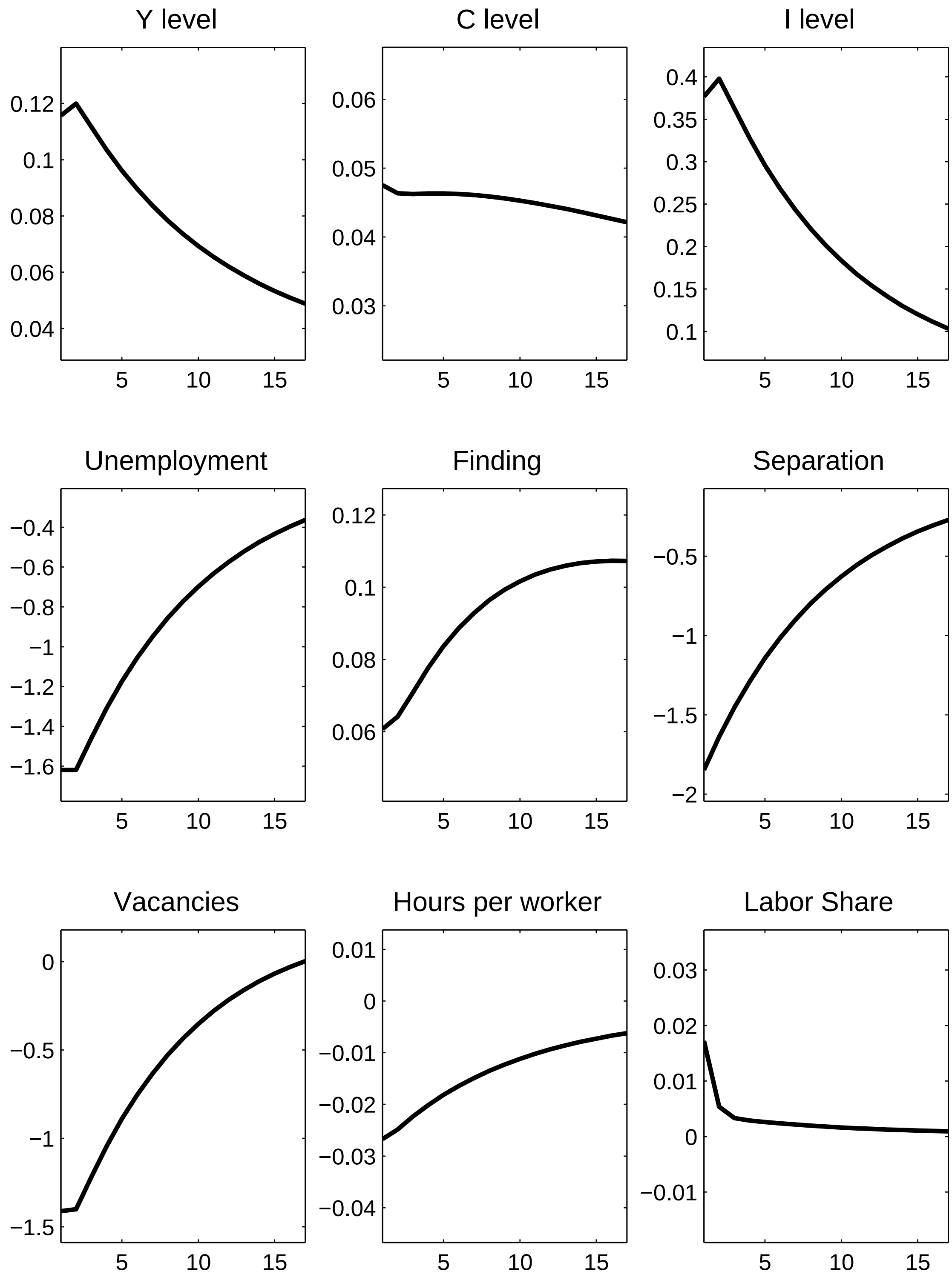


Figure 7: IRF to Discount Factor

