

Central bank's two-way communication with the public and inflation dynamics*

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Abstract

Using a model of island economy where financial markets aggregate dispersed information of the public, we analyze how two-way communication between the central bank and the public affects inflation dynamics. When inflation target is observable and credible to the public, markets provide the bank with information about the aggregate state of the economy, and hence the bank can stabilize inflation. However, when inflation target is unobservable or less credible, the public updates their perceived inflation target and the information revealed from markets to the bank becomes less perfect. The degree of uncertainty facing the bank crucially depends on how two-way communication works.

Keywords: Monetary policy, central bank communication, inflation target.

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1 Introduction

The literature on monetary policy emphasizes that good communication from central bank to the public as an important part of monetary policy in practice. [Woodford \(2005\)](#) argues that managing expectations is crucial because effects of monetary policy depend not only on the current policy stance but also on expectations of the future course of monetary policy. There is vast literature emerged in this decade on central bank communication. [Blinder et al. \(2008\)](#) survey this literature and conclude that communication can be important because it has ability to move financial markets, to enhance the predictability of monetary policy decisions and potentially to help achieve central bank's macroeconomic objectives.

While academic literature has focused on central bank's communication to the public through financial markets, communication is not one-way. The bank is subject to a wide variety of uncertainty, and one way to cope with it is to observe financial markets. As Chairman of the FRB (then Governor) [Bernanke \(2004b\)](#) explains, the bank can extract information about inflation expectations and the economic fundamentals from financial markets because markets can aggregate a wide range of dispersed information. This direction of communication is less emphasized in the academic literature on monetary policy, even though its importance is well recognized in the central-banking community. Many central bankers, such as [Dodge \(2001\)](#) (former Governor of the Bank of Canada), [Fukui \(2007\)](#) (former Governor of the Bank of Japan), [Macklem \(2005\)](#) (Deputy Governor of the Bank of Canada), [Kohn \(2008\)](#) (Vice Chairman of the FRB) emphasize that the communication between the central bank and the public is *two-way* communication.

The objective of this paper is to analyze theoretically how the two-way communication works between them and affects inflation dynamics. We consider a particular form of communication about monetary policy objectives: whether or not inflation target is made observable to the public and credible.¹ What we mean by communication from financial markets to central

¹According to [Blinder et al. \(2008\)](#), central banks communicate about four different aspects of monetary policy: policy objectives and strategy, the motives behind a particular policy decision; the economic outlook; and future monetary policy decisions. In our model presented below the central bank announces its functional form of monetary policy rule and economic assessment. Announcement of monetary policy rule is useful for agents to understand monetary policy strategy and to predict future monetary policy decisions.

bank is information revealed by asset prices. In order to analyze the above issue, we construct a model of island economy in which information is dispersed. It is shown that the information revealed from markets depends on whether or not the bank's inflation target is observable to the public and credible. In our model, when inflation target is observable to the public, the degree of uncertainty facing the bank about aggregate state of the economy becomes smaller. As a result, the central bank can stabilize inflation around its target value by responding to the aggregate economic shocks. On the contrary, when inflation target is not observable to the public —imperfect communication from central bank to the public—, we found that there are two equilibria. One is the same as the equilibrium that would arise when the target is made observable to the public. In the other equilibrium, information about the aggregate state of the economy is revealed by less to the bank. This multiplicity of equilibria is different from the multiplicity that would arise when monetary policy does not satisfy the Taylor Principle. The multiplicity in our model results from the interaction between uncertainty facing the public and uncertainty facing the bank. In the latter equilibrium inflation is persistence and volatile through two channels. One channel is private agents' uncertainty about inflation target. The private-agent learning about the target creates inflation persistence, as is shown in [Erceg and Levin \(2003\)](#). The other, which is our main focus, is the central bank's uncertainty about the aggregate economy. When equilibrium fails to reveal information about the aggregate state of the economy, the bank fails to stabilize inflation. The bank's learning process also adds inflation persistence. Our analysis shows that the communication from financial markets to the central bank depends crucially on the communication from the bank to markets.

