

# The Demand for Liquid Assets and International Capital Flows<sup>1</sup>

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## **Abstract**

In the recent decade, capital outflows from emerging economies, in the form of a demand for liquid assets, have played a key role in the context of global imbalances. In this paper, we present a two-country dynamic model, with an emerging country and an industrial country, where excess saving may be generated by a demand for liquidity. This demand comes from constrained entrepreneurs in the emerging country. We show that the implications of this model for the international transmission of shocks are totally different from standard models. For example, a permanent productivity increase in the emerging country leads to a capital outflow, a lower world interest rate and a positive spillover to the industrial country. On the other hand, shocks in the industrial country have little impact on the other country and on capital flows, which is consistent with their resilience during the recent crisis.

# 1 Introduction

The recent emergence of developing countries as major actors in international financial markets is a challenge to economic analysis. That the fastest-growing countries export capital instead of attracting it is a puzzle, as pointed out by Lucas (1990), and more recently by Gourinchas and Jeanne (2007). Moreover, a remarkable feature is that these flows take mostly the form of liquid assets, mainly in US dollars and that they have remained relatively stable during the recent financial crisis. Given their significance in the context of global imbalances, it is important to understand both the nature and the implications of these flows. The recent literature has proposed two explanations for the net capital outflows from emerging markets. First, emerging markets have a lower level of financial development, in particular a limited supply of liquid assets (e.g., Dooley et al. (2005), Matsuyama (2007), Ju and Wei (2006, 2007), Caballero et al. (2008) and Aguiar and Amador (2009) ). Second, net capital outflows result from precautionary saving due to idiosyncratic risk (e.g. Mendoza et al. (2009), Sandri (2010)). However, the literature has given little attention to the implications of these flows for the international transmission of shocks. For example, it has not explained the resilience of these flows during the crisis.

The objective of this paper is twofold. First, we offer an alternative explanation for the net capital outflow based on a demand for liquidity which arises even in the absence of risk. Second, we examine the implications of such a demand for the international transmission of shocks in a two-country dynamic general equilibrium framework. We consider an asymmetric world economy composed of an industrial country and an emerging country, the difference being their level of financial development. We show that, due to the lower level of financial development, the emerging country has a demand for liquidity that can generate capital outflows. Moreover, we show that this demand is less sensitive to interest rate changes than capital flows based on other motives. In this context, we show that shocks occurring in the industrial country have little effect on net capital flows and on the emerging country, while emerging country shocks have a significant impact on capital flows and on the industrial country. For example, an increase in productivity in the emerging country leads to a net capital flow to the industrial country (implying a "saving glut") and a lower interest rate. On the other hand, a

negative productivity shock in the industrial country has little impact on capital flows, while it also decreases the interest rate. The exchange rate impact is also muted. This is consistent with the limited reaction of net capital flows and exchange rates in the wake of the financial crisis.

The demand for liquid assets comes from infinitely lived credit constrained entrepreneurs who have investment projects that last two periods. Entrepreneurs need to install their capital one period before producing, so capital is a long-term asset while bonds are short-term assets. In the period where entrepreneurs install their capital, they anticipate a need for funds (working capital) to operate their firms, e.g. to hire labor. If entrepreneurs are credit-constrained for their working capital, they will need to save in a liquid asset at the same time as they invest in capital. Investment and the demand for bonds are thus complements. In contrast, if entrepreneurs are unconstrained, they can borrow their working capital and have no need for liquidity. This liquidity motive is generated by a production structure, with a time-to-build technology and with working capital, that can be naturally incorporated in a dynamic macroeconomic model.<sup>1</sup> We assume that entrepreneurs have an investment project every other period and that at each period half the entrepreneurs have a new project.

While our model is built to study macroeconomic questions which have hardly been addressed in the literature, it shares many features with the vast literature on liquidity. In particular, as in Holmstrom and Tirole (2001), the lack of pledgeability of future output is crucial to generate a demand for liquid assets.<sup>2</sup> In a dynamic macro context, our demand for liquidity is in the spirit of Woodford (1990), where entrepreneurs receive high productivity projects on alternate dates. It is also in the spirit of Kiyotaki and Moore (2008), where entrepreneurs have a fifty percent probability of receiving a high productivity shock. Our production structure is different and does not assume productivity heterogeneity across agents. The only source of heterogeneity is the existence of two groups of entrepreneurs who start projects at alternative dates.

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<sup>1</sup>The assumptions of time-to-build and working capital are often made in macroeconomic models. For example, see Gilchrist and Williams (2000) for multi-period investment projects and Christiano et al. (2010) working capital to pay for the wage bill.

<sup>2</sup>Most of the literature following Holmstrom and Tirole (2001) is cast in a microeconomic setup with two or three periods. However, Aghion et al. (2010) present a dynamic macroeconomic model where entrepreneurs hoard in the perspective of future liquidity shocks.

Our contribution is related to a growing literature introducing credit market imperfections in open economy models.<sup>3</sup> In particular, Song et al. (2009) model a capital outflow with firm heterogeneity specific to the Chinese economy. However, their focus is on growth and they do not introduce a demand for liquidity.

It is useful to compare the mechanism leading to liquidity demand to the mechanism leading to precautionary saving in the presence of risk, as in Mendoza et al. (2009). A major difference is that with the liquidity motive bonds and investment are complements instead of being substitutes. In precautionary saving models, the demand for bonds comes from a preference for safe assets as opposed to risky capital, but bonds and capital are still substitutes. For example, with a liquidity need a net capital outflow will be associated with higher productivity and higher investment.<sup>4</sup> To draw a sharp contrast with the impact of precautionary saving, we consider a model without uncertainty.

To better understand the model's mechanism we first examine the behavior of entrepreneurs in partial equilibrium when they are constrained and unconstrained. We show that credit-constrained entrepreneurs have a demand for liquidity and examine the properties of this demand. We show in particular that it is insensitive to the interest rate, and thus to foreign shocks. Then we incorporate these entrepreneurs in a two-country general equilibrium model, assuming that entrepreneurs in one country, the Emerging country, are constrained and those in the other country, the Industrial country, are unconstrained. We derive analytical results for log utility and Cobb-Douglas production functions and then provide numerical results in more general cases. An important aspect of the analysis is the asymmetric impact of shocks. Shocks originating in the Industrial country have little impact on the Emerging country so that they impact the Industrial country as if it were a closed economy. On the other hand, shocks originating in the Emerging country affects both countries. However, the impact of the shocks is different from standard models. While the standard response to a positive productivity shock is a capital inflow and an increase in interest rates, in our model the opposite occurs. An increase in productivity leads to a capital outflow, a decrease in the interest rate and an

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<sup>3</sup>Earlier contributions include Aghion et al. (2004).

