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Banking Competition, Collateral Constraints and Optimal Monetary Policy

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Abstract

We analyze optimal monetary policy in a model with two distinct financial frictions. First, borrowing is subject to collateral constraints. Second, credit flows are intermediated by monopolistically competitive banks, thus giving rise to endogenous lending spreads. We show that, up to a second order approximation, welfare maximization is equivalent to stabilization of four goals: inflation, output gap, the consumption gap between constrained and unconstrained agents, and the distribution of the collateralizable asset between both groups. Following both productivity and financial shocks, the optimal monetary policy commitment implies a short-run trade-off between stabilization goals. Such policy trade-offs become amplified as banking competition increases, due to the fall in lending spreads and the resulting increase in financial leveraging.

Keywords: banking competition, lending spreads, collateral constraints, monetary policy, linear-quadratic method

JEL codes: E32, E52, G10, G21

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1 Introduction

In this paper we provide a theoretical framework for the analysis of the optimal conduct of monetary policy in the presence of financial frictions. Both optimal monetary policy and the macroeconomic effects of financial frictions have attracted much attention in recent times. However, much less effort has been devoted to exploring the connections between both fields in the context of modern dynamic stochastic general equilibrium (DSGE) models.

Here we perform such an exploration in the framework of a model economy featuring two distinct financial frictions: collateral constraints and endogenous bank-lending spreads. In this way, our setup addresses two of the most prominent hypotheses in the macroeconomic literature on financial frictions: one that emphasizes the role of endogenous collateral constraints and fluctuations in asset prices (Kiyotaki and Moore, 1997; Iacoviello, 2005), and another one that stresses the role of endogenous lending spreads, as exemplified by the "financial accelerator" literature (Bernanke and Gertler, 1989; Bernanke, Gertler and Gilchrist, 1999) and by the recent literature on banking and macroeconomics (Goodfriend and McCallum, 2007).

Specifically, we consider an economy in which consumers are divided into households and entrepreneurs, where the former are assumed to be relatively more patient and therefore act as savers. Entrepreneurs face credit constraints that limit their borrowing capacity to a certain fraction (the *pledgeability ratio*) of the expected value of their real estate holdings, which are assumed to be the only collateralizable asset. Real estate can also be used by entrepreneurs as commercial property for production purposes, and by households in the form of residential property. Unlike in most of the literature, savers do not lend directly to borrowers. Instead, they provide banks with deposits that are then used to make loans to entrepreneurs. Banks are assumed to have some monopolistic power in the loans market. In particular, following Andrés and Arce (2008) we assume that a fixed number of identical banks compete to attract investors as in the spatial competition model of Salop (1979). In this framework, each bank is able to charge a positive lending spread on the deposit rate. In equilibrium, lending spreads depend negatively on the expected evolution of real estate prices, the pledgeability ratio and the degree of banking competition, and positively on the policy rate, due to their respective effects on the elasticity of demand for collateralized loans. Finally, our economy features two familiar nominal frictions: nominal (non-state-contingent) debt and staggered nominal price adjustment à la Calvo (1983), both of which open two additional channels of influence for monetary policy.

Our main objective is to understand the nature of optimal monetary policy in this framework. With this aim we follow the welfare-based linear-quadratic approach pioneered by Rotemberg and Woodford (1997).[1] The central bank's optimal monetary policy commitment is the one that maximizes the quadratic approximation of the welfare criterion subject to the linear approximation of the equilibrium constraints. We first show that the central bank's quadratic welfare criterion features four stabilization goals: inflation, the output gap, the difference in consumption between households and entrepreneurs (or *consumption gap*) and the inefficiency in the distribution of real estate between both groups (or *housing gap*). The first two, inflation and the output gap, are related to the existence of staggered price adjustment and are therefore standard in the New Keynesian literature. The last two are novel and are directly related to the existence of financial frictions. Regarding the consumption gap, collateral constraints prevent constrained consumers from smoothing their consumption the way unconstrained consumers do.[2] This gives rise to inefficient risk sharing between both consumer types. Regarding the housing gap, the distribution of real estate between both groups will generally be inefficient, because entrepreneurs' demand for real estate is distorted by its role as collateral. We then use the linear equilibrium constraints to illustrate some of the trade-offs among stabilization goals. Our analysis reveals that the presence of collateral constraints makes it impossible for the central bank to stabilize all four goals simultaneously.[3] Therefore, the optimal policy commitment must yield a compromise between stabilization goals.

In order to illustrate the nature of optimal monetary policy, we calibrate our model economy and simulate the effects of a fall in TFP, as an example of non-financial shock, and a fall in the pledgeability ratio, as an example of financial shock. The latter represents a tightening of borrowing constraints, and can be thus interpreted as a *credit crunch*. In both cases, and relative to a simple policy of strict inflation targeting, we find that the optimal policy engineers a sharper cut in the policy rate, which banks pass on to nominal loan rates. This contains the fall in investment in commercial real estate (hence narrowing the housing gap) and in real estate prices. This results in an improvement in entrepreneur net worth which, in turn, narrows the consumption gap. The more aggressive policy response comes at the cost of generating temporary increases in inflation and the output gap, but these are very short-lived thanks to the improved output-inflation trade-off allowed for by the smaller consumption gap. Intuitively, by reacting rapidly and aggressively under the optimal rule, the central bank tries to avoid a

[1] See Woodford (2003), and Benigno and Woodford (2003, 2008) for extensive applications of the linear-quadratic approach to the study of optimal monetary policy.

[2] In particular, we show that in equilibrium entrepreneurs consume a constant fraction of their net worth, which includes their real estate wealth.

[3] In particular, we show that the consumption gap arises as a cost-push term in the New Keynesian Phillips curve. That is, collateral constraints and the resulting inefficient risk-sharing create an *endogenous trade-off* between inflation and output-gap stabilization. Also, we show that narrowing the consumption gap requires the central bank to accept inefficient fluctuations in inflation and the output gap.

large initial fall in entrepreneurs' net wealth, the effects of which are very persistent due to the presence of borrowing constraints.

We also look for a simple and robust targeting rule that approximates well the optimal policy. In searching for such a simple rule, we first note that in our model the price of the collateralizable asset (real estate) has an important effect on the transmission of shocks. On the one hand, entrepreneurs' expenditure decisions are very sensitive to *current* fluctuations in real estate prices, due to their effect on their current net worth. On the other hand, *expected* changes in real estate prices have a potentially important effect on equilibrium lending margins, through their effect on the elasticity of demand for loans. The previous argument suggests considering simple targeting rules that capture the central bank's concern for stabilizing the actual and expected evolution of asset prices, together with the usual concern for inflation stabilization. In particular, we find that a simple targeting rule that relates current inflation negatively with current and one-period ahead expected changes in real estate prices performs fairly well in the face of both financial and non-financial shocks, in the sense that the implied welfare losses are close to those obtained under the optimal policy. This analysis suggests that, to the extent that fluctuations in the price of collateralizable assets cause large distortions in the consumption and investment decisions of constrained agents, then the monetary authority has a rationale for taking into account such asset price fluctuations in its policy decisions.

Finally, we are interested in understanding how the degree of banking competition affects the severity of the trade-offs just discussed. With this purpose, we consider a counter-factual scenario in which the banking sector becomes perfectly competitive, and then compare the associated welfare losses in this and our baseline scenario. We find that welfare losses are *higher* with perfect banking competition, both under the optimal commitment and (especially) under suboptimal policy rules, such as inflation or output-gap targeting. The reason is that, as lending spreads fall, entrepreneurs become more leveraged. This makes their net worth more sensitive to fluctuations in real estate prices, which in turn amplifies fluctuations in the consumption and housing gaps. This mechanism operates regardless of the nature of shocks. However, in the case of financial shocks it is counteracted by an opposing force. When banks have maket power, exogenous variations in the pledgeability ratio have a direct countercyclical effect on lending margins. Under perfect competition, lending margins become zero, and so the amplifying effect of their counter-cyclical response disappears.

Our analysis is related to recent work by Cúrdia and Woodford (2008; CW, for short), who focus on the design of optimal monetary policy rules in a model in which a positive spread exists between lending and deposit rates. CW assume that the lending spread is determined by an ad-hoc function of banks' loan volume, aimed at capturing the costs of originating and monitoring loans. We differ from CW in two important respects regarding the nature of credit frictions. First, we model credit spreads as arising endogenously in an environment in which banks enjoy some monopolistic power in the loans market. Second, we subject borrowers to endogenous collateral constraints. As in CW, we cast our optimal policy problem in a linear-quadratic representation, motivated by its potential for delivering analytical results. Importantly, both CW and we find that cyclical fluctuations in lending spreads have small quantitative effects on the nature of optimal monetary policy design. However, in our model the average level of lending spreads has non-negligible effects that arise from the interaction between spreads and collateral constraints, a channel which is missing in CW.

Also related is the work of Monacelli (2007), who analyzes the Ramsey optimal monetary policy in a model with collateral constraints and quadratic price adjustment costs. He finds a trade-off between stabilizing inflation and relaxing collateral constraints. In addition to collateral constraints, our model also incorporates a monopolistically competitive banking sector, which gives rise to endogenous lending margins. Also, we follow the linear-quadratic approach to monetary policy analysis, which allows us to obtain intuitive expressions for the central bank's stabilization goals and trade-offs. De Fiore, Teles and Tristani (2009) explore optimal monetary rules in a model where firms' assets and liabilities are denominated in nominal terms and predetermined. Our model also incorporates predetermined nominal debt that, as in their model, works to amplify or dampen shocks. However, our focus is rather on the consequences of other forms of financial frictions, especially collateral constraints and endogenous lending spreads, for the optimal conduct of monetary policy. Finally, De Fiore and Tristani (2009) have recently explored optimal monetary policy in an environment where lending spreads are the result of costly state verification problems, as in the financial accelerator theory in Bernanke and Gertler (1989).

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 analyzes the efficient equilibrium, which provides a helpful normative benchmark. In section 4, we derive the central bank's quadratic welfare criterion and discuss some of the trade-offs among stabilization goals. In section 5 we calibrate our model and perform a number of quantitative exercises, in order to illustrate the working of optimal and suboptimal monetary rules. Section 6 concludes.

2 Model

In this section we describe a model economy that relies on Iacoviello (2005) and Andrés and Arce (2008). The population of consumers, whose size is normalized to 1, is composed of two types of agents: there is a fraction ω of households and a fraction $1-\omega$ of entrepreneurs. The latter are assumed to be more impatient than the former. This produces credit flows between both groups which are intermediated by monopolistically competitive banks. A sector of monopolistic final goods producers transforms the homogenous intermediate good produced by the entrepreneurs into differentiated final goods, which are then sold to consumers. We now analyze the problem of each type of agent.