An intuition behind our result can be obtained from the following example. Consider an economy in which the bank wishes to keep track of the natural interest rate, which is the equilibrium real interest rate under flexible prices. By keeping track of the natural rate and controlling the interest rate, the bank can offset the effects of changes in the natural rate on inflation. Suppose now that the bank observes an increase in (long-term) nominal interest rates. There are two possible reasons. One reason is that the private agents may have revised their inflation expectations. The other reason is that the future natural interest rate may have increased. When inflation target is observable to the public and is credible, then inflation expectations can be

pinned down by the target. If the bank knows this fact, it could infer the natural interest rate from nominal interest rates. However, when the inflation target is not observable to the public, the bank cannot tell if the observed increase in the nominal rate is due to a revision in inflation expectations or due to a change in the natural rate. Inaccurate information about the natural rate makes the bank difficult to offset the effects of the natural rate on inflation, and hence destabilizes inflation. With this intuition, we show that under imperfect two-way communication the learning process of both the public and the central bank causes higher order beliefs to become relevant, thus increasing the persistence and volatility of inflation. This mechanism is capable of generating high persistence and volatility even though the underlying shocks are purely transitory.

Our model is related to the literature on monetary policy under data uncertainty facing the bank. For example, [Orphanides \(2001\)](#) emphasizes the importance of the measurement problem in monetary policy and inflation dynamics. This strand of literature regards the degree of data uncertainty as exogenous and given to the bank. In this paper we show that the degree of data uncertainty is endogenously determined by the communication problem. Improving communication from the bank can make the measurement problem less serious. There is a growing literature on the roles of higher order beliefs in monetary models. [Woodford \(2002\)](#) and [Amato and Shin \(2003\)](#) consider higher order beliefs among firms under strategic complementarity and how this setting generates persistent effect of monetary policy. Using a similar framework, [Lorenzoni \(2008\)](#) shows that the bank can affect the way agents respond to their dispersed information by choosing its monetary policy rule appropriately.² Compared with those papers, we abstract from strategic complementarity among firms and focus on the interaction of beliefs between private agents and the central bank. We analyze how the bank's communication can affect information revealed to the bank in equilibrium.

The structure of the paper is as follows. The next section layouts the model. Section 3 and 4 derive equilibrium, and Section 5 analyzes inflation dynamics under imperfect two-way communication and draws policy implications. Section 6 concludes.

²See, also, [Angeletos and Pavan \(2008\)](#). In the earlier literature, [King \(1982\)](#) show that in the framework of [Lucas \(1972\)](#) monetary policy can change the information content of prices and thus affect real allocation.

2 Model

The model is based on [Aoki \(2006\)](#), which is an island model with stochastic endowment under flexible prices. The assumption of endowment economy means that there is no real effects of monetary policy. However, this framework allows us to analyze in a tractable way how information revelation depends on monetary policy. The economy consists of continuum of islands with mass 1. In each island there is mass 1 of Lucas trees that produce island-specific goods. Stochastic fluctuations in production represent supply shock. Agents in each island visits a discrete number n of islands and consume goods and hold the claims to trees in those islands. They do not observe the variables of the islands they do not visit. This assumption captures the idea that information is dispersed across agents. There are two kinds of financial assets in the economy: claims to the trees and risk-free nominal bonds. There is central bank which sets the nominal interest rate on the risk-free bonds. The model can be interpreted as an island-economy version of the model of price-level determination in Chapter 2 of [Woodford \(2003\)](#).

2.1 Structural equations

Each island is indexed by $i \in [0, 1]$. In each island, there are measure 1 of agents and Lucas trees. The trees in island i produce goods i , and production at time t per unit of tree is denoted by $Y_t(i)$. All agents in island i are assumed to be identical, and in what follows we call those agents “agent i ”. Agent i consumes his consumption basket that contains a discrete number n kinds of goods. Its consumption basket is denoted by J_i . It is assumed that J_i is constant over time.

The preference of agent i is defined by

$$E_0^i \sum_{t=0}^{\infty} \beta^t \log C_t^i, \quad 0 < \beta < 1, \quad (1)$$

where C_t^i is agent i 's consumption aggregator

$$C_t^i = \frac{1}{n} \prod_{j \in J_i} C_t^i(j)^{1/n}. \quad (2)$$

Operator E_t^i is the expectation operator conditional on the information set of agent i at time t . We will define the information set in detail in Section 2.2. Agent i holds trees of island $j \in J_i$ and the risk-free nominal bonds.³ His flow budget constraint is given by

$$\sum_{j \in J_i} P_t(j) C_t^i(j) + \sum_{j \in J_i} S_{t+1}^i(j) Q_t(j) + B_{t+1}^i = \sum_{j \in J_i} S_t^i(j) [Q_t(j) + P_t(j) Y_t(j)] + R_{t-1} B_t^i \equiv W_t^i, \quad (3)$$

where $P_t(j)$ is the price of good j , $Q_t(j)$ is the price of tree j , B_t^i is the holdings of the nominal bonds of agent i at the beginning of period t , R_t is the risk-free nominal interest rate between time t and $t+1$, and $S_t^i(j)$ is the holdings of tree j of agent i at the beginning of time t . $P_t(j) Y_t(j)$ represents nominal dividend per unit of tree j . Ownership of a tree at the beginning of time t entitles the owner to receive the dividend in period t and to have the right to sell the tree at price Q_t in period t . W_t^i represents the total wealth of agent i at time t . At time 0, it is assumed that agent i is endowed with one unit of tree i , that is, $S_0^i(i) = 1$ and $S_0^i(j) = 0, i \neq j$.