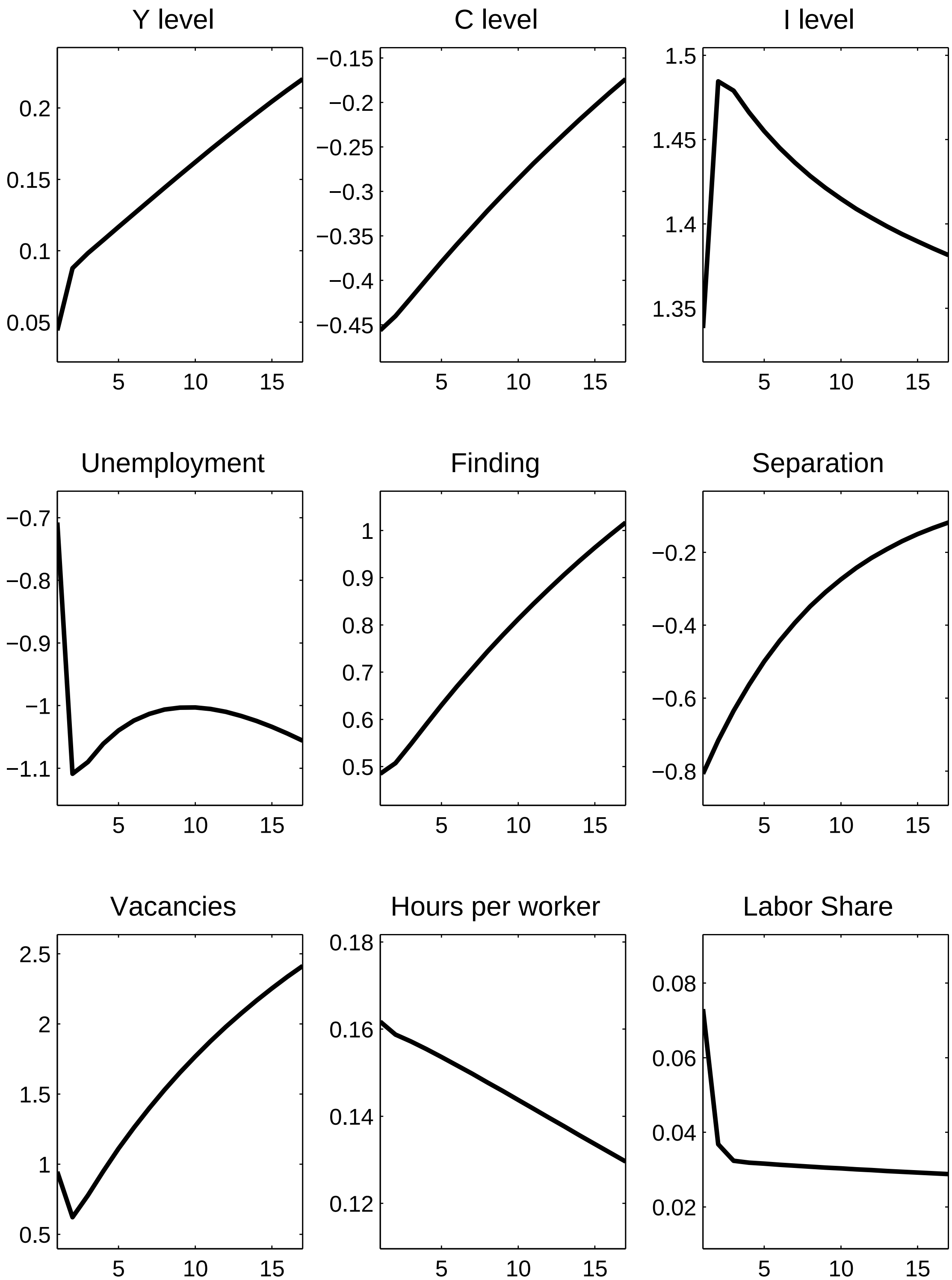


Figure 8: IRF to