Equilibrium Business Cycles, Equity Prices, and the Term Structure of Interest Rates*

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Abstract

We show that a fruitful way to account for prominent business cycle and asset pricing facts consists of introducing segmented markets into an otherwise standard business-cycle model with isoelastic preferences and elastic capital and labor supply. The key feature of our model is that households rebalance their financial portfolio allocations infrequently, as they face a fixed cost of transferring cash across accounts. The model generates reasonable business cycle properties and is consistent with the mean and volatility of the equity premium and risk-free rate, a nominal yield curve that is upward sloping on average, and the high volatility and predictability of excess returns on equity and bonds. In this framework, long-term nominal debt is riskier than short-term debt, because the systematic response of monetary policy to current inflation induces long-run inflation risk, contributing to high and volatile term premia.

*The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

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1 Introduction

Evidence from the macrofinance literature suggests the need for a mutual explanation of asset markets and business cycles. Both risk premia on equity and bonds move countercyclically, and as a result excess returns on these assets over Treasury bills are predicted by variables that are correlated with the business cycle. Furthermore, there is evidence that equity returns and yield spreads have enjoyed some success in predicting business cycles.

Understanding the relationship between asset markets and business cycles has proven to be a difficult task. While recent research using equilibrium business cycle models has made considerable progress in overcoming some of the pitfalls associated with these models, it remains a major challenge to jointly account for business cycle fluctuations, equity, and bond prices. Several facts that have proven especially difficult for these models to explain include the mean and volatility of the equity premium and risk-free rate, a nominal yield curve that is upward sloping on average, and excess returns for both bonds and equity that are both highly volatile and predictable. Ideally, a model that explains these facts should generate reasonable business cycle properties.

In this paper, we show that a fruitful way to explain these facts consists of introducing segmented markets into an otherwise standard business-cycle model with isoelastic preferences and endogenous capital and labor supply. Market segmentation arises in the model, because it is costly for households to transfer funds between these markets. Accordingly, households may only infrequently update their desired allocation of cash between a checking account devoted to purchasing goods and a brokerage account used for financial transactions. The optimal decision by an individual household to rebalance their cash holdings is a state-dependent one, reflecting that doing so involves paying a fixed cost in the presence of uncertainty. Households are heterogenous in this fixed cost, and only those households that rebalance their portfolios during the current period matter for determining asset prices.

Our model is able to match the observed means and volatilities on equity and interest rates with a power utility function that implies constant relative risk aversion. The model generates an upward sloping average yield curve, and both bond and equity premia move countercyclically, helping the model explain the predictability of excess stock and bond returns. We show that to match these moments, the average fraction of households that reallocates funds across markets can not be too large. For the benchmark calibration, less than 10 percent of households, on average, rebalance their portfolios in a quarter. Underlying this average fraction of rebalancing, there is a considerable degree of heterogeneity, with some households rebalancing every period and another fraction rarely rebalancing away from their initial allocation.

Recent microdata on household finance provides strong support for infrequent portfolio rebalancing. For instance, in two recent papers, Calvet, Campbell, and Sodini (2009a) and Calvet, Campbell, and Sodini (2009b) document that, while there is little rebalancing of the financial portfolios of stockholders by the average household, there is a great deal of heterogeneity at the micro level with some households rebalancing these portfolios very frequently. In addition, using information on asset holdings from the PSID, Bilias, Georgarakos, and Haliassos (2008) and Brunnermeier and Nagel (2008) provide evidence that household portfolio allocation display substantial inertia. Surveys conducted by the Investment Company Institute (ICI) and the Securities Industry Association (SIA) also suggest that households rebalance their portfolios infrequently. For instance, in 2004, the median number of total equity transactions for an individual was four. In addition, sixty percent of equity investors did not conduct any equity transactions during 2004. Finally, in a 2005 survey, the ICI reports that more than two-thirds of the time the proceeds from the sales of stocks by households are fully reinvested.[1]

While the segmentation of asset and goods is an important ingredient in the success of the model, it is not sufficient. We demonstrate that the model's ability to explain prominent asset market fact crucially hinges on how markets are segmented. In particular, the benchmark version of the model allows all households to set up an initial non-state contingent plan that transfers a predetermined amount of funds across their brokerage and checking accounts at future dates. At future dates, households can alter this financial plan by making state-contingent transfers only by incurring their fixed costs. Without this plan, the model is similar to the segmented market models of Alvarez, Atkeson, and Kehoe (2009) and Khan and Thomas (2007), as households

[1] See, Figure F.7, "Disposition of Proceeds from most recent sale of individual stocks", in ICI 2005.

that do not rebalance simply consume out of their wage income. In this case, aggregate shocks lead to relatively high consumption volatility of these households, lowering the consumption volatility of the households that rebalance. The non-state contingent plan provides a means for the households that do not rebalance to smooth their consumption, driving up the volatility of consumption of the households that rebalance. Because the consumption of the latter households matters for asset pricing, they demand higher compensation to hold risky assets. Without this financial plan, aggregate risk is more evenly spread out across households, resulting in smaller average equity and bond premia. Thus, the financial plan by providing insurance to some households raises the amount of risk borne by households that actively rebalance. We show that this mechanism is crucial in quantitatively accounting for prominent asset pricing facts.

Another important ingredient to the success of the model is imperfect substitutability between investment and installed capital. As emphasize in Jermann (1997) and Boldrin, Christiano, and Fisher (2001), this feature reduces the elasticity of capital supply, inhibiting the ability of households that actively rebalance to smooth their consumption. With this property and market segmentation, the model generates reasonable asset pricing implications and consistent and performs as well as a standard model without segmentation in matching key business cycle statistics.

A third ingredient that is important in explaining movements in the nominal yield curve is the systematic component of monetary policy. Nominal bonds are risky in the model, because their real payoffs decline in bad times. In response to a negative supply shocks, inflation increases while the consumption of households that rebalance falls, and thus these households demand higher yields to compensate for holding nominal bonds. Long-term nominal debt is riskier than short-term debt, because the systematic response of monetary policy to current inflation causes long-run inflation risk. By responding to short-run inflation, monetary policy induces a highly persistent response of inflation to technology shocks. This persistence is directly related to the inertia in the real interest rate coming from the precautionary savings of the households that actively rebalance. Under an alternative policy in which money growth is kept constant, there would not be as much long-run inflation risk, because under such a rule the effect of real shocks on inflation is transient; and, as a result, inflation risk does not contribute to an upward sloping average yield curve.[2]

The rest of this paper proceeds as follows. The next section describes the model and its calibration. Section 3 discusses the results in detail. Section 4 presents several extensions of the model, and Section 5 reviews the related literature. Section 6 concludes.

2 The Model

The economy is populated by infinitely-lived households and firms as well as a government sector. Firms own the economy's capital stock which they augment through investment, while households endogenously supply their labor to the economy's firms. Asset markets are segmented from goods markets, since households face fixed costs of transferring funds from a checking account used to purchase goods and a brokerage account used to purchase financial securities. The only source of uncertainty in our economy are aggregate shocks to production, θ_t. We let θ_t index the aggregate event in period t with θ_0 given, and let $\theta^t = (\theta_1, ..., \theta_t)$ denote the state, which consists of the aggregate shocks that have occurred through period t.[3]

2.1 Firms

There is a large number of perfectly competitive firms, which each have access to the following technology for converting capital, $K(\theta^{t-1})$, and labor, $L(\theta^t)$, into output, $Y(\theta^t)$ at dates $t \geq 1$:

$$Y(\theta^t) = K(\theta^{t-1})^\alpha \left[\exp(\theta_t + \eta t) L(\theta^t)\right]^{1-\alpha}. \tag{1}$$

The variable η determines the economy's growth rate and we assume that θ_t follows a first-order Markov process.

[2] This implication of the model is consistent with the evidence discussed in Atkeson and Kehoe (2008) and Cochrane (2008), who emphasize how changes in systematic monetary policy have been an important determinant of the nominal yield curve.

[3] Although through the main bulk of the paper we focus on technology shocks, later on we will introduce alternative exogenous sources of business cycle fluctuations.