Agent i maximizes (1) subject to (2) and (3). As is well known, the optimal consumption decision for each good is given by

$$P_t(j) C_t^i(j) = \frac{1}{n} P_t^i C_t^i, \quad (4)$$

where

$$P_t^i = \prod_{j \in J_i} P_t(j)^{1/n}. \quad (5)$$

Because of Cobb-Douglas specification (2), the expenditure share of each good is equal to $1/n$.

The two Euler equations are

$$\frac{1}{C_t^i} = \beta E_t^i \left[\frac{1}{C_{t+1}^i} R_t \frac{P_t^i}{P_{t+1}^i} \right], \quad (6)$$

³For simplicity of notation, we assume that the consumption basket and portfolio of assets are both equal to J_i . It is possible to relax this assumption without changing the results below.

$$\frac{1}{C_t^i} = \beta E_t^i \left[\frac{1}{C_{t+1}^i} \frac{Q_{t+1}(j) + P_{t+1}(j)Y_{t+1}(j)}{Q_t(j)} \frac{P_t^i}{P_{t+1}^i} \right] \quad \forall j \in J_i. \quad (7)$$

Since utility is logarithmic (equation (1)), the agents spends a fraction $1 - \beta$ of its total wealth on current consumption⁴

$$P_t^i C_t^i = (1 - \beta) W_t^i. \quad (8)$$

Now we characterize equilibrium. Assume for symmetry that each island respectively receives customers from n islands. Let I_i be the set of islands whose agents consume good i and hold tree i . Since there is a fixed supply 1 of Lucas trees in each island and nominal bond is zero in net supply, the equilibrium conditions for the asset markets are

$$\sum_{j \in I_i} S_t^j(i) = 1 \quad \forall i \quad (9)$$

and

$$\int_0^1 B_t^i di = 0. \quad (10)$$

Next, we construct the market clearing condition for each good. The total demand for good i is given by $\frac{1}{n} \sum_{j \in I_i} P_t^j C_t^j$. Therefore the market clearing condition is given by

$$P_t(i) Y_t(i) = \sum_{j \in I_i} \frac{1}{n} P_t^j C_t^j \quad \forall i, t. \quad (11)$$

In equilibrium, $\{C_t^j(i), P_t(i), Q_t(i), S_t^i(j), B_t^i\}$ are determined in order to satisfy the market clearing conditions ((9), (10) and (11)) and the optimality conditions ((4), (6) and (7)), given the sequence of nominal interest rate that is specified by the Central Bank. Appendix A shows that the relative prices of any goods i and j is given by

$$\frac{P_t(i)}{P_t(j)} = \frac{Y_t(j)}{Y_t(i)}, \quad (12)$$

⁴See, for example, Sargent (1987), Chapter 3.

and the price of tree i is given by

$$Q_t(i) = \frac{\beta}{1-\beta} P_t(i) Y_t(i). \quad (13)$$

Equations (12) and (13) imply that $Q_t(i) = Q_t(j)$ for any i, j . Equation (12) stems from the assumption of the Cobb-Douglas preference (2). Equation (13) is an implication of log utility.⁵ Because (12) implies that the nominal dividends of all trees are identical, the portfolio decision regarding the claims to trees is indeterminate as long as it satisfies the market clearing condition (9). In equilibrium, the optimal holding of the nominal bond is given by $B_t^i = 0$ for all agents. Therefore, (3), (12) and (9) imply that

$$P_t^i C_t^i = P_t(i) Y_t(i). \quad (14)$$

Substituting (14) into the Euler equation (6), we obtain

$$E_t^i \beta \left[\frac{P_t(i) Y_t(i)}{P_{t+1}(i) Y_{t+1}(i)} \right] R_t = 1. \quad (15)$$

Equation (15) represents the expectational IS equation for island i . By substituting (13), one obtains

$$E_t^i \beta (Q_{t+1}/Q_t) R_t = 1, \quad \forall i. \quad (16)$$

In the next section we log-linearize the model around the steady state, and specify monetary policy.

