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The Great Escape? A Quantitative Evaluation of the Fed's Non-Standard Policies

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The Great Escape?

A Quantitative Evaluation of the Fed's Non-Standard Policies*

– Preliminary draft –

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Abstract

This paper extends the model in Kiyotaki and Moore (2008) to include nominal wage and price frictions and explicitly incorporates the zero bound on the short-term nominal interest rate. We subject this model to a shock which arguably captures the 2008 US financial crisis. Within this framework we ask: Once interest rate cuts are no longer feasible due to the zero bound, what are the effects of non-standard open market operations in which the government exchanges liquid government liabilities for illiquid private assets? We find that the effect of this non-standard monetary policy can be large at zero nominal interest rates. We show model simulations in which these policy interventions prevented a repeat of the Great Depression in 2008-2009.

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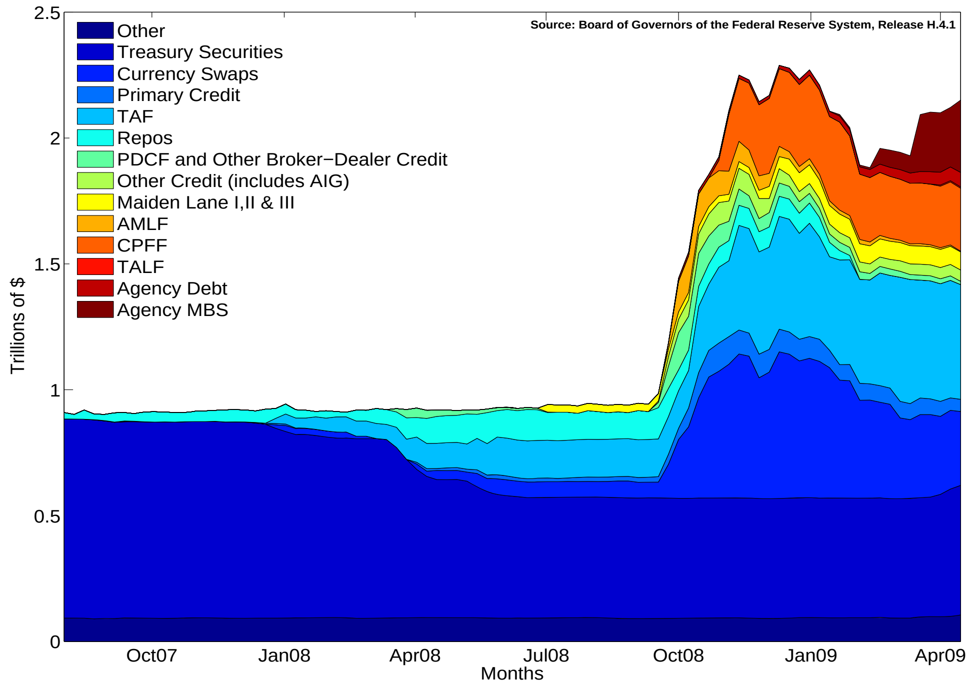

Figure 1: Asset Side of Fed's Balance Sheet

1 Introduction

In 2008, the Federal Funds rates collapsed to zero. Standard monetary policy through interest rate cuts had reached its limit. Around the same time, the Federal Reserve expanded its balance sheet by about 1 trillion dollars, or 7% of U.S. GDP (see figure 1). This expansion mostly involved the Federal Reserve exchanging government liquidity, that is money or government debt, for private assets (through direct purchases of these assets or taking them as collateral in short term loans). This was achieved through the operation of various "facilities", such as the Term Auction Facility (TAF) or the Primary Dealer Credit Facility (PDCF). In broad terms, the operation of these facilities can be thought of as "nonstandard" open market operations, whereby the government exchanges highly liquid government liabilities for less liquid private assets. Alternatively, one can think of them, broadly speaking, as non-standard "discount window" lending which also involve giving government liquidity in exchange for private assets (as collateral in that case).[1] This paper is about the quantitative effect of this policy. Our main result is that it can be large, especially at zero interest rates.

Ever since Wallace (1982) famous irrelevance result, the benchmark for many macroeconomists is that non-standard open market operations in private assets are irrelevant. This result was extended by Eggertsson and Woodford (2003) to show that it also applies to standard open market operations, i.e. printing money for debt, in models with nominal and monetary frictions, provided that the interest rate is zero. There is no role for "liquidity" in these models, or most other standard models with various types of frictions, such as Rotemberg and Woodford (1997), Christiano, Eichenbaum and Evans (2005), or Smets and Wouters (2007). The price of any private security – be it stock or corporate bond – depends on what it will pay out in various states of the world. Both the supply and the ownership of these securities are irrelevant if they do not change their state contingent payoffs. Why should it matter if the government or a private agent holds a particular stock or security, holding constant the revenue stream that they are expected to generate? In fact, in a well known recent paper, Taylor and Williams (2009) argue that in the context of the recent crisis the Federal Reserve's facilities, and the TAF in particular, had no material impact. Their prior was presumably informed by modern general equilibrium theory which embeds Wallace's irrelevance result.

In this paper we break Wallace's irrelevance result in a straightforward fashion. We incorporate a particular form of credit frictions, proposed by Kiyotaki and Moore (2008) (henceforth, KM). Our objective is twofold: First, we investigate whether a liquidity shock modeled as in KM can quantitatively generate movements in *both* macro and financial variables similar to those observed in the current recession. Second, we explore what credit frictions of the form suggested by KM mean for the quantitative effect of the Federal Reserve facilities during the crisis of 2008 and 2009. The KM credit frictions are of two distinct forms. First, a firm (or a bank) that faces an investment opportunity can only borrow up to a fraction of the net present return of its investment. This is a

[1] In fact, one of the original motivation of the earlier facilities was to eliminate the negative "stigma" associated with discount window lending.

relatively standard borrowing constraint.[2] Second, there is a resaleability constraint in the model. A firm that faces an investment opportunity can only sell a certain fraction of its "illiquid" assets in each period. These illiquid assets correspond to equity in other firms. More generally, we interpret these illiquid assets as privately issued commercial paper, loans of banks, stocks, mortgages, and so on. KM argue for this resaleability constraint on ground of imperfect information. We do not underpin its microfoundations in any way, but instead take it as given and explore its quantitative implication. In contrast to private liquidity that is subject to a resaleability constraint, we follow KM and assume that government issued paper, i.e. money and bonds, is not subject to this constraint. This gives government debt and money a primary role as "liquidity" that lubricates transactions. In this world, Wallace's irrelevance result no longer applies because the relative quantity of liquid and illiquid assets in the hands of the private sector affects the equilibrium, and the government can change this composition. This gives a natural story for the crisis of 2008 and the ensuing Fed's response in the context of the KM model.

The shock we then consider as the source of the crisis of 2008 is a shock to the resaleability of private assets. Suddenly, the private market for credit freezes. We think of this shock as capturing central aspects of the crisis of 2008. For our quantitative analysis, we can calibrate this shock by matching a new observable variable that we construct from the flow of funds data and captures the "liquidity share"− the share of liquid assets in the economy. In addition to using this observable to calibrate the shock in the model, we also use the 1 trillion dollar intervention to calibrate the non-standard policy reaction-function of the government. This allows us not only to explore the quantitative effect of the crisis shock, but also illustrate to counterfactual evolution of the economy had the Federal Reserve not intervened.

We embed the KM credit frictions in a relatively standard dynamic stochastic general equilibrium (DSGE) model along the lines of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). This model has standard frictions, such as wage and price rigidities and aggregate capital adjustments costs. Standard monetary policy is then

[2] The quantitative impact of this friction has been explored in recent work by Liu et al. (2010).

variations in the nominal interest rate. Non-standard policy is open market operations in private assets that increase the overall level of liquidity in the economy.

Our first main result is that neither the financial shock nor the 1 trillion dollar intervention have a large quantitative effect in the absence of price and wage rigidities. Our second main result is that if we calibrate the other frictions in the model to values consistent with the existing literature, and monetary policy follows a standard Taylor rule (i.e., interest rate react more than one to one to inflation), then both the financial shock and non-standard policy have a significant effect. Our third result is that once the zero bound on the short-term nominal interest rate is introduced, and in the absence of the intervention, the economy may suffer a Great Depression-style collapse. In contrast, with intervention, the model exhibits similar response as now observed in the US economy. This is the "Great Escape" referred to in the title of the paper, since our numerical example illustrates that in the absence of non-standard policy the US economy could have suffered the second coming of the Great Depression. The reason why the effect of non-standard policy can be especially large at zero interest rates is similar to what is found in Christiano, Eichenbaum and Rebelo (2009) and Eggertsson (2009). They report that the "multiplier of government spending" is unusually large at zero interest rates.

This paper belongs to the strand of literature introducing financial frictions in monetary DSGE models, such as Bernanke, Gertler and Gilchrist (1999), Christiano, Motto and Rostagno (2003, 2009), Goodfriend and McCallum (2007) and Curdia and Woodford (2009*a*). Gertler and Karadi (2009), Gertler and Kiyotaki (2009) and Curdia and Woodford (2009*b*) also analyze the role of non-conventional central bank policies in the current recession.

Before going further let us emphasize important limitations of the analysis. Our main objective is to understand if non-standard policy can have quantitative important implications, given the Wallace benchmark that they are irrelevant. To cast light on this question we have chosen a particular form of liquidity constraints proposed by Kiyotaki and Moore (2008) which we believe is natural as a first cut. It is worth stressing, however, that these liquidity constraints are "reduced form" in certain respects, which means that our model is not helpful in its current form to address longer-term question that should also be high on the research agenda. In particular our approach is silent on whether interventions of the kind the Federal Reserve has conducted can have an effect the incentive structure of the private sector going forward, which may endogenously change the reduced form liquidity constraints we impose and take as given. More generally, we do not model the costs of intervening, which can be many. Therefore this is not a normative paper, but a positive one: we show that non-standard monetary policy can be quantitatively important for macroeconomic stability in the short-run, and this suggest that understanding their consequences should be high on the research agenda.

2 The Model

The model consists of six different economic actors: entrepreneurs, capital producers, retail good producers, final good producers, workers, and government. We start the model's description with the entrepreneurs, as these are the least standard, and are at the heart of our model. Their problem is identical to that in Kiyotaki and Moore (2008), aside from the specification of the "liquid" assets, which in this paper consist of standard real and nominal bonds. The rest of the model is similar to a several recent DSGE studies.

2.1 Entrepreneurs

The economy is populated by a continuum of entrepreneurs $e \in (0,1)$, whose objective is given by:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} log(c_s(e)) \tag{1}$$

where $c_t(e)$ is the entrepreneur's consumption. Entrepreneurs have random investment opportunities, and need to acquire resources to take advantage of it. Specifically, they accumulate capital according to the law of motion:

$$k_{t+1}(e) = \begin{cases} \lambda k_t(e) + i_t(e) & \text{with probability } \varkappa \\ \lambda k_t(e) & \text{with probability } 1 - \varkappa \end{cases}, \tag{2}$$

where $0 < \lambda < 1$ is one minus the depreciation rate, and $i_t(e)$ represents investment. We assume that $\beta > \lambda$. Investment opportunities are i.i.d. across time and entrepreneurs. Entrepreneurs rent out capital to firms and earn the rental rate r_t^k. Equity represents claims on the future stream of such rental rates, and its per-unit value expressed in units of the consumption good is q_t. The main problem of the entrepreneurs is that they may not have enough money to finance this investment.

Financial market imperfections are modeled as constraints on the evolution of the balance sheet of the entrepreneur. The entrepreneur's liabilities consist of claims issued on her own capital, $n_t^I(e)$. Her assets consist of the residual claims on her own capital, $k_t(e) - n_t^I(e)$, and on claims on the equity of other entrepreneurs $n_t^O(e)$. Moreover, the entrepreneur owns real bonds $l_t(e)$ – a government issued risk-free bond that pays a gross return r_t in terms of units of the consumption good. Finally, she has an amount $b_t(e)$ of risk-free nominal government bond that pay a gross nominal interest rate R_t. The value of 1 \$ of nominal government bonds in terms of output is $1/P_t$, where P_t is the price level. The entrepreneur's balance sheet at the end of period t looks as follows:

Assets		Liabilities	
real bonds	$l_{t+1}(e)$	*own* equity issued	$q_t n_{t+1}^I(e)$
nominal bonds	$b_{t+1}(e)/P_t$		
equity of *other* entrepreneurs	$q_t n_{t+1}^O(e)$		
capital stock	$q_t k_{t+1}(e)$	net worth	$q_t n_{t+1}(e)$ $+l_{t+1}(e) + b_{t+1}(e)/P_t$

where $n_t(e) \equiv n_t^O(e) + (k_t(e) - n_t^I(e))$.

The constraints on the evolution of the entrepreneur's balance sheet are as follows. The entrepreneur's *Borrowing Constraint (BC)* implies that the entrepreneur can issue new equity only up to a fraction θ of investment $i_t(e)$. The *Resaleability Constraint on "own" equity (RCi)* implies that in any given period the entrepreneur can sell only a fraction ϕ_t^I of his equity holdings $k_t(e) - n_t^I(e)$. These two constraints together imply that the evolution of $n_t^I(e)$ is subject to the following inequality:

$$n_{t+1}^I(e) - \lambda n_t^I(e) \leq \underbrace{\phi_t^I \lambda (k_t(e) - n_t^I(e))}_{\text{RCi}} + \underbrace{\theta_t i_t(e)}_{\text{BC}}. \tag{3}$$

Finally, the *Resaleability Constraint on "other" equity (RCo)* implies that the entrepreneur can sell only a fraction ϕ_t^O of his equity stake in other entrepreneurs $n_t^O(e)$:

$$-(n_{t+1}^O(e) - \lambda n_t^O(e)) \leq \phi_t^O \lambda n_t^O(e). \tag{4}$$

Assume that $\phi_t^I = \phi_t^O = \phi_t$ and add (3) and (4) together to obtain:

$$n_{t+1}(e) \ \geq \ (1-\phi_t)\lambda n_t(e) + (1-\theta_t) i_t(e). \tag{5}$$

Note that RCi and BC are similar in spirit: the former is a resaleability constraint on existing equity, the latter on new equity. In addition, the entrepreneur cannot hold negative liquid assets or reserves:

$$m_{t+1}(e) \ \geq \ 0, \tag{6}$$

$$b_{t+1}(e) \ \geq \ 0. \tag{7}$$

The entrepreneur's intertemporal budget constraint (flow of funds) is given by:

$$\begin{aligned} & c_t(e) + p_t^I i_t(e) + q_t(n_{t+1}(e) - \lambda n_t(e) - i_t(e)) + l_{t+1}(e) - r_{t-1} l_t(e) \\ & + \tfrac{b_{t+1}(e) - R_{t-1} b_t(e)}{P_t} \ \leq \ r_t^k n_t(e), \end{aligned} \tag{8}$$

where p_t^I is the price of investment in terms of general output, and where we used the law of motion of capital (2) and the definition of $n_t(e)$.

2.2 Capital Goods Producers

Capital producers transform the consumption goods into capital goods and operate in a national market. They choose the amount of investment goods produced I_t so to maximize profits

$$C(I_t) = p_t^I I_t - I_t[1 + S(\frac{I_t}{I_*})], \tag{9}$$

taking the price of capital in terms of consumption goods p_t^I as given. The adjustment cost function depends on steady state aggregate investment I^* and we assume that $S(1) = S'(1) = 0$ and $S''(1) > 0$. Capital producers' profits, which can differ from zero outside of steady state, are rebated to workers.

