Stationary equilibrium distributions in economies with limited commitment

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ESSIM 2010

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$\Rightarrow$ Need models of partial insurance, over and above self-insurance

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     - Positive income shocks lead to jump in consumption, negative income shocks are smoothed

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   - Both focus on marginal cross-sectional distributions.

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Outline

1. Environment
2. Planner's Problem
3. Analytical Results
   3.1 Characterisation of c, y distribution
   3.2 Existence and uniqueness
4. Quantitative model
   - Non-parametric comparison with CEX data
   - GMM estimation
   - Model comparison
   - Sensitivity

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- Large number of agents $i \in \mathbb{I}$ of mass 1

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- Preferences

  $U = E_0 \sum_0^\infty \beta^t u(c_{i,t})$

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- NB: $A(s')$ restricts borrowing against future high income

Partial insurance

- Assumption 1: Perfect insurance infeasible

$$W(z^1) > \sum_0^\infty \beta^t u(Y) \tag{1}$$

- Assumption 2: Autarky not an equilibrium

$$\beta > \frac{u'(z^1)}{u'(z^N)} \tag{2}$$

Recursive formulation of Household's Problem (Alvarez and Jermann 2000)

$$V(z(s), a(s)) = max_{c,\{a(s')\}}\{u(c) + \beta E_s V(z', a(s'))\}$$

$$s.t.\ c + \sum_s a(s')q(s') \leq a(s) + z(s)$$

$$a(s') \geq A(s')$$

$$A(s') = min\{\alpha(s') : V(z(s'), \alpha(s')) \geq W(z(s'))\}$$

Competitive equilibrium

- Prices and decision rules with associated $V(z, a)$ such that
  1. V is the maximum value function associated to the household problem given $q(s')$
  2. V is attained by $c(.), a'(s', .)$
  3. The market for Arrow-Debreu Securities clears $\sum_i a(s) = 0, \ \forall s$

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- Solution to constrained social planner's problem with initial weights $\mu_{i,0} \in \{\mu^1, ..., \mu^K\}$ in a social welfare function
- Solved by Marcet and Marimon (2009) method:
  Planner increases individual weights of
  participation-constrained agents $\mu_i' = \mu_i + \gamma_i$

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- $\Rightarrow$ $R = \frac{1}{\beta}$ implies perfect insurance and vice versa

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- CRRA preferences

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  3. Frequency mass Φ declines geometrically at rate q
  4. With $\Phi(c_{min}) \approx 0$

     $c_{max} =$
     $[\frac{(1-\beta(p+q)+\beta^2(1-p-q)(1-\beta q(\beta R)^{\frac{1-\sigma}{\sigma}}))}{1+(1-p-q)(1-\beta q(\beta R)^{\frac{1-\sigma}{\sigma}})}]^{\frac{1}{1-\sigma}}[(1-\sigma)W(y_0+\epsilon)^{\frac{1}{1-\sigma}}]$

Proposition 1 - Intuition

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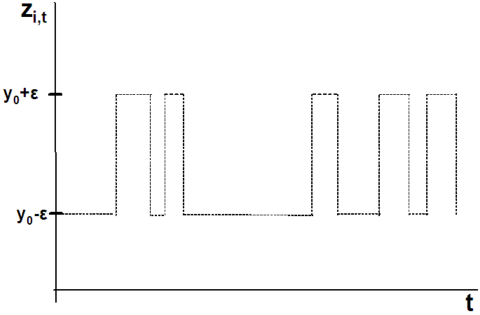
1. Characterisation of joint c, y distribution for given R
2. Existence and uniqueness of market-clearing R