2.1 Households

The representative household maximizes the following welfare criterion,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \frac{\left(l_t^s\right)^{1+\varphi}}{1+\varphi} + \vartheta_t \log h_t \right),$$

where c_t are units of a Dixit-Stiglitz basket of final consumption goods, l_t^s is labor supply, h_t are units of housing, $\vartheta_t = \vartheta \exp(z_t^h)$ is an exogenously time-varying weight on utility from housing services (where z_t^h follows a zero-mean AR(1) process) and $\beta \in (0,1)$ is the household's subjective discount factor. Maximization is subject to the following budget constraint expressed in real terms,

$$w_t l_t^s + \frac{\Omega_t^b + \Omega_t^f}{P_t} + s_t + \frac{R_{t-1}^D}{\pi_t} d_{t-1} = c_t + p_t^h \left[\left(1+\tau^h\right) h_t - h_{t-1} \right] + d_t,$$

where w_t is the hourly wage, Ω_t^b and Ω_t^f are lump-sum nominal profits from the banking and final goods sectors, respectively, and s_t are lump-sum subsidies from the government. We assume that nominal, risk-free, one-period bank deposits are the only financial asset available to households, where d_t is the real value of deposits at the end of period t, R_t^D is the gross nominal deposit rate and $\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate, where P_t is the Dixit-Stiglitz aggregate price index. Households can also buy and sell real estate at a unit price p_t^h (measured in terms of consumption goods). End-of-period housing wealth is taxed at the rate τ^h (the role of which is discussed later on). We assume that real estate does not depreciate. The first order conditions of this problem can be expressed as

$$w_t = c_t \left(l_t^s\right)^{\varphi}, \tag{1}$$

$$\frac{1}{c_t} = \beta R_t^D E_t \left\{ \frac{1}{c_{t+1}} \frac{P_t}{P_{t+1}} \right\}, \tag{2}$$

$$\frac{\left(1+\tau^h\right) p_t^h}{c_t} = \frac{\vartheta_t}{h_t} + \beta E_t \frac{p_{t+1}^h}{c_{t+1}}. \tag{3}$$

2.2 Entrepreneurs (intermediate good producers)

Entrepreneurs produce a homogenous intermediate good that is sold under perfect competition to a final goods sector. They operate a Cobb-Douglas production technology,

$$y_t = e^{a_t} \left(l_t^d\right)^{1-\nu} \left(h_{t-1}^e\right)^{\nu}, \tag{4}$$

where y_t is output of the intermediate good, l_t^d is labor demand, h_{t-1}^e is the stock of commercial real estate and a_t is a zero-mean AR(1) exogenous productivity process. Entrepreneurs also demand consumption goods and loans. The budget constraint of the representative entrepreneur is given by

$$b_t + (1-\tau^e)\left(p_t^I y_t - w_t l_t^d\right) = c_t^e + p_t^h (h_t^e - h_{t-1}^e) + \frac{R_{t-1}^L}{\pi_t} b_{t-1}, \tag{5}$$

where b_t is the real value of one-period nominal loans at the end of period t, R_t^L is the gross nominal loan rate, p_t^I is the real price of the intermediate good, τ^e is a tax rate on entrepreneur profits (the role of which is explained below) and c_t^e is entrepreneur consumption. Banks impose a collateral constraint on entrepreneurs: the nominal loan gross of interest payments cannot exceed a certain fraction (the *pledgeability ratio*) of the expected nominal resale value of the entrepreneur's real estate holdings. The collateral constraint can be expressed in real terms as,

$$b_t \leq m_t E_t \frac{\pi_{t+1}}{R_t^L} p_{t+1}^h h_t^e, \tag{6}$$

where $m_t = m \exp(z_t^m)$ is the exogenously time-varying pledgeability ratio and z_t^m is a zero-mean AR(1) process. In order to obtain a loan, the entrepreneur must first travel to a bank, incurring a utility cost which is proportional to the distance between hers and the bank's location. We assume that entrepreneurs and banks are uniformly distributed on a circle of length one. Subject to (4), (5) and (6), an entrepreneur located at point $k \in (0,1]$ maximizes

$$E_0 \sum_{t=0}^{\infty} (\beta^e)^t \left(\log c_t^e - \alpha d_t^{k,i}\right),$$

where $d_t^{k,i}$ is the distance between the entrepreneur and the lending bank (denoted by i), and α is the utility cost per distance unit.[4] Entrepreneurs are assumed to be more impatient than savers: $\beta^e < \beta$. The first order conditions of this problem are

$$w_t = p_t^I (1-\nu) \frac{y_t}{l_t^d}, \tag{7}$$

$$\frac{1}{c_t^e} = \beta^e R_t^L E_t \left\{ \frac{1}{c_{t+1}^e} \frac{P_t}{P_{t+1}} \right\} + \xi_t, \tag{8}$$

$$\frac{p_t^h}{c_t^e} = E_t \frac{\beta^e}{c_{t+1}^e} \left\{ (1-\tau^e) p_{t+1}^I \nu \frac{y_{t+1}}{h_t^e} + p_{t+1}^h \right\} + \xi_t m_t E_t \frac{\pi_{t+1}}{R_t^L} p_{t+1}^h, \tag{9}$$

where ξ_t is the Lagrange multiplier on the collateral constraint and $p_t^I \nu y_t / h_{t-1}^e$ is the marginal revenue product of commercial real estate. When binding ($\xi_t > 0$), the collateral constraint has two effects on the entrepreneur's decisions: first, it prevents them from smoothing their consumption the way households do (equation 8); second, it increases the marginal value of real estate due to its role as collateral (equation 9).

Equations (8) and (2) imply that in the steady state the borrowing constraint is binding ($\xi_{ss} > 0$, where the *ss* subscript denotes steady state values) if and only if $\beta R_{ss}^D > \beta^e R_{ss}^L$, which holds under our subsequent calibration. Provided that the fluctuations in the relevant variables around their steady state are sufficiently small, the borrowing constraint will also bind along the dynamics; that is, equation (6) will hold with equality. In that case, it is possible to show that entrepreneur consumption equals[5]

$$c_t^e = (1-\beta^e) \left[(1-\tau^e) \nu p_t^I y_t + p_t^h h_{t-1}^e - \frac{R_{t-1}^L}{\pi_t} b_{t-1} \right]. \tag{10}$$

That is, the entrepreneur always consumes a fraction $1-\beta^e$ of her real *net worth*, which is the sum of after-tax real profits, $(1-\tau^e) \nu p_t^I y_t$, and commercial real estate wealth, $p_t^h h_{t-1}^e$, minus real debt repayments, $R_{t-1}^L b_{t-1} / \pi_t$.

[4]This simple device is meant to motivate the existence of some monopoly power on the part of banks. See Andrés and Arce (2008) for a discussion on the foundations of this assumption.

[5]See the proof in the Appendix.

2.3 Banks

Banks are assumed to intermediate all credit flows between households (savers) and entrepreneurs (borrowers). We assume that banks are perfectly competitive on the deposits market, and so they take as given the nominal deposit rate, R_t^D, which is set by the central bank. However, competition in the loans market is imperfect, so that each bank enjoys some monopolistic power. In order to model imperfect competition in the loans market we use a version of Salop's (1979) circular-city model. A discrete number n of banks are located symmetrically on the unit circle and their position is time-invariant, whereas entrepreneurs' locations vary each period according to an *iid* stochastic process.[6] Bank $i \in \{1, 2, ..., n\}$ chooses the gross nominal interest rate on its loans, $R_t^L(i)$, to maximize

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{c_t}{c_{t+s}} \frac{\Omega_{t+s}(i)}{P_{t+s}}$$

where $\beta^s c_t / c_{t+s}$ is the time $t+s$ stochastic discount factor of the households (who are assumed to own the banks) and $\Omega_{t+s}(i)$ is the bank's nominal profit flow. Denoting by $B_t(i)$ and $D_t(i)$ the nominal stock of loans and deposits of bank i at the end of time t, respectively, we can write its flow of funds constraint as

$$\Omega_t(i) + B_t(i) + R_{t-1}^D D_{t-1}(i) = R_{t-1}^L(i) B_{t-1}(i) + D_t(i).$$

Further, bank i must also obey the balance-sheet identity, $D_t(i) = B_t(i)$. This implies that period t nominal profits are simply $\Omega_t(i) = \left(R_{t-1}^L(i) - R_{t-1}^D\right) B_{t-1}(i)$. To solve for the bank's optimal loan rate, it is convenient to express its loan volume in real terms as

$$\frac{B_t(i)}{P_t} = b_t(i)\tilde{b}_t(i),$$

where $b_t(i)$ is the *intensive* business margin (the size of each loan) and $\tilde{b}_t(i)$ is the *extensive* business margin (the number of customers, or market share).[7] The first order condition of this problem can be written as

$$R_t^L(i) = R_t^D + \frac{1}{\Lambda_t(i) + \tilde{\Lambda}_t(i)}, \tag{11}$$

[6] This last assumption removes the possibility that banks exploit strategically the knowledge about the current position of each entrepreneur to charge higher rates in the future.

[7] See Andrés and Arce (2008) for analytical derivations of both margins.

where $\Lambda_t(i) \equiv \left[-\partial b_t(i)/\partial R_t^L(i)\right]/b_t^i$ and $\tilde{\Lambda}_t(i) \equiv [-\partial \tilde{b}_t(i)/\partial R_t^L(i)]/\tilde{b}_t(i)$ are the semi-elasticities of the intensive and the extensive business margins, respectively. Thus the spread between the lending and the deposit rate is a negative function of the bank's market power, as measured by the semi-elasticities of individual loan size and market share.

As shown in Andrés and Arce (2008), in a symmetric equilibrium (i.e. $R_t^L(i) = R_t^L \; \forall i$), the optimal nominal loan rate can be expressed as

$$R_t^L = R_t^D + \frac{R_t^D - m_t E_t\left(\pi_{t+1} p_{t+1}^h / p_t^h\right)}{\eta m_t E_t\left(\pi_{t+1} p_{t+1}^h / p_t^h\right) - R_t^D} R_t^D, \tag{12}$$

where

$$\eta \equiv 1 + \frac{n}{\alpha} \frac{\beta^e}{1-\beta^e}.$$

Therefore, the lending spread is decreasing in expected nominal house price inflation, $E_t\left(\pi_{t+1} p_{t+1}^h / p_t^h\right)$, the pledgeability ratio, m_t, and the degree of banking competition, as captured by the ratio n/α; and it is increasing in the nominal deposit rate, R_t^D. The intuition for these effects is the following. An increase in expected house price inflation or in the pledgeability ratio increase entrepreneurs' borrowing capacity. As their indebtedness rises, their demand for loans becomes more elastic, which reduces banks' market power and compresses lending spreads. Similarly, as entrepreneurs become more indebted, the utility cost of servicing the debt becomes more important in the choice of bank relative to the distance utility cost. As a result, small changes in loan rates lead to large flows of customers in search for the lowest loan rate. This reduces the elasticity of the extensive business margin and hence pushes lending spreads down. Similar intuitions can be provided regarding the effects of the nominal deposit rate. Finally, an increase in the degree of banking competition (i.e. a rise in n/α) compresses lending spreads through an increase in the elasticity of banks' market share with respect to the lending rate.

In the symmetric equilibrium, each bank has $\tilde{b}_t(i) = (1-\omega)/n$ customers and each loan equals $b_t(i) = b_t$ in real terms, for all $i \in \{1, 2, ..., n\}$. Therefore, aggregate real profits in the banking sector equal $\Omega_t^b / P_t = \left(R_{t-1}^L - R_{t-1}^D\right) b_t (1-\omega)$.

2.4 Final goods producers

There exist a measure-one continuum of firms that purchase the intermediate good from entrepreneurs and transform it one-for-one into differentiated final good varieties. For these firms, the real price of the intermediate good, p_t^I, represents the real marginal cost. Cost minimization by consumers implies that each final good producer $j \in [0,1]$ faces the following demand curve for its product variety,

$$y_t^f(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} y_t^f, \tag{13}$$

where $P_t(j)$ is the firm's nominal price, $\varepsilon > 1$ is the elasticity of substitution between final good varieties and

$$y_t^f = \omega c_t + (1-\omega)\, c_t^e \tag{14}$$

is the aggregate demand for final goods. As is standard in the New Keynesian literature, we assume staggered nominal price adjustment à la Calvo (1983). Letting θ denote the constant probability of non-adjustment, the optimal price decision of price-setting firms is given by

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^s \frac{c_t}{c_{t+s}} \left\{ (1+\tau)\frac{\tilde{P}_t}{P_{t+s}} - \frac{\varepsilon}{\varepsilon-1} p_{t+s}^I \right\} P_{t+s}^{\varepsilon} y_{t+s}^f = 0, \tag{15}$$

where $\tau > 0$ is a subsidy rate on the revenue of final goods producers (the role of which is explained below) and $\tilde{P}_t$ is the optimal price decision. Under Calvo price adjustment, the aggregate price index evolves as follows,

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\,\tilde{P}_t^{1-\varepsilon}\right]^{1/(1-\varepsilon)}. \tag{16}$$

Aggregate nominal profits in the final goods sector equal $\Omega_t^f = \int_0^1 \left[(1+\tau)\, P_t(j) - P_t p_t^I\right] y_t^f(j) dj$.