2.2 Log-linearized model and monetary policy

We log-linearize the model around the steady state in which $Y_t(i) = Y$ and $P_t(i)$ is constant for all t and all i . Then equation (15) implies that $R = \beta^{-1}$. Define $r_t \equiv \log(R_t/R)$, $y_t(i) \equiv \log(Y_t(i)/Y)$, $q_t \equiv \log Q_t$, $p_t(i) \equiv \log P_t(i)$, $\pi_t(i) \equiv p_t(i) - p_{t-1}(i)$. The log-linear approxima-

⁵See, for example, Sargent (1987), Chapter 3.

tion of the IS equation (15) for island i is given by

$$\begin{aligned} r_t &= E_t^i \pi_{t+1}(i) + E_t^i [y_{t+1}(i) - y_t(i)] \\ &\equiv E_t^i \pi_{t+1}(i) + \bar{r}_t(i), \end{aligned} \quad (17)$$

where

$$\bar{r}_t(i) \equiv E_t^i [y_{t+1}(i) - y_t(i)] \equiv E_t^i \Delta y_{t+1}(i) \quad (18)$$

represents the natural interest rate for island i .⁶ From (13), one obtains

$$\Delta q_t = \pi_t(i) + \Delta y_t(i), \quad \forall i, \quad (19)$$

where $\Delta q_t \equiv \log(Q_t/Q_{t-1})$. Therefore, the Euler equation can be written as

$$r_t = E_t^i \Delta q_{t+1}, \quad \forall i. \quad (20)$$

Next we consider how island-specific variables are related to aggregate variables. We assume that output in each island consists of aggregate and idiosyncratic components:

$$y_t(i) = y_t + \varepsilon_t(i). \quad (21)$$

Term $\varepsilon_t(i)$ represents idiosyncratic supply shock in island i . Furthermore we assume that those are *i.i.d.* with zero mean across islands and across time, so that $\int_0^1 \varepsilon_t(i) di = 0$. Then we have

$$y_t = \int_0^1 y_t(i) di, \quad p_t = \int_0^1 p_t(i) di, \quad \pi_t = \int_0^1 \pi_t(i) di,$$

where y_t , p_t and π_t respectively represent aggregate output, price level and inflation.

Finally let us discuss monetary policy. The bank wishes to stabilize aggregate inflation around its inflation target. However, the aggregate state of the economy, including true aggregate inflation π_t , is not directly observable to the central bank. The underlying assumption is

⁶See Woodford (2003) for this concept.

that it can visit only a subset of islands to collect data. It chooses the nominal interest rate by following the simple monetary policy rule:

$$r_t = \phi(E_t^c \pi_t - \bar{\pi}_t) + \bar{\pi}_t + E_t^c \bar{r}_t, \quad \phi > 1, \quad (22)$$

where E_t^c is the expectation operator conditional on the bank's information set at time t , and $\bar{\pi}_t$ is the inflation target at time t , and

$$E_t^c \bar{r}_t \equiv E_t^c \Delta y_{t+1} \quad (23)$$

represents the bank's estimate of *aggregate* natural interest rate. The information set of the bank is specified in Section 2.3. Equation (22) assumes that the bank reacts to the deviation of its best estimate of aggregate inflation from the target. Equation (22) also assumes that the bank tries to keep track of the path of the natural interest rate \bar{r}_t . We interpret this term as representing the bank's stabilization policy. As is shown in Section 3, it can offset the effects of the changes in \bar{r}_t on inflation by keeping track of \bar{r}_t . Finally, by assuming $\phi > 1$, the monetary policy rule (22) satisfies the so-called Taylor principle. Following Erceg and Levin (2003), we assume that the inflation target consists of the long-run component ($\bar{\pi}$), and the transitory shock (e_t):

$$\bar{\pi}_t = \bar{\pi} + e_t, \quad (24)$$

where e_t is i.i.d. with mean zero. Notice that the monetary policy rule (22) can be written as

$$r_t = \phi(E_t^c \pi_t - \bar{\pi}) + \bar{\pi} + E_t^c \bar{r}_t + u_t, \quad (25)$$

where $u_t \equiv (1 - \phi)e_t$ can be interpreted as monetary policy shock in the empirical literature on policy rules.

2.3 Information structure

Now let us describe the information structure. We assume that the structure of the economy and parameter values are known to all agents and the central bank, and that this fact is common knowledge. However different agents have different and imperfect information about the state of the economy.