2.3 Final Good Producers

Final goods producers operate in monopolistic competition. They buy intermediate goods $y_t(i)$, where $i \in (0,1)$, on the market at a price $p_t(i)$, expressed in units of the final good (that is , $p_t(i) = P_t(i)/P_t$, where $P_t(i)$ and P_t are the Dollar prices of intermediate good i and the final good, respectively). They package the final good y_t according to:

$$y_t = \left[\int_0^1 y_t(i)^{\frac{1}{1+\lambda_f}} di\right]^{1+\lambda_f}, \tag{10}$$

where $\lambda_f > 0$ and resell it to consumers. Their problem is therefore:

$$\max_{y_t, y_t(i)} y_t - \int_0^1 p_t(i) y_t(i) di, \text{ subject to } (10) \tag{11}$$

taking $p_t(i)$ as given. The maximization problem of the final goods producers gives rise to the following demand function for the intermediate good:

$$y_t(i) = p_t(i)^{-\frac{1+\lambda_f}{\lambda_f}} y_t, \tag{12}$$

and the zero profit condition implies that the aggregate price level is given by:

$$P_t = [\int_0^1 P_t(i)^{-\frac{1}{\lambda_f}} di]^{-\lambda_f}.$$

2.4 Intermediate Goods Producers

Intermediate goods producing firms are perfectly competitive. They use the technology:

$$y_t(i) = A_t k_t(i)^{\gamma} h_t(i)^{1-\gamma}, \tag{13}$$

with $0 < \gamma < 1$, where $k_t(i)$ is the firm's capital input and $h_t(i)$ is labor input, and productivity A_t is common across firms. Labor and capital are hired in competitive markets at the real wage w_t and rental rate r_t^k, respectively. The firm's profit, written in unit of the final good, is given by:

$$\mathcal{P}_t(i) = p_t(i) y_t(i) - w_t h_t(i) - r_t^k k_t(i), \tag{14}$$

Prices are sticky as in Calvo (1983). Specifically, each firm can readjust nominal prices with probability $1-\zeta_p$ in each period. The firm's objective is to maximize the present discounted value of profits:

$$max_{\{p_t(i),y_t(i),k_t(i),h_t(i)\}_1^\infty} E_t \sum_{t=1}^{\infty} \beta^t \Xi_t^p \mathcal{P}_t(i), \text{ subject to (12) and (13)}, \tag{15}$$

where Ξ_t^p is the marginal utility of consumption of workers, who own the firms.

2.5 Workers

The economy is populated by a continuum of workers $\omega \in (0,1)$, all belonging to the same family, but supplying differentiated labor $h_t'(\omega)$. Consumption c_t', real bonds l_t', nominal bonds b_t', and equity n_t' are going to be the same for all workers within the family. The family's flow of funds is given by:

$$\begin{aligned} c_t' + q_t(n_{t+1}' - \lambda n_t') + l_{t+1}' - r_{t-1} l_t' + \frac{b_{t+1}' - R_{t-1} b_t'}{P_t} \\ \leq \; r_t^k n_t' + \int \frac{W_t(\omega)}{P_t} h_t'(\omega) d\omega + C(I_t) + \int \mathcal{P}(i) di + \tau_t, \end{aligned} \tag{16}$$

where $W_t(\omega)$ are nominal wages and τ_t are lump-sum transfers from the government, and where we assume that workers receive the profit from the intermediate firms and the capital producers. The family planner's objective function is given by:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} U[c_s' - \int \frac{\omega_0}{1+\nu} h_s(\omega)'^{\;1+\nu} d\omega], \tag{17}$$

where $U[.]$ is increasing and strictly concave, $\omega_0 > 0$, $\nu > 0$. Workers cannot borrow:

$$n_{t+1}' \geq 0, \tag{18}$$

$$l_{t+1}' \geq 0. \tag{19}$$

$$b_{t+1}' \geq 0. \tag{20}$$

The workers' differentiated labor $h_t'(\omega)$ is packed by competitive labor aggregators into a composite:

$$h_t' = \left[\int h_t'(\omega)^{\frac{1}{1+\lambda_{w,t}}} d\omega \right]^{1+\lambda_{w,t}}. \tag{21}$$

Since labor packers are competitive:

$$W_t h'_t = \int W_t(\omega) h'_t(\omega) d\omega$$

where $W_t = w_t P_t$ is the aggregate nominal wage in the economy. The worker is subject to nominal rigidities as in Erceg, Henderson, and Levin (2003). Specifically, the worker can readjust wages with probability $1 - \zeta_w$ in each period. For those that cannot adjust wages, $W_t(\omega)$ remains fixed.

2.6 The Government

The central bank's policy amounts to setting directly the nominal interest rate by an interest rate rule:

$$R_t = R_* \left(\pi_t / \pi^*\right)^{\psi}, \tag{22}$$

where π^* is the inflation target chosen by the central bank. In absence of intervention we assume that the amount of real and nominal debt issued by the government are constant:

$$L_{t+1} = L_*, \tag{23}$$

$$B_{t+1} = 0. \tag{24}$$

where for simplicity we have assumed that nominal bonds are in zero supply. The government budget constraint is:

$$L_{t+1} - r_{t-1} L_t = \tau_t, \tag{25}$$

where τ_t represents transfers to the the workers.

In case of intervention, the amount of equity $\tilde{N}_t^g$ purchased by the government at the beginning of period t follows:

$$\tilde{N}_t^g = K_* \xi (\frac{\phi_t}{\phi_*} - 1). \tag{26}$$

Differently from Kiyotaki and Moore (2008) we assume that the intervention occurs after the liquidity shock is revealed, but before entrepreneurs make their investment and saving decision (and before the idiosyncratic investment opportunity shock occurs). We model the intervention as a swap of liquid (real bonds) for illiquid assets (and viceversa, when the shock abates), which for each entrepreneur is proportional to her holdings of capital at the end of period $t-1$. In the aggregate, the intervention takes the form:

$$q_t(\tilde{N}_t^g - N_t^g) = \tilde{L}_t - L_t. \tag{27}$$

where $\tilde{N}_t^g$ is given by (26), and $\tilde{L}_t$ represents the amount of liquid assets issued to pay for the capital acquisition. Note that while the intervention is forced, it occurs at time t (hence, post shock) prices, so that in Dollar terms the entrepreneurs' balance sheet at the time of the intervention is unaffected, but now she has more liquid assets.

From the time of the intervention to the end of period (t) the government intertemporal budget constraint is:

$$q_t N_{t+1}^g - L_{t+1} + \tau_t = (r_t + q_t\lambda)\tilde{N}_t^g - r_{t-1}\tilde{L}_t + r_t^k \tilde{N}_t^g, \tag{28}$$

In principle when financial markets open the government could choose to further intervene and acquire a quantity of capital N_{t+1}^g different from $\tilde{N}_t^g$ (and correspondingly issue an amount of debt L_{t+1} different from $\tilde{L}_t$).[3] Our timing protocol precludes this from happening so that:

$$N_{t+1}^g = \tilde{N}_t^g \tag{29}$$

$$L_{t+1} = \tilde{L}_t. \tag{30}$$

2.7 Resource Constraints and Equilibrium

An equilibrium can now be defined as a sequence for $\{q_t, w_t, p_t^I, P_t, r_t^k, r_t, R_t\}$ such that: (i) Entrepreneurs choose $\{c_t(e),\, i_t(e),\, k_{t+1}(e)\ , n_{t+1}(e), l_{t+1}(e), b_{t+1}(e)\}$ to maximize (1) subject to the law of motion of capital (2), the resaleability/borrowing constraint (5), non-negativity of real and nominal bond holdings (6) and (7), and the flow-of-funds constraint (8); (ii) Capital producers choose I_t to maximize (9); (iii) Intermediate firms

[3] This is the form of the intervention in Kiyotaki and Moore (2008), that is, N_{t+1}^g is determined by (26), L_{t+1} is determined by the intertemporal budget constraint, while no intervention occurs before markets open ($\tilde{N}_t^g = N_t^g$ and $\tilde{L}_t = L_t$).

choose $\{p_t(i), y_t(i), k_t(i), h_t(i)\}$ to maximize (15) subject to the production function (13) and demand by the final goods producers; (iv) Final goods producers solve (11); (v) Workers choose $\{c'_t(\omega),\ l'_t(\omega), n'_{t+1}(\omega), m'_{t+1}(\omega), W_t(\omega), h_t(\omega)\}$ to maximize (17) subject to the flow-of-funds constraint (16), non-negativity of m holdings (19) and b holdings (20), non-negativity of equity holdings (18), and demand by the labor packers. (vi) Labor packers choose $\{h_t, h_t(\omega)\}$ subject to (21); (vii) Zero profit conditions for labor packers and final good producers hold; (viii) Markets for labor, output, liquid assets, nominal government bonds, and equity clear:

$$\int_i h_t(i) = h'_t, \tag{31}$$

$$Y_t = I_t[1 + S(\frac{I_t}{I_*})] + \int_e c_t(e) + \int_\omega c'_t(\omega), \tag{32}$$

$$L_t = \int_e l_t(e) + \int_\omega l'_t(\omega), \tag{33}$$

$$0 = \int_e b_t(e) + \int_\omega b'_t(\omega), \tag{34}$$

$$\int_e k_t(e) = \int_\omega n'_t(\omega) + \int_e n_t(e) + \tilde{N}^g_t. \tag{35}$$

where

$$I_t = \int_e i_t(e). \tag{36}$$

3 Characterizing a Solution

The Appendix show the first order conditions of the maximization problems listed in the equilibrium definition. It also shows how they can be aggregated so the equilibrium can be characterized in terms of aggregate variables. We take this set of equations and find a steady state of the model, also shown in the appendix. We then approximate the solution around this steady state. The resulting set of equation is linear, apart from the that we keep track of that the nominal interest rate cannot be lower than zero. Our main focus will be on solving the model when it is subject to a shock to ϕ_t. We will see that the zero bound will be binding in this case. We use the solution method described in Eggertsson (2008) to take account of the zero bound.

4 Calibration

Table 1

β	$=$	0.991		Discount factor
$\varkappa$	$=$	0.06		Probability of investment opportunity
λ	$=$	0.975		$1-$ Depreciation rate
$\phi = \theta$	$=$	0.15		Resaleability and borrowing constraint
$L_*/4Y_*$	$=$	0.40		Steady state liquidity/GDP
γ	$=$	0.33		Capital share
ψ	$=$	1.5		Taylor rule coefficient
$\zeta_p = \zeta_w$	$=$	0.66		Price/Wage Calvo probability
$\lambda_p = \lambda_p$	$=$	0.1		Price/Wage steady state markup
$S''(1)$	$=$	3		Adjustment cost parameter
ν	$=$	1		Inverse Frisch elasticity
		Baseline	Great Escape	
ϕ_L	$=$	-0.820	-0.547	Size of the liquidity shock
ζ_{ZB}	$=$	0.125	0.031	Probability of exiting the zero bound
ξ	$=$	-0.035	-0.040	Government intervention coefficient

We calibrate the model at quarterly frequency. A key variable for our model, and hence for our calibration, is the liquidity share:

$$ls_t \equiv \frac{L_{t+1}}{L_{t+1} + q_t K_{t+1}}.$$

which measures the relative quantity of liquid and illiquid assets in the economy. We use data from the U.S. Flow of Funds between 1952q1 and 2008q4 to construct the empirical counterpart of the liquidity share in the model. Our measure of liquid assets consists of all liabilities of the Federal Government, that is, Treasury securities (L.106 line 17) net of holdings by the monetary authority (L.106 line 12) and the budget agency (L.209 line 20) plus reserves (L.108 line 26), vault cash (L.108 line 27) and currency (L.108 line 28) net of remittances to the Federal Government (L.108 line 29). All other assets in the U.S. economy fall into our notion of capital stock. We consolidate the balance sheet of households, non-corporate and corporate sector to obtain the market value of aggregate capital. For households, we sum real estate (B.100 line 3), equipment and software of non-profit organizations (B.100 line 6) and consumer durables (B.100 line 7). For the non-corporate sector, we sum real estate (B.103 line 3), equipment and software (B.103 line 6) and inventories (B.103 line 9). For the corporate sector, we obtain the market value of the capital stock by summing the market value of equity (B.102 line 35) and liabilities (B.102 line 21) net of financial assets (B.102 line 6). We then subtract from the market value of capital for the private sector the government credit market instruments (B.106 line 5), TARP (B.106 line 10) and trade receivables (B.106 line 11). Two qualifications are in order. First, no data is available for the physical capital stock of the financial sector. Second, in our calculations we do not net out liquid and illiquid assets held by the rest of the world. It turns out that if we do net out both liquid and illiquid assets held by the rest of the world the numbers are not very different, since the rest of the world, on net, holds both liquid (government liabilities) and illiquid (private sector liabilities) in roughly the same proportion. The liquidity share calculated taking into account the foreign sector averages 10.56% over the sample period and exhibits very similar dynamics. We fix the steady state liquidity share in the model at its sample mean, equal to 12.64%. At the apex of the financial crisis (i.e. right after Lehman's collapse) the liquidity share jumped to 15.48% in 2008q4 from a value of 12.23% in 2008q3 consistent with its historical average.

Figure 2 shows the evolution of the liquidity share over the sample. We use this variable for two purposes. First, we use it to quantify the magnitude of the liquidity shock during the crisis. As the liquidity shocks hits, liquid assets become relatively more valuable than illiquid ones since entrepreneurs can resell the former but not the latter, thereby raising the liquidity share, ceteris paribus. At the same time we have a government intervention that increases the amount of liquid assets L_t. Since we know the size of this intervention (1 trillion shown in Figure 1), we can use the liquidity share to obtain a measure of the size of the shock. The next section describes this procedure in more detail. Second, we use the average liquidity share in the data to calibrate the steady state values of the financial friction parameters θ_* and ϕ_*, as we discuss below.

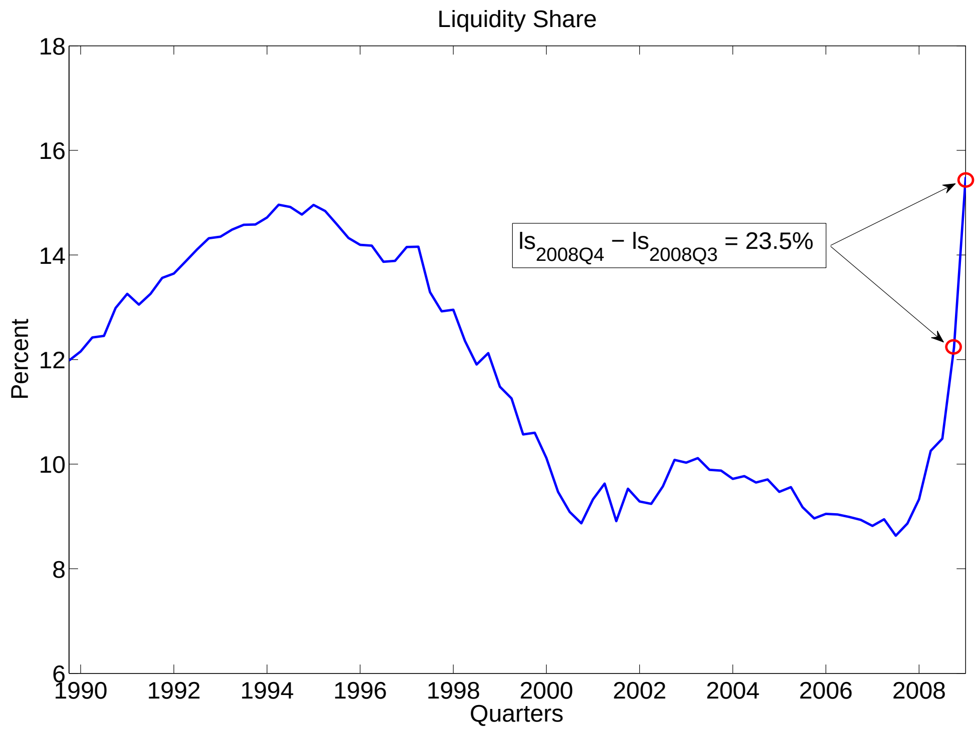

Figure 2: The liquidity share in the data.

