Monetary Policy and the Equity Premium

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Motivation

- Monetary policy affects the macroeconomy primarily through financial markets.

- Two possible channels:
  - Monetary policy affects the (unconditional mean of the) real rate, which in turn affects real economy.
  - Monetary policy affects economic risk (conditional variances).

- Standard models (i.e. CIA and NK) abstract from the second channel.

- Evidence that economic risk is important in accounting for asset prices.

- We develop a DSGE model with both channels to explain the response of equity prices to monetary shocks.
We extend the neoclassical CIA model to allow for **segmented** goods and asset markets.

**Fixed costs** of transferring funds from a brokerage to a checking account.

Households only **infrequently** update their desired allocation of cash across these two accounts. Households are **heterogenous in their fixed cost** of transferring funds.

Recent micro evidence on household finance provides strong support for **infrequent portfolio rebalancing** with considerable heterogeneity across households.

- *Brunnermeier & Nagel (2008)* and *Haliassos et al. (2008)*
- *Calvet, Campbell, and Sodini (2008, 2009)*
Using high frequency data *Bernanke & Kuttner (2005)* show that:

1. Stock prices rise 1 percent to a 25 bp surprise in federal funds rate.
2. An important part of the increase is due to changes in equity premia.

- Standard models can not capture the second fact.
- We show that a model with infrequent portfolio rebalancing can account for this evidence.
Related Literature

  - We differ by emphasizing the endogenous asset segmentation along an intensive margin (i.e. rebalancing) rather than the extensive margin (i.e. participation).

- Abel, Eberly, and Panageas (2007) study infrequent adjustment of funds between assets and goods markets in a partial equilibrium setting with rational inattention.
  - We study monetary policy in a general equilibrium framework.
Agents: (i) households, (ii) firms, and (iii) the government.

In $t = 0$, there is trade in asset markets, and no trade in goods markets.

In $t \geq 1$,
- In the asset markets, households trade a complete set of state-contingent claims and equity in the firms.
- In the goods markets, households use money to buy goods subject to a CIA constraint.

There are two sources of uncertainty: aggregate shocks to technology, $z_t$, and to money growth, $\mu_t$.

We index the states at date $t$ by $s_t = (z_t, \mu_t)$, and $s^t = (s_1, \ldots, s_t)$ denote history through period $t$. 
Market Segmentation

- At date $t = 0$, households decide on a costless, non-state contingent amount of cash to transfer from assets to goods markets in periods $t \geq 1$.
- The initial non-state-contingent allocation plan (i.e., the annuity) ensures that all agents participate in financial markets.
- At $t \geq 1$, make state-contingent transfers between these markets, households pay a fixed cost.
  - This cost is constant over time but varies across households.
  - If pays this cost, HH actively rebalances portfolio. If not, then inactive.
Firms

- Technology:

\[ Y(s^t) = K(s^{t-1})^\alpha \left[ \exp(z_t)L(s^t) \right]^{1-\alpha}, \]

with \( z_t = \rho_z z_{t-1} + \epsilon_{zt}, \) and \( \epsilon_{zt} \sim N(0, \sigma^2_z). \)

- Firms have a one-period planning horizon. To operate capital in period \( t + 1, \) a firm must purchase it by issuing equity.

- In equilibrium:

\[
\begin{align*}
\omega(s^{t+1}) &= (1 - \alpha) \frac{Y(s^{t+1})}{L(s^{t+1})}, \\
1 + r^e(s^{t+1}) &= \left[ \alpha \frac{Y(s^{t+1})}{K(s^t)} + p_k(s^{t+1}) \right] \frac{1}{p_k(s^t)}
\end{align*}
\]

- Fixed factor supply: \( K(s^t) = 1, \) \( L(s^t) = 1 \)
Issues one-period state-contingent bonds and controls the economy’s money stock, $M_t$.

At date $t = 0$, the government also issues an annuity at price, $P_A$, which has a constant payoff $A_0$ in units of consumption.

Budget constraints at date $t = 0$:

$$\bar{B} = \int_{s_1} q(s_1)B(s_1)ds_1 + P_A A_0,$$

$\bar{B}$ is given, $q(s_1)$ is the price of the state-contingent bond, $B(s_1)$.