1. Stationary consumption distribution at given R

Example income path of individual i



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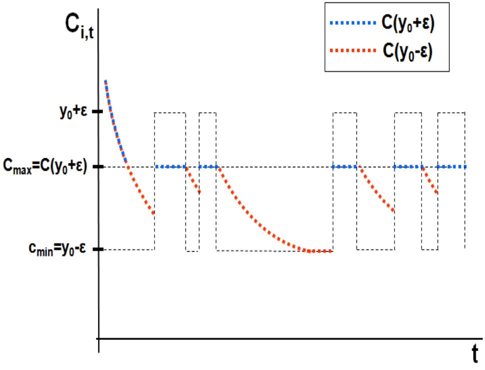
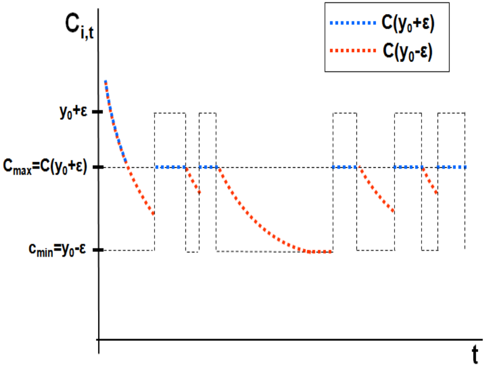
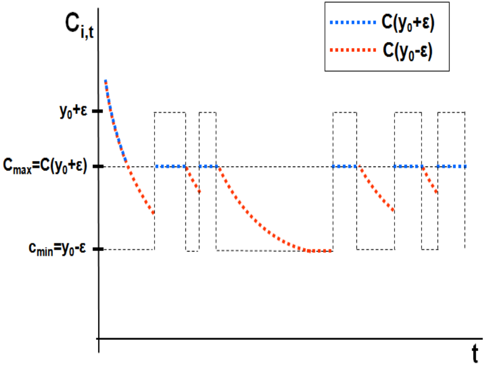


Minimum consumption



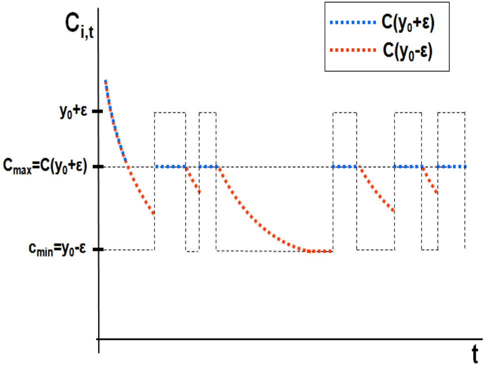
- $c_{min} = y - \epsilon$, solves
  $W(y_0 - \epsilon) = U(c_{min}) + \beta[qW(y_0 - \epsilon) + (1 - q)W(y_0 + \epsilon)]$

Maximum consumption



- c_{max} solves participation constraint at $y_0 + \epsilon$ given LOM for c and c_{min}

Consumption distribution



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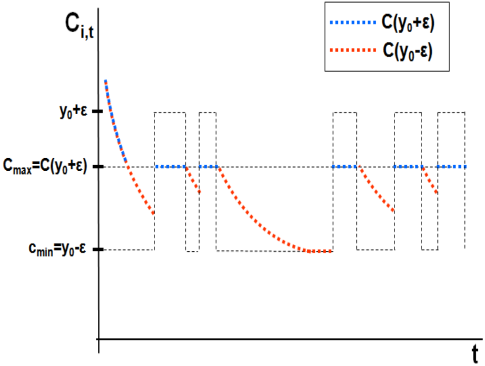

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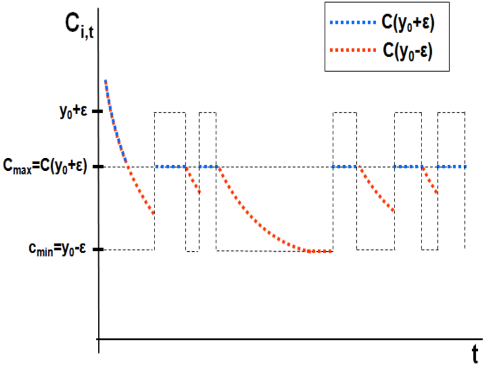
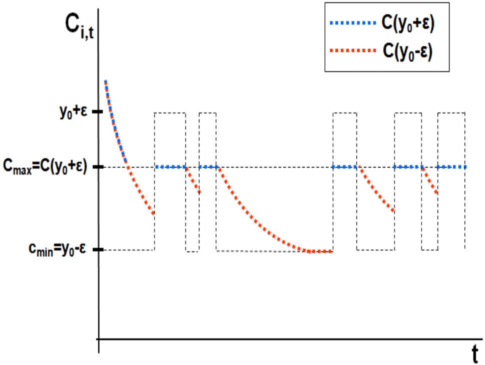