<sup>4</sup>In Mendoza et al. (2009) and especially Mendoza et al. (2007), global imbalances are not associated with an increase in investment. On the contrary, excess saving generated by risk is diverted from domestic capital to foreign assets which leads to a decrease in investment.

increase in foreign investment.

We show that productivity shocks can capture the recent developments in the world economy. First, a permanent increase in the productivity of the Emerging country leads to global imbalances with a lower interest rate, a booming world economy and a delayed appreciation of the Emerging country currency. Second, a temporary decrease in productivity in the Industrial country (capturing the initial crisis in the US) has little impact on capital flows and implies only a limited depreciation of the Industrial country currency.

In the next section we describe the mechanism leading to the demand for liquidity by credit-constrained entrepreneurs. Section 3 presents the two-country general equilibrium model and Section 4 analyze the impact of productivity shocks. Section 5 will develop several extensions and Section 6 concludes.

## **2 Entrepreneurs and the Demand for Liquidity**

We first consider entrepreneurs in a partial equilibrium setup. This allows us to clearly understand the mechanism behind the demand for liquid assets. There are basically three ingredients in the model that are necessary to generate a demand for liquidity. First, production takes time: capital needs one installation period before it can be used in the production process. Second, workers have to be paid before output is available to entrepreneurs. This generates a need for funds. The third assumption is that entrepreneurs face credit constraints. This implies that entrepreneurs are not always able to borrow all the funds needed to hire labor for production. Consequently, when they invest in capital, entrepreneurs need to keep liquid assets.

The fact that liquid assets are used to finance a production factor (here, labor) that is imperfectly substitutable with capital generates a complementarity between these assets and capital. In the following sections, this complementarity in the entrepreneurs' portfolio will appear essential to explain both global imbalances and the international transmission of shocks.

In this section, we focus on the demand for liquidity by entrepreneurs. In particular, we study how they allocate their savings between capital and liquidity. We first describe

the optimal behavior of entrepreneurs in a general setup. We then focus on a benchmark case that allows us to derive analytical results. In that case we derive the demand for liquidity and compare it to the case of unconstrained entrepreneurs, where no demand for liquidity arises. We show that the response of the demand for liquidity, and thus of capital flows, is very different from standard models. At the end of the section we also examine how the results are affected when we deviate from the benchmark case.

## 2.1 The production process

Entrepreneurs are infinitely lived and maximize the present value of their utility. They have two-period production projects as it takes one period to install capital before producing. An entrepreneur starting a project at time  $t$  invests  $K_{t+1}$ . At  $t+1$ , once capital is installed, he hires labor  $l_{t+1}$  and pays a wage  $w_{t+1}$  to produce  $F(K_{t+1}, l_{t+1})$ .<sup>5</sup> This production is available only at  $t+2$ . At  $t+2$ , the entrepreneur gets another investment opportunity. The entrepreneur also consumes  $c_t$  each period and can borrow or lend short-term bonds  $B_t$  with a gross interest rate  $r_t$ .

In this setup, entrepreneurs can use part of the proceeds from previous production to invest  $K_{t+1}$  at  $t$ . At  $t+1$ , however, they have no income to pay  $w_{t+1}l_{t+1}$  for the workers.<sup>6</sup> Consequently, they have an incentive to borrow  $-B_{t+2}$ . When an entrepreneur is credit constrained, however, he will not be able to borrow the desired amount to pay for the wage bill. He will therefore have a demand for liquidity at time  $t$  in the form of a positive demand for bonds,  $B_{t+1}$ . When the entrepreneur is unconstrained, there is no need for liquidity at time  $t$ .

## 2.2 Optimal Behavior

Entrepreneurs maximize:

$$\sum_{s=0}^{\infty} \beta^s u(c_s) \tag{1}$$

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<sup>5</sup>The production function  $F$  is concave and satisfies the Inada conditions.

<sup>6</sup>Workers are fully paid in advance. We could assume that only a proportion of wages are paid in advance, but this would not qualitatively alter our results.

Consider an entrepreneur who invests every other period, starting at time  $t$ . His budget constraint at  $t$  and  $t + 1$  are:

$$r_t B_t + A_{t-1} F(K_{t-1}, l_{t-1}) = c_t + K_{t+1} + B_{t+1} \quad (2)$$

$$r_{t+1} B_{t+1} = c_{t+1} + w_{t+1} l_{t+1} + B_{t+2} \quad (3)$$

The income of the entrepreneur at date  $t$  is made of the production process initiated at date  $t - 2$ ,  $A_{t-1} F(K_{t-1}, l_{t-1})$ , and of the return from bond holdings,  $r_t B_t$ . This income is allocated to consumption,  $c_{t+1}$ , investment in a new project,  $K_{t+1}$ , and bond holdings  $B_{t+1}$ . In the following period, at  $t + 1$ , the only income is the bond return,  $r_{t+1} B_{t+1}$ . This has to pay for consumption  $c_{t+1}$  and the wage bill  $w_{t+1} l_{t+1}$ . Typically the entrepreneur will borrow, so that at the optimum  $B_{t+2} \leq 0$ .

The entrepreneur might face a credit constraint at date  $t + 1$ . Due to standard moral hazard arguments, a fraction  $0 \leq \phi \leq 1$  of capital has to be used as collateral for bond repayments:<sup>7</sup>

$$r_{t+2} B_{t+2} \geq -\phi K_{t+1} \quad (4)$$

Let  $\lambda_{t+1}$  denote the multiplier associated with this constraint. The entrepreneur's program yields the following first-order conditions:

$$A_{t+1} F_k(K_{t+1}, l_{t+1}) = r_{t+1} r_{t+2} \left( 1 + \frac{\lambda_{t+1}}{\beta u'(c_{t+2})} \left( 1 - \frac{\phi}{r_{t+1} r_{t+2}} \right) \right) \quad (5)$$

$$A_{t+1} F_l(K_{t+1}, l_{t+1}) = r_{t+2} w_{t+1} \left( 1 + \frac{\lambda_{t+1}}{\beta u'(c_{t+2})} \right) \quad (6)$$