Firstly, let us define the information set of the central bank. The bank knows inflation target ($\bar{\pi}$ and e_t), and it observes the interest rate r_t and asset price q_t . It also collects data from a subset of islands. Let J_c be the set of islands the central bank visits, which contains a discrete number m of islands. The bank observes output $y_t(i)$, inflation $\pi_t(i)$ in island i in J_c . Those are used to construct the bank's noisy aggregate data. For example, the noisy measure of the aggregate output is given by

$$y_t^o \equiv \frac{1}{m} \sum_{i \in J_c} y_t(i). \quad (26)$$

Superscript 'o' stands for 'observable'. From equations (21) and (26), we obtain

$$y_t^o = y_t + \varepsilon_t^o, \quad (27)$$

where

$$\varepsilon_t^o \equiv \frac{1}{m} \sum_{i \in J_c} \varepsilon_t(i) \quad (28)$$

represents the bank's measurement error of the aggregate output. Similarly, we can define the bank's measure of inflation:

$$\pi_t^o \equiv \frac{1}{m} \sum_{i \in J_c} \pi_t(i). \quad (29)$$

In reality Consumer Price Index (CPI) corresponds to π_t^o . CPI is not necessarily equal to the true inflation because CPI covers only a subset of goods. Equation (22) assumes that the bank does not just respond to CPI but uses all available information to estimate the underlying true aggregate inflation and reacts to it. In reality, $E_t^c y_t$ and $E_t^c \pi_t$ correspond to the bank's assessment of economic activity after taking account of other information such as information from financial markets. (If the bank had perfect information, then $E_t^c \pi_t = \pi_t$ and it would

stabilize the true aggregate inflation.) We are aware that some inflation targeting countries such as the United Kingdom define their policy regime in terms of a particular price index. Rather, our assumption is closer to countries such as Japan and the United States, where monetary policy regime is not defined in terms of a specific price index.

Secondly, let us define the information set of agent i . The island-specific variables are inflation $\pi_t(i)$ and output $y_t(i)$. We assume that the agent i observes their own variables ($\pi_t(i)$, $y_t(i)$) and the prices (inflation) and quantities of goods they consume ($\pi_t(j)$, $y_t(j)$) for $j \in J_i$). Without loss of generality, assume that $i \in J_i$. We assume that the agents cannot directly distinguish aggregate and idiosyncratic part of shocks in each variable (21). Asset price q_t and the nominal interest rate r_t are observable to the agents. We assume that the bank's noisy measures of output and prices (y_t^o , π^o) are made observable to the agents. Also, $E_t^c \pi_t$ which is in the monetary policy rule is observable. More specifically, we assume that $E_t^c p_t$ and $E_t^c p_{t-1}$ are released by the bank (note that $E_t^c \pi_t \equiv E_t^c (p_t - p_{t-1})$). The underlying assumption is that the Bank publishes its best estimate of the aggregate inflation to which its policy reacts. Since $q_t = p_t + y_t = E_t^c p_t + E_t^c y_t$, $E_t^c y_t$ is also observable to the agents. Then (22) implies that $\bar{\pi}_t$ also becomes observable. However, following Erceg and Levin (2003), we assume that the private agents cannot directly observe the underlying components of $\bar{\pi}_t$ unless the bank announces it explicitly and it becomes credible.

To summarize, the bank's information set at time t , Ω_t^c , is defined by

$$\Omega_t^c = \left\{ (\pi_s^o, y_s^o), (q_s, r_s), (\bar{\pi}, e_s) \right\}_{s=0}^t.$$

The bank's expectations operator, E_t^c is conditional on Ω_t^c .⁷ The information set of agent i , Ω_t^i ,

⁷One may wonder if the bank has survey measures of economic activities, such as survey measures of inflation expectations. While it would be interesting to extend our analysis to incorporate survey measures, we abstract from those indicators and focus on how equilibrium prices and quantities traded in the markets reveal information. The bank may also observe break even inflation rates that are derived from the nominal bonds and inflation-indexed bonds. While such measure can be informative, Bernanke (2004b) argues that the bank should use it with caution because of time-varying inflation risk premium and liquidity premium. See, for example, Sack (2000), Shen and Corning (2001) and Sack and Elsasser (2004). A useful analysis that includes inflation-indexed bonds would have to consider the time-varying premia, and we will leave it for future research.