Aggregate Demand

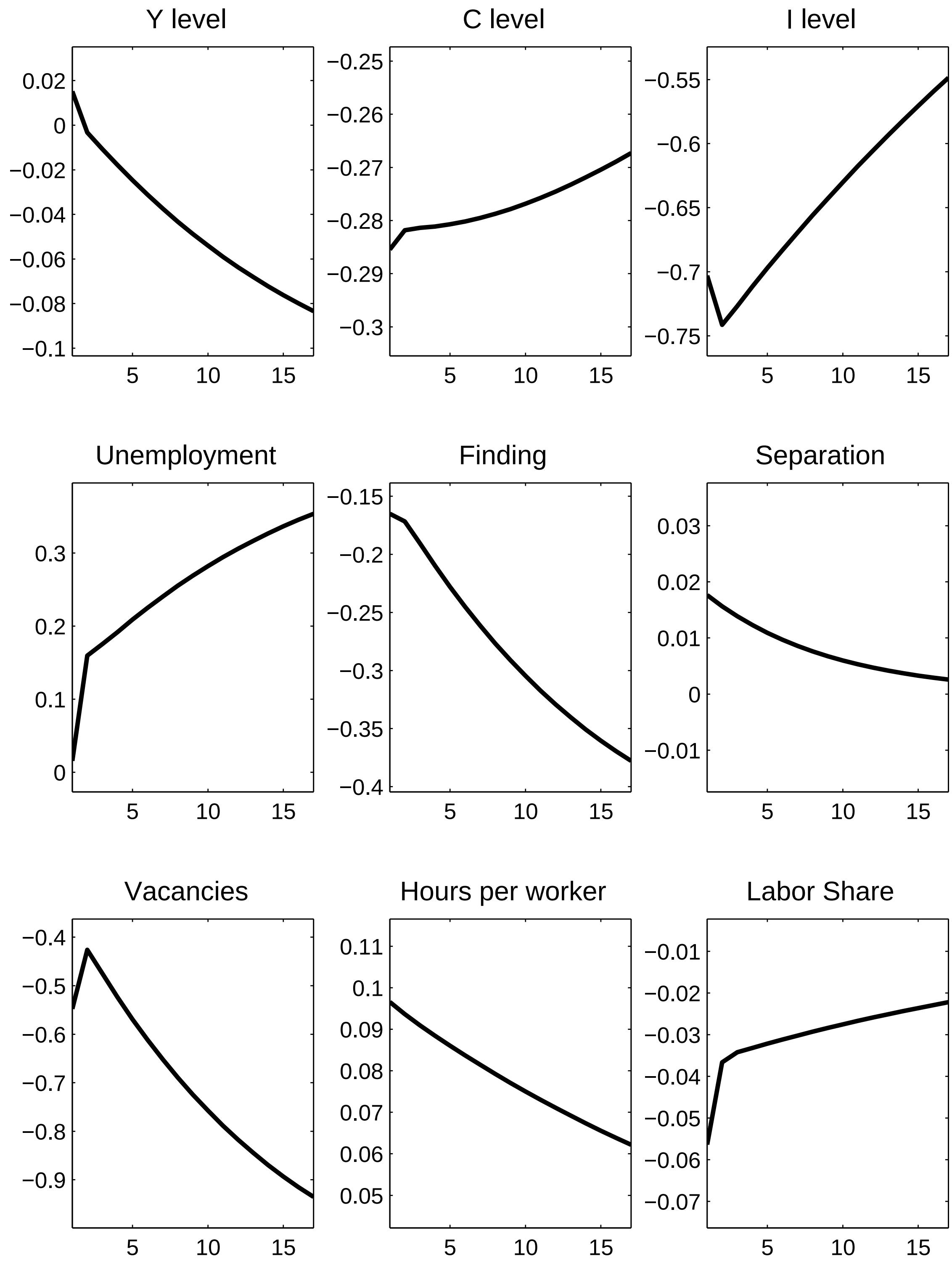


Figure 9: IRF to Matching

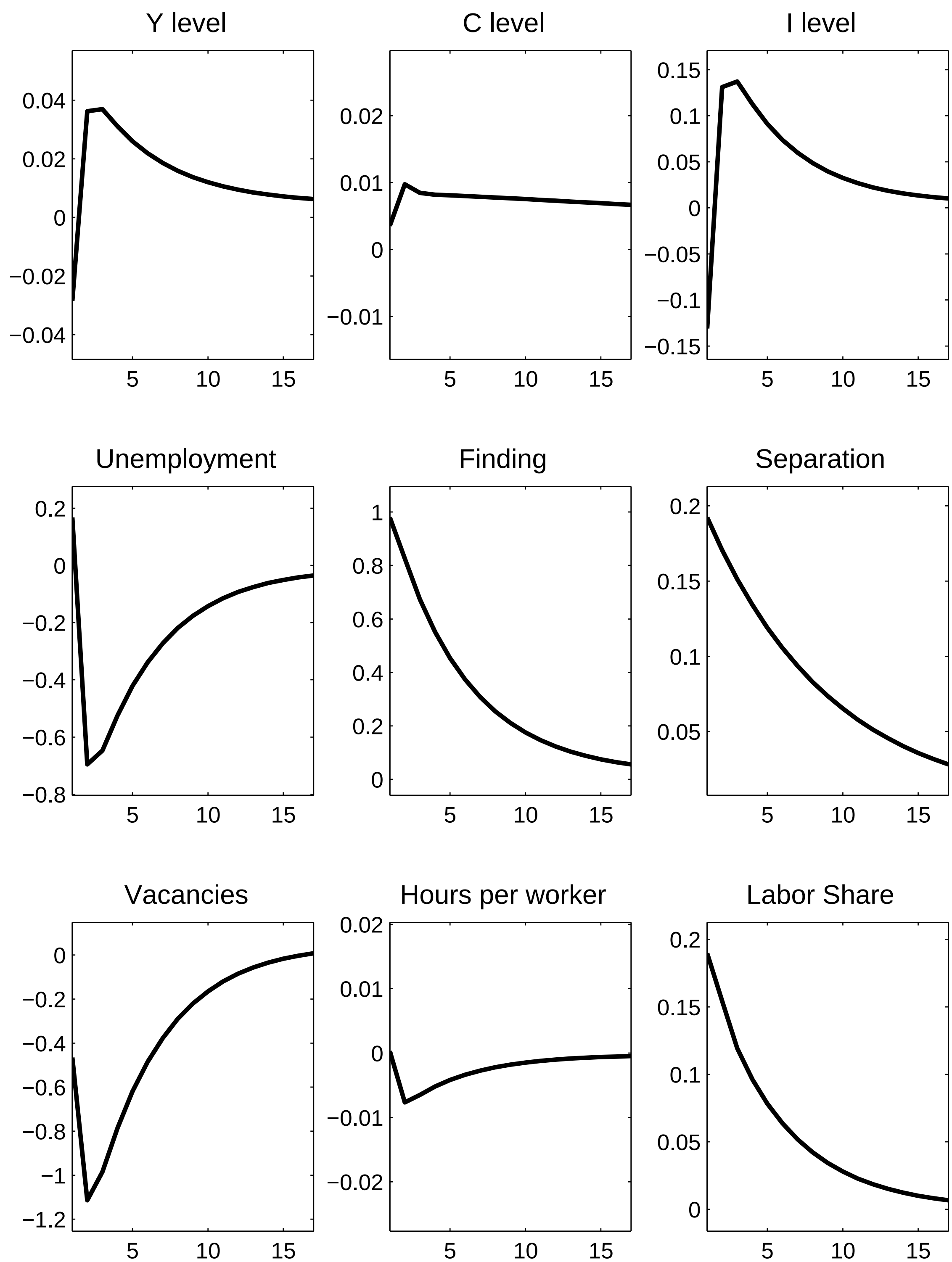
