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Can International Macroeconomic Models Explain Low-Frequency Movements of Real Exchange Rates?

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Abstract

We first show that most of the volatility of real effective exchange rates of major currencies can be allocated to low frequencies (i.e., below business cycle frequencies). This makes the analysis of real exchange rates at all frequencies a more sound exercise than the typical business cycle one, which compares actual data and simulated data after the Hodrick-Prescott filter is applied. Then, we challenge the conventional wisdom that a simple two-country, two-good model, as described in Heathcote and Perri (2002) cannot generate either enough volatility or enough persistence in real exchange rates when compared to the data. When analyzing real exchange rate of the U.S. dollar at all frequencies, the model explains most of its standard deviation and assigns most of its volatility to low frequencies, as in the data. In fact, the puzzle is that the model generates too much persistence of real exchange rate instead of too little, as the business cycle analysis asserts. Part of the success of our model resides in its ability to exhibit hump-shaped dynamics of RER. This is not surprising, but comforting, after Steinsson (2008) showed that real shocks in a two-country sticky-price model yields RER hump-shaped dynamics.

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1. Introduction

This paper challenges the conventional wisdom that a baseline international real business cycle (IRBC) two-country, two-good model, such as the one described in Heathcote and Perri (2002), cannot generate either enough volatility or enough persistence in the real exchange rate (RER) when compared with the data. In particular, we show that when the object of interest is RER fluctuations at all frequencies, instead of just business cycle (BC) ones, this model can explain 60 percent of the standard deviation of the U.S. dollar RER. In addition, we argue that analyzing RER fluctuations at all frequencies is a more compelling exercise than just studying the BC ones. Using spectral analysis, we show that most of the volatility of the RER in the data can be assigned to low-frequency movements (about 70 percent), while movements at BC frequencies account for a small share of the RER fluctuations (just 25 percent). We show that the model can also account for this fact. Actually, the main discrepancy between the model and the data is that when fluctuations at all frequencies are taken into account, the model generates too much persistence instead of too little, as the BC analysis asserts.

Since the seminal works of Backus, Kehoe, and Kydland (1992) and Baxter and Crucini (1995), the IRBC literature has been preoccupied with explaining the international transmission of shocks, the cyclical comovement of variables across countries, and the behavior of relative prices, such as the terms of trade or the RER. As does most of the real business cycle (RBC) literature, the IRBC literature mainly concentrates on explaining the BC fluctuations in the data. The success of the model is measured by its ability to reproduce selected second moments of Hodrick-Prescott (HP) filtered data, which removes trends and low-frequency movements in the data.[1] The researcher compares the second moments of the detrended data with those implied by artificial data generated by the model after the same detrending procedure has been applied.

One of the most relevant facts in the HP-filtered data is that relative international prices are both much more volatile than either output or consumption and highly persistent. IRBC models with reasonable calibrations have a hard time reproducing these features. In earlier work Backus, Kehoe and Kydland (1994) and Stockman and Tesar (1995) showed that IRBC models cannot match the volatility of the HP-filtered terms of trade, while in a more recent contribution Heathcote and Perri (2002) have pointed out the inability of the standard IRBC model to explain the volatility and persistence of the HP-filtered RER.

In this paper, we first argue that analyzing only the BC fluctuations of the RER leads re-

[1] Other papers use instead the band-pass filter, as described in Baxter and King (1999) or Christiano and Fitzgerald (2003), that also removes high-frequency movements in the data (higher than business cycle frequencies).

searchers to miss a large part of the story. The reason is as follows. The top panel of Figure 1 plots the (log) U.S. dollar real effective exchange rate (REER) along with its implied HP filter "trend" using a bandwidth of 1600. Visibly, most of the relevant movements in the U.S. dollar REER have been movements at low-frequencies. This observation is confirmed by the spectral analysis to be found in the next Section: most of the variation of the RER in the data is at frequencies lower than BC fluctuations (between 60 to 75 percent depending on the currency we examine). These low-frequencies are removed by HP filtering.[2,3]

Second, we propose to analyze the fluctuations of the RER at all frequencies instead. In order to do that, we need to consider a model able to generate low-frequency fluctuations in the data. Our benchmark model is an extension of the two-country, two-good model of Heathcote and Perri (2002) in which stochastic processes for total factor productivity (TFP) are non-stationary but cointegrated across countries.[4] We show that in such a model it is possible to closely match the estimated spectrum of the U.S. dollar RER when the TFP processes are estimated from the data following a vector error correction (VECM) model. In particular, the model can explain 50 percent of the standard deviation of the RER in the data and concentrates fluctuations in low-frequencies while closely matching the volatility of output growth. It is also the case that the model-generated RER concentrates more fluctuations at low-frequencies than in the data; i.e., the model-generated RER is too persistent when compared with the data. This last claim is reinforced when we look at the impulse response function (IRF) for the model-based RER (see Figure 11 in Section 6.3). In our model the RER peaks 13 quarters after a TFP shock and the half-life of the response is about 10 years, while Steinsson (2008) estimates a half-life of 4.5 years with a peak at around 9 quarters for the U.S. dollar RER. Hence, the puzzle is that the model generates too much persistence and not too little as the IRBC literature reports. Finally, we study an extension of the model in which we include and estimate cointegrated investment-specific technology (IST) shocks across countries (see Raffo, 2009 and Mandelman et. al 2009) and show that they improve the model's ability to match the spectrum of the RER in the data.

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[2] We would like to thank Jon Steinsson for informal conversations that motivated us to conduct the current research. In addition, we were particularly interested in further pursuing the following claim in his 2008 American Economic Review paper: *Figure 4 plots the U.S. real exchange rate along with its HP-filter "trend." According to the HP-filter, most of the large movements in the U.S. real exchange rate over the last 30 years - such as the large appreciation and subsequent depreciation in the 1980's - have been movements in the "trend."*

[3] Many RER series have a trend, in particular those of emerging economies that experience higher productivity growth rates than advanced economies. In that case, the use of a trend/cycle decomposition would be justified. However, the focus of most of the IRBC literature is to explain the real exchange rate of the US dollar vis-a-vis other industrialized countries. In that case RERs do not have a trend.

[4] Rabanal et. al (2009) use this setup to analyze BC fluctuations of several international variables showing that cointegrated shocks improve the model's ability to explain the data fluctuations at BC frequencies.

After introducing IST shocks, the model can explain 60 percent of the standard deviation of the U.S. dollar RER and, as before, concentrates fluctuations in low-frequency movements.

Thus, the main conclusion of our paper is that, contrary to conventional wisdom, when the analysis of the RER is carried out at all frequencies, a baseline two-country, two-good model does a good job in explaining fluctuations in the RER.

2. Spectral Analysis of Main Currencies' Real Exchange Rate

In this Section we study the spectral density for the (log) real effective exchange rate (REER) of six main currencies: the U.S. dollar, the euro, the UK pound sterling, the Japanese yen, and the Canadian and Australian dollars. Except for the U.S. dollar, all of our data are from the IMF's International Financial Statistics database and our sample goes from 1980:Q1 to 2009:Q2.[5,6] For better comparison purposes, we build our own U.S. dollar RER series by recomputing the REER against the same set of countries that we will use to construct our TFP series for the ROW later in Section 5.1. In particular, we use the currency weights from the Broad Index of the Foreign Exchange Value of the dollar calculated by the U.S. Federal Reserve.[7]

It is convenient for our purposes to examine the spectral density. The spectral density contains the same information as auto-correlations and it allows us to decompose the total variation of the RER across different frequencies. In order to estimate the spectrum we use the modified Bartlett kernel methodology described in Section 6.4 of Hamilton (1994).

In Figures 1-6 we present the RERs along with their implied HP filter "trend," and their estimated spectrum density for the above-described currencies. From the first panel of Figures 1-6 we can observe that RERs do not have an evident time trend and they are highly persistent. In the bottom panel of Figures 1-6 we present the estimated spectral density. For all the currencies, the estimated spectral densities tell us that most of the fluctuations occur at low-frequencies.

In Table 1 we decompose the estimated variance of each RER into BC frequencies (8 to 32 quarters), lower than BC frequencies (more than 32 quarters) and higher than BC frequencies (less than 8 quarters). The first thing to notice is that most of the variation of the RER, between 60 to 75 percent, is concentrated at low-frequencies. Therefore, the literature that tries to explain BC frequency fluctuations of RERs misses a large part of the picture that resides in the low-frequency

[5] In this section we will always analyze the log of the REER. In any case, to avoid cumbersome language, we refer to it as the RER.

[6] In this section we define the RER as the real effective exchange rate as reported by the IMF.

[7] For a description see http://www.federalreserve.gov/releases/H10/Weights/.

end of the spectrum.

Table 1: Variance Decomposition			
	Low	BC	High
U.S.	70.0	24.9	5.0
Euro Area	60.0	33.5	6.5
U.K.	68.0	25.5	6.6
Japan	72.4	21.8	5.6
Australia	66.3	27.0	6.7
Canada	75.5	20.4	4.1

Note that the fact that most of the volatility of the RER is concentrated at low-frequencies can be related to two well-documented facts. First, the large half-life of estimated IRFs of the RER (Rogoff, 1996; Murray and Papell, 2002; and Steinsson, 2008) and second, its hump-shaped dynamics (Huizinga, 1987; Eichenbaum and Evans, 1995; Cheung and Lai, 2000; and Steinsson, 2008). Both the large half-life and the dynamic non-monotonic response pattern are closely related to the high persistence of RERs in the data. As we will see later, part of the success of our model resides in its ability (in both specifications) to exhibit hump-shaped dynamics of RER. This is not surprising, but comforting, after Steinsson (2008) showed that real shocks in a two-country sticky-price model yields RER hump-shaped dynamics.

3. Relationship to the Literature

In this paper we also bridge the gap between empirical models and dynamic stochastic general equilibrium (DSGE) models in explaining RER fluctuations. The empirical literature since the seminal work of Meese and Rogoff (1983) has mostly used univariate and multivariate time series to model the (log) of the nominal or the real exchange rate. This analysis is mostly performed at all frequencies, that is, using non-filtered data. In a recent paper, Steinsson (2008) follows a large literature that models the linear univariate empirical properties of the (log) RER. Other univariate nonlinear time series approaches are reviewed in Sarno (2003). In the multivariate setup, Clarida and Gali (1994), Faust and Rogers (2003) and Monacelli and Perotti (2009) have used VAR models to explain the response of exchange rates (both real and nominal) to several shocks. Other authors examine the relationship between exchange rates (both real and nominal) and fundamentals derived from open economy macro models, such as Engel and West (2005), and Cheung, Chinn and Garcia-Pascual (2005).

However, most calibrated DSGE models are typically concerned with explaining the BC fluctuations of the RER and hence analyze its HP-filtered series. Since Heathcote and Perri (2002), the literature has been energetically trying to reconcile the discrepancy between theory and HP-filtered RER data, with some success. For example, Chari, Kehoe and McGrattan (2002) show that a monetary economy with monopolistic competition and sticky prices can explain HP-filtered RER volatility if a high degree of risk aversion is assumed and Corsetti, Dedola and Leduc (2008a and 2008b) show that introducing nontraded goods also helps reconcile theory with data. Although such models do a better job explaining the volatility of the HP-filtered RER, they still cannot match its persistence. A number of related papers have tried to tackle this issue in the context of monetary models (for example, see Bergin and Feenstra, 2001, Benigno, 2004, or Bouakez, 2005) without completely addressing it.

In this paper we fill the breach between the two approaches by comparing the properties of the (log) RER in the DSGE model and in the data, without applying any filtering method. It is also worth noting that a few recent exceptions to this filtering practice arise in the literature that estimates open economy DSGE models with Bayesian methods. Adolfson et al. (2007) and Rabanal and Tuesta (2009) include the (log) of the RER in the set of observable variables, while Nason and Rogers (2008) use the (log) of the nominal exchange rate between the U.S. and Canadian dollars in their estimated model.

4. The Benchmark Model

In this Section, we present our benchmark framework. We use a two-country, two-good model similar to the one described in Heathcote and Perri (2002) with a main difference: (log) TFP shocks are assumed to be non-stationary but cointegrated across countries. In other words, they follow a VECM process. This cointegration assumption has strong and testable implications for the data. The empirical evidence supporting our assumption will be presented in Section 5. Later, we will extend the model to also consider non-stationary but cointegrated (log) IST shocks.

In each country, a single final good is produced by a representative competitive firm that uses intermediate goods from both countries in the production process. These intermediate goods are imperfect substitutes for each other and can be purchased from representative competitive producers of intermediate goods in both countries. Intermediate goods producers use domestic capital and labor in the production process and face a domestic TFP shock. The final good can only be locally consumed or invested in by consumers. Thus, all trade of goods between countries occurs at the intermediate goods level. In addition, consumers trade across countries an uncontingent international riskless bond denominated in units of domestic intermediate goods. No other financial asset is available. In each period of time t, the economy experiences one of many finite events s_t. We denote by $s^t = (s_0, ..., s_t)$ the history of events up through period t. The probability, as of period 0, of any particular history s^t is $\pi(s^t)$ and s_0 is given.

4.1. Households

In this subsection, we describe the problem faced by home-country households. The problem faced by foreign-country households is similar, and hence it is not presented. The representative household of the home country solves

$$\max_{\{C(s^t),L(s^t),X(s^t),K(s^t),D(s^t)\}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi\left(s^t\right) \frac{\left\{C\left(s^t\right)^{\tau}\left[1-L\left(s^t\right)\right]^{1-\tau}\right\}^{1-\sigma}}{1-\sigma}$$

subject to the following budget constraint

$$\begin{aligned} P\left(s^t\right)\left[C\left(s^t\right)+X\left(s^t\right)\right]+P_H\left(s^t\right)\overline{Q}\left(s^t\right)D\left(s^t\right) \leqslant P\left(s^t\right)\left[W\left(s^t\right)L\left(s^t\right)+R\left(s^t\right)K\left(s^{t-1}\right)\right] \\ +P_H\left(s^t\right)\left\{D\left(s^{t-1}\right)-\Phi\left[D\left(s^t\right)\right]\right\} \end{aligned} \quad (1)$$

and the law of motion for capital

$$K\left(s^t\right) = (1-\delta)\,K\left(s^{t-1}\right)+X\left(s^t\right).$$

The following notation is used: $\beta \in (0,1)$ is the discount factor, $L\left(s^t\right) \in (0,1)$ is the fraction of time allocated to work in the home country, $C\left(s^t\right) \geq 0$ are units of consumption of the final good, $X\left(s^t\right) \geq 0$ are units of investment, $K\left(s^t\right) \geq 0$ is the capital level in the home country at the beginning of period $t+1$, $P\left(s^t\right)$ is the price of the home final good, which will be defined below, $W\left(s^t\right)$ is the hourly wage in the home country, and $R\left(s^t\right)$ is the home-country rental rate of capital, where the prices of both factor inputs are measured in units of the final good. $P_H\left(s^t\right)$ is the price of the home intermediate good, $D\left(s^t\right)$ denotes the holdings of the internationally traded riskless bond that pays one unit of home intermediate good (minus a small cost of holding bonds, $\Phi[\cdot]$) in period $t+1$ regardless of the state of nature, and $\overline{Q}\left(s^t\right)$ is its price, measured in units of the home intermediate good. Finally, the function $\Phi[\cdot]$ is the arbitrarily small cost of holding bonds measured in units of the home intermediate good.[8] We assume that $\Phi[\cdot]$ takes the

[8] The $\Phi[\cdot]$ cost is introduced to ensure stationarity of the level of $D(s^t)$ in log-linearized IRBC models with

following functional form: $\Phi\left[D\left(s^t\right)\right] = \frac{\phi}{2}A(s^{t-1})\left[\frac{D\left(s^t\right)}{A(s^{t-1})}\right]^2$. Why do we need to normalize $D\left(s^t\right)$? As we will see below, $D\left(s^t\right)$ grows at the rate of growth of $A(s^{t-1})$ along the balanced growth path, making the ratio $\frac{D\left(s^t\right)}{A(s^{t-1})}$ stationary.

