Booms and Busts in Asset Prices

Klaus Adam
Mannheim University & CEPR

Albert Marcet
London School of Economics & CEPR

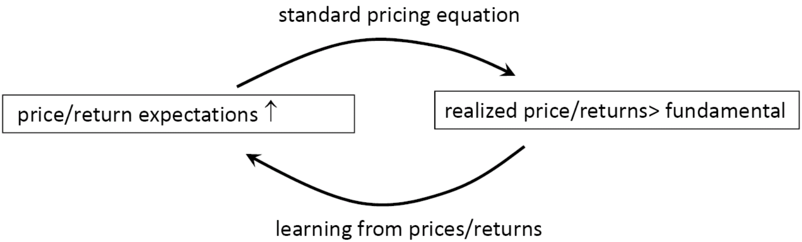
May 2010

Introduction

- Present a simple asset pricing model
  - with **rationally investing infinitely lived agents**
  - in which **Bayesian learning**.gives rise to
  - **low frequency booms & busts** in asset prices.
- Learning model replicates the low frequency behavior of PD ratio of US stocks 1926-2006
- Consistent with survey evidence that return expectations correlate positively with market valuation (unlike RE models)

Introduction

- Learning about price/return & long-standing asset price puzzles



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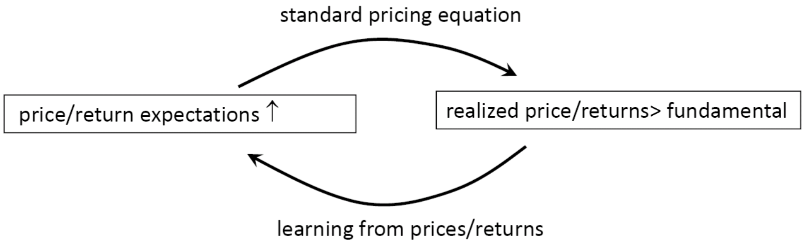
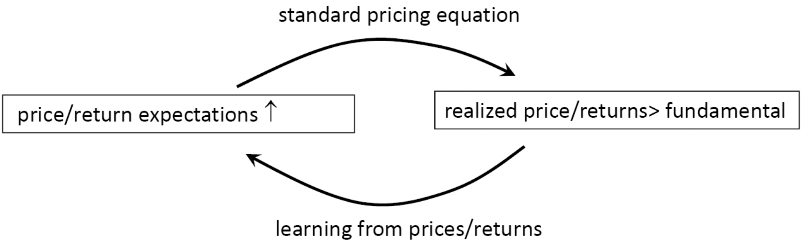


- Implications for the behavior of price/return expectations...

- Learning about price/return & long-standing asset price puzzles



- Implications for the behavior of price/return expectations...
- Implications for efficiency of asset price movements...

Relation to earlier work

- "*Internal Rationality, Imperfect Market Knowledge and Asset Prices*", Adam & Marcet, 2009

  Decision theoretic foundations

  Risk neutral agents, heterogeneity & slight forms of market incompleteness

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- **Internal Rationality**

  agents maximize IH utility under uncertainty
  consistent probability beliefs about payoff-relevant *external variables*: prices & dividends

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  Risk neutral agents, heterogeneity & slight forms of market incompleteness

- **Internal Rationality**

  agents maximize IH utility under uncertainty
  consistent probability beliefs about payoff-relevant *external variables*: prices & dividends

- **External Rationality**

  agents hold correct prior beliefs (RE) about external variables: prices & dividends

Relation to earlier work

- **Relax External Rationality slightly**

  subjective prior beliefs close to objective priors assumed under RE

  requires relaxing singularity in joint beliefs about prices&dividends

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- **Relax External Rationality slightly**

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- **Equilibrium asset price: one-step ahead equation**

$$P_t = \delta E_t^{\mathcal{P}^{m_t}} \left[P_{t+1} + D_{t+1} \right]$$

(& no dividend discount formula...)

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  (& no dividend discount formula...)

- Price beliefs not restricted by dividend beliefs + internal rationality:

  learning about price a potentially important source of price volatility!

