Optimal Policy with Long Bonds

Albert Marcet

London School of Economics

joint with

Elisa Faraglia

Institut d'Anàlisi Econòmica CSIC and London Business School

Andrew Scott

London Business School

What we want to do

Formulate a model to study optimal fiscal policy and government's optimal portfolio choice jointly

What maturities?, nominal or indexed?

Fiscal Policy and Debt Management jointly determined:

Why we want to do it

Choice of long/short bonds often ignored in models of policy analysis

Very relevant in ... crisis.

Markets seem to appreciate a certain type of portfolio

What should be the aim of Debt Management?

minimize cost or fiscal insurance?

It seemed easy at first ...

Just combine

- optimal policy with incomplete markets
- portfolio choice with heterogeneous agents

But the computer told us not so easy

Also, some in the profession thought there was no need to go away from complete markets.

In this (long run) project

1. Look at data on portfolio of gov't debt

2. We first reexamine effectively complete markets approach

   (a) justify using incomplete markets

   (b) learn some properties of the model in an easier model

3. Set up a model with long bonds

4. Study optimal debt management under incomplete markets

Standard basic framework

Rational Expectations

Full Commitment

Benevolent government

Full Information

Flexible prices

Gov't knows mapping from fiscal policy to equilibrium quantities

Debt Management with Effectively Complete Markets

Angeletos (2002) QJE

Buera and Nicolini (2004) JME

Assume government issues uncontingent debt at various maturities.

Same number of possible realizations of underlying shocks as number of maturities.

So gov't can effectively complete the markets.

Their results:

- Govt't issue debt at long maturities ($b_t^M > 0$)
- Gov't save on private debt at short maturities ($b_t^1 < 0$)

Barro (2004), Nosbusch (2008) extend these results.

Farhi (2007) argues gov't should hold private equity.

Problems with effectively completing markets

Buera and Nicolini (2004)