4.2. Firms

The final good in the home country, $Y\left(s^t\right)$, is produced using both home intermediate goods, $Y_H\left(s^t\right)$, and foreign intermediate goods, $Y_F\left(s^t\right)$, with the following technology:

$$Y\left(s^t\right) = \left[\omega^{\frac{1}{\theta}} Y_H\left(s^t\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_F\left(s^t\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{2}$$

where ω denotes the fraction of home intermediate goods that are used for the production of the home final good and θ controls the elasticity of substitution between home and foreign intermediate goods. We will calibrate ω to have home bias; this is the only source of RER fluctuations in our model.

Therefore, the representative final goods producer in the home country solves the following problem:

$$\max_{Y(s^t)\geq 0, Y_H(s^t)\geq 0, Y_F(s^t)\geq 0} P\left(s^t\right) Y\left(s^t\right) - P_H\left(s^t\right) Y_H\left(s^t\right) - P_F^*\left(s^t\right) Y_F\left(s^t\right)$$

subject to the production function (2). The representative intermediate goods producer in the home country uses home labor and capital in order to produce home intermediate goods and sells her product to both the home and foreign final good producers. Taking prices of all goods and factor inputs as given, she maximizes profits. Hence, she solves:

$$\underset{L(s^t)\geq 0, K(s^{t-1})\geq 0}{Max} P_H\left(s^t\right)\left[Y_H\left(s^t\right) + Y_H^*\left(s^t\right)\right] - P\left(s^t\right)\left[W\left(s^t\right)L\left(s^t\right) + R\left(s^t\right)K\left(s^{t-1}\right)\right]$$

subject to the production function

$$Y_H\left(s^t\right) + Y_H^*\left(s^t\right) = A\left(s^t\right) K\left(s^{t-1}\right)^{\alpha} L\left(s^t\right)^{1-\alpha}$$

where $Y_H\left(s^t\right)$ is the amount of home intermediate goods sold to the home final goods producers, $Y_H^*\left(s^t\right)$ is the amount of home intermediate goods sold to the foreign final goods producers, and $A\left(s^t\right)$ is a stochastic process affecting TFP of home intermediate goods producers to be defined

incomplete markets, as discussed by Heathcote and Perri (2002). We choose the cost to be numerically small, so it does not affect the dynamics of the rest of the variables.

below.

4.3. A VECM for TFP

We assume that (log) TFP shocks are non-stationary but cointegrated (we will show later that this assumption cannot be rejected by the data). Let us now described the process for $\log A\left(s^t\right)$ and $\log A^*\left(s^t\right)$:

$$\begin{pmatrix} \Delta \log A\left(s^t\right) \\ \Delta \log A^*\left(s^t\right) \end{pmatrix} = \begin{pmatrix} c_A \\ c_A^* \end{pmatrix} + \sum_{k=1}^{K} \rho_A(k) \begin{pmatrix} \Delta \log A\left(s^{t-k}\right) \\ \Delta \log A^*\left(s^{t-k}\right) \end{pmatrix}$$
$$+ \begin{pmatrix} \kappa_A \\ \kappa_A^* \end{pmatrix} \left[\log A\left(s^{t-1}\right) - \gamma_A \log A^*\left(s^{t-1}\right) - \log \xi_A\right] + \begin{pmatrix} \varepsilon^A\left(s^t\right) \\ \varepsilon^{A,*}\left(s^t\right) \end{pmatrix}$$

where $\rho_A(k)$ are 2×2 coefficient matrices, K is the number of lags, $(1, -\gamma_A)$ is the cointegrating vector, ξ_A is the constant in the cointegrating relationship, $\varepsilon^A\left(s^t\right) \sim N\left(0, \sigma^A\right)$ and $\varepsilon^{A,*}\left(s^t\right) \sim N\left(0, \sigma^{A,*}\right)$, $\varepsilon^A\left(s^t\right)$ and $\varepsilon^{A,*}\left(s^t\right)$ are correlated. This VECM representation implies that deviations of today's log differences of TFP with respect to its mean value depend not only on lags of home and foreign log differences of TFP but also on a function of the ratio of lag home and foreign TFP, $A\left(s^{t-1}\right) / \left[\xi_A A^*\left(s^{t-1}\right)^{\gamma_A}\right]$. Thus, if the ratio $A\left(s^{t-1}\right) / A^*\left(s^{t-1}\right)^{\gamma_A}$ is larger than its long-run value, ξ_A, then $\kappa_A < 0$ and $\kappa_A^* > 0$ will imply that $\Delta \log A\left(s^t\right)$ would fall and $\Delta \log A^*\left(s^t\right)$ would rise, driving both series toward their long-run equilibrium values. The VECM representation also implies that $\Delta \log A\left(s^t\right)$, $\Delta \log A^*\left(s^t\right)$, and $\log A\left(s^{t-1}\right) - \gamma_A \log A^*\left(s^{t-1}\right) - \log \xi_A$ are stationary processes.

4.4. Market Clearing

The model is closed with the following market clearing conditions in the final goods markets

$$C\left(s^t\right) + X\left(s^t\right) = Y\left(s^t\right)$$

and

$$C^*\left(s^t\right) + X^*\left(s^t\right) = Y^*\left(s^t\right)$$

and the bond markets

$$D\left(s^t\right) + D^*\left(s^t\right) = 0.$$

4.5. Equilibrium

Let us now describe the equilibrium conditions for this economy. Before doing that, it is useful to define the following relative prices: $\widetilde{P}_H(s^t) = \frac{P_H(s^t)}{P(s^t)}$, $\widetilde{P}_F^*(s^t) = \frac{P_F^*(s^t)}{P^*(s^t)}$ and $RER(s^t) = \frac{P^*(s^t)}{P(s^t)}$. Note that $\widetilde{P}_H(s^t)$ is the price of home intermediate goods in terms of home final goods, $\widetilde{P}_F^*(s^t)$ is the price of foreign intermediate goods in terms of foreign final goods, and $RER(s^t)$ is the RER between the home and foreign countries. In our model the law of one price holds; hence, we have that $P_H(s^t) = P_H^*(s^t)$ and $P_F(s^t) = P_F^*(s^t)$.

We now describe the equilibrium conditions implied by the first-order conditions of households, intermediate and final goods producers in the home country (the foreign country ones are identical and not reported here for space reasons), as well as the relevant laws of motion, production functions, and market clearing conditions. The marginal utility of consumption and the labor supply are given by:

$$U_C\left[C\left(s^t\right), L\left(s^t\right)\right] = \lambda\left(s^t\right) \tag{3}$$

and

$$\frac{U_L\left[C\left(s^t\right), L\left(s^t\right)\right]}{U_C\left[C\left(s^t\right), L\left(s^t\right)\right]} = W\left(s^t\right), \tag{4}$$

where U_x denotes the partial derivative of the utility function U with respect to variable x. The first-order condition with respect to investment and capital delivers:

$$\lambda\left(s^t\right) = \mu\left(s^t\right) \tag{5}$$

and

$$\mu\left(s^t\right) = \beta \sum_{s^{t+1}} \pi\left(s^{t+1}|s^t\right)\left\{R\left(s^{t+1}\right)\lambda\left(s^{t+1}\right) + \mu\left(s^{t+1}\right)(1-\delta)\right\} \tag{6}$$

respectively, where $\pi\left(s^{t+1}|s^t\right) = \frac{\pi\left(s^{t+1}\right)}{\pi\left(s^t\right)}$ is the conditional probability of s^{t+1} given s^t. The law of motion of capital is:

$$K\left(s^t\right) = (1-\delta)K\left(s^{t-1}\right) + X\left(s^t\right). \tag{7}$$

The optimal choice for riskless bonds delivers the following expression for the price of the riskless bond:

$$\overline{Q}\left(s^t\right) = \beta \sum_{s^{t+1}} \pi\left(s^{t+1}|s^t\right)\frac{\lambda\left(s^{t+1}\right)}{\lambda\left(s^t\right)}\frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)} - \frac{\Phi'\left[D\left(s^t\right)\right]}{\beta}. \tag{8}$$

The risk-sharing condition is given by the optimal choice of the households of both countries for the riskless bond:

$$\sum_{s^{t+1}} \pi\left(s^{t+1}|s^{t}\right)\left[\frac{\lambda^{*}\left(s^{t+1}\right)}{\lambda^{*}\left(s^{t}\right)} \frac{\widetilde{P}_{H}\left(s^{t+1}\right)}{\widetilde{P}_{H}\left(s^{t}\right)} \frac{RER\left(s^{t}\right)}{RER\left(s^{t+1}\right)}-\frac{\lambda\left(s^{t+1}\right)}{\lambda\left(s^{t}\right)} \frac{\widetilde{P}_{H}\left(s^{t+1}\right)}{\widetilde{P}_{H}\left(s^{t}\right)}\right]=-\frac{\Phi^{\prime}\left[D\left(s^{t}\right)\right]}{\beta}. \tag{9}$$

From the intermediate goods producers' maximization problems, we obtain the result that labor and capital are paid their marginal product:

$$W\left(s^{t}\right)=(1-\alpha) \widetilde{P}_{H}\left(s^{t}\right) A\left(s^{t}\right) K\left(s^{t-1}\right)^{\alpha} L\left(s^{t}\right)^{-\alpha} \tag{10}$$

and

$$R\left(s^{t}\right)=\alpha \widetilde{P}_{H}\left(s^{t}\right) A\left(s^{t}\right) K\left(s^{t-1}\right)^{\alpha-1} L\left(s^{t}\right)^{1-\alpha}. \tag{11}$$

From the final goods producers' maximization problem, we obtain the demands of intermediate goods, which depend on their relative price:

$$Y_{H}\left(s^{t}\right)=\omega \widetilde{P}_{H}\left(s^{t}\right)^{-\theta} Y\left(s^{t}\right) \tag{12}$$

and

$$Y_{H}^{*}\left(s^{t}\right)=(1-\omega)\left(\frac{\widetilde{P}_{H}\left(s^{t}\right)}{RER\left(s^{t}\right)}\right)^{-\theta} Y^{*}\left(s^{t}\right). \tag{13}$$

Good, input, and bond markets clear. Thus:

$$C\left(s^{t}\right)+X\left(s^{t}\right)=Y\left(s^{t}\right), \tag{14}$$

$$Y\left(s^{t}\right)=\left[\omega^{\frac{1}{\theta}} Y_{H}\left(s^{t}\right)^{\frac{\theta-1}{\theta}}+(1-\omega)^{\frac{1}{\theta}} Y_{F}\left(s^{t}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{15}$$

$$Y_{H}\left(s^{t}\right)+Y_{H}^{*}\left(s^{t}\right)=A\left(s^{t}\right) K\left(s^{t-1}\right)^{\alpha} L\left(s^{t}\right)^{1-\alpha}, \tag{16}$$

and

$$D\left(s^{t}\right)+D^{*}\left(s^{t}\right)=0. \tag{17}$$

The law of motion of the level of debt

$$\widetilde{P}_{H}\left(s^{t}\right) \overline{Q}\left(s^{t}\right) D\left(s^{t}\right)=\widetilde{P}_{H}\left(s^{t}\right) Y_{H}^{*}\left(s^{t}\right)-\widetilde{P}_{F}^{*}\left(s^{t}\right) RER\left(s^{t}\right) Y_{F}\left(s^{t}\right)+\widetilde{P}_{H}\left(s^{t}\right) D\left(s^{t-1}\right)-\widetilde{P}_{H}\left(s^{t}\right) \Phi\left[D\left(s^{t}\right)\right] \tag{18}$$

is obtained using (1) and the fact that intermediate and final goods producers at home make zero profits. Finally, the (log) TFP shocks follow the VECMs described above.

4.6. Balanced Growth and the Restriction on the Cointegrating Vector

Equations (3) to (18) together with the VECM process for (log) TFP characterize the equilibrium in this model. Since we assume that both $\log A(s^t)$ and $\log A^*(s^t)$ are integrated processes, we need to normalize the equilibrium conditions in order to obtain a stationary system more amenable to study. Following King, Plosser, and Rebelo (1988) we basically divide the home-country variables that have a trend by the lagged domestic level of TFP, $A(s^{t-1})$, and the foreign-country variables that have a trend by the lagged foreign level of TFP, $A^*(s^{t-1})$. In appendix A, we detail the full set of normalized equilibrium conditions for the benchmark model.

When non-stationary TFP shocks are present, for the model to have balanced growth, we require some restrictions on preferences, production functions, and the law of motion of productivity shocks. The restrictions on preferences and technology of King, Plosser, and Rebelo (1988) are sufficient for the existence of balanced growth in a closed-economy RBC model. However, in our two-country model, an additional restriction on the cointegrating vector is needed if the model is to exhibit balanced growth. In particular, we need the ratio $A(s^{t-1})/A^*(s^{t-1})$ to be stationary. Our VECM implies that the ratio between $A(s^{t-1})$ and $A^*(s^{t-1})^{\gamma_A}$ is stationary. Therefore, a sufficient condition for balanced growth in this model is that the parameter γ_A equals one or, equivalently, that the cointegrating vector equals $(1, -1)$.

5. Estimation of the VECM for TFP

In this Section, after describing our constructed (log) TFP series for the U.S. and the "rest of the world" (ROW), we perform three exercises. First, we show that our assumption that the (log) TFP processes are cointegrated of order C(1,1) cannot be rejected in the data. Second, we also show that the restriction imposed by balanced growth, i.e., that the parameter γ_A is equal to one, cannot be rejected in the data either. Finally, we estimate the parameters driving our VECM in order to simulate the model in the next Section.