Relation to earlier work

- "*Stock Market Volatility & Learning*", Adam, Marcet & Nicolini, 2009

  Explore the **quantitative performance** of a learning model

  Learning moves (Lucas) AP model strongly in direction of data!

  - strong return volatility amplification
  - persistent, volatile and mean reverting PD ratio
  - large equity premium

Quantitative Performance of Learning Model

	US Data		Model (OLS)	
Statistics		std		t-ratio
$E(r^s)$	2.41	0.45	2.41	0.01
$E(r^b)$	0.18	0.23	0.48	-1.30
$E(PD)$	113.20	15.15	95.93	1.14
σ_{r^s}	11.65	2.88	13.21	-0.54
σ_{PD}	52.98	16.53	62.19	-0.56
$\rho_{PD,-1}$	0.92	0.02	0.94	-1.20
c_5^2	-0.0048	0.002	-0.0067	0.92
R_5^2	0.1986	0.083	0.3012	-1.24
Parameters:	$\delta = .999, \quad 1/\alpha_1 = 0.015$			

Coefficient of rel risk aversion: $\sigma = 5$.

Relation to earlier work

- Unsatisfactory aspects => could not discuss asset price booms & their end!
- Model with risk neutrality / exogenous stoch discount factor: IES infinite & too much momentum in price dynamics
- Projection facility: stop asset price booms in exogenous ways
  Ability to match unconditional second moments unaffected but conditionally strong effects
- What ends price booms? Should policy prevent booms? When? How?
  Exogenous projection facility subject to Lucas critique

Relation to earlier work

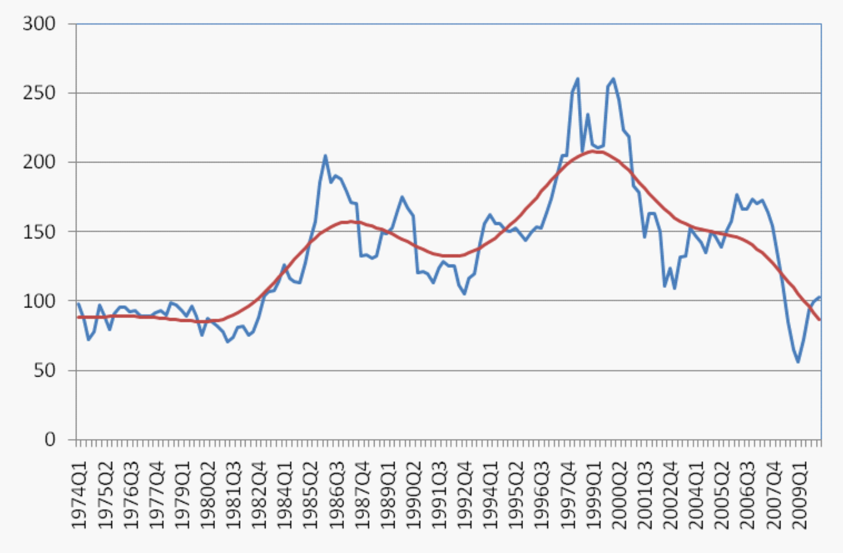
- This paper: non-linear utility & endogenous discount factor
- IES finite and no need for projection facility
- Technically more demanding:

