Can International Macroeconomic Models Explain Low Frequency Movements of Real Exchange Rates?

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Motivation

- The conventional wisdom is that a baseline IRBC two-country, two-good model (Backus et al., 1994, Heathcote and Perri, 2002) cannot generate neither:

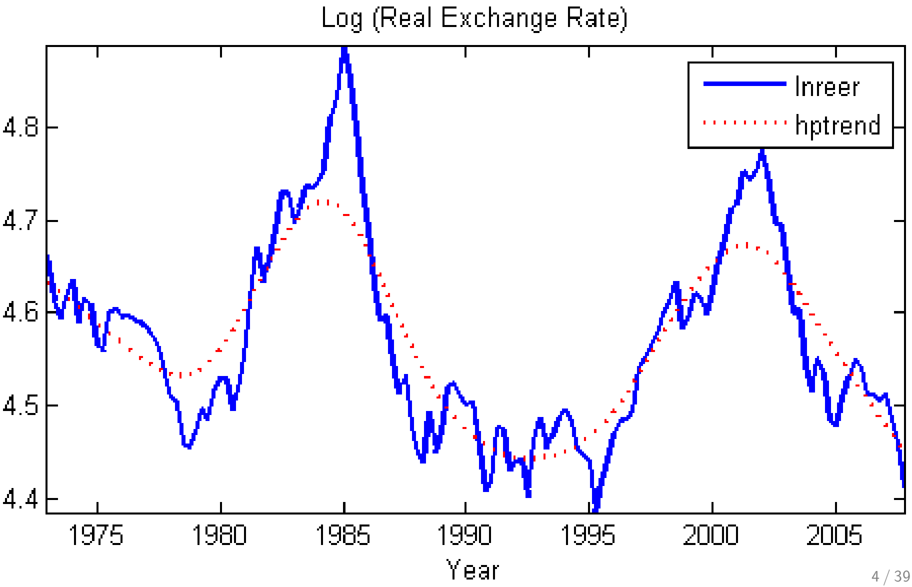
    1. enough volatility nor
    2. enough persistence in the RER when compared with the data.

- This paper challenges this conventional wisdom.

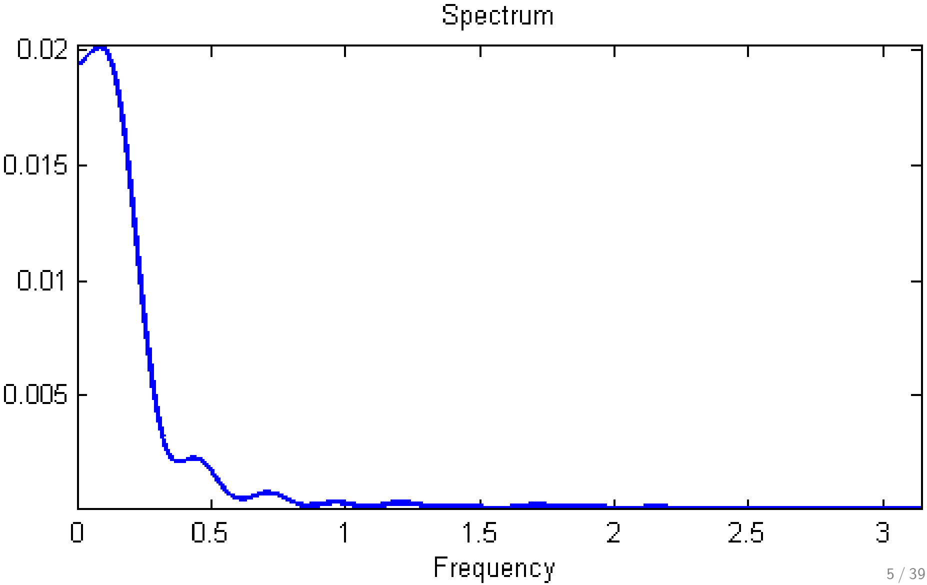
Motivation

- Where does the claim that the baseline IRBC model cannot account for volatility and persistence of the RER come from?
- Most (if not all) of the literature has tried to explain business cycle (BC) fluctuations of RER.
- Here we first argue that **ONLY** analyzing the BC fluctuations of the RER misses a large part of the story.
- Most of the movements in the U.S. dollar RER have been movements at low frequencies.