In our model the steady state is a function of the overall amount of liquid assets present in the economy. Using the flow of funds data described above we compute the average ratio of liquid assets to GDP in the US economy for post-war data ($L_*/4Y_*$), which is about 40%.[4] For given amount of liquid assets L_*, Figure 3 shows that there is a monotonic relationship between the level of ϕ_* and the following variables: 1) liquidity share, 2) the level of q_*, 3) the expected return on the liquid asset, and 4) the average difference in expected returns between the illiquid and liquid assets (the equity premium). These monotonic relationships are fairly intuitive. As ϕ_* drops and financial frictions worsen, the value of installed capital q_* increases (upper right chart). For a given amount of liquid assets L, this leads to a decline in the liquidity share $L_*/(L_* + q_*K_*)$ (upper left chart). At the same time, as ϕ_* drops liquid assets become more valuable, hence the

[4] We also computed this figure using CBO estimates for the overall amount of government liabilities, and found the same figure.

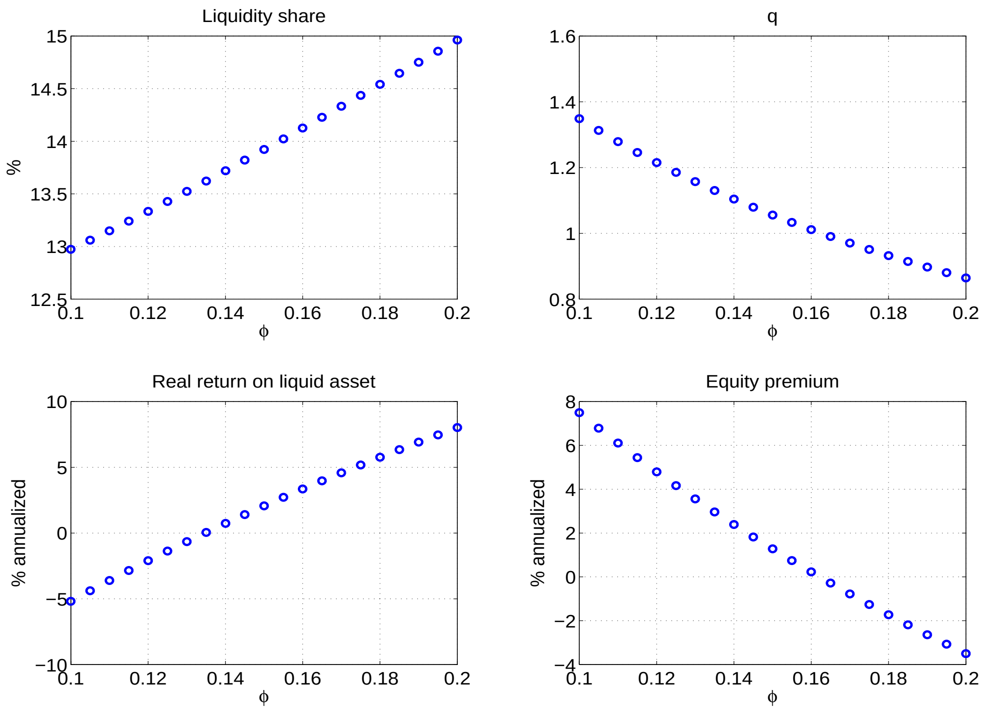

Figure 3: The relation between the resaleability constraint and some key endogenous variables.

return agents demand for holding them decreases (lower left chart). Together with the fact that capital is scarce in the constrained economy, and hence the return to capital is higher, this leads to an increase in the equity premium (lower right chart).

We do not have measures of q_* from the data and, given our broad definition of capital, we arguably do not have good measures of the equity premium. We do however have a (possibly imperfect) measure of the liquidity share as discussed above. Moreover, we do have measures of the average ex-post return on government liabilities. These range from 1.72 to 2.57 for short (one year) and long (ten years) maturities, respectively. Of course these relationships depend also on the other model parameters, and in particular on θ_*.[5] Since ϕ_* and θ_* both measure resaleability – θ_* on new capital and ϕ_* on existing

[5] They depend also on the degree of depreciation and the discount rate, which we discuss below.

one – we tie our hands and set $\theta_* = \phi_*$.[6] We set $\theta_* = \phi_* = .15$, which implies a liquidity share close to .14 and a real return on liquid assets of 2%. This value for the real return on government liabilities is achieved with the subjective discount factor β set to 0.991.

The literature on investment spikes suggests between 20% (Doms and Dunne, 1998) and 40% (Cooper, Haltiwanger and Power, 1999) of U.S. manufacturing plants adjust their capital each year. Based on this evidence, we calibrate the probability of receiving an investment opportunity in each quarter $\varkappa$ to 6%. This value is probably close to an upper bound for the average frequency of investment to the extent that in the data very few plants adjust their capital stock more than once a year.

The remaining parameters correspond to standard values in the business cycle literature. We set the inverse Frisch elasticity of labor supply ν to 1. We choose a capital share γ of 1/3, an annual depreciation rate of 10% ($\lambda = 0.975$) and a moderate degree of investment adjustment costs ($S''(1) = 3$). The average duration of price and wage contracts is equal to 3 quarters ($\zeta_p = \zeta_w = 2/3$). This value lies roughly in the middle of the estimates of Bils and Klenow (2004) and Nakamura and Steinsson (2008) for prices and possibly on the low side for wages. We also calibrate symmetrically the degree of monopolistic competition in labor and product markets assuming a steady state markup of 10% ($\lambda_p = \lambda_w = 0.1$).

Finally, we set the feedback coefficient on inflation in the interest rate rule (22) to 1.5, a value commonly used in the literature. The quantitative results depend crucially on the expected duration of the shock, whose process is discussed more in the next section. We choose the expected duration of the shock to be 8 quarters, i.e. agents expect the zero bound to be binding for 8 quarters, as will be further discussed. This value is consistent with survey evidence of market participants during the crisis. Later, we present results based on more extreme expectations of financial disruption.

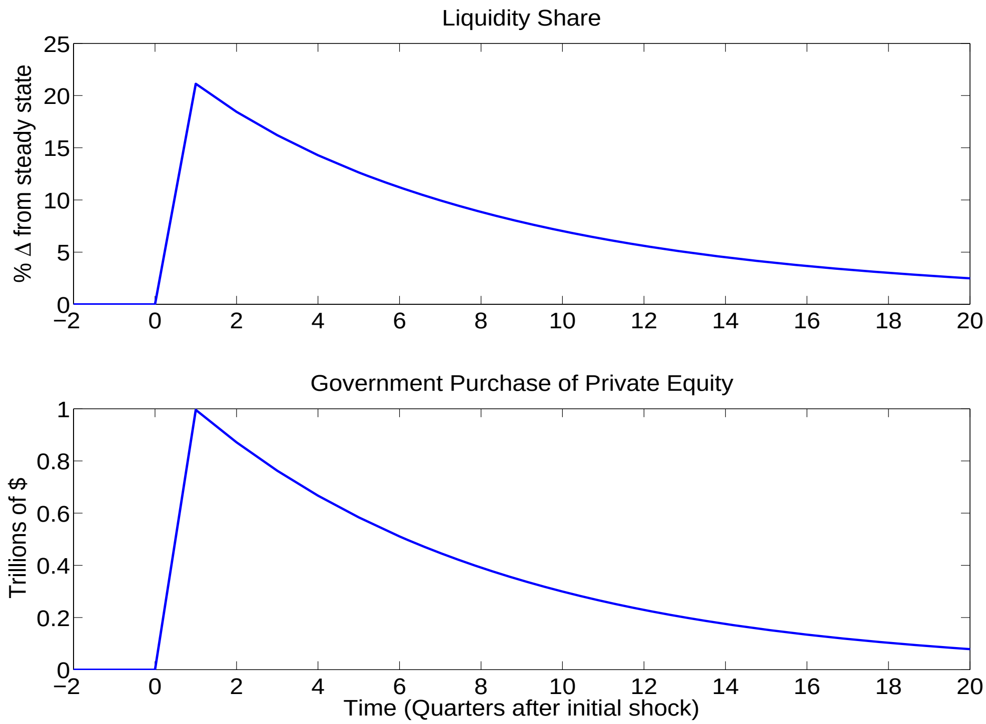

Figure 4: Calibrated intervention and liquidity share.

5 Simulating the Financial Crisis: Two Examples

This section describes how we capture the financial crisis and the response of macroeconomic and financial variables. To solve the model we assume that the shock variable ϕ_t follows a two state Markov Process. There is one "crisis" state, $\hat{\phi}_L < 0$ and one "normal" state $\hat{\phi}_H = 0$ where hat indicates deviations from steady state. In period 1 we assume that the crisis state hits, starting from steady state in period 0. There is then a transition probability ζ_{ZB} that the shock goes back to normal (set at 0.125 in the baseline calibration, i.e., the expected duration of the shock is 8 quarters). Once the shock returns to normal (at some stochastic date τ) we assume it stays there forever.

Two parameters remain to be chosen, the size of the crisis $\hat{\phi}_L < 0$ and the government

[6] We also conducted experiments where this is the case for the shock as well, and the results do not change much.

response to the crisis in terms of liquidity provision ξ. The parameter ξ measures how much liquidity (government debt/money) the Fed pumps into the system in response to the shock $\hat{\phi}_{L.}$ We pick these two parameters $(\hat{\phi}_L, \xi)$ simultaneously to match two features of the data, (i) An increase in the liquidity share of about 20%, consistent with the evidence between 2008q3 and 2008q4; (ii) A government intervention of about $1 trillion, consistent with the increase in the asset side of the Fed's balance sheet after the collapse of Lehman Brothers. This results in $\hat{\phi}_L = -0.82$, i.e. there is a 82 percent decline in the resaleability of equity in the secondary market. Figure 4 shows the impulse response from the perspective of period 0 in the model for the liquidity share.

We also report the results from a calibration that is a bit less conservative than described above. A key parameter is the persistence of the shock, i.e. the probability of the crisis being over, ζ_{ZB}. We calibrated this parameter based on survey evidence, such that the expected duration for crisis was 8 quarters. The Great Depression in the US and the "Great Recession" in Japan were also characterized by zero interest rates, deflationary pressures and weakness in the financial sector. Common reference has been made to these episodes in popular and academic discussion. For example, Time magazine claimed that Ben Bernanke had "prevented another Great Depression" when choosing him as the man of the year 2009. These episodes, however, lasted much longer than 2 years so we regard our baseline as relatively "conservative". For comparison, we therefore also consider a shock with an expected lifetime of 8 years in later sections, a calibration we coin "extreme" while the two year is the "baseline".

6 Impact on Macroeconomic and Financial Variables with Intervention

The main question we are interested in is quantitative. In this section, we investigate whether the model can account for a significant portion of the movement in macroeconomic and financial variables observed in the data according to the baseline calibration. We start with the following headline variables: output, inflation, nominal interest rates, the stock market and the spread between equity and government bonds. In this first

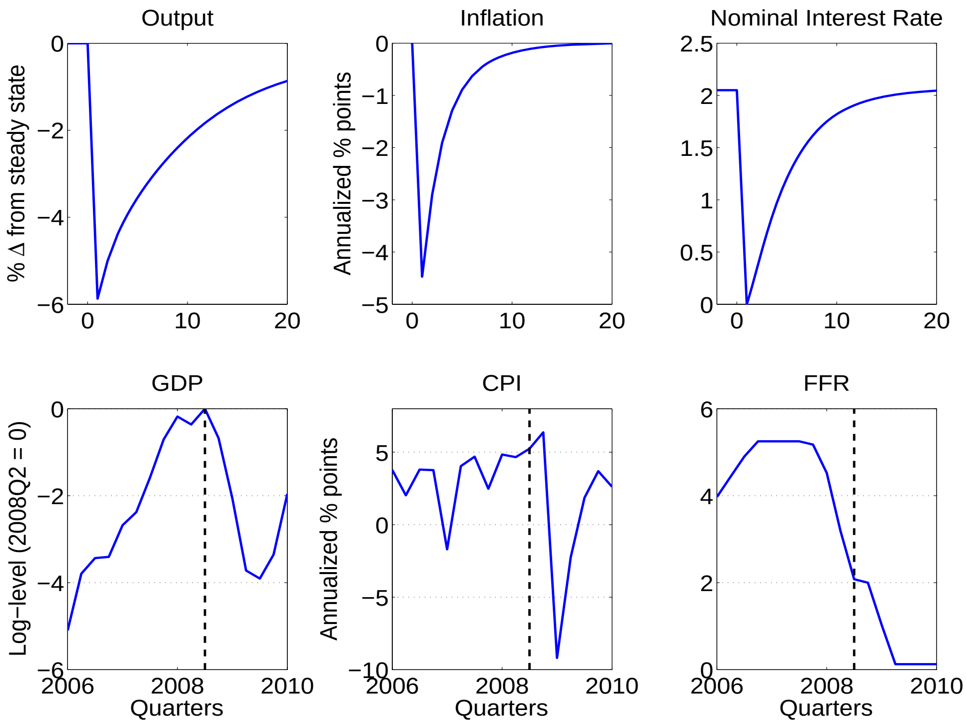

Figure 5: Response of key macro variables to a shock to resaleability of assets (with interventions).

set of results we show the evolution of these variables according to the model, taking the intervention into account. The next section asks what would have happened in the absence of intervention.

Figure 5 shows output inflation and the nominal interest rate in the data and compares it with the impulse responses from the model. We see that the model can explain a simultaneous drop in output inflation and interest rates, roughly of the same order as in the data. The model output, again, is an impulse response function, i.e. we show the expected path of each of the variable conditional on shock hitting in period 0.