is defined by

$$\Omega_t^i = \left\{ (\pi_s(j), y_s(j), \text{ for } j \in J_i), (q_s, r_s), (\pi_s^o, y_s^o, E_s^c \pi_s, E_s^c y_s, \bar{\pi}_s) \right\}_{s=0}^t.$$

The expectation operator E_t^i is conditional on Ω_t^i . In what follows, we assume the output and monetary policy shock are *i.i.d.* normal and independent from each other. More specifically, we assume

$$y_t \sim N(0, \gamma_y^{-1}), \quad \varepsilon_t(i) \sim N(0, \gamma_\varepsilon^{-1}), \quad e_t \sim N(0, \gamma_e^{-1}). \quad (30)$$

Here $\gamma_y, \gamma_\varepsilon, \gamma_e$ are respectively precision of y_t, ε_t and e_t .⁸ It is assumed that the distributions are known and common knowledge. Under this assumption, (23) implies that the bank's estimate of the aggregate natural interest rate is

$$E_t^c \bar{r}_t = -E_t^c y_t. \quad (31)$$

This implies that $E_t^c \bar{r}_t$ in equation (22) is also observable to private agents.

2.4 Inflation and expectations

In equilibrium, the endogenous variables $\{i_t, \pi_t\}_{t=0}^\infty$ satisfies equations (15) and (22), and expectations of each of the bank and the private agents are rational. Although expectations are determined endogenously in equilibrium, it is useful to see how inflation depends on the expectations of the central bank and the private agents. For the subsequent analysis it is convenient to rewrite (22). Since integration of (19) over i yields

$$\Delta q_t = \pi_t + \Delta y_t, \quad (32)$$

equation (22) can be written as

$$r_t = \phi \Delta q_t + (1 - \phi) \bar{\pi}_t - \phi E_t^c \Delta y_t + E_t^c \Delta y_{t+1}. \quad (33)$$

⁸Precision is defined as the inverse of variance.

From (20) and (33), we can construct a first-order different equation with respect to Δq_t :

$$\Delta q_t = \phi^{-1} E_t^i \Delta q_{t+1} + (1 - \phi^{-1}) \bar{\pi}_t + E_t^c \Delta y_t - \phi^{-1} E_t^c \Delta y_{t+1}. \quad (34)$$

Equation (34) holds for any agent i . Solve this difference equation forward:⁹

$$\Delta q_t = (1 - \phi^{-1}) \bar{\pi}_t + \phi^{-1} \tilde{\pi}_t(i) + E_t^c \Delta y_t + E_t^i \sum_{s=1}^{\infty} \phi^{-s} [E_{t+s}^c \Delta y_{t+s} - E_{t+s-1}^c \Delta y_{t+s}], \quad \forall i, \quad (35)$$

where

$$\tilde{\pi}_t(i) \equiv E_t^i \bar{\pi}$$

denotes the perceived inflation target of agent i . Then, by using (32), one can show that the equilibrium inflation satisfies

$$\pi_t = (1 - \phi^{-1}) \bar{\pi}_t + \phi^{-1} \tilde{\pi}_t(i) + (E_t^c \Delta y_t - \Delta y_t) + E_t^i \sum_{s=1}^{\infty} \phi^{-s} [E_{t+s}^c \Delta y_{t+s} - E_{t+s-1}^c \Delta y_{t+s}]. \quad (36)$$

By noticing that

$$E_{t+s}^c \Delta y_{t+s} - E_{t+s-1}^c \Delta y_{t+s} = (E_{t+s}^c \Delta y_{t+s} - \Delta y_{t+s}) - (E_{t+s-1}^c \Delta y_{t+s} - \Delta y_{t+s}),$$

equation (36) is written as

$$\begin{aligned} \pi_t = & (1 - \phi^{-1}) \bar{\pi}_t + \phi^{-1} \tilde{\pi}_t(i) + (E_t^c \Delta y_t - \Delta y_t) \\ & + E_t^i \sum_{s=0}^{\infty} \phi^{-s} [E_{t+s}^c \Delta y_{t+s} - \Delta y_{t+s}] \\ & - E_t^i \sum_{s=0}^{\infty} \phi^{-s-1} [E_{t+s}^c \Delta y_{t+s+1} - \Delta y_{t+s+1}]. \quad (37) \end{aligned}$$

⁹In deriving equation (35) we use the fact that

$$E_t^c \Delta y_{t+1} = -E_t^c y_t = -E_t^i E_t^c y_t = E_t^i E_t^c \Delta y_{t+1}.$$

Here the first equality uses the assumption that y_t is *i.i.d.* with zero mean, and the second equality uses the fact that $E_t^c y_t$ is observable to agents.