At dates $t \geq 1$, the government’s budget constraint:

$$B(s^t) + M_{t-1} + P(s^t)A_0 = M_t + \int_{s_{t+1}} q(s^t, s_{t+1})B(s^t, s_{t+1})ds_{t+1},$$

Finally, the government injects cash:

$$\frac{M_t}{M_{t-1}} = \mu_t = (1 - \rho_\mu)\bar{\mu} + \rho_\mu \mu_{t-1} + \epsilon_\mu, \quad \epsilon_\mu \sim N(0, \sigma^2_\mu).$$
Households’ Fixed Costs

- A household can purchase and sell bonds and stocks in asset markets.

- Asset and goods markets are segmented, so that a household must pay a real fixed cost, $\gamma$, to transfer cash between them. Hence, households are indexed by $\gamma$.

- This cost is constant for a household but differs across households according to the distribution $F(\gamma)$ with density $f(\gamma)$. 
Households and Cash in the Goods Market

- CIA in goods market for household $\gamma$:

\[ P(s^t)c(s^t, \gamma) = M(s^{t-1}, \gamma) + P(s^t)A(\gamma) + P(s^t)x(s^t, \gamma)z(s^t, \gamma). \]

- At dates $t \geq 1$, HH receives a non-state contingent transfer of cash, $A(\gamma)$, from the annuity purchased at date $t = 0$.

- Households pay fixed cost $\gamma$ of making state contingent transfer, $x(s^t, \gamma)$, between checking and brokerage accounts.

- If HH pays fixed cost $\gamma$, then $z(s^t, \gamma) = 1$.

\[ z(s^t, \gamma) = 0, \text{ otherwise}. \]
HH with different fixed costs of transferring \( x(s^t, \gamma) \) will demand different \( A(\gamma) \).

Through the choice of \( A(\gamma) \), all HHs participate in financial markets.

- If HH cannot make non-state contingent transfers (i.e., \( A(\gamma) = 0 \)), there is limited participation in asset markets (AAK (2007)).

Household cash constraint in asset markets at dates \( t \geq 1 \):

\[
B(s^t, \gamma) + (1 + R^e(s^t))S(s^{t-1}, \gamma) = \int_{s_{t+1}} q(s^t, s_{t+1})B(s^t, s_{t+1}, \gamma)ds_{t+1} + S(s^t, \gamma) + P(s^t)[x(s^t, \gamma) + \gamma]z(s^t, \gamma).
\]
Households maximize: 

\[ \sum_{t=1}^{\infty} \beta^t \int_{s^t} U(c(s^t, \gamma)) g(s^t) ds^t \]

where \( g(s^t) \) denotes the probability distribution over history \( s^t \), and:

\[ U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \]
Households Budget Constraint

- Household budget constraint simplifies to:
  \[ M(s^t, \gamma) = P(s^t)w(s^t) \]

- Beginning of period cash is independent of \( \gamma \).
- This requires:
  \[
  \int_{s_{t+1}} \beta \frac{U'[c(s_{t+1}, \gamma)]}{U'[c(s^t, \gamma)]} \frac{P(s^t)}{P(s_{t+1})} g(s_{t+1}) ds_{t+1} < 1
  \]

- For HHs that always rebalance, this implies:
  \[ R(s^t) > 1 \]
Timing in the Two Markets

**Asset Markets**

Starting Assets

\[ B(s) + (1 + R^e)S_{-1} \]

\[ s = (\mu, z) \text{ are observed} \]

Asset Market Constraint

If cash transferred:

\[ \int q(s') B(s') + S + P \left[ x(s) + \gamma \right] \]

If no transfer:

\[ \int q(s') B(s') + S \]

Ending Assets

\[ B(s') + S \]

If transfer \( x(s) \), pay fixed cost \( \gamma \)

Cash-in-Advanced Constraint: Consumption

If cash transferred:

\[ c = n + x(s) + A(\gamma) \]

If no transfer:

\[ c = n + A(\gamma) \]

