Equilibrium Business Cycles, Equity Prices, and the Term Structure of the Interest Rate

Christopher Gust   David López-Salido

Federal Reserve Board

May 25, 2010

Motivation

- Evidence that business cycles and asset prices linked.
- Understanding the link between asset prices and business cycles has proven to be a difficult task.
- Lots of progress has been made though (e.g., Campbell and Cochrane (1999), Boldrin, Christiano, and Fisher (2001), and Guvenen (2009).)
- We examine the ability of fully-specified DSGE model that incorporates market segmentation between asset and goods markets to explain business cycles, equity prices, and the term structure of interest rates.

Key Model Feature: Infrequent Portfolio Rebalancing

- Embed market segmentation into an otherwise standard business cycle model with isoelastic preferences and endogenous capital and labor supply.
- At date $t = 0$ a household sets up a non-state contingent plan allocating funds between a checking and a brokerage account.
- At $t \geq 1$, HH can alter this plan by making state-contingent transfers. To do so, HH pays a fixed cost, γ (only source of HH heterogeneity).
- Non-state contingent plan intended to capture predetermined, automatic transfers set up between accounts (401K, mutual funds, annuities).

Recent Evidence on Household's Finance

- Infrequent portfolio rebalancing, with substantial heterogeneity.
- Some households rebalance very frequently (i.e. active HHs).
- A significant fraction remains inactive for a long period of time.
- Growing literature: Bilias et al. (2005), Brunnermeier and Nagel (2008), Calvet, Campbell, and Sodini (2008, 2009), and Bachetta and van Wincoop (2009), Bonaparte and Cooper (2009).

Preview of the Results

- Benchmark model does a good job in quantitatively accounting for prominent business cycle and asset pricing facts.
- Market segmentation by itself is not sufficient.
  - The form of segmentation matters: The non-state contingent plan is crucial in accounting for the average equity premium and risk free rate. Intuition: By providing insurance to inactive households, this plan raises the amount of risk borne by active households.
  - Imperfect substitutability between installed capital and investment is also necessary.
  - Systematic monetary policy is important in generating an upward sloping yield curve.

Firms

- Technology:

$$
\begin{aligned}
Y(\theta^t) &= K(\theta^{t-1})^{\alpha} \left[\exp(\theta_t + \eta t) L(\theta^t)\right]^{1-\alpha} \\
K(\theta^t) &= \left\{ a_1 \left[(1-\delta) K(\theta^{t-1})\right]^{\psi} + a_2 I(\theta^t)^{\psi} \right\}^{\frac{1}{\psi}}
\end{aligned}
$$

- In equilibrium, price of a new unit of capital is given by:

$$
p_K(\theta^t) = \frac{1}{a_2} \left(\frac{I(\theta^t)}{K(\theta^t)} \right)^{1-\psi}
$$

Households: Brokerage Account

- In this account, HHs trade all financial securities including equity, real and nominal zero-coupon bonds.
- At date $t = 0$, HH sets up a non-state contingent transfer plan to checking account for dates $t \geq 1$.

$$\overline{B}(\gamma) = \int_{\theta_1} q(\theta^1) B(\theta^1) d\theta_1$$

- Household cash constraint in asset markets at dates $t \geq 1$:

$$B(\theta^t, \gamma) = \int_{\theta_{t+1}} q(\theta^t, \theta_{t+1}) B(\theta^t, \theta_{t+1}, \gamma) d\theta_{t+1} + \exp(\eta t) A(\theta_0, \gamma) +$$
$$[x(\theta^t, \gamma) + \exp(\eta t)\gamma] z(\theta^t, \gamma)$$

- HHs pay γ of making state contingent transfer, $x(\theta^t, \gamma)$, between brokerage and checking accounts. If HHs pay γ, then $z(\theta^t, \gamma) = 1$.

Checking Account, Preferences, and Labor Supply

- Checking Account

$$c(\theta^t,\gamma) = w(\theta^t)L(\theta^t,\gamma) + x(\theta^t,\gamma)z(\theta^t,\gamma) + \exp(\eta t)A(\theta_0,\gamma), \ \ \forall\gamma$$

- Preferences

$$U(c_t, L_t) = \frac{v(c_t, L_t)^{1-\sigma}}{1-\sigma}, \qquad v(c_t, L_t) = c_t - \exp(\eta t)\chi_0 \frac{L_t^{1+\chi}}{1+\chi}$$

- Each household has the same labor supply function:

$$w(\theta^t) = \exp(\eta t)\chi_0 L(\theta^t,\gamma)^{\chi}, \longrightarrow\longrightarrow\longrightarrow L(\theta^t,\gamma) = L(\theta^t), \ \forall\gamma$$

Equilibrium Allocations

- Consumption of *inactive rebalancers*:

$$c_I(\theta^t, \gamma) = w(\theta^t)L(\theta^t, \gamma) + \exp(\eta t)A(\theta_0, \gamma).$$

- Complete risk-sharing among *active rebalancers*:

$$c_A(\theta^t, \gamma) = c_A(\theta^t), \quad \forall\, \gamma$$

- Initial transfer plan provides *insurance* for *inactive* types:

$$\sum_{t=1}^{\infty} \int_{\theta^t} \beta^t \left[U_c^A\left(\theta^t\right) - U_c^I\left(\theta^t, \gamma\right) \right] \left(1 - z(\theta^t, \gamma)\right) g(\theta^t) d\theta^t = 0$$

- If no access to transfer plan (i.e. $A(\gamma) = 0, \forall \gamma$), this is similar to the endogenous segmentation models of AAK (2002, 2008) and Khan and Thomas (2008).