$b_t^M \simeq 6 \cdot GDP$

$b_t^1 \simeq -5 \cdot GDP$

Figure 1: Money Market Instruments (% of Debt) 1993-2003

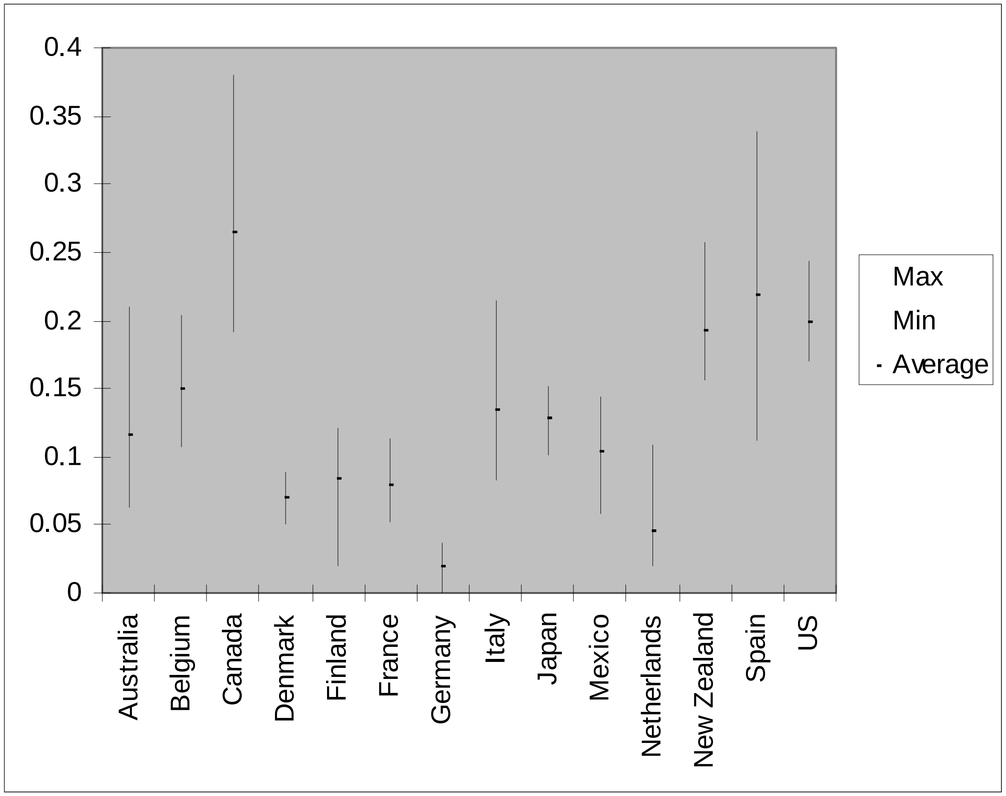


Figure 2: Short Term Debt (% of Debt) 1993-2003

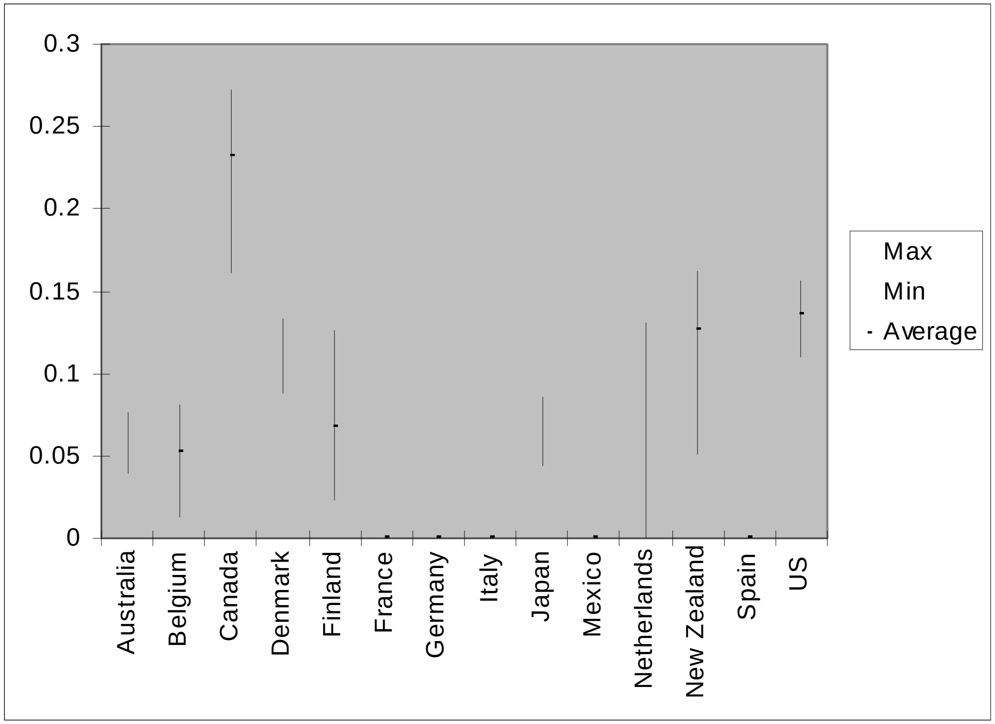


Figure 3: Medium Term Debt (% of Debt) 1993-2003

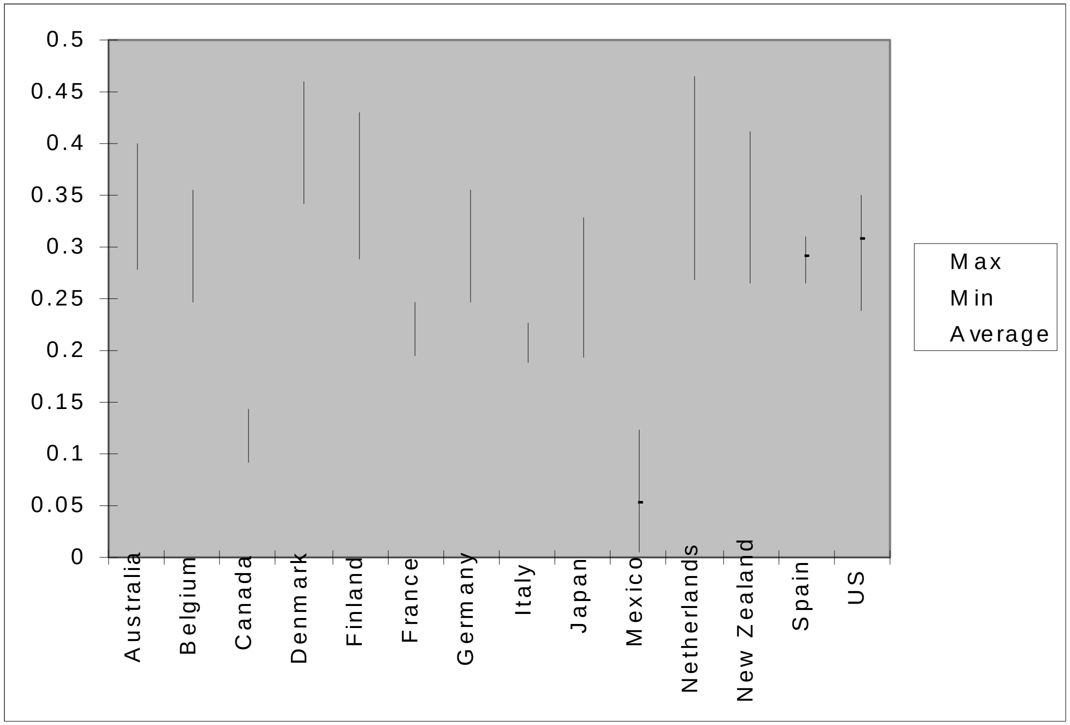


Figure 4: Long Term Debt (% of Debt) 1993-2003

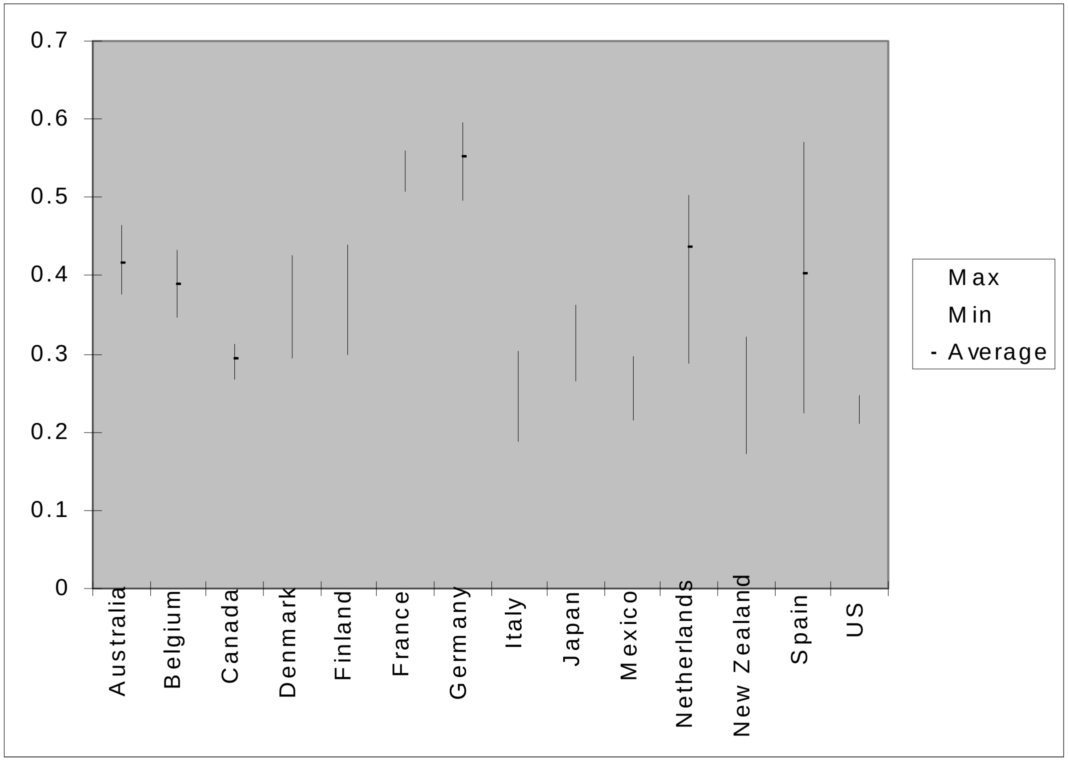
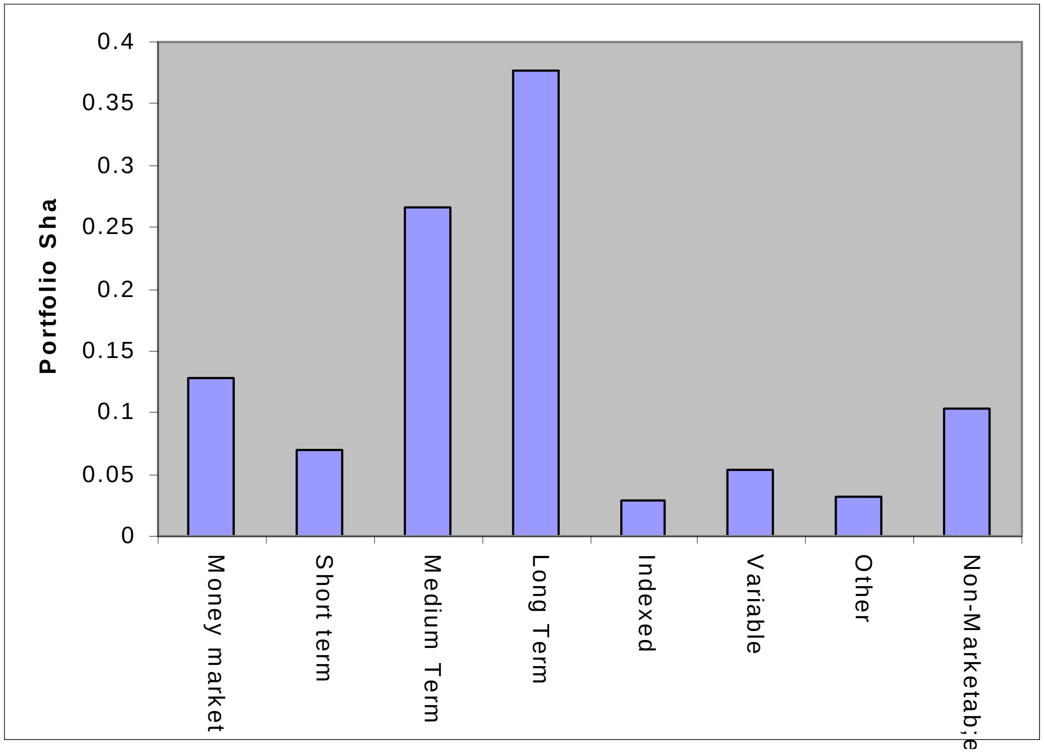