- Support: c_{max}, $c_j = (R\beta)^{\frac{j}{\sigma}} c_{max} \geq c_{min} = y - \epsilon$
- Mass: $\Phi(c_{max}) = \Phi(y + \epsilon)$, $\Phi(c_j) = \Phi(y + \epsilon)(1 - p)q^{j-1}$, $j = 1, \ldots$

Joint moments



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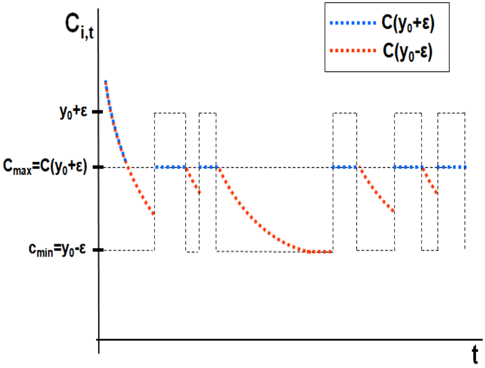

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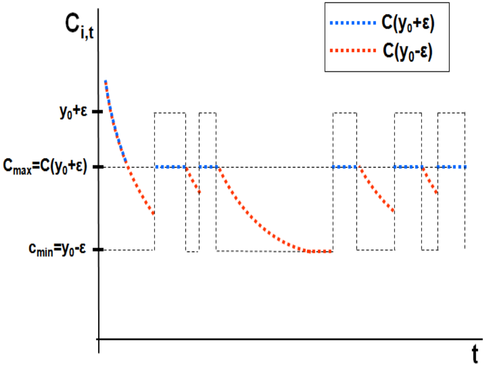

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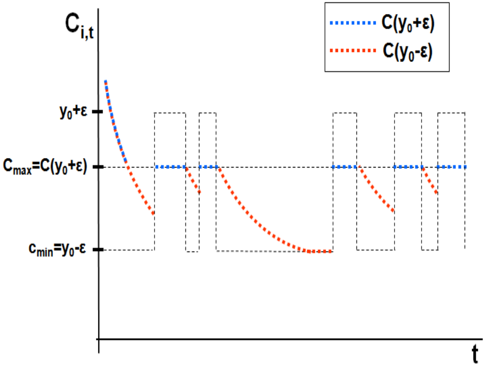

- $Cov(c, y) > 0$, Mean consumption increases in y
- Variance of c decreases in y
- Left-skew of c, right-skew of dc

Corollary

With CRRA preferences and 2 income values, the following is true:

1. The cross-sectional covariance between income and consumption is positive. The covariances between income and both financial returns and wealth are negative.
2. The mean of consumption increases in income. Its conditional variance decreases.

Generalisation of the results

- More than 2 income values ($N > 2$)
- Non-CRRA preferences

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- This paper: Characterisation of consumption distribution based on primal Marcet and Marimon approach
- Defines "excess demand mapping"
  $\Psi : [R^{aut} = \frac{u'(z^1)}{\beta u'(z^N)}, \frac{1}{\beta}[\longrightarrow \mathbb{R}$ as

$$\Psi = \sum c_i \Phi_i - Y \tag{3}$$

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- Shows:

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- Defines "excess demand mapping"
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3. Since R^{-i} is decreasing (increasing) for $R > 1$ ($R < 1$) it underweighs the positive (negative) elements of $\frac{dc_i}{dR}$. So unweighted sum $\frac{d\Psi}{dR} = \sum \frac{dc_i}{dR}\Phi_i$ is positive (negative).

Proof: Maths

1. PC: $V(c_0^m, z^m) - W(z^m) =$
   $\sum_{i=0}^n \beta^i [\pi_{i|m} u(c_i^m) - \sum_j \pi_{ij|m} u(z_{ij})] = 0$
2. Differentiate to get:
   $0 = \sum_{i=0}^n \beta^i \pi_{i|m} u'(c_i) dc_i = u'(c_0^m) \sum_{i=0}^n \pi_{i|m} R^{-i} dc_i$
3. Differentiate LOM $U_c(c_{i-1}) = \beta R U_c(c_i)$

   $$\frac{dc_i}{dR} = \frac{\frac{u''(c_{i-1})}{u'(c_{i-1})}}{\frac{u''(c_i)}{u'(c_i)}} \frac{dc_{i-1}}{dR} - \frac{u'(c_i)}{u''(c_i)} \frac{1}{R} \doteq \alpha_1(c_i, c_{i-1}) \frac{dc_{i-1}}{dR} + \alpha_2(c_i), \alpha_2 >$$