As in Christiano and Fisher (1995), we assume that the technology for producing end of period t capital, $K(\theta^t)$ is given by:

$$K(\theta^t) = \left\{ a_1 \left[(1-\delta) K(\theta^{t-1}) \right]^{\psi} + a_2 I(\theta^t)^{\psi} \right\}^{\frac{1}{\psi}}, \tag{2}$$

where $I(\theta^t)$ denotes investment, $a_1 = ((1-\delta)\exp(-\eta))^{1-\psi}$, and $a_2 = (1-(1-\delta)\exp(-\eta))^{1-\psi}$.

The parameter ψ plays an important role in the analysis as $\frac{1}{1-\psi}$ determines the capital supply elasticity. With $\psi < 1$, a large increase in investment will be less effective in augmenting the capital stock than a small increase, since in that case the marginal product of investment is a decreasing function of investment. With $\psi = 1$, the marginal product of investment is constant, investment and previously installed capital are perfect substitutes, and this CES function is the conventional linear technology for augmenting capital. In this case, there is no difference between the price of newly installed capital and investment, and as a result, there will be no variation in Tobin's 'Q'. In contrast, with $\psi < 1$, the price of newly installed capital differs from the price of investment and there can be variation in Tobin's 'Q', where $\frac{1}{1-\psi}$ corresponds to the elasticity of Tobin's 'Q' with respect to the investment-capital ratio.

At each date, a firm hires workers, $L(\theta^t)$, and purchases investment goods. It pays for these expenditures using the proceeds from selling its good and its profits or dividends at date t are:

$$D(\theta^t) = Y(\theta^t) - w(\theta^t)L(\theta^t) - I(\theta^t), \tag{3}$$

where $w(\theta^t)$ denotes the real wage.

A firm chooses $\{K(\theta^t), L(\theta^t), I(\theta^t), Y(\theta^t)\}_{t=1}^{\infty}$ to maximize its discounted stream of profits:

$$\sum_{t=1}^{\infty} \int_{\theta^t} Q(\theta^t) D(\theta^t) d\theta^t, \tag{4}$$

subject to equations (1)-(3) taking real wages, the level of technology and $K(\theta_0)$ as given. A firm also takes its stochastic discount factor, $Q(\theta^t)$, as given, and later a firm's discount factor is related to the marginal rate of substitution of its owners.

2.2 Households

There are a large number of households indexed by γ. At date 0, a household sets up a plan allocating funds from a brokerage account to a checking account. This plan specifies a non-state contingent path of funds that will be transferred between these two accounts at future dates, $t \geq 1$. A household can deviate from this plan at $t \geq 1$ and make a state contingent transfer by incurring a fixed cost, γ. This fixed cost is constant over time but differs across households according to the probability density function $f(\gamma)$.

Brokerage Account. In her brokerage account, a household trades a complete set of one-period contingent claims issued by the government, shares in the economy's firms, and real and nominal zero-coupon government bonds issued. Let $q(\theta^t)$ denote the price of one-period contingent claims, then arbitrage between the shares in the firms and the one-period contingent claims implies:

$$1 = \int_{\theta_{t+1}} q(\theta^t, \theta_{t+1}) \left[1 + r^e(\theta^t, \theta_{t+1})\right] d\theta_{t+1}, \tag{5}$$

where $1 + r^e(\theta^{t+1})$ denotes the return on equity for $t > 0$. Similarly, the price of a zero-coupon real bond for maturity $\tau > 1$ can be determined from:

$$p_\tau(\theta^t) = \int_{\theta_{t+1}} q(\theta^t, \theta_{t+1}) p_{\tau-1}(\theta^{t+1}) d\theta_{t+1}, \tag{6}$$

and the risk-free rate, $r^f(\theta^t)$, is given by:

$$p_1(\theta^t) = \int_{\theta_{t+1}} q(\theta^t, \theta_{t+1}) d\theta_{t+1} = (1 + r^f(\theta^t))^{-1}. \tag{7}$$

The price of a zero-coupon nominal bond for maturity $\tau > 1$ can be determined from:

$$P_\tau(\theta^t) = \int_{\theta_{t+1}} \frac{q(\theta^t, \theta_{t+1})}{1 + \pi(\theta^{t+1})} P_{\tau-1}(\theta^{t+1}) d\theta_{t+1}, \tag{8}$$

where $\pi(\theta^t)$ denotes the economy's inflation rate at date t. For $\tau = 1$, it is convenient to define the one-period nominal interest rate as:

$$P_1(\theta^t) = \int_{\theta_{t+1}} \frac{q(\theta^t, \theta_{t+1})}{1 + \pi(\theta^{t+1})} d\theta_{t+1} = (1 + R^1(\theta^t))^{-1}. \tag{9}$$

Given these arbitrage conditions, the flow of funds in a household's brokerage account can be written using only the one period contingent claims without loss of generality. Arbitrage also implies that a firm's stochastic discount factor satisfies $Q(\theta^t) = \prod_{j=1}^{t} q(\theta^t)$.

At date 0, a household engages in an initial round of trade in the asset markets after learning her type and planning her non-state contingent transfers between accounts. The flow of funds in a household's brokerage account at date 0 is given by:

$$\overline{B}(\gamma) = \int_{\theta_1} q(\theta^1)B(\theta^1)d\theta_1, \tag{10}$$

where $\overline{B}(\gamma)$ denotes the initial asset position of household γ.

For dates $t \geq 1$, a household's brokerage account evolves according to:

$$\begin{aligned} B(\theta^t, \gamma) = \int_{\theta_{t+1}} & q(\theta^t, \theta_{t+1})B(\theta^t, \theta_{t+1}, \gamma)d\theta_{t+1} + \exp(\eta t)A(\theta_0, \gamma) + \\ & [x(\theta^t, \gamma) + \exp(\eta t)\gamma]z(\theta^t, \gamma), \end{aligned} \tag{11}$$

where $\exp(\eta t)A(\theta_0, \gamma)$ is the non-state contingent transfer of funds from a household's brokerage account to checking account chosen as part of the household's plan at date 0.[4] A household can alter this initial transfer plan by choosing $x(\theta^t, \gamma) \neq 0$, which requires paying the fixed cost γ. Accordingly, $z(\theta^t, \gamma)$ is an indicator variable equal to one if a household opts to pay her fixed cost and make a state-contingent transfer and equal to zero if a household does not.

We view the fixed cost, γ, as reflecting cognitive costs associated with collecting and processing information necessary to recompute the optimal portfolio allocation in response to shocks.[5] Our approach is similar to Gabaix and Laibson (2002) and Bacchetta and van Wincoop (2009); however, we emphasize that the decision to reoptimize portfolio holdings is state dependent

[4] In principle, a household could choose a time-varying, non-state contingent plan at date 0 and would do so if the model allowed for finite-lived agents with life-cycle considerations. For example, a household born with low initial assets would save some of her wage income by setting up a plan that at first transferred a fixed amount of funds from her checking account to her brokerage account. Later, the transfers would be reversed at her expected retirement date, when cash transfers from the brokerage to checking account become her primary source of cash for consumption.While including financial planning over a household's life cycle would be more realistic, we abstract from such considerations to keep the analysis tractable and simply focus on the non-state contingent nature of $A(\gamma)$.

[5] We follow Alvarez, Atkeson, and Kehoe (2002) and model this cost as a monetary cost instead of one that reduces an agent's utility.

rather than time dependent. Our approach also shares similarities with Reis (2006), who derives inattentive behavior on the part of households who infrequently adjust their consumption and saving plans.

Checking Account. For $t \geq 1$, a household purchases goods for consumption, $c(\theta^t, \gamma)$ using her labor income, $w(\theta^t)L(\theta^t, \gamma)$, and funds transferred from her brokerage account:

$$c(\theta^t, \gamma) = w(\theta^t)L(\theta^t, \gamma) + x(\theta^t, \gamma)z(\theta^t, \gamma) + \exp(\eta t)A(\theta_0, \gamma). \tag{12}$$

If a household deems her labor income and planned transfer low relative to her preferred level of consumption in period t, a household can opt to deviate from her plan and choose $x(\theta^t, \gamma) > 0$ by incurring her fixed cost.[6] Similarly, a household can rebalance her funds across accounts by paying the fixed cost and choosing $x(\theta^t, \gamma) < 0$.[7]

A household's problem is to choose $A(\theta_0, \gamma)$ and $\{c(\theta^t, \gamma), L(\theta^t, \gamma), x(\theta^t, \gamma), z(\theta^t, \gamma), B(\theta^t, \gamma)\}_{t=1}^{\infty}$ to maximize:

$$\sum_{t=1}^{\infty} \int_{\theta^t} \beta^t U(c(\theta^t, \gamma), L(\theta^t, \gamma))g(\theta^t)d\theta^t, \tag{13}$$

subject to equations (10), (11), and (12) taking wages, asset prices and her initial holdings of bonds as given. In equation (13), the function $g(\theta^t)$ denotes the probability distribution over history θ^t.