Figure 5: **Average OECD: Debt Composition 1993-2003**



Data

Data shows U-shaped portfolios

Positive holdings at most maturities.

Large failure to match data

This could be because

- micro fundamentals (utility, technology) are wrong
- financial frictions play a role
- some other failure of the model (gov't is not benevolent, or lazy...)

Faraglia, Marcet and Scott (2009) "In Search of a Theory of Debt Management", working paper

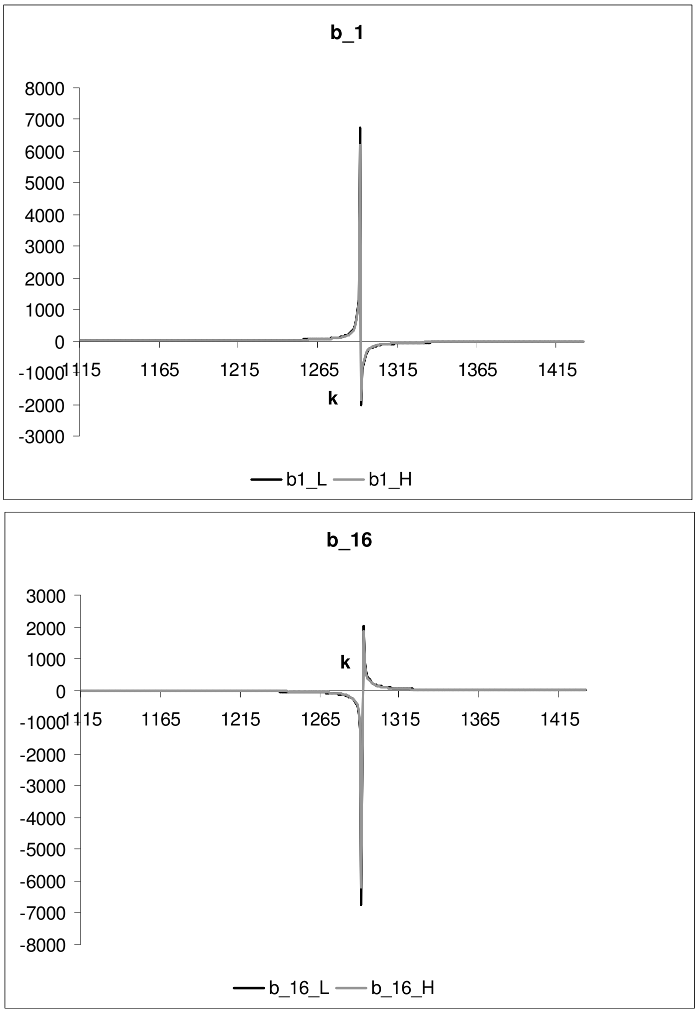
consider more general technology and utility

- Add capital
- habits
- no buyback of gov't bonds

Outcome: it gets worse all the time

- even larger positions
- no longer long bonds should be issued
- huge changes in sign from one period to next

Figure 2: Policy Functions for Debt Issuance - Capital Accumulation and Persistent Technology Shocks



Reason: for any properly calibrated model yield curve does not move too much

Difficult to implement complete markets with bonds.

ONE PROBLEM DETECTED: very non-linear laws of motion

Debt Management under INcomplete Markets

The Model

Technology

$$c_t + g_t \leq 1 - x_t$$

x_t leisure

c_t private consumption

g_t government expenditure shock

Consumer

Preferences

$$E_0 \sum_{t=0}^{\infty} \delta^t \left[u\left(c_t\right) + v\left(x_t\right) \right]$$

Government

Levies taxes at a tax rate τ_t.

Chooses taxes.

Issues uncontingent (real) bonds at several maturities.