5.1. Data

In order to construct our (log) TFP series we combine data on output, employment, and capital for the U.S. and ROW to obtain the Solow residual. Hence, we have

$$\log A\left(s^t\right) = \log Y\left(s^t\right) - (1-\alpha)\log L\left(s^t\right) - \alpha \log K\left(s^{t-1}\right)$$

and

$$\log A^*\left(s^t\right) = \log Y^*\left(s^t\right) - (1-\alpha)\log L^*\left(s^t\right) - \alpha \log K^*\left(s^{t-1}\right)$$

for the U.S. and for the ROW respectivelly, where α is the capital share of output and takes a value of 0.36 and capital is constructed using a perpetual inventory method with quarterly real investment and a quarterly depreciation rate of $\delta = 0.025$.[9] All of our data in this Section are from 1970:1 to 2007:4. For the U.S., we use quarterly nominal gross domestic product (GDP) as reported by the IMF's *International Financial Statistics* (IFS) and employment is defined as civilian employment from the OECD's Main Economic Indicators.[10] Real GDP is obtained by deflating nominal gross domestic product by the GDP deflator (base 2000). The investment series used to construct capital for the U.S. is nominal gross fixed capital formation deflated by the GDP deflator. Both the nominal gross fixed capital formation and the GDP deflator series come from the IFS.[11]

The ROW aggregates data for the 15 countries of the Euro Area, the United Kingdom, Canada, Japan, and Australia. This group accounts for about 50 percent of the basket of currencies that the Federal Reserve uses to construct the RER for the U.S. dollar. For the Euro Area we use real GDP and real investment as defined in the Area Wide Model (AWM) dataset, in domestic currency at 1995 fixed prices. Employment is the number of employees from Eurostat. For the United Kingdom, we use real GDP, real investment (base 2000 in local currency) and employment from Eurostat. For Canada, real GDP and real investment are from Statistics Canada (CANSIM) in domestic currency, base 2002. Employment is also from CANSIM. For Japan we construct real GDP and real investment using nominal GDP and nominal gross fixed capital formation (both in local currency) deflated using the GDP deflator (base 2000) all from the IFS. Employment is the number of employees from the G10. For Australia, we use real GDP and real gross fixed capital formation from the Australian Bureau of Statistics in domestic currency (base 2006). Employment is the number of employees from the G10.

To aggregate series for the ROW we first transform real GDP and real investment series to base 2000 in local currency (the United Kingdom and Japan are already in this base so we skip this step for these two countries). We do that using the GDP deflator (base 1995) from the AWM for the Euro Area, the GDP deflator (base 2000) from the IFS for Canada and the GDP deflator (base 2000) from the IFS for Australia. Second, we construct capital series for each country using

[9] Details about how the perpetual inventory method is implemented are provided in Appendix B.

[10] Details on IFS codes are provided in the Appendix C.

[11] We deseasonalized all the series in this section when a significant seasonal component was discovered, so we do seasonally adjust our data using X-12.

our investment series. Third, we convert each national real output and capital series to U.S. dollars (base 2000) using the implied national PPP exchange rate for the year 2000.[12] All of the exchange rates are obtained from the WEO. Finally, we add up output, capital, and employment to get output, capital, and employment for the ROW. Figure 7 shows both series.

Backus, Kehoe, and Kydland (1992) and Heathcote and Perri (2002, 2008) use a similar approach when constructing (log) TFP series for the United States and a ROW aggregate. It is typical in the IRBC literature to rely on output and labor data only in order to build the (log) TFP series. Since capital fluctuates little over BC frequencies, omitting capital should not affect the BC analysis. Since we want to explain fluctuations at all frequencies, we need to add capital to the computation of (log) TFP shocks.

5.2. Integration and Cointegration Properties

In this Section, we present evidence supporting our assumption that the (log) TFP processes for the U.S. and the ROW are cointegrated of order C(1,1). First, we will empirically support the unit root assumption for the univariate processes. Second, we will test for the presence of cointegrating relationships using the Johansen (1991) procedure. Univariate analysis of the (log) TFP processes for the U.S. and the ROW indicates that both series can be characterized by unit root processes with drift. Table 2 presents results for the (log) TFP processes for the U.S. and ROW using the following commonly applied unit root tests: augmented Dickey-Fuller (ADF) (Dickey and Fuller, 1979, and Said and Dickey 1984); the DF-GLS and the optimal point statistic (P_TGLS), both of Elliott et al. (1996); and the modified MZ_α, MZ_t, and MSB of Ng and Perron (2001). The lag length is chosen using the Schwarz criterion. In each case a constant and a trend are included in the specification. None of the tests can reject the null hypothesis of unit root at the 5 percent critical value. This is the case for the U.S. and the ROW. Using the same tests, there is also strong evidence that the first difference of the (log) TFP processes for the U.S. is stationary. All the tests reject the null hypothesis of unit root at the 5 percent critical value. For the ROW the results are somewhat weaker. Only the ADF and P_TGLS reject the null hypothesis of unit root at the 5 percent critical value.

[12] For the Euro Area we use the nominal exchange rate, since PPP exchange rates were not available for the whole period in the AWM.

Table 2: Unit Root Tests for TFP								
	U.S. TFP $[log(A)]$				ROW TFP $[log(A^*)]$			
	Level		First Difference		Level		First Difference	
Method	c. value	t-Stat.	c. value	t-Stat.	c. value	t-Stat.	c. value	t-Stat.
ADF	-3.44	-1.88	-3.44	-11.36	-3.44	-2.55	-3.44	-10.65
DF-GLS	-2.99	-1.49	-2.98	-11.30	-2.99	-0.10	-2.99	-1.86
P_T-GLS	5.65	21.74	5.65	1.28	5.65	277.61	5.65	2.01
MZ_α	-17.3	-4.41	-17.3	-74.58	-17.3	0.12	-17.3	-4.06
MZ_t	-2.91	-1.46	-2.91	-6.09	-2.91	0.09	-2.91	-1.41
MSB	0.16	0.33	0.16	0.08	0.16	0.72	0.16	0.34

Notes: t-Stat. stands for t-Statistic and c. value are 5 % critical values. Critical values for the DF-GLS and P_T-GLS tests are as in Elliott-Rothenberg-Stock (1996, Table 1). Critical values for MZ_α, MZ_t, and MSB are as reported in Ng-Perron (2001) Table 1.

Having presented evidence that indicates that the (log) TFP for the U.S. and the ROW is well characterized by integrated processes of order one, we now focus on presenting evidence supporting our assumption that the processes are cointegrated. We estimated an unrestricted VAR with one lag and a deterministic trend for the bivariate system $[\log A(s^t), \log A^*(s^t)]$ for the sample period 1970:1 to 2007:4 where the number of lags was chosen using the Schwarz criterion. We found that the two eigenvalues' modulus are 1.00 and 0.97. If $\log A(s^t)$ and $\log A^*(s^t)$ share one common stochastic trend (balanced growth), the estimated VAR has to have a single eigenvalue equal to one and all other eigenvalues have to be less than one. Our point estimates are in accord with this prediction: the highest eigenvalue equals one, while the second highest is less than one. But this is not a formal test of cointegration. Table 3 reports results from the unrestricted cointegration rank test using the trace and the maximum eigenvalue methods as defined by Johansen (1991). We assume both an intercept in the VAR and a constant in the cointegration relationship and zero lags. Clearly, the data strongly support a single eigenvalue.

Table 3: Cointegration Statistics II: Johansen's test				
Number of Vectors	Trace Statistic	5 % critical value	Max-Eigenvalue Statistic	5 % critical value
0	37.68	15.49	37.47	14.26
1	0.21	3.84	0.21	3.84

5.3. The Estimated VECM Model

In the last subsection, we presented evidence that $\log A(s^t)$ and $\log A^*(s^t)$ are cointegrated of order C(1,1). In this subsection we show that the null hypothesis of $\gamma_A = 1$ cannot be rejected by the data. This is very important because a cointegrating vector $(1, -1)$ implies that the balanced growth path hypothesis cannot be rejected. In fact, if we estimate a VECM with zero lags ($\rho_A = 0$) and intercept both in the VAR and in the cointegration relationship as suggested by the Johansen (1991) test reported above, the likelihood ratio test cannot reject the joint null hypothesis $\gamma_A = 1$ and $\kappa_A = 0$ at the 5 percent critical value. This indicates that there is joint evidence for balanced growth and spillovers of the U.S. to the ROW but not the other way around. The likelihood ratio (LR) test for the joint restriction is distributed as Chi-squared with two degrees of freedom and takes a value of 5.70, smaller than the 5 percent critical value of 5.99. Conditional on the restriction the VECM estimates are reported in the next table.

Table 4: The VECM Model

c_A	c_A^*	κ_A^*
0.0020 (3.51)	0.0035 (10.55)	0.0263 (5.92)

Notes: t-statistics in parenthesis.

Finally, the standard deviations of the innovations $\varepsilon^A(s^t)$ and $\varepsilon^{A,*}(s^t)$ (σ^A and $\sigma^{A,*}$) are estimated to be 0.0072 and 0.0042, respectively. In the simulation, we will assume that $\varepsilon^A(s^t)$ and $\varepsilon^{A,*}(s^t)$ are uncorrelated, since this null hypothesis cannot be rejected in the data.

6. Results

We now present the results for our benchmark model. We first describe the parameterization. Then we show how our model matches the spectrum of the RER. Finally, we give some intuition for our results using the IRFs implied by our model.

6.1. Baseline Parameterization

Our baseline parameterization follows that in Heathcote and Perri (2002) closely. The discount factor β is set equal to 0.99, which implies an annual rate of return on capital of 4 percent. We set the consumption share, τ, equal to 0.34 and the coefficient of risk aversion, σ, equal to 2. Backus, Kehoe, and Kydland (1992) assume the same value for the latter parameter. We assume a cost of bond holdings, ϕ, of 1 basis point (0.01). Parameters on technology are fairly standard in the literature. Thus, the depreciation rate, δ, is set to a quarterly value of 0.025, the capital share of output is set to $\alpha = 0.36$, and home bias for domestic intermediate goods is set to $\omega = 0.9$, which implies the actual import/output ratio in steady state. We calibrate the elasticity of substitution between intermediate goods to be $\theta = 0.85$. The value is based on Heathcote and Perri (2002).

6.2. Matching Real Exchange Rate Spectra

In Figure 9 we present the model-based spectra for the (log) RER under a baseline calibration and compare it with the estimated spectra for the (log) U.S. dollar RER.[13] In order to compute the model-based spectra, we simulate 500 series of 148 observations (as in the data) from the model and compute the average spectrum frequency by frequency. The grey area represents the BC frequencies. Two observations. First, the model replicates an important portion of the standard deviation of the U.S. dollar RER. Second, as in the data, the model assigns a large share of the RER's volatility to movements below BC frequencies. Table 5 puts numbers to these two claims. Table 5 first shows the average of the model-based RER and output growth standard deviations. Second, it also reports the average share of the volatility of the model-based RER assigned to low, BC and high-frequency movements. All these moments are computed using the simulated 500 series of 148 observations. Finally, it compares these moments with those in the data. The U.S. dollar RER has a standard deviation of 10.64 percent, while the model-generated RER has an average standard deviation of 5.15 percent and assigns most of the RER variation to low-frequency fluctuations, as do the data. In fact, on average the model overpredicts the fraction of low-frequency movements in the data (about 75 percent in the model while it is 70 percent in the data). Also, the model produces an average standard deviation of real GDP growth very similar to the one in the data (0.77 versus 0.80 percentage point). Table 5 also reports the standard deviation (in parenthesis) of our model-based moments. These standard deviations do not change our main message.

[13] In this section we will always analyze the log of the RER. In any case, to avoid cumbersome lenguage, we will refer to the log of the RER as RER.

Table 5: Implications of the Model with Only TFP					
	Standard Deviation		Frequency decomposition of RER		
	RER	GDP growth	Low	BC	High
U.S. Data	10.64	0.8	70.0	24.9	5.0
Baseline	5.15 (1.40)	0.77 (0.05)	76.2 (9.2)	18.6 (7.4)	5.2 (2.1)
$\theta = 0.62$	8.44 (2.44)	0.69 (0.04)	72.8 (10.4)	20.8 (7.9)	6.3 (2.9)
$\theta = 1.00$	3.81 (1.11)	0.81 (0.06)	76.6 (8.3)	18.4 (6.7)	5.0 (2.0)

Note: RER denotes the log of the RER. Growth rates are computed taking the first differences of the logs.

Thus, in our baseline calibration, the model (1) explains 50 percent of the volatility of the U.S. dollar RER, while it does even better on the GDP growth front and (2) follows closely the data assigning most of the RER movements to low-frequency fluctuations. In fact, the model overweights the fraction of low-frequency movements; i.e., the model-based RER is too persistent when compared with the data. Since, as Chari, Kehoe and McGrattan (2002) mention, "The central puzzle in international ... is that fluctuations in RER are volatile and persistent," we see these two results as very important. Once we analyze the RER fluctuations at all frequencies (and not only at BC frequencies as most of the literature does) the basic model described in Heathcote and Perri (2002) modified to consider cointegrated (log) TFP shocks does reasonably well at explaining unconditional volatility and generates too much persistence of RER (instead of too low, as commonly believed).

A key parameter is the elasticity of substitution between home and foreign goods, θ. The lower this elasticity, the easier it is for the model to explain the volatility of relative prices (such as the RER) with respect to quantities. Heathcote and Perri (2002) estimate this value to be $\theta = 0.85$, which is what we take as our baseline. When trying to fit aggregate data, Lubik and Schorfheide (2005) estimate a much lower number of 0.43, using a two-country model and Bayesian methods.[14] Hence, we also try an alternative lower value of $\theta = 0.62$, which has been used by Corsetti, Dedola and Leduc (2008b) and Coeurdacier, Kollmann and Martin (2007). When we allow for this lower elasticity of substitution, Table 6 shows that the model does even better matching the standard deviation of the U.S. dollar RER. However, the model still overpredicts the fraction of low-frequency movements in the RER. For completeness we also report results for the case of $\theta = 1$. As expected, the model-implied volatility of the RER is lower, but it still

[14] However, estimates of the elasticity of substitution using disaggregated data are much higher, in the range of 5 to 10. See, for instance, Imbs and Mejean (2009).

concentrates most of its volatility in low-frequencies. Figure 10 reports the spectra of the RER associated with the three values considered for θ and compares them with the spectral density of the data. The figure confirms the moments reported in Table 5. As the elasticity of substitution gets lower, the area below the spectrum gets bigger while the share of volatility assigned to movements below BC frequencies remains high for all three values of θ.