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta r_{t+1} \quad (7)$$

$$\frac{u'(c_{t+1})}{u'(c_{t+2})} = \beta r_{t+2} \left( 1 + \frac{\lambda_{t+1}}{\beta u'(c_{t+2})} \right) \quad (8)$$

The credit constraint (4) introduces three wedges in the optimal decisions. First, from equation (5), when  $\lambda_{t+1} = 0$ , the marginal return of capital invested at  $t$  should be equal to the return of one unit invested over two periods in the bond, as capital is immobile for two periods. But when  $\lambda_{t+1} > 0$ , the constraint is binding at  $t + 1$ , which implies that the entrepreneur is unable to finance the wage bill associated with the first-best capital stock. This creates a wedge between the return of capital and the bond

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<sup>7</sup>There could be a similar constraint at date  $t$ , but one can show that it is never binding, precisely because of the demand for liquidity.



return. Moreover, this wedge is decreasing in  $\frac{\phi}{r_{t+1}r_{t+2}}$ , which is the relative liquidity value of capital as compared to the bond. Second, from equation (6), when  $\lambda_{t+1} = 0$ , the marginal return of labor should be equal to its cost, which is given by the wage rate multiplied by the interest rate, because production is available only in the following period. When  $\lambda_{t+1} > 0$ , the entrepreneur has exhausted his financing capacities before hiring the first-best level of labor. Finally, when  $\lambda_{t+1} > 0$ , it is more difficult to transfer consumption between period  $t + 1$  and  $t + 2$ : there are excess saving at  $t + 1$ , as equation (8) suggests.

### 2.3 The Demand for Liquidity in the Benchmark Case

To derive simple analytical results for the constrained entrepreneur ( $\lambda_{t+1} > 0$ ), we consider a benchmark where we make three specific assumptions: i) utility is logarithmic:  $u(c) = \log(c)$ ; ii) the production function is Cobb-Douglas:  $F(K, l) = K^\alpha l^{1-\alpha}$ ; iii) entrepreneurs cannot borrow:  $\phi = 0$ . We examine analytically the implications of relaxing these assumptions in Section 2.4. We also look at more general cases in the numerical simulations.

With log utility, it can be shown that an entrepreneur who invests at  $t$  consumes a fixed fraction of his revenue:

$$c_t = (1 - \beta)[r_t B_t + A_{t-1} F(K_{t-1}, l_{t-1})] \quad (9)$$

Using the Euler equation (7) at  $t$ , we get the following rule for consumption at  $t + 1$ :

$$c_{t+1} = \beta(1 - \beta)r_{t+1}[r_t B_t + A_{t-1} F(K_{t-1}, l_{t-1})] \quad (10)$$

From (2) and (9), total saving at  $t$  are:

$$S_{t+1} = B_{t+1} + K_{t+1} = \beta[r_t B_t + A_{t-1} F(K_{t-1}, l_{t-1})] \quad (11)$$

Equation (11) states that total saving at  $t$  is a constant fraction of total revenues. When the constraint at  $t + 1$  is binding, the availability of funds to finance the wage bill at  $t + 1$  is limited. The fraction of saving allocated to liquidity  $B_{t+1}$  therefore depends on the liquidity needs at  $t + 1$ ,  $w_{t+1}l_{t+1}$ . These needs are related to the amount of capital  $K_{t+1}$  invested at  $t$ , since  $K_{t+1}$  and  $l_{t+1}$  are imperfect substitutes.

Since  $\phi = 0$ , the first-order conditions (6) and (5) give a straightforward relationship between the liquidity needs  $w_{t+1}l_{t+1}$  and capital  $K_{t+1}$ :

$$w_{t+1}l_{t+1} = \frac{1 - \alpha}{\alpha} r_{t+1} K_{t+1} \quad (12)$$

To determine  $K_{t+1}$  we use (3), (10), (11) with (12) to get:

$$K_{t+1} = \alpha\beta^2 [A_{t-1} F(K_{t-1}, l_{t-1})] \quad (13)$$

Replacing in (11), we obtain:

$$B_{t+1} = \beta(1 - \alpha\beta) [A_{t-1} F(K_{t-1}, l_{t-1})] \quad (14)$$

Moreover, since  $\phi = 0$ ,  $B_t = B_{t+2} = 0$ .

A striking result is that the demand for liquidity,  $B_{t+1}$ , is independent of  $A_{t+1}$  and  $r_{t+1}$  (and so is  $K_{t+1}$ ). This is because the constraint is binding, which means that  $K_{t+1}$  is not optimal, which disconnects both  $K_{t+1}$  and  $B_{t+1}$  from  $A_{t+1}$  and  $r_{t+1}$ . Since  $B_t = B_{t+2} = 0$ , they are obviously also independent of  $A_{t+1}$  and  $r_{t+1}$ . The absence of impact of  $r_{t+1}$  will have important implications in a general equilibrium framework as it means that a constrained economy is isolated from foreign shocks. This will have important implications in terms of the international transmission of shocks.

Another important consequence of (13) and (14) is that the ratio between  $B_{t+1}$  and  $K_{t+1}$  is constant:

$$\frac{B_{t+1}}{K_{t+1}} = \frac{1 - \alpha\beta}{\alpha\beta}$$

This equation implies that, contrary to the standard models, capital and bonds are complements, because bonds are needed to finance the wage bill, which is proportional to capital. Indeed, the bond-capital ratio is decreasing in  $\alpha$ , the share of capital in the value added. The higher  $\alpha$ , the lower the amount of bonds needed to finance labor. An important consequence of this result is that growth in  $K$  will naturally generate growth in  $B$ , leading to so-called “global imbalances”.

This behavior, that is the inelasticity of the demand for liquidity and the complementarity between liquidity and capital, is in sharp contrast with the case where entrepreneurs are unconstrained.

## 2.4 Unconstrained Entrepreneurs

When entrepreneurs are unconstrained, the optimal demand for bonds at  $t + 1$  is still given by (11), but  $K_{t+1}$  is unconstrained. According to (5), a productivity shock has a negative effect on capital:

$$\frac{\partial K_{t+1}}{\partial A_{t+1}} = \frac{K_{t+1}}{(1 - \alpha)A_{t+1}l_{t+1}} > 0$$

Consequently, from (11) we conclude that the productivity shock has a negative impact on bonds at  $t$ , i.e.  $\partial B_{t+1}/\partial A_{t+1} = -\partial K_{t+1}/\partial A_{t+1} < 0$ .