2.5 Market clearing

Total supply of the intermediate good equals $(1-\omega)\, y_t$. Total demand from final good producers equals $\int_0^1 y_t^f(j)dj$, where each firm's demand is given by (13). Equilibrium in the intermediate good market therefore requires

$$(1-\omega)\, y_t = \Delta_t y_t^f, \tag{17}$$

where $\Delta_t \equiv \int_0^1 (P_t(j)/P_t)^{-\varepsilon} dj$ is a measure of price dispersion in final goods. Notice that price dispersion increases the amount of the intermediate good that must be produced in order to satisfy a certain level of final consumption demand.

Equilibrium in the real estate market requires

$$\bar{h} = \omega h_t + (1-\omega)\, h_t^e, \tag{18}$$

where $\bar{h}$ is the fixed aggregate stock of real estate. The labor market equilibrium condition is

$$\omega l_t^s = (1-\omega)\, l_t^d. \tag{19}$$

2.6 Fiscal and monetary authorities

The fiscal authority passively rebates its flow surplus to households in a lump-sum manner (if such surplus is negative, then it represents a lump-sum tax). Letting s_t denote the surplus per household, the aggregate fiscal surplus equals

$$\omega s_t = \tau^h \omega p_t^h h_t + \tau^e (1-\omega) \left(p_t^I y_t - w_t l_t^d\right) - \tau \int_0^1 P_t(j) y_t^f(j) dj.$$

The model is closed by means of a monetary policy rule. The latter can be a simple rule, such as strict inflation targeting, or a policy that is optimal with respect to some criterion. Sections 4 and 5 below are devoted to characterizing different types of policy rules and their effects on equilibrium allocations.

3 Efficient equilibrium

In this section we analyze the efficient equilibrium in our model, which will be the normative benchmark for the monetary authority. We assume that, when maximizing aggregate welfare, the social planner assigns to entrepreneurs the same discount factor as that of households, β.[8] The social planner therefore maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \omega \left[\log(c_t) - \frac{(l_t^s)^{1+\varphi}}{1+\varphi} + \vartheta_t \log(h_t) \right] + (1-\omega) \log(c_t^e) \right\}$$

subject to the aggregate resource constraints for real estate, equation (18), and for consumption goods,

$$(1-\omega)\, e^{a_t} \left(h_{t-1}^e\right)^{\nu} \left(\frac{\omega}{1-\omega} l_t^s\right)^{1-\nu} = \omega c_t + (1-\omega)\, c_t^e, \tag{20}$$

[8] Otherwise, the social planner equilibrium would assign less and less consumption to the impatient consumers (entrepreneurs) relative to the patient ones (households) as time went by. This would make the equilibrium allocation dependent on the time elapsed since the implementation of the social planner allocation. The resulting system of equations would not have a recursive representation, thus making standard solution techniques inapplicable.

where we have used equation (19) to substitute for l_t^d in the production function. Using equations (20) and (18) to solve for c_t and h_t, respectively, the social-planner problem simplifies to the choice of the optimal state-contingent path of c_t^e, h_t^e and l_t^s. The first-order conditions of this problem can be expressed as

$$c_t = c_t^e, \tag{21}$$

$$\beta E_t \frac{1}{c_{t+1}} \nu \frac{e^{a_{t+1}} \left(h_t^e\right)^{\nu} \left[\omega l_{t+1}^s / \left(1-\omega\right)\right]^{1-\nu}}{h_t^e} = \frac{\vartheta_t}{h_t}, \tag{22}$$

$$c_t \left(l_t^s\right)^{\varphi} = \frac{1-\omega}{\omega} \left(1-\nu\right) \frac{y_t}{l_t^s}. \tag{23}$$

Notice that equations (20) and (21) jointly imply $c_t = (1-\omega) e^{a_t} \left(h_{t-1}^e\right)^{\nu} \left[\omega l_t^s / (1-\omega)\right]^{1-\nu}$. Using this in equation (22), we have that the efficient distribution of the stock of real estate is given by

$$\frac{h_t^e}{h_t} = \frac{\beta \nu}{\left(1-\omega\right) \vartheta_t}. \tag{24}$$

This, combined with equation (18), implies the following solution for aggregate housing,

$$\omega h_t = \frac{\omega \vartheta_t}{\omega \vartheta_t + \beta \nu} \bar{h}. \tag{25}$$

Using equations (20) and (21) in equation (23), we obtain the following solution for efficient labor supply,

$$l_t^s = \left(\frac{1-\nu}{\omega}\right)^{1/(1+\varphi)} \equiv l^{s,*}. \tag{26}$$

The efficient level of output is then given by

$$y_t = e^{a_t} \left(h_{t-1}^e\right)^{\nu} \left(\frac{\omega}{1-\omega} l^{s,*}\right)^{1-\nu} \equiv y_t^*. \tag{27}$$

To summarize, the efficient equilibrium is characterized by full consumption risk sharing between households and entrepreneurs (equation 21), a distribution of real estate that changes only with shocks to preferences for housing (equation 24) and a constant level of labor supply (equation 26).[9] These features will help us understand the stabilization goals and trade-offs of monetary policy. We turn to this now.

[9] The fact that neither labor hours nor the distribution of real estate are affected by productivity shocks in the efficient equilibrium is due to our assumption of logarithmic utility of consumption. Deviating from the latter assumption would complicate the algebra without adding much to our main insights about the nature of optimal monetary policy.

4 Optimal monetary policy

In order to analyze optimal monetary policy, we follow the welfare-based *linear-quadratic* approach pioneered by Rotemberg and Woodford (1997). This method consists of deriving a log-quadratic approximation of aggregate welfare (which represents the objective function of the central bank) and a log-linear approximation of the equilibrium conditions (which are the constraints on the central bank's optimization problem). As is well known, this method is helpful at clarifying the stabilization goals faced by the central bank and the various trade-offs among those goals. Indeed, the application of this method in our setup delivers a set of analytical results that facilitate greatly the interpretation of the subsequent numerical results.

4.1 Quadratic loss function

As emphasized by Benigno and Woodford (2008), the approximation of the aggregate welfare criterion must be purely quadratic in order for the linear-quadratic approach to provide a correct welfare ranking (with an accuracy of up to second order) of alternative monetary policy rules. Derivation of a purely quadratic approximation is greatly simplified by the assumption of an efficient steady state for the welfare-relevant variables. As shown in the Appendix, steady-state efficiency for such variables can be implemented in our framework by making the following three assumptions.

Assumption 1 *The subsidy rate on the revenue of final goods producers is given by*

$$\tau = \frac{\varepsilon}{\varepsilon - 1} - 1 > 0.$$

Assumption 2 *The tax rate on entrepreneur profits is given by*

$$\tau^e = 1 - \frac{1-\omega}{(1-\beta^e)\,\nu} \frac{1-\beta^e - m\left(1/R_{ss}^L - \beta^e\right)}{1 - m/R_{ss}^L}.$$

Assumption 3 *The tax rate on housing wealth is given by*

$$\tau^h = \frac{\beta}{\beta^e} \frac{1-\beta^e - m\left(1/R_{ss}^L - \beta^e\right)}{1-\tau^e} - 1 + \beta.$$

The first assumption eliminates the monopolistic distortion in final goods markets, such that steady-state real marginal costs are unity ($p_{ss}^I = 1$). The second assumption guarantees efficient risk-sharing between households and entrepreneurs in the steady state ($c_{ss} = c_{ss}^e$). The third one implements the efficient steady-state distribution of real estate between commercial and residential uses ($h^e_{ss}/h_{ss} = \beta\nu/\left[(1-\omega)\,\vartheta\right]$). Under assumptions 1 to 3, aggregate welfare can be approximated by[10]

$$\sum_{t=0}^{\infty}\beta^t\left\{\omega\left[\log c_t+\vartheta_t\log h_t-\frac{(l^s_t)^{1+\varphi}}{1+\varphi}\right]+(1-\omega)\log c^e_t\right\}=-\sum_{t=0}^{\infty}\beta^t L_t+t.i.p.+O^3,$$

where $t.i.p.$ are terms independent of policy, O^3 are terms of order third and higher, and

$$L_t=\lambda_\pi\hat{\pi}^2_t+\lambda_y\left(\hat{y}_t-\hat{y}^*_t\right)^2+\lambda_c\left(\hat{c}_t-\hat{c}^e_t\right)^2+\lambda_h\left(\hat{h}_t-\hat{h}^*_t\right)^2 \tag{28}$$

is a purely quadratic period loss function, where hats denote log-deviations from steady state and weight coefficients are given by

$$\lambda_\pi\equiv\frac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)},\lambda_y\equiv\frac{1+\varphi}{1-\nu},\lambda_c\equiv\omega\left(1-\omega\right),\lambda_h\equiv\omega\vartheta\frac{\omega\vartheta+\beta\nu}{\beta\nu}.$$

The loss function illustrates the existence of four stabilization goals for the central bank. The first one is inflation. As is well known, under staggered price adjustment inflation creates inefficient price dispersion and hence a welfare loss. The second goal is the output gap, which is the difference between the actual and the efficient level of output. The latter is defined as

$$\hat{y}^*_t\equiv a_t+\nu\hat{h}^e_{t-1},$$

which is simply the log-linear version of equation (27). Nominal price rigidities produce inefficient fluctuations in output, which generates in turn inefficient fluctuations in labor hours. These first two goals are standard in the New Keynesian model.

The third and fourth goals are directly related to the existence of financial frictions in this model. The third goal is the (log)difference in per capita consumption between households and entrepreneurs, i.e. between unconstrained and constrained consumers, which we may refer to as the *consumption gap*. This term captures the aggregate welfare losses produced by inefficient risk sharing between households and entrepreneurs, which is in turn the result of collateral constraints on entrepreneurs. The fourth goal is the (log)difference between the actual and the

[10] See the proof in the Appendix.

efficient level of housing, or *housing gap*, where

$$\hat{h}_t^* \equiv \frac{\beta\nu}{\omega\vartheta + \beta\nu} z_t^h \tag{29}$$

is efficient housing (see equation 25). Notice that, given the fixed supply of real estate, an inefficient level of housing is equivalent to an inefficient distribution of real estate between residential and commercial uses. In our framework, inefficiency in the real estate distribution can arise for two reasons. First, the demand for commercial property by entrepreneurs is distorted by its role as collateral in loan agreements. Second, since households and entrepreneurs differ in their degree of impatience and their consumption, they also use different stochastic discount factors for pricing future state-contingent payoffs from housing.