A key difference between the households in our model and earlier models of endogenous market segmentation is that we allow households to set up non-state contingent transfer plans. As we show below, the main reason we introduce this plan is that without it the model is unable to generate average equity and term premia in line with the data. This feature also has

[6] We have abstracted from the possibility that a household may want to save extra cash in their checking by imposing that equation (12) always binds. In the benchmark calibraiton of the model, we find that there is a very small measure of households that have a precautionary demand for cash. Gust and López-Salido (2009) explicitly allow the constraints to be occasionally binding in the case in which there are only a finite number of household types and show that the equity pricing implications of the model are robust to this possibility. In addition, in section 4, we consider a version of the model with a finite number of household types. In this case, the constraints are always binding, and there is very little difference between the results of that model and the benchmark model.

[7] Note that a household is free to choose $A(\theta_0, \gamma) < 0$, although in equilibrium $A(\theta_0, \gamma) > 0$, $\forall \gamma > 0$.

an attractive interpretation: a household's decision to revise its plans in response to shocks is equivalent to a rebalancing of funds between a liquid and illiquid account. Accordingly, it is broadly consistent with micro evidence that many households adjust their portfolio allocations very infrequently.[8]

We have focused on transfers only between a checking account (i.e., more liquid assets) and a brokerage account (i.e., less liquid assets). In practice, a household has access to a wider range of financial products such as credit cards and other "near-money" assets that blur this distinction. In principle, one could incorporate such near-money assets by having an additional account whose assets can not directly be used to purchase goods but whose transaction cost is smaller than for the financial assets in the brokerage account. Although in principle this is an interesting extension, the current version of the model illustrates how a segmented market model can quantitatively accounts for key asset pricing puzzles and business cycles fluctuations in a parsimonious framework.

2.3 Monetary Policy and Market Clearing

The government issues the economy's one-period state-contingent bonds and chooses the nominal interest according to:

$$R^1(\theta^t) = \omega + \omega_\pi \pi(\theta^t), \tag{14}$$

where ω and ω_π are policy parameters that affect the economy's average nominal interest rate and the responsiveness of the nominal interest rate to inflation. This interest rate rule and equation (9) determine the economy's inflation rate and short-term nominal interest rate.

At date 0, the government's budget constraint is:

$$\bar{B} = \int_{\theta_1} q(\theta_1) B(\theta^1) d\theta_1, \tag{15}$$

where the government's initial assets, $\bar{B}$, are given. At dates $t \geq 1$, its budget constraint is:

$$B(\theta^t) = m(\theta^t) - \frac{m(\theta^{t-1})}{1+\pi(\theta^t)} + \int_{\theta_{t+1}} q(\theta^t, \theta_{t+1}) B(\theta^t, \theta_{t+1}) d\theta_{t+1}, \tag{16}$$

[8] See, for example, Souleles (2003) and Ameriks and Zeldes (2004).

where $m(\theta^t)$ denotes real money balance and $m(\theta_0) > 0$ is given. With monetary policy characterized by an interest rate rule, money evolves endogenously to satisfy $m(\theta^t) = Y(\theta^t) - I(\theta^t)$.[9] Finally, the zero-coupon nominal bonds and real bonds for $\tau > 1$ are assumed to be in zero net supply at each date, so they do not need to be included in the government's budget constraints.

The economy's resource constraint is:

$$Y(\theta^t) = c(\theta^t) + I(\theta^t) + \int_0^\infty \exp(\eta t)\gamma z(\theta^t, \gamma) f(\gamma) d\gamma, \tag{17}$$

where aggregate consumption is given by $c(\theta^t) = \int_0^\infty c(\theta^t, \gamma) f(\gamma) d\gamma$. Labor market clearing implies:

$$L(\theta^t) = \int_0^\infty L(\theta^t, \gamma) f(\gamma) d\gamma. \tag{18}$$

2.4 Equilibrium Characterization

The consumption of an inactive household (i.e., one that sets $z(\theta^t, \gamma) = 0$) is given by:

$$c_I(\theta^t, \gamma) = w(\theta^t) L(\theta^t, \gamma) + \exp(\eta t) A(\theta_0, \gamma). \tag{19}$$

Optimization by the economy's firms implies that the real wage satisfies $w(\theta^t) = (1-\alpha)\frac{Y(\theta^t)}{L(\theta^t)}$. By increasing the real wage, an unanticipated increase in technology tends to boost the consumption of inactive household. However, the consumption of active households can potentially rise even more, because an active household not only receives higher wage income but can transfer additional funds above the amount scheduled by her initial plan from her brokerage account to checking account. By doing so, the consumption of an active household (unlike an inactive one) will reflect that the technology shock boosts the capital income derived from ownership of the economy's firms. As can be seen from equation (19), an important factor affecting the consumption of the inactive households is their labor supply decision.

[9] The condition relating real money balances to output net of investment can be derived from the money market clearing condition:

$$m(\theta^t) = \int_0^\infty \left\{ w(\theta^t) L(\theta^t, \gamma) + \left[x(\theta^t, \gamma) + \exp(\eta t)\gamma \right] z(\theta^t, \gamma) + \exp(\eta t) A(\theta_0, \gamma) \right\} f(\gamma) d\gamma.$$

To understand the interaction between consumption, labor supply, and asset pricing, we adopt preferences commonly used for business cycle analysis. The first specification, introduced by Greenwood, Hercowitz, and Huffman (1988), is:

$$U\left(c_t, L_t\right) = \frac{v(c_t, L_t)^{1-\sigma}}{1-\sigma}, \tag{20}$$

where the utility is defined over the consumption-labor bundle, $v(c_t, L_t) = c_t - \exp(\eta t)\chi_0 \frac{L_t^{1+\chi}}{1+\chi}$.

This specification has the convenient feature that each household has the same labor supply function:

$$w(\theta^t) = \exp(\eta t)\chi_0 L(\theta^t, \gamma)^{\chi}, \tag{21}$$

so that $L(\theta^t, \gamma) = L(\theta^t)\ \forall\gamma$. However, these preferences have the drawback that there is no wealth effect on labor supply, requiring that the nonstochastic trend affect the disutility of leisure in order to render these preferences consistent with balanced growth. To understand the role of this assumption and endogenous labor supply in general, two other cases are considered. First, as in Jermann (1997), we assume that labor is supplied inelastically by the households (i.e., $L(\theta^t, \gamma) = 1$, $\forall\gamma$), which corresponds to $\chi_0 = 0$. Second, the preferences of King, Plosser, and Rebelo (1988), which allow for a wealth effect on labor supply, are used.

Given the attractive property of the Greenwood, Hercowitz, and Huffman (1988) preferences that all the households make the same labor supply decision and perfect risk-sharing amongst active households, the initial asset holdings of the households, $\bar{B}(\gamma)$, can be chosen so that:

$$c_A(\theta^t, \gamma) = c_A(\theta^t). \tag{22}$$

Accordingly, with these preferences, the consumption of active households is independent of γ. To further characterize, the consumption of active and inactive households, we need to determine the flow of transfers from a household's initial plan. A household's choice of $A(\theta_0, \gamma)$ satisfies:

$$\sum_{t=1}^{\infty} \int_{\theta^t} \beta^t \left[U_c^A\left(\theta^t\right) - U_c^I\left(\theta^t, \gamma\right)\right]\left(1 - z(\theta^t, \gamma)\right) g(\theta^t) d\theta^t = 0, \tag{23}$$

where $U_c^A\left(\theta^t\right) = U_c\left(c_A(\theta^t), L(\theta^t)\right)$, $U_c^I\left(\theta^t, \gamma\right) = U_c\left(c_I(\theta^t, \gamma), L(\theta^t)\right)$. This latter condition implies that in states of the world in which a household is inactive (i.e., $z(\theta^t, \gamma) = 0$), the household chooses $A(\theta_0, \gamma)$ to equate her expected discounted value of marginal utility of its consumption to the expected discounted value of the marginal utility of consumption of the active households. Accordingly, the non-state contingent transfer plan provides some consumption insurance to households with large fixed costs.