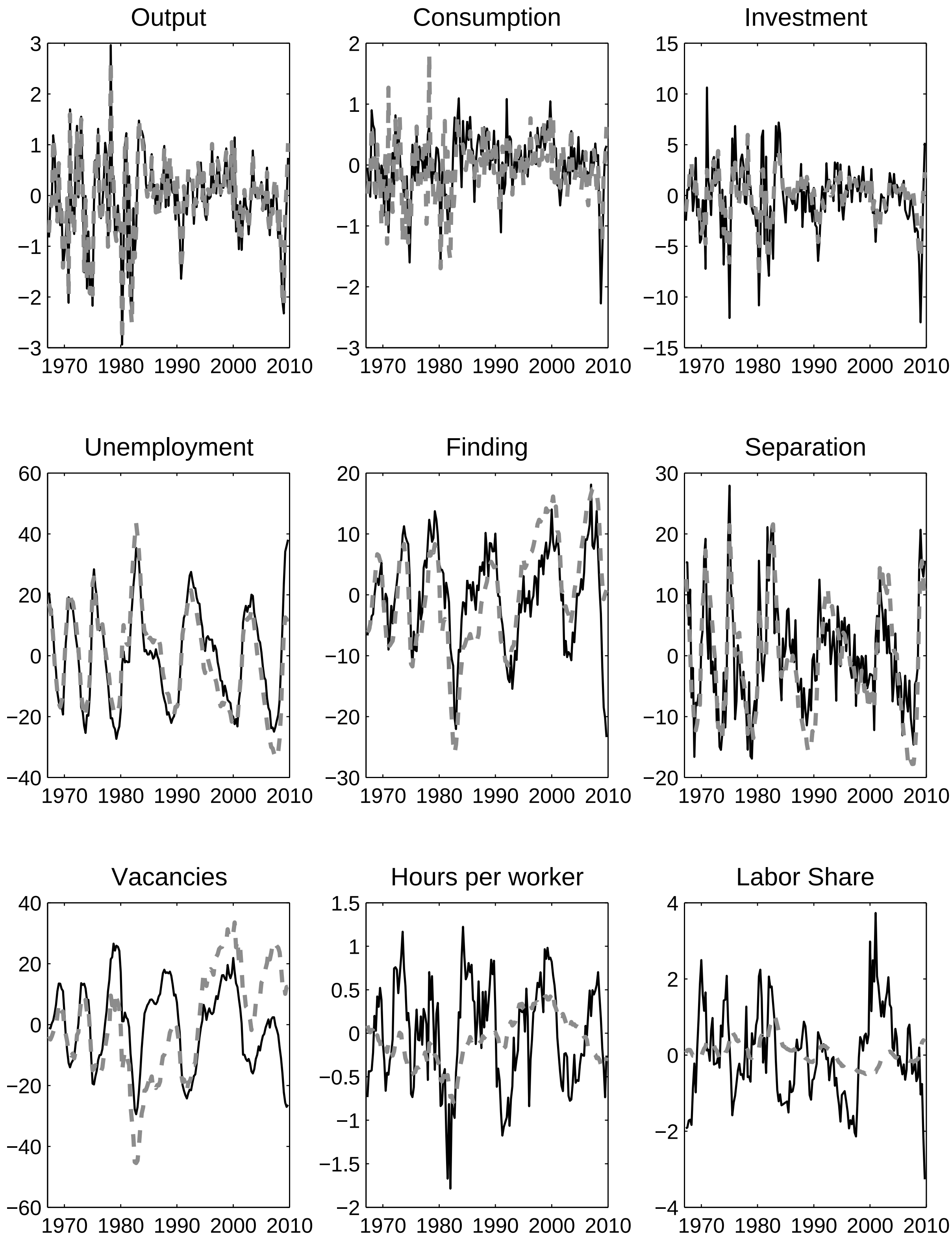
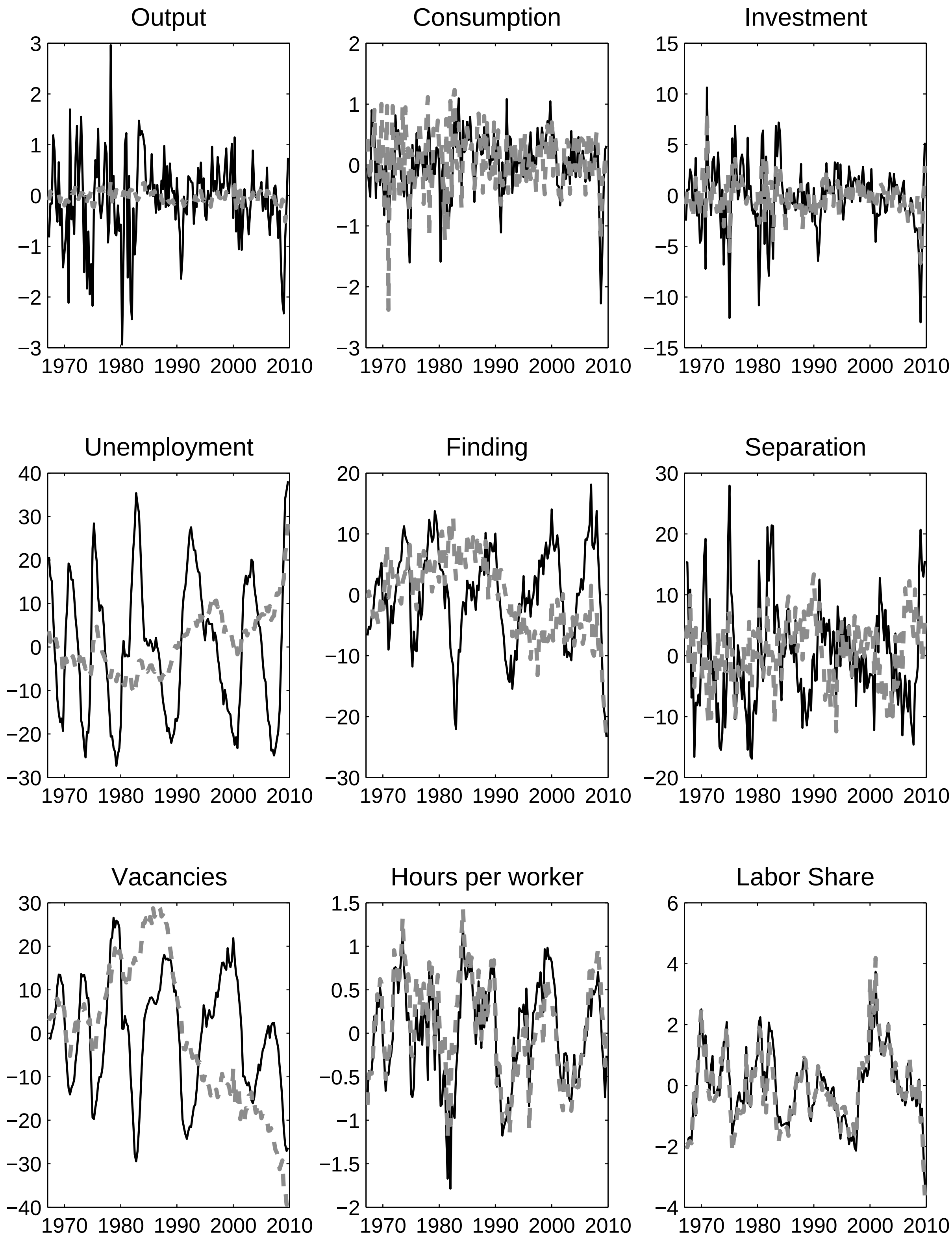