  solve for optimal consumption plans implied by agents' beliefs

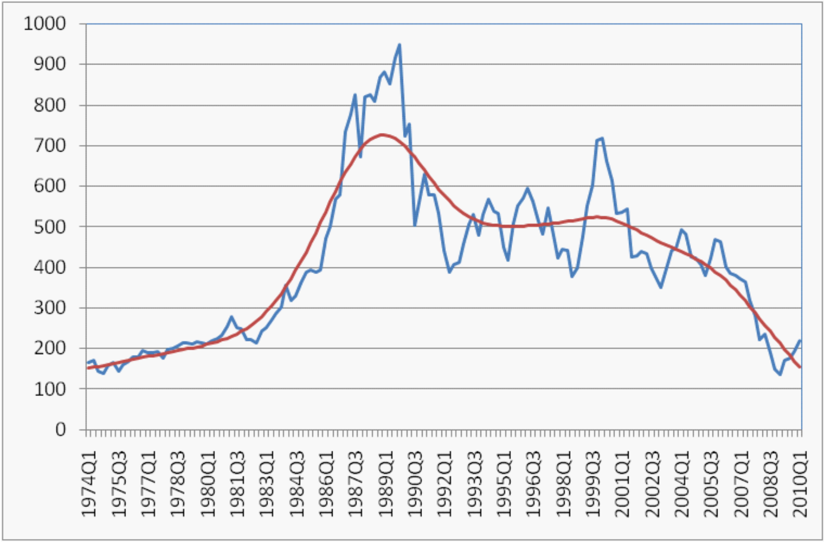
  non-linear optimization problem in which beliefs are state variables & other complications
- Provide closed form solution for the case with vanishing risk.

Fact 1:
Stock market valuation (PD ratio) displays low frequency mean reversion

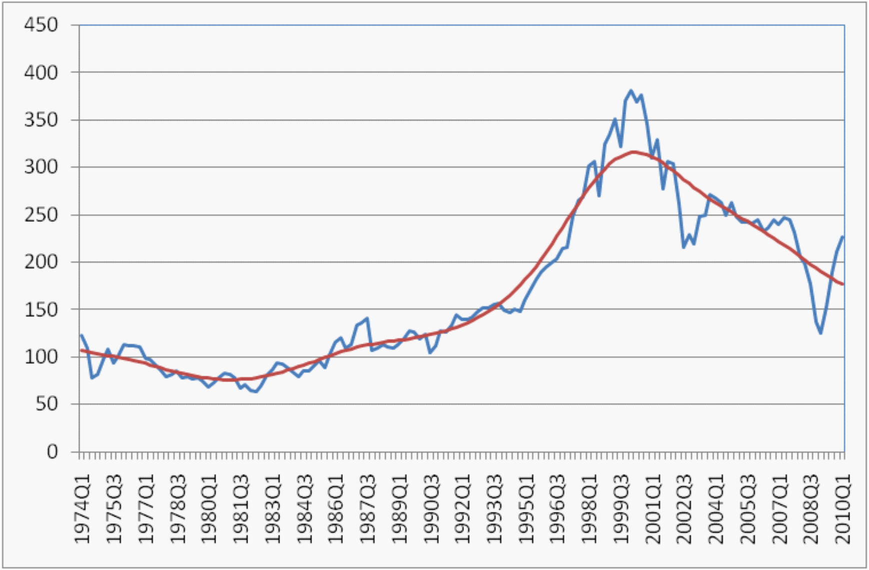
Euro Area: Quarterly Price Dividend Ratio



Japan: Quarterly Price Dividend Ratio



U.S.: Quarterly Price Dividend Ratio



Learning: why relevant for boom and bust behavior?

Fact 2: Dividend growth is largely unpredictable

Facts 1 + 2 =>

High market valuation predicts low future (excess) stock returns

	EMU(84-06)		U.S.(74-06)		Japan(74-06)	
k	c_1	R^2	c_1	R^2	c_1	R^2
4	-0.20 (0.06)	0.06	-0.0426 (0.02)	0.06	-0.20 (0.035)	0.21
8	-0.16 (0.06)	0.11	-0.0422 (0.01)	0.10	-0.21 (0.02)	0.44
12	-0.16 (0.02)	0.24	-0.0432 (0.01)	0.16	-0.19 (0.01)	0.54

Learning: why relevant for boom and bust behavior?

- **Facts 1 + 2 & External Rationality (RE):**

  Investors' return expectations low when market valuation (PD) is high

- Available evidence suggests the opposite...

Learning: why relevant for boom and bust behavior?

UBS/Gallup Survey, Source: Vissing-Jorgensen (2003 NBER Macro Annual).

Learning: why relevant for boom and bust behavior?