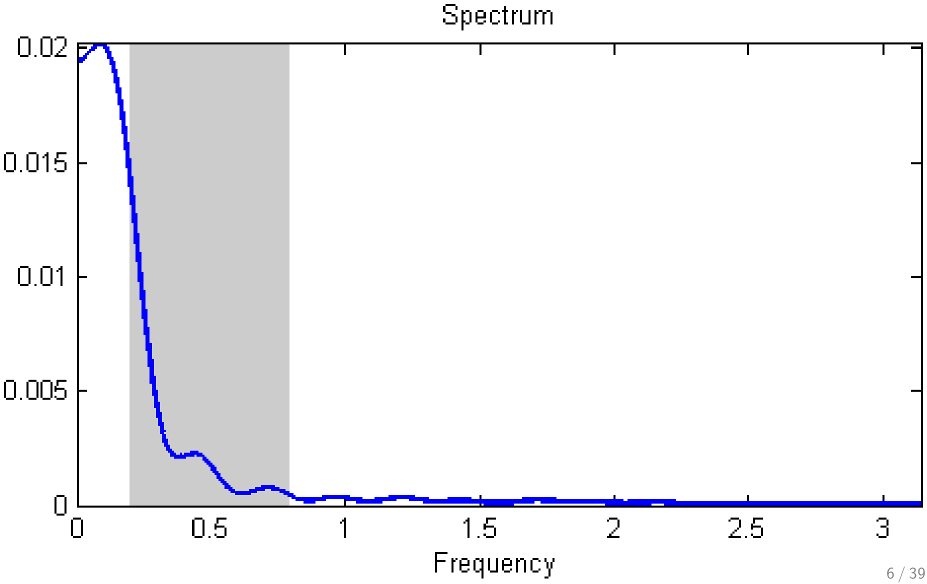
Motivation


Log (Real Exchange Rate)
lnreer
hptrend
4.8
4.7
4.6
4.5
4.4
1975
1980
1985
1990
1995
2000
2005
Year

Motivation



Motivation



Motivation

- Not only for the U.S. dollar.
- We decompose the estimated variance of several RER in BC frequencies (8 to 32 quarters), lower than BC frequencies (more than 32 quarters) and higher than BC frequencies (less than 8 quarters) for several countries:

Table 1: Variance Decomposition

	Low	BC	High
U.S.	70.0	24.9	5.0
E.M.U.	60.0	33.5	6.5
U.K.	68.0	25.5	6.6
Japan	72.4	21.8	5.6
Australia	64.3	28.5	7.2
Canada	72.7	22.6	4.7

What do we do?

- In this paper, we consider RER fluctuations at all frequencies.
- We need to have a model able to generate low frequency movements.
- We (slightly) modify the IRBC two-country, two-good model to do that.
- In particular, our model is an extension of the Heathcote and Perri (2002) model where:
  1. Stochastic processes for total factor productivity (TFP) are non-stationary but cointegrated across countries.
  2. Extension: We include and estimate cointegrated investment specific technological (IST) shocks.

Results

- Our model:
  1. Explains more than 60 percent of the volatility of the RER and
  2. concentrates fluctuations in low frequencies.
- The model-generated RER concentrates more fluctuations at low frequencies than the actual RER.
- Hence, the puzzle is that the model generates too much persistence and not too little as the literature analyzing the BC frequencies reports.