   If $\frac{dc_{i-1}}{dR} < (>) 0$ then $\frac{dc_i}{dR} > \frac{dc_{i-1}}{dR} (> 0)$. So $\frac{dc_{i-1}}{dR}$ crosses 0 line once from below.
4. Implies

   $$\frac{dC^m}{dR} = \nu \sum_{i=0}^{m} \pi_i dc_i < (>) \nu \sum_{i=0}^{m} \pi_i R^{-i} dci = 0 \textit{ for } R < 1 \; (R > 1)$$

Analytical results - Summary

1. Marginal consumption distribution is geometric
2. Variance of consumption declines with income.
3. LOM $U_c(c_{i-1}) = \beta R U_c(c_i) \longrightarrow$ all unconstrained agents at the floor of consumption growth.
4. Participation constraints $\longrightarrow$ some individuals experience very large growth in consumption

Quantitative results

Quantitative results

1. Non-parametric comparison of joint c,y,w distribution with CEX data
2. GMM estimation
3. Model Comparison
4. Sensitivity

Income process: Krueger and Perri (2006)

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- $log(z_{i,t}) = m_{i,t} + \varepsilon_{i,t} \; \varepsilon_{i,t} \sim N(0, \sigma^2_{\varepsilon,t})$
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- $\rho = 0.9989$
- Discretisation:
  $m_{i,t}$: 7-state Markov Chain (Tauchen and Hussey 1991)
  $\varepsilon_{i,t}$: binary process

Asset markets

- As before but with saving at equilibrium interest rate after default

I. Comparison of joint c,y,w distribution with US data

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- Parameters
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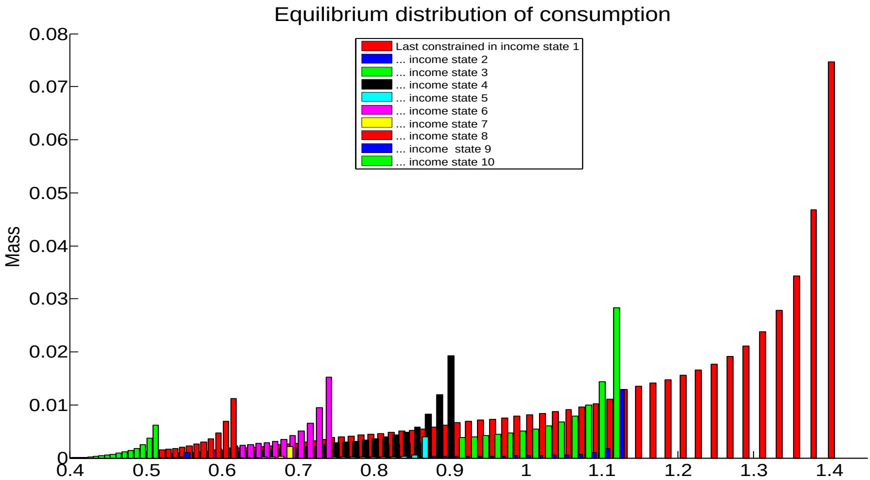
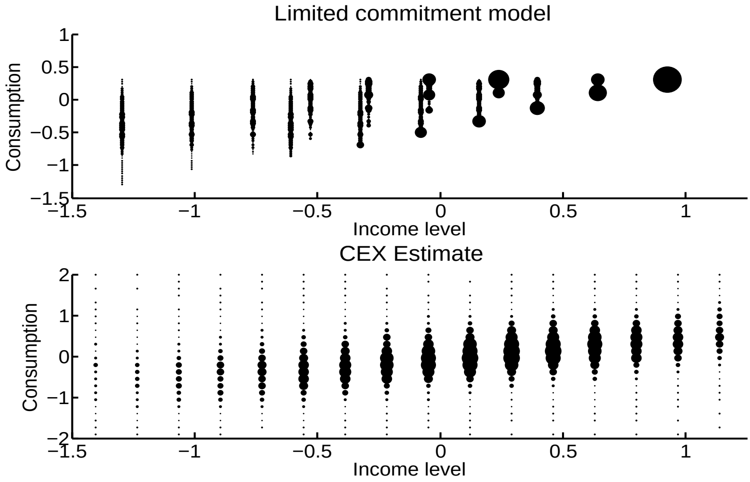
- Consumption and Income from CEX 2003 (KP 06)
- Wealth data from 2004 SCF