- Data suggests: boom & bust behavior (at least partly) driven by investors' return expectations
- Investors observe high returns & become optimistic about future returns
- Return optimism drives up prices (IES$>$1) $=>$ high realized returns
- Return optimism *positively* associated with market valuation
- What causes mean reversions? Consumption plans...

Model Sketch

Investors' maximization problem:

$$\max_{\left\{C_t^i \geq 0, S_t^i \in \left[0, \overline{S}\right]\right\}_{t=0}^{\infty}} E_0^{\mathcal{P}^i} \left[\sum_{t=0}^{\infty} \delta^t \frac{C_t^{i\,1-\gamma}}{1-\gamma} \right] \quad (1)$$

$$\begin{aligned} & s.t. \\ S_t^i P_t + C_t &= S_{t-1}^i \left(P_t + D_t\right) \text{ for all } t \geq 0 \\ S_{-1}^i &> 0 \text{ given} \end{aligned}$$

Sufficient intertemporal elasticity of substitution (IES): $\gamma^{-1} > 1$

Standard dividend process:

$$\ln D_t = \ln D_{t-1} + \ln \beta^D + \ln \varepsilon_t^D$$

Model Sketch

- $\mathcal{P}^i$: agents' subjective probability measure defined over space of outcomes Ω
- $\omega \in \Omega$: $\omega = \{P_0, D_0, P_1, D_1, P_2, D_2, ...\}$
- ω^t : history of ω up to period t
- Ω^t : set of possible histories up to t
- Agents make fully contingent plans:

$$S_t^i : \Omega^t \rightarrow [0, \overline{S}]$$

- $\mathcal{P}^i$: will be generated from some 'perceived law of motion' + prior beliefs about unknown parameters in the law of motion

Rational Expectations Equilibrium

REE returns:

$$\ln R_t = \ln \overline{R} + \ln \varepsilon_t^D \tag{2}$$

with

$$\overline{R} = \frac{\delta^{-1}\left(\beta^D\right)^{\gamma}}{e^{-\gamma(1-\gamma)\frac{\sigma_D^2}{2}}}$$

Relaxing External Rationality

- Keep rational dividend expectations

  => all price effects from subjective beliefs about price behavior

- Differentiates us from Bayesian RE learning literature:

  only subjective beliefs about dividends.

Relaxing External Rationality

- Relax agents' prior beliefs

$$\ln R_t = \ln \overline{R}_t + \ln \varepsilon_t \tag{3}$$

ε_t : transitory (iid) return component

$\overline{R}_t$: persistent & time varying return component

$$\ln \overline{R}_t = \ln \overline{R}_{t-1} + \ln \nu_t$$

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- Note: no mean reversion incoorporated into beliefs!

Relaxing RE Beliefs

- Learning....

$$\ln R_t = \underbrace{\ln \overline{R}_t + \ln \varepsilon_t}_{\text{observed jointly}}$$

... & need to attribute returns to persistent & transitory components.

- Prior beliefs

$$\ln \overline{R}_0 \sim N(\ln m_0, \sigma_0^2) \tag{4}$$

updated using Bayes rule.

$\sigma_0 > 0$ is stationary variance under the Kalman filter.

The Evolution of Beliefs

- Bayesian updating of beliefs

$$\ln m_t = \ln m_{t-1} + g\left(\ln R_t + \frac{\sigma_\varepsilon^2 + \sigma_v^2}{2} - \ln m_{t-1}\right)$$

In which sense a 'small deviation from RE priors'?

- RE prior

$$\ln R_t = \ln \overline{R} + \ln \varepsilon_t^D$$

- Agents' prior beliefs

$$\begin{aligned} \ln R_t &= \ln \overline{R}_t + \ln \varepsilon_t && (5) \\ \ln \overline{R}_t &= \ln \overline{R}_{t-1} + \ln \nu_t && (6) \end{aligned}$$