Ramsey Equilibrium

A Peek at Debt Management

Government issues bonds of maturity 1 and of maturity M

b_t^M government bonds, pay one unit of consumption at $t + M$.

p_t^M competitive price of long bonds

b_t^1 short bonds

p_t^1 price short bonds

primary deficit $d_t = g_t - \tau_t w_t (1 - x_t)$

Gov't budget constraint,

$$p_t^1 \ b_t^1 + p_t^M \ b_t^M = d_t + p_t^{M-1} \ b_{t-1}^M + b_{t-1}^1$$

Uncertainty in only one period.

Consider special case where

$$\begin{aligned}\tilde{g} &= g_0 = g_2 = g_3 = \ldots \\ & g_1 \quad \text{random}\end{aligned}$$

Add transaction costs

$$p_t^1\ b_t^1 + p_t^M\ b_t^M + TC(b_t^1, b_t^M) = d_t + p_t^{M-1}\ b_{t-1}^M + b_{t-1}^1$$

3 bonds, mat 1 5 30

α	$\frac{b^1}{y}$	$\frac{b^5}{y}$	$\frac{b^{30}}{y}$	$\frac{tot\ \text{TC}}{y}$	Welf loss (rel to no TC)
0	-0.9455	-9.3157	10.2612	0	0.
10^{-7}	-0.8833	0.6233	0.2599	10^{-6}	$5 \cdot 10^{-8}$
10^{-6}	-0.1763	0.1243	0.0519	10^{-7}	10^{-7}
10^{-5}	-0.0196	0.0138	0.0057	10^{-8}	10^{-7}

ANOTHER PROBLEM DETECTED: Still huge and sensitive positions.

With very small transaction costs reasonable positions.

So frictions a big part of the story if we are to get reasonable positions

Simplify the model for now and look only at long bonds

Long Bonds

Government issues bonds of maturity M.

$$p_t^M \ b_t^M = d_t + p_t^{M-1} \ b_{t-1}^M$$

Aiyagari, Marcet, Sargent and Seppala (2002), assumed $M = 1$.

Useful because

- We can address some technical difficulties that will also be present in debt management

- of interest to compare with $M = 1$

- We can gain some intuition about the role of commitment in optimal taxes with incomplete markets

Two technical difficulties:

- Recursive formulation non-standard

  Apply recursive Contracts as in Marcet and Marimon.

- Many state variables.

  Design a technique to reduce dimension of state space.

A recursive formulation

$$p_t^M \ b_t^M = d_t + p_t^{M-1} \ b_{t-1}^M$$

Plugging equilibrium prices

$$\delta^M \frac{E_t u'(c_{t+M})}{u'(c_t)} \ b_t^M = d_t + \delta^{M-1} \frac{E_t u'(c_{t+M-1})}{u'(c_t)} \ b_{t-1}^M$$

λ_t lagrange multiplier of this constraint.

Lagrangean

$$
\begin{aligned}
L &= E_0 \sum_{t=0}^{\infty} \delta^t \{ u(c_t) + v(x_t) \\
&\quad + \lambda_t \left[\delta^M E_t u'(c_{t+M})\ b_t^M - d_t u'(c_t) - \delta^{M-1} E_t u'(c_{t+M-1})\ b_{t-1}^M \right] \}
\end{aligned}
$$

$$
\begin{aligned}
L &= E_0 \sum_{t=0}^{\infty} \delta^t \{ u\left(c_t\right) + v\left(x_t\right) \\
&\quad + \lambda_t \left[\delta^M E_t u'(c_{t+M})\ b_t^M - d_t u'(c_t) - \delta^{M-1} E_t u'(c_{t+M-1})\ b_{t-1}^M \right] \}
\end{aligned}
$$

$$
\begin{aligned}
L &= E_0 \sum_{t=0}^{\infty} \delta^t \{ u\left(c_t\right) + v\left(x_t\right) \\
&\quad + \lambda_t \left[\delta^M u'(c_{t+M})\ b_t^M - d_t u'(c_t) - \delta^{M-1} u'(c_{t+M-1})\ b_{t-1}^M \right] \}
\end{aligned}
$$

$$
\begin{aligned}
L &= E_0 \sum_{t=0}^{\infty} \delta^t \{ u\left(c_t\right) + v\left(x_t\right) \\
&\quad + \lambda_t \left[\delta^M u'(c_{t+M})\ b_t^M - d_t u'(c_t) - \delta^{M-1} u'(c_{t+M-1})\ b_{t-1}^M \right] \}
\end{aligned}
$$

$$
\begin{aligned}
L &= E_0 \sum_{t=0}^{\infty} \delta^t \{ u\left(c_t\right) + v\left(x_t\right) - \lambda_t d_t u'(c_t) \\
&\quad + \lambda_{t-M} u'(c_t)\ b_{t-M}^M - \lambda_{t-M+1} u'(c_t)\ b_{t-M}^M \}
\end{aligned}
$$

$$L = E_0 \sum_{t=0}^{\infty} \delta^t \{ u(c_t) + v(x_t) - \lambda_t d_t u'(c_t) + \lambda_{t-M} u'(c_t)\ b_{t-M}^M - \lambda_{t-M+1} u'(c_t)\ b_{t-M}^M \}$$

$$L = E_0 \sum_{t=0}^{\infty} \delta^t \{ u(c_t) + v(x_t) - \lambda_t d_t u'(c_t) + (\lambda_{t-M} - \lambda_{t-M+1})\ b_{t-M}^M\ u'(c_t) \}$$

for

$$\lambda_{-1} = ... = \lambda_{-M} = 0$$

$$L = E_0 \sum_{t=0}^{\infty} \delta^t \{u(c_t) + v(x_t) - \lambda_t d_t u'(c_t)$$
$$+ s_{t-M} \; u'(c_t)\}$$

For

$$s_t = (\lambda_{t-1} - \lambda_t) \; b_{t-1}^M$$

As long as

$$s_{-1} = ... = s_{-M} = 0$$

So, optimal choice

$$\begin{bmatrix} \tau_t \\ b_t^M \\ \lambda_t \\ c_t \end{bmatrix} = F(g_t, s_{t-1}, ..., s_{t-M}, b_{t-1}^M)$$

$$s_{-1} = ... = s_{-M} = 0, \quad \text{given } b_{-1}^M$$

So $M + 2$ state variables

In Aiyagari et al. (2002) case $M = 1$ state variables are

$$(g_t, \theta_t, \lambda_{t-1}, b_{t-1}^1)$$

Promised utility approach (APS):

For $M = 1$ APS can be used as follows:

assume g can take two values g^H, g^L

Budget constraint of gov't

$$d_t + b^1_{t-1} = \delta \frac{E_t(u'(c_{t+1}))}{u'(c_t)} b^1_t$$

$$d_t + b^1_{t-1} = \delta \frac{\pi u'(c^H_{t+1}) + (1-\pi) u'(c^L_{t+1})}{u'(c_t)} b^1_t$$

Sufficient state variables decided at t are

$$c^H_{t+1}, c^L_{t+1}$$

These variables are decided at t

Realized shock is a state at $t+1$.