The other difference between constrained and unconstrained entrepreneurs is the reaction of bonds demand to the interest rate. Differentiating (11) and (5) with respect to  $r_{t+1}$ , we obtain:

$$\frac{\partial B_{t+1}}{\partial r_{t+1}} = \frac{-\partial K_{t+1}}{\partial r_{t+1}} = \frac{K_{t+1}}{(1 - \alpha)r_{t+1}l_{t+1}} > 0 \quad (15)$$

Contrary to the benchmark case, the demand for bonds in  $t+1$ ,  $B_{t+2}$  is unconstrained, and it therefore reacts to the future interest rate  $r_{t+2}$ .  $B_{t+2}$  can be found using (3), (10), (11), and the zero profit condition ( $A_{t+1}F(K_{t+1}, l_{t+1}) = r_{t+1}r_{t+2}K_{t+1} + r_{t+2}w_{t+1}l_{t+1}$ ):

$$B_{t+2} = \beta^2 r_{t+1} [r_t B_t + A_{t-1} F(K_{t-1}, l_{t-1})] - \frac{A_{t+1} F(K_{t+1}, l_{t+1})}{r_{t+2}} \quad (16)$$

Differentiating (16) with respect to  $r_{t+2}$ , we obtain:

$$\frac{\partial B_{t+2}}{\partial r_{t+2}} = \frac{r_{t+1}}{1 - \alpha} + \frac{A_{t+1} F(K_{t+1}, l_{t+1})}{r_{t+2}^2} > 0 \quad (17)$$

From (16) we also clearly have  $\partial B_{t+2}/\partial r_{t+3} = 0$  and  $\partial B_{t+2}/\partial A_{t+2} = 0$ .

Thus, contrarily to the constrained case, the demand for bonds is elastic to both productivity and the interest rate, in both stages of production. Besides, capital and the demand for bonds are substitutes. Indeed, capital is determined by (5), and the demand for bonds is determined by the amount of saving that is not used for capital.

## 2.5 Deviations from the Benchmark Case

In the benchmark case, we found the strong result that the demand for liquidity is totally insensitive to contemporaneous interest rate or productivity shocks. It is interesting to analyze in what direction results change when we deviate from the benchmark. We show

below that with a more general production function the demand for bonds reacts to the interest rate with a sign that depends on the substitutability between labor and capital. We also show that when the credit constraint is less extreme, i.e.  $\phi > 0$ , the demand for bonds responds more to the interest rate.

### 2.5.1 Positive $\phi$

Assume that  $\phi > 0$ . This implies a capital inflow in production periods, i.e.,  $B_t = -\phi K_{t-1}/r_t$  and  $B_{t+2} = -\phi K_{t+1}/r_{t+2}$ . Moreover, combining (5) and (6) gives:

$$w_{t+1}l_{t+1} = \frac{(1 - \alpha) \left( r_{t+1} - \frac{\phi}{r_{t+2}} \right) K_{t+1}}{\alpha - \frac{\phi k_{t+1}^{1-\alpha}}{A_{t+1}}} \quad (18)$$

We use this equation to replace  $w_{t+1}l_{t+1}$  in (3) and (10) to replace  $c_{t+1}^i$ .

After rearranging, we obtain:

$$K_{t+1} \left( 1 - \frac{\phi}{r_{t+1}r_{t+2}} \right) \left( 1 + \frac{1 - \alpha}{\alpha - \frac{\phi K_{t+1}^{1-\alpha}}{A_{t+1}}} \right) = \beta^2 \left( A_{t-1}f(K_{t-1}) - \frac{\phi K_{t-1}}{r_t} \right) \quad (19)$$

From (19) we find that  $\partial K_{t+1}/\partial r_{t+1} < 0$  and  $\partial K_{t+1}/\partial r_{t+2} < 0$  so that  $\partial B_{t+1}/\partial r_{t+1} > 0$  and  $\partial B_{t+1}/\partial r_{t+2} > 0$ . There is now an additional effect due to a positive  $\phi$ . From (5), we see that the liquidity premium is decreasing in  $\frac{\phi}{r_{t+1}r_{t+2}}$ . An increase in interest rates implies a higher premium for bonds and thus an increase in demand.

### 2.5.2 More general production function

Assume a CES production function:

$$F(K, l) = \left( \alpha K^{1-\frac{1}{\rho}} + (1 - \alpha) l^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}}$$

where  $\rho$  is the elasticity of substitution between capital and labor. When  $\rho = 1$ ,  $F$  boils down to the Cobb-Douglas function used in the benchmark case. When  $\rho = 0$ ,  $F$  is a Leontief function.

In that case, (6) and (5) yield:

$$\alpha w_{t+1} K_{t+1}^{\frac{-1}{\rho}} = (1 - \alpha) r_{t+1} l_{t+1}^{\frac{-1}{\rho}} \quad (20)$$

which gives the demand for labor:

$$l_{t+1} = K_{t+1} \left( \frac{1 - \alpha}{\alpha} \frac{r_{t+1}}{w_{t+1}} \right)^\rho \quad (21)$$

We use (21) to replace  $l_{t+1}$  in (3) and (10) to replace  $c_{t+1}^i$ . We also set  $B_t^h = B_{t+2}^h = 0$  because the credit constraint is binding. After rearranging, we obtain:

$$K_{t+1} + \left( \frac{1 - \alpha}{\alpha} \right)^\rho K_{t+1} r_{t+1}^{\rho-1} w_{t+1}^{-\rho} = \beta^2 [A_{t-1} F(K_{t-1}, l_{t-1})]$$

Differentiating with respect to  $r_{t+1}$ , we can show that:

$$\frac{\partial B_{t+1}}{\partial r_{t+1}} = (\rho - 1) \frac{w_{t+1} l_{t+1}}{r_{t+1}^2 \left( 1 + \frac{w_{t+1} l_{t+1}}{r_{t+1} K_{t+1}} \right)}$$