The Literature

- Stockman and Tesar (1995), Backus, Kehoe, Kydland (1994), Heathcote and Perri (2002).
- Chari, Kehoe and McGrattan (2002), Corsetti, Dedola and Leduc (2008).
- Steinsson (2008), Huizinga (1987), Cheung and Lai (2000).
- Rabanal, Rubio-Ramírez, and Tuesta (2009).
- Raffo (2009) and Mandelman, Rabanal, Rubio-Ramírez, and Vilán (2009).
- Adolfson et al. (2007), Nason and Rogers (2008), Rabanal and Tuesta (2010).

The Model: Households

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left\{ C\left(s^t\right)^{\tau} \left[1 - L\left(s^t\right)\right]^{1-\tau} \right\}^{1-\sigma}}{1-\sigma}$$

s.t.

$$P\left(s^t\right)\left[C\left(s^t\right) + X\left(s^t\right)\right] + P_H\left(s^t\right)\overline{Q}\left(s^t\right)D\left(s^t\right) \leqslant P_H\left(s^t\right)D\left(s^{t-1}\right) + P\left(s^t\right)\left[W\left(s^t\right)L\left(s^t\right) + R\left(s^t\right)K\left(s^{t-1}\right)\right] - P_H\left(s^t\right)\left[\Phi D\left(s^t\right)\right],$$

and

$$K\left(s^t\right) = (1-\delta)K\left(s^{t-1}\right) + X\left(s^t\right),$$

The Model: Production

- Final Good

$$\max P\left(s^t\right) Y\left(s^t\right) - P_H\left(s^t\right) Y_H\left(s^t\right) - P_F^*\left(s^t\right) Y_F\left(s^t\right)$$

s.t.

$$Y\left(s^t\right) = \left[\omega^{\frac{1}{\theta}} Y_H\left(s^t\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_F\left(s^t\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

- Intermediate Good

$$\max\left\{\begin{array}{c} P_H\left(s^t\right)\left[Y_H\left(s^t\right) + Y_H^*\left(s^t\right)\right] - \\ P\left(s^t\right)\left[W\left(s^t\right) L\left(s^t\right) + R\left(s^t\right) K\left(s^{t-1}\right)\right] \end{array}\right\}$$

s.t.

$$Y_H\left(s^t\right) + Y_H^*\left(s^t\right) = A\left(s^t\right) K\left(s^{t-1}\right)^{\alpha} L\left(s^t\right)^{1-\alpha}$$

The Model: VECM for TFP

$$\begin{pmatrix} \Delta \log A\left(s^{t}\right) \\ \Delta \log A^{*}\left(s^{t}\right) \end{pmatrix} = \begin{pmatrix} c_A \\ c_A^{*} \end{pmatrix} + \sum_{k=1}^{K} \rho_A(k) \begin{pmatrix} \Delta \log A\left(s^{t-k}\right) \\ \Delta \log A^{*}\left(s^{t-k}\right) \end{pmatrix}$$

$$+ \begin{pmatrix} \kappa_A \\ \kappa_A^{*} \end{pmatrix} \left[\log A\left(s^{t-1}\right) - \gamma_A \log A^{*}\left(s^{t-1}\right) - \log \xi_A\right] + \begin{pmatrix} \varepsilon^{A}\left(s^{t}\right) \\ \varepsilon^{A,*}\left(s^{t}\right) \end{pmatrix}$$

- Implies that:
  - $\Delta \log A\left(s^{t}\right)$
  - $\Delta \log A^{*}\left(s^{t}\right)$, and
  - $\log A\left(s^{t-1}\right) - \gamma_A \log A^{*}\left(s^{t-1}\right)$ are stationary processes.