With one-period bonds this means c_t is a state variable at t:

$$\begin{bmatrix} b_t^1 \\ c_{t+1}^H \\ c_{t+1}^L \end{bmatrix} = F(g_t, c_t, b_{t-1}^1)$$

Same number of state variables as lagrangean approach for $M = 1$.

Standard (Big) Problem with APS:

In fact these are sets of feasibla future consumptions $\mathcal{C}^H(b_t^1), \mathcal{C}^L(b_t^1)$.

Then impose additional constraints

$$c_{t+1}^H \in \mathcal{C}^H(b_t^1), \qquad c_{t+1}^L \in \mathcal{C}^L(b_t^1)$$

Very difficult.

No such problem with Lagrangean approach

continuation problem always well defined

Recall for $M = 1$ recursive formulation with lagrangean approach

$$\begin{bmatrix} b_t \\ c_t \\ \lambda_t \end{bmatrix} = F(g_t, \lambda_{t-1}, b_{t-1})$$

All we need is to impose

$$\lambda_{t-1} \in R_+$$

The "continuation problem" is always well defined

Recall

$$L = E_0 \sum_{t=0}^{\infty} \delta^t \{ u(c_t) + v(x_t) - \lambda_t d_t u'(c_t) + (\lambda_{t-1} - \lambda_t) u'(c_t)\ b_{t-1}^1 \}$$

In new version of Marcet Marimon (2008) we show full commitment amounts to re-solving at t

$$\max_{\left\{ b_{t+j}^1, c_{t+j} \right\}} E_t \sum_{j=0}^{\infty} \delta^j \left[u\left(c_{t+j}\right) + v\left(x_{t+j}\right)\right] + \lambda_{t-1} u'(c_t)\ b_{t-1}^1$$

s.t. CE constraints

given optimal λ_{t-1}.

So continuation problem changes objective function

Well defined for all λ_{t-1}.

APS unfeasible for large M

$$d_t + b^M_{t-M} = \delta^M \frac{E_t(u'(c_{t+M}))}{u'(c_t)} b^M_t$$

$$d_t + b^M_{t-1} = \delta \frac{\sum_{\widetilde{g}\in\Omega^M} u'(c_{t+M}(g^t,\widetilde{g}))\ P(g^{t+M} = (g^t,\widetilde{g}))}{u'(c_t)} b^M_t$$

Even if $g_t \in (g^H, g^L)$ there are $2^M + 3$ state variables!.

Recall with Lagragean approach only $M + 3$ state variables.

Second Technical Difficulty

$M + 3$ state variables, still many variables

In many economic models, many state variables are nearly redundant

Try to introduce only "relevant" combinations of state variables

Related to Smolyak polynomials

Related to Sims' reduction of state space.

We could start with few variables,

then add variables one by one,

and claim victory when one new variable makes little difference

But this could easily overlook that globally the remaining variables may be relevant.

We try to give the best chance to all remaining variables when we refine the solution.

Recall state variables are

$$(g_t, \theta_t, s_{t-1}, ..., s_{t-M-1}, b_{t-1}^M) \equiv X_t$$

Split state variables in "core" variables and "remaining" variables.

$$X_t^{core} \subset X_t$$
$$X_t^{out} \text{ remaining variables in } X_t$$

Replace X_t^{out} by linear combinations

$$(\alpha^1 \cdot X_t^{out}, ..., \alpha^N \cdot X_t^{out})$$

where each $\alpha^j \in R^{M+3}$ is chosen so as to maximize "relevance" of this linear combination relative to previous solution.

More precisely.

Step 1. Choose $X_t^{core} \subset X_t$

Find approximate solution with only X_t^{core}.

In our case we set $X_t^{core} \equiv (g_t, \theta_t, s_{t-1}, b_{t-1}^M)$

Step 2. Solve the model with a time-invariant function of X_t^{core}

In our case we use PEA

Approximate

$$E_t \left\{ u' \left(c_{t+M} \right) \right\} = \beta \cdot X_t^{core}.$$

Converge on β. Call it β^1.

Step 3. Add linear combinations of X_t^{out}.

Find α^1 by fitting the Euler equation residual on all remaining state variables

In our case

Run a regression of X_t^{out} on the core variables:

$$X_t^{out} = B^1 \cdot X_t^{core}$$

Find the residuals

$$X_t^{res,1} = X_t^{out} - B^1 \cdot X_t^{core}.$$

Given solution for c, X found with core variables, find α^1 such that

$$\alpha^1 = \arg\min_{\alpha} \sum_{t=1}^{T} \left(u'\left(c_{t+1}\right) - \beta^1 \cdot X_t^{core} - \alpha \cdot X_t^{res,1} \right)^2$$

If

$$\beta^1 \cdot X_t^{core} + \alpha^1 \cdot X_t^{res,1} \cong \beta^1 \cdot X_t^{core}$$

for every t and every realization stop here.

Solve the model using as state variables $(X_t^{core}, \alpha^1 \cdot X_t^{res,1})$

Notice, new fixed point problem is well conditioned, initial condition $(\beta^1, 1)$

If the solution changes very little, stop here.

If not, find $X_t^{res,2}$ and so on.

In our problem we only need one linear combination and it changes very, very little the solution.

Comments:

- We can also use this to add higher-order terms in non-linear approximation

- Do not revise past B or α at each iteration.