**Goods Markets**

Starting Cash: \( (P_{-1} w) \)

Real Balances \( n = P_{-1} w / P \)

Ending Cash: \( (P w) \)

Wages sold for cash: \( P w \)

Shopper

Worker
Characterizing Equilibrium Allocations

- Consumption of *inactive rebalancers*:
  \[ c_I(s^t, \gamma) = \frac{w(s^t)}{\mu_t} + A(\gamma) \]

- Complete risk-sharing among *active rebalancers*:
  \[ c_A(s^t, \gamma) = c_A(s^t), \ \forall \gamma \]

- HH annuity provides insurance for inactive types:
  \[
  \sum_{t=1}^{\infty} \beta^t \int_{s^t} \left[ U'(c_A(s^t)) - U'(c_I(s^t, \gamma)) \right] (1 - z(s^t, \gamma)) g(s^t) ds^t = 0.
  \]

- To get proceeds from equity, inactive types set \( A(\gamma) \geq 0 \).
The Marginal Rebalancer

- $\tilde{\gamma}(s^t)$ is the fixed cost of the marginal rebalancer determined by:
  \[
  U[c_A(s^t)] - U[c_\gamma(s^t, \tilde{\gamma}(s^t))] = U'[c_A(s^t)] [c_A(s^t) - c_\gamma(s^t, \tilde{\gamma}(s^t)) + \tilde{\gamma}(s^t)],
  \]

  where $c_\gamma(s^t, \tilde{\gamma}(s^t)) = \frac{w(s^t)}{\mu_t} + A(\tilde{\gamma}(s^t))$.

- Consumption of rebalancers is given by:
  \[
  F(\tilde{\gamma}(s^t))c_A(s^t) + \int_{\tilde{\gamma}(s^t)}^{\infty} \left[ \frac{w(s^t)}{\mu_t} + A(\gamma) \right] f(\gamma) d\gamma = Y(s^t) - \int_{0}^{\tilde{\gamma}(s^t)} \gamma f(\gamma) d\gamma,
  \]
  where $F(\gamma)$ and $f(\gamma)$ are the cdf and pdf of $\gamma$: $\log \gamma \sim N(\tilde{\gamma}_m, \sigma^2_\gamma)$. 
State-dependent Rebalancing

The Demand Schedule for the Annuity

Regions of Active and Inactive Rebalancing
Pricing kernel depends on the consumption of rebalancers:

\[ m(s^t, s_{t+1}) = \beta \left( \frac{c_A(s^t)}{c_A(s_{t+1})} \right)^\sigma \]

Risk-free rate:

\[ [1 + r^f_t]^{-1} = E_t m_{t,t+1} \]

Return on equity:

\[ 1 + r^e_{t+1} = \frac{[\alpha \exp((1 - \alpha)z_{t+1}) + p_{k,t+1}]}{p_{k,t}} \]

The equity premium:

\[ 1 + r^{ep}_t = \frac{E_t[1 + r^e_{t+1}]}{1 + r^f_t} = 1 - \text{cov}_t \left( \beta \left[ \frac{c_{A,t}}{c_{A,t+1}} \right]^\sigma, 1 + r^e_{t+1} \right) \]
Global Model Solution

- Given $A(\gamma)$, the resource constraint and equilibrium condition for the marginal rebalancer determine $c_A(s^t)$ and $\bar{\gamma}(s^t)$.

- Following Tauchen and Hussey (1991) and Judd (1998), we use the linear Fredholm integral equations (Type 2) and quadrature to determine the price of capital from the stochastic difference equation:

$$p_k(s^t) = \int_{s_{t+1}} m(s^t, s_{t+1}) \left[ \alpha \exp[(1 - \alpha)z_{t+1}] + p_k(s^{t+1}) \right] g(s_{t+1}|s^t) \, ds_{t+1}$$

- Similar approach to determine $A(\gamma)$ for a fixed value of $\gamma$. Then $A(\gamma)$ is approximated using piecewise linear interpolation.
Calibration

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<th>Parameter Values</th>
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<td>$\sigma_\mu(%)$</td>
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The endogenous rebalancing model reduces to a representative agent model:

\[ c_A = c_I = (1 - \alpha)\mu^{-1} + A \]

The model only becomes interesting in the presence of uncertainty.