The Model: Equilibrium Conditions

Let us define

$$\widetilde{P}_H\left(s^t\right) = P_H\left(s^t\right) / P\left(s^t\right),$$

$$\widetilde{P}_F^*\left(s^t\right) = P_F^*\left(s^t\right) / P^*\left(s^t\right),$$

and

$$RER\left(s^t\right) = P^*\left(s^t\right) / P\left(s^t\right),$$

Risk sharing

$$\mathbb{E}_t \left[\frac{\lambda^*\left(s^{t+1}\right)}{\lambda^*\left(s^t\right)} \frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)} \frac{RER\left(s^t\right)}{RER\left(s^{t+1}\right)} - \frac{\lambda\left(s^{t+1}\right)}{\lambda\left(s^t\right)} \frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)} \right] = \frac{\Phi'\left[D\left(s^t\right)\right]}{-\beta}$$

Price of bond

$$\overline{Q}\left(s^t\right) = \beta \mathbb{E}_t \frac{\lambda\left(s^{t+1}\right)}{\lambda\left(s^t\right)} \frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)} - \frac{\Phi'\left[D\left(s^t\right)\right]}{\beta},$$

where

$$U_C\left(s^t\right) = \lambda\left(s^t\right).$$

The Model: Equilibrium Conditions

$$\frac{U_L\left(s^t\right)}{U_C\left(s^t\right)} = W\left(s^t\right),$$

$$\lambda\left(s^t\right) = \beta \mathbb{E}_t\left\{\lambda\left(s^{t+1}\right)\left[R\left(s^{t+1}\right) + (1-\delta)\right]\right\},$$

The Model: Equilibrium Conditions

$$W\left(s^t\right) = (1-\alpha)\widetilde{P}_H\left(s^t\right)A\left(s^t\right)K\left(s^{t-1}\right)^{\alpha}L\left(s^t\right)^{-\alpha},$$

$$R\left(s^t\right) = \alpha\widetilde{P}_H\left(s^t\right)A\left(s^t\right)K\left(s^{t-1}\right)^{\alpha-1}L\left(s^t\right)^{1-\alpha},$$

$$Y_H\left(s^t\right) = \omega\widetilde{P}_H\left(s^t\right)^{-\theta}Y\left(s^t\right),$$

$$Y_F\left(s^t\right) = (1-\omega)\left(\widetilde{P}_F^*\left(s^t\right)RER\left(s^t\right)\right)^{-\theta}Y\left(s^t\right),$$

The Model: Equilibrium Conditions

$$C\left(s^t\right) + X\left(s^t\right) = Y\left(s^t\right),$$

$$Y\left(s^t\right) = \left[\omega^{\frac{1}{\theta}} Y_H\left(s^t\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_F\left(s^t\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

$$Y_H\left(s^t\right) + Y_H^*\left(s^t\right) = A\left(s^t\right) K\left(s^{t-1}\right)^{\alpha} L\left(s^t\right)^{1-\alpha},$$

and

$$D\left(s^t\right) + D^*\left(s^t\right) = 0.$$

The Model: Balanced Growth

- Model satisfies balanced growth path restrictions for a closed economy.
- Basic Idea: Domestic variables normalized by $A\left(s^{t-1}\right)^{\frac{1}{1-\alpha}}$ and foreign ones by $A^{*}\left(s^{t-1}\right)^{\frac{1}{1-\alpha}}$.
- But in the open economy we need an additional restriction

$$\widehat{Y}_F\left(s^t\right) = (1-\omega)\left[\widetilde{P}_F^{*}\left(s^t\right) RER\left(s^t\right)\right]^{-\theta} \widehat{Y}\left(s^t\right)\left[\frac{A\left(s^{t-1}\right)}{A^{*}\left(s^{t-1}\right)}\right]^{\frac{1}{1-\alpha}}$$

where $\widehat{Y}_F\left(s^t\right) = \frac{Y_F\left(s^t\right)}{A^{*}\left(s^{t-1}\right)^{\frac{1}{1-\alpha}}}$ and $\widehat{Y}\left(s^t\right) = \frac{Y\left(s^t\right)}{A\left(s^{t-1}\right)^{\frac{1}{1-\alpha}}}$.