4.2 Policy trade-offs

The second step of the linear-quadratic approach consists of log-linearizing the equilibrium conditions around the steady state. For brevity, the complete list of log-linear equations is deferred to the Appendix.[11] Here, we restrict our attention to those equations that are helpful for understanding the trade-offs among stabilization goals. We start by log-linearizing and combining equations (15) and (16), which yields

$$\hat{\pi}_t = \frac{(1-\theta)(1-\beta\theta)}{\theta} \hat{p}_t^I + \beta E_t \hat{\pi}_{t+1}. \tag{30}$$

In order to find an expression for the real marginal cost, $\hat{p}_t^I$, we first log-linearize equations (1), (7) and (19), and combine them to get

$$\hat{c}_t + \varphi \hat{l}_t^s = \hat{p}_t^I + \hat{y}_t - \hat{l}_t^s. \tag{31}$$

That is, the labor supply schedule (the marginal rate of substitution between consumption and leisure) must intersect the labor demand schedule (the marginal revenue product of labor).

[11] Simulation results not reported here indicate that the Ramsey optimal long-run gross rate of inflation is $\pi_{ss} = 1$, regardless of whether the steady state is assumed to be efficient or not. Therefore, our log-linearization is performed around a zero net inflation steady state. The reason for this result is essentially the same as the reason why the optimal long-run net rate of inflation is zero in the standard New Keynesian model, namely that the welfare losses of committing to positive inflation rates in the future outweigh the welfare gains of exploiting the short-run output-inflation trade-off when output is inefficiently low (see e.g. Woodford, 2003, ch. 6).

Second, we log-linearize the production function and solve for labor hours, obtaining

$$\hat{l}_t^s = \frac{1}{1-\nu}\left(\hat{y}_t - \hat{a}_t - \nu \hat{h}_{t-1}^e\right) = \frac{1}{1-\nu}\left(\hat{y}_t - \hat{y}_t^*\right), \tag{32}$$

where in the second equality we have used the definition of efficient output. Third, we log-linearize the equilibrium conditions in the final goods and intermediate good markets, equations (14) and (17) respectively, and combine them into

$$\hat{y}_t = \omega \hat{c}_t + (1-\omega)\,\hat{c}_t^e, \tag{33}$$

where we have used the fact that $(1-\omega)\,y_{ss} = c_{ss} = c_{ss}^e$ and that $\hat{\Delta}_t$ is actually a second-order term (see the appendix). Combining equations (31) to (33), we can express real marginal costs as

$$\hat{p}_t^I = \frac{1+\varphi}{1-\nu}\left(\hat{y}_t - \hat{y}_t^*\right) + (1-\omega)\left(\hat{c}_t - \hat{c}_t^e\right). \tag{34}$$

Using this in equation (30) yields the following *New Keynesian Phillips curve*,

$$\hat{\pi}_t = \kappa \frac{1+\varphi}{1-\nu}\left(\hat{y}_t - \hat{y}_t^*\right) + \beta E_t \hat{\pi}_{t+1} + \kappa (1-\omega)\left(\hat{c}_t - \hat{c}_t^e\right), \tag{35}$$

where $\kappa \equiv (1-\theta)(1-\beta\theta)/\theta$. Equation (35) has the same form as the standard New Keynesian Phillips curve, with the exception of the last term on the right hand side, which is proportional to the consumption gap. Therefore, collateral constraints and the resulting inefficient risk-sharing create an endogenous trade-off between the output gap and inflation. The reason is the following. From equation (31), real marginal costs $\hat{p}_t^I$ depend on labor hours and the difference between aggregate demand and household consumption. Because of inefficient risk sharing, fluctuations in aggregate demand and household consumption will be unequal. As a result, keeping labor hours constant (that is, closing the output gap) is not enough to prevent fluctuations in real marginal costs and hence in inflation.

From the preceding analysis, it follows that closing the consumption gap has several beneficial effects on aggregate welfare. First, it improves the trade-off between inflation and output gap. Second, since the consumption gap is itself a stabilization goal, closing it directly improves welfare. An additional normative reason for closing the consumption gap is that it reduces the distortionary effects of collateral constraints on entrepreneurs' stochastic discount factor and hence on their valuation of future income streams from commercial property. This makes the real estate distribution more efficient over the cycle and thus helps closing the housing gap.

While desirable, consumption gap stabilization requires itself inefficient fluctuations in other stabilization goals. To see this, consider the log-linear approximation of the expression for entrepreneur consumption (equation 10) around the efficient steady state,

$$\hat{c}_t^e = (1-\beta^e)\left[\frac{(1-\tau^e)\,\nu}{1-\omega}\left(\hat{p}_t^I + \hat{y}_t\right) + \frac{p_{ss}^h h_{ss}^e}{c_{ss}^e}\left(\hat{p}_t^h + \hat{h}_{t-1}^e\right) - \frac{b_{ss} R_{ss}^L}{c_{ss}^e}\left(\hat{R}_{t-1}^L + \hat{b}_{t-1} - \hat{\pi}_t\right)\right], \quad (36)$$

where both sides have been normalized by c_{ss}^e and the steady-state condition $c_{ss}^e/y_{ss} = 1-\omega$ has been used. The binding collateral constraint (equation 6 holding with equality) can be approximated by $\hat{R}_t^L + \hat{b}_t = z_t^m + E_t\hat{p}_{t+1}^h + \hat{h}_t^e + E_t\hat{\pi}_{t+1}$. Substituting this into equation (36), using $b_{ss}R_{ss}^L = mp_{ss}^h h_{ss}^e$, and rearranging terms, we obtain

$$\begin{aligned}\hat{c}_t^e &= (1-\beta^e)\frac{(1-\tau^e)\,\nu}{1-\omega}\left(\hat{p}_t^I + \hat{y}_t\right) \\ &\quad + (1-\beta^e)\frac{p_{ss}^h h_{ss}^e}{c_{ss}^e}\left[\left(\hat{p}_t^h - mE_{t-1}\hat{p}_t^h\right) + (1-m)\,\hat{h}_{t-1}^e + m\left(\hat{\pi}_t - E_{t-1}\hat{\pi}_t\right) - mz_{t-1}^m\right].\end{aligned} \quad (37)$$

Therefore, entrepreneur profits $(\hat{p}_t^I + \hat{y}_t)$, quasi-surprises in real estate prices $(\hat{p}_t^h - mE_{t-1}\hat{p}_t^h)$, the stock of commercial property $(\hat{h}_{t-1}^e)$ and inflation surprises $(\hat{\pi}_t - E_{t-1}\hat{\pi}_t)$ are the endogenous determinants of entrepreneur consumption. The latter will therefore differ from household consumption, which is driven exclusively by intertemporal substitution considerations. In response to unexpected shocks, it is however possible for the central bank to bring entrepreneur and household consumption closer to each other. First, the central bank can use its interest rate policy to indirectly affect the path of real estate wealth. Second, it can engineer inflation surprises so as to alter the real value of debt repayments. Third, notice that entrepreneur profits can be expressed in terms of stabilization goals as follows,

$$\hat{p}_t^I + \hat{y}_t = \left(\frac{1+\varphi}{1-\nu} + 1\right)(\hat{y}_t - \hat{y}_t^*) + (1-\omega)\,(\hat{c}_t - \hat{c}_t^e) + \hat{y}_t^*,$$

where we have used equation (34) to substitute for $\hat{p}_t^I$. Therefore, the central bank can also affect the output-gap in order to narrow the consumption gap.

To summarize, optimal monetary policy will involve a trade-off between all four stabilization goals in response to macroeconomic shocks. We now turn to the quantitative analysis of these trade-offs.

5 Quantitative analysis

5.1 Calibration

We calibrate our model to quarterly US data. The calibration is largely based on Andrés and Arce (2008) and Iacoviello (2005). The household discount factor, $\beta = 0.993$, is chosen such that the annual real interest rate equals 3%. The entrepreneur discount factor is set to 0.95, within the range of values for constrained consumers typically used in the literature. The elasticity of production with respect to commercial housing, ν, is set to generate a steady-state ratio of commercial real estate wealth to annual output of 62%. Similarly, the weight on housing utility, ϑ, is chosen to match an average ratio of housing wealth to annual output of 140%. Regarding the banking parameters, what matters for the steady-state level of lending spreads is the ratio n/α. We arbitrarily set the number of banks to 10, and then set the distance utility parameter, α, to obtain a steady-state annualized lending spread of 2.5%. The size of the household population, $\omega = 0.979$, is chosen such that the tax rate on entrepreneur profits that implements the efficient steady state is zero.[12] The loan-to-value ratio is set to $m = 0.85$, as in Iacoviello (2005). The labor supply elasticity is set to one half, which is broadly consistent with micro evidence. The elasticity of demand curves is set to 6, which would imply a monopolistic mark-up of 20% in the absence of subsidies. The Calvo parameter implies a mean duration of price contracts of 3 quarters, consistent with recent micro evidence (Bils and Klenow, 2004, Nakamura and Steinsson, 2008). The tax rate on housing wealth that implements the efficient steady state is $\tau^h = 0.012$. The structural parameters imply weights (normalized by their sum) of $\lambda_\pi = 91.2\%$, $\lambda_y = 7.9\%$, $\lambda_c = 0.1\%$ and $\lambda_h = 0.8\%$ in the loss function. Finally, the autocorrelation coefficient of a_t is set to a standard value of 0.95, whereas that of z_t^m is set to 0.75, implying a half-life of four quarters for shocks to the pledgeability ratio.

[12] Alternatively, we could have calibrated ω empirically and then obtained τ^e from the formula in section 4.1. Since the share of entrepreneurs in the population is fairly small, this alternative approach would produce a very similar calibration.

Table 1. Baseline calibration.

	Value	Target	Description
β	0.993	$R_{ss}^D/\pi_{ss} = (1.03)^{1/4}$	household discount factor
β^e	0.95	standard	entrepreneur discount factor
ν	0.05	$p_{ss}^h h_{ss}^e/(4y_{ss}) = 0.62$	elasticity of output wrt real estate
ϑ	0.11	$\omega p_{ss}^h h_{ss}/\left[4\left(1-\omega\right)y_{ss}\right] = 1.40$	relative weight on housing utility
n, α	10, 6.32	$4(R_{ss}^L - R_{ss}^D) = 2.5\%$	number of banks, distance cost
ω	0.979	$\tau^e = 0$	household share of population
m	0.85	standard	pledgeability ratio
φ	2	$1/\varphi = 0.5$	(inverse of) labor supply elasticity
ε	6	$\varepsilon/\left(\varepsilon-1\right) = 1.20$	intratemporal elasticity of subst.
θ	0.67	$1/(1-\theta) = 3$ qrts.	Calvo parameter

5.2 Impulse-response analysis

In order to further investigate the nature of optimal monetary policy in this framework, we now analyze the economy's response to shocks under the optimal commitment. We consider both productivity shocks as well as *credit-crunch* shocks, the latter in the form of shocks to the pledgeability ratio.[13] We also analyze the impulse-responses under a policy of strict inflation targeting ($\hat{\pi}_t = 0$). Such a policy has been shown to be optimal in the standard New Keynesian model (see e.g. Goodfriend and King, 2001, and Woodford, 2003). By comparing both policies, we can illustrate the trade-offs that render inflation targeting suboptimal in this framework.

5.2.1 Productivity shocks

Figure 1 plots the economy's response to a 1% negative productivity shock. Let us focus first on the case of strict inflation targeting (dotted lines). The fall in total factor productivity reduces the marginal product of commercial real estate. Entrepreneurs respond by reducing their demand for commercial property, which leads to a persistent decline in real estate prices. Lower expected asset prices means that real estate is less valuable as a collateral, which further reduces demand for commercial real estate. Since productivity shocks do not affect the efficient real estate distribution, the fall in commercial property is mirrored by a symmetric increase in

[13] For brevity, we omit the results regarding the effects of shocks to the utility of housing services (ϑ_t). These results are available upon request from the authors.

the housing gap. Also, lower profits and lower real estate wealth trigger a large reduction in entrepreneur net worth and therefore in entrepreneur consumption. Household consumption falls too, but it does so by a relatively small amount, thanks to households' better ability to smooth consumption. As a result, the consumption gap increases sharply on impact. In addition to lowering welfare directly, the consumption gap also shifts the New Keynesian Phillips curve upwards. In order to keep inflation at zero, the central bank is obliged to engineer a (small) drop in the output gap. To summarize, strict inflation targeting requires inefficient fluctuations in the output gap and, especially, in the consumption and housing gaps.