Role of commitment fiscal policy under incomplete markets

First look at certainty, Lucas and Stokey ($M = 1$)

$$\sum_{t=0}^{\infty} \beta^t \frac{u'(c_t)}{u'(c_0)} d_t = -b^1_{-1}$$

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) d_t = -b^1_{-1}\, u'(c_0)$$

Then optimal fiscal policy is to lower interest rates:

$$c_t < c_0$$

$$\tau_t > \tau_0$$

With a long bond

$$\sum_{t=0}^{\infty} \beta^t \frac{u'(c_t)}{u'(c_0)} d_t = -b_{-1}^M \; p_t^{M-1}$$
$$\sum_{t=0}^{\infty} \beta^t \frac{u'(c_t)}{u'(c_0)} d_t = -b_{-1}^M \; \delta^{M-1} \frac{u'(c_{M-1})}{u'(c_0)}$$
$$\sum_{t=0}^{\infty} \beta^t u'(c_t) d_t = -b_{-1}^M \; \delta^{M-1} \; u'(c_{M-1})$$

Here the government sets

$$c_t < c_{M-1}$$
$$\tau_t > \tau_{M-1}$$

so as to lower interest rates

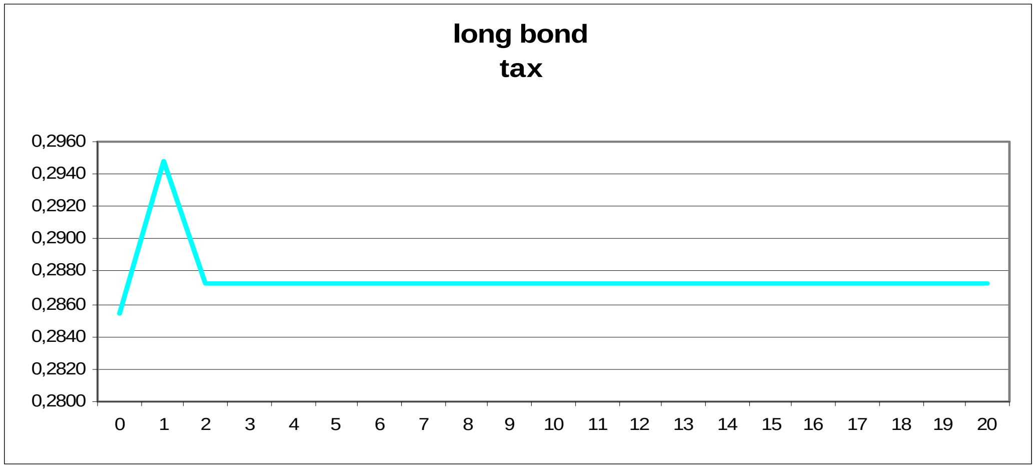
Uncertainty in only one period.

If g_1 high

- increases taxes at τ_1
- commit to lowering taxes in M periods to reduce today's higher debt burden

Time Inconsistency: at $t = M + 1$ government would prefer to smooth taxes.

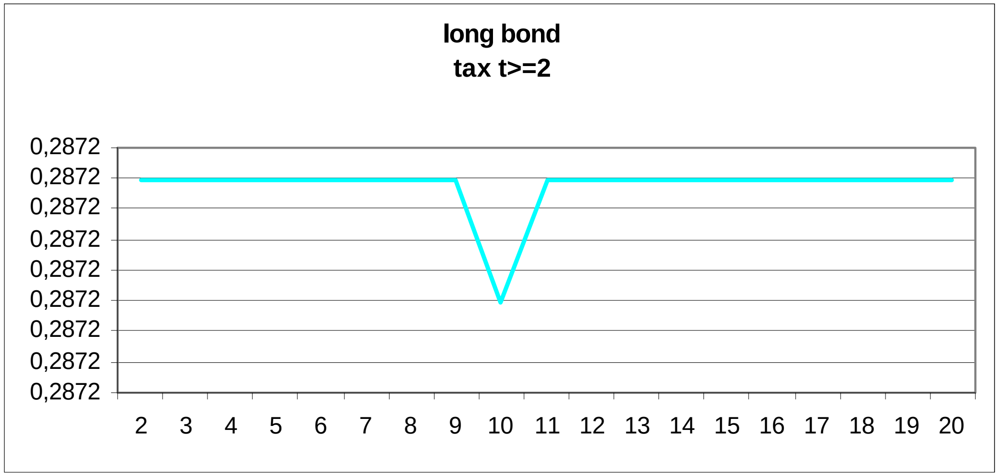
In Aiyagari et al. both effects take effect simultaneously, less apparent