- $\frac{A\left(s^{t-1}\right)}{A^{*}\left(s^{t-1}\right)}$ is stationary if $\gamma_A = 1$.

Estimation of the VECM for TFP: Data

- We construct (log) TFP series with output, employment, and capital for the U.S. and ROW.

- Hence, we have

$$\log A\left(s^t\right) = \log Y\left(s^t\right) - (1-\alpha)\log L\left(s^t\right) - \alpha \log K\left(s^{t-1}\right)$$

  for the U.S. and

$$\log A^*\left(s^t\right) = \log Y^*\left(s^t\right) - (1-\alpha)\log L^*\left(s^t\right) - \alpha \log K^*\left(s^{t-1}\right)$$

  for the ROW.

- Where α is the capital share of output and takes a value of 0.36.

- Capital is constructed using a perpetual inventory method with quaterly real investment and depreciation rate of 0.025.

Estimation of the VECM for TFP: Data

- All our data is from 1970:1 to 2007:4.
- For the U.S., we use
  - Quarterly nominal gross domestic product
  - Employment (civilian employment).
  - Quarterly nominal gross fixed capital formation.
  - GDP deflator (base 2000).
- The ROW aggregates data for the countries of the Euro Area, the United Kingdom, Canada, Japan, and Australia.
- To aggregate series for the ROW we first transform nominal output and investment series to base 2000 in local currency using GDP deflator.
- Second, we construct capital series for each country using our investment series.
- Third, we convert each national real output and capital series to U.S. dollars (base 2000) using implied PPP exchange rate.

Estimation of the VECM for TFP: Unit Root tests

- Unit root tests: we cannot reject a unit root for the level of (log) TFP processes.
- We can reject a unit root for their first difference. TFP's are I(1).
- Using Johansen's test, we cannot reject the existence of one cointegrating relationship. Hence, the TFP processes are C(1,1).
- We estimate the VECM with 0 lags and cannot reject that $\gamma_A = 1$ (joint with $\kappa_A = 0$).
- Why 0 lags?

Estimation of the VECM for TFP: Point Estimates

Table 4: The VECM model

c_A	c_A^*	κ_A^*
0.0020 (3.51)	0.0035 (10.55)	0.0263 (5.92)

Results: Baseline Parameterization

- Our baseline parameterization follows that in Heathcote and Perri (2002) closely.
- The discount factor β is set equal to 0.99, which implies an annual rate of return on capital of 4 percent.
- We set the consumption share, τ, equal to 0.34
- The coefficient of risk aversion, σ, equal to 2.
- We assume a cost of bond holdings, ϕ, of 1 basis points (0.01).
- The depreciation rate, δ, is set to a quarterly value of 0.025.
- The capital share of output is set to $\alpha = 0.36$.
- Home bias for domestic intermediate goods is set to $\omega = 0.9$.
- We calibrate the elasticity of substitution between intermediate goods to be $\theta = 0.85$. The value is based on Heathcote and Perri (2002).

Results: Moments

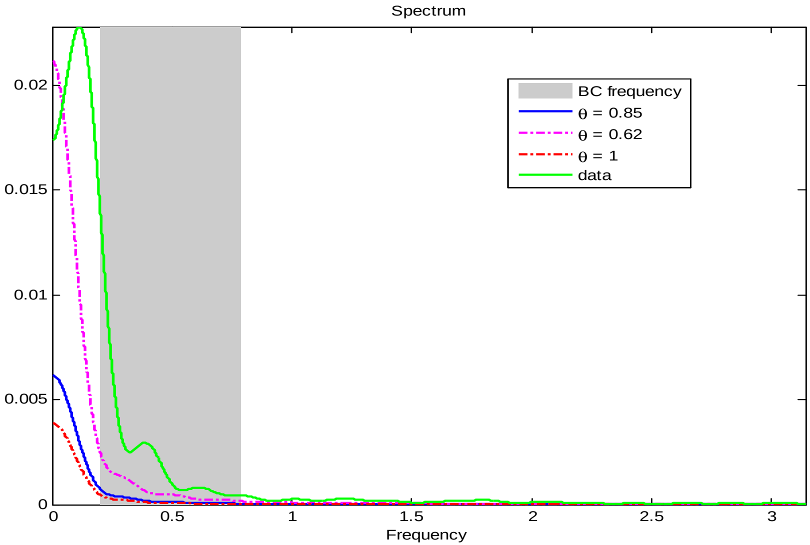
- We simulate 500 samples of 148 periods from our economy.