The liquidity shock also generates a non-negligible response of financial variables in the model. Figure 6 displays the spread of equity versus the liquid asset, defined as the

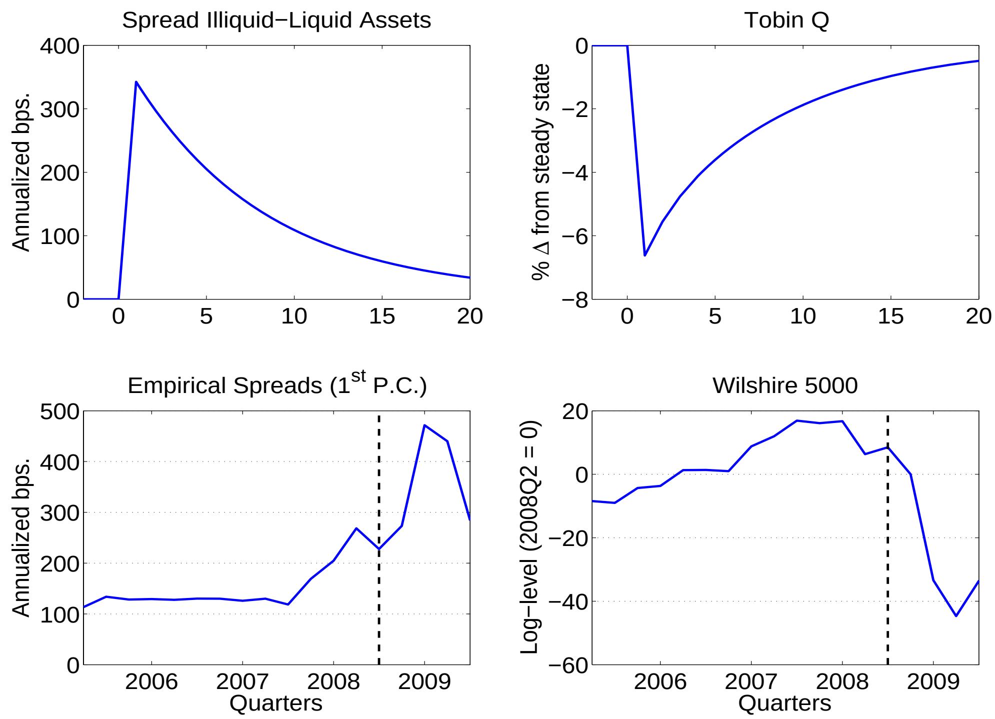

Figure 6: Response of financial variables to the resellability of assets (with intervention).

difference in expected returns between the two type of assets

$$\Psi_t^{km} \equiv \mathbb{E}_t \left(\frac{r_{t+1}^k + \lambda q_{t+1}}{q_t} - r_t \right). \tag{37}$$

The liquidity shock causes this spread to increase by about than 350 annualized basis points.

The spread in the model has no immediate empirical counterpart. However, the bottom panel of figure 6 shows that our results are consistent with the behavior during the crisis of the first principal component extracted from a number of spreads between U.S. Treasury securities at various maturities and corporate bonds of different ratings.[7]

The model also produces a drop in the stock market, measured in terms of the price of capital, of about 7 percent. This is of slightly lower order than in the data. We do not want to make too much out of this discrepancy, since a proper definition of the stock

[7] Thanks to Tobias Adrian for providing us with the data.

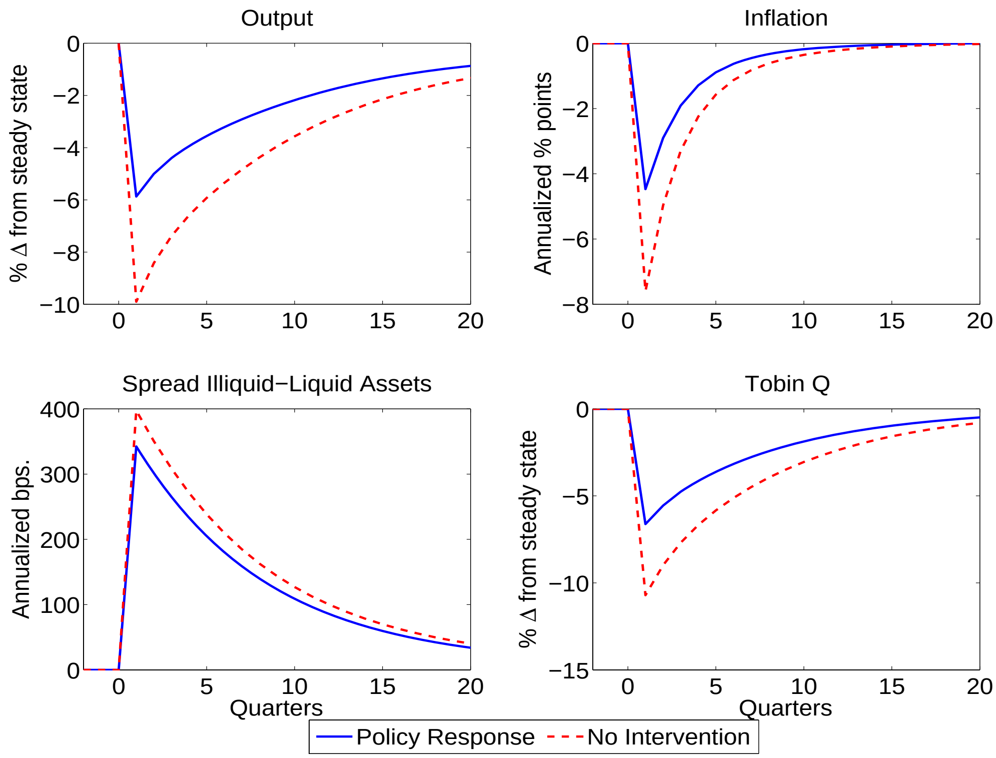

Figure 7: The effect of policy intervention.

market in our model should also measure the change in net present value of profits for the intermediate firms, an extension we leave for future revisions. The bottom line is that our simulated crisis generates movements in macroeconomic and financial variables following a liquidity shock in line with their empirical counterparts during the last months of 2008.

7 The Great Escape? Did the Fed Prevent a Second Great Depression?

Table 2		
	Baseline	Great Escape
Full model	0.643	2.240
No zero bound constraint	0.247	0.309
No nominal rigidities	0.064	0.052

We now consider the following question: What would have happened in the absence of a policy intervention? Figure 7 shows the answer to this question. In the absence of intervention the output drop would have almost doubled, going from a contraction of about 6 percent to 10 percent. A similar story can be told about the stock market and inflation. We also see the effect of the intervention on the spreads. The intervention reduces the spreads by about 50 basis points.

One interesting way of getting a feel for the quantitative significance of the intervention is to compute a "balance sheet multiplier" defined as

$$M_{B,0} = \frac{E_0 \sum_{t=0}^{\infty}(\hat{Y}_t^I - \hat{Y}_t^N)}{E_0 \sum_{t=0}^{\infty} \hat{N}_{t+1}^g}$$

where $\hat{Y}_t^I$ is output when there is intervention and $\hat{Y}_t^N$ is output without an intervention. The variable $\hat{N}_{t+1}^g$ measures the intervention (it is zero in its absence). This statistic answers the following question. By how much should output increase (in expected terms) for an expected dollar increase in liquidity? Our measure of the increase in output is the the area between the impulse response for output with and without intervention. The denominator, on the other hand, is the area under the impulse response of the Fed's balance sheet in the model. This multiplier is equal to 0.643. In expected terms a one dollar increase in the balance sheet increases aggregate output by about 60 cents. It is interesting to compare this number to the "multiplier of government spending". In the case of government spending one is usually measuring the effect of consuming some real resources and computing its impact on output. In case of our intervention, however, we are simply considering increasing government liquidity for some time but against this the government is holding privately issued equity, without consuming any real resources. In the model, in fact, not only is the private sector better off because of this, the government makes money on the transaction.

We now consider the equilibrium outcome under a more extreme scenario: What if the shock is expected to last for 8 years instead of 2, i.e. be of similar duration as the shocks perturbing the Japanese economy during the Great Recession or the US during the Great Depression? Figure 8 shows the equilibrium outcome in this case. Without intervention the equilibrium is a disaster. Output collapses by about 20 percent and deflation reaches double digits. In short, the equilibrium outcome starts looking a bit like the Great Depression. What is the effect of policy in this case? As shown by the solid line the policy response essentially creates a similar outcome as before, i.e. a recession of similar order as seen in the data. Hence the same amount of intervention (in dollar terms) becomes even more effective. The statistic for the balance sheet "multiplier" is now 2.24, i.e., the multiplier is three and a half times bigger than before. Experimenting with the model we have found this to be the case in all of the numerical results considered, i.e. as the outcome becomes more unstable, then policy tends to become more effective. This is a similar conclusion as reached by Christiano et al (2009) and Eggertsson (2009) with respect to the government spending multiplier, what Christiano et al (2009) have coined "the divine coincidence", i.e., policy happens to be most effective for the parameter configurations which create the biggest disasters.

8 The Role of the Zero Bound

An impulse response only shows the expected path. It does not show a particular history in the model. This is particularly important to keep in mind when interpreting the impulse response for the nominal interest rate. The impulse response function in figure 5 shows that the nominal interest rate is expected to be above zero after the first period. Underlying this calculation are several "contingencies" in which the zero bound stops being binding once the shock is over. It is important to stress, however, that as long as

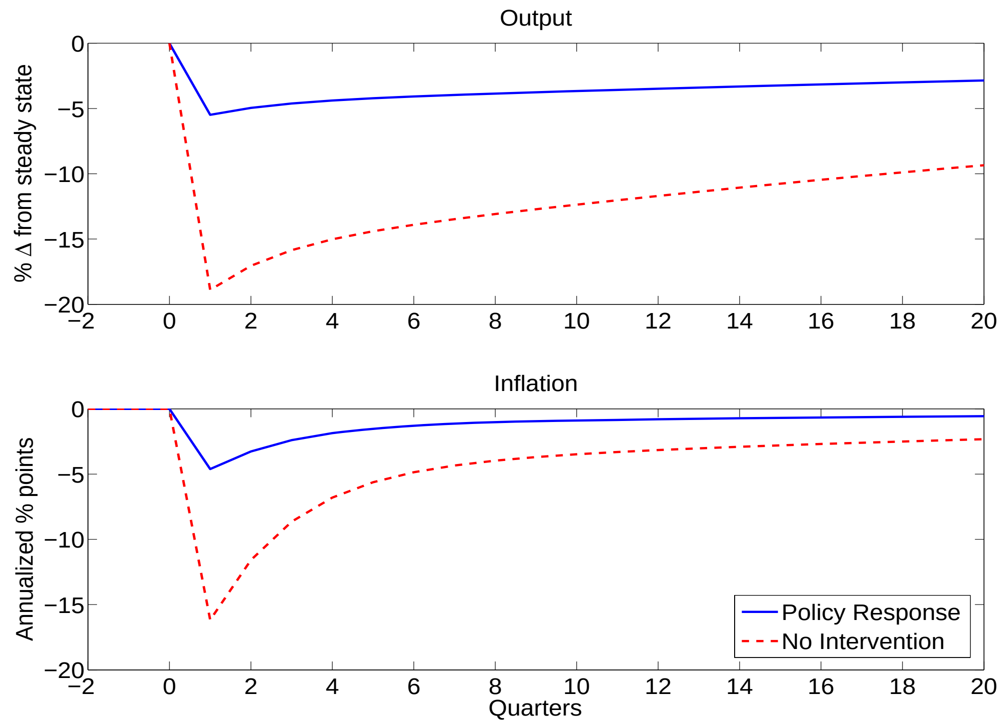

Figure 8: The Great Escape?

the shock is in its "low state", i.e., ϕ_L, *the zero bound remains binding.* Figure 9 shows all possible state contingent paths for the nominal interest rate. The first dashed line shows the evolution if the shock reverses to steady state in period 1, the second if it reverses in period 2 and so on. We see that when the shock is over the nominal interest rates will temporarily rise above its steady state. Note that the zero bound is always binding as long as the shock in the low state. The impulse response is then derived by weighting each possible history with the probability of it occurring. The same, of course, underlies the derivation of all the impulse response functions we have shown for various variables.

How important is the fact that the zero bound is binding whenever $\phi_t = \phi_L$? Figure 10 shows the effect of the shocks with and without interventions when the government follows the Taylor rule but does not respect the zero bound. We see that the interest rate becomes negative on impact by about 3.5 percent (without intervention). As a conse-

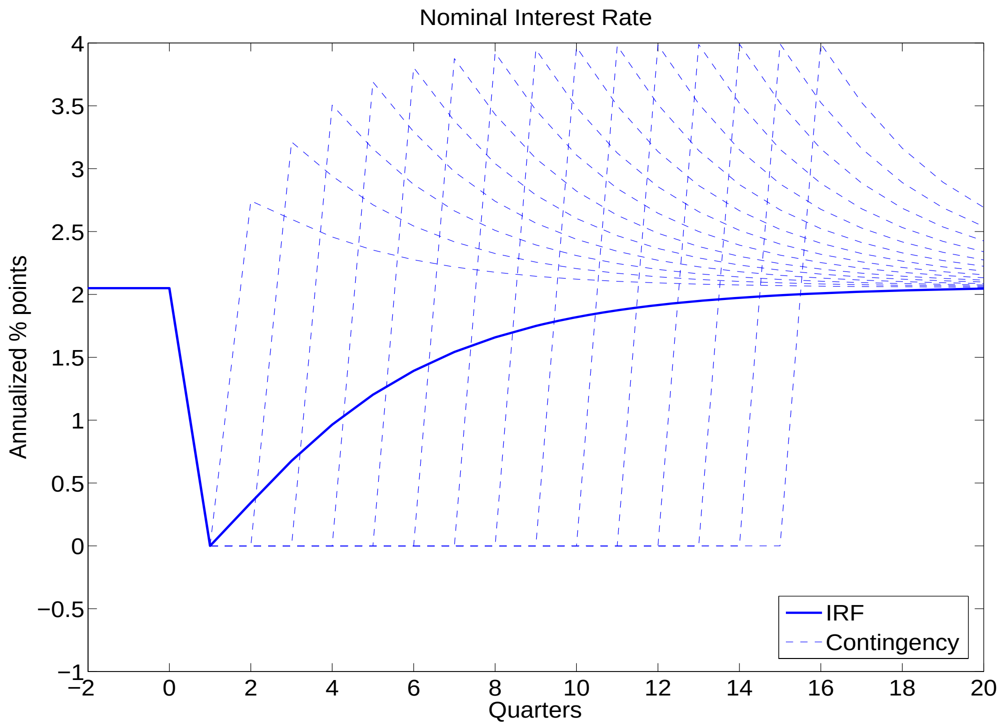

Figure 9: State contingent paths for the nominal interest rate.

quence the economy contracts by much less. The effect of the balance sheet expansion is much smaller in this case. The balance sheet multiplier is now about three times smaller, only about 0.25 as shown in Table 2. As emphasized in the literature on the zero bound, what is going on here is not only the fact that the zero bound is binding in a given period. What is even more important is that people *expect it to be binding in the future*, which then leads to lower expectations of future income and expected deflation, both of which contract demand. One way of seeing the importance of this force is to observe that the difference between the solution with and without the zero bound becomes even larger in our "extreme Great Depression" calibration. In this case people expect the shock to last longer, and hence the zero bound plays an even bigger role due to the interaction of expectations and demand. In this case the balance sheet multiplier is almost 8 times larger once the zero bound is taken into account as seen in Table 2, as opposed to being 3 times larger in the baseline calibration. The reason for this larger difference is that if

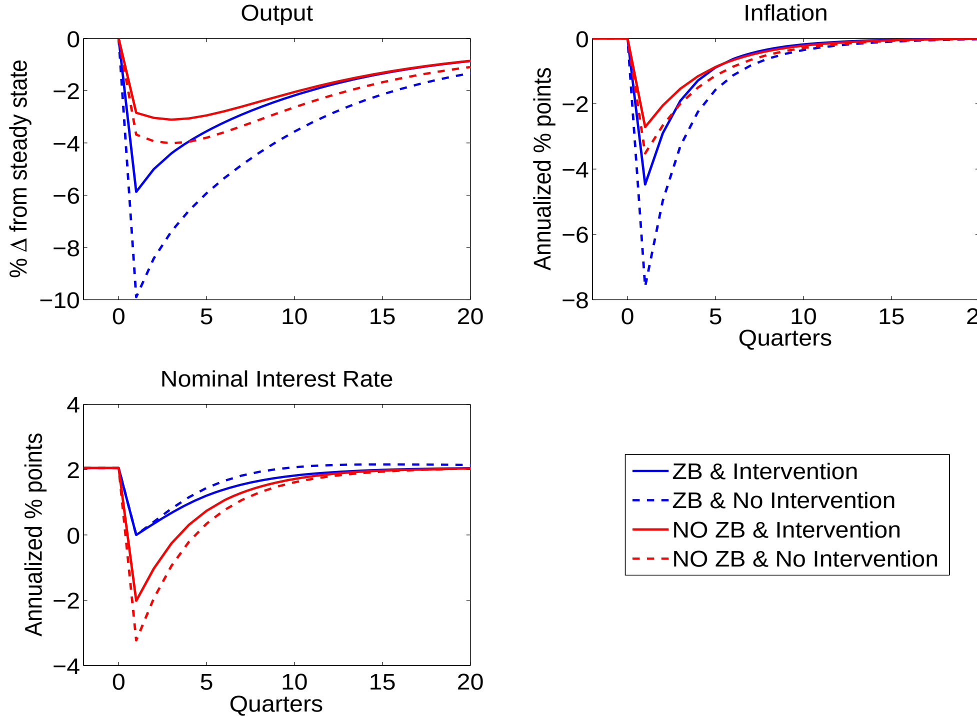

Figure 10: The role of the zero bound

people expect the crisis to last for a long time, the role of expectations of future balance sheet expansion in states of the world in which the zero bound is binding becomes ever more important.