FIGURE 1 HERE

Relative to the situation under inflation targeting, the optimal policy (solid line) can improve matters by cutting the nominal interest rate more sharply on impact. This way, it undoes part of the reduction in entrepreneurs' demand for commercial real estate, thus narrowing the housing gap. Thanks to the smaller drop in real estate prices and in commercial property, entrepreneurs' net worth and consumption fall substantially less, which produces a significant reduction in the consumption gap. Notice that the more aggressive policy response leads to an increase in the output gap and inflation on impact. This lowers welfare directly, but it also helps at reducing the consumption gap, by increasing entrepreneur profits and reducing their real debt burden. Calculations not reported here show however that most of the reduction in the consumption gap is due to the smaller drop in real estate wealth. The contraction in the consumption gap contains the upward shift in the New Keynesian Phillips curve, thus improving the trade-off between inflation and output gap. Indeed, both variables return to zero very quickly. Finally, the endogenous response in lending spreads (not shown in the figure) is very small under both policy regimes, with peak drops of 1 and 0.2 basis points, respectively. The reason is that the reaction of the two endogenous determinants of lending spreads (expected inflation in real estate prices, and the policy rate) tend to cancel each other out.[14]

5.2.2 Credit crunch shocks

Figure 2 plots the impulse-responses to a 1% negative shock to the pledgeability ratio. Again, we focus first on the case of inflation targeting (dotted lines). The fall in the pledgeability ratio reduces the marginal value of commercial real estate by reducing its value as collateral. Entrepreneurs respond by decreasing their demand for commercial real estate. This produces

[14] For further discussion of these opposite-sign effects and the resulting low responsiveness of spreads following a productivity shock, see Andrés and Arce (2008).

again a symmetric increase in the housing gap, because the efficient real estate distribution is independent of the pledgeability ratio. Regarding the other stabilization goals, the responses are of minor importance. First, the absolute deviations of the consumption gap from its efficient value (zero) are much smaller than in the case of a productivity shock. The reason is that the credit crunch has two opposing effects on entrepreneur net worth. On the one hand, the fall in the price and the quantity of real estate reduces entrepreneurial net worth but, on the other hand, the credit crunch lowers their real debt burden in subsequent periods, thus improving their net worth (as the center right panel shows, the latter effect becomes dominant from the third period onwards). Second, since what matters for inflation dynamics is the present-discounted sum of consumption gaps and the latter sum responds relatively little, the shift in the New Keynesian Phillips curve is small, such that a tiny fall in the output gap is enough to keep inflation at zero.

FIGURE 2 HERE

Therefore, the optimal policy (solid lines) is primarily aimed at reducing the housing gap. In order to achieve this, the monetary authority resorts again to a sharper reduction in the policy rate. This way, it counteracts the negative effect of the credit crunch on entrepreneurs' demand for commercial property. As in the case of productivity shocks, this policy comes at the cost of increases in inflation and the output gap on impact, followed by transitory negative deviations from their efficient levels (zero). Notice finally that lending margins experience a non-negligible impact increase of 15 basis points in annualized terms (lower left panel), which contrasts with their negligible response to productivity shocks. This is due to the fact that lending margins depend negatively on the pledgeability ratio, through the latter's effect on the elasticity of demand for funding. This countercyclical response of lending margins has the property of amplifying the negative effect of the credit crunch under both policy scenarios.

5.3 Welfare analysis

The previous section characterized the responses of the stabilization goals to productivity and credit-crunch shocks. These goals however enter with different weights in the loss function of the central bank, and therefore have different quantitative effects on welfare. We are ultimately concerned with the welfare implications of alternative monetary policy rules. This subsection quantifies the welfare losses that arise under different such rules.

5.3.1 Welfare losses under the baseline calibration

The first four columns of Table 2 display the standard deviation of the four stabilization goals, conditional on productivity shocks (with a standard deviation of 1%). As in the analysis of impulse responses, we consider the cases of inflation targeting and the optimal policy commitment. We also include output gap targeting ($\hat{y}_t = \hat{y}_t^*$), which is equivalent to inflation targeting in the standard New Keynesian model (in the absence of an exogenous output-inflation trade-off) but not in our framework, due to the presence of the consumption gap in the New Keynesian Phillips curve. The last column displays the implied average welfare loss, as a percent of steady-state consumption.

Table 2. Standard deviation of stabilization goals and welfare loss, productivity shocks

Policy rule	$4\hat{\pi}_t$	$\hat{y}_t - \hat{y}_t^*$	$\hat{c}_t - \hat{c}_t^e$	$\hat{h}_t - \hat{h}_t^*$	Welfare loss
inflation targeting	0	0.08	11.67	4.73	0.11
output gap targeting	0.85	0	9.79	3.97	0.09
optimal policy	0.81	0.49	3.49	1.41	0.03
simple targeting rule**	0.95	0.31	5.17	2.10	0.04

Note: standard deviations in %, welfare loss as a % of steady-state consumption
** optimal weight: $\zeta = 0.212$

As the table makes clear, a policy of strict inflation targeting implies large fluctuations in the consumption and housing gaps. Fluctuations in the output gap are rather small. These volatilities, together with the weights in the loss function, imply an average welfare loss of 0.11% of steady-state consumption. Regarding the case of output gap targeting, fluctuations in the consumption and housing gaps are of similar magnitude, whereas (annualized) inflation has a standard deviation of 85 basis points. The implied average welfare loss (0.09%) is close to the one under inflation targeting. Finally, the optimal monetary policy balances all the trade-offs among goals, producing a welfare loss of just 0.03% of steady-state consumption.

Table 3 shows the standard deviation of the stabilization goals and the implied average welfare losses, conditional on shocks to the pledgeability ratio (with a standard deviation of 1%). Again, the larger fluctuations take place in the consumption and housing gaps. Under inflation and output-gap targeting, housing gaps are more volatile than consumption gaps. Since the former have a larger weight in the loss function, the optimal policy focuses on reducing fluctuations in the housing gap. This comes at the cost of larger fluctuations in all other goals. Intuitively, exogenous changes in the pledgeability ratio directly affect both the value of real estate as collateral and the lending margins charged to entrepreneurs; as a result, the policy of sustaining demand for commercial real estate becomes less effective and hence more costly in terms of the other goals. This implies that, in terms of average welfare losses, inflation and output-gap targeting are much closer to the optimal policy than in the case of productivity shocks.

Table 3. Standard deviation of stabilization goals and welfare loss, credit-crunch shocks

Policy rule	$4\hat{\pi}_t$	$\hat{y}_t - \hat{y}_t^*$	$\hat{c}_t - \hat{c}_t^e$	$\hat{h}_t - \hat{h}_t^*$	Welfare loss
inflation targeting	0	0.01	2.19	3.37	0.04
output gap targeting	0.31	0	2.08	3.45	0.05
optimal policy	0.36	0.23	4.94	2.21	0.03
simple targeting rule**	0.31	0.11	2.85	2.98	0.04

Note: standard deviations in %, welfare loss as a % of steady-state consumption

** optimal weight: $\zeta = 0.262$

5.3.2 Optimal simple targeting rules

In the standard New Keynesian model without an exogenous trade-off between inflation and output (the so-called "cost-push" shocks), the targeting rule that implements the optimal monetary policy commitment is simply $\hat{\pi}_t = 0$, that is, strict inflation targeting. In the presence of cost-push shocks, the corresponding targeting rule is a simple and intuitive expression linking inflation and the output gap (see e.g. Woodford 2003, ch. 7). In our model, due to its larger scale and the presence of financial frictions, the optimal targeting rule is too complex to be implemented in practice. In order to make the optimal monetary policy operational, we look for a simple targeting rule that approximates well the optimal policy. An important feature of our analysis is that the real price of the collateralizable asset (in our case, real estate) has an important effect on the transmission of shocks to the economy. On the one hand, consumption of the constrained agents is very sensitive to fluctuations in real estate wealth, and hence in real estate prices. On the other hand, the expected evolution of real estate prices has a potentially important effect on equilibrium lending margins, through their effect on the elasticity of demand for loans. In response to an adverse shock that persistently depresses real estate prices, both effects work towards amplifying the negative effects of the shock: actual reductions in asset prices lower entrepreneurial net worth and thus widen the consumption gap, whereas expected reductions in the latter prices cause countercyclical increases in lending margins.

The previous argument suggests considering simple targeting rules that capture the central bank's concern for stabilizing the actual and expected evolution of asset prices, together with the usual concern for inflation stabilization. In particular, we consider the following family of simple targeting rules,

$$\hat{\pi}_t + \zeta \left(\hat{p}_t^h - \hat{p}_{t-1}^h + E_t \hat{p}_{t+1}^h - \hat{p}_t^h \right) = 0.$$

According to this rule, the central bank targets a weighted average of inflation, on the one hand, and the sum of current and expected growth rates in the real price of real estate, on the other.[15] In the special case of $\zeta = 0$, the rule collapses to strict inflation targeting. For each shock, we find the value of ζ that minimizes the average welfare loss. The last line of tables 2 and 3 display the results under our proposed rule, which is labelled as 'simple targeting rule'. Notice first that the optimal coefficient ζ is positive conditional on either type of shock. The intuition is simple. Following for instance adverse shocks (financial or non-financial), strict inflation targeting produces an excessively large fall in demand for commercial property (i.e. a positive housing gap), together with persistent drops in real estate prices. In order to foster entrepreneurs' demand for real estate, and to reduce to some extent the drop in asset prices, the central bank finds it optimal to implement a comparatively more expansionary monetary policy, the by-product of which is to create inflation.[16] That is, actual and expected deflation in real estate prices coincide in time with positive consumer price inflation.

Regarding the actual welfare losses, our simple rule is very close to the optimal policy in the case of productivity shocks (table 2). Indeed, the rule succeeds in reducing the volatility of both the consumption and housing gaps, relative to the other two suboptimal policies. Conditional on credit crunch shocks (table 3), welfare losses under our simple rule are very similar to (though not higher than) those under the other suboptimal rules, although this is not surprising given the small welfare differences between the different policies in this case. The simple targeting rule does share with the optimal policy the feature of reducing the volatility of the housing gap

[15] We also considered a policy that set to zero the weights on the consumption and housing gaps in the quadratic loss function and minimized the resulting 'myopic' loss function. Although such a policy does not deliver a simple expression for the targeting rule either, it can shed light on the extent to which monetary policy should worry about the non-standard stabilization goals. We found that such a policy was much closer to inflation or output gap targeting than to the optimal policy in terms of welfare loss. This strongly suggests that the central bank should not obviate the goals arising from financial frictions in the conduct of monetary policy.

[16] For brevity of exposition we do not display here the impulse-responses under the optimal simple targeting rule. The latter are available upon request from the authors.

while increasing fluctuations in the other goals. Interestingly, the optimal value of the weight coefficient, ζ, is very similar for both shocks, which guarantees that the same rule with a similar coefficient performs well also unconditionally.[17]

These results may shed some light on the recent debate as to whether central bankers should pay attention to asset prices when conducting monetary policy.[18] Our analysis suggests that, to the extent that fluctuations in the real price of collateralizable assets cause large distortions in the consumption and investment decisions of collateral-constrained agents, then the monetary authority has a rationale for taking into account such asset price fluctuations in its policy decisions.