Figure XX: Neutral and Job Destructions shocks only

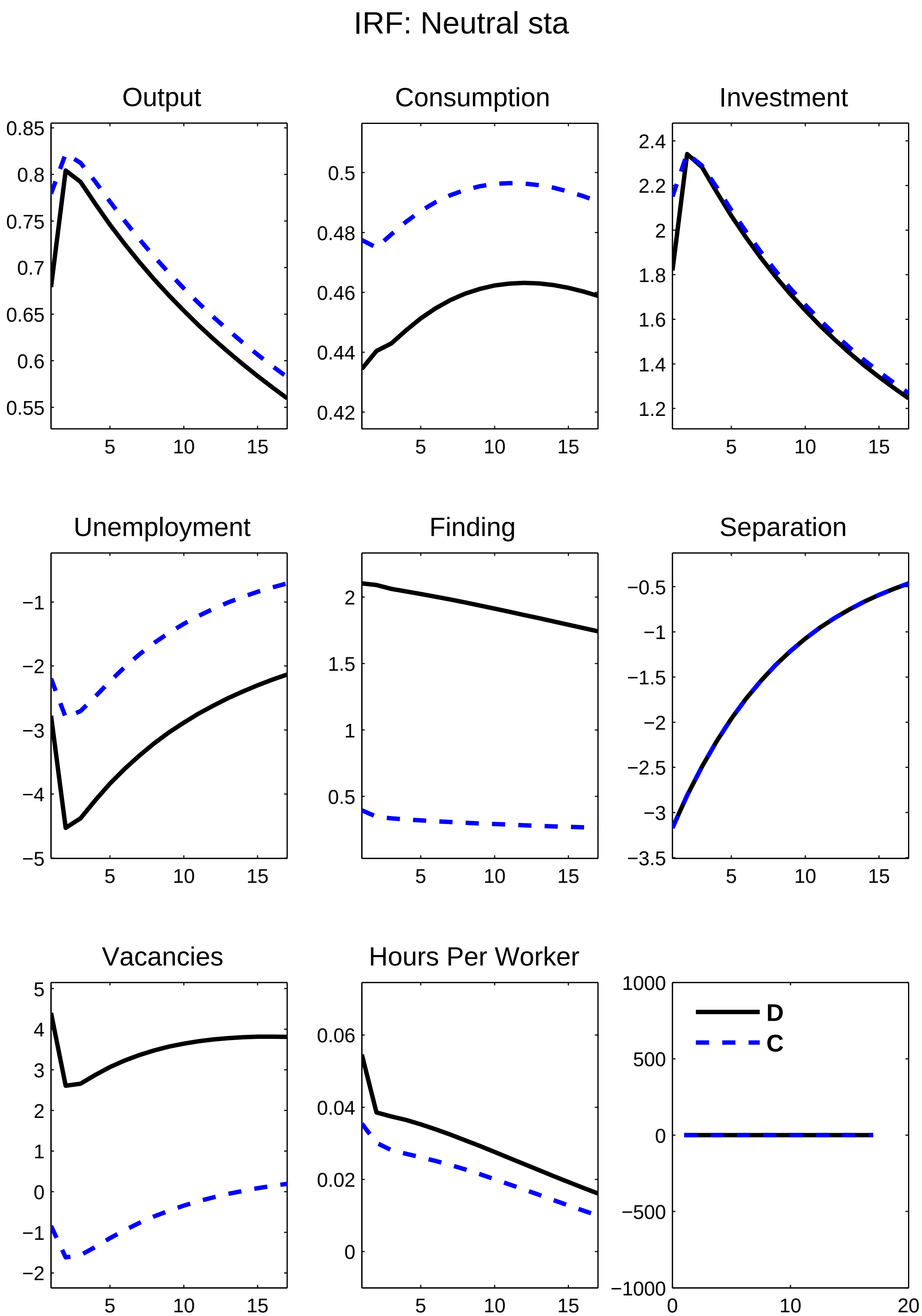


Data (solid black) and counterfactual (grey dashed)