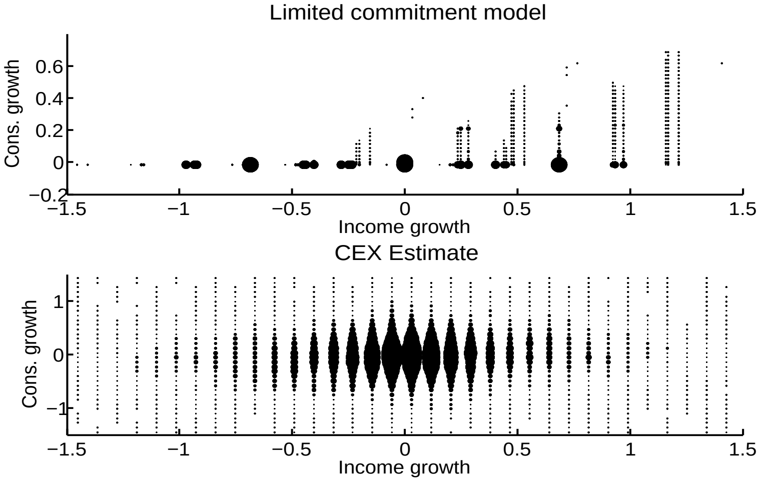
The marginal consumption distribution



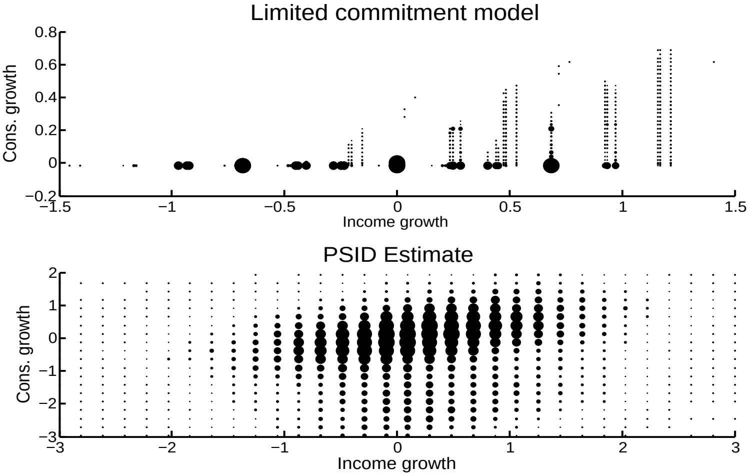
Joint distribution of consumption and income



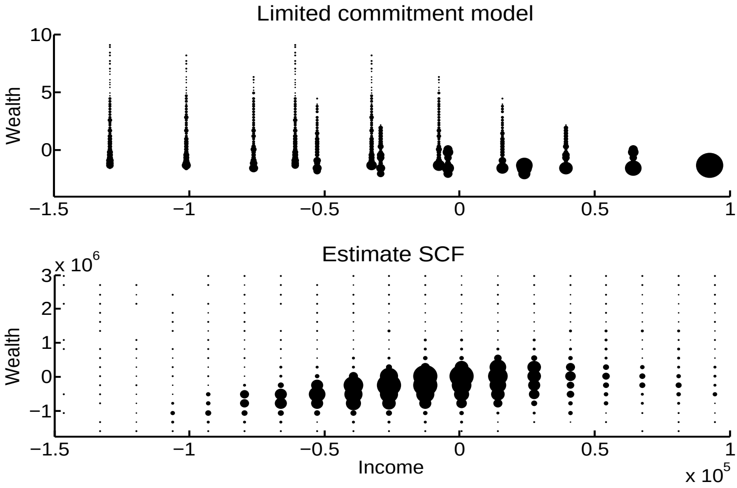
Joint distribution of consumption and income changes