The Marginal Rebalancer

- The marginal rebalancer is determined by:

$$U^A(\theta^t)\text{-}U^I(\theta^t) = U_c^A(\theta^t)\left[c_A(\theta^t)\text{-}c_I(\theta^t,\bar{\gamma}(\theta^t))+\bar{\gamma}(\theta^t)\right],$$

- Implies state-dependent cutoff rule in which households with $\gamma \leq \bar{\gamma}(\theta^t)$ are active rebalancers ($z(\theta^t,\gamma) = 1$).

- Aggregate resource constraint:

$$Y(\theta^t) = c(\theta^t) + I(\theta^t) + \exp(\eta t)\int_0^\infty \gamma z(\theta^t,\gamma) f(\gamma) d\gamma,$$

Equity Return and the Risk-Free Rate

- Pricing kernel depends on the consumption of rebalancers:

$$m(\theta^t, \theta_{t+1}) = \beta \left[\frac{v_A(\theta^t)}{v_A(\theta^{t+1})} \right]^{\sigma}.$$

- Risk-free rate:

$$[1 + r_t^f]^{-1} = \mathrm{E}_t m_{t,t+1}.$$

- Return on equity:

$$(1 + r^e(\theta^{t+1})) = \frac{\alpha \frac{Y(\theta^{t+1})}{K(\theta^t)} + (1-\delta) a_1 \left(\frac{K(\theta^{t+1})}{(1-\delta)K(\theta^t)} \right)^{1-\psi} p_K(\theta^{t+1})}{p_K(\theta^t)}$$

- The equity premium:

$$1 + r_t^{ep} = \frac{\mathrm{E}_t[1 + r_{t+1}^e]}{1 + r_t^f} = 1 - \mathrm{cov}_t \left(\beta \left[\frac{v_{A,t}}{v_{A,t+1}} \right]^{\sigma}, 1 + r_{t+1}^e \right).$$

Benchmark Calibration

- We set $\sigma = 6$, $\alpha = 0.36$, $\delta = 0.02$.
- Technology shock θ_t ($\rho_\theta = 0.98$). We set $\eta = 0.0037$, implying the economy grows at an annualized rate near 1.5%.
- $F(\gamma)$ is uniformly distributed so that, on average, about 6 percent of households rebalance their portfolios in a quarter with some households rebalancing frequently and a large mass of households rarely rebalancing.
- Elasticity of the investment to capital ratio with respect to Tobin's 'Q', $\frac{1}{1-\psi} = 0.4$.

Table 1. Equity Premium and the Risk-Free Rate

	US Data		Rep. Agent	No HH Plans	Benchmark Model
$E(r^e - r^f)$	6.17	(1.99)	2.17	1.93	6.01
$\sigma(r^e - r^f)$	19.4	(1.41)	17.47	16.50	22.95
$\frac{E(r^e\text{-}r^f)}{\sigma(r^e\text{-}r^f)}$	0.32	(0.11)	0.12	0.12	0.26
$E(r^f)$	1.94	(0.54)	7.66	7.79	2.77
$\sigma(r^f)$	5.44	(0.62)	1.91	1.82	2.17
$\sigma(\Delta c_A)/\sigma(\Delta c)$			1	0.95	1.8
$E(\Delta c_A)/E(\Delta c)$			1	1.8	1.12
$E(F(\gamma))$			100	6.68	5.92
$\sigma(F(\gamma))$			0	0.03	0.03
Avg. Cost (%Y)			0	12.52	0.05

Table 2. The Determinants of Risk-Free Rate

$$E_t(r_t^f) \approx -\log(\beta) + \sigma E_t(\Delta v_{t+1}^A) - \frac{\sigma^2}{2}\mathrm{var}_t(\Delta v_{t+1}^A)$$

	Mean Risk-Free Rate	
	Rep. Agent	Benchmark
1. $-\log(\beta)$	0.40	0.40
2. Intertemporal Substitution	8.95	8.95
3. Precautionary Savings	1.69	6.58
4. Average Real Rate (1+2-3)	7.66	2.77
	Risk-Free Rate Volatility	
	Rep. Agent	Benchmark
1. Intertemporal Substitution	4.48	12.98
2. Precautionary Savings	0.03	7.44
3. Covariance Term	0.87	15.70
4. Risk-Free Rate Variance (1+2-3)	3.64	4.72
5. Risk-Free Rate Std. Dev.	1.91	2.17