Equation (37) has the standard property that equilibrium inflation depends on expectations about future monetary policy. This is one of the reasons why communication is considered to be an important part of monetary policy, as is shown in Woodford (2005). In equation (37) inflation depends on the agents' expectations about the bank's mismeasurement of the economy. Note that $\pi_t - E_t^c \pi_t = (E_t^c \Delta y_t - \Delta y_t)$. Therefore the second line of equation (37) represents the expected discounted sum of the bank's current and future estimation error of inflation. Similarly, noticing that $E_t^c \bar{r}_t \equiv E_t^c \Delta y_{t+1}$, the third line is the expected discounted sum of the current and future estimation error of the aggregate natural interest rate. Intuitively speaking, the bank's information problem is the inability to decompose aggregate nominal variable (q_t) into quantity (y_t) and prices (p_t), and prices depend on agents' perceived inflation target. An imprecise estimate of perceived inflation target results in an imprecise estimate of y_t . In the subsequent Sections we analyze how information revealed by financial markets may help the bank do this decomposition.

3 Equilibrium with perfect two-way communication

In this section we analyze the rational expectations equilibrium when the bank's inflation target is made observable to the public and credible. In that case $\bar{\pi}$ and e_t are known and common knowledge, implying $\tilde{\pi}_t(i) = \bar{\pi}$. In this case, it is shown that the rational expectations equilibrium is fully revealing.¹⁰ We can prove this by guess-and-verify. First, guess that y_t is revealed in equilibrium at all t . Then, equation (37) reduces to

$$\pi_t = \bar{\pi} + (1 - \phi^{-1})e_t. \quad (38)$$

¹⁰The result that the equilibrium is fully revealing depends on the assumption that there is only one aggregate shock that is not directly observable to the bank. If there are other kinds of unobservable shocks, such as aggregate demand shock, then the equilibrium is not necessarily fully revealing. See Aoki (2006). However, the general results discussed below would go through — namely, the degree of uncertainty facing the bank becomes smaller under perfect two-way communication than under imperfect communication.

Now we verify y_t is revealed by asset price when inflation is given by (38). Since $\Delta q_t = \pi_t + \Delta y_t$, (38) implies

$$\Delta q_t - \bar{\pi} - (1 - \phi^{-1})e_t = \Delta y_t. \quad (39)$$

Since the left hand side is observable to both the central bank and the public, Δy_t is indeed revealed in equilibrium. This means that, given y_{t-1} is revealed in time $t - 1$, y_t is revealed in time t . Therefore we have confirmed that information is fully revealed to the bank even though the bank cannot directly observe the aggregate state of the economy. In other words, when $\bar{\pi}$ is made observable to the public and credible, financial markets aggregate information on behalf of the bank, and as a result, the bank is not subject to uncertainty regarding the measurement of the natural rate. Thus, communication from markets to the central bank works perfectly when the bank communicates its inflation target.

Equation (38) implies that inflation fluctuation does not involve any persistence when e_t is white noise. The other disturbance, \bar{r}_t , does not affect inflation fluctuations because the bank fully offsets its effects on inflation. In this sense, the bank's stabilization policy works perfectly.

4 Equilibrium when long-run inflation target is not directly observable to the public

In this section we analyze the case in which the bank's long-run inflation target $\bar{\pi}$ is not credibly observable to the public. It is shown that there are two equilibria: one in which information is fully revealed and the other in which it is not. This multiplicity of equilibria is not the same as indeterminacy of rational expectations equilibria when monetary policy does not satisfy the so-called Taylor principle. As is shown below, the multiplicity is due to the interaction between uncertainty facing the bank and uncertainty facing the public.

4.1 Fully revealing equilibrium

Even if $\bar{\pi}$ is not directly observable to the public, equilibrium inflation (38) is still an equilibrium. To show this, suppose that inflation is given by (38). Then Δq_t is given by (39). Recall

that Δq_t is observable to both the bank and agents. Similar to Section 3, the bank can infer Δy_t by looking at (39). This implies $E_t^c y_t = y_t$, and $E_t^c \pi_t = \pi_t$. How about the agents? Since it is assumed that the bank announces $E_t^c \pi_t$ and $\Delta q_t = \pi_t + \Delta y_t$, Δy_t is also revealed to the agents when $E_t^c \pi_t = \pi_t$. Then equation (39) can be written as

$$\Delta q_t - \Delta y_t - \bar{\pi}_t = -\phi^{-1} e_t.$$

Since the left hand side is directly observable to the agents, they can identify e_t and hence they can infer $\bar{\pi}$ by $\bar{\pi} = \bar{\pi}_t - e_t$.