We now characterize a household's decision for $z(\theta^t, \gamma)$ given optimal decisions for $c(\theta^t, \gamma)$, $x(\theta^t, \gamma)$, $L(\theta^t, \gamma)$ and $A(\theta_0, \gamma)$. A household will choose to be active if $\gamma \leq \bar{\gamma}(\theta^t)$ where $\bar{\gamma}(\theta^t)$ is defined by:

$$U\left(c_A(\theta^t), L(\theta^t)\right) - U\left(c_I(\theta^t, \bar{\gamma}(\theta^t)), L(\theta^t))\right) = U_c^A\left(\theta^t\right)\left[c_A(\theta^t) - c_I(\theta^t, \bar{\gamma}(\theta^t)) + \bar{\gamma}(\theta^t)\right], \qquad (24)$$

and inactive otherwise. Equation (24) implies that there is a marginal household with fixed cost $\bar{\gamma}(\theta^t)$ whose net gain of rebalancing is equal to the cost of transferring funds across the two markets. The net gain, $U\left(c_A(\theta^t), L(\theta^t)\right) - U\left(c_I(\theta^t, \bar{\gamma}(\theta^t)), L(\theta^t))\right)$, is simply the difference in the level of utility from being active as opposed to inactive. The net cost of making the state-contingent transfer comprises the fee γ and the amount transferred by the household, since $x(\theta^t, \gamma) = c_A(\theta^t) - c_I(\theta^t, \gamma)$.[10]

The price of the one-period state contingent bond depends on the marginal utility of consumption of the active rebalancers and is given by:

$$q(\theta^{t+1}) = \frac{\beta U_c^A\left(\theta^{t+1}\right)}{U_c^A\left(\theta^t\right)} g(\theta^t, \theta_{t+1}). \qquad (25)$$

This expression can be used to determine the equilibrium prices of bonds using equations (6)-(8).

Optimization by the firms and arbitrage imply that the return on equity satisfies:

$$(1 + r^e(\theta^{t+1})) = \frac{\alpha \frac{Y(\theta^{t+1})}{K(\theta^t)} + (1-\delta) a_1 \left(\frac{K(\theta^{t+1})}{(1-\delta)K(\theta^t)}\right)^{1-\psi} p_K(\theta^{t+1})}{p_K(\theta^t)}, \qquad (26)$$

where $p_K(\theta^t)$ is the price of a new unit of capital, or equivalently Tobin's 'Q' in our setting. According to equation (26) the return on equity is comprised of the marginal product of capital

[10] The property that equation (24) is static for a given value of $A(\theta_0, \gamma)$ follows from the assumption that equation (12) always binds. See Gust and López-Salido (2009) for a version of the model in which this is not the case.

and a change in the price of new capital (inclusive of capital adjustment costs and depreciation). The price of a new unit of capital is:

$$p_K(\theta^t) = \frac{1}{a_2}\left(\frac{I(\theta^t)}{K(\theta^t)}\right)^{1-\psi}. \tag{27}$$

From equation (27), it is clear that $\frac{1}{1-\psi}$ also has the interpretation as the elasticity of the investment to capital ratio with respect to Tobin's 'Q'. With $\psi < 1$, an unanticipated increase in technology tends to increase the ratio of investment to the capital stock, and as a result, Tobin's Q rises.

In this economy money is neutral and thus does not affect real quantities. However, monetary policy, through systematic changes in the nominal interest rate, can introduce inflation risk into the economy, which has important implications for the nominal yield curve.

2.5 Parameter Values and Numerical Solution

The parameter values are chosen to be consistent with a quarterly model. The depreciation rate, $\delta = 0.02$, and the economy's capital share, $\alpha = 0.36$. The growth rate of technology, η, was set to 0.0037, which implies an annualized growth rate of 1.5 percent, while the money growth rate, μ, was chosen to imply an annualized money growth rate of 4 percent. The parameter, ψ, which governs the elasticity of the investment to capital ratio with respect to Tobin's 'Q', was set so that $\frac{1}{1-\psi} = 0.4$. This value for the elasticity is lower than used by King and Watson (1996) but higher than the value used by Jermann (1997).

For the preference parameters, we set $\beta = 0.999$, which for the calibration of the benchmark economy, implies a quarterly real interest rate of about 2.5 percent on an annualized basis. Following the discussion in Guvenen (2009), the relative risk aversion coefficient, σ, was set to 6, which is low relative to values typically used in the macrofinance literature (e.g., Campbell and Cochrane (1999)). The labor supply elasticity, $\frac{1}{\chi}$, was set to 1, which is a compromise between the low estimates based on micro evidence and the higher estimates based on macro evidence. The parameter χ_0 is normalized so that in the model's nonstochastic steady state $L = 1$. For the distribution of households, $F(\gamma)$, it is assumed that there is a mass of households with zero fixed costs. The remaining distribution, $1 - F(0)$ is uniformly distributed, $U(0, \gamma_U)$, with $F(0) = 0.0525$ and $\gamma_U = 2000$. For our benchmark model, this distribution implies that, on average, the fixed cost are negligible – less than 0.05 percent of GDP – and that, on average, about 6 percent of households rebalance their portfolios each quarter with a large mass rarely rebalancing. Such a calibration is broadly in line with evidence that household portfolio allocation between liquid and illiquid assets displays substantial inertia.[11] In the monetary policy rule, $\omega = 0$ and $\omega_\pi = 2$, which imply an average annualized inflation rate of 3 percent and a standard deviation of inflation of 2.1 percent.

For the technology shock, we follow Tauchen (1986) and Flodén (2008), who show how to construct the transition probabilities of a first-order Markov process to approximate an AR(1) process. The technology shock, θ_t follows a nine-state, symmetric Markov chain in which the associated transition probabilities imply that technology shocks are highly persistent with a first-order autocorrelation of 0.98. The nodes for the technology shock are equally-spaced between [-0.17,0.17] and, in line with U.S. data, lead to a standard deviation of HP-filtered output of about 2 percent.

The model is solved using the global methods discussed in Judd (1999) and Christiano and Fisher (2000). The algorithm involves approximating policy functions for $p_K(\theta^t)$, as a function of the state variables, $K(\theta^{t-1})$, and θ^t. These policy rules are determined using collocation and Chebychev polynomials. Collocation and piecewise polynomial interpolation are used to determine $A(s_0, \gamma)$.

3 Quantitative Analysis

In this section, different versions of the model are evaluated based on their ability to account for the average equity premium and risk-free rate as well as important business cycle statistics such as the volatility of output, labor, consumption, and investment. The model's implications for the nominal and real term structure are discussed with a focus on the model's ability to

[11] See, for example, Brunnermeier and Nagel (2008) and Bonaparte and Cooper (2009).

replicate the average slope of the yield curve and the volatility of excess bond returns. Given the compelling evidence from the empirical finance literature that equity and bond premia are predictable, the ability of the different model versions to account for these observations is also investigated.

3.1 Non-Stochastic Steady State

With access to this plan, all the households have the same level of consumption and no household incurs the fixed cost in a deterministic setting. This result follows from equation (23), which in a deterministic setting requires that consumption be equalized across all households. Households with $\gamma > 0$ achieve this level of consumption by choosing $A(\gamma) = A$ such that $\tilde{c}_A = \tilde{c}_I = (1-\alpha)\tilde{Y} + A = \tilde{Y} - \tilde{I}$, where the tilde over a variable reflects that this variable has been scaled by $\exp(\eta t)$. These households will not deviate from their initial plan. The households with $\gamma = 0$ will be indifferent between their choice of $A(0)$ and $\tilde{x}(0)$.