5.4 The effects of banking competition

The welfare analysis can also shed some light on the importance of the intensity of competition in the banking sector.[19] To isolate the effects of banking competition on the policy trade-offs, we consider the limiting case of perfect banking competition ($\alpha = 0$, or $n \to \infty$). In the latter scenario, the steady-state loan rate, R_{ss}^L, falls from its baseline value to $R_{ss}^D = 1/\beta$, and the interest rate spread becomes zero. The effect of this structural change on the fluctuations of the stabilization goals and the associated welfare losses are summarized in Tables 4 and 5.

[17]Indeed, once both shocks are taken into account (with an equal standard deviation of 1%), we find an optimal weight coefficient of $\zeta = 0.214$. Welfare losses in this case equal 0.08% of steady-state consumption, which is close to those under the optimal policy (0.06%) and substantially lower than those under inflation or output gap targeting (0.15% and 0.14%, respectively).

[18]See for instance Bernanke and Gertler (2001) and Cecchetti et al. (2003) for opposing views on this issue.

[19]Andrés and Arce (2008) find that variations in the level of banking competition have a moderate effect on the short term impulse response dynamics of the main variables, but a sizeable one in the medium term, due to the interaction between lending spreads and persistent net worth effects.

Table 4. Banking competition and welfare loss, productivity shocks

Banking regime	$4\hat{\pi}_t$	$\hat{y}_t - \hat{y}_t^*$	$\hat{c}_t - \hat{c}_t^e$	$\hat{h}_t - \hat{h}_t^*$	Welfare loss
Inflation targeting					
baseline calibration	0	0.08	11.67	4.73	0.11
perfect competition	0	0.10	14.24	5.74	0.16
Output gap targeting					
baseline calibration	0.85	0	9.79	3.97	0.09
perfect competition	1.22	0	11.44	4.61	0.14
Optimal policy					
baseline calibration	0.81	0.49	3.49	1.41	0.03
perfect competition	0.90	0.54	3.27	1.32	0.04
Simple targeting rule**					
baseline calibration	0.95	0.31	5.17	2.10	0.04
perfect competition	1.12	0.35	5.33	2.15	0.05

Note: standard deviations in %, welfare loss as a % of steady-state consumption

** optimal weights: $\zeta = 0.212$ (baseline) and $\zeta = 0.251$ (perfect competition)

Conditional on productivity shocks (table 4), the policy trade-offs clearly worsen when the banking industry is more competitive. For both inflation and output-gap targeting, all stabilization goals other than the one being targeted become more volatile. As a result, average welfare losses increase. Regarding the optimal policy, the increase in inflation and output-gap volatility is partially offset by smaller fluctuations in the consumption and housing gaps. However, the large weights of the former two goals in the loss function (91.2% and 7.9%, respectively) imply an increase in average welfare loss. This increase is small however. The same can be said about the simple targeting rule based on inflation and real estate prices, which remains very close to the optimal policy in terms of welfare loss. We conclude that, conditional on productivity shocks, the transit to perfect banking competition makes the policy trade-offs more severe, but especially so for the suboptimal policy rules.

To understand these results, it is helpful to consider the steady-state effects of an increase in banking competition. We start by defining the *leverage ratio* as the fraction of borrowers' asset holdings that are financed with debt, $b_t / \left(p_t^h h_t^e\right)$. Binding collateral constraints imply that, in the steady state, the leverage ratio equals $b_{ss} / \left(p_{ss}^h h_{ss}^e\right) = m / R_{ss}^L$. Therefore, the fall in loan rates produced by stronger banking competition increases the steady-state leverage ratio. Now consider equation (37), which determines the cyclical fluctuations in entrepreneur consumption. In the latter equation, the term capturing the sensitivity of entrepreneur consumption to real estate prices can be expressed as

$$(1-\beta^{e})\,\frac{p_{ss}^{h}h_{ss}^{e}}{c_{ss}^{e}}=\frac{\beta^{e}}{1-b_{ss}/\left(p_{ss}^{h}h_{ss}^{e}\right)},$$

where we have used the fact that $c_{ss}^{e}=(1-\beta^{e})\left(p_{ss}^{h}h_{ss}^{e}-b_{ss}\right)/\beta^{e}$, which in turn stems from the fact that entrepreneurs devote a fraction $1-\beta^{e}$ of their real net worth to consumption, c_{ss}^{e}, and the remaining fraction β^{e} to financing the part of real estate holdings that exceeds the amount of borrowing, $p_{ss}^{h}h_{ss}^{e}-b_{ss}$. Therefore, the increase in the steady-state leverage ratio amplifies the effect of fluctuations in real estate prices on entrepreneur consumption. Since household consumption is not affected by collateral constraints, the increased volatility of entrepreneur consumption carries over to the consumption gap. As we have seen, this has a direct negative effect on welfare, but it also worsens the output-inflation trade-off (by causing larger shifts in the Phillips curve) and amplifies the distortions in the distribution of real estate through its effects on entrepreneurs' stochastic discount factor. Taking all these effects together, we have that the increase in banking competition tends to exacerbate the trade-offs of monetary policy and the associated welfare losses.

Table 5 shows the effects of stronger banking competition on welfare losses conditional on credit-crunch shocks. The main message from the table is that the volatility of the different stabilization goals tends to increase but it does so by very small amounts, and in some cases such volatilities actually fall. As a result, the effect on average welfare losses is virtually inexistent. The intuition for this can be found in the behavior of the lending spread. The latter responds countercyclically to credit crunch shocks when banks have market power, thus amplifying the effects on the economy. However, under perfect banking competition lending spreads are zero, and so their amplifying role disappears. As the table makes clear, this basically neutralizes the amplifying effect of the leverage ratio on welfare losses.

Table 5. Banking competition and welfare loss, credit crunch shocks

Banking regime	$4\hat{\pi}_t$	$\hat{y}_t - \hat{y}_t^*$	$\hat{c}_t - \hat{c}_t^e$	$\hat{h}_t - \hat{h}_t^*$	Welfare loss
		Inflation targeting			
baseline calibration	0	0.01	2.19	3.37	0.04
perfect competition	0	0.02	2.63	3.33	0.04
		Output gap targeting			
baseline calibration	0.31	0	2.08	3.45	0.05
perfect competition	0.42	0	2.41	3.45	0.05
		Optimal policy			
baseline calibration	0.36	0.23	4.94	2.21	0.03
perfect competition	0.28	0.18	5.38	2.42	0.03
		Simple targeting rule**			
baseline calibration	0.31	0.11	2.85	2.98	0.04
perfect competition	0.27	0.10	3.37	3.00	0.04

Note: standard deviations in %, welfare loss as a % of steady-state consumption

** optimal weights: $\zeta = 0.262$ (baseline) and : $\zeta = 0.226$ (perfect competition)

6 Conclusions

In this paper we provide a theoretical framework for the analysis of the optimal conduct of monetary policy in the presence of financial frictions, in the form of collateral constraints and a monopolistically competitive banking sector. In our economy consumers are divided into households and entrepreneurs, with different time preferences. The resulting credit flows are intermediated by banks, which have some monopolistic power in the loans market and set optimal lending rates accordingly. There is only one collateralizable asset, real estate, which also yields utility to households and productive returns to entrepreneurs. The latter face endogenous credit limits that link their borrowing capacity to the expected value of their real estate holdings.

We have shown that, under the assumption of steady state efficiency in the welfare-relevant variables and up to a second order approximation, welfare maximization is equivalent to stabilization of four goals: inflation, output gap, the consumption gap between households and entrepreneurs, and the distribution of real estate between both groups (or housing gap). Following both productivity and credit-crunch shocks (the latter in the form of exogenous changes in collateral requirements), the optimal monetary policy commitment implies a short-run trade-off between stabilization goals. Relative to strict inflation targeting, the optimal policy commitment changes the nominal policy rate more aggressively so as to avoid large fluctuations in the consumption and housing gaps, at the cost of inducing inefficient fluctuations in inflation and the output gap. The welfare gain of pursuing optimal policies is comparatively larger under productivity-driven fluctuations. Credit-crunch shocks have a direct effect both on lending margins and on the collateral value of real estate, such that policies aimed at sustaining entrepreneurs' demand for real estate (and hence at preventing the emergence of large housing gaps) become less effective.

We also find that a simple targeting rule that relates current inflation negatively with current and one-period ahead expected changes in real estate prices performs remarkably well in the face of both TFP or financial shocks. From a policy perspective, this suggests that, to the extent that fluctuations in the price of collateralizable assets cause large distortions in the expenditure decisions of collateral-constrained agents, then the monetary authority has a rationale for taking into account such asset price fluctuations in its policy decisions

Finally, we have compared the nature of these trade-offs under alternative assumptions about the degree of competition in the banking industry. We find that, both under optimal and suboptimal policies, welfare losses due to cyclical fluctuations are amplified as banking competition increases. This amplification is negligible in financially driven (credit-crunch) fluctuations, but is substantial if productivity shocks dominate. Key to this different effect of banking compettion is the interplay between two endogenous mechanisms at work in our model: financial leveraging and lending margins. As banking competition increases, entrepreneurs become more leveraged and this amplifies the response of their net worth (and hence consumption) to asset prices. This latter feature worsens the aforementioned trade-offs, especially under suboptimal policy rules, thus making the use of the optimal policy more compelling. The countercyclical response of lending margins, which is naturally stronger in less competitive environments, aggravates the policy trade-offs. The first mechanism is equally important regardless of the nature of the shocks, whereas the second one is very weak under productivity shocks and significant if credit-crunch shocks are the driving force behind fluctuations.

References

[1] Andrés, J. and O. Arce (2008): "Banking Competition, Housing Prices and Macroeconomic Stability," Banco de España WP no. 830.

[2] Benigno, P. and M. Woodford (2003): "Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach," *NBER Macroeconomics Annual*, 18, 271-364.

[3] Benigno, P. and M. Woodford (2008): "Linear-Quadratic Approximation of Optimal Policy Problems". Working paper.

[4] Bernanke, B. and M. Gertler (1989), "Agency Costs, Net Worth, and Business Fluctuations", *American Economic Review*, March, Vol. 79(1), pp. 14-31.

[5] Bernanke, B. and M. Gertler (2001), "How Should Central Bankers Respond to Asset Prices?". *American Economic Review Papers and Proceedings*, 91(2), 253-257.

[6] Bernanke, B., M. Gertler and S. Gilchrist (1999), "The Financial Accelerator in a Quantitative Business Cycle Framework," in J.B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, vol. 1C. Amsterdam: Elsevier Science, North-Holland.

[7] Bils, M., and P. J. Klenow (2004), "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 112(5), 947-985.

[8] Calvo, G. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics* 12: 383-398.

[9] Cecchetti, S. G., H. Genberg, and S. Wadhwani (2002), "Asset Prices in a Flexible Inflation Targeting Framework", NBER Working Paper 8970

[10] Cúrdia, V. and M. Woodford (2008), "Credit Frictions and Optimal Monetary Policy," mimeo.

[11] De Fiore, F., and O. Tristani (2009), "Optimal Monetary Policy in a Model of the Credit Channel", European Central Bank, working paper series no. 1043.

[12] De Fiore, F., P. Teles, and O. Tristani (2009), "Monetary Policy and the Financing of Firms," CEPR DP no. 7419.

[13] Goodfriend, M., and R. G. King (2001). "The Case for Price Stability," NBER Working Papers 8423.

[14] Goodfriend, M. , B. McCallum (2007): "Banking and interest rates in monetary policy analysis: A quantitative exploration", *Journal of Monetary Economics* 54 1480–1507.