9 The Role of Nominal Frictions

Our model is a full scale DSGE model with a variety of frictions. Apart from the financial frictions, of greatest importance are the nominal friction, i.e., that both prices and wages are set only at stochastic intervals. This means that labor and output are demand determined, i.e. the firms and workers commit to supply goods and labor as demanded at the price they post. How important are these rigidities to explain the collapse in output and the consequences of the Fed's balance sheet intervention? Answer: Very important.

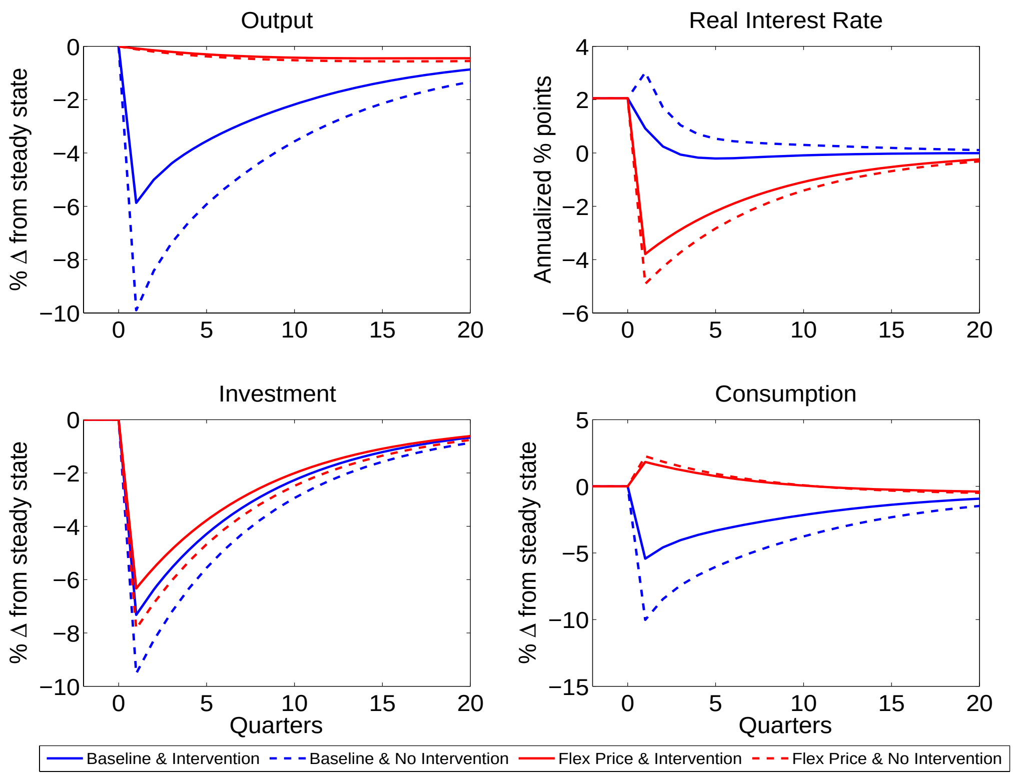

Figure 11: The role of sticky prices and wages.

Figure 11 shows output under the baseline and then also when prices and wages are perfectly flexible. This model is almost identical to the original Kiyotaki-Moore (2008) model. Overall, we see that under flexible prices/wages there is almost no drop in output as a result of the shock. What is the reason for this? It is helpful to study the composition of aggregate output to understand the answer to this question. Output consists of two components, consumption and investment spending. The negative shock causes a big drop in investment with or without flexible prices as we can see in the third panel in Figure 11 . The reason for this is that the shock makes it harder for an investing entrepreneurs to sell their existing assets. Thus they will be able to finance less investment. Because every entrepreneurs invests as much as they can when they have an investment opportunity, this means that aggregate investment will go down. We see that the assumption of price frictions does not matter much in determining by how much investment collapses. They key difference lies in the response of the other component of output, that is, consumption. The reason why there is so little drop in output under flexible prices is that the drop in investment is met *with a large boom in consumption.* In contrast, when prices/wages are rigid, consumption collapses with investment.

Why the difference? It is helpful to understand this by looking at what happens to the real interest rate under flexible prices, which will be equal to the difference between the nominal interest rates and expected inflation, i.e.

$$\hat{r}_t = \hat{R}_t - E_t\pi_{t+1}$$

As we can see in Figure 11, the real interest rate becomes negative in the flexible price/wage variation of the model. This happens so as to induce people to consume more to make up for the collapse in investment. This change in the real interest rate is hard to achieve when prices are rigid. If all prices were fixed, then $E_t\pi_{t+1} = 0$. Negative real interest rates could only be achieved with negative nominal interest rates, which is obviously a problem. With some price flexibility, the problem becomes even harder. The contraction is associated with *expected deflation* in the model, which makes the real interest rate even higher, thus going in the opposite direction of what the flexible price allocation calls for. That explains the disaster we see under rigid prices. While negative real interest rates are required, the interaction of the zero bound and price frictions lead to very high real interest rate (due to expected deflation) which causes consumption to collapse with investment. The result can be a Great Depression scenario.

10 Conclusions

In this paper we studied the effect on non-standard monetary policy using a theory of credit friction proposed by Kiyotaki and Moore (2008). We found that non-standard policy can have large effects. This is particularly true at zero interest rates. We showed a numerical example in which the model economy generates a collapse of the same order as the Great Depression in the absence of non-standard government policies.

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11 Appendix

Entrepreneur's problem: The entrepreneur's problem depends on whether the constraint (5) is binding. Under the following parametric configuration:

Condition 1 $(1-\lambda)\theta + \varkappa\lambda\phi > (1-\lambda)(1-\varkappa)$,

the credit and resaleability constraints are loose enough that the constraint is not binding, at least in the neighborhood of steady state. Note that this condition makes sure that constraint 5 is not bounding at steady state (for $I_t = \int_0^{\varkappa} i_t(e) = (1-\lambda)K_*$, and $\int_0^{\varkappa} n_t(e) = \varkappa K_*$).

Claim 1 *Under condition 1 the following holds in a neighborhood of the steady state: (i) the resource allocation is first best; (ii)* $q_t = 1$*; (iii) Return to capital approximately equals the time preference rate minus depreciation:* $r_t^k \sim \frac{1}{\beta} - \lambda$*.*

Note that since Tobin's q is one, there is no advantage from having investment opportunities: entrepreneurs are indifferent between buying equity in the market and producing it. We want to restrict attention to an equilibrium where q_t is greater than one. Hence we make the following assumption:

Assumption 1 *:* θ_*, ϕ_* *are such that* $q_* > 1$*.*

Under assumption 1 $q_t > 1$ in a neighborhood of the steady state.

Claim 2 *Under assumption 1 the following holds in a neighborhood of the steady state: (i) the resaleability constraint (5) is binding for an entrepreneur with investment opportunities; (ii) the resaleability constraint (5) is not binding for an entrepreneur without investment opportunities; (iii) An entrepreneur with investment opportunities (*$e^i \in (0, \varkappa)$*, where* i *is for saver) will choose not to hold the liquid assets:* $l_{t+1}(e^i) = 0,\ b_{t+1}(e^i) = 0$*;*

We prove these claims at the end of the appendix.

Since for an entrepreneur with an investing opportunity constraint (5) binds we can then make her problem simpler and substitute for $n_{t+1}(e)$ in (5) into the intertemporal budget constraint (8), and obtain:

$$c_t(e^i) + (p_t^I - q_t\theta_t)i_t(e^i) + l_{t+1}(e^i) + \frac{b_{t+1}(e^i)}{P_t} \leq r_{t-1}l_t(e^i) + \frac{R_{t-1}b_t(e)}{P_t} + (r_t^k + q_t\phi_t\lambda)n_t(e^i). \tag{K.38}$$

The right hand side indicates the resources available to the entrepreneur: income from equity, the fraction of equity that can be sold in the market, plus previous period money holdings. The left hand side indicates that the investing entrepreneur can use these resources to consume or to finance the fraction of investment for which she cannot borrow. Alternatively, we can substitute for $i_t(e)$ in (5) into (8) and obtain:

$$c_t(e^i)+q_t^R n_{t+1}(e^i)+l_{t+1}(e^i)+\frac{b_{t+1}(e^i)}{P_t} \leq r_{t-1}l_t(e^i)+\frac{R_{t-1}b_t(e^i)}{P_t}+[r_t^k+((1-\phi_t)q_t^R+\phi_t q_t)\lambda]n_t(e^i), \tag{K.39}$$

where

$$q_t^R = \frac{p_t^I - \theta_t q_t}{(1-\theta_t)} \tag{K.40}$$

is the *effective replacement cost of equity* for the investing entrepreneur: she needs a downpayment of $p_t^I - \theta_t q_t$ output units to obtain $(1-\theta_t)$ of inside equity. Note that $q_*^R < 1$ whenever $q_* > 1$, since $p_*^I = 1$. For an entrepreneur without an investing opportunity $e^s \in (\varkappa, 1)$ (where s is for saver) the intertemporal budget constraint is:

$$c_t(e^s) + q_t n_{t+1}(e^s) + l_{t+1}(e^s) + \frac{b_{t+1}(e^s)}{P_t} \ \leq \ r_{t-1}l_t(e^s) + \frac{R_{t-1}b_t(e^s)}{P_t} + (r_t^k + q_t\lambda)n_t(e^s). \tag{K.41}$$

The Lagrangian for an investing entrepreneur is:

$$\begin{aligned}
\mathcal{L} = \quad & [log(c_t(e^i)) - \eta_t(e^i)(c_t(e^i) + q_t^R n_{t+1}(e^i) + l_{t+1}(e^i) + \tfrac{b_{t+1}(e^i)}{P_t} + \dots) \\
& + z_{m,t}(e^i)l_{t+1}(e^i) + z_{b,t}(e^i)b_{t+1}(e^i)] + \varkappa\beta E_t[log(c_{t+1}(e^i)) - \eta_{t+1}(e^i)(c_{t+1}(e^i) + \dots \\
& \dots - [r_{t+1}^k + ((1-\phi_{t+1})q_{t+1}^R + \phi_{t+1}q_{t+1})\lambda]n_{t+1}(e^i) - r_t l_{t+1}(e^i) + \tfrac{R_t b_{t+1}(e^i)}{P_{t+1}})] \\
& + (1-\varkappa)\beta E_t[log(c_{t+1}(e^s)) - \eta_{t+1}(e^s)(c_{t+1}(e^s) + \dots - (r_{t+1}^k + q_{t+1}\lambda)n_{t+1}(e^i) - r_t l_{t+1}(e^i) \\
& + \tfrac{R_t b_{t+1}(e^i)}{P_{t+1}})] + \dots
\end{aligned}$$

where $\eta_t(e^i)$, $\eta_t(e^s)$, $z_{m,t}(e^i)$, and $z_{b,t}(e^i)$ are the multipliers associated with constraints (K.39), (K.41), (6),and (7), respectively. The first order conditions are:

$$
\begin{aligned}
&(\partial c_t(e^i)) && \tfrac{1}{c_t(e^i)} = \eta_t(e^i) \\
&(\partial n_{t+1}(e^i)) && q_t^R \eta_t(e^i) = \varkappa\beta E_t[\eta_{t+1}(e^i)[r_{t+1}^k + ((1-\phi_{t+1})q_{t+1}^R + \phi_{t+1}q_{t+1})\lambda]] \\
& && \quad + (1-\varkappa)\beta E_t[\eta_{t+1}(e^s)[r_{t+1}^k + q_{t+1}\lambda]] \\
&(\partial l_{t+1}(e^s)) && \eta_t(e^i) - z_{m,t}(e^i) = \varkappa\beta E_t[\eta_{t+1}(e^i) r_t] + (1-\varkappa)\beta E_t[\eta_{t+1}(e^s) r_t] \\
&(\partial b_{t+1}(e^s)) && \eta_t(e^i) - z_{b,t}(e^i) = \varkappa\beta E_t[\eta_{t+1}(e^i) R_t \tfrac{P_t}{P_{t+1}}] + (1-\varkappa)\beta E_t[\eta_{t+1}(e^s) R_t \tfrac{P_t}{P_{t+1}}]
\end{aligned}
$$