Joint distribution of consumption and income changes



Joint distribution of wealth and income



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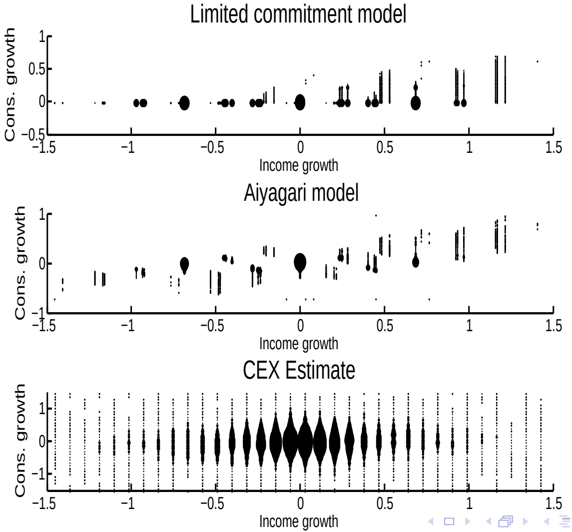
- Use GMM to estimate preference parameters and to show how model fits different moments
- Problem: $ln(c') = ln(c) + \frac{ln(\beta)+ln(R)}{\sigma}$
  $\Rightarrow \beta$ and σ affect consumption heterogeneity similarly
- Restrict attention to economies with market-clearing interest rate $R^{\star} = 1.03$

Results

	β	σ	$\frac{Var(C)}{VAR(Y)}$	$\beta_{dc,dy}$	$\frac{Var(dc)}{VAR(dy)}$	$Skew(dc)$	Chi^2
CEX 2003			0.25	0.05	0.28	0.07	
$W^{opt}, 2mts$	0.95	0.7	0.39	0.077	(0.030)	(3.4)	9.6
$\mathbb{I}, 2mts$	0.95	1.14	0.25	0.039	(0.013)	(4.65)	
$W^{opt}, 4mts$	0.945	0.685	0.41	0.087	0.034	3.3	$1.3\ e^6$
$\mathbb{I}, 4mts$	-	-	-	-	-	-	-

III. Model Comparison

dc and dz: Limited Commitment vs. Self-insurance



Model Comparison: Limited Commitment (LC) vs. Self-insurance (SI)

	β	σ	$\frac{Var(C)}{VAR(Y)}$	$\beta_{dc,dy}$	$\frac{Var(dc)}{VAR(dy)}$	$Skew(dc)$	R
CEX 2003			0.25	0.05	0.28	0.07	
LC	0.96	1	0.24	0.037	0.012	4.7	1.024
SI	0.96	1	0.68	0.22	0.12	0.65	1.0129