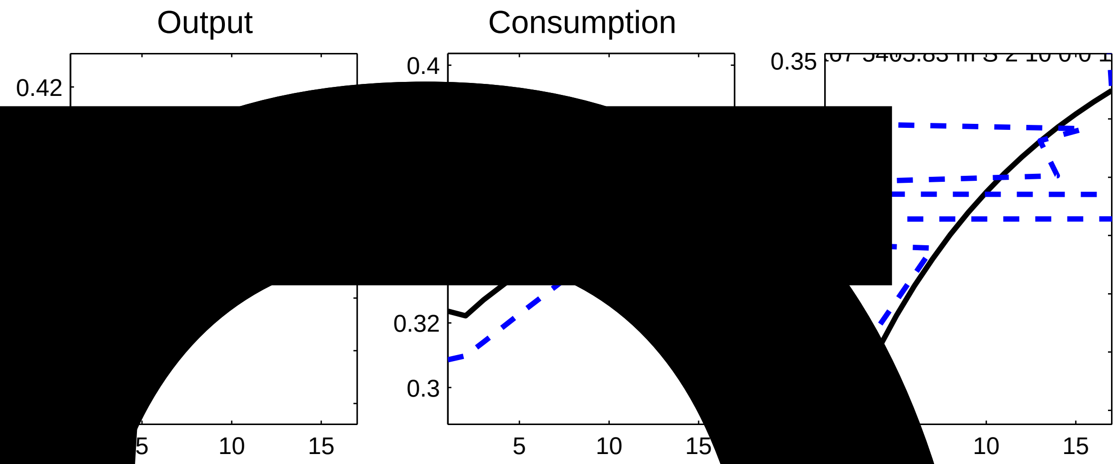
Figure XX: All Other shocks



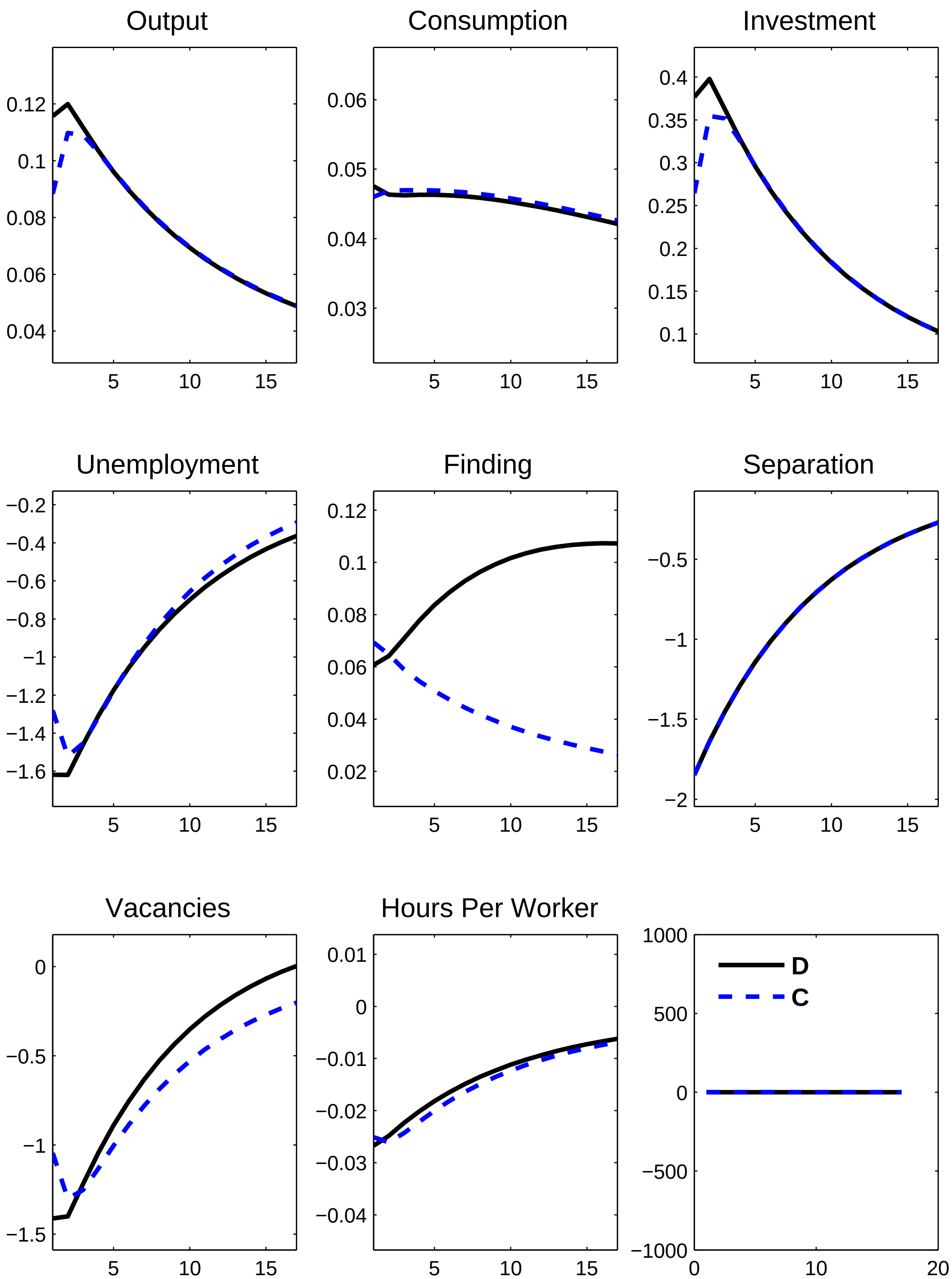
Data (solid black) and counterfactual (grey dashed)