The interest impact on capital and therefore on bond demand depends on the elasticity of substitution between capital and labor. This is due to the presence of a revenue and a substitution effect. The intuition is as follows. When the interest rate increases, fewer bonds are necessary to fulfil the liquidity needs, for a given wage bill. This is the negative revenue effect on the bond demand. But when the interest rate increases, the entrepreneur substitutes labor for capital, which means that the liquidity needs are exacerbated. This leads to a positive substitution effect on the bond demand. In the benchmark case,  $\rho = 1$ , there is no impact as these two effects cancel each other. When  $\rho < 1$ , the revenue effect dominates: as  $r_{t+1}$  increases, the income from bonds increases so that more labor can be hired. With low substitutability, this implies a higher need for capital so that  $K_{t+1}$  increases and  $B_{t+1}$  decreases. In contrast, when  $\rho > 1$ , the substitution effect dominates: as  $r_{t+1}$  increases, it is desirable to hire more labor, which reduces the demand for capital.<sup>8</sup>

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<sup>8</sup>At the level of the entrepreneur, labor supply is totally elastic. However, as we move to the aggregate level, the elasticity of labor supply plays a role. It can be shown that a reduced labor supply elasticity decreases the impact of an interest change on  $K_{t+1}$  when the production function is not Cobb-Douglas. The reason is simply that an increase in labor demand raises the wage rate, which offsets either the revenue or the substitution effects mentioned above. In the limit, when labor supply is totally inelastic, wage increases fully offset the other effects and the demand for bonds is inelastic as with the Cobb-Douglas production function.

### 3 A Two-country General Equilibrium Model

The entrepreneurs described above are incorporated in a two-country general equilibrium model. We consider an asymmetric world composed of an Emerging country with constrained entrepreneurs and an Industrial country with unconstrained entrepreneurs. We will denote Industrial country variables with an asterisk. The industrial country has a high level of financial development ( $\phi^*$  is large), so that entrepreneurs are never constrained. The emerging country has a low level of financial development; in the benchmark  $\phi = 0$ . The two countries are linked through the bond market as they can trade one-period bonds.

In each country there are two types of entrepreneurs and workers. We describe these in turn and examine the properties of the Emerging and Industrial country in the benchmark case. We then consider equilibrium in the two-country setup. We are particularly interested in the steady-state demand for bonds and in the international transmission of a productivity shock originated in the industrial country. In the next section, we solve the model numerically in a more general case.

#### 3.1 Labor supply

Labor is supplied domestically by a continuum of impatient workers of mass one who maximize their utility, which has the following form:

$$\sum_{s=0}^{\infty} \beta_w^s u(c_s^w - v(l_s)) \quad (22)$$

with  $v$  increasing and convex and  $\beta_w < \beta$ . Workers do not have access to the production technology, but they can buy bonds, so their budget constraint has the following form:

$$r_t B_t^w + w_t l_t = c_t^w + B_{t+1}^w \quad (23)$$

Workers have no collateral, so they face a no-borrowing constraint:

$$r_{t+1} B_{t+1}^w \geq 0 \quad (24)$$

Denote by  $\mu_t^w$  the multiplier associated with this credit constraint. The worker's program yields the following first-order conditions:

$$\frac{u'(c_t^w)}{u'(c_{t+1}^w)} = \beta_w r_{t+1} \left( 1 + \frac{\mu_t^w}{\beta_w u_{t+1}^w} \right) \quad (25)$$

$$w_t = v'(l_t) \quad (26)$$

We focus on equilibria where  $r_{t+1} < 1/\beta_w$ , so that workers are always constrained, which implies  $B_t^w = 0$  and  $c_t^w = w_t l_t$ . Condition (26) gives the following labor supply equation:

$$l_t = l(w_t) \quad (27)$$

where  $l = v'^{-1}$  is an increasing function.

For the rest of this section, we will assume that the labor supply is completely inelastic and normalized to one:  $l_t = 1$ . This assumption is used for the benchmark case and will be relaxed in the numerical analysis. We can then work with the production function  $f(K_t) = F(K_t, 1)$ . With Cobb-Douglas, output per capita at  $t + 1$  is simply  $A_t K_t^\alpha$ . The same assumptions are made for the Industrial country, so that Industrial output per capita is  $A_t^* K_t^{*\alpha}$ .

### 3.2 Two Types of Entrepreneurs

Each entrepreneur has access to a project every two periods. There are two types of entrepreneurs, each with a mass of one, with overlapping projects. Type 1 entrepreneurs get a project in odd periods, while Type 2 get a project in even periods. The analysis of a single entrepreneur, described in the previous section, can be easily extended by changing time subscripts. For example, if time  $t$  is an odd period, the demand for bonds (of constrained entrepreneurs) in the Emerging country is (based on (14)):<sup>9</sup>

$$B_{t+1}^1 = \beta(1 - \alpha\beta)A_{t-1}f(K_{t-1}) \quad (28)$$

$$B_{t+1}^2 = 0 \quad (29)$$

and the total demand for bonds is:  $B_{t+1} = B_{t+1}^1 + B_{t+1}^2$ .

In the Industrial country the demand for bonds is (from (11) and (16)):

$$B_{t+1}^{1*} = \beta^*[r_t B_t^{1*} + A_{t-1}^* f(K_{t-1}^*)] - K_{t+1}^* \quad (30)$$

$$B_{t+1}^{2*} = \beta^{*2} r_t [r_{t-1} B_{t-1}^{2*} + A_{t-2}^* f(K_{t-2}^*)] - \frac{A_t^* f(K_t^*)}{r_{t+1}} \quad (31)$$

and naturally  $B_{t+1}^* = B_{t+1}^{1*} + B_{t+1}^{2*}$ .

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<sup>9</sup>We do not need to use group superscripts for production variables, as they correspond to only one type of entrepreneurs in each period.

### 3.3 World Equilibrium

The interest rate will adjust such that equilibrium in the bonds market holds. It is given by:

$$B_{t+1} + B_{t+1}^* = 0 \quad (32)$$

### 3.4 Steady-State Liquidity Demand

First consider the unconstrained Industrial country in the benchmark case. Stationarity in the Industrial country's consumption implies that the steady state interest rate is  $\bar{r} = 1/\beta^*$ . As a result, the stationary capital stock in the absence of constraint,  $\bar{K}^*$ , is given by (5):

$$\bar{K}^* = (A\alpha\beta^{*2})^{\frac{1}{1-\alpha}} \quad (33)$$

The dynamics of the Emerging constrained economy are given by (13), which can be written as:

$$K_{t+1} = \alpha\beta^2 A_{t-1} K_{t-1}^\alpha \quad (34)$$

The stationary capital  $\bar{K}$  is then:

$$\bar{K} = (A\alpha\beta^2)^{\frac{1}{1-\alpha}} \quad (35)$$

If  $\beta = \beta^*$ , the Emerging country converges to the unconstrained steady state. When we assume that Emerging is constrained, it is either because it is still on its transition path or because  $\beta^* > \beta$ .