The Lagrangian for an entrepreneur without an investing opportunity is:

$$
\begin{aligned}
\mathcal{L} = \;& [log(c_t(e^s)) - \eta_t(e^s)(c_t(e^s) + q_t n_{t+1}(e^s) + l_{t+1}(e^s) + \tfrac{b_{t+1}(e^s)}{P_t} + \dots)] \\
& + \varkappa\beta E_t[log(c_{t+1}(e^i)) - \eta_{t+1}(e^i)(c_{t+1}(e^i) + \dots \\
& - [r_{t+1}^k + ((1-\phi_{t+1})q_{t+1}^R + \phi_{t+1}q_{t+1})\lambda] n_{t+1}(e^s) - r_t l_{t+1}(e^s) + \tfrac{R_t b_{t+1}(e^s)}{P_{t+1}})] \\
& + (1-\varkappa)\beta E_t[log(c_{t+1}(e^s)) - \eta_{t+1}(e^s)(c_{t+1}(e^s) + \dots - (r_{t+1}^k + q_{t+1}\lambda) n_{t+1}(e^s) \\
& - r_t l_{t+1}(e^s) + \tfrac{R_t b_{t+1}(e^s)}{P_{t+1}}(e^s))] + \dots
\end{aligned}
$$

where $\eta_t(e^i)$ and $\eta_t(e^s)$ are the multipliers associated with constraints (K.39) and (K.41), respectively. The first order conditions are:

$$
\begin{aligned}
&(\partial c_t(e^s)) && \tfrac{1}{c_t(e^s)} = \eta_t(e^s) \\
&(\partial n_{t+1}(e^s)) && q_t \eta_t(e^s) = \varkappa\beta E_t[\eta_{t+1}(e^i)[r_{t+1}^k + ((1-\phi_{t+1})q_{t+1}^R + \phi_{t+1}q_{t+1})\lambda]] \\
& && \quad + (1-\varkappa)\beta E_t[\eta_{t+1}(e^s)[r_{t+1}^k + q_{t+1}\lambda]] \\
&(\partial l_{t+1}(e^s)) && \eta_t(e^s) = \varkappa\beta E_t[\eta_{t+1}(e^i) r_t] + (1-\varkappa)\beta E_t[\eta_{t+1}(e^s) r_t] \\
&(\partial b_{t+1}(e^s)) && \eta_t(e^s) = \varkappa\beta E_t[\eta_{t+1}(e^i) R_t \tfrac{P_t}{P_{t+1}}] + (1-\varkappa)\beta E_t[\eta_{t+1}(e^s) R_t \tfrac{P_t}{P_{t+1}}]
\end{aligned}
$$

By equating the right hand side of the first order condition with respect to $l_{t+1}(e^s)$ and $n_{t+1}(e^s)$ for the entrepreneur without an investing opportunity:

$$
\begin{aligned}
&\varkappa E_t[\tfrac{1}{c_{t+1}(e^i)} r_t] + (1-\varkappa) E_t[\tfrac{1}{c_{t+1}(e^s)} r_t] = \\
&\varkappa E_t[\tfrac{1}{c_{t+1}(e^i)} \tfrac{r_{t+1}^k + ((1-\phi_{t+1})q_{t+1}^R + \phi_{t+1}q_{t+1})\lambda}{q_t}] + (1-\varkappa) E_t[\tfrac{1}{c_{t+1}(e^s)} \tfrac{r_{t+1}^k + q_{t+1}\lambda}{q_t}].
\end{aligned} \tag{K.42}
$$

Another arbitrage condition is between liquid assets and reserves:

$$
\begin{aligned}
&\varkappa E_t[\tfrac{1}{c_{t+1}(e^i)} r_t] + (1-\varkappa) E_t[\tfrac{1}{c_{t+1}(e^s)} r_t] = \\
&\varkappa E_t[\tfrac{1}{c_{t+1}(e^i)} \tfrac{R_t P_t}{P_{t+1}}] + (1-\varkappa) E_t[\tfrac{1}{c_{t+1}(e^s)} \tfrac{R_t P_t}{P_{t+1}}].
\end{aligned} \tag{K.43}
$$

Define $\mathcal{W}_t(e^i)$ and $\mathcal{W}_t(e^s)$ as the right hand side of (K.39) and (K.41) respectively (i.e., $\mathcal{W}_t(e^i) = r_{t-1}l_t(e^i) + \frac{R_{t-1}b_t(e^i)}{P_t} + [r_t^k + ((1-\phi_t)q_t^R + \phi_t q_t)\lambda]n_t(e^i)$ and $\mathcal{W}_t(e^s) = r_{t-1}l_t(e^s) + \frac{R_{t-1}b_t(e^s)}{P_t} + (r_t^k + q_t\lambda)n_t(e^s))$. We conjecture that the policy functions $c_t(e^i) = b\mathcal{W}_t(e^i)$ and $c_t(e^s) = b\mathcal{W}_t(e^s)$. Substituting these into the first order conditions one can see that these are indeed the solutions with $b = 1-\beta$. Hence we have:

$$c_t(e^i) = (1-\beta)(\ r_{t-1}l_t(e^i) + \frac{R_{t-1}b_t(e^i)}{P_t} + [r_t^k + ((1-\phi_t)q_t^R + \phi_t q_t)\lambda]n_t(e^i)\ ). \tag{K.44}$$

$$\text{and } c_t(e^s) = (1-\beta)(\ r_{t-1}l_t(e^s) + \frac{R_{t-1}b_t(e^s)}{P_t} + (r_t^k + q_t\lambda)n_t(e^s)\ ). \tag{K.45}$$

From (K.38) it follows that:

$$(p_t^I - q_t\theta_t)i_t(e^i) = \beta(\ r_{t-1}l_t(e^i) + \frac{R_{t-1}b_t(e^i)}{P_t} + (r_t^k + q_t\phi_t\lambda)n_t(e^i)\ ) - (1-\beta)(1-\phi_t)q_t^R\lambda n_t(e^i). \tag{K.46}$$

Note that since $q_t^R < 1 < q_t$, then $c_t(e^i) < c_t(e^s)$ and $\frac{r_{t+1}^k + ((1-\phi_{t+1})q_{t+1}^R + \phi_{t+1}q_{t+1})\lambda}{q_t} < \frac{r_{t+1}^k + q_{t+1}\lambda}{q_t}$. Therefore equity is "risky" relative to money in that it pays a low return when consumption is low.

Since $l_{t+1}(e^i) = 0$ the quantity $n_{t+1}(e^i)$ can be found by substituting (K.44) into (K.39) and obtain:

$$q_t^R n_{t+1}(e^i) = \beta(\ r_{t-1}l_t(e^i) + [r_t^k + ((1-\phi_t)q_t^R + \phi_t q_t)\lambda]n_t(e^i)\ ). \tag{K.47}$$

For the saving entrepreneur the overall amount of savings is given by:

$$q_t n_{t+1}(e^s) + l_{t+1}(e^s) + \frac{b_{t+1}(e^i)}{P_t} \;=\; \beta(\ r_{t-1}l_t(e^s) + \frac{R_{t-1}b_t(e^s)}{P_t} + (r_t^k + q_t\lambda)n_t(e^s)\ ).$$

Its composition between money, reserves, and equity holdings is such that:

Claim 3 *All non-investing entrepreneurs have the same equity and reserves to money holdings ratios.*

We prove the above claim at the end of the appendix.

The first order conditions for the <u>capital producers</u> imply:

$$p_t^I = 1 + S(\frac{I_t}{I_*}) + S'(\frac{I_t}{I_*})\frac{I_t}{I_*}. \tag{K.48}$$

Final goods producers' problem: The first order conditions for the final goods producers are:

$$(\partial y_t) \qquad 1 = \mu_{f,t} \tag{K.49}$$

$$(\partial y_t(i)) \quad -p_t(i) + \mu_{f,t}(1+\lambda_{f,t})[\dots]^{\lambda_{f,t}} y_t(i)^{-\frac{\lambda_{f,t}}{1+\lambda_{f,t}}} = 0 \tag{K.50}$$

Note that $[\dots]^{\lambda_{f,t}} = y_t^{\frac{\lambda_{f,t}}{1+\lambda_{f,t}}}$. From the first order conditions one obtains:

$$y_t(i) = (p_t(i))^{-\frac{1+\lambda_{f,t}}{\lambda_{f,t}}} y_t$$

Combining this condition with the zero profit condition (these firms are competitive) one obtains the expression

$$1 = \left[\int_0^1 p_t(i)^{-\frac{1}{\lambda_{f,t}}} d\right]^{-\lambda_{f,t}}. \tag{K.51}$$

Firm's problem: Cost minimization subject to 13 yields the conditions:

$$\begin{aligned}
(\partial h_t(i)) \quad & \mathcal{V}_t(i)(1-\gamma)A_t k_t(i)^{\gamma} h_t(i)^{-\gamma} = w_t \\
(\partial k_t(i)) \quad & \mathcal{V}_t(i)\gamma A_t k_t(i)^{\gamma-1} h_t(i)^{1-\gamma} = r_t^k
\end{aligned}$$

where $\mathcal{V}_t(i)$ is the Lagrange multiplier associated with 13. In turn, these conditions imply:

$$\frac{k_t(i)}{h_t(i)} = \frac{\gamma}{1-\gamma}\frac{w_t}{r_t^k}. \tag{K.52}$$

If we integrate both sides of the equation with respect to di and define $K_t = \int k_t(i)di$ and $H_t = \int h_t(i)di$ we obtain a relationship between aggregate labor and capital:

$$K_t = \frac{\gamma}{1-\gamma}\frac{w_t}{r_t^k} H_t. \tag{K.53}$$

Total cost, expressed in terms of the final good, can be rewritten as:

$$\begin{aligned}
\text{Cost} &= (w_t + r_t^k \tfrac{k_t(i)}{h_t(i)})h_t(i) \\
&= (w_t + r_t^k \tfrac{k_t(i)}{h_t(i)})y_t(i)A_t^{-1}\left(\tfrac{k_t(i)}{h_t(i)}\right)^{-\gamma}.
\end{aligned}$$

The marginal cost mc_t (which coincides with $\mathcal{V}_t(i)$, the Lagrange multiplier associated with 13) is the same for all firms and equal to:

$$\begin{aligned}
mc_t &= (w_t + w_t \tfrac{k_t(i)}{h_t(i)})A_t^{-1}\left(\tfrac{k_t(i)}{h_t(i)}\right)^{-\gamma} \\
&= \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)} w_t^{1-\gamma} r_t^{k\ \gamma} A_t^{-1}.
\end{aligned} \tag{K.54}$$

For those firms that can adjust prices, the problem is to choose a price level $\tilde{P}_t(i)$ that maximizes the expected present discounted value of profits in all states of nature where the firm is stuck with that price in the future:

$$\begin{aligned}\max_{\tilde{P}_t(i)} \quad & \Xi_t^p \left(\tilde{P}_t(i) - MC_t\right) y_t(i) \\ & + E_t \textstyle\sum_{s=1}^{\infty} \zeta_p^s \beta^s \Xi_{t+s}^p \left(\tilde{P}_t(i) \left(\Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}\right) - MC_{t+s}\right) y_{t+s}(i) \\ s.t. \quad y_{t+s}(i) = & \left(\frac{\tilde{P}_t(i) \left(\Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}\right)}{P_{t+s}}\right)^{-\frac{1+\lambda_{f,t+s}}{\lambda_{f,t+s}}} y_{t+s},\end{aligned} \tag{K.55}$$

where $\beta^s \Xi_{t+s}^p$ is the discount rate used by firms in discounting future. This expression can be rewritten in real terms as:

$$\begin{aligned}\max_{\tilde{p}_t} \quad & \Xi_t^p \left(\tilde{p}_t - mc_t\right) y_t(i) \\ & + E_t \textstyle\sum_{s=1}^{\infty} \zeta_p^s \beta^s \Xi_{t+s}^p (\Pi_{l=1}^s \pi_{t+l}) \left(\tilde{p}_t \frac{\Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}}{\Pi_{l=1}^s \pi_{t+l}} - mc_{t+s}\right) y_{t+s}(i) \\ s.t. y \quad {}_{t+s}(i) = & \left(\tilde{p}_t \frac{\Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}}{\Pi_{l=1}^s \pi_{t+l}}\right)^{-\frac{1+\lambda_{f,t+s}}{\lambda_{f,t+s}}} y_{t+s},\end{aligned} \tag{K.56}$$

where since all firms readjusting prices face an identical problem and we consider only the symmetric equilibrium in which all firms that can readjust prices will choose the same $p_t(i)$, we denote this price as $\tilde{p}_t$. The first order condition for the firm is:

$$\begin{aligned}& \frac{\xi_t}{\lambda_{f,t}} \tilde{p}_t^{-\frac{(1+\lambda_{f,t})}{\lambda_{f,t}}-1} \left(\tilde{p}_t - (1+\lambda_{f,t}) mc_t\right) y_t(i) \\ & + E_t \textstyle\sum_{s=1}^{\infty} \zeta_p^s \beta^s \frac{\xi_{t+s}}{\lambda_{f,t+s}} \left(\frac{\tilde{p}_t}{\Pi_{l=1}^s \pi_{t+l}}\right)^{\frac{(1+\lambda_{f,t+s})}{\lambda_{f,t+s}}-1} \left(\Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}\right)^{\frac{(1+\lambda_{f,t+s})}{\lambda_{f,t+s}}} \\ & \left(\tilde{p}_t \frac{\Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}}{\Pi_{l=1}^s \pi_{t+l}} - (1+\lambda_{f,t+s}) mc_{t+s}\right) y_{t+s}(i) = 0.\end{aligned} \tag{K.57}$$

Expression (K.51) becomes:

$$1 = [(1-\zeta_p)\tilde{p}_t^{-\frac{1}{\lambda_{f,t}}} + \zeta_p(\pi_{t-1}^{\iota_p} \pi_*^{1-\iota_p} \pi_t^{-1})^{-\frac{1}{\lambda_{f,t}}}]^{-\lambda_{f,t}}. \tag{K.58}$$

Worker's problem: We discuss the worker's consumption/saving decision first, and the wage setting decision later. The Lagrangian for the worker's intertemporal problem in terms of n'_{t+1}, l'_{t+1}, and b'_{t+1} is:

$$\begin{aligned}\mathcal{L} = \quad & [U(c'_t + \dots) - \eta'_t (c'_t + q_t n_{t+1} + r_{t-1} l'_{t+1} + \tfrac{b_{t+1}(e^s)}{P_t} + \dots) \\ & + z_{n,t}(\omega) n_{t+1} + z_{m,t}(\omega) l_{t+1} + z_{b,t}(\omega) b_{t+1}] \\ & + \beta E_t [U(c'_{t+1} + \dots) - \eta'_{t+1}(c'_{t+1} + \dots - [r_{t+1}^k + q_{t+1}\lambda] n_{t+1} + r_t l'_{t+1} + \tfrac{R_t b'_{t+1}}{P_{t+1}})] + \dots\end{aligned}$$

where η'_t, $z_{n,t}(\omega)$, and $z_{m,t}(\omega)$ are the multipliers associated with constraints (16), (18) (19),, and (20), respectively. The first order conditions are:

$$\begin{array}{ll} (\partial c'_t) & U'(c'_t + \dots) = \eta'_t \\ (\partial n'_{t+1}) & q_t \eta'_t + z_{n,t}(\omega) = \beta E_t[\eta'_{t+1}[r^k_{t+1} + q_{t+1}\lambda]] \\ (\partial l'_{t+1}) & p_t \eta'_t + z_{m,t}(\omega) = \beta E_t[\eta'_{t+1} p_{t+1}] \\ (\partial b'_{t+1}) & \eta'_t + z_{m,t}(\omega) = \beta E_t[\eta'_{t+1} \frac{R_t P_t}{P_{t+1}}] \end{array}$$

together with the slack conditions on the constraints (18) and (19). We postulate that the solution to the worker's consumption/saving problem is:

$$c'_t = \int \frac{W_t(\omega)}{P_t} h'_t(\omega) d\omega + \int \mathcal{P}(i) d \ + C(I_t) + \tau_t, \tag{K.59}$$

$$n'_{t+1} = 0, \tag{K.60}$$

$$l'_{t+1} = 0, \tag{K.61}$$

$$b'_{t+1} = 0. \tag{K.62}$$

Now to the wage setting decision. From the first order conditions of the labor packers one obtains the labor demand from each worker:

$$h_t(\omega) = \left(\frac{W_t(\omega)}{W_t} \right)^{-\frac{1+\lambda_w}{\lambda_w}} h'_t \tag{K.63}$$

where W_t represents aggregate nominal wages. Combining this condition with the zero profit condition one obtains an expression for the aggregate nominal wage:

$$W_t = \left[\int_0^1 W_t(\omega)^{\frac{1}{\lambda_{w,t}}} di \right]^{\lambda_{w,t}}. \tag{K.64}$$