It is important to check that while cointegrated TFP shocks help to match the volatility and persistence of the RER in the data, they do not affect the model's ability to explain other commonly analyzed moments in the IRBC literature. Table 6 compares some data- and model-based moments under a baseline calibration for output growth, consumption growth, investment growth, and employment. The moments are computed using a first-differenced series of (log) consumption, output, and investment and the (log) employment, instead of an HP-filtered series as is commonly done in the IRBC literature. In any case, for the data-based reported moments, using first differences or HP-filtered series does not produce very different outcomes.[15] Like Heathcote and Perri (2002), we are able to match the relative volatility of consumption and investment growth with respect to output growth (although they look at HP-filtered data). However, we are able to match the relative volatility of employment with respect to output growth, while Heathcote and Perri (2002) find that HP-filtered employment is much less volatile than HP-filtered real output. As in Heathcote and Perri (2002) this version of the model does a very bad job of explaining the contemporaneous correlations between output growth, consumption growth, investment growth, and employment.

Finally, all the puzzles related to the international comovement of real variables also show up in our model. In particular, consumption growth rates are more correlated than in the data, real GDP growth rates are less correlated than in the data, and factor input growth rates are negatively correlated in the model and mildly positively correlated in the data. The most relevant difference with Heathcote and Perri (2002) in this dimension is that the cross-country correlation between the U.S. and ROW series is slightly stronger when using HP-filtered data. In general, when looking at Table 6, there are no large significant differences between Heathcote and Perri's (2002) reported results and ours, although we do somewhat better on the employment dimension.

[15] Moments based on HP-filtered series in the data and in the model are available upon request.

Table 6: Other Moments of the Model with TFP				
	Std. Dev.	Relative Std. Dev. with respect to GDP growth		
	GDP growth	Cons. growth	Inv. growth	Empl.
U.S. Data	0.80	0.87	2.42	0.59
Model	0.77 (0.04)	0.67 (0.06)	1.97 (0.11)	0.56 (0.13)
Correlations with GDP Growth				
		Cons. growth	Inv. growth	Empl.
U.S. Data		0.60	0.73	0.58
Model		0.99 (0.00)	0.99 (0.00)	0.33 (0.03)
Cross-Country Correlations				
	GDP growth	Cons. growth	Inv. growth	Empl.
Data	0.29	0.29	0.12	0.25
Model	-0.06 (0.07)	0.68 (0.05)	-0.47 (0.06)	-0.25 (0.28)

Notes: Growth rates are computed taking first difference of the logs.

6.3. Impulse Response Functions and the Half-Life of the RER

Next, in Figures 11 and 12, we look at the IRFs of the RER to a TFP shock in the U.S. and ROW economies. As we can see in Figure 11, after a U.S. TFP shock, the model implies a depreciation in the RER. When TFP increases in the U.S., U.S. households consume and invest more, while labor rises because of the increase in the marginal productivity of labor. Provided that the elasticity of substitution between home and foreign intermediate goods is low enough, the demand for the ROW intermediate good rises. Therefore, the ROW intermediate good becomes relatively scarce with respect to the U.S. intermediate good, and the terms of trade and the RER depreciate. This depreciation is larger when a VECM is present because labor and investment decline in the ROW. Why is this the case? After a TFP shock in the U.S. a "news" channel arises in the ROW: because TFP shocks are correlated, ROW households know their productivity will increase soon and, because of an income effect, they supply less labor and capital, making the ROW intermediate good even more scarce. In Figure 12 we present the same results for a ROW TFP shock. Since the U.S. does not experience the "news" channel (remember that κ_A is estimated to be zero), hours, investment, and output increase in the U.S. to satisfy the foreign demand for U.S. intermediate goods. Hence, although the RER drops (appreciates), it moves less than in the case of a TFP shock in the U.S. As we can observe from Figures 11 and 12, the model generates a hump-shaped and highly persistent response of the RER. The model-based RER peaks around 3 years after the shock and the half-life of the shock is 10 years. The first fact is comforting, after Steinsson (2008) showed that real shocks in a two-country sticky-price model yield RER hump-shaped dynamics. The second fact reinforces the results reported in the last Section: when compared with the data, the model-based RER is too persistent. Steinsson (2008) estimates a half-life of 4.5 years with a peak at around 2.25 years for the U.S. dollar RER.

6.4. Conclusion to This Section

When we look at all of the frequencies instead of concentrating on BC fluctuations, the baseline two-country, two-good IRBC model works better than previously reported in replicating RER volatility and persistence. In particular, this model with cointegrated TFP shocks does a reasonable job of explaining the volatility of the RER while not affecting its ability to match other commonly analyzed moments. The model also generates persistent deviations of the RER, even more persistent than in the data. Hence, the puzzle is that the model-generated RER series are too persistent with respect the actual ones, not the other way around, as formerly thought. In our benchmark calibration, we can explain about 50 percent of the volatility of the RER when a reasonable elasticity of substitution across types of goods is assumed and about 80 percent when the elasticity of substitution is low. In the next Section, we study the role of introducing IST shocks in the model. Raffo (2009), Rabanal et al. (2009), and Mandelman et al. (2009) have studied the role of IST shocks in explaining HP-filtered properties of the data. Here, we study their role in helping the model fit the spectrum of the of U.S. dollar RER.

7. A Model with IST Shocks

In this Section we extend our benchmark framework to consider IST shocks. As was the case with (log) TFP shocks, in this Section we assume that (log) IST shocks are also non-stationary but cointegrated: they follow a VECM process. This assumption has strong and testable implications for the data. The empirical evidence supporting our assumption will be presented in Section 7.3. IST shocks affect the rate of transformation of consumption goods into capital goods (Fisher, 2006; Greenwood et al., 1988). Only two equations are affected with respect to the benchmark model described in Section 4. The law of motion for capital is now described by

$$K\left(s^{t}\right)=(1-\delta)\,K\left(s^{t-1}\right)+V\left(s^{t}\right)X\left(s^{t}\right) \tag{19}$$

where $V(s^t)$ is the IST shock in the home country. Foreign-country households face an equivalent shock, $V^*(s^t)$. Also the cost of holding bonds, $\Phi[\cdot]$, takes the new following functional form $\Phi[D(s^t)] = \frac{\phi}{2} Z(s^{t-1}) \left[\frac{D(s^t)}{Z(s^{t-1})}\right]^2$, where $Z(s^{t-1})$ is a weighted geometric mean of both the TFP and IST shocks to be defined later. Equation (19) implies that $V(s^t)$ can also be interpreted as the ratio of consumption prices with respect to investment prices in the home country (an equivalent interpretation is also valid for $V^*(s^t)$). This interpretation will be crucial later when we build our home and foreign countries IST series.

7.1. A VECM for IST Shocks

We assume that the (log) IST shocks are non-stationary but cointegrated. Hence, the process for $\log V(s^t)$ and $\log V^*(s^t)$ are described by the following equation:

$$\begin{pmatrix} \Delta \log V(s^t) \\ \Delta \log V^*(s^t) \end{pmatrix} = \begin{pmatrix} c_V \\ c_V^* \end{pmatrix} + \sum_{k=1}^{K} \rho_V(k) \begin{pmatrix} \Delta \log V\left(s^{t-k}\right) \\ \Delta \log V^*\left(s^{t-k}\right) \end{pmatrix}$$
$$+ \begin{pmatrix} \kappa_V \\ \kappa_V^* \end{pmatrix} \left[\log V\left(s^{t-1}\right) - \gamma_V \log V^*\left(s^{t-1}\right) - \log \xi_V\right] + \begin{pmatrix} \varepsilon^V(s^t) \\ \varepsilon^{V,*}(s^t) \end{pmatrix}$$

where $\rho_V(k)$ are 2×2 coefficient matrices, K is the number of lags, $(1, -\gamma_V)$ is the cointegrating vector, ξ_V is the constant in the cointegrating relationship, $\varepsilon^V(s^t) \sim N\left(0, \sigma^V\right)$ and $\varepsilon^{V,*}(s^t) \sim N\left(0, \sigma^{V,*}\right)$, $\varepsilon^V(s^t)$ and $\varepsilon^{V,*}(s^t)$ are correlated. As before, this VECM representation implies that deviations of today's log differences of IST shocks with respect to its mean value depend not only on lags of home and foreign log differences of IST shocks but also on a function of the ratio of lag home and foreign IST shocks, $V(s^{t-1}) / \left[\xi_V V^*(s^{t-1})^{\gamma_V}\right]$. The VECM representation also implies that $\Delta \log V(s^t)$, $\Delta \log V^*(s^t)$, and $\log V(s^{t-1}) - \gamma_V \log V^*(s^{t-1}) - \log \xi_V$ are stationary processes.

7.2. Equilibrium and Balanced Growth

In our benchmark model, equations (3) to (18) and the VECM process for (log) TFP fully characterize the equilibrium. When we consider IST shocks, two equilibrium conditions change. In particular, equations (5) and (7) are to be substituted by the following two new first order conditions:

$$\lambda\left(s^t\right) = V\left(s^t\right) \mu\left(s^t\right) \tag{20}$$

and

$$K\left(s^{t}\right)=(1-\delta)\,K\left(s^{t-1}\right)+V\left(s^{t}\right)X\left(s^{t}\right) \tag{21}$$

respectively. Hence, the equilibrium is now characterized by (3) to (18) (where (20) and (21) substitute for (5) and (7)) and the two VECM processes: one for the (log) TFP shocks and another for the (log) IST shocks. Since both (log) TFP shocks and (log) IST shocks are integrated, we need to normalize the equilibrium conditions. The basic idea is to divide most of the home-country variables by $Z\left(s^{t-1}\right)$, where $Z\left(s^{t}\right)=A\left(s^{t}\right)^{\frac{1}{1-\alpha}}V\left(s^{t}\right)^{\frac{\alpha}{1-\alpha}}$, and the foreign-country variables by $Z^{*}\left(s^{t-1}\right)$. One exception is the capital stock series, which are instead divided by: $Z\left(s^{t-1}\right)V\left(s^{t-1}\right)$ in the home country and $Z^{*}\left(s^{t-1}\right)V^{*}\left(s^{t-1}\right)$ in the foreign country. In appendix D, we detail the full set of normalized equilibrium conditions for the model with both shocks. In this case, for the model to have balanced growth, we require the ratio $Z\left(s^{t-1}\right)/Z^{*}\left(s^{t-1}\right)$ to be stationary. A sufficient condition to guarantee the stationarity of $Z\left(s^{t-1}\right)/Z^{*}\left(s^{t-1}\right)$ is to check for the stationarity of both $A\left(s^{t-1}\right)/A^{*}\left(s^{t-1}\right)$ and $V\left(s^{t-1}\right)/V^{*}\left(s^{t-1}\right)$. In Section 5.3 we have shown that the first ratio is stationary. In what follows, we focus the analysis on the IST shocks.

7.3. Estimation of the VECM for IST

In this subsection, we proceed in a similar way as when we estimated a VECM for TFP shocks. First, we explain how we construct measures for IST shocks for the U. S. and the ROW. Second, we present unit root tests for each series and we also present evidence of cointegration. Finally, we present the parameter estimates of the VECM.

7.3.1. Data

In order to estimate our VECM for (log) IST shocks we use data for the U.S. and an aggregate for the ROW. The ROW is composed of the most significant trading partners of the U.S.: the 15 countries of the Euro Area, Canada, Japan, the United Kingdom, Australia and South Korea.[16] Our sample period goes from 1982:4 to 2007:4. Both for the U.S. and for the ROW, we aim to obtain the relative price of investment goods with respect to consumption goods, which will be measured as an investment deflator divided by a consumption deflator. In particular, for the U.S. the shock $V\left(s^{t}\right)$ is defined as PCE_{t}^{US}/PI_{t}^{US}, where PCE_{t}^{US} is the personal consumption expenditures deflator, and PI_{t}^{US} is the investment deflator. For the ROW aggregate, we define

[16] Note that for ISTs we have included South Korea to maximize the number of countries in the measure of ISTs. Excluding it does not affect the main results of this section.

$V^*(s^t)$ as:

$$\sum_i w_t^i * \left(\frac{PCE_t^i}{PI_t^i}\right)$$

where i identifies the country in the set {15 countries of the Euro Area, Canada, Japan, the United Kingdom, Australia and South Korea} and w_t^i is the trade weight of a particular country i at time t. The weights are the currency weights used in the Broad Index of the Foreign Exchange Value of the dollar calculated by the U.S. Federal Reserve.

The particular deflators being used are now described. For the U.S. we use the personal consumption expenditure (PCE) deflator as our consumption deflator and the gross domestic investment deflator as our investment deflator. Both series are derived directly from the National Income and Product Accounts (NIPA) and provided by the Bureau of Economic Analysis (BEA). For Japan, we employ the private final consumption expenditure and the private-sector capital formation deflator series obtained from the Cabinet Office. In the case of Canada, we use the personal expenditure on consumer goods and services deflator and the business gross fixed capital formation deflator series. Both series can be obtained from Canada's statistical agency, Statistics Canada. For the UK, we use the final consumption expenditure deflator and the gross fixed capital formation deflator provided by the UK national statistics. The deflators for Australia are derived from the Australian Bureau of Statistics. The particular series used were the households final consumption expenditure and the gross fixed capital formation implicit price deflators. For South Korea we use the final consumption expenditure deflator and gross capital formation deflator series retrieved from the Navi-Data database provided by the Korean National Statistical Office. Finally, for the Euro Area countries, we employ the consumption deflator and the gross investment deflator from the AWM Database constructed by the European Central Bank. Figure 8 shows both series.

7.3.2. Integration and Cointegration Properties

In this Section, we present evidence supporting our assumption that the (log) IST processes for the U.S. and the ROW are cointegrated of order C(1,1). As was the case for the (log) TFP shocks, we will first empirically support the unit root assumption for the univariate processes and then we will test for the presence of cointegrating relationships using the Johansen (1991) procedure.

Table 7 presents results for the (log) IST processes for the U.S. and ROW using the same unit root tests as in Section 5.2. As before, the lag length is chosen using the Schwarz criterion. In each case a constant and a trend are included in the specification. None of the tests can reject the null hypothesis of unit root at the 5 percent critical value.[17] This is the case for the U.S. and the ROW. Using the same tests, there is also strong evidence that the first difference of the (log) IST processes for the U.S. is stationary. All of the tests reject the null hypothesis of unit root at the 5 percent critical value. For the ROW the evidence of stationarity of the first difference is weaker. Only the ADF test rejects clearly at the 5 percent and the DF-GLS test marginally does not reject at the 10 percent critical value. The rest of the tests cannot reject. So, there is strong evidence that the (log) IST process for the ROW is integrated, but it is hard to clarify whether it is integrated of order one or two. Given that there is strong evidence that the (log) IST processes for the U.S. are integrated of order one and, as we show below, there is also strong evidence of a cointegration relationship between the (log) IST processes for the U.S. and the ROW, we take the evidence presented here as evidence in favor of the (log) IST process for the ROW being integrated of order one.