The steady-state demand for liquidity in the Emerging country is obtained by combining (28) and (35):

$$\bar{B} = \beta(1 - \alpha\beta)A\bar{K}^\alpha \quad (36)$$

In the two-country equilibrium, the Industrial country's debt is equal to the Emerging's demand for liquidity  $-\bar{B}^* = \bar{B}$ . Therefore, Industrial's debt, as a ratio of output, depends on the ratio between the Emerging and Industrial outputs:

$$\frac{\bar{B}^*}{A^*\bar{K}^{*\alpha}} = \beta(1 - \alpha\beta)\frac{A\bar{k}^\alpha}{A^*\bar{K}^{*\alpha}} = \beta(1 - \alpha\beta)\left(\frac{\beta}{\beta^*}\right)^{\frac{2\alpha}{1-\alpha}}\left(\frac{A}{A^*}\right)^{\frac{1}{1-\alpha}} \quad (37)$$



It is clear from (37) that TFP growth in the Emerging country,  $A$ , generates “global imbalances”. The reason is that, in the Emerging country, capital and bonds are complements: any increase in production generates both extra investment in capital and extra liquidity needs through the wage bill.

### 3.5 Productivity Shock in the Industrial Country

In this section we examine analytically the impact of a productivity shock in the Industrial country, focusing on the benchmark case. In the next section, we examine numerically the impact of shocks in the more general case.

To examine the impact of a shock to  $A_{t+1}^*$  on the interest rate and on capital flows, we simply differentiate (32) with respect to  $A_{t+1}^*$ :

$$\frac{\partial B_{t+1}^*}{\partial A_{t+1}^*} + \frac{\partial B_{t+1}^*}{\partial r_{t+1}} \frac{\partial r_{t+1}}{\partial A_{t+1}^*} + \frac{\partial B_{t+1}^*}{\partial r_{t+2}} \frac{\partial r_{t+2}}{\partial A_{t+1}^*} = - \left( \frac{\partial B_{t+1}}{\partial r_{t+1}} \frac{\partial r_{t+1}}{\partial A_{t+1}^*} + \frac{\partial B_{t+1}}{\partial r_{t+2}} \frac{\partial r_{t+2}}{\partial A_{t+1}^*} \right)$$

The total effect of the productivity shock on the Industrial country demand for bonds (LHS), which includes the direct effect and the indirect effect through the adjustment of the interest rates at  $t+1$  and  $t+2$ , depends on the elasticity of Emerging bond demand to interest rates. Using the fact that  $\frac{\partial B_{t+1}}{\partial r_{t+2}} = \frac{\partial B_{t+1}}{\partial r_{t+2}} = 0$  in the benchmark case, it is straightforward that the total effect on capital flows in the Industrial country is equal to zero.

$$\frac{\partial B_{t+1}^*}{\partial A_{t+1}^*} + \frac{\partial B_{t+1}^*}{\partial r_{t+1}} \frac{\partial r_{t+1}}{\partial A_{t+1}^*} + \frac{\partial B_{t+1}^*}{\partial r_{t+2}} \frac{\partial r_{t+2}}{\partial A_{t+1}^*} = 0$$

This implies that  $B_{t+1}^*$  is unaffected by  $A_{t+1}^*$  in equilibrium: the interest rate increases to offset the direct capital inflow effect caused by a surge in  $A_{t+1}^*$ . In other words, the Industrial country reacts as if it were a closed economy.

As a corollary of the absence of capital flows, the productivity shock in the Industrial country has no effect on investment. Indeed, saving is a function of past variables, so the total effect of  $A_{t+1}^*$  on  $K_{t+1}^*$  is equal to the opposite of the total effect on bond holdings:

$$\frac{\partial K_{t+1}^*}{\partial A_{t+1}^*} + \frac{\partial K_{t+1}^*}{\partial r_{t+1}} \frac{\partial r_{t+1}}{\partial A_{t+1}^*} + \frac{\partial K_{t+1}^*}{\partial r_{t+2}} \frac{\partial r_{t+2}}{\partial A_{t+1}^*} = - \left( \frac{\partial B_{t+1}^*}{\partial A_{t+1}^*} + \frac{\partial B_{t+1}^*}{\partial r_{t+1}} \frac{\partial r_{t+1}}{\partial A_{t+1}^*} + \frac{\partial B_{t+1}^*}{\partial r_{t+2}} \frac{\partial r_{t+2}}{\partial A_{t+1}^*} \right) = 0$$

In the benchmark case, the equilibrium  $K_{t+1}^*$  is unaffected by changes in  $A_{t+1}^*$ . Obviously, it is interesting to look at the dynamic response of a shock, but we leave this for the following section.

## 4 The International Transmission of Shocks and Global Imbalances

We now consider the dynamic impact of productivity shocks. Our main objective is to analyze the impact and the international transmission of shocks in a model with liquidity demand. We compare the asymmetric two-country model, with constrained entrepreneurs in the Emerging country, with a two-country model where both countries are unconstrained. As indicated in the previous sections, the impact may be quite different. We consider two shocks: 1) a permanent positive shock in the constrained Emerging country; 2) a temporary negative shock in the Industrial country. The first shock illustrates how higher productivity in emerging markets creates a larger demand for liquidity and leads to global imbalances. The second shock is meant to illustrate the impact of the US financial crisis and shows the resilience of the demand for liquidity.

We first present the results for a given set of parameters, that we call the numerical benchmark as opposed to the benchmark in the previous sections. The numerical benchmark shares several assumptions from the previous benchmark. We still assume a Cobb-Douglas production function and a log utility function. On the other hand, we assume that  $\phi = 0.1$  (instead of 0) and that the labor supply is somewhat elastic with  $v(l) = l^{\eta+1}/(\eta+1)$  and  $\eta = 1$ . The parameters of the numerical benchmark are given in Table 1. We look at the dynamics of the shocks starting from a steady state. When we compare with the unconstrained case, we normalize variables so that the initial steady state has the same value.<sup>10</sup>

We consider the impact of shocks on international net asset positions  $B_t = -B_t^*$ , on capital stocks  $K_t$  and  $K_t^*$ , on the world interest rate  $r_t$ , and on the real exchange rate  $w_t/w_t^*$ . The relative wage measures the real exchange based on labor costs and an increase in this ratio means a real appreciation of the Emerging country.