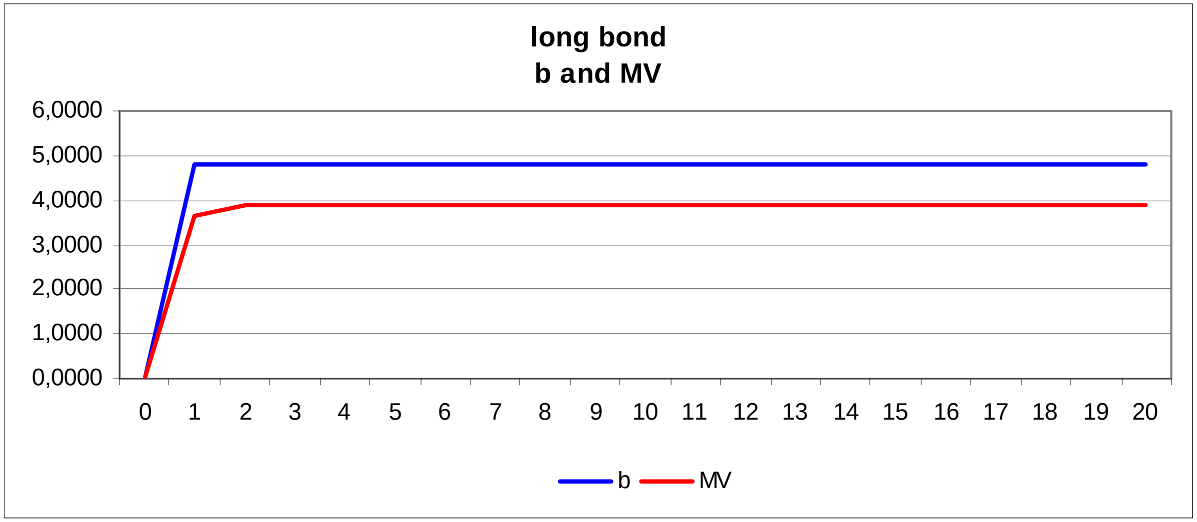


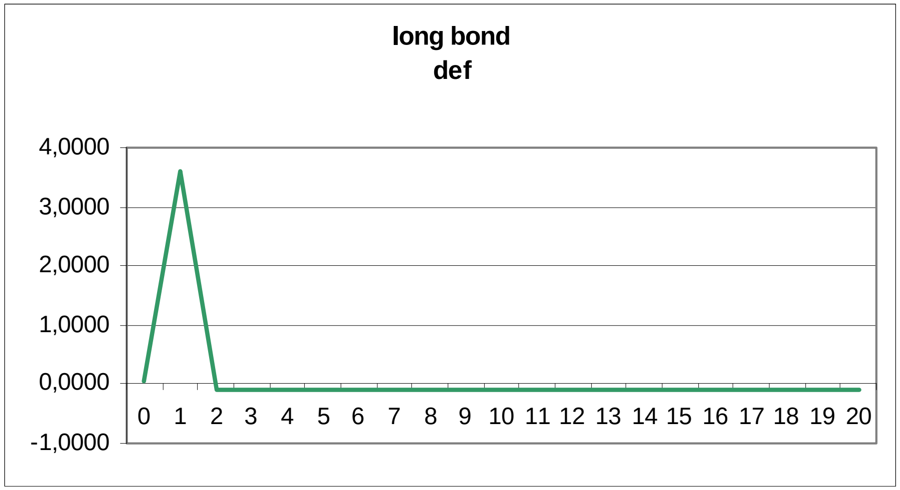
Uncertainty in all periods. ONE long bond.

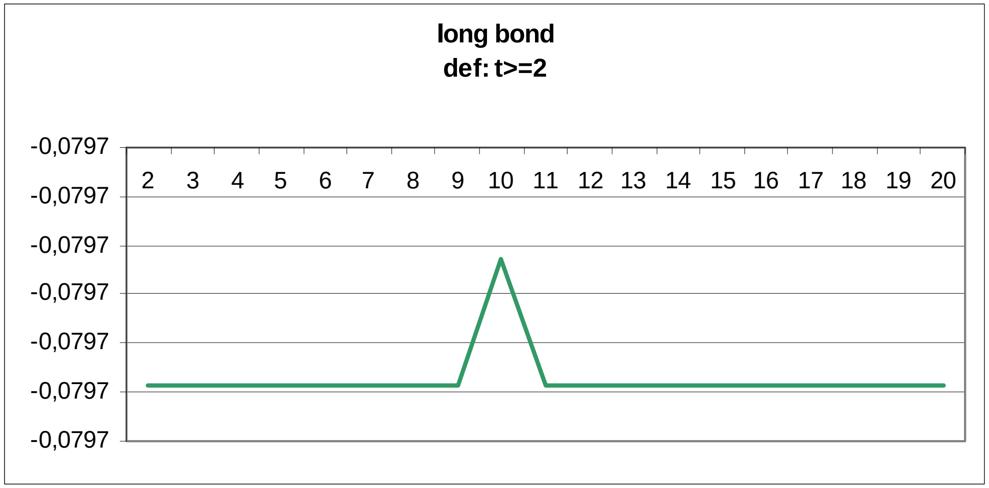
The algorithm for state space reduction works very well.

Only one linear combination needs to be added.


long bond
tax t>=2
0,2872
0,2872
0,2872
0,2872
0,2872
0,2872
0,2872
0,2872
0,2872
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20


long bond
b and MV
6,0000
5,0000
4,0000
3,0000
2,0000
1,0000
0,0000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
b
MV


long bond
def
4,0000
3,0000
2,0000
1,0000
0,0000
-1,0000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20


long bond
def: t>=2
-0,0797
-0,0797
-0,0797
-0,0797
-0,0797
-0,0797
-0,0797
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

With only one bond

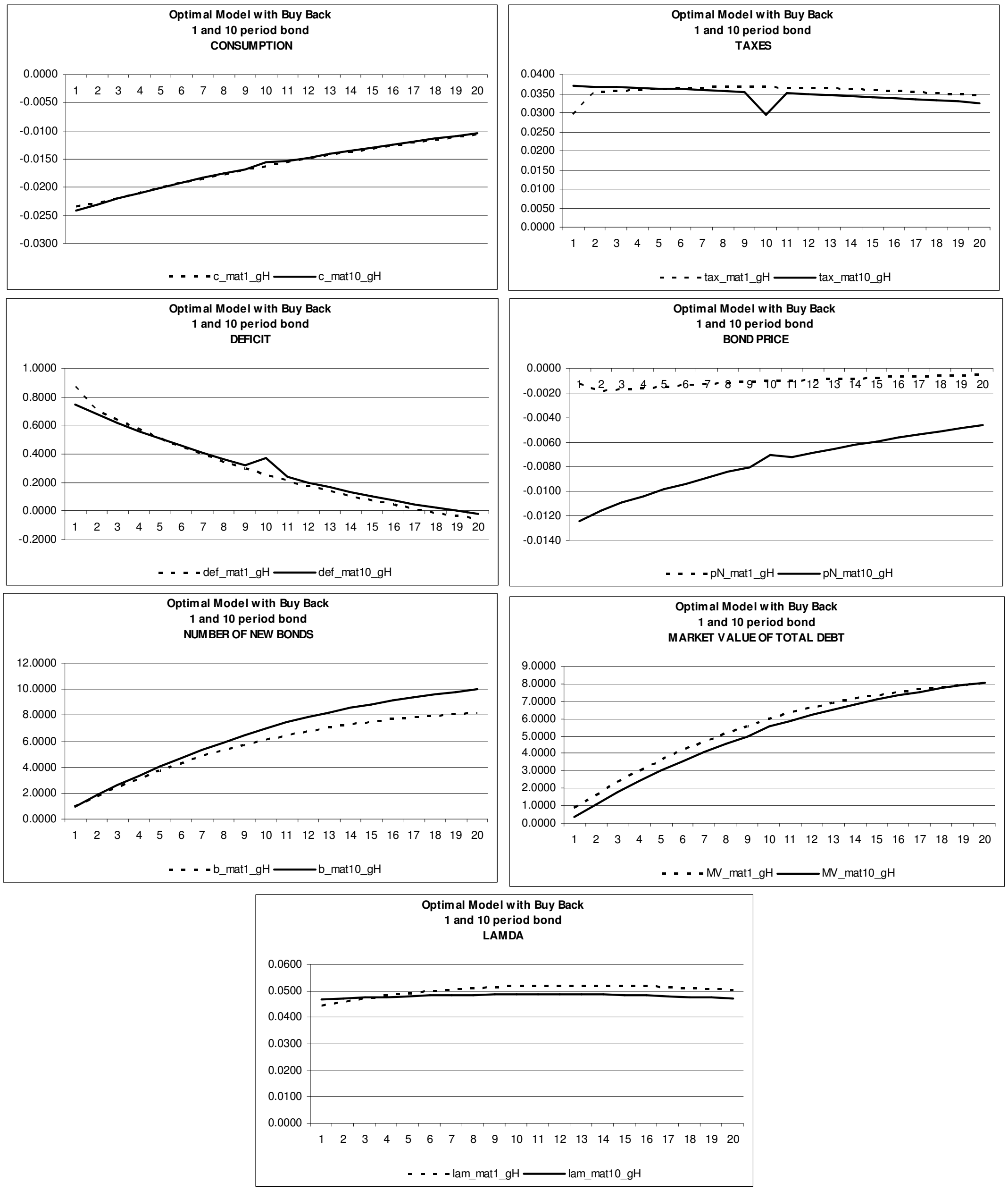
- Debt is used as a buffer stock, as in Aiyagari et al. (2002)