Table 7: Unit Root Tests for IST Shocks

	U.S. TFP $[log(V)]$				ROW TFP $[log(V^*)]$			
	Level		First Difference		Level		First Difference	
Method	c. value	t-Stat.	c. value	t-Stat.	c. value	t-Stat.	c. value	t-Stat.
ADF	-3.45	1.53	-3.45	-7.85	-3.45	-0.48	-3.45	-3.60
DF-GLS	-3.03	0.02	-3.03	-7.10	-3.03	-1.04	-3.03	-2.45
P_T-GLS	5.64	103.86	5.64	2.37	5.64	35.42	5.64	30.03
MZ_α	-17.3	-0.02	-17.3	-44.37	-17.3	-5.18	-17.3	-1.90
MZ_t	-2.91	0.00	-2.91	-4.70	-2.91	-1.38	-2.91	-0.95
MSB	0.17	0.34	0.17	0.11	0.17	0.27	0.17	0.50

Notes: t-Stat. stands for t-Statistic and c. value are 5% critical values. Critical values for the DF-GLS and P_T-GLS tests are as in Elliott-Rothenberg-Stock (1996, Table 1). Critical values for MZ_α, MZ_t, and MSB are as reported in Ng-Perron (2001) Table 1.

Once we have presented evidence that indicates that the (log) IST for the U.S. and the ROW is well characterized by integrated processes of order one, we now focus on presenting evidence supporting our assumption that the processes are cointegrated. We estimated an unrestricted VAR with one lag and a deterministic trend for the two-variable system $[\log V(s^t), \log V^*(s^t)]$. The two eigenvalues are 0.98 and 0.88, where the number of lags was chosen using the Schwarz criterion. If $\log V(s^t)$ and $\log V^*(s^t)$ share one common stochastic trend, the estimated VAR has

[17] The ADF test marginally rejects at 5 percent but does not reject at 10 percent for the ROW.

to have a single eigenvalue equal to one and all other eigenvalues have to be less than one. But this is not a formal test of cointegration.

Table 8 reports results from the unrestricted cointegration rank test using the trace and the maximum eigenvalue methods as defined by Johansen (1991). We assume no VAR intercept but a constant in the cointegration relationship and zero lags.[18] Clearly, the data strongly support a single cointegration vector.

Table 8: Cointegration Statistics II: Johansen's Test

Number of Vectors	Trace	p-value	Max-Eigenvalue	p-value
0	62.61	0.00	56.75	0.00
1	5.85	0.20	5.85	0.21

7.3.3. The Estimated VECM Model

In the last subsection, we presented evidence that $\log V(s^t)$ and $\log V^*(s^t)$ are cointegrated of order C(1,1). In this subsection we show that the null hypothesis of $\gamma_V = 1$ cannot be rejected by the data. This is very important because a cointegrating vector $(1, -1)$ implies that the balanced growth path hypothesis cannot be rejected. In fact, the LR test for the null hypothesis $\gamma_V = 1$ is distributed as a Chi-squared with one degree of freedom and takes a value of 1.1, clearly smaller than the 5 percent critical value of 3.84. Conditional on these restrictions and assuming zero lags, the VECM estimates are reported in following table.[19]

Table 9: The VECM Model

κ_V	κ_V^*
-0.035 (-8.26)	-0.017 (-3.68)

Notes: t-statistics in parenthesis.

Finally, the standard deviation of the innovations $\varepsilon^V(s^t)$ and $\varepsilon^{V,*}(s^t)$ (σ^V and $\sigma^{V,*}$) are estimated to be 0.0047 and 0.0052, respectively. In the simulation, we will assume that $\varepsilon^V(s^t)$ and $\varepsilon^{V,*}(s^t)$ are uncorrelated, since this null hypothesis cannot be rejected in the data.

[18] The Johansen (1991) test rejects the existence of a cointegration relationship if we allow for a trend in the VAR or we do not allow for a constant in the cointegration relationship.

[19] We normalize the (log) IST shocks so that the constant takes a value equal to zero. Hence, we do not report it.

7.4. Results

Using the same parameterization described in Section 6.1, in Figure 13 we present the model-based spectra for the (log) RER and compare them with the data-estimated one. As was the case before, in order to compute the model-based spectra (or any other model-based moment presented in this Section), we simulate 500 series of 148 observations from the model and compute the average spectra frequency by frequency. Introducing IST shocks has an effect similar to lowering the elasticity of substitution across types of goods: the spectrum moves upward (i.e. the model generates more RER volatility) but still most of the weight is at the low-frequency end of the spectrum (i.e., the model-generated RER is too persistent when compared with the data). In Table 10 we can see the implications of the model for the same variables of interest analyzed in Section 6.

Table 10: Implications of the Model with TFP and IST

	Standard Deviation		Frequency decomposition of RER		
	RER	GDP growth	Low	BC	High
U.S. Data	10.64	0.8	70.0	24.9	5.0
Baseline	6.05 (1.7)	0.81 (0.05)	75.6 (8.5)	18.9 (6.7)	5.6 (2.1)
$\theta = 0.62$	9.55 (2.9)	0.75 (0.05)	73.1 (10.3)	20.6 (8.1)	6.4 (2.5)
$\theta = 1.00$	4.01 (1.14)	0.85 (0.05)	76.3 (8.0)	18.5 (6.6)	5.2 (1.7)

Notes: RER denotes the log of the RER. Growth rates are computed taking logs and first differences.

The model now (1) explains around 60 percent of the volatility of the U.S. dollar RER and perfectly matches the volatility of GDP growth and (2) assigns most of the RER movements to low-frequency fluctuations. As was the case with only TFP shocks, the model overweights the fraction of low-frequency movements. In fact the model-based allocations of fluctuations across frequencies are almost identical to the ones in Section 6. When the elasticity of substitution is reduced to $\theta = 0.62$, the model gets very close to explaining the volatility of the U.S. dollar RER. Table 11 shows that adding cointegrated IST shocks slightly improves the model's ability to explain other commonly analyzed moments. Table 11 is equivalent to Table 6 when both TFP and IST shocks are considered. As shown in Mandelman et al. (2009), by comparing both tables we see that the model is now doing better in terms of explaining the contemporaneous correlations between output, consumption, and investment growth, and employment. This is a considerable improvement with respect to Heathcote and Perri (2002). With IST shocks, it is also possible to explain the relative volatility of investment growth with respect to GDP growth, but at the cost of obtaining a volatility of employment that is too large. Finally, it is worth noting that introducing IST shocks does not help the model fit the cross-country dimension of the data better, and the same puzzles that we referred to when commenting on Table 6 remain.

Table 11: Other Moments of the Model with TFP and IST				
	Std. Dev.	Relative Std. Dev. with respect to GDP growth		
	GDP growth	Cons. growth	Inv. growth	Empl.
U.S. Data	0.80	0.87	2.42	0.59
Model	0.79 (0.05)	0.70 (0.03)	2.59 (0.10)	0.98 (0.23)
Correlations with GDP Growth				
		Cons. growth	Inv. growth	Empl.
U.S. Data		0.60	0.73	0.58
Model		0.79 (0.03)	0.88 (0.02)	0.25 (0.06)
Cross-Country Correlations				
	GDP growth	Cons. growth	Inv. growth	Empl.
Data	0.29	0.29	0.12	0.25
Model	-0.08 (0.08)	0.62 (0.05)	-0.41 (0.06)	-0.05 (0.31)

Notes: Growth rates are computed taking first differences of the log.

8. Concluding Remarks

In this paper, we have taken a different approach to studying fluctuations of the RER. We have presented evidence that most fluctuations of the RER in the data can be attributed to low-frequencies, and hence studying only its BC fluctuations may lead researchers to miss a large part of the story. Then, we have studied the properties of the RER in a benchmark two-country, two-good model where TFP and IST shocks are cointegrated between the U.S. and a sample of advanced industrialized countries. In related work, Rabanal et al. (2009) and Mandelman et al. (2009) have shown that this model performs well in explaining the volatility of the RER at BC frequencies. Here, we have shown that the same model also does well in explaining the RER at all frequencies and matching the shape of the spectrum of the U.S. dollar RER. Actually, we find that the main discrepancy between the model and the data is that when fluctuations at all frequencies are taken into account, the model generates too much persistence in the RER instead of too little, as the BC analysis asserts.

Although results are not reported in the paper, we have also studied other extensions of the model. Results for the model-based RER for a version of the model with monopolistic competition and sticky prices do not differ much from what we have presented here (they do affect the behavior of other variables, but not the RER). Adding monetary policy shocks to a model with sticky prices also does not change the results. This is because the estimated size of the monetary policy shocks (i.e., the standard deviation of the residuals of an estimated Taylor rule with interest rate smoothing) is too small to make a difference.

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A. Appendix: Normalized Equilibrium Conditions for the Benchmark Model (Not for Publication)

Equations (3) to (18) characterize equilibrium in this model. Since both $\log A\left(s^t\right)$ and $\log A^*\left(s^t\right)$ are integrated, we now normalize the above-described system in order to get a stationary system more amenable to study. Additional restrictions on the VECM defining the law of motion of the technological processes are required if the model is to exhibit balanced growth. Those restrictions are described in the next subsection.

Let us first define the following normalized variables $\widehat{Y}_H\left(s^t\right) = \frac{Y_H\left(s^t\right)}{A\left(s^{t-1}\right)}$, $\widehat{Y}_H^*\left(s^t\right) = \frac{Y_H^*\left(s^t\right)}{A\left(s^{t-1}\right)}$, $\widehat{Y}_F\left(s^t\right) = \frac{Y_F\left(s^t\right)}{A^*\left(s^{t-1}\right)}$, $\widehat{Y}_F^*\left(s^t\right) = \frac{Y_F^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}$, and $\widehat{K}\left(s^{t-1}\right) = \frac{K\left(s^{t-1}\right)}{A\left(s^{t-1}\right)}$, $\widehat{K}^*\left(s^{t-1}\right) = \frac{K^*\left(s^{t-1}\right)}{A^*\left(s^{t-1}\right)}$, $\widehat{Y}\left(s^t\right) = \frac{Y\left(s^t\right)}{A\left(s^{t-1}\right)}$, $\widehat{Y}^*\left(s^t\right) = \frac{Y^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}$, $\widehat{C}\left(s^t\right) = \frac{C\left(s^t\right)}{A\left(s^{t-1}\right)}$, $\widehat{C}^*\left(s^t\right) = \frac{C^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}$, $\widehat{X}\left(s^t\right) = \frac{X\left(s^t\right)}{A\left(s^{t-1}\right)}$, $\widehat{X}^*\left(s^t\right) = \frac{X^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}$, $\widehat{W}\left(s^t\right) = \frac{W\left(s^t\right)}{A\left(s^{t-1}\right)}$, $\widehat{W}^*\left(s^t\right) = \frac{W^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}$, $\widehat{D}\left(s^t\right) = \frac{D\left(s^t\right)}{A\left(s^{t-1}\right)}$, $\widehat{D}^*\left(s^t\right) = \frac{D^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}$, $\widehat{\lambda}\left(s^t\right) = \lambda\left(s^t\right) A\left(s^{t-1}\right)^{1-\gamma(1-\sigma)}$, and $\widehat{\lambda}^*\left(s^t\right) = \lambda^*\left(s^t\right) A^*\left(s^{t-1}\right)^{1-\gamma(1-\sigma)}$. Then, the stationary first order conditions are

$$U_C\left[\widehat{C}\left(s^t\right), L\left(s^t\right)\right] = \widehat{\lambda}\left(s^t\right), \tag{22}$$

$$\frac{U_L\left[\widehat{C}\left(s^t\right), L\left(s^t\right)\right]}{U_C\left[\widehat{C}\left(s^t\right), L\left(s^t\right)\right]} = \widehat{W}\left(s^t\right), \tag{23}$$

$$\widehat{\lambda}\left(s^t\right) = \widehat{\mu}\left(s^t\right) \tag{24}$$

$$\left(\frac{A\left(s^t\right)}{A\left(s^{t-1}\right)}\right)^{1-\gamma(1-\sigma)} \widehat{\mu}\left(s^t\right) = \beta \sum_{s^{t+1}} \pi\left(s^{t+1}|s^t\right) \widehat{\lambda}\left(s^{t+1}\right)\left(\widehat{\lambda}\left(s^{t+1}\right) R\left(s^{t+1}\right) + \widehat{\mu}\left(s^{t+1}\right)(1-\delta)\right), \tag{25}$$

$$\widehat{K}\left(s^t\right) = (1-\delta)\widehat{K}\left(s^{t-1}\right)\frac{A\left(s^{t-1}\right)}{A\left(s^t\right)} + \widehat{X}\left(s^t\right)\frac{A\left(s^{t-1}\right)}{A\left(s^t\right)}, \tag{26}$$

$$\overline{Q}\left(s^t\right) = \beta \sum_{s^{t+1}} \pi\left(s^{t+1}/s^t\right) \frac{\widehat{\lambda}\left(s^{t+1}\right)}{\widehat{\lambda}\left(s^t\right)}\left(\frac{A\left(s^{t-1}\right)}{A\left(s^t\right)}\right)^{1-\gamma(1-\sigma)} \frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)} - \Phi'\left(D\left(s^t\right)\right), \tag{27}$$

$$\sum_{s^{t+1}} \pi\left(s^{t+1}/s^t\right)\left[\begin{array}{c} \frac{\widehat{\lambda}^*\left(s^{t+1}\right)}{\widehat{\lambda}^*\left(s^t\right)} \frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)} \frac{RER\left(s^t\right)}{RER\left(s^{t+1}\right)}\left(\frac{A\left(s^t\right)}{A\left(s^{t-1}\right)} \frac{A^*\left(s^{t-1}\right)}{A^*\left(s^t\right)}\right)^{1-\gamma(1-\sigma)} \\ -\frac{\widehat{\lambda}\left(s^{t+1}\right)}{\widehat{\lambda}\left(s^t\right)} \frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)} \end{array}\right] = -\frac{\Phi'\left(D\left(s^t\right)\right)}{\beta}, \tag{28}$$

$$\widehat{W}\left(s^t\right) = (1-\alpha)\widetilde{P}_H\left(s^t\right)\widehat{K}\left(s^{t-1}\right)^{\alpha} L\left(s^t\right)^{-\alpha}\left(\frac{A\left(s^t\right)}{A\left(s^{t-1}\right)}\right)^{1-\alpha}, \tag{29}$$

$$R\left(s^t\right) = \alpha\widetilde{P}_H\left(s^t\right)\widehat{K}\left(s^{t-1}\right)^{\alpha-1} L\left(s^t\right)^{1-\alpha}\left(\frac{A\left(s^t\right)}{A\left(s^{t-1}\right)}\right)^{1-\alpha}, \tag{30}$$