Without this plan, the model's steady state is quite different, as the consumption of active households will exceed the consumption of inactive households. Some households, with $\gamma > 0$, will opt to incur the fixed transfer cost. Households who do not make the transfer do not receive any capital income so that $\tilde{c}_I = (1-\alpha)\tilde{Y}$, leaving all of the capital income net of the cost of the investment good to the other households. For households with a relatively low γ, this implies that the benefits of making a transfer relative to the costs, as implied by equation (24), are high so that the marginal rebalancer has a positive fixed cost (i.e., $\bar{\gamma} > 0$). Using the resource constraint, it can be shown that active households (i.e., those with $\gamma < \bar{\gamma}$) receive:

$$\tilde{c}_A = (1-\alpha)\tilde{Y} + F(\bar{\gamma})^{-1}\left[\alpha\tilde{Y} - \tilde{I} - \int_0^{\bar{\gamma}} \gamma f(\gamma) d\gamma\right], \tag{28}$$

where the first term reflects an active household's steady state wage income and the second term reflects their steady state capital income. For the functional forms of $F(\gamma)$ and $U(c, L)$ discussed above, it is possible to show that equations (24) and (28) imply that $\tilde{c}_A > \tilde{c}_I$. The baseline parameters imply $I/Y = 0.2$, $c_A/c_I = 1.9$, $c_A/Y = 1.2$, $F(\bar{\gamma}) = 7\%$, and the average fixed cost incurred by the average household is about 12% of the economy's output. In comparison, with household access to the financial plan, only the investment to output ratio remains unchanged. In this case, as noted above, consumption is equal across households, no fixed cost is incurred, $F(\bar{\gamma}) \leq F_L = 5.25\%$, and $c_A/Y = 0.8$. As discussed later, these different properties are important in understanding why the model with the financial plan is more successful in explaining asset prices.

3.2 The Equity Premium and the Risk-Free Rate

Table 1 displays several statistics of interest from alternative versions of the model and compares them with their empirical counterparts taken from Guvenen (2009). As a reference point, the third column of the table reports the results from the economy with a single representative household.[12] For the baseline calibration, with a relatively low coefficient of relative risk aversion, the representative agent model is unable to replicate prominent asset pricing features: the average equity premium is 2% and the average (real) risk-free rate is 7.6% on an annualized basis.[13] The lack of success of the model in explaining the average equity premium underlies the model's low Sharpe ratio. As discussed in Weil (1989), it is possible to increase the equity premium in this version of the model by increasing σ; however, this comes at the cost of raising an already counterfactually high risk-free rate.

The fourth column of Table 1 shows the results from the benchmark segmented market model. This model is consistent with the high average equity premium and the model's Sharpe ratio at 0.26 is just below the point estimate based on U.S. data. It implies a reasonable value for the average risk-free rate, while the volatility of the risk-free rate is on the low side relative to U.S. data. The model's ability to generate a relatively low and smooth risk-free rate is remarkable in light of the difficulty of other production-based asset pricing models in matching these observations. For instance, both Boldrin, Christiano, and Fisher (2001) and Jermann (1997) report an excessively volatile risk-free rates in their models with habit formation, while Guvenen (2009) remarks that his model with limited stock market participation and

[12] We use the calibrated parameters discussed above except that there is a single household that always rebalances (i.e., $F_L = 1$).

[13] See Mehra and Prescott (1985).

heterogeneous household preferences would generate a risk-free rate above 5% if the model incorporated trend growth.

3.2.1 The Average Equity Premium

There are two key ingredients that allow the model to be consistent with the observed equity premium and Sharpe ratio. The first ingredient is allowing households to set up the non-state contingent financial plan in the context of segmented markets. The second ingredient is the imperfect substitutability between installed capital and investment. Both of these features impair the ability of active households to reduce fluctuations in their consumption. Without either one of these features, the segmented market model is unable to replicate the observed equity premium.

To understand the role of the non-state contingent plan, the fifth column of Table 1 shows the results for the version of the model when households do not have access to this plan (i.e., $(A(\theta_0, \gamma) = 0 \; \forall \gamma)$. In this case, the model is similar to the segmented market models of Alvarez, Atkeson, and Kehoe (2002) or Khan and Thomas (2007), and the average equity premium is only 1.9% while the average risk-free rate is nearly 8%. The primary difference in accounting for the average equity premium across the two models is the volatility in $c_A(\theta^t)$, which translates into a more volatile marginal utility of consumption.

The financial plan has very different implications for a household with a low γ, who is more likely to deviate from her non-state contingent plan, and a household with a high γ, who is less likely to deviate. Without this plan, the consumption of an individual with a high enough γ would be equal to her wage income, inheriting both its mean level and its volatility. However, with access to the financial plan, this household will choose $A(\theta_0, \gamma) > 0$, because doing so provides her with funds derived from the capital income of firms that are readily available for consumption. Accordingly, her consumption increases relative to the case in which she does not have access to this plan and her consumption is smoother given the plan's non-state contingent nature.

Interestingly, by smoothing the consumption of households that do not rebalance, this plan tends to increase the volatility of the consumption of active households (i.e., those with a relatively low γ). Using the resource constraint and defining capital income as $r_K K = \alpha Y$, the consumption of an active household is given by:

$$c^A = wL + F(\bar{\gamma})^{-1}\left[r_K K - I - \int_{\bar{\gamma}}^{\gamma_U} A(\gamma)f(\gamma)d\gamma - \int_0^{\bar{\gamma}} \gamma f(\gamma)d\gamma\right], \tag{29}$$

where the variables dependence on θ^t and technological growth have been suppressed for convenience. The presence of the non-state contingent plan (i.e., the $A(\gamma)$ term in the above equation) raises the sensitivity of the consumption of active households to capital income and hence to fluctuations in equity markets. To see this more explicitly, it is helpful to hold holding I and $\bar{\gamma}$ fixed and take the derivative, $d\ln(c_a)/d\ln(r_K K) = F(\bar{\gamma})^{-1}\alpha Y/c_a$. Evaluating this derivative at the non-stochastic steady state of the version of the model in which households do not have access to the financial plan implies $F(\bar{\gamma}) = 7\%$, $\alpha = 0.36$, $c_a/Y = 1.2$, and $d\ln(c_a)/d\ln(r_K K) = 4.1$. In the version with financial planning, $c_a/Y = 0.8$ and $F(\bar{\gamma}) = 5.25\%$ implying $d\ln(c_a)/d\ln(r_K K) = 7.8$, which is nearly twice as large.

The above calculation suggests that it may be possible to improve the performance of the version of the model without financial planning through a calibration in which c_A is small relative c_I and Y and/or the cost borne by the marginal rebalancer, $\bar{\gamma}$ is low and hence $F(\bar{\gamma})$ is low. However, this does not work, because the marginal number of rebalancers, $F(\bar{\gamma})$, is also determined from equation (24). This condition limits the ability to increase the sensitivity of active consumption to changes in capital income through a low value of $\bar{\gamma}$ and low fixed costs, because c_A is high in that case, raising the incentive to rebalance, putting upward pressure on $\bar{\gamma}$. Alternatively, attempting to increase the sensitivity of active consumption through high fixed costs is limited, because high fixed costs lower c_A relative to c_I, reducing the incentive to rebalance and the number of households willing to pay that fixed cost. As shown in the appendix, the inability of the model without the financial plan to generate a reasonable average equity premium and risk-free rate holds across a wide range of parameter values.

By providing the inactive households with consumption insurance from volatility in capital markets, the financial plan pushes that volatility onto active households. It is this key feature that allows the segmented market model to account for the average equity premium and risk-free rate. In the benchmark model, the volatility of consumption growth of active households is more than 3 times as high as aggregate consumption growth. Why is a household with better access to financial markets willing to tolerate greater consumption volatility than the average household? Households that rebalance more frequently do so, because this raises their average level of consumption. In the benchmark model, the level of consumption of active households is more than 10 percent higher, on average, than the level of aggregate consumption.[14] Accordingly, by altering their initial financial plan with state contingent transfers, a household is trading off lower consumption volatility in order to gain a higher average level of consumption.