The endogenous participation model (strong incentive to participate):

\[ c_A > c_I (= (1 - \alpha)\mu^{-1}) \]
Endogenous Rebalancing and the Equity Premium

![Diagram showing the relationship between risk-free rate and equity premium with different rebalancing fractions and standard CIA models.]

- Avg. Fraction of Rebalancers = 11% ($\gamma_m = 3.5\%$)
- Avg. Fraction of Rebalancers = 24% ($\gamma_m = 0.5\%$)
- Avg. Fraction of Rebalancers = 11% ($\sigma = 2$)
- Avg. Fraction of Rebalancers = 15% ($\sigma = 4$)
- Avg. Fraction of Rebalancers = 13% ($\gamma_m = 2\%, \sigma = 3$)
- Standard CIA Model ($\sigma = 15$)
- Standard CIA Model ($\sigma = 3$)

U.S. Data
Endogenous Participation

U.S. Data
Avg. Participation Rate = 18% (α = 0.01)

Endogenous Rebalancing
Avg. Participation Rate = 100% (α = 0.36)

Endogenous Participation

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Equity Premium
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Understanding the Mechanism

**The Demand Schedule for the Annuity**

- Higher $\gamma \Rightarrow$ less frequency $\Rightarrow$ Higher demand $A$

**Endogenous Rebalancing Model**

- GM Schedule: $\bar{\gamma}cA = e^{(1-\alpha)z} - \bar{\gamma}^2$  
- MR Schedule: $(cA - cI)^2 = cI\bar{\gamma}$, $cI = (1-\alpha)\mu + A$

**Endogenous Participation Model**

- GM Schedule: $\bar{\gamma}cA = e^{(1-\alpha)z}$  
- MR Schedule: $cI = (1-\alpha)\mu$
Understanding the Mechanism

- Higher $\gamma \Rightarrow$ less frequency $\Rightarrow$ Higher demand $A$
- Insurance against consumption losses
Understanding the Mechanism

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- Understanding the mechanism ($A(\gamma) \cong A$)
Understanding the Mechanism

- Higher $\gamma \Rightarrow$ less frequency $\Rightarrow$ Higher demand $A$
- Insurance against consumption losses
- Understanding the mechanism ($A(\gamma) \equiv A$)
- GM schedule:

$$\frac{\tilde{\gamma} c_A}{\gamma_u} + (1 - \frac{\tilde{\gamma}}{\gamma_u}) c_I = e^{(1-\alpha)z} - \frac{\tilde{\gamma}^2}{2\gamma_u}$$

The Demand Schedule for the Annuity

Higher $\gamma$ implies less frequency, leading to higher demand for the annuity. Insurance against consumption losses is also considered. Understanding the mechanism is key, denoted by $A(\gamma) \equiv A$. The GM schedule is given by:

$$\frac{\tilde{\gamma} c_A}{\gamma_u} + (1 - \frac{\tilde{\gamma}}{\gamma_u}) c_I = e^{(1-\alpha)z} - \frac{\tilde{\gamma}^2}{2\gamma_u}$$
Understanding the Mechanism

- **Higher** $\gamma \Rightarrow$ less frequency $\Rightarrow$ Higher demand $A$
- Insurance against consumption losses
- Understanding the mechanism ($A(\gamma) \approx A$)
- **GM schedule:**
  \[
  \frac{\bar{\gamma}c_A}{\gamma_u} + (1-\bar{\gamma})c_I = e^{[(1-\alpha)z]} - \frac{\bar{\gamma}^2}{2\gamma_u}
  \]

- **MR schedule:**
  \[
  (c_A-c_I)^2 = c_I \bar{\gamma}, \quad c_I = (1-\alpha)\mu^{-1} + A
  \]
A Technology Improvement

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Heterogeneity in portfolio rebalancing, with a large fraction showing inertia.
Individual Heterogeneity

- Heterogeneity in portfolio rebalancing, with a large fraction showing inertia.