- Small noise limit: $\sigma^2_{\ln \varepsilon^D} \rightarrow 0, \sigma^2_{\ln v}, \sigma^2_{\ln \varepsilon} \rightarrow 0$, $\sigma^2_{\ln v} \rightarrow 0$
- As long as $\lim \sigma^2_{\ln \varepsilon} / \sigma^2_{\ln v} \in]0, \infty[$ learning well defined in the limit
- If initial beliefs $\ln m_0 = \ln \overline{R}$, as we assume, then agents' $\mathcal{P}$ **arbitrarily close** to PF-REE beliefs (in distribution)

Asset Demand under Learning about Return Behavior

- Determine asset demand function

$$S(S_{1-1}, \frac{P_t}{D_t}, \ln m_t)$$

solves the FOCs of the investment problem under the perceived state dynamics

$$\begin{aligned}
S_t &= S(S_{1-1}, \frac{P_t}{D_t}, \ln m_t) \\
\frac{P_{t+1}}{D_{t+1}} &= \frac{R_{t+1} P_t - D_{t+1}}{D_{t+1}} \\
\ln m_{t+1} &= \ln m_t + g\left(\ln R_{t+1} + \frac{\sigma_\varepsilon^2 + \sigma_v^2}{2} - \ln m_t\right)
\end{aligned}$$

Equilibrium Prices under Learning Behavior

- If all agents hold the same beliefs, the market clearing condition is

$$1 = S(1, (P_t / D_t)^*, \ln m_t)$$

which defines equilibrium asset price as a function of beliefs

$$PD^*(\ln m_t)$$

- Sufficient intertemporal elasticity ($\gamma^{-1} > 1$): PD increases with return expectations

Analytical Solution with Vanishing Risk

Proposition

Proposition: *Under vanishing uncertainty, i.e.,* $\left(\sigma_{\varepsilon}^2, \sigma_{\nu}^2, \sigma_{\varepsilon^D}^2\right) \to 0$, *the equilibrium PD is given by*

$$\frac{P_t}{D_t} + 1 = \sum_{j=0}^{\infty} \left(\left(\delta^{\frac{1}{\gamma}}\right)^j \prod_{i=1}^{j} \left(E_t^{\mathcal{P}} R_{t+i}\right)^{\frac{1-\gamma}{\gamma}} \right) \tag{7}$$

Holds for all beliefs $\mathcal{P}$ whether externally rational or not.

Analytical Solution with Vanishing Risk

For the specific beliefs above, realized returns:

$$R_t = \frac{1 - \left(\delta \left(m_{t-1}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}}{1 - \left(\delta \left(m_t\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}} \frac{1}{\left(\delta \left(m_{t-1}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}} \frac{D_t}{D_{t-1}} \tag{8}$$

- Constant beliefs: $m_{t-1} = m_t$
- Changing beliefs: $m_{t-1} \neq m_t$

Booms & Busts in the Learning Model

- Simulate the learning model: low frequency boom and bust dynamics?
- Baseline parameterization (quarterly model):

$$
\begin{aligned}
\delta &= 0.995 \\
\beta^D &= 1.0035 \\
\gamma &= 0.8 \\
g &= 0.014
\end{aligned}
$$

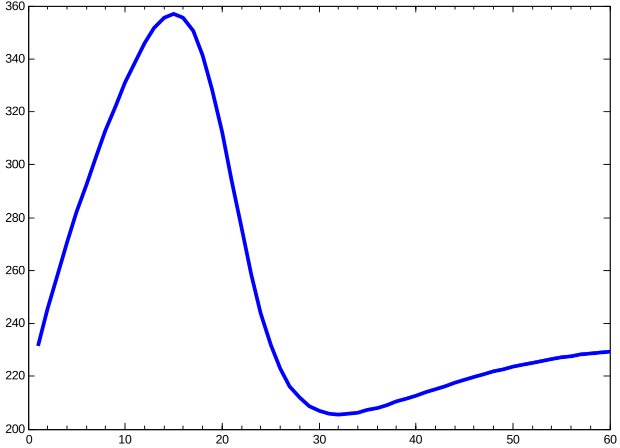
- Agents attribute 1,4% of any return observation to the persistent component and 98,6% to the transitory component.