The second key ingredient in generating a reasonable average equity premium is the imperfect substitutability between installed capital and investment in the capital accumulation process. To highlight the role of this assumption, the sixth column of Table 1 displays the results when $\psi = 0.98$ so that the supply of capital is very elastic. In this case, the average equity premium is only 2.3% and the average risk-free rate is more than 6%. The intuition for why a low capital supply elasticity is useful is similar to Jermann (1997), who emphasizes how capital adjustment costs make it more costly for the representative agent to smooth consumption through changing the capital shock. However, in our case, it applies only to the active households, since these are the only households who are subject to consumption risk through capital markets. As implied by equation (29), the consumption of active households tends to be more volatile as investment responds more sluggishly to shocks. Accordingly, the volatility of active consumption is markedly higher for the benchmark calibration than for the case with $\psi = 0.98$, and this higher volatility of active consumption translates into greater volatility of the consumption-labor bundle.

The final column of Table 1 displays the results from the economy in which labor is supplied inelastically by the households and $\chi_0 = 0$. In this case, the average equity premium and risk-free rate are 7.4% and 2.4%, respectively. Accordingly, the incorporation of endogenous labor supply using Greenwood, Hercowitz, and Huffman (1988) preferences only marginally reduces the average equity premium.

[14] This implication is in line with evidence of Parker and Vissing-Jorgensen (2009) provided that 'high consumption' households are in fact more likely to rebalance. In particular, these authors find that the exposure to changes in aggregate consumption growth of households in the top 10 percent of the consumption distribution is about five times that of households in the bottom 80 percent.

3.2.2 The Average Risk-Free Rate

The benchmark model does reasonably well in generating a low and smooth risk-free rate. To understand this result, it is useful to take a second-order Taylor series expansion of equation (7) around the nonstochastic steady state, which yields:

$$E_t(r_t^f) \approx -\log(\beta) + \sigma E_t(\Delta v_{t+1}^A) - \frac{\sigma^2}{2}\mathrm{var}_t(\Delta v_{t+1}^A), \tag{30}$$

where E_t and var_t denote the conditional mean and variance of a variable at date t, respectively, and Δv_{t+1}^A denotes the log-growth rate of the consumption-labor bundle of the active households.[15] Equation (30) relates the average risk-free rate to the discount factor, a term reflecting intertemporal smoothing of the marginal utility of consumption, and a term reflecting precautionary savings by active households. Taking the unconditional expectation of this expression, columns two and three of Table 2 decompose the average risk-free rate into a mean component, corresponding to $\sigma\eta - \log(\beta)$, and a variance component, $0.5\sigma^2\mathrm{var}(\Delta v_{t+1}^A)$. In the benchmark economy, the average real rate is low, because the level effect associated with intertemporal smoothing is offset by the precautionary savings of active households. In the version of the model with a representative agent, the level of precautionary savings is small; hence, the model fails to explain the average risk-free rate. A low level of precautionary saving also accounts for the high average risk-free rates shown in Table 2 for the economies with either low capital adjustment costs or no financial planning.

Using equation (30), the unconditional variance of the risk-free rate can be expressed as:

$$\mathrm{var}(r^f) \approx \sigma^2\mathrm{var}(E_t\Delta v_{t+1}^A) + \frac{\sigma^4}{4}\mathrm{var}\left(\mathrm{var}_t(\Delta v_{t+1}^A)\right) - \sigma^3\mathrm{cov}\left(E_t\Delta v_{t+1}^A, \mathrm{var}_t(\Delta v_{t+1}^A)\right), \tag{31}$$

where cov denotes the unconditional covariance between two variables. Thus, the variance of the risk-free rate reflects the volatilities associated with intertemporal smoothing and precautionary savings as well as the covariance between these two effects. Table 2 shows that nearly all of the volatility of the risk-free rate in the representative agent model is driven by the volatility

[15] There is also an approximation error associated with this equation. However, this error is small, which can be seen by comparing the average risk-free rate in Table 2 to the value reported in Table 1.

associated with intertemporal smoothing. In contrast, in the benchmark model, both the variance of the precautionary savings term and the covariance between precautionary savings and intertemporal smoothing have a large effect on the volatility of the risk-free rate. In that model, the negative covariance between the intertemporal smoothing and precautionary savings term is important in accounting for the smooth risk-free rate. The covariance is negative, because after a technological improvement an active household's desire to save for purely intertemporal smoothing motives increases, as her consumption rises more today than in the future. At the same time, a household's desire to save for precautionary reasons declines in response to the technological improvement, implying that intertemporal smoothing and precautionary savings motives have counterbalancing effects on the volatility of the risk-free rate.

3.3 Equity Returns Predictability

An important finding in the empirical finance literature is that equity returns are predictable. The second and third columns of Table 3 displays the results from regressing the price-dividend ratio on excess returns at different horizons using U.S. stock market data.[16] As in Fama and French (1989), low equity prices today forecast higher excess returns in the future. Moreover, the fraction of variation in returns that is predictable increases at longer horizons.

Table 3 also shows the results from running these regressions based on simulated data from the benchmark model.[17] As in the data, excess returns are predictable, with the R^2 from the regression increasing with the horizon. Notably, the amount of predictability explained by the model is roughly in line with the estimates based on U.S. data, as the R^2 at the five year horizon in the model is near its empirical counterpart. In contrast, Guvenen (2009) notes that his model of heterogenous preferences and limited stock market participation generates a smaller amount of predictability than observed in the data.

The price-dividend ratio in the model forecasts future excess returns, because an increase in technology has a persistent effect on the price-dividends ratio and reduces expected excess

[16] Describe data and how excess returns were constructed at different horizons.
[17] We use $p_K(\theta^t)$ as the model's measure of equity prices and scale dividends, $D(\theta^t)$, by a firm's capital stock, to construct the model's price-dividend ratio.

returns. Figure 1 shows that the price-dividend ratio rises more than 2 percent on impact in response to a one percent increase in technology. Due to endogenous capital accumulation and the persistence of the shock, the price-dividend ratio remains high well after the initial shock. On impact, the price of risk and the equity premium fall 40 and 20 basis points, respectively. The equity premium and the price of risk move countercyclically in the model mainly due to the curvature of the marginal utility of consumption: the marginal utility of active consumption is less responsive to changes in $v^A(\theta^t)$ at higher values of the consumption-labor bundle of active households, $v^A(\theta^t)$. As shown in Figure 1, an increase in technology leads to a relatively large increase in the consumption of active households and hence the consumption-labor bundle. With the marginal utility of active consumption becoming less responsive to changes in $v^A(\theta^t)$, its volatility declines as does the equity premium and the price of risk.

An additional factor contributing to countercyclical movements in risk is that an increase in technology tends to drive up the number of households that actively rebalance funds between their brokerage and checking accounts. A positive technology shock raises the consumption of active rebalancers more than inactive households, reflecting that the consumption of the active households rises in response to the increase in capital gains. This jump in capital income induces more households to rebalance their cash allocation. Accordingly there is a greater degree of risk-sharing amongst active households, which lowers the price of risk and the equity premium.

3.4 Business Cycle Properties

Table 4 displays pertinent business cycle statistics from the US data as well as their counterparts from different versions of the model. As shown in the last column of the table, the benchmark model (by construction) matches the output volatility observed in the data. The model is qualitatively consistent with the facts that the volatilities of consumption and labor are low relative to output volatility, while investment volatility is relatively high. In addition, the volatility of Tobin's 'Q', as measured by stock prices, is in line with the value observed in the data.

Table 4 shows that the imperfect substitutability between installed capital and investment is important in accounting for the volatility of Tobin's 'Q'. By comparing columns four and six, the table also demonstrates that the financial plan helps smooth the consumption of inactive households and thus aggregate consumption. Still, the volatility of aggregate consumption in the benchmark model is high relative to the data. This high volatility reflects that inactive consumption volatility is largely influenced by their wage income, which moves proportionally with the economy's output, given the competitive nature of labor markets in the model.[18]

Although the benchmark model does not display a positive correlation between current consumption growth and lagged consumption growth, it is broadly consistent with the autocorrelation of aggregate consumption growth at greater lags.[19] This latter finding is important in light of the criticism of Piazzesi (2002) of Gabaix and Laibson (2002), whose model of sluggish consumption adjustment requires consumption growth to be autocorrelated at longer lags to explain the equity premium puzzle. This criticism is not applicable to our model, whose asset pricing implications are based on the marginal utility of consumption of only the active households.