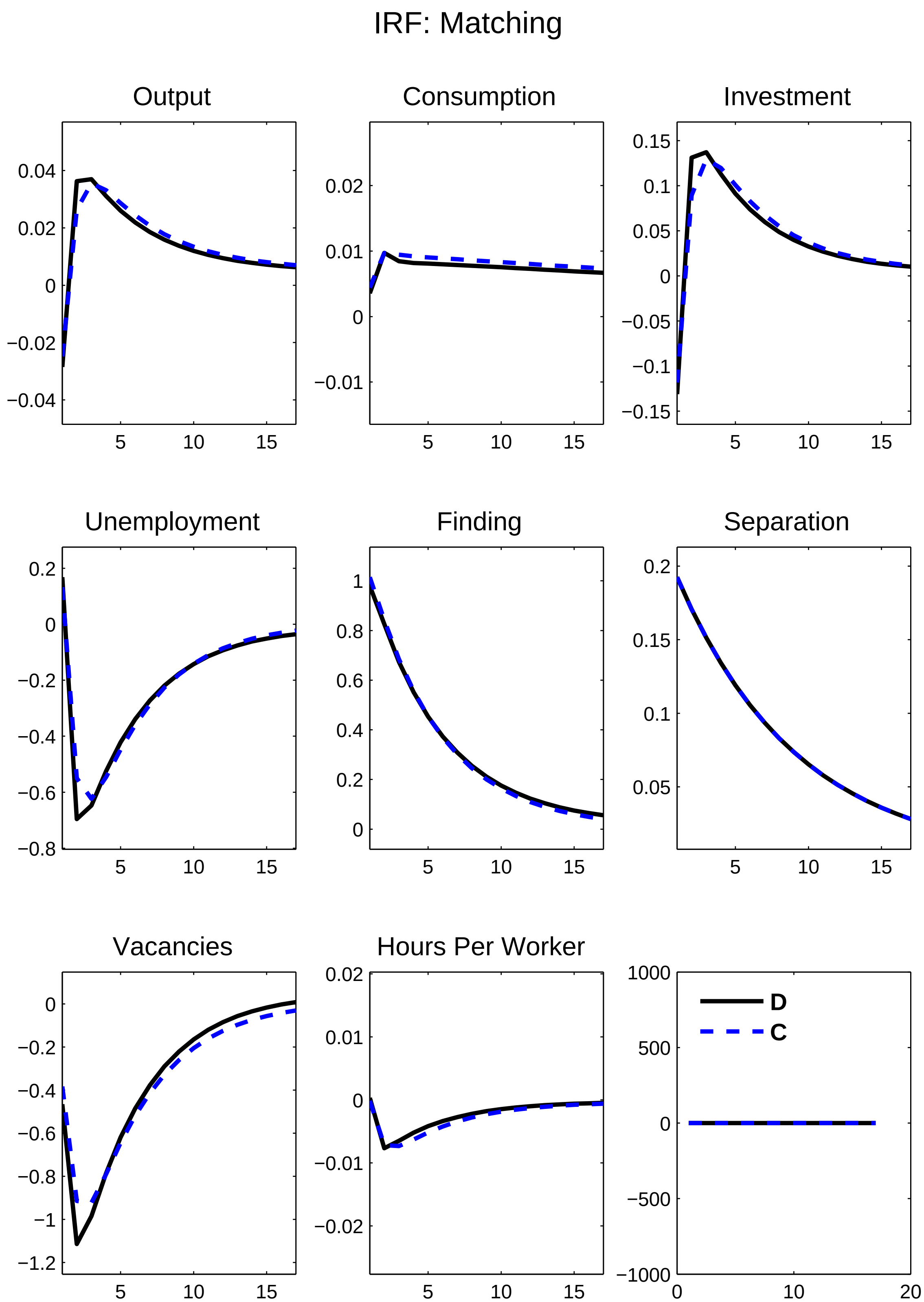
O:\PROJ_LIB\LJM\exog\udsregmebnd\lindtr\1trend z terragona mode\CvsD


Output
Consumption
0.42
0.4
0.32
0.3
0.35
5
10
15

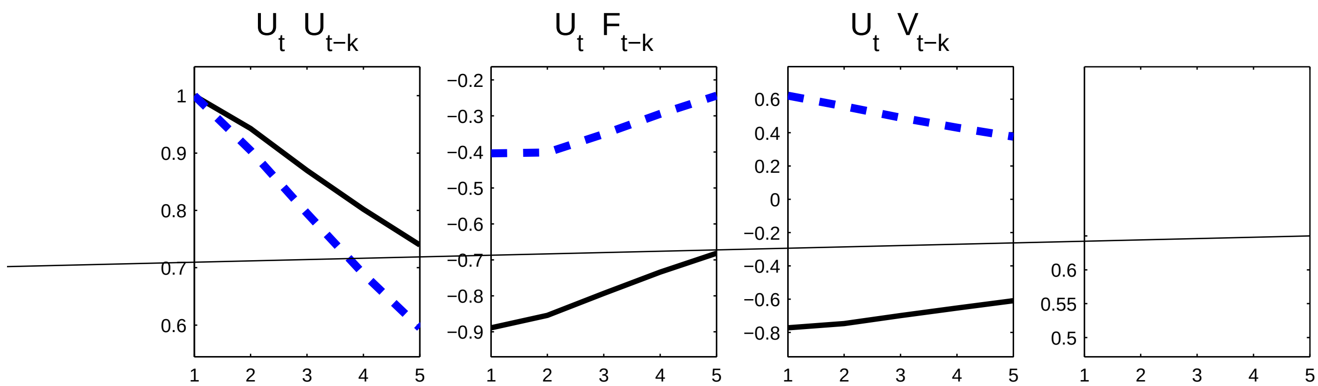
IRF: Job destruction



O:\PROJ_LIB\LJM\exog\udsregmebnd\lindtr\1trend z terragona mode\CvsD



O:\PROJ_LIB\LJM\exog\udsregmebnd\lindtr\1trend z terragona mode\CvsD



U_t U_{t−k}
U_t F_{t−k}
U_t V_{t−k}