Booms & Busts in the Learning Model

- Impulse response to a 10 basis points change of the quarterly real return expectations from its PF-REE value (78 bp)
- Positive and negative impulse responses: non-linear model.
- Plausibly sized impulse given the data: generated by observation of +8% (-7%) quarterly real return

Booms & Busts in the Learning Model

IR of the PD Ratio to a 10bp increase in permanent return exp (m_t)



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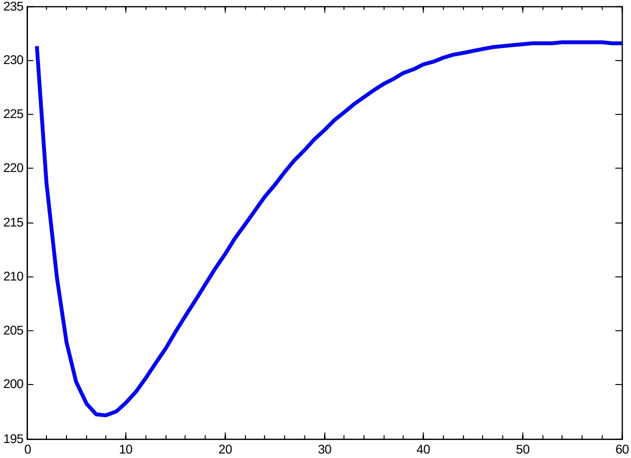

Booms & Busts in the Learning Model

- At the peak of the boom: return expectations very optimistic
- Positive comovement between market valuation & return expectations consistent with survey data
- Agents make very optimistic consumption plans

$$1 = \delta E_t^{\mathcal{P}^i} \left[\frac{R_{t+1}}{\left(\frac{C_{t+1}}{C_t}\right)^{\gamma}} \right]$$

Matching the US PD Ratio & Survey Expectations

- Assess the ability of the learning model to replicate
  - the time series of US PD Ratio 1926-2006
  - the survey data 1998-2003
- Learning model parameters

$$
\begin{aligned}
g &= 0.014 \\
m_0 &: \text{ to match PD ratio in 1926:4} \\
\delta &= 0.988 \\
\gamma &= 0.72
\end{aligned}
$$

- Historical return process & initial belief $m_0 \Rightarrow$ time series of model-implied beliefs $\{m_t\}$
- Compare PD ratio in the data to the model implied PD ratio:

$$P_t / D_t = \left(\delta \left(m_t\right)^{1-\gamma}\right)^{\frac{1}{\gamma}} / \left(1 - \left(\delta \left(m_t\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}\right)$$

Matching the US PD Ratio & Survey Expectations

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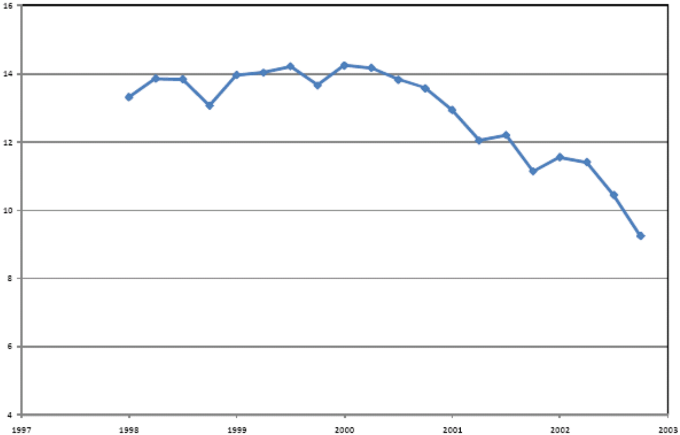

Figure 9: Model Implied Return Expectations

Summary

- Endogenous expectations driven boom and bust cycles in a model with rationally investing agents
- Agents are 'internally rational' but have insufficient knowledge about the equilibrium behavior of returns/market discount factor
- Bayesian learning about return behavior causes booms + busts: momentum and mean reversion
- Asset prices comove positively with return expectations, unlike in RE models
- Price fluctuations potentially inefficient (but no welfare implications in current model)