III. Sensitivity

Including Production

Parameters

1. Technology

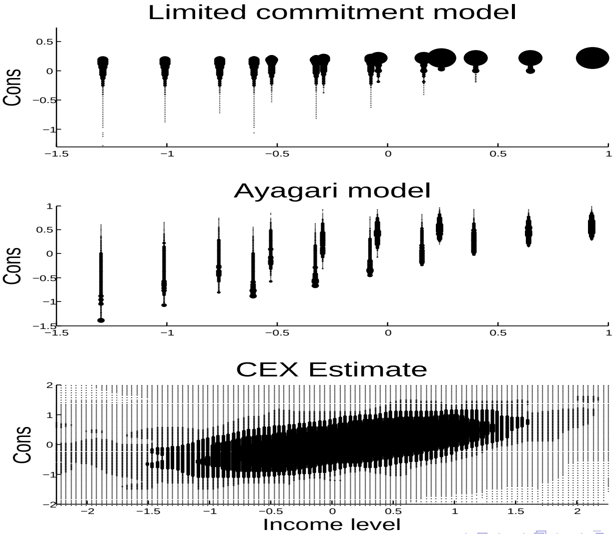
$$Y = AK^{\alpha}L^{1-\alpha}$$

- $\alpha = 0.3$
- $\delta = 0.08$

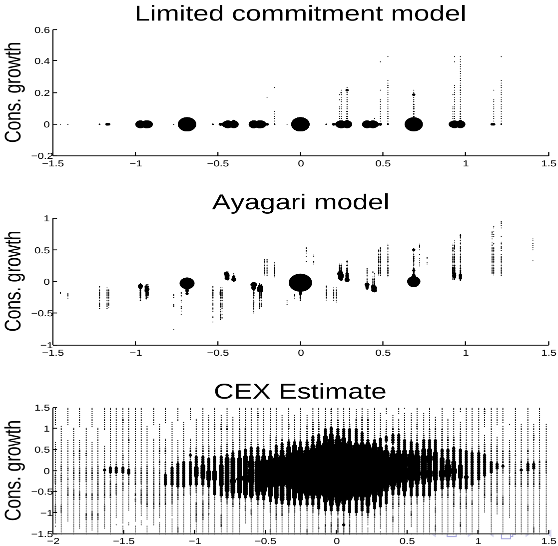
2. Preferences

- $u(c) = log(c)$
- $\beta = 0.96$

Joint distribution of c and z with capital



Joint distribution of **dc** *and* **dz** *with capital*



Model Comparison with Capital

	R	$\frac{Var(C)}{VAR(Y)}$	$\beta_{dc,dy}$	$\frac{Var(dc)}{VAR(dy)}$	$Skew(dc)$
CEX 2003		0.25	0.05	0.28	0.07
LC	1.386	0.05	0.007	0.0016	10.1
SI	1.322	0.58	0.11	0.046	0.91

A limited commitment model with heterogeneity in discount factors (Broer 2009)

- Chose heterogeneity in β on uniform grid $\{0.9, 0.92, 0.94, 0.96, 0.98\}$ to match US Gini coefficient on wealth
- yields 50/50 "spenders/savers" (Mankiw)

Model Comparison

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CEX 2003		0.25	0.05	0.28	0.07
LC	1.039	0.05	0.007	0.0016	10.1
SI	1.032	0.58	0.11	0.046	0.91
LC^{het}	1.02	0.52	0.10	0.046	1.18

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Future Work

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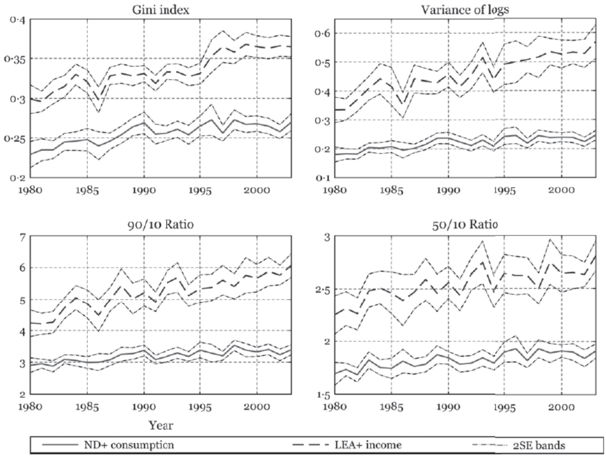
4. Combining bufferstock saving with insurance