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<sup>10</sup>Notice that in the unconstrained case the level of net foreign asset is undetermined. We simply set it equal to the constrained steady state level.

#### 4.1 A Permanent Productivity Increase in the Emerging Country

Figure 1 shows the impact of a permanent increase in  $A_t$  of 1 percent from its steady state. The broken line shows the impact of the shock when both countries are unconstrained, while the continuous line shows the model with constrained entrepreneurs in the Emerging country. In standard international business cycle models a productivity increase leads to a capital inflow, an increase in the domestic capital stock, an increase in the interest rate and a decrease in the foreign capital stock. This is exactly what the broken line shows. The decline in  $B_t$  (or  $-B_t^*$ ) illustrates the capital inflow. In addition, there is an immediate real appreciation. Notice that the unconstrained model yields the standard results even with the special production structure with two-period investment and working capital.

When there is a demand for liquidity, the impact of the shock is totally different. Instead of a capital inflow, there is an outflow from the Emerging country and the net asset position is permanently higher. Capital also increases in the Emerging country, but more slowly due to the credit constraint. Capital increases in the Industrial country, in contrast with the unconstrained case. This is because of a lower interest rate, implied by the capital inflow into the Industrial country. As for the real appreciation, it is significantly delayed. There are two reasons for a smaller appreciation. First, investment in the Emerging country is slower due to the constraints. Second, investment in the Industrial country also increases, which drives up the wage in the Industrial country.

To summarize, a persistent productivity increase in the Emerging country is consistent with a world of global imbalances. An increased productivity implies an increased demand for liquidity and thus a capital outflow. This outflow pushes down the world interest rate and has a positive effect on the Industrial country. This is in contrast with the standard unconstrained model, where such a productivity increase would have a negative spillover.

#### 4.2 A Temporary Productivity Decrease in the Industrial Country

Figure 2 shows the impact of a temporary decline in the Industrial country. The shock is a 1 percent decline in  $A_t^*$  with a persistence of 0.9 and can be seen in the upper

left panel.<sup>11</sup> Here again the broken line represents the unconstrained case and yields standard results. The decline in Industrial country productivity is accompanied by a decline in Industrial country investment, a decline in capital inflow into the Industrial country, and a spike in Emerging investment associated with a drop in the interest rate. There is also a sharp real depreciation of the Industrial country currency.

Again, results are very different in presence of a demand for liquidity. Instead of a capital inflow to the Industrial country, the net asset position hardly changes. The decline in Industrial country capital is delayed and there is even a slight initial increase. The delayed decline in  $K_t^*$  occurs precisely because capital inflows do not decline. Instead the interest rate decreases more than in the unconstrained case. As shown in the previous section, the Emerging economy is not much affected by this shock and  $K_t$  is almost unchanged. Finally, there is no initial depreciation of the Industrial country currency. This reflects the delayed increase in  $K_t^*$  and the absence of reaction in  $K_t$ .

The outcome of this shock is in line with what happened during the recent financial crisis where we observed a resilience of the demand for liquidity and the net capital inflow to the US. It is also consistent with the absence of a real appreciation in the US dollar.

### 4.3 Robustness Tests

Figure 1 and 2 shows the results with the parameters specified in Table 1. It is interesting to see how the results are affected by considering differences in the elasticity of substitution between capital and labor, in the level of the credit constraint or in the elasticity of labor supply. While the main patterns are notably robust to changes in parameters, there are some interesting differences. TBC

## 5 Extensions

The model has been kept simple to illustrate the mechanism behind the demand for liquidity and analyze its implications for the transmission of shocks. Several extensions from this basic structure will be investigated. For example, it is straightforward to extend

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<sup>11</sup>More precisely, we assume that  $A_t^* = \bar{A}^*(1 - \delta) + \delta A_{t-1}^*$ , with  $A_0^* = \bar{A}^* * 1.01$  and  $\delta = 0.9$ .

the model to include capital controls and a central bank playing the role of intermediary for the private sector. This would explain the Chinese experience. Another simple extension is to introduce a positive supply of a liquid asset in the Industrial country to match the US situation. It should be clear, however, that this positive supply is not crucial for the results.

Another obvious extension is to introduce uncertainty in the level of productivity, idiosyncratic or aggregate. This will add a precautionary demand, but it will not affect the basic mechanism. We will also examine the impact of growth.