Table 3. Long Run Regressions on Price Dividends

$$\log(ER_{t\rightarrow t+k}) = a + \beta \ \log(\frac{P}{D})_t$$

	US Data		Benchmark Model	
Horizon (Years)	β	R^2	β	R^2
1	-0.22	0.09	-0.14	0.06
2	-0.39	0.14	-0.27	0.11
3	-0.47	0.15	-0.40	0.16
5	-0.77	0.26	-0.64	0.24

Impulse Responses to a Positive Technology Shock

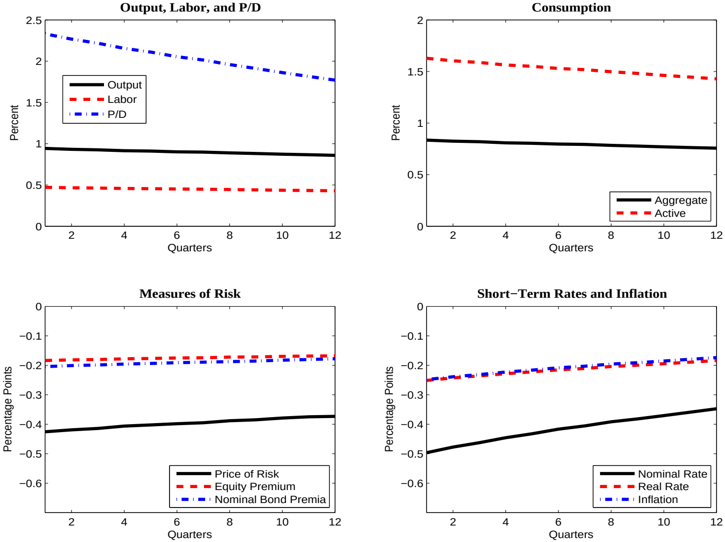

Table 4. Business Cycle

	US Data		Rep. Agent	No HH Plans ($A(\gamma) = 0$)	Benchmark Model
$\sigma(y)$	1.9	(0.20)	1.99	2.00	2.03
$\frac{\sigma(c)}{\sigma(y)}$	0.5	(0.04)	0.97	0.99	0.87
$\frac{\sigma(i)}{\sigma(y)}$	2.4	(0.06)	1.12	1.06	1.46
$\frac{\sigma(n)}{\sigma(y)}$	0.8	(0.05)	0.5	0.5	0.5
$\frac{\sigma(p_k)}{\sigma(y)}$	4.5	(0.68)	2.82	2.67	3.67

Monetary Policy and Nominal Yield Curve

- Compare the effects of a constant money growth rule and an interest rate rule (in logs):

$$R^1(\theta^t) = \omega + \omega_\pi \pi(\theta^t)$$

- Average return on nominal debt:

$$E\left(R_t^\tau\right) \approx r^f + E\pi_t - \frac{\sigma^2}{2\tau} var\left(\sum_{j=1}^{\tau} \Delta v_{t+j}^A\right) - \frac{1}{2\tau} var\left(\sum_{j=1}^{\tau} \pi_{t+j}\right) - \frac{\sigma}{\tau} cov\left(\sum_{j=1}^{\tau} \Delta v_{t+j}^A, \sum_{j=1}^{\tau} \pi_{t+j}\right)$$

Table 7. Determinants of Average Nominal Rates

Contribution of:	Quarterly Rate: Benchmark Model	Quarterly Rate: Const. Money Growth	5-Year Rate: Benchmark Model	5-Year Rate: Const. Money Growth
1. $-\log(\beta)$	0.40	0.40	0.40	0.40
2. Int. Substitution	8.95	8.95	8.95	8.95
3. Prec. Savings	6.58	6.58	5.53	5.53
4. Real Rate (1+2-3)	2.77	2.77	3.82	3.82
5. Inflation Rate	2.95	2.95	2.95	2.95
6. Inflation Risk	0.16	0.86	0.91	0.65
Nom. Rate (4+5-6)	5.88	6.58	7.68	7.42

Table 8. Fama-Bliss Excess Return Regressions

$$ER_{t+1}^{(\tau)} = a + \beta(f_t^{(\tau)} - y_t^{(1)})$$

	US Data		Benchmark Model	
Horizon (Years)	β	R^2	β	R^2
2	0.99	0.16	1.26	0.15
3	1.35	0.17	1.40	0.28
4	1.61	0.18	1.31	0.42
5	1.27	0.09	1.23	0.56

Conclusions

- The Neoclassical model with market segmentation is a good starting place for understanding the relationship between business cycles and asset prices.
- We're currently working on the last section of the paper. Extensions:
  - Alternative Timing for wage income
  - Alternative Preferences