Table 5: Implications of the Model with Only TFP

	Standard Deviation			Frequency decomposition of RER		
	RER	GDP growth		Low	BC	High
U.S. Data	10.64	0.8		70.0	24.9	5.0
Baseline	5.15 (1.40)	0.77 (0.05)		76.2 (9.2)	18.6 (7.4)	5.2 (2.1)
$\theta = 0.62$	8.44 (2.44)	0.69 (0.04)		72.8 (10.4)	20.8 (7.9)	6.3 (2.9)
$\theta = 1.00$	3.81 (1.11)	0.81 (0.06)		76.6 (8.3)	18.4 (6.7)	5.0 (2.0)

- With baseline calibration, the model explains 50 percent of the RER std. deviation.
- As in the data, the model asigns most of the RER variation to low frequency fluctuations.
- The model overpredicts the fraction of low frequency movements in the data.

Results: Spectrum



Results: Moments

Table 6: Other Moments of the Model with TFP				
	Std. Dev.	Relative Std. Dev. with respect to GDP growth		
	GDP growth	Cons. growth	Inv. growth	Empl.
U.S. Data	0.80	0.87	2.42	0.59
Model	0.77 (0.04)	0.67 (0.06)	1.97 (0.11)	0.56 (0.13)
Correlations with GDP Growth				
		Cons. growth	Inv. growth	Empl.
U.S. Data		0.60	0.73	0.58
Model		0.99 (0.00)	0.99 (0.00)	0.33 (0.03)
Cross-Country Correlations				
	GDP growth	Cons. growth	Inv. growth	Empl.
Data	0.29	0.29	0.12	0.25
Model	-0.06 (0.07)	0.68 (0.05)	-0.47 (0.06)	-0.25 (0.28)

Results: Conclusion

- Heathcote and Perri (2002) model with cointegrated TFP shocks explains 50 percent of the volatility of the RER and
- its relative volatility with respect to output growth.
- However, the model places too much weight at low frequency movements of the RER.
- Finally, the puzzle is that the model generated RER series are too persistent with respect the observed ones, not the other way around as formerly thought.
- Can we improve the model?

An Extension: A Model with IST

- In this section we extend our benchmark framework to consider investment specific technological (IST) shocks.
- As (log) TFP shocks, (log) IST shocks are also assumed to be non-stationary but cointegrated: they follow a VECM process.
- IST shocks affect the rate of transformation of consumption goods into capital goods.
- The law of motion for capital is now described by

$$K\left(s^{t}\right)=(1-\delta)\,K\left(s^{t-1}\right)+V\left(s^{t}\right)X\left(s^{t}\right)$$

where $V\left(s^{t}\right)$ is the IST shock in the home country.

VECM for IST

$$\begin{pmatrix} \Delta \log V\left(s^t\right) \\ \Delta \log V^*\left(s^t\right) \end{pmatrix} = \begin{pmatrix} c_V \\ c_V^* \end{pmatrix} + \sum_{k=1}^{K} \rho_V(k) \begin{pmatrix} \Delta \log V\left(s^{t-k}\right) \\ \Delta \log V^*\left(s^{t-k}\right) \end{pmatrix}$$

$$+ \begin{pmatrix} \kappa_V \\ \kappa_V^* \end{pmatrix} \left[\log V\left(s^{t-1}\right) - \gamma_V \log V^*\left(s^{t-1}\right) - \log \xi_V\right] + \begin{pmatrix} \varepsilon^V\left(s^t\right) \\ \varepsilon^{V,*}\left(s^t\right) \end{pmatrix}$$

- As before, it implies that:
  - $\Delta \log V\left(s^t\right)$
  - $\Delta \log V^*\left(s^t\right)$, and
  - $\log V\left(s^{t-1}\right) - \gamma_V \log V^*\left(s^{t-1}\right)$ are stationary processes.