## **6 Conclusion**

TBC

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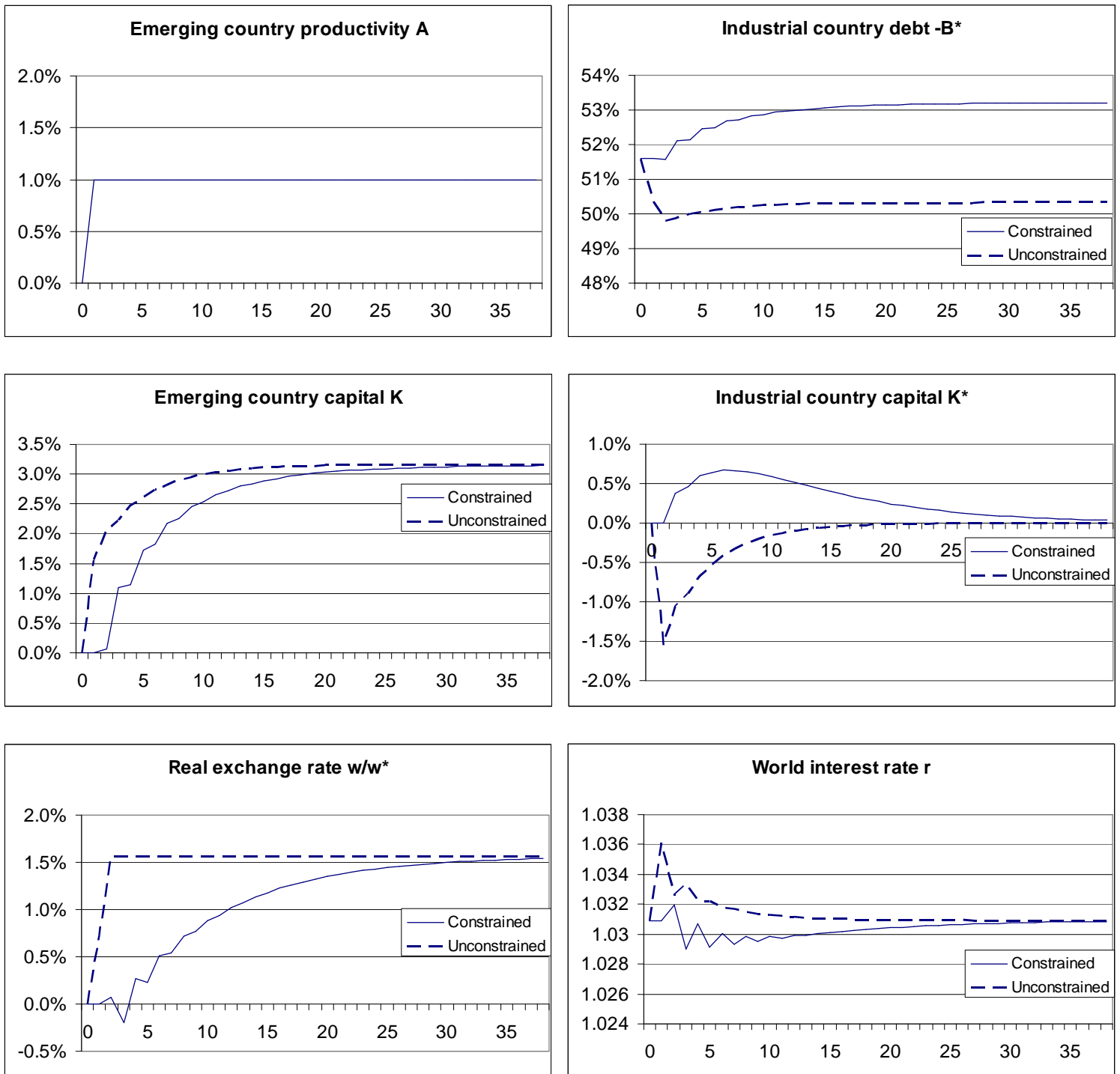
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Table 1: Parameter values	
Numerical benchmark	
$\alpha$	0.36
$\beta$	0.95
$\beta^*$	0.97
$\phi$	0.1
$\rho$	1
$\eta$	1

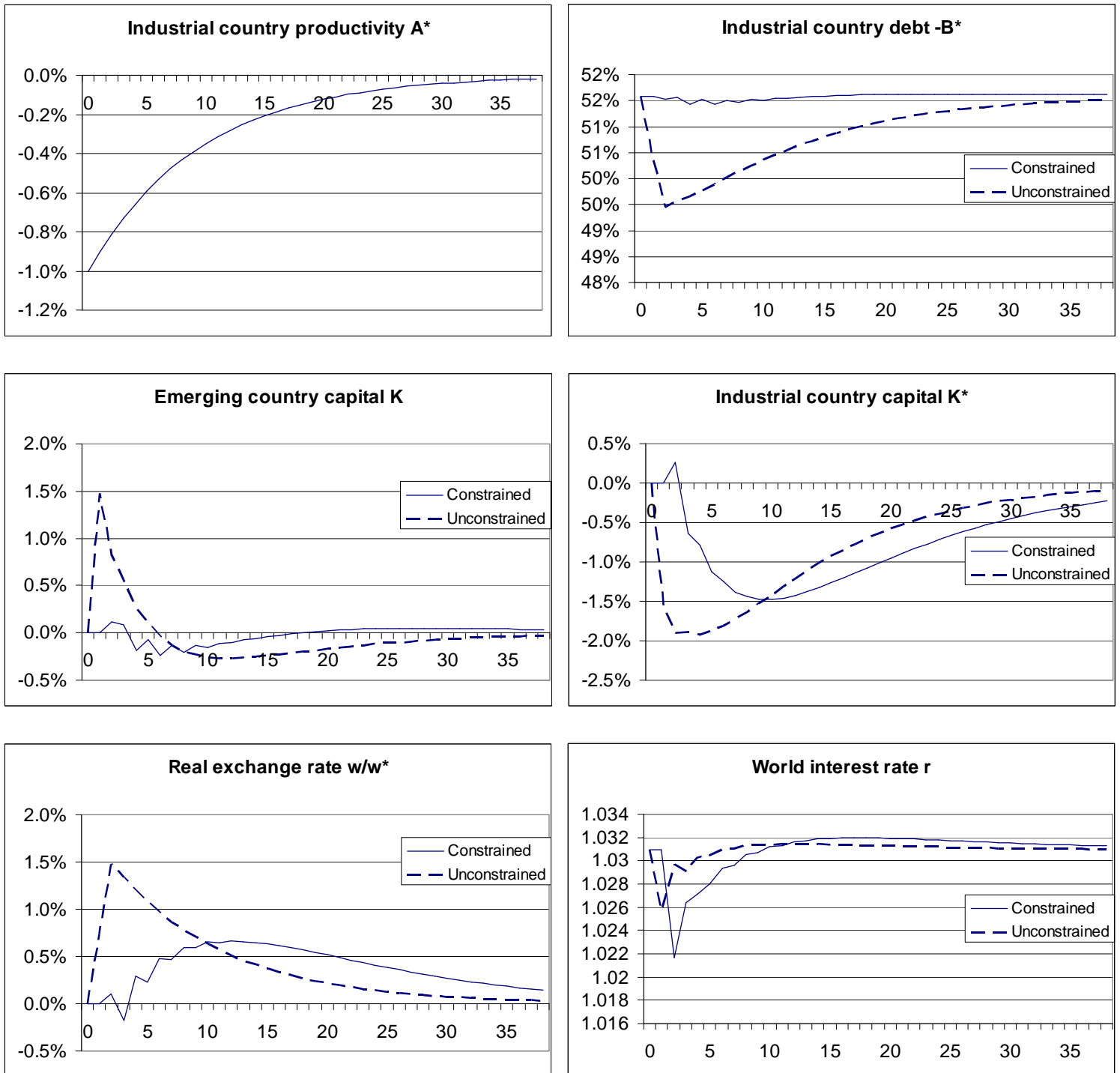


**Figure 1: Global imbalances – Positive and permanent productivity shock in the Emerging country**



Note: all variables are in percentage deviation from the initial steady state, except the industrial country's debt, which is in percentage of initial output, and the world interest rate, which is in level.

**Figure 2: The crisis – Negative and temporary productivity shock in the Industrial country**



Note: all variables are in percentage deviation from the steady state, except the industrial country's debt, which is in percentage of initial output, and the world interest rate, which is in level.