- gov't promises to lower interest rates if debt becomes high (spike)

- payoff of long bond closer to complete markets.

- reasonable positions.

Figure 3b: 1 and 10 Period Bond: Impulse Response - gH - $b_{N,-1} = \frac{0.5y^*}{\beta^N}$



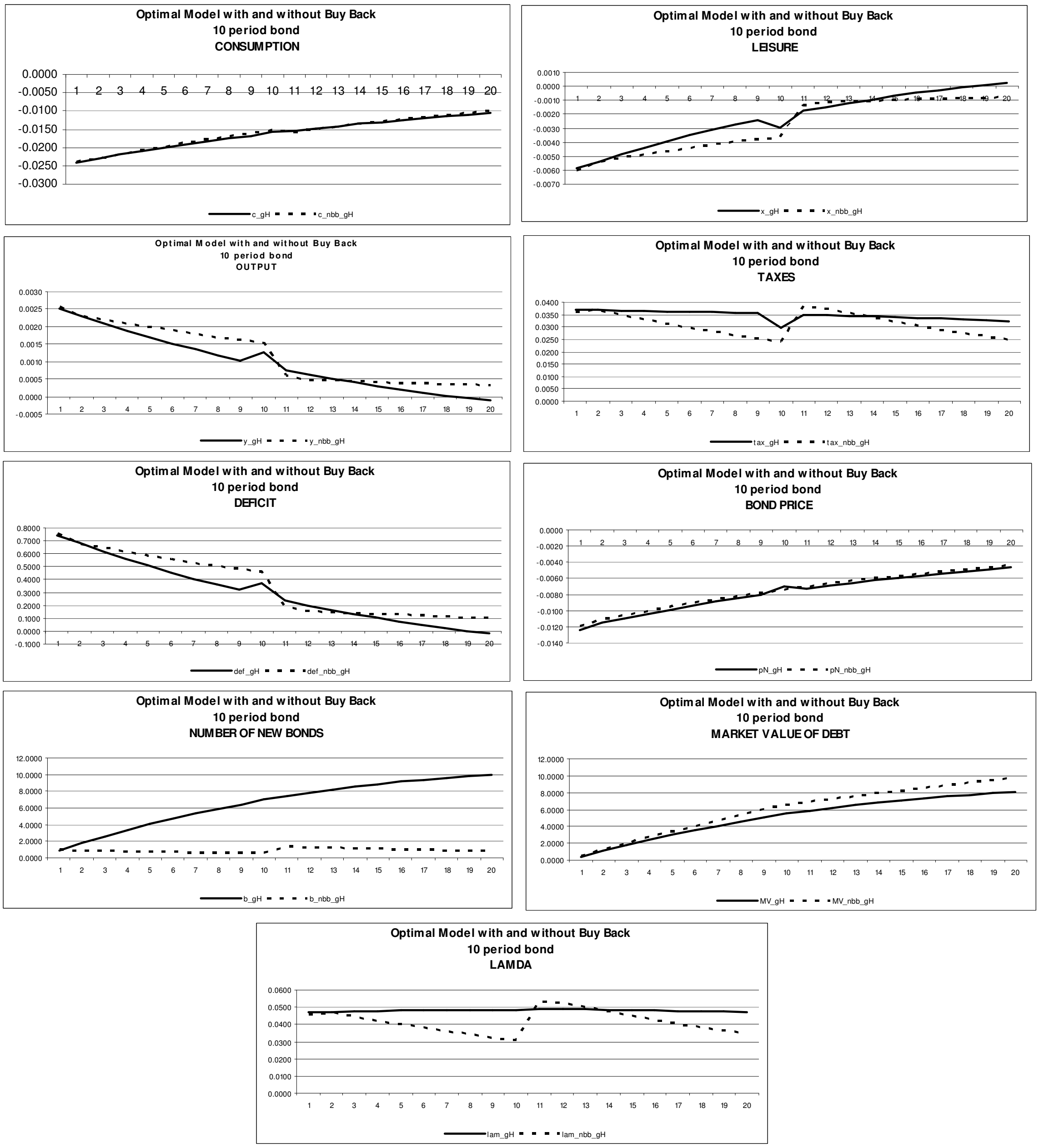
No buyback

$$p_t^M \; b_t^M = d_t + b_{t-M}^M$$

Now if g high, promise lower taxes for next M periods

No spike.

Figure 8a Standard Model with and without Buy Back: Impulse Response - gH- $b_{N,-1} = \frac{0.5y^*}{\beta^N}$ **and** $b_N^{nBB} = b_{N-1}^{nBB} = ... = b_{-1}^{nBB} = \frac{0.5y^*}{\sum_{i=1}^{N} \beta^i}$



For the future

Finish this paper

Model with two bonds, add "relevant" frictions

1. potentially large welfare gain from using debt optimally under incomplete markets.

2. Add private default risk.

3. Add rollover risk