- HH are trading off consumption volatility against higher average consumption.
Individual Heterogeneity

- Heterogeneity in portfolio rebalancing, with a large fraction showing inertia.

- HH are trading off consumption volatility against higher average consumption.

- Higher volatility of $c_A$ translates into high return on equity.
They find that a broad index of stock prices registers a one-day gain of 1 percent in reaction to a 25 basis point easing of the federal funds rate.

They decompose the response of stock prices into three components:
- Current and expected changes in the real rate
- Expected future excess equity returns or equity premia
- Current and expected changes in dividends,

They conclude that an important channel by which stock prices increase occurs through changes in the equity premium.
Impulse Response to a Monetary Policy Shock

- An IRF of $y(s^t)$ to $\mu_1$ is defined as the revision in expectations from a variable’s conditional mean (Hamilton (1994)):

$$E[\log (y(s^t)) | \mu_1, z_0] - E[\log (y(s^t)) | \mu_0, z_0]$$

where $\mu_0 = \bar{\mu}$ and $z_0 = \bar{z}$.

- We use Monte Carlo integration to compute the conditional expectation, which involves multidimensional integrals.
IRFs to a Money Growth Shock

- Nominal Interest Rate
- Stock Price
- Real Interest Rate ($r_t$)
- Equity Premium
- Consumption of Active Types
- Fraction of Active Types

- Endogenous Rebalancing
- Exogenous Rebalancing
- Endogenous Participation (42%)

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IRFs to a Money Growth Shock (cont.)

- Nominal Interest Rate
- Real Interest Rate ($r_t$)
- Stock Price
- Equity Premium
- Consumption of Active Types
- Fraction of Active Types

- $\rho_{\mu} = 0.9$
- $\rho_{\mu} = 0.68$

GLS (Bank of Spain)
Why Does a Monetary Expansion Lower Risk?

- Higher $\mu \Rightarrow$ higher $c_A$ and $\frac{\bar{\gamma}}{\gamma_u}$.
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- Higher $\mu \Rightarrow$ higher $c_A$ and $\frac{\gamma}{\gamma_u}$.
- Final Equilibrium $\Rightarrow$ less volatile $c_A$. Translates into lower equity premium.
Why Does a Monetary Expansion Lower Risk?

- Higher $\mu \Rightarrow$ higher $c_A$ and $\frac{\tilde{\gamma}}{\gamma_u}$.
- Final Equilibrium $\Rightarrow$ less volatile $c_A$. Translates into lower equity premium.
- Lower volatility because $\frac{\tilde{\gamma}}{\gamma_u}$ is higher. In the limit, as $\frac{\tilde{\gamma}}{\gamma_u} \to 1$, $c_A$ is unaffected by $\mu$. 
Higher $\mu \Rightarrow$ higher $c_A$ and $\frac{\tilde{\gamma}}{\gamma_u}$.

Final Equilibrium $\Rightarrow$ less volatile $c_A$. Translates into lower equity premium.

Lower volatility because $\frac{\tilde{\gamma}}{\gamma_u}$ is higher. In the limit, as $\frac{\tilde{\gamma}}{\gamma_u} \rightarrow 1$, $c_A$ is unaffected by $\mu$.

Changes in risk reflect that $c_A$ is increasing and concave in $\mu$. 
A Reduction in Money Growth

![Graph of A Small Monetary Contraction](image1)

![Graph of A Big Monetary Contraction](image2)

- **A Small Monetary Contraction**
  - Initial Equilibrium ($μ_0$)
  - Final Equilibrium ($μ_1 < μ_0$)

- **A Big Monetary Contraction**
  - Initial Equilibrium ($μ_0$)
  - Final Equilibrium ($μ_1 < μ_0$)
We have developed a DSGE model with infrequent portfolio rebalancing where monetary policy affects the economy through changes in risk.

The model is helpful in accounting for the average equity premium and the response of the equity prices to monetary policy shocks.

Future research:

- Feedback from changes in risk to the policy instrument.
- Endogenous capital and labor supply to jointly analyze asset prices and business cycles.
- Address how endogenous movements in risk affect optimal monetary policy.