The Model with IST: Balanced Growth

- Model satisfies balanced growth path restrictions for a closed economy.

- Basic Idea: Domestic variables normalized by $Z\left(s^{t-1}\right) = A\left(s^{t}\right)^{\frac{1}{1-\alpha}} V\left(s^{t}\right)^{\frac{\alpha}{1-\alpha}}$ and $Z^{*}\left(s^{t}\right) = A^{*}\left(s^{t}\right)^{\frac{1}{1-\alpha}} V^{*}\left(s^{t}\right)^{\frac{\alpha}{1-\alpha}}$.

- But in the open economy we need an additional restriction

$$\widehat{Y}_{F}\left(s^{t}\right) = (1-\omega)\left[\widetilde{P}_{F}^{*}\left(s^{t}\right) RER\left(s^{t}\right)\right]^{-\theta} \widehat{Y}\left(s^{t}\right) \frac{Z\left(s^{t-1}\right)}{Z^{*}\left(s^{t-1}\right)}$$

where $\widehat{Y}_{F}\left(s^{t}\right) = \frac{Y_{F}\left(s^{t}\right)}{Z^{*}\left(s^{t-1}\right)}$ and $\widehat{Y}\left(s^{t}\right) = \frac{Y\left(s^{t}\right)}{Z\left(s^{t-1}\right)}$.

- Sufficient Condition for $\frac{Z\left(s^{t-1}\right)}{Z^{*}\left(s^{t-1}\right)}$ to be stationary $\rightarrow \gamma_{A} = 1$ and $\gamma_{V} = 1$.

Estimation of the VECM for IST: Data

- In order to estimate our VECM for (log) IST shocks we use data for the U.S. and an aggregate for the ROW.
- ROW is comprised by the U.S. most significant trading partners: the Euro Area, Canada, Japan, the United Kingdom, Australia, and South Korea.
- Both for the U.S. and for the ROW, we aim to obtain the relative price of investment goods with respect to consumption goods.
- For the U.S. the shock $V\left(s^t\right)$ is defined as

$$PCE_t^{US} / PI_t^{US}$$

- For the ROW $V^*\left(s^t\right)$ is defined as

$$\sum_i w_t^i * \left(\frac{PCE_t^i}{PI_t^i}\right)$$

Estimation of the VECM for IST: Data

- All our data is from 1982:4 to 2007:4.
- For the U.S., we use
  - The Personal Consumption Expenditure (PCE) deflator
  - and the Gross Domestic Investment deflator as our investment deflator.
- For Japan, the Private final consumption expenditure and the Private sector capital formation deflator series
- For Canada, the Personal expenditure on consumer goods and services deflator and the Business gross fixed capital formation deflator series.
- For the UK, we use the Final consumption expenditure deflator and the Gross fixed capital formation deflator.
- For Australia the Households final Consumption Expenditure and the Gross Fixed Capital formation implicit price.
- For South Korea we use the Final Consumption Expenditure deflator and Gross Capital Formation deflator series.
- For the E.M.U. the Consumption Deflator and the Gross Investment deflator.

Estimation of the VECM for IST: Unit Root tests

- Unit root tests: we cannot reject a unit root for the level of (log) IST processes.
- We can reject a unit root for their first difference. IST's are I(1).
- Using Johansen's test, we cannot reject the existence of one cointegrating relationship. Hence, the IST processes are C(1,1).
- We estimate the VECM with 0 lags and cannot reject that $\gamma_V = 1$.
- Why 0 lags?