For those workers that can adjust their wage, the problem is to choose a wage $\tilde{W}_t(\omega)$ that maximizes utility in all states of nature where the household is stuck with that wage in the future:

$$\begin{array}{l} \max_{\tilde{W}_t(\omega)} \quad E_t \sum_{s=0}^{\infty} (\zeta_w \beta)^s U[c'_s - \int \frac{\omega}{1+\nu} h_s(\omega)'^{\ 1+\nu} d\omega] \\ \text{s.t. K.63 for } s = 0, \dots, \infty, \\ c'_{t+s} \dots \leq \ \dots + \int \frac{W_{t+s}}{P_{t+s}} h'_{t+s}(\omega) d\omega \\ \ \text{for } s = 0, \dots, \infty, \text{ and } W_{t+s}(\omega) = \mathcal{X}_{t,s} \tilde{W}_t(\omega) \text{ for } s = 1, \dots, \infty \end{array} \tag{K.65}$$

where we have rewritten the worker's budget constraint in terms of nominal wages, and where

$$\mathcal{X}_{t,s} = \begin{cases} 1 \quad \text{if } s = 0 \\ \Pi_{l=1}^{s} \pi_*^{1-\iota_w} \pi_{t+l-1}^{\iota_w} \quad \text{otherwise.} \end{cases}$$

The first order condition for this problem with respect to $\tilde{W}_t(\omega)$ are:

$$\begin{aligned} & E_t \textstyle\sum_{s=0}^{\infty} (\zeta_w \beta)^s U'[c'_{t+s} + ...] \left(\frac{\mathcal{X}_{t,s} \tilde{W}_t(\omega)}{W_{t+s}} \right)^{-\frac{1+\lambda_w}{\lambda_w} - 1} \frac{\mathcal{X}_{t,s}}{W_{t+s}} h'_{t+s} \\ & \left[\frac{\mathcal{X}_{t,s} \tilde{W}_t(\omega)}{P_{t+s}} - (1+\lambda_w) \omega h_{t+s}(\omega)'^{\ \nu} \right] = 0. \end{aligned} \tag{K.66}$$

The planner faces an identical problem for all agents readjusting wages, and we will consider only the symmetric equilibrium in which she chooses the same wage for all agents that can readjust their wage $\tilde{W}_t(\omega) = \tilde{W}_t$. From K.64 it follows that:

$$W_t = [(1-\zeta_w) \tilde{W}_t^{\frac{1}{\lambda_w}} + \zeta_w (\pi_*^{1-\iota_w} \pi_{t-1}^{\iota_w} W_{t-1})^{\frac{1}{\lambda_w}}]^{\lambda_w}. \tag{K.67}$$

Since both W_t and $\tilde{W}_t$ are trending because of inflation, we need to express both (K.66) and (K.67) in terms of the stationary variables $w_t = \frac{W_t}{P_t}$ and $\tilde{w}_t = \frac{\tilde{W}_t}{P_t}$:

$$w_t = [(1-\zeta_w) \tilde{w}_t^{\frac{1}{\lambda_w}} + \zeta_w (\pi_*^{1-\iota_w} \pi_{t-1}^{\iota_w} \frac{w_{t-1}}{\pi_t})^{\frac{1}{\lambda_w}}]^{\lambda_w}. \tag{K.68}$$

$$E_t \sum_{s=0}^{\infty} (\zeta_w \beta)^s U'[...] \left(\frac{\tilde{\mathcal{X}}_{t,s} \tilde{w}_t}{w_{t+s}} \right)^{-\frac{1+2\lambda_w}{\lambda_w}} \frac{\tilde{\mathcal{X}}_{t,s}}{w_{t+s}} h'_{t+s} \left[\tilde{\mathcal{X}}_{t,s} \tilde{w}_t - (1+\lambda_w) \omega h_{t+s}(\omega)'^{\ \nu} \right] = 0. \tag{K.69}$$

where

$$\tilde{\mathcal{X}}_{t,s} = \begin{cases} 1 \quad \text{if } s = 0 \\ \frac{\Pi_{l=1}^{s} \pi_*^{1-\iota_w} \pi_{t+l-1}^{\iota_w}}{\Pi_{l=1}^{s} \pi_{t+l}} \quad \text{otherwise.} \end{cases}$$

Aggregation and Equilibrium conditions:

Aggregating expression (K.46) we obtain:

$$(p_t^I - q_t \theta_t) I_t = \beta(\varkappa r_{t-1} L_t + (r_t^k + q_t \phi_t \lambda) \varkappa K_t) - (1-\beta)(1-\phi_t) q_t^R \lambda \varkappa K_t, \tag{K.70}$$

where we used the fact that reserves are in zero aggregate supply from (24), and that resource constraint for capital (35) implies:

$$K_t = N_t$$

since no worker holds equity and the assignment between investor and savers at the beginning of time t is random.

The resource constraint for output (32) implies:

$$r_t^k K_t + w_t H_t + \int \mathcal{P}(i)d \ = I_t[1 + S(\frac{I_t}{I_*})] + \int_{e^i} c_t(e) + \int_{e^s} c_t(e) + c_t'.$$

Using (K.44), (K.45), and (K.59) this condition can be rewritten as:

$$r_t^k K_t \ = \ I_t[1+S(\frac{I_t}{I_*})]+\tau_t+(1-\beta)\left\{r_{t-1}L_t + [r_t^k + (1-\varkappa+\varkappa\phi_t)q_t\lambda + \varkappa(1-\phi_t)q_t^R\lambda]K_t\right\}. \tag{K.71}$$

By Claim 2 Tobin's q is greater than one and, as a consequence, the investing entrepreneur's credit constraint (5) is always binding, hence:

$$N_{t+1}^i \equiv \int_{e^i} n_{t+1}(e) \ = \ (1-\phi_t)\lambda\varkappa K_t + (1-\theta)I_t. \tag{K.72}$$

Since workers hold no equity, and since the law of motion of aggregate capital is (from aggregating expression (2)):

$$K_{t+1} = \lambda K_t + I_t, \tag{K.73}$$

non-investing entrepreneurs must hold the rest:

$$N_{t+1}^s \equiv \int_{e^s} n_{t+1}(e) \ = \ (1-\varkappa)\lambda K_t + \phi_t\lambda\varkappa K_t + \theta_t I_t. \tag{K.74}$$

Using the fact that all non-investing entrepreneurs have the same equity and reserves to money holdings ratios (claim 3), we can rewrite conditions (K.42) and (K.43) using aggregate variables:

$$\begin{aligned} \varkappa E_t\left[\frac{\left(r_t-\frac{r_{t+1}^k+((1-\phi_{t+1})q_{t+1}^R+\phi_{t+1}q_{t+1})\lambda}{q_t}\right)}{r_tM_{t+1}+[r_{t+1}^k+((1-\phi_{t+1})q_{t+1}^R+\phi_{t+1}q_{t+1})\lambda]N_{t+1}^s}\right] &= \\ (1-\varkappa)E_t\left[\frac{\left(\frac{r_{t+1}^k+q_{t+1}\lambda}{q_t}-r_t\right)}{r_tM_{t+1}+(r_{t+1}^k+q_{t+1}\lambda)N_{t+1}^s}\right]&. \end{aligned} \tag{K.75}$$

$$\begin{aligned} E_t\Big[\left(r_t-\frac{R_t}{\pi_{t+1}}\right)\Big(&\frac{\varkappa}{r_tM_{t+1}+[r_{t+1}^k+((1-\phi_{t+1})q_{t+1}^R+\phi_{t+1}q_{t+1})\lambda]N_{t+1}^s} \\ &-\frac{(1-\varkappa)}{r_tM_{t+1}+(r_{t+1}^k+q_{t+1}\lambda)N_{t+1}^s}\Big)\Big] = 0. \end{aligned} \tag{K.76}$$

Other equilibrium conditions are the interest rule is given by (22), the formula for marginal costs (K.54), the definition of the aggregate price level (K.58), the price-setting decision of the firm (K.57), the relationship determining the capital/labor ratio (K.53), the definition of the aggregate wages (K.68), and the wage-setting decision of the firm (K.69).

Consumption of workers, entrepreneurs, aggregate output, and aggregate consumption are given by:

$$Y_t = A_t K_t^{\gamma} H_t^{1-\gamma}, \tag{K.77}$$

$$C_t = Y_t - I_t[1 + S(\frac{I_t}{I_*})], \tag{K.78}$$

$$C_t^e = r_t^k K_t - I_t[1 + S(\frac{I_t}{I_*})] - \tau_t, \tag{K.79}$$

$$C_t^w = Y_t - r_t^k K_t + \tau_t. \tag{K.80}$$

where in aggregating expression (13) across intermediate goods producers we used the fact that they all use the same capital labor ratio. The above conditions are block exogenous to the rest of the system.

Steady state: At steady state condition (K.73) implies:

$$I_* = (1 - \lambda) K_*. \tag{K.81}$$

Using the steady state first order conditions of firms and workers we obtain the steady state real rental rate on capital:

$$r_*^k = a_* K_*^{\alpha - 1}, \tag{K.82}$$

where

$$a_* = \left(\frac{1}{1 + \lambda_{f,*}}\right)^{\frac{\gamma(1+\nu)}{\gamma+\nu}} A_*^{\frac{1+\nu}{\gamma+\nu}} \gamma \left(\frac{1-\gamma}{\omega(1 + \lambda_{w,*})}\right)^{\frac{1-\gamma}{\gamma+\nu}}$$

and

$$\alpha = \frac{\gamma(1+\nu)}{\gamma+\nu} \in (0, 1).$$

Condition (K.70) becomes

$$\beta \varkappa a_* K_*^{\alpha-1} + \beta\varkappa \frac{r^* L_*}{K_*} = (1 - q_*\theta_*)\left(1 - \lambda + \varkappa\lambda(1-\beta)\frac{1-\phi_*}{1-\theta_*}\right) - \varkappa\lambda\beta q_*\phi_*. \tag{K.83}$$

Condition (K.71) becomes

$$\beta a_* K_*^{\alpha-1} - (1-\beta)\frac{r^* L_*}{K_*} = 1 - \lambda + \frac{\tau_*}{K_*} + (1-\beta)\lambda\left[(1 - \varkappa\frac{1-\phi_*}{1-\theta_*})q_* + \varkappa\frac{1-\phi_*}{1-\theta_*}\right]. \tag{K.84}$$

Using (K.74), at steady state $\chi \equiv \frac{N_*^s}{K_*} = (1-\varkappa)\lambda + \phi_*\lambda\varkappa + \theta(1-\lambda)$. Condition (K.75) becomes:

$$a_*K_*^{\alpha-1} + \lambda q_* - \frac{q_*}{p_*} = \varkappa\lambda\frac{1-\phi_*}{1-\theta_*}(q_*-1)\frac{\frac{q_*}{p_*} + \frac{r^*L_*}{\chi K_*}}{a_*K_*^{\alpha-1} + \lambda\frac{1-\phi_*}{1-\theta_*} + \lambda\frac{\phi_*-\theta_*}{1-\theta_*}q_* + \frac{r^*L_*}{\chi K_*}} \quad \text{(K.85)}$$

Steady state transfers are given by:

$$\tau_* = (1-r_*)L_*. \quad \text{(K.86)}$$

and steady state inflation is determined by (K.76)

$$\frac{\pi_*}{p_*} = R_*, \quad \text{(K.87)}$$

Intervention: L for K Swap

The intervention is described in section 2.6. Following the intervention, the market clearing condition for capital (35) becomes

$$K_{t+1} = N_{t+1} + N_{t+1}^g. \quad \text{(K.88)}$$

and also (at the beginning of the period):

$$K_t = \tilde{N}_t + \tilde{N}_t^g. \quad \text{(K.89)}$$

Equity held by investing and non investing entrepreneurs are respectively:

$$N_{t+1}^i = (1-\phi_t)\varkappa\lambda\tilde{N}_t + (1-\theta_t)I_t, \quad \text{(K.90)}$$

$$\begin{aligned} N_{t+1}^s &= (1-\varkappa+\phi_t\varkappa)\lambda\tilde{N}_t + \theta_t I_t + \lambda\tilde{N}_t^g - N_{t+1}^g \\ &= (1-\varkappa+\phi_t\varkappa)\lambda\tilde{N}_t + \theta_t I_t - (1-\lambda)N_{t+1}^g \end{aligned}. \quad \text{(K.91)}$$

where we used the law of motion of aggregate capital (K.73) to derive the last expression.

The four conditions (K.70), and (K.71), (**??**), and (**??**)become:

$$(p_t^I - q_t\theta_t)I_t = \beta(\varkappa r_{t-1}\tilde{L}_t + (r_t^k + q_t\phi_t\lambda)\varkappa\tilde{N}_t) - (1-\beta)(1-\phi_t)q_t^R\lambda\varkappa\tilde{N}_t, \quad \text{(K.92)}$$

$$r_t^k K_t = I_t[1+S(\frac{I_t}{I_*})]+\tau_t+(1-\beta)\left\{r_{t-1}\tilde{L}_t + [r_t^k + (1-\varkappa+\varkappa\phi_t)q_t\lambda + \varkappa(1-\phi_t)q_t^R\lambda]\tilde{N}_t\right\}, \quad \text{(K.93)}$$

$$C_t^{si} = r_{t-1}\tilde{L}_t + [r_t^k + ((1-\phi_t)q_t^R + \phi_t q_t)\lambda]N_t^s, \quad \text{(K.94)}$$

$$C_t^{ss} = r_{t-1}\tilde{L}_t + (r_t^k + q_t\lambda)N_t^s, \tag{K.95}$$

The steady state is the same as without intervention.