Stationary equilibrium distributions in economies with limited commitment

Tobias Broer
Institute for International Economic Studies,
Stockholm University

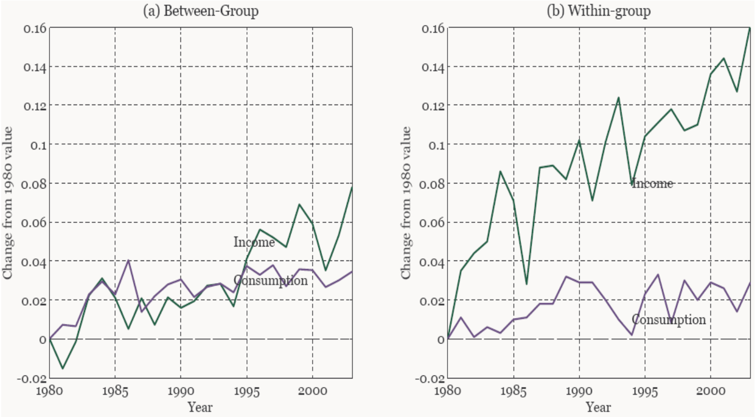
ESSIM 2010

US income and consumption inequality - CEX data



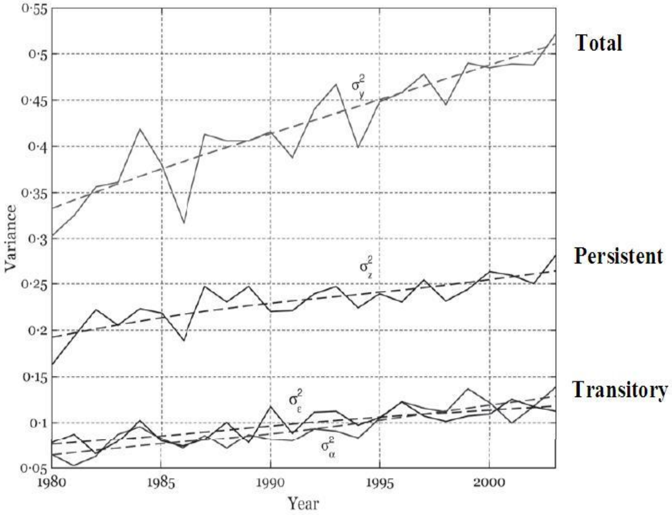
Source: Krueger and Perri (2006)

Within vs. between group inequality in the US (log-variance)



Source: Krueger and Perri (2006)

US income inequality: Permanent and transitory shocks



Source: Krueger and Perri (2006)

Planner's Problem

$$
\begin{aligned}
& max_{\{c_{i,t}\}} \int_{\mathbb{I}} \mu_{i,0} \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \\
s.t. \quad & \int_{\mathbb{I}} c_{i,t} = \int_{\mathbb{I}} z_{i,t}, \ \forall t \\
& V_{i,t} \geq W(z_{i,t}), \ \forall t, i
\end{aligned}
$$

Recursive Planner's Problem (Marcet and Marimon 1998)

$$\mathbb{VV}(\Phi_{\mu,z}) =$$

$$inf_{\{\gamma_i \geq 0\}} max_{\{c_i\}} \int_{\mathbb{I}} [(\mu_i + \gamma_i) u(c_i) - \gamma_i W(z_i)] + \beta E[\mathbb{VV}(\Phi_{\mu',z'})]$$

$$s.t. \int_{\mathbb{I}} c_i = \int_{\mathbb{I}} z_i$$

$$\mu_i' = \mu_i + \gamma_i, \ \forall i$$

First order conditions with CRRA preferences

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- LOM for unconstrained individuals: $c_i' = (R\beta)^{\frac{1}{\sigma}} c_i$

First order conditions with CRRA preferences

- LOM for unconstrained individuals: $c_i' = (R\beta)^{\frac{1}{\sigma}} c_i$
- Definition of R:
  $R\beta = \frac{\lambda}{\lambda'} = [\frac{C'}{C}]^{\sigma} \frac{(\sum_i \mu_i'^{1/\sigma})^{\sigma}}{[\sum_i (\mu_i' + \gamma_i')^{1/\sigma}]^{\sigma}}$

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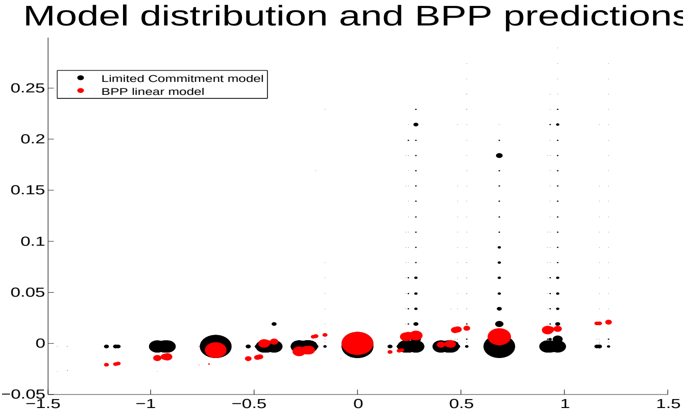
Consumption and income growth - actual and predicted by BPP coefficients



Model Comparison: Limited Commitment (LC) vs. Self-insurance (SI), GMM

	β	σ	$\frac{Var(C)}{VAR(Y)}$	$\beta_{dc,dy}$	$\frac{Var(dc)}{VAR(dy)}$	$Skew(dc)$	Chi^2
CEX 2003			0.25	0.05	0.28	0.07	
2 moments							
LC	0.95	0.7	0.39	0.077	(0.030)	(3.4)	9.6
SI	0.958	1.04	0.75	0.13	(0.07)	(0.98)	124
4 moments							
LC	0.945	0.685	0.41	0.087	0.034	3.3	$1.3\ e^6$
SI	0.96	0.8	0.75	0.14	0.07	0.99	$1.7e^6$