$$\widehat{Y}_H\left(s^t\right) = \omega \widetilde{P}_H\left(s^t\right)^{-\theta} \widehat{Y}\left(s^t\right), \tag{31}$$

$$\widehat{Y}_H^*\left(s^t\right) = (1-\omega)\left(\frac{\widetilde{P}_H\left(s^t\right)}{RER\left(s^t\right)}\right)^{-\theta} \widehat{Y}^*\left(s^t\right)\frac{A^*\left(s^{t-1}\right)}{A\left(s^{t-1}\right)}, \tag{32}$$

$$\widehat{C}\left(s^t\right) + \widehat{X}\left(s^t\right) = \widehat{Y}\left(s^t\right), \tag{33}$$

$$\widehat{C}^*\left(s^t\right) + \widehat{X}^*\left(s^t\right) = \widehat{Y}^*\left(s^t\right), \tag{34}$$

$$\widehat{Y}\left(s^t\right) = \left[\omega^{\frac{1}{\theta}} \widehat{Y}_H\left(s^t\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} \widehat{Y}_F\left(s^t\right)^{\frac{\theta-1}{\theta}} \left(\frac{A^*\left(s^{t-1}\right)}{A\left(s^{t-1}\right)}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{35}$$

$$\widehat{Y}^*\left(s^t\right) = \left[\omega_F^{\frac{1}{\theta}} \widehat{Y}_F^*\left(s^t\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} \widehat{Y}_H^*\left(s^t\right)^{\frac{\theta-1}{\theta}} \left(\frac{A\left(s^{t-1}\right)}{A^*\left(s^{t-1}\right)}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{36}$$

$$\widehat{Y}_H\left(s^t\right) + \widehat{Y}_H^*\left(s^t\right) = \widehat{K}\left(s^{t-1}\right)^{\alpha}\left(\widehat{L}\left(s^t\right)\frac{A\left(s^t\right)}{A\left(s^{t-1}\right)}\right)^{1-\alpha}, \tag{37}$$

$$\widehat{Y}_F\left(s^t\right) + \widehat{Y}_F^*\left(s^t\right) = \widehat{K}^*\left(s^{t-1}\right)^{\alpha}\left(\widehat{L}^*\left(s^t\right)\frac{A^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}\right)^{1-\alpha}, \tag{38}$$

$$\widehat{D}\left(s^t\right) + \widehat{D}^*\left(s^t\right)\frac{A^*\left(s^{t-1}\right)}{A\left(s^{t-1}\right)} = 0, \tag{39}$$

and

$$\begin{aligned}\widetilde{P}_H\left(s^t\right)\overline{Q}\left(s^t\right)\widehat{D}\left(s^t\right) &= \widetilde{P}_H\left(s^t\right)\widehat{Y}_H^*\left(s^t\right) - \widetilde{P}_F^*\left(s^t\right)RER\left(s^t\right)\widehat{Y}_F\left(s^t\right)\frac{A^*\left(s^{t-1}\right)}{A\left(s^{t-1}\right)} \\ &+\widetilde{P}_H\left(s^t\right)\widehat{D}\left(s^{t-1}\right)\frac{A\left(s^{t-2}\right)}{A\left(s^{t-1}\right)} - \widetilde{P}_H\left(s^t\right)\frac{\Phi\left(D\left(s^t\right)\right)}{A\left(s^{t-1}\right)}.\end{aligned} \tag{40}$$

Finally, the productivity shocks do not need to be normalized. Also, note that our functional form $\Phi\left[D\left(s^t\right)\right] = \frac{\phi}{2}A(s^{t-1})\left[\frac{D\left(s^t\right)}{A\left(s^{t-1}\right)}\right]^2$ implies that $\Phi\left[\widehat{D}\left(s^t\right)\right]/A\left(s^{t-1}\right)$ and $\Phi'\left[\widehat{D}\left(s^t\right)\right]$ are stationary.

B. Appendix: Constructing the Capital Series (Not for Publication)

In order to calculate capital series we use the perpetual inventory method. In order to calculate the initial capital stock using quarterly data, we assume that the average capital stock to GDP ratio in the period 1970.1 to 1970.4 equals the average capital stock to GDP ratio in the period 1971.1 to 1979.4. Hence:

$$\frac{1}{4}\sum_{1970.1}^{1970.4}\frac{K_i}{Y_i} = \frac{1}{36}\sum_{1971.1}^{1979.4}\frac{K_i}{Y_i}$$

We use the capital law of motion to replace capital on the right-hand side of the last expression. In this way we can rewrite the equation in terms of capital in the first quarter of 1970 and the time series of output and investment. With one equation and one unknown, capital in the first quarter is uniquely pinned down.

C. Appendix: Data for Constructing TFP shocks (Not for Publication)

For the U.S., data are from the IFS: Gross Fixed Capital Formation (11193E.CZF...), Gross Domestic Product (11199B.CZF...) and GDP Deflator (11199BIRZF...). Employment is from the OECD.

For the Euro Area, Gross Domestic Product, Gross Fixed Capital Formation, and GDP deflator are from Area Wide Model Dataset (AWM) in domestic currency at 1995 fixed prices. Employment is the number of employees from Eurostat. Nominal Exchange rate -National Currency per US Dollars- is from the AWM Database.

For Canada, Real Gross Domestic Product, Employment, and Real Gross Fixed Capital Formation are from Statistics Canada (CANSIM) in domestic currency, while GDP deflator is from IFS (15699BIRZF...). Employment is number of employees from Statistics Canada (CANSIM). PPP exchange rates -National Currency per US Dollars- are from the WEO (W156PPPEX).

For Japan, Gross Domestic Product (15899B.CZF...), Gross Fixed Capital Formation (15893E.CZF...), and GDP deflator (15899BIRZF...) are from IFS. Employment is the number of employees from the G10. PPP exchange rates -National Currency per US Dollars- are from the WEO (W158PPPEX).

For the U.K., we use Gross Domestic Product and Investment from Eurostat, and GDP deflator from the IFS (11299BIRZF...). Employment is number of employees from Eurostat. PPP exchange rates -National Currency per US Dollars- are from the WEO (W112PPPEX).

For Australia, we use Real GDP, employment and real Gross Fixed Capital Formation from from the Australian Bureau of Statistics in domestic currency. GDP deflator is from the IFS (19399BIRZF...). Employment is the number of employees from the G10. PPP exchange rates -National Currency per US Dollars- are from the WEO (W193PPPEX).

D. Appendix: Normalized Equilibrium Conditions for the IST Model (Not for Publication)

Equations (3) to (18) characterize equilibrium in this model. Since both $\log A\left(s^t\right)$ and $\log A^*\left(s^t\right)$ are integrated, we now normalize the above-described system in order to get a stationary system more amenable to study. Additional restrictions on the VECM defining the law of motion of the technological processes are required if the model is to exhibit balanced growth. Those restrictions are described in the next subsection.

Let us first define the following normalized variables $\widehat{Y}_H\left(s^t\right) = \frac{Y_H\left(s^t\right)}{A\left(s^{t-1}\right)}$, $\widehat{Y}_H^*\left(s^t\right) = \frac{Y_H^*\left(s^t\right)}{A\left(s^{t-1}\right)}$, $\widehat{Y}_F\left(s^t\right) = \frac{Y_F\left(s^t\right)}{A^*\left(s^{t-1}\right)}$, $\widehat{Y}_F^*\left(s^t\right) = \frac{Y_F^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}$, and $\widehat{K}\left(s^{t-1}\right) = \frac{K\left(s^{t-1}\right)}{A\left(s^{t-1}\right)}$, $\widehat{K}^*\left(s^{t-1}\right) = \frac{K^*\left(s^{t-1}\right)}{A^*\left(s^{t-1}\right)}$, $\widehat{Y}\left(s^t\right) = \frac{Y\left(s^t\right)}{A\left(s^{t-1}\right)}$, $\widehat{Y}^*\left(s^t\right) = \frac{Y^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}$, $\widehat{C}\left(s^t\right) = \frac{C\left(s^t\right)}{A\left(s^{t-1}\right)}$, $\widehat{C}^*\left(s^t\right) = \frac{C^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}$, $\widehat{X}\left(s^t\right) = \frac{X\left(s^t\right)}{A\left(s^{t-1}\right)}$, $\widehat{X}^*\left(s^t\right) = \frac{X^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}$, $\widehat{W}\left(s^t\right) = \frac{W\left(s^t\right)}{A\left(s^{t-1}\right)}$, $\widehat{W}^*\left(s^t\right) = \frac{W^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}$, $\widehat{D}\left(s^t\right) = \frac{D\left(s^t\right)}{A\left(s^{t-1}\right)}$, $\widehat{D}^*\left(s^t\right) = \frac{D^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}$, $\widehat{\lambda}\left(s^t\right) = \lambda\left(s^t\right) A\left(s^{t-1}\right)^{1-\gamma(1-\sigma)}$, and $\widehat{\lambda}^*\left(s^t\right) = \lambda^*\left(s^t\right) A^*\left(s^{t-1}\right)^{1-\gamma(1-\sigma)}$. Then, the stationary first order conditions are

$$U_C\left[\widehat{C}\left(s^t\right), L\left(s^t\right)\right] = \widehat{\lambda}\left(s^t\right), \tag{41}$$

$$\frac{U_L\left[\widehat{C}\left(s^t\right), L\left(s^t\right)\right]}{U_C\left[\widehat{C}\left(s^t\right), L\left(s^t\right)\right]} = \widehat{W}\left(s^t\right), \tag{42}$$

$$\widehat{\lambda}\left(s^t\right) = \widehat{\mu}\left(s^t\right)\frac{V\left(s^t\right)}{V\left(s^{t-1}\right)} \tag{43}$$

$$\frac{V\left(s^t\right)}{V\left(s^{t-1}\right)}\left(\frac{Z\left(s^t\right)}{Z\left(s^{t-1}\right)}\right)^{1-\gamma(1-\sigma)}\widehat{\mu}\left(s^t\right) = \beta\sum_{s^{t+1}}\pi\left(s^{t+1}|s^t\right)\left(\widehat{\lambda}\left(s^{t+1}\right)R\left(s^{t+1}\right) + \widehat{\mu}\left(s^{t+1}\right)(1-\delta)\right), \tag{44}$$

$$\widehat{K}\left(s^t\right)\frac{V\left(s^t\right)}{V\left(s^{t-1}\right)}\frac{Z\left(s^t\right)}{Z\left(s^{t-1}\right)} = (1-\delta)\widehat{K}\left(s^{t-1}\right) + \widehat{X}\left(s^t\right)\frac{V\left(s^{t-1}\right)}{V\left(s^t\right)}, \tag{45}$$

$$\overline{Q}\left(s^t\right) = \beta\sum_{s^{t+1}}\pi\left(s^{t+1}/s^t\right)\frac{\widehat{\lambda}\left(s^{t+1}\right)}{\widehat{\lambda}\left(s^t\right)}\left(\frac{Z\left(s^{t-1}\right)}{Z\left(s^t\right)}\right)^{1-\gamma(1-\sigma)}\frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)} - \Phi'\left(D\left(s^t\right)\right), \tag{46}$$

$$\sum_{s^{t+1}}\pi\left(s^{t+1}/s^t\right)\left[\begin{array}{c}\frac{\widehat{\lambda}^*\left(s^{t+1}\right)}{\widehat{\lambda}^*\left(s^t\right)}\frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)}\frac{RER\left(s^t\right)}{RER\left(s^{t+1}\right)}\left(\frac{Z\left(s^t\right)}{Z\left(s^{t-1}\right)}\frac{Z^*\left(s^{t-1}\right)}{Z^*\left(s^t\right)}\right)^{1-\gamma(1-\sigma)} \\ -\frac{\widehat{\lambda}\left(s^{t+1}\right)}{\widehat{\lambda}\left(s^t\right)}\frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)}\end{array}\right] = -\frac{\Phi'\left(D\left(s^t\right)\right)}{\beta}, \tag{47}$$

$$\widehat{W}\left(s^t\right) = (1-\alpha)\widetilde{P}_H\left(s^t\right)\widehat{K}\left(s^{t-1}\right)^{\alpha}L\left(s^t\right)^{-\alpha}\left(\frac{A\left(s^t\right)}{A\left(s^{t-1}\right)}\right)^{1-\alpha}, \tag{48}$$

$$R\left(s^t\right) = \alpha \widetilde{P}_H\left(s^t\right) \widehat{K}\left(s^{t-1}\right)^{\alpha-1} L\left(s^t\right)^{1-\alpha} \left(\frac{A\left(s^t\right)}{A\left(s^{t-1}\right)}\right)^{1-\alpha}, \tag{49}$$

$$\widehat{Y}_H\left(s^t\right) = \omega \widetilde{P}_H\left(s^t\right)^{-\theta} \widehat{Y}\left(s^t\right), \tag{50}$$

$$\widehat{Y}_H^*\left(s^t\right) = (1-\omega)\left(\frac{\widetilde{P}_H\left(s^t\right)}{RER\left(s^t\right)}\right)^{-\theta} \widehat{Y}^*\left(s^t\right) \frac{Z^*\left(s^{t-1}\right)}{Z\left(s^{t-1}\right)}, \tag{51}$$

$$\widehat{C}\left(s^t\right) + \widehat{X}\left(s^t\right) = \widehat{Y}\left(s^t\right), \tag{52}$$

$$\widehat{C}^*\left(s^t\right) + \widehat{X}^*\left(s^t\right) = \widehat{Y}^*\left(s^t\right), \tag{53}$$

$$\widehat{Y}\left(s^t\right) = \left[\omega^{\frac{1}{\theta}} \widehat{Y}_H\left(s^t\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} \widehat{Y}_F\left(s^t\right)^{\frac{\theta-1}{\theta}} \left(\frac{Z^*\left(s^{t-1}\right)}{Z\left(s^{t-1}\right)}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{54}$$

$$\widehat{Y}^*\left(s^t\right) = \left[\omega_F^{\frac{1}{\theta}} \widehat{Y}_F^*\left(s^t\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} \widehat{Y}_H^*\left(s^t\right)^{\frac{\theta-1}{\theta}} \left(\frac{Z\left(s^{t-1}\right)}{Z^*\left(s^{t-1}\right)}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{55}$$

$$\widehat{Y}_H\left(s^t\right) + \widehat{Y}_H^*\left(s^t\right) = \widehat{K}\left(s^{t-1}\right)^{\alpha} \left(\widehat{L}\left(s^t\right) \frac{A\left(s^t\right)}{A\left(s^{t-1}\right)}\right)^{1-\alpha}, \tag{56}$$

$$\widehat{Y}_F\left(s^t\right) + \widehat{Y}_F^*\left(s^t\right) = \widehat{K}^*\left(s^{t-1}\right)^{\alpha} \left(\widehat{L}^*\left(s^t\right) \frac{A^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}\right)^{1-\alpha}, \tag{57}$$

$$\widehat{D}\left(s^t\right) + \widehat{D}^*\left(s^t\right) \frac{Z^*\left(s^{t-1}\right)}{Z\left(s^{t-1}\right)} = 0, \tag{58}$$

and

$$\begin{aligned}\widetilde{P}_H\left(s^t\right) \overline{Q}\left(s^t\right) \widehat{D}\left(s^t\right) &= \widetilde{P}_H\left(s^t\right) \widehat{Y}_H^*\left(s^t\right) - \widetilde{P}_F^*\left(s^t\right) RER\left(s^t\right) \widehat{Y}_F\left(s^t\right) \frac{Z^*\left(s^{t-1}\right)}{Z\left(s^{t-1}\right)} \\ &+ \widetilde{P}_H\left(s^t\right) \widehat{D}\left(s^{t-1}\right) \frac{Z\left(s^{t-2}\right)}{Z\left(s^{t-1}\right)} - \widetilde{P}_H\left(s^t\right) \frac{\Phi\left(D\left(s^t\right)\right)}{Z\left(s^{t-1}\right)}.\end{aligned} \tag{59}$$

Finally, the productivity shocks do not need to be normalized. Also, note that our functional form $\Phi\left[D\left(s^t\right)\right] = \frac{\phi}{2} Z(s^{t-1}) \left[\frac{D\left(s^t\right)}{Z\left(s^{t-1}\right)}\right]^2$ implies that $\Phi\left[\widehat{D}\left(s^t\right)\right] / Z\left(s^{t-1}\right)$ and $\Phi'\left[\widehat{D}\left(s^t\right)\right]$ are stationary.