[15] Iacoviello, M. (2005): "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," *American Economic Review* 95, no. 3: 739-764.

[16] Kiyotaki, N. and J. H. Moore (1997). "Credit Cycles," *Journal of Political Economy,* 105, 211-248.

[17] Monacelli, T. (2007): "Optimal Monetary Policy with Collateralized Household Debt and Borrowing Constraints", in Asset Prices and Monetary Policy, J. Campbell (ed.), University of Chicago Press.

[18] Nakamura, E., and J. Steinsson (2008). "Five Facts about Prices: A Reevaluation of Menu Cost Models," *Quarterly Journal of Economics*, 123(4), 1415-1464.

[19] Rotemberg, J. and M. Woodford (1997): "An Optimization-Based Econometric Model for the Evaluation of Monetary Policy". *NBER Macroeconomics Annual*, 12, 297-346.

[20] Salop, S. (1979): "Monopolistic Competition with Outside Goods," *Bell Journal of Economics* 10: 141-156.

[21] Woodford, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

7 Appendix

7.1 The entrepreneur's consumption decision

Equations (8) and (9) in the text can be combined as follows,

$$\frac{p_t^h - \chi_t}{c_t^e} = \beta^e E_t \left\{ \frac{(1-\tau^e)\,\nu p_{t+1}^I y_{t+1}/h_t^e + p_{t+1}^h - R_t^L \chi_t/\pi_{t+1}}{c_{t+1}^e} \right\}, \tag{38}$$

where $\chi_t \equiv m_t E_t \pi_{t+1} p_{t+1}^h / R_t^L$. The latter definition allows us in turn to write the collateral constraint (equation 6) as

$$b_t = \chi_t h_t^e. \tag{39}$$

Define real net worth, nw_t, as the sum of after-tax real profits and beginning-of-period real estate wealth, minus real debt repayments,

$$\begin{aligned} nw_t &\equiv (1-\tau^e)\left(p_t^I y_t - w_t l_t^d\right) + p_t^h h_{t-1}^e - \frac{R_{t-1}^L}{\pi_t} b_{t-1} \\ &= (1-\tau^e)\,\nu p_t^I y_t + p_t^h h_{t-1}^e - \frac{R_{t-1}^L}{\pi_t}\chi_{t-1} h_{t-1}^e \\ &= \left[(1-\tau^e)\,\nu p_t^I y_t / h_{t-1}^e + p_t^h - \frac{R_{t-1}^L}{\pi_t}\chi_{t-1}\right] h_{t-1}^e, \end{aligned} \tag{40}$$

where in the first equality we have used (7) to substitute for $w_t l_t^d$ and (39) to substitute for b_{t-1}. We now guess that the entrepreneur consumes a fraction $1-\beta^e$ of her real net worth,

$$c_t^e = (1-\beta^e)\, nw_t. \tag{41}$$

Using (40) and (41) in equation (38), the latter collapses to

$$\frac{p_t^h - \chi_t}{c_t^e} = \frac{\beta^e}{1-\beta^e}\frac{1}{h_t^e}. \tag{42}$$

At the same time, the definition of real net worth and equation (39) allow us write the entrepreneur's budget constraint (equation 5) as

$$\chi_t h_t^e + nw_t = c_t^e + p_t^h h_t^e,$$

Combining the latter with equation (42), we finally obtain equation (41), which verifies our guess.

7.2 Implementation of the efficient steady state

Equations (1), (7) and (19) in the steady state jointly imply

$$c_{ss}\left(l_{ss}^{s}\right)^{\varphi}=\frac{1-\omega}{\omega}\left(1-\nu\right)p_{ss}^{I}\frac{y_{ss}^{s}}{l_{ss}^{s}}.$$

The latter corresponds to its efficient counterpart (the steady state of equation 23) only if $p_{ss}^{I}=1$. In the zero-inflation steady state, equation (15) becomes $1+\tau=\left[\varepsilon/\left(\varepsilon-1\right)\right]p_{ss}^{I}$, where τ is the subsidy rate on the revenue of final goods producers. Therefore, steady-state efficiency requires setting the subsidy rate to

$$\tau=\frac{\varepsilon}{\varepsilon-1}-1.$$

On the other hand, the steady-state counterpart of equation (10), rescaled by y_{ss}, is given by

$$\frac{c_{ss}^{e}}{y_{ss}}=\left(1-\beta^{e}\right)\left[\left(1-\tau^{e}\right)\nu+\left(1-m\right)\frac{p_{ss}^{h}h_{ss}^{e}}{y_{ss}}\right],\tag{43}$$

where we have imposed $p_{ss}^{I}=1$ and we have used the collateral constraint in the steady state, $R_{ss}^{L}b_{ss}=mp_{ss}^{h}h_{ss}^{e}$. Similarly, the steady-state counterparts of equations (9) and (8) jointly imply

$$\frac{p_{ss}^{h}h_{ss}^{e}}{y_{ss}}=\beta^{e}\left[\left(1-\tau^{e}\right)\nu+\frac{p_{ss}^{h}h_{ss}^{e}}{y_{ss}}\right]+\left(\frac{1}{R_{ss}^{L}}-\beta^{e}\right)m\frac{p_{ss}^{h}h_{ss}^{e}}{y_{ss}},$$

which implies the following steady-state ratio of entrepreneurial real estate wealth over output,

$$\frac{p_{ss}^{h}h_{ss}^{e}}{y_{ss}}=\frac{\beta^{e}\left(1-\tau^{e}\right)\nu}{1-\beta^{e}-m\left(1/R_{ss}^{L}-\beta^{e}\right)}.\tag{44}$$

Using (44) to substitute for $p_{ss}^{h}h_{ss}^{e}/y_{ss}^{s}$ in (43), and imposing the steady-state efficiency requirement that $c_{ss}^{e}/y_{ss}=1-\omega$ (as a result of $c_{ss}=c_{ss}^{e}$ and $\left[1-\omega\right]y_{ss}=\omega c_{ss}+\left[1-\omega\right]c_{ss}^{e}$), we can solve for the tax rate on profits that is consistent with an efficient allocation,

$$\tau^{e}=1-\frac{1-\omega}{\left(1-\beta^{e}\right)\nu}\frac{1-\beta^{e}-m\left(1/R_{ss}^{L}-\beta^{e}\right)}{1-m/R_{ss}^{L}}.$$

Finally, equation (3) implies that, in the steady state,

$$\frac{p_{ss}^{h}h_{ss}}{c_{ss}}=\frac{\vartheta}{1-\beta+\tau^{h}}.$$

Combining this with equation (44) and the efficiency requirement $c_{ss} = c^e_{ss} = (1-\omega)\, y_{ss}$, we have that the steady-state distribution of real estate in the decentralized economy is given by

$$\frac{h_{ss}}{h^e_{ss}} = \frac{(1-\omega)\,\vartheta}{\beta^e\,(1-\tau^e)\,\nu}\frac{1-\beta^e - m\left(1/R^L_{ss} - \beta^e\right)}{1-\beta+\tau^h}.$$

The latter coincides with the efficient steady-state distribution, $(1-\omega)\,\vartheta/\,(\beta\nu)$, only if

$$\tau^h = \frac{\beta}{\beta^e}\frac{1-\beta^e - m\left(1/R^L_{ss} - \beta^e\right)}{1-\tau^e} - (1-\beta)\,.$$

7.3 Derivation of the quadratic loss function

We start by performing a second order approximation (in logs) of the period utility function around the steady-state,

$$\begin{aligned} U_t &\equiv \omega\left[\log(c_t) + \vartheta_t \log(h_t) - \frac{\left(l^s_t\right)^{1+\varphi}}{1+\varphi}\right] + (1-\omega)\log(c^e_t) \\ &= \omega\hat{c}_t + (1-\omega)\,\hat{c}^e_t - \omega\left(l^s_{ss}\right)^{1+\varphi}\left(\hat{l}_t + \frac{1+\varphi}{2}\hat{l}^2_t\right) + \omega\vartheta\left(\hat{h}_t + z^h_t\hat{h}_t\right) + t.i.p. + O^3, \quad (45)\end{aligned}$$

where hats denote log-deviations from steady state, the subscript ss indicates steady state values, $t.i.p.$ are terms independent of policy and O^3 collects all terms of order third and higher in the size of the shocks.

The aggregate resource constraint in goods markets, $(1-\omega)\,y_t/\Delta_t = \omega c_t + (1-\omega)\,c^e_t$, can be approximated by

$$(1-\omega)\left(\hat{y}_t + \frac{1}{2}\hat{y}^2_t - \hat{\Delta}_t\right) = \omega\frac{c_{ss}}{y_{ss}}\left(\hat{c}_t + \frac{1}{2}\hat{c}^2_t\right) + (1-\omega)\,\frac{c^e_{ss}}{y_{ss}}\left(\hat{c}^e_t + \frac{1}{2}\left(\hat{c}^e_t\right)^2\right) + O^3, \quad (46)$$

where we have used the fact that $\hat{\Delta}_t$ is already a second-order term (see below). Equation (46) implies that

$$\hat{y}^2_t = \left(\frac{\omega}{1-\omega}\frac{c_{ss}}{y_{ss}}\right)^2\hat{c}^2_t + \left(\frac{c^e_{ss}}{y_{ss}}\right)^2\left(\hat{c}^e_t\right)^2 + 2\frac{\omega}{1-\omega}\frac{c_{ss}}{y_{ss}}\frac{c^e_{ss}}{y_{ss}}\hat{c}_t\hat{c}^e_t + O^3.$$

Using this to substitute for $\hat{y}_t^2$ in (46) and rearranging terms, we obtain

$$\begin{aligned}\hat{y}_t &= \frac{\omega}{1-\omega}\frac{c_{ss}}{y_{ss}}\hat{c}_t + \frac{c_{ss}^e}{y_{ss}}\hat{c}_t^e + \hat{\Delta}_t + O^3 \\ &+\frac{1}{2}\left[\frac{\omega}{1-\omega}\frac{c_{ss}}{y_{ss}}\left(1-\frac{\omega}{1-\omega}\frac{c_{ss}}{y_{ss}}\right)\hat{c}_t^2 + \frac{c_{ss}^e}{y_{ss}}\left(1-\frac{c_{ss}^e}{y_{ss}}\right)\left(\hat{c}_t^e\right)^2 - 2\frac{\omega}{1-\omega}\frac{c_{ss}}{y_{ss}}\frac{c_{ss}^e}{y_{ss}}\hat{c}_t\hat{c}_t^e\right]. \quad (47)\end{aligned}$$

We now make use of our assumption of efficient steady state. This implies $c_{ss} = c_{ss}^e = (1-\omega)\, y_{ss}$. Using this in (47) yields

$$\hat{y}_t = \omega\hat{c}_t + (1-\omega)\,\hat{c}_t^e + \hat{\Delta}_t + \frac{\omega(1-\omega)}{2}\left(\hat{c}_t - \hat{c}_t^e\right)^2 + O^3. \quad (48)$$

The production function, $y_t = e^{a_t}\left[\omega l_t^s/(1-\omega)\right]^{1-\nu}\left(h_{t-1}^e\right)^{\nu}$, admits the following exact log-linear representation,

$$\hat{y}_t = a_t + (1-\nu)\,\hat{l}_t^s + \nu\hat{h}_{t-1}^e. \quad (49)$$

Using (48) and (49) to substitute for $\omega\hat{c}_t + (1-\omega)\,\hat{c}_t^e$ and $\hat{l}_t^s$ respectively in (45), we obtain

$$\begin{aligned}U_t &= \hat{y}_t - \hat{\Delta}_t - \omega\left(l_{ss}^s\right)^{1+\varphi}\left[\frac{\hat{y}_t - \nu\hat{h}_{t-1}^e}{1-\nu} + \frac{1+\varphi}{2}\left(\frac{\hat{y}_t - a_t - \nu\hat{h}_{t-1}^e}{1-\nu}\right)^2\right] \\ &\quad -\frac{\omega(1-\omega)}{2}\left(\hat{c}_t - \hat{c}_t^e\right)^2 + \omega\vartheta\left(\hat{h}_t + z_t^h\hat{h}_t\right) + t.i.p. + O^3. \quad (50)\end{aligned}$$