Summary of log-linearized equilibrium conditions: Define $\hat{x}_t \equiv \log(x_t/x_*)$ where x_* is the steady state value of x_t. In order to describe the log-linear conditions we will use a slightly different notation than in the rest of the paper, namely:

$$r_{t-1} = \frac{1}{p_t} \tag{K.96}$$

$$M_t = r_{t-1}L_t. \tag{K.97}$$

$$\hat{K}_{t+1} - (1-\lambda)\,\hat{I}_t - (1-\lambda)\,\hat{p}_t^I = \lambda\hat{K}_t. \tag{K.98}$$

$$\left(\frac{q_*^R}{q_*}\right)\hat{q}_t^R + \left(\frac{\theta}{1-\theta}\right)\hat{q}_t + \left(\frac{\theta}{1-\theta}\right)\left(1-\frac{q_*^R}{q_*}\right)\hat{\theta}_t - \frac{1}{(1-\theta)q_*}\hat{p}_t^I \;=\; 0. \tag{K.99}$$

$$\begin{aligned}(\frac{1}{q_*}-\theta)(1-\lambda)\hat{I}_t - (\theta(1-\lambda)+\beta\varkappa\phi\lambda)\hat{q}_t + (1-\beta)(1-\phi)\lambda\varkappa\frac{q_*^R}{q_*}\hat{q}_t^R \\ -\left(\beta+(1-\beta)\frac{q_*^R}{q_*}\right)\lambda\varkappa\phi\hat{\phi}_t - \beta\varkappa\frac{r_*^k}{q_*}\hat{r}_t^k - \theta(1-\lambda)\hat{\theta}_t - \frac{1}{q_*}(1-\lambda)\hat{p}_t^I - \beta\varkappa\frac{M_*}{q_*K_*}\widehat{\widetilde{M}}_t = \\ \left(\beta(\frac{r_*^k}{q_*}+\phi\lambda)-(1-\beta)(1-\phi)\frac{q_*^R}{q_*}\lambda\right)\varkappa\hat{\tilde{N}}_t.\end{aligned} \tag{K.100}$$

$$\begin{aligned}\beta r_*^k\hat{r}_t^k - (1-\lambda)\hat{I}_t - (1-\beta)\lambda\varkappa\phi(q_*-q_*^R)\hat{\phi}_t \\ -(1-\beta)\lambda(1-\varkappa+\varkappa\phi)q_*\hat{q}_t - (1-\beta)\lambda\varkappa(1-\phi)q_*^R\hat{q}_t^R - (1-\beta)\frac{M_*}{K_*}\widehat{\widetilde{M}}_t - \hat{\tau}_t \\ = -r_*^k\hat{K}_t + \left[(1-\beta)\left(r_*^k+(1-\varkappa+\varkappa\phi)q_*\lambda+\varkappa(1-\phi)q_*^R\lambda\right)\right]\hat{\tilde{N}}_t.\end{aligned} \tag{K.101}$$

$$\begin{aligned}\left(\frac{1-\varkappa}{C^{ss}}\right)\left(\frac{1}{p_*}-\frac{r_*^k+q_*\lambda}{q_*}\right)(E_t\hat{C}_{t+1}^{si}-E_t\hat{C}_{t+1}^{ss}) \\ +\frac{\varkappa^C}{p_*}\hat{q}_t - \lambda\left(\varkappa^C-\frac{(1-\phi)\varkappa}{C_*^{si}}\right)E_t\hat{q}_{t+1} - \varkappa^C\frac{r_*^k}{q_*}E_t\hat{r}_{t+1} \\ -\frac{\varkappa\lambda(1-\phi)}{C_*^{si}}\frac{q_*^R}{q_*}E_t\hat{q}_{t+1}^R + \frac{\varkappa\lambda\phi}{C_*^{si}}(\frac{q_*^R}{q_*}-1)E_t\hat{\phi}_{t+1} - \frac{\varkappa^C}{p_*}\hat{p}_t = 0,\end{aligned} \tag{K.102}$$

$$\begin{aligned}\frac{C_*^{si}}{q_*N_*^s}\hat{C}_t^{si} - \frac{r_*^k}{q_*}\hat{r}_t - \lambda(1-\phi)\frac{q_*^R}{q_*}\hat{q}_t^R - \lambda\phi(1-\frac{q_*^R}{q_*})\hat{\phi}_t - \lambda\phi\hat{q}_t \\ -\frac{M_*}{q_*N_*^s}\widehat{\widetilde{M}}_t = \left[\frac{r_*^k}{q_*}+\left((1-\phi)\frac{q_*^R}{q_*}+\phi\right)\lambda\right]\hat{N}_t^s,\end{aligned} \tag{K.103}$$

$$\frac{C_*^{ss}}{q_* N_*^s}\hat{C}_t^{ss} - \frac{r_*^k}{q_*}\hat{r}_t - \lambda\hat{q}_t - \frac{M_*}{q_* N_*^s}\widehat{\tilde{M}}_t = (\frac{r_*^k}{q_*} + \lambda)\hat{N}_t^s. \tag{K.104}$$

$$-\hat{p}_t^I = S''(1)\hat{I}_t. \tag{K.105}$$

$$\widehat{m}c_t \;=\; (1-\gamma)\;\widehat{w}_t + \gamma\;\widehat{r}_t^k - \widehat{A}_t. \tag{K.106}$$

$$\begin{aligned}\widehat{\pi}_t \;&= \frac{(1-\zeta_p\beta)(1-\zeta_p)}{(1+\iota_p\beta)\zeta_p}\left[\widehat{m}c_t + \frac{\lambda_f}{1+\lambda_f}\widehat{\lambda}_{f,t}\right] \\ &+ \frac{\iota_p}{1+\iota_p\beta}\widehat{\pi}_{t-1} + \frac{\beta}{1+\iota_p\beta}E_t[\widehat{\pi}_{t+1}]\end{aligned} \tag{K.107}$$

where so far ι_p was set to 0.

$$\widehat{K}_t \;=\; \widehat{w}_t \;-\; \widehat{r}_t^k \;+\; \widehat{H}_t. \tag{K.108}$$

$$\hat{w}_t = (1-\zeta_w)\widehat{\tilde{w}}_t + \zeta_w(\hat{w}_{t-1} + \iota_w\hat{\pi}_{t-1} - \hat{\pi}_t). \tag{K.109}$$

$$\begin{aligned}&(1 + (1-\zeta_w\beta)\nu\frac{1+\lambda_w}{\lambda_w})\widehat{\tilde{w}}_t - (1-\zeta_w\beta)\nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_t \\ &\qquad = (1-\zeta_w\beta)\nu\hat{h}_t + \zeta_w\beta E_t\left(\widehat{\tilde{w}}_{t+1} + \hat{\pi}_{t+1} - \iota_w\hat{\pi}_t\right).\end{aligned} \tag{K.110}$$

$$\hat{R}_t = \psi_1\hat{\pi}_t + \epsilon_t^r, \tag{K.111}$$

$$\hat{R}_t - E_t\widehat{\pi}_{t+1} = -\widehat{p}_t. \tag{K.112}$$

$$\hat{N}_{t+1}^g = \xi_1\hat{\phi}_t. \tag{K.113}$$

$$M_*(\hat{M}_{t+1} - \hat{M}_t) = q_* K_*(\hat{\tilde{N}}_t^g - \hat{N}_t^g). \tag{K.114}$$

$$\hat{K}_{t+1} = \hat{N}_{t+1} + \hat{N}_{t+1}^g. \tag{K.115}$$

$$\hat{K}_t = \hat{\tilde{N}}_t + \hat{N}_{t+1}^g. \tag{K.116}$$

$$\begin{aligned}&\frac{N_*^s}{K_*}\hat{N}_{t+1}^s - \theta(1-\lambda)\hat{I}_t - \theta(1-\lambda)\hat{\theta}_t \\ &\qquad - \lambda\phi\varkappa\hat{\phi}_t - \lambda\left(1 - \varkappa(1-\phi)\right)\hat{\tilde{N}}_t + (1-\lambda)\hat{N}_{t+1}^g = 0.\end{aligned} \tag{K.117}$$

$$\hat{\tau}_t = (r_*^k - (1-\lambda)q^*)\hat{N}_{t+1}^g + p_* M_*\hat{p}_t - (1-p_*)M_*\hat{M}_{t+1}. \tag{K.118}$$

Other formulas of interest are:

$$\hat{Y}_t = \hat{A}_t + \gamma\hat{K}_t + (1-\gamma)\hat{H}_t, \tag{K.119}$$

$$\frac{C_*}{Y_*}\hat{C}_t = \hat{Y}_t - \frac{I_*}{Y_*}\hat{I}_t, \tag{K.120}$$

$$\frac{C^e_*}{K_*}\hat{C}^e_t = r^k_*\hat{r}^k_t + r^k_*\hat{K}_t - (1-\lambda)\hat{I}_t - \hat{\tau}_t, \tag{K.121}$$

$$\frac{C^w_*}{K_*}\hat{C}^w_t = \frac{Y_*}{K_*}\hat{Y}_t - r^k_*\hat{r}^k_t - r^k_*\hat{K}_t + \hat{\tau}_t, \tag{K.122}$$

$$\hat{R}^q_t = \frac{\frac{r^k_*}{q_*}}{\frac{r^k_*}{q_*} + \lambda} E_t\hat{r}^k_{t+1} + \frac{\lambda}{\frac{r^k_*}{q_*} + \lambda} E_t\hat{q}_{t+1} - \hat{q}_t, \tag{K.123}$$

$$\hat{R}^m_t = -\widehat{p}_t. \tag{K.124}$$

Proofs: **Proof of claim 2-(i)**: *the resaleability constraint (5) is binding for an entrepreneur with investment opportunities.* The Lagrangian for an investing entrepreneur is:

$$\mathcal{L} = \quad log(c_t(e^i)) - \zeta_t(e^i)(\cdots + (1-\theta)i_t(e^i) + \dots) - \eta_t(e^i)(\cdots + i_t(e^i) - q_t i_t(e^i) + \dots) + \dots$$

where $\zeta_t(e^i)$ and $\eta_t(e^s)$, and $z_{m,t}(e^i)$ are the multipliers associated with constraints (5) and (8), and where we ignore all terms in the Lagrangian that do not depend on $i_t(e^i)$. The first order condition with respect to $i_t(e^i)$ is:

$$-\zeta_t(e^i)(1-\theta) + \eta_t(e^i)(q_t - 1) = 0.$$

Since the intertemporal budget constraint is binding ($\eta_t(e^i) > 0$), then $q_t > 1$ implies $\zeta_t(e^i) > 0$ (conversely if $q_t = 1$ then $\zeta_t(e^i) = 0$).

Proof of claim 2-(ii): *the resaleability constraint (5) is not binding for an entrepreneur without investment opportunities.* Recall that $i_t(e^s) = 0$. The resaleability constraint (5) can be rewritten as:

$$q_t n_{t+1}(e^s) \;\geq\; q_t(1-\phi_t)\lambda n_t(e^s).$$

Using the flow of funds (8) we can substitute for $q_t n_{t+1}(e^s)$ into the above expression and obtain:

$$(r_t^k + \phi_t q_t \lambda)n_t(e^s) + [r_{t-1}l_t(e^s) + \frac{R_{t-1}b_t(e^s)}{P_t}] \;\geq\; c_t(e^s) + l_{t+1}(e^s) + \frac{b_{t+1}(e^s)}{P_t}$$

Substituting for $c_t(e^s)$ from (K.45) we obtain:

$$(r_t^k+\phi_t q_t\lambda)n_t(e^s)+\beta[r_{t-1}l_t(e^s)+\frac{R_{t-1}b_t(e^s)}{P_t}] \;\geq\; (1-\beta)(r_t^k+q_t\lambda)n_t(e^s)+l_{t+1}(e^s)+\frac{b_{t+1}(e^s)}{P_t}$$

Proof of claim 2-(iii): *An entrepreneur with investment opportunities will choose not to hold the liquid asset:* $l_{t+1}(e^i) = 0,\; b_{t+1}(e^i) = 0$. If $l_{t+1}(e^i)$ were different from zero, then $z_{m,t}(e^i) = 0$ and we would also obtain:

$$\begin{aligned}&\varkappa E_t[\tfrac{1}{c_{t+1}(e^i)}r_t] + (1-\varkappa)E_t[\tfrac{1}{c_{t+1}(e^s)}r_t] = \\ &\varkappa E_t[\tfrac{1}{c_{t+1}(e^i)}\tfrac{r_{t+1}^k+((1-\phi_{t+1})q_{t+1}^R+\phi_{t+1}q_{t+1})\lambda}{q_t^R}] + (1-\varkappa)E_t[\tfrac{1}{c_{t+1}(e^s)}\tfrac{r_{t+1}^k+q_{t+1}\lambda}{q_t^R}].\end{aligned} \qquad \text{(K.125)}$$

The left hand side of (K.42) and (K.125) are the same. But the right hand side of (K.42) is lower than that of (K.125) since $q_t > 1 > q_t^R$: the return to equity is greater for the investing entrepreneur (since she can acquire equity cheap) than for the saver. Hence arbitrage conditions (K.42) and (K.125) cannot hold at the same time. We must have that the return to money is dominated by the return to equity for the investing entrepreneur, hence $l_{t+1}(e^i) = 0$. Given the arbitrage condition between the liquid asset and reserves (K.43), it must also be that $b_{t+1}(e^i) = 0$.

Proof of claim 3: *All non-investing entrepreneurs have the same equity and reserves to money holdings ratios.* Substituting (K.44) and (K.45) into (K.42) we obtain:

$$\varkappa E_t \left[\frac{\left(r_t - \frac{r^k_{t+1} + ((1-\phi_{t+1})q^R_{t+1} + \phi_{t+1} q_{t+1})\lambda}{q_t} \right)}{r_t l_{t+1}(e^s) + [r^k_{t+1} + ((1-\phi_{t+1})q^R_{t+1} + \phi_{t+1} q_{t+1})\lambda] n_{t+1}(e^s) + \frac{R_t b_{t+1}(e^s)}{P_{t+1}}} \right] = (1-\varkappa) E_t \left[\frac{\left(\frac{r^k_{t+1} + q_{t+1}\lambda}{q_t} - r_t \right)}{r_t l_{t+1}(e^s) + (r^k_{t+1} + q_{t+1}\lambda) n_{t+1}(e^s) + \frac{R_t b_{t+1}(e^s)}{P_{t+1}}} \right],$$

or

$$\varkappa E_t \left[\frac{\left(r_t - \frac{r^k_{t+1} + ((1-\phi_{t+1})q^R_{t+1} + \phi_{t+1} q_{t+1})\lambda}{q_t} \right)}{r_t + [r^k_{t+1} + ((1-\phi_{t+1})q^R_{t+1} + \phi_{t+1} q_{t+1})\lambda] \frac{n_{t+1}(e^s)}{l_{t+1}(e^s)} + \frac{R_t}{P_{t+1}} \frac{b_{t+1}(e^s)}{l_{t+1}(e^s)}} \right] = (1-\varkappa) E_t \left[\frac{\left(\frac{r^k_{t+1} + q_{t+1}\lambda}{q_t} - r_t \right)}{r_t + (r^k_{t+1} + q_{t+1}\lambda) \frac{n_{t+1}(e^s)}{l_{t+1}(e^s)} + \frac{R_t}{P_{t+1}} \frac{b_{t+1}(e^s)}{l_{t+1}(e^s)}} \right], \tag{K.126}$$

where we could divide both sides by $l_{t+1}(e^s)$ since this variable is known at the end of time t. From (K.42) we have:

$$\varkappa E_t \left[\frac{\left(r_t - \frac{R_t P_t}{P_{t+1}} \right)}{r_t + [r^k_{t+1} + ((1-\phi_{t+1})q^R_{t+1} + \phi_{t+1} q_{t+1})\lambda] \frac{n_{t+1}(e^s)}{l_{t+1}(e^s)} + \frac{R_t}{P_{t+1}} \frac{b_{t+1}(e^s)}{l_{t+1}(e^s)}} \right] = (1-\varkappa) E_t \left[\frac{\left(\frac{R_t P_t}{P_{t+1}} - r_t \right)}{r_t + (r^k_{t+1} + q_{t+1}\lambda) \frac{n_{t+1}(e^s)}{l_{t+1}(e^s)} + \frac{R_t}{P_{t+1}} \frac{b_{t+1}(e^s)}{l_{t+1}(e^s)}} \right], \tag{K.127}$$

The two expressions above show that the ratios $\frac{n_{t+1}(e^s)}{l_{t+1}(e^s)} = \kappa^1_{t+1}$ and $\frac{b_{t+1}(e^s)}{l_{t+1}(e^s)} = \kappa^2_{t+1}$ is the same across saving entrepreneurs (2 equations in 2 unknowns).