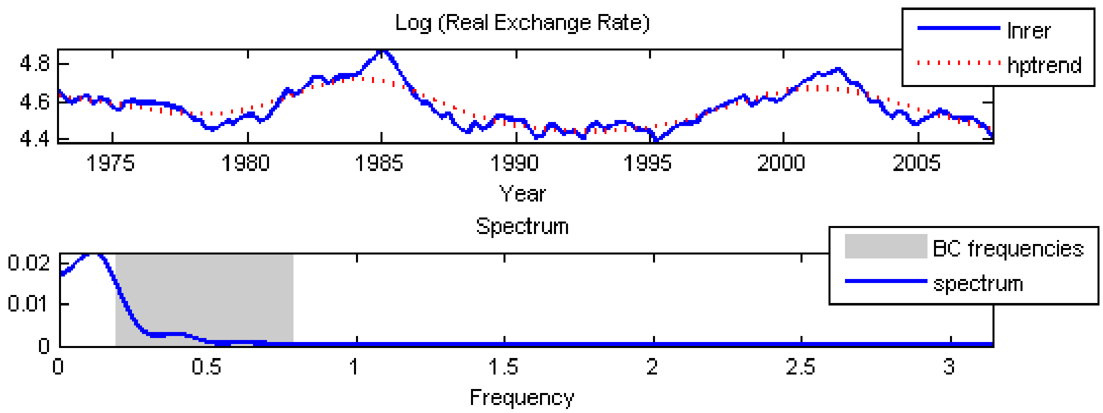

Figure 1: Log RER, autocorrelation function, and spectral density of the U.S. dollar

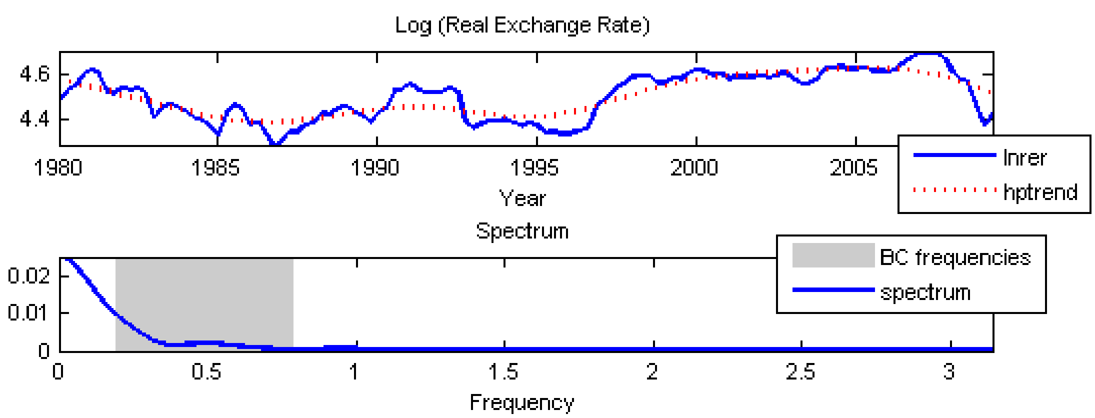

Figure 2: Log RER, autocorrelation function, and spectral density of the U.K. pound

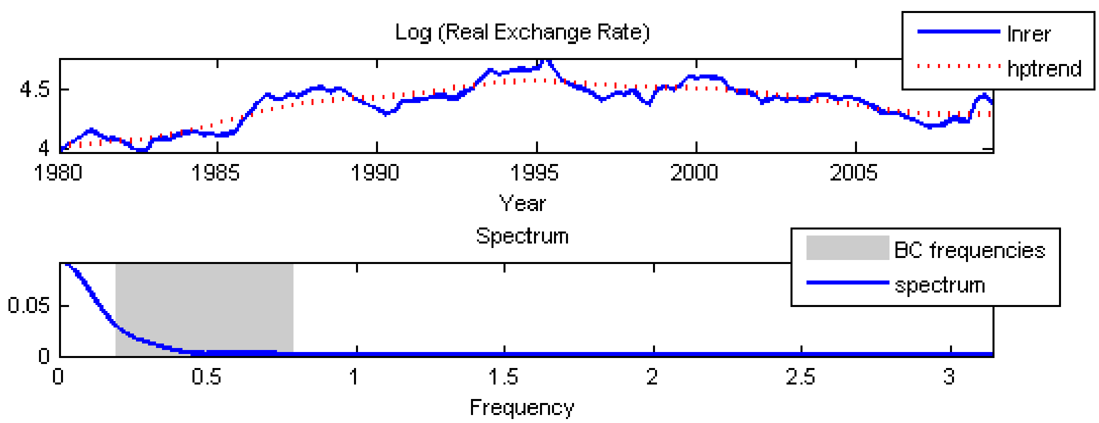

Figure 3: Log RER, autocorrelation function, and spectral density of the Japanese yen

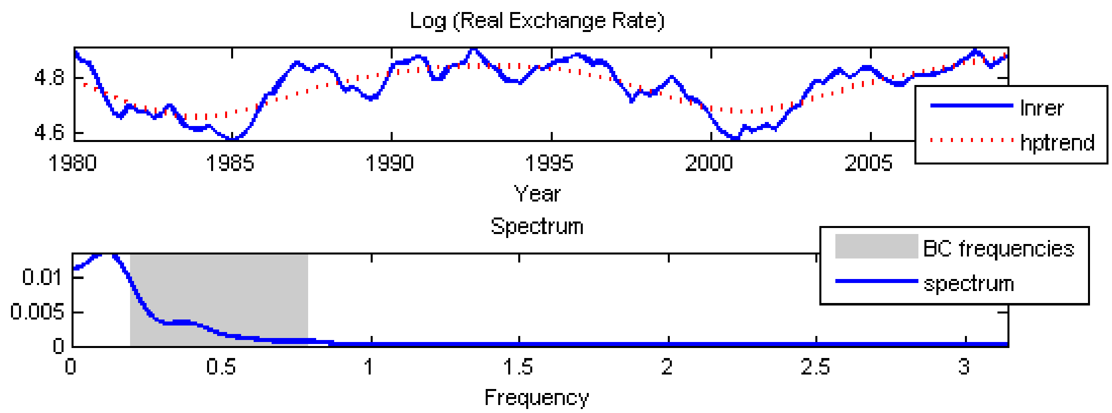

Figure 4: Log RER, autocorrelation function, and spectral density of the euro

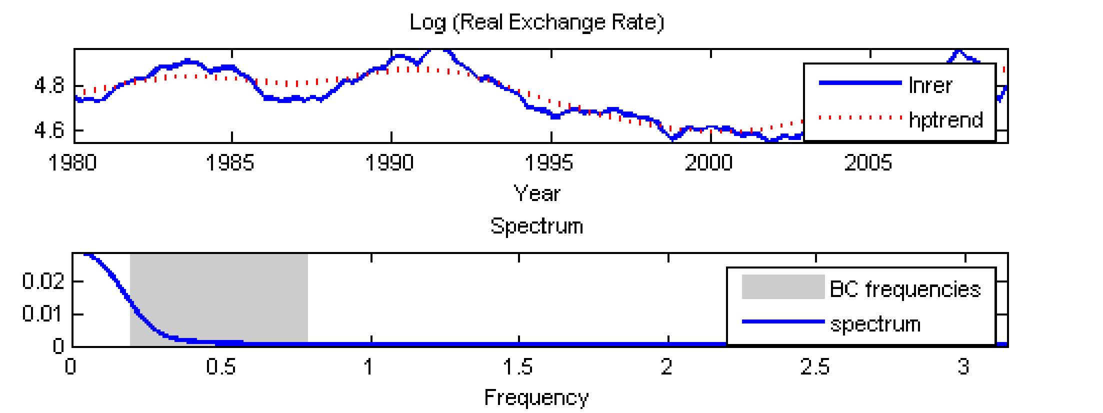

Figure 5: Log RER, autocorrelation function, and spectral density of the Canadian dollar

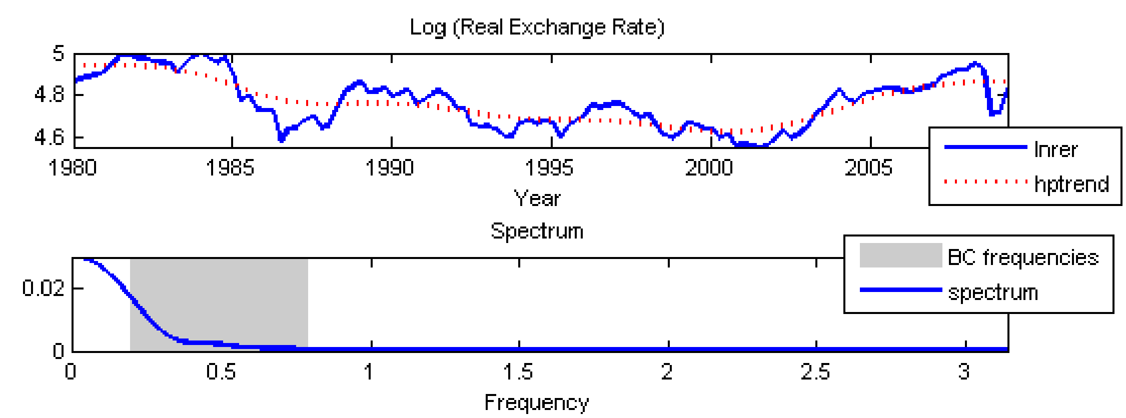

Figure 6: Log RER, autocorrelation function, and spectral density of the Australian dollar