Overall, we conclude that the benchmark model performs reasonably well in accounting for several prominent features of equity prices, real interest rates, and business cycles. While its fit could be improved by incorporating labor market frictions, the benchmark model's performance is impressive given its relatively parsimonious nature.

3.5 Term Structure of Interest Rates

Table 5 shows the implications of alternative versions of the model for the means and standard deviations of yield spreads and excess returns. As in the data, the benchmark model generates an upward sloping average yield curve. However, yield spreads, on average, are somewhat higher in the model than in the data. The model generates reasonable volatilities in spreads and replicates the fact that longer maturity bonds are less volatile than shorter maturity bonds. To evaluate

[18] Incorporating labor market frictions as in Uhlig (2007) should improve the performance of the model. Moreover, to the extent that they help smooth the consumption of inactive households and increase the volatility of active households, they should also improve the asset pricing implications.

[19] In section 4, we investigate a version of the model that helps account for the first-order autocorrelation of consumption growth.

the model's performance in terms of bond premia, Table 5 defines the excess return over the quarterly nominal rate from buying a τ-period bond at time t and selling it at $t+1$ as:

$$\mathrm{er}_{t+1}^{\tau} = \log(P_{t+1}^{\tau-1}) - \log(P_t^{\tau}) + \log(P_t^1). \tag{32}$$

As in the data, the expectations theory of the term structure does not hold in the benchmark model. Rather, excess returns are positive and highly volatile, with the benchmark model generating more volatility in excess returns than observed in the data.

Table 6 displays the benchmark model's implications for real yields. Real yields account for a little more than half of the upward sloping nominal yield curve; hence, both inflation and real risk are important factors in explaining nominal yields. To understand the contribution of both of these factors, it is useful to take a second-order Taylor series expansion and write the the average nominal yield of a τ-maturity bond as:

$$E\left(R_t^{\tau}\right) \approx r^f + E\pi_t - \frac{\sigma^2}{2\tau}\mathrm{var}\left(\sum_{j=1}^{\tau}\Delta v_{t+j}^A\right) - \frac{1}{2\tau}\mathrm{var}\left(\sum_{j=1}^{\tau}\pi_{t+j}\right) - \frac{\sigma}{\tau}\mathrm{cov}\left(\sum_{j=1}^{\tau}\Delta v_{t+j}^A, \sum_{j=1}^{\tau}\pi_{t+j}\right), \tag{33}$$

where $r^f = -\log(\beta) + \sigma\eta$. The variance term involving the sum of changes to the consumption-labor bundle reflects the effect of precautionary savings on real interest rates, while the last two terms reflect the effect of inflation risk on interest rates.

Table 7 presents the decomposition of both quarterly and 5-year nominal rates according to the determinants in equation (33). The average real yield curve is upward sloping in the benchmark model, because as shown in Table 7, precautionary savings of active households are less important at longer horizons than shorter horizons.[20] This reflects the dynamic response of v_t^A, which, as shown in the lower right panel of Figure 1, jumps in response to a technology shock before gradually returning to its unshocked level. Hence, the cumulative change in the consumption-labor bundle of active households at longer horizons is smaller than at shorter

[20] The model's upward sloping real yield curve is consistent with the evidence of Kim and Wright (2005) and Gurkaynak, Sack, and Wright (2010) for the United States. Kim and Wright (2005), for example, document real term premia that are, on average, nearly twice as large as inflation risk premia based on U.S. data from 1990-2005. For the United Kingdom, real term premium appear to be smaller. Using U.K. data from 1985-2004, Rivadeneyra (2008) documents that the real yield curve, on average, is only marginally upward sloping. Similarly, Ang, Bekaert, and Wei (2008) estimate that the average real yield curve in the United States is flat.

horizons.

Nominal bonds are risky in the benchmark model, because their real payoffs decline in bad times. In response to a negative technology shock, inflation increases while active consumption falls, and thus active households demand higher yields to compensate for holding nominal bonds. As shown in Table 7, inflation risk contributes about 15 basis points, on average, to the quarterly rate and over 90 basis points to the 5-year rate. Moreover, inflation risk accounts for about 40 percent of the 5-year spread in the benchmark model. The reason why long-term nominal debt is riskier than short-term reflects the influence of systematic monetary policy. In particular, the systematic response of monetary policy to current inflation induces long-run inflation risk. By responding to short-run inflation, monetary policy generates highly persistent movements in inflation. With $\omega_\pi > 1$, inflation inertia can be directly related to the persistent response of the real interest rate. Figure 1 shows that the real interest rate remains well below its pre-shocked level due to a persistent decline in precautionary savings. As a result, the upward sloping nominal yield curve reflects both long-run inflation risk and real term premia.

To highlight the role of monetary policy in inducing long-run inflation risk, Table 7 displays the decomposition of the quarterly and 5-year nominal rates for a constant money growth rate rule. For this rule, the average rate of money growth is chosen so that, as in the benchmark version of the model, the average rate of inflation is close to 3 percent. By holding money growth constant, monetary policy ensures that technology shocks only induce transitory changes in inflation and will not translate into much long-run inflation risk. In this case, inflation risk contributes about 90 basis points to the quarterly rate and 65 basis points to the 5-year rate. Accordingly, the upward sloping average yield curve entirely reflects real yields.

Table 7 also documents that the capital supply elasticity is an important determinant of real term premia. With a higher elasticity of capital ($\psi = -0.5$), active consumption becomes less volatile and hence the contribution from precautionary savings to both short and long-term rates is smaller than in the benchmark model. Thus, there is an upward shift in the entire yield curve. The impact on active consumption volatility is greater in the short run, which reduces the need for precautionary savings in the short run than in the long run. As a result, the 5-year spread is only about 30 basis points, compared to 180 basis points in the benchmark model and 110 basis points in the data. Using this mechanism to lower average yield spreads also implies a lower average equity premium of 4.5 percent, 150 basis points less than the benchmark model but still within a 95 percent confidence interval of the point estimates shown in Table 1.

Table 5 compares the mean and volatility of excess bond returns under a constant money growth rate rule to the interest rate rule. By inducing more long-run inflation risk, the interest rate rule that targets inflation is contributing substantially to both the average and volatility of excess returns at longer maturities. Table 5 also shows that the mean and volatility of excess returns falls with a higher capital supply elasticity.

In classic papers, Fama and Bliss (1987) and Campbell and Shiller (1991) demonstrate that excess returns on long-maturity bonds are forecastable. Table 8 reproduces the results of Fama-Bliss from Cochrane and Piazzesi (2005) in which the excess returns are regressed against the same maturity forward spread. In the data, the slope coefficients are all greater than one and the R^2's indicate that the excess bond returns are predictable, contradicting the expectations hypothesis. Table 8 also shows the results from running regressions using simulated data from the benchmark model. As in the data, excess returns move more than one-for-one with forward spreads. However, the amount of predictability in the model is generally too high relative to the data.

4 Extensions

In this section, we first analyze the case in which there are two types of households (i.e., $\gamma = 0$ or $\gamma = \infty$) so that markets are still segmented but there are not endogenous fluctuations in the number of rebalancers. We will show that the endogenous movements in the number of rebalancer are not the key driver of our results. This simplified version of the model allows us to explore several interesting extensions. First, we can modify preferences and use the ones developed in Jamovich and Rebelo which nest both the GHH and Cobb-Douglas preferences and allow us to explore the role of the wealth effect on labor supply in affecting the results. Second, we plan to explore an alternative source of business fluctuations coming from news about future productivity. Finally, both real wage income and the non-state contingent plan are invariant to changes in invariant to inflation in the model. If households consume based on their previous period wages or the non-state contingent was not indexed to inflation, monetary policy would have important redistribute effects. In addition, this model has more interesting dynamics and can potentially induce the output predictability of yield spreads.

To be completed.

5 Relationship with the Literature

To be completed.

6 Conclusions

To be completed.