In an efficient steady state, labor market equilibrium implies $c_{ss}\left(l_{ss}^s\right)^{\varphi} = (1-\nu)\, y_{ss}^s/\left(\frac{\omega}{1-\omega}l_{ss}^s\right)$, which combined with $c_{ss} = (1-\omega)\, y_{ss}$ implies $\omega\left(l_{ss}^s\right)^{1+\varphi} = 1-\nu$. Using this in (50), we have

$$U_t = \nu\hat{h}_{t-1}^e + \omega\vartheta\left(\hat{h}_t + z_t^h\hat{h}_t\right) - \frac{1+\varphi}{2(1-\nu)}\left(\hat{y}_t - \hat{y}_t^*\right)^2 - \frac{\omega(1-\omega)}{2}\left(\hat{c}_t - \hat{c}_t^e\right)^2 - \hat{\Delta}_t + t.i.p. + O^3, \quad (51)$$

where we have used the definition of efficient output (in log-deviations), $\hat{y}_t^* \equiv a_t + \nu\hat{h}_{t-1}^e$. Taking the present discounted sum of (51), we have

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t U_t &= -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\frac{1+\varphi}{1-\nu}\left(\hat{y}_t - \hat{y}_t^*\right)^2 + \omega(1-\omega)\left(\hat{c}_t - \hat{c}_t^e\right)^2\right] \\ &\quad + \sum_{t=0}^{\infty}\beta^t\left[\beta\nu\hat{h}_t^e + \omega\vartheta\left(\hat{h}_t + z_t^h\hat{h}_t\right)\right] - \sum_{t=0}^{\infty}\beta^t\hat{\Delta}_t + t.i.p. + O^3, \quad (52)\end{aligned}$$

where we have used the fact that $\hat{h}_{-1}^e$ and $\hat{\Delta}_{-1}$ are independent of policy as of time 0. The equilibrium condition in the real estate market, $\bar{h} = \omega h_t + (1-\omega) h_t^e$, can be approximated as follows,

$$\omega h_{ss}\left(\hat{h}_t + \frac{\hat{h}_t^2}{2}\right) + (1-\omega) h_{ss}^e\left(\hat{h}_t^e + \frac{\left(\hat{h}_t^e\right)^2}{2}\right) = O^3. \tag{53}$$

The latter equation implies that $\left(\hat{h}_t^e\right)^2 = [\omega/(1-\omega)]^2 (h_{ss}/h_{ss}^e)^2 \hat{h}_t^2 + O^3$. Using this and the efficient distribution of real estate in the steady state, $h_{ss}/h_{ss}^e = (1-\omega)\vartheta/(\beta\nu)$, equation (53) becomes

$$\omega\vartheta\hat{h}_t + \beta\nu\hat{h}_t^e = -\frac{\omega\vartheta}{2}\frac{\beta\nu+\omega\vartheta}{\beta\nu}\hat{h}_t^2 + O^3.$$

This implies

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\left[\beta\nu\hat{h}_t^e + \omega\vartheta\left(\hat{h}_t + z_t^h\hat{h}_t\right)\right] &= -\frac{\omega\vartheta}{2}\sum_{t=0}^{\infty}\beta^t\left(\frac{\beta\nu+\omega\vartheta}{\beta\nu}\hat{h}_t^2 - 2z_t^h\hat{h}_t\right) + O^3 \\ &= -\frac{\omega\vartheta}{2}\frac{\beta\nu+\omega\vartheta}{\beta\nu}\sum_{t=0}^{\infty}\beta^t\left(\hat{h}_t - \hat{h}_t^*\right)^2 + t.i.p. + O^3, \end{aligned} \tag{54}$$

where in the second equality we have used the definition of the efficient level of housing, $\hat{h}_t^* \equiv [\beta\nu/(\omega\vartheta+\beta\nu)]\, z_t^h$.

It is possible to show (see e.g. Woodford, 2003) that

$$\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_t = \frac{\varepsilon}{2}\frac{\theta}{(1-\theta)(1-\beta\theta)}\sum_{t=0}^{\infty}\beta^t\hat{\pi}_t^2 + t.i.p. + O^3. \tag{55}$$

Using (54) and (55) in (52), we finally obtain

$$\sum_{t=0}^{\infty}\beta^t U_t = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t L_t + t.i.p. + O^3, \tag{56}$$

where

$$L_t = \frac{1+\varphi}{1-\nu}\left(\hat{y}_t - \hat{y}_t^*\right)^2 + \omega(1-\omega)\left(\hat{c}_t - \hat{c}_t^e\right)^2 + \omega\vartheta\frac{\beta\nu+\omega\vartheta}{\beta\nu}\left(\hat{h}_t - \hat{h}_t^*\right)^2 + \frac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)}\hat{\pi}_t^2.$$

QED.

7.4 Log-linear equations

All variables in log-deviations from the steady state. The log-linear constraints of the central bank's problem are the following.

1. Household's consumption Euler equation,

$$\hat{c}_t = E_t \hat{c}_{t+1} - E_t \left(\hat{R}_t^D - \hat{\pi}_{t+1} \right).$$

2. Household's demand for housing,

$$\left(1 + \tau^h\right) \left(\hat{p}_t^h - \hat{c}_t\right) = \left(1 + \tau^h - \beta\right) \left(z_t^h - \hat{h}_t\right) + \beta E_t \left(\hat{p}_{t+1}^h - \hat{c}_{t+1}\right).$$

3. Entrepreneur's borrowing constraint,

$$\hat{b}_t = z_t^m + E_t \hat{p}_{t+1}^h + \hat{h}_t^e - \left(\hat{R}_t^L - E_t \hat{\pi}_{t+1} \right).$$

4. Entrepreneur's consumption Euler equation,

$$\hat{c}_t^e = \beta^e R_{ss}^L E_t \left(\hat{c}_{t+1}^e - \hat{R}_t^L + \hat{\pi}_{t+1} \right) - \left(1 - \beta^e R_{ss}^L\right) \hat{\xi}_t.$$

5. Entrepreneur's demand for real estate,

$$\begin{aligned} \hat{p}_t^h - \hat{c}_t^e &= \beta^e E_t \left\{ \frac{(1-\tau^e)\,\nu}{s_h^e} \left(\hat{y}_{t+1} + \hat{p}_{t+1}^I - \hat{h}_t^e \right) + \hat{p}_{t+1}^h - \left[\frac{(1-\tau^e)\,\nu}{s_h^e} + 1 \right] \hat{c}_{t+1}^e \right\} \\ &\quad + m \left(\frac{1}{R_{ss}^L} - \beta^e \right) \left[z_t^m + \hat{\xi}_t + E_t \hat{p}_{t+1}^h - \left(\hat{R}_t^L - E_t \hat{\pi}_{t+1} \right) \right], \end{aligned}$$

where $s_h^e \equiv p_{ss}^h h_{ss}^e / y_{ss}$.

6. Entrepreneur consumption,

$$\hat{c}_t^e = \frac{1-\beta^e}{1-\omega} \left[(1-\tau^e)\,\nu \left(\hat{y}_t + \hat{p}_t^I \right) + s_h^e \left(\hat{p}_t^h + \hat{h}_{t-1}^e \right) - s_h^e m \left(\hat{R}_{t-1}^L + \hat{b}_{t-1} - \hat{\pi}_t \right) \right].$$

7. Bank lending margin,

$$\hat{R}_t^L = \hat{R}_t^D$$

$$+\frac{\beta R_{ss}^L - 1}{\beta R_{ss}^L}\left[\frac{\hat{R}_t^D + \hat{p}_t^h - m\beta E_t\left(\hat{\pi}_{t+1} + \hat{p}_{t+1}^h + z_t^m\right)}{1 - m\beta} - \frac{\eta m\beta E_t\left(\hat{\pi}_{t+1} + \hat{p}_{t+1}^h + z_t^m\right) - \left(\hat{R}_t^D + \hat{p}_t^h\right)}{\eta m\beta - 1}\right].$$

8. New Keynesian Phillips curve,

$$\hat{\pi}_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{p}_t^I + \beta E_t\hat{\pi}_{t+1}.$$

9. Real marginal costs,

$$\hat{p}_t^I = \hat{c}_t - \hat{y}_t + \frac{1+\varphi}{1-\nu}\left(\hat{y}_t - a_t - \nu\hat{h}_{t-1}^e\right).$$

10. Equilibrium in goods markets,

$$\hat{y}_t = \omega\hat{c}_t + (1-\omega)\,\hat{c}_t^e.$$

11. Equilibrium in the real estate market,

$$\hat{h}_t = -\frac{\beta\nu}{\omega\vartheta}\hat{h}_t^e.$$

Figure 1: Impulse-responses to a negative productivity shock (%)

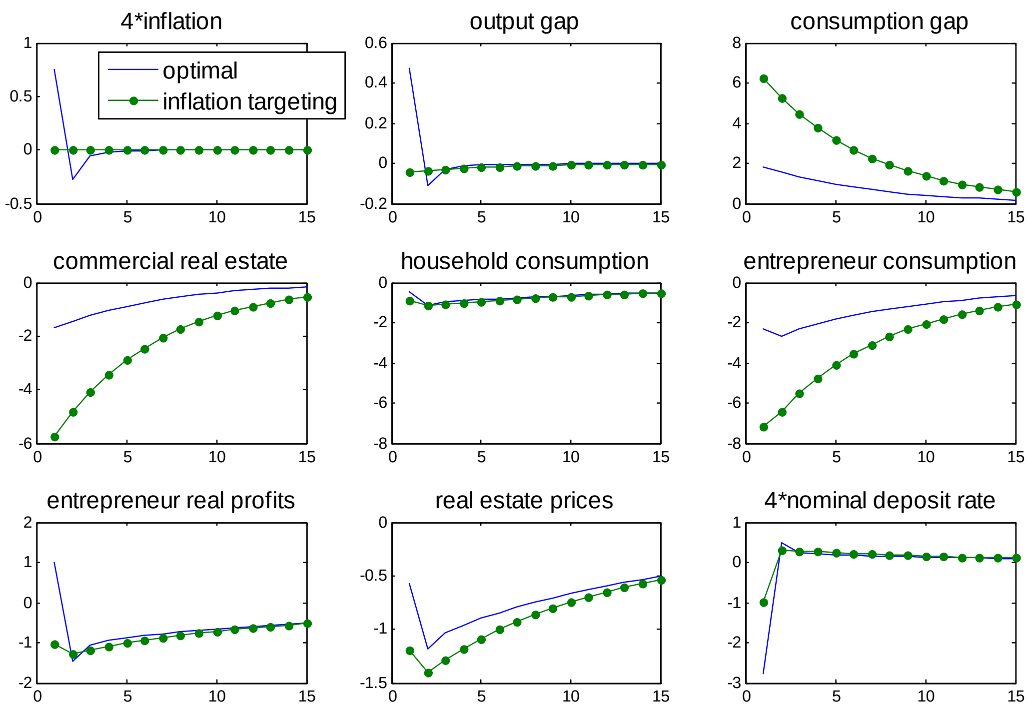

Figure 2: Impulse-responses to a credit-crunch shock (%)