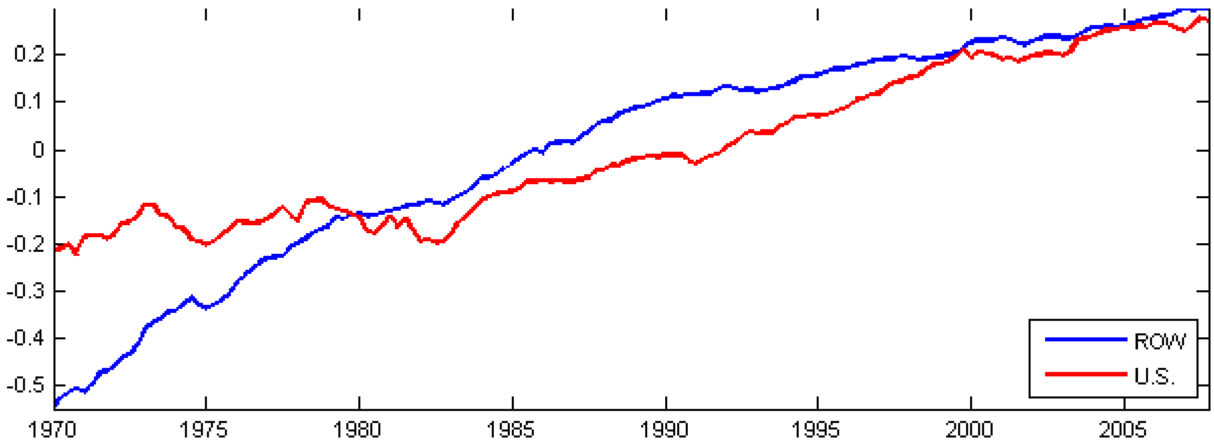

Figure 7: Log TFP for the U.S. and the ROW

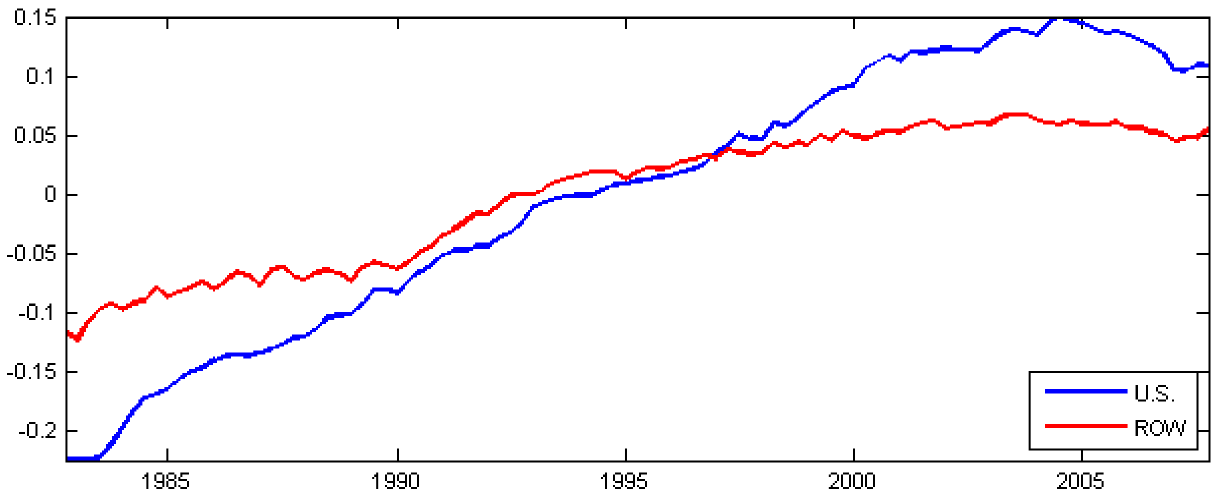

Figure 8: Log IST for the U.S. and the ROW

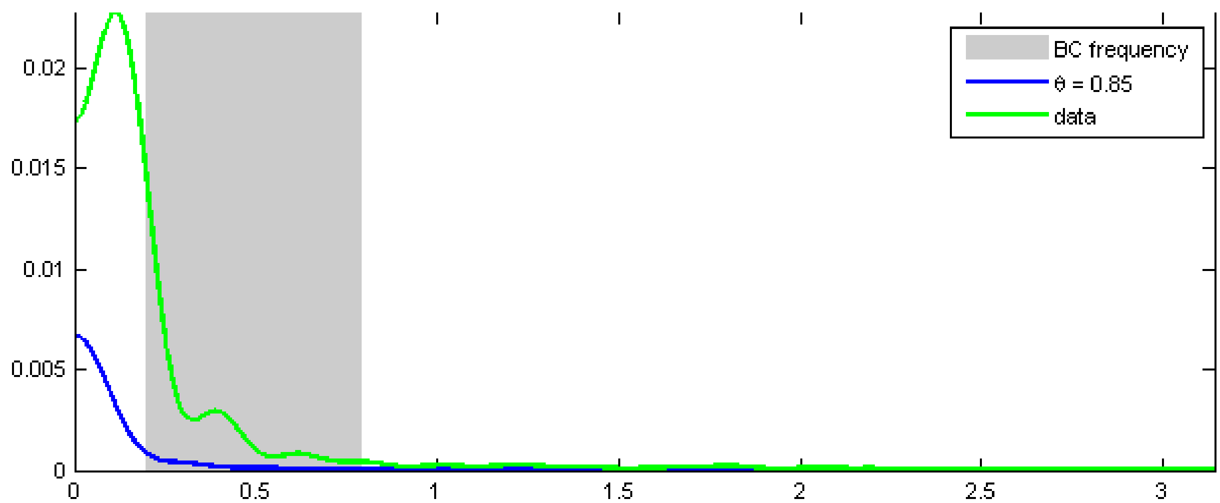

Figure 9: Comparing the spectrum of the model with only TFP shocks for $\theta = 0.85$ with the spectrum of the US dollar

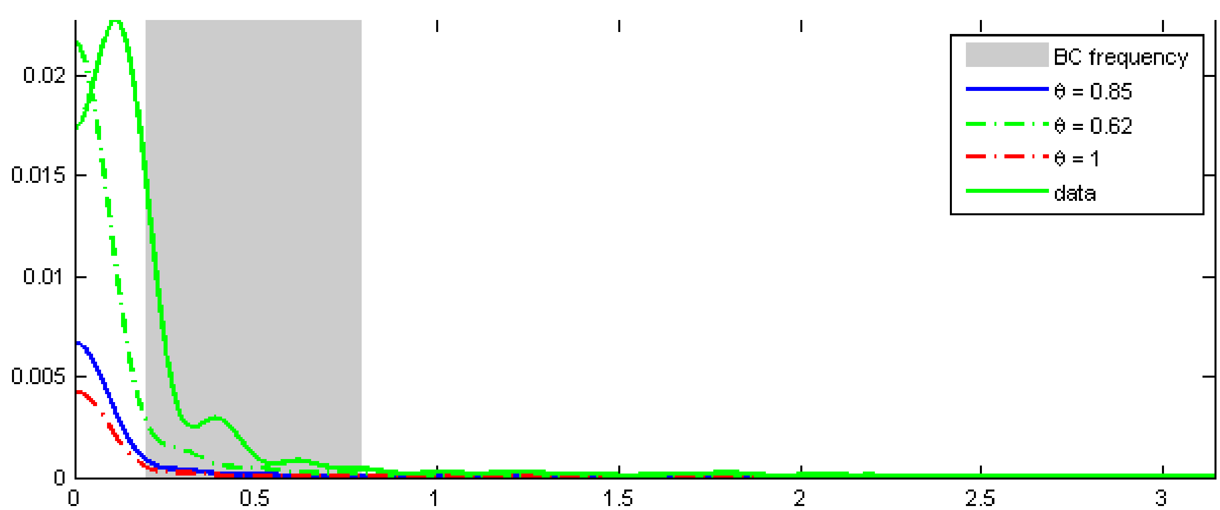

Figure 10: Comparing the spectrum of the model with only TFP shocks for different values of θ with the spectrum of the US dollar

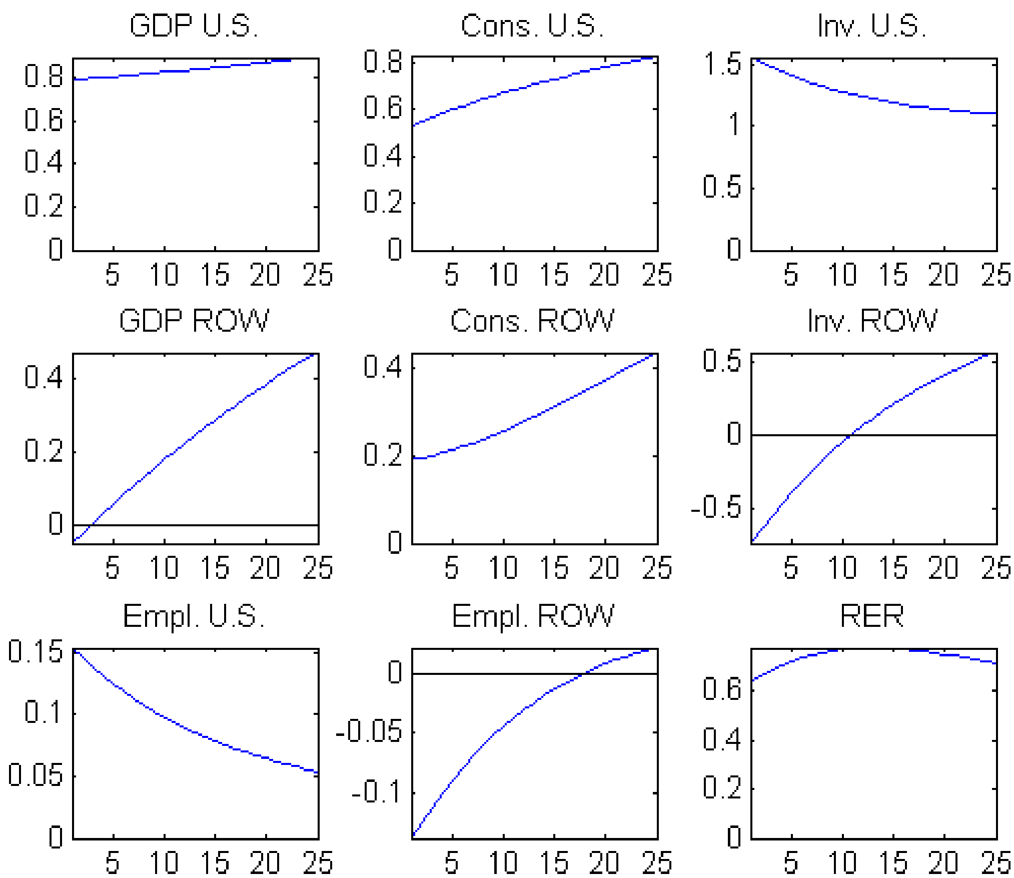

Figure 11: IRF to a home country TFP shock

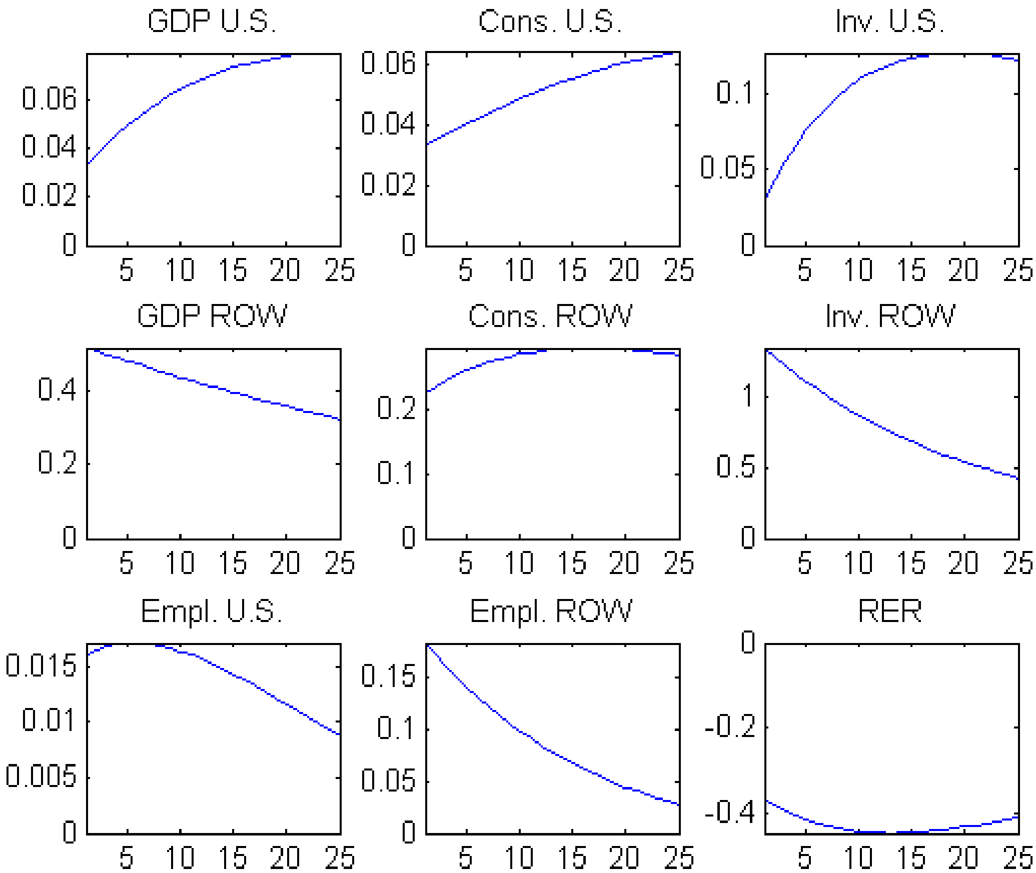

Figure 12: IRF to a foreign country TFP shock

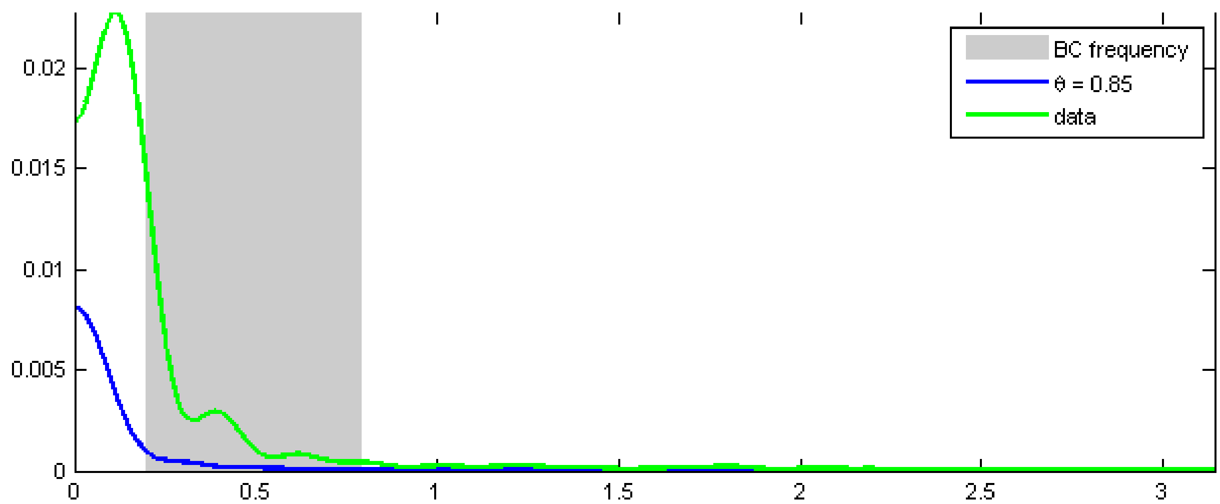

Figure 13: Comparing the spectrum of the model with both TFP and IST shocks for $\theta = 0.85$ with the spectrum of the US dollar

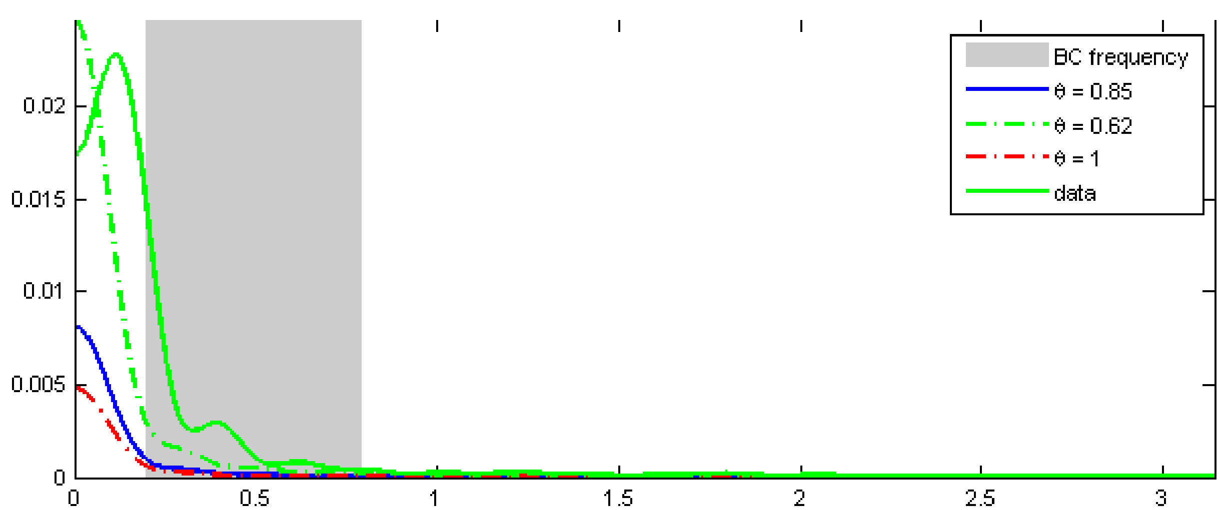

Figure 14: Comparing the spectrum of the model with both TFP and IST for different values of θ with the spectrum of the US dollar