Estimation of the VECM for IST: Point Estimates

- Note that we do not include a constant in the VECM (the Johansen's test does not reject the existence of a cointegration relationship only in this case)

Table 9: The VECM Model

κ_V	κ_V^*
-0.035 (-8.26)	-0.017 (-3.68)

Results: Moments

- Using the same calibration, we simulate 500 samples of 148 periods.

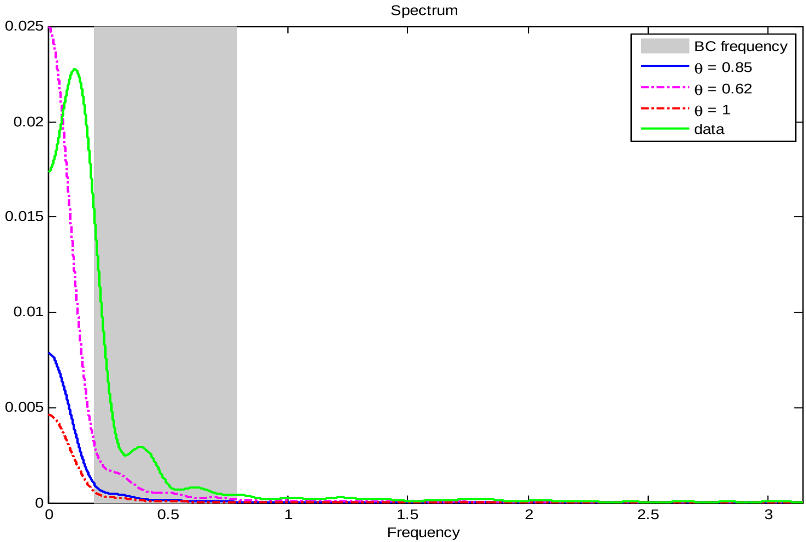
Table 10: Implications of the Model with TFP and IST

	Standard Deviation			Frequency decomposition of RER		
	RER	GDP growth		Low	BC	High
U.S. Data	10.64	0.8		70.0	24.9	5.0
Baseline	6.05 (1.7)	0.81 (0.05)		75.6 (8.5)	18.9 (6.7)	5.6 (2.1)
$\theta = 0.62$	9.55 (2.9)	0.75 (0.05)		73.1 (10.3)	20.6 (8.1)	6.4 (2.5)
$\theta = 1.00$	4.01 (1.14)	0.85 (0.05)		76.3 (8.0)	18.5 (6.6)	5.2 (1.7)

Results: Moments

Table 11: Other Moments of the Model with TFP and IST				
	Std. Dev.	Relative Std. Dev. with respect to GDP growth		
	GDP growth	Cons. growth	Inv. growth	Empl.
U.S. Data	0.80	0.87	2.42	0.59
Model	0.81 (0.05)	0.70 (0.03)	2.59 (0.10)	0.98 (0.23)
Correlations with GDP Growth				
		Cons. growth	Inv. growth	Empl.
U.S. Data		0.60	0.73	0.58
Model		0.79 (0.03)	0.88 (0.02)	0.25 (0.06)
Cross-Country Correlations				
	GDP growth	Cons. growth	Inv. growth	Empl.
Data	0.29	0.29	0.12	0.25
Model	-0.08 (0.08)	0.62 (0.05)	-0.41 (0.06)	-0.05 (0.31)

Results: Spectrum



Conclusion

- This paper challenges the conventional wisdom that the simple IRBC model cannot generate neither:
  1. enough volatility nor
  2. enough persistence in RER when compared with the data.
- Spectral analysis shows that most of the observed volatility of RER is below BC frequencies.
- This makes the analysis of RER at all frequencies a more sound exercise than the typical BC one.
- When fluctuations at all frequencies are considered, the model explains:
  1. most of the observed standard deviation of RER and
  2. assigns most of its volatility to below BC frequency.