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Table 1: Financial Statistics in Alternative Models*

	US Data		Rep. Agent	No HH Plans ($A(\gamma)=0$)	High Capital Supply Elast. ($\psi = 0.98$)	Benchmark Model
$E(r^e\text{-}r^f)$	6.17	(1.99)	2.17	1.93	2.27	6.01
$\sigma(r^e\text{-}r^f)$	19.4	(1.41)	17.47	16.50	0.98	22.95
$\frac{E(r^e\text{-}r^f)}{\sigma(r^e\text{-}r^f)}$	0.32	(0.11)	0.12	0.12	2.31	0.26
$E(r^f)$	1.94	(0.54)	7.66	7.79	6.57	2.77
$\sigma(r^f)$	5.44	(0.62)	1.91	1.82	1.08	2.17
$E(F(\gamma))$			100	6.68	5.95	5.92
$\sigma(F(\gamma))$			0	0.03	0.03	0.03
Avg. Fixed Cost (% of GDP)			0	12.52	0.08	0.05

*Entries are expressed in percent on an annualized basis. Standard errors reported in parentheses. U.S. financial statistics are taken from Guvenen (2009).

Table 2: Determinants of the Risk-Free Rate

	Mean Risk-Free Rate	
	Rep. Agent	Benchmark
Contribution of:		
1. $-\log(\beta)$	0.40	0.40
2. Intertemporal Substitution	8.95	8.95
3. Precautionary Savings	1.69	6.58
4. Average Real Rate (1+2-3)	7.66	2.77
	Risk-Free Rate Volatility	
	Rep. Agent	Benchmark
Contribution of:		
1. Intertemporal Substitution	4.48	12.98
2. Precautionary Savings	0.03	7.44
3. Covariance Term	0.87	15.70
4. Risk-Free Rate Variance (1+2-3)	3.64	4.72
5. Risk-Free Rate Std. Dev.	1.91	2.17

Table 3: Long Run Regressions on Price Dividends

	US Data		Models: Benchmark		Models: Two Agent	
Horizon (Years)	β	R^2	β	R^2	β	R^2
1	-0.22	0.09	-0.14	0.06	-0.15	0.06
2	-0.39	0.14	-0.27	0.11	-0.29	0.12
3	-0.47	0.15	-0.40	0.16	-0.42	0.17
5	-0.77	0.26	-0.64	0.24	-0.68	0.27

Table 4: Business Cycle Statistics in Alternative Models*

	US Data		Rep. Agent	No HH Plans ($A(\gamma) = 0$)	High Capital Supply Elast. ($\psi = 0.98$)	Benchmark Model
$\sigma(y)$	1.9	(0.20)	1.99	2.00	1.99	2.03
$\frac{\sigma(c)}{\sigma(y)}$	0.5	(0.04)	0.97	0.99	0.85	0.87
$\frac{\sigma(i)}{\sigma(y)}$	2.4	(0.06)	1.12	1.06	1.59	1.46
$\frac{\sigma(n)}{\sigma(y)}$	0.8	(0.05)	0.5	0.5	0.5	0.5
$\frac{\sigma(p_k)}{\sigma(y)}$	4.5	(0.68)	2.82	2.67	0.08	3.67
$\rho(\Delta c_t, \Delta c_{t-1})$	0.20	(0.08)	0.0	0.0	0.0	0.0

*Entries are expressed in percent on an annualized basis. Standard errors reported in parentheses. U.S. financial statistics are taken from Guvenen (2009).

Table 5: Nominal Yield Statistics for Alternative Models*

	Long Bond Maturity (Quarters)				
	4	8	12	20	40
Average Yield Spreads					
Data	0.39	0.68	0.84	1.10	1.39
Benchmark	0.34	0.76	1.14	1.79	2.91
Constant Money	0.16	0.35	0.53	0.84	1.42
Higher Capital Supply Elast.	0.06	0.13	0.20	0.31	0.53
Standard Deviation of Yield Spreads					
Data	0.42	0.73	0.84	1.12	1.38
Benchmark	0.26	0.51	0.70	1.03	1.72
Constant Money	0.13	0.25	0.33	0.48	0.80
Higher Capital Supply Elast.	0.07	0.13	0.17	0.23	0.36
Standard Deviation of Yields					
Data	2.97	2.91	2.81	2.70	2.56
Benchmark	4.04	3.87	3.70	3.36	2.62
Constant Money	1.93	1.85	1.77	1.62	1.28
Higher Capital Supply Elast.	0.76	0.72	0.68	0.61	0.46
Average Excess Returns					
Data	0.88	2.04	2.72	3.84	1.59
Benchmark	0.64	1.38	2.00	2.97	4.28
Constant Money	0.29	0.63	0.94	1.44	2.19
Higher Capital Supply Elast.	0.11	0.24	0.36	0.55	0.82
Standard Deviation of Excess Returns					
Data	2.56	5.39	7.66	11.7	20.86
Benchmark	3.75	8.13	11.99	18.41	28.71
Constant Money	1.79	3.88	5.73	8.86	14.04
Higher Capital Supply Elast.	0.76	1.61	2.33	3.49	5.25

*The higher capital supply elasticity case has a constant money growth rate rule and raises the capital supply elasticity from 0.4 in the benchmark case to 0.667.

Table 6: Real Yield Statistics for Alternative Models

	Long Bond Maturity (Quarters)				
	4	8	12	20	40
Average Yield Spreads					
Benchmark	0.18	0.40	0.61	0.98	1.66
Constant Money	0.18	0.40	0.61	0.98	1.66
Higher Capital Supply Elast.	0.07	0.16	0.25	0.40	0.68
Standard Deviation of Yield Spreads					
Benchmark	0.16	0.29	0.39	0.55	0.87
Constant Money	0.16	0.29	0.39	0.55	0.87
Higher Capital Supply Elast.	0.09	0.17	0.22	0.30	0.43
Standard Deviation of Yields					
Benchmark	2.06	1.98	1.90	1.74	1.38
Constant Money	2.06	1.98	1.90	1.74	1.38
Higher Capital Supply Elast.	0.88	0.84	0.80	0.72	0.55
Average Excess Returns					
Benchmark	0.33	0.73	1.09	1.68	2.57
Constant Money	0.33	0.73	1.09	1.68	2.57
Higher Capital Supply Elast.	0.14	0.31	0.46	0.71	1.08
Standard Deviation of Excess Returns					
Benchmark	1.96	4.22	6.22	9.60	15.20
Constant Money	1.96	4.22	6.22	9.60	15.20
Higher Capital Supply Elast.	0.93	1.94	2.81	4.22	6.39

*The higher capital supply elasticity case has a constant money growth rate rule and raises the capital supply elasticity from 0.4 in the benchmark case to 0.667.

Table 7: Determinants of Average Nominal Rates

	Quarterly Rate			5-Year Rate		
	Bench.	Constant Money Growth		Bench.	Constant Money Growth	
Contribution of:	Model	($\psi = -1.5$)	($\psi = -0.5$)	Model	($\psi = -1.5$)	($\psi = -0.5$)
1. $-\log(\beta)$	0.40	0.40	0.4	0.40	0.40	0.40
2. Int. Substitution	8.95	8.95	8.95	8.95	8.95	8.95
3. Prec. Savings	6.58	6.58	5.10	5.53	5.53	4.77
4. Real Rate (1+2-3)	2.77	2.77	4.25	3.82	3.82	4.68
5. Inflation Rate	2.95	2.95	2.95	2.95	2.95	2.95
6. Inflation Risk	0.16	0.86	0.76	0.91	0.65	0.67
Nom. Rate (4+5-6)	5.88	6.58	7.96	7.68	7.42	8.30

Table 8: Fama-Bliss Excess Return Regressions

	US		Models			
	Data		Benchmark		Two Agent	
Horizon (Years)	β	R^2	β	R^2	β	R^2
2	0.99	0.16	1.26	0.15	1.21	0.16
3	1.35	0.17	1.40	0.28	1.47	0.30
4	1.61	0.18	1.31	0.42	1.34	0.45
5	1.27	0.09	1.23	0.56	1.25	0.59

Figure 1: Impulse Response to a Technology Shock in the Benchmark Model (Deviation from Date 0 Expectation of a Variable)