Therefore we have established that when equilibrium inflation is given by (38) then y_t is revealed to the bank and $\bar{\pi}$ is revealed to the agents. On the other hand, when the agents know $\bar{\pi}$ and the bank knows y_t , it is straightforward to show that equilibrium inflation is given by (38). Thus the fully revealing equilibrium is still an equilibrium even when $\bar{\pi}$ is not directly observable to the public.

4.2 Equilibrium with imperfect two-way communication

While the equilibrium analyzed in Section 4.1 is an equilibrium, it is not the only equilibrium. Here we construct an equilibrium in which the two-way communication does not work perfectly. Unlike the case of perfect two-way communication (equations (38) and (39)), equations (35) and (36) imply that the bank may not be able to decompose Δq_t into π_t and Δy_t when the perceived inflation target is not directly observable. In other words, financial markets may not provide accurate information about y_t when $\bar{\pi}$ is not directly observable to the agents. Similarly, agents may not be able to infer $\bar{\pi}$ by observing q_t , and therefore they have to estimate it. Assume that agent i has the following prior distribution about the target

$$\bar{\pi} \sim N(\tilde{\pi}_{-1}, (\tau_{-1}^p)^{-1}), \quad (40)$$

where $\tilde{\pi}_{-1}$ is agent i 's initial prior about $\bar{\pi}$, and τ_{-1}^p is the initial precision. Here we consider the simplest case in which the initial prior and precision are identical to all agents, and assume

that it is common knowledge.¹¹ Under this assumption, there is an equilibrium in which: $\bar{\pi}$ is not revealed immediately to the agents; and both y_t and perceived inflation target are not revealed immediately to the central bank. Here we report our main results and the details of the derivation are given in Appendix B.

Since Δq_t is observable to the bank (i.e., $E_t^c \Delta q_t = \Delta q_t$), a useful expression can be obtained by taking E_t^c of equation (35):

$$\Delta q_t = (1 - \phi^{-1})\bar{\pi}_t + \phi^{-1}E_t^c \tilde{\pi}_t(i) + E_t^c \Delta y_t. \quad (41)$$

Here we used the fact that, in equilibrium,

$$E_t^c E_t^i E_{t+s}^c \Delta y_{t+s} = E_t^c \Delta y_{t+s}, \quad \forall s \geq 1, \quad (42)$$

$$E_t^c E_t^i E_{t+s}^c \Delta y_{t+s+1} = E_t^c \Delta y_{t+s+1}, \quad \forall s \geq 0. \quad (43)$$

In Appendix B.3, we prove that equation (42) and (43) indeed hold under information revealed in equilibrium. By substituting (32) into (41), one obtains

$$\pi_t = \bar{\pi} + (1 - \phi^{-1})e_t + \phi^{-1}(E_t^c \tilde{\pi}_t(i) - \bar{\pi}) + (E_t^c \Delta y_t - \Delta y_t). \quad (44)$$

Equation (44) shows that π_t depends on the second-order belief, namely, the bank's belief about agent i 's perceived inflation target $\tilde{\pi}_t(i)$.

Firstly, let us derive the evolution of the agents' belief. In each period, the agents update their perceived long-run inflation target $\bar{\pi}$. Since monetary policy rule (22) implies that the agents can identify $\bar{\pi}_t$, their observation equation is given by equation (24). Under assumption

¹¹This may be a strong assumption because in reality different agents may have different perceived inflation target. However, since the agents have the same observation equation (24) in our model, their learning process from time 0 on is identical, and their perceived inflation target will converge with each other even if they start with different initial priors. As is shown below, by assuming that the initial prior is the same across agents the dynamic path of the perceived inflation target becomes identical from time 0 on. Modelling heterogeneity in perceived inflation target is left for future research.

(30), the distribution of $\tilde{\pi}_t$ is given by

$$\tilde{\pi}_t \sim N(\bar{\pi}, \gamma_e^{-1}). \quad (45)$$

The filtering problem of the agents is to distinguish the transitory component e_t from the constant term $\bar{\pi}$. This is a classic inference problem from a normal distribution with unknown mean and known variance. Then the posterior mean after t observations is given by (see DeGroot (1970))

$$\tilde{\pi}_t = a_t \tilde{\pi}_{t-1} + (1 - a_t) \left(\frac{1}{t+1} \sum_{s=0}^t \tilde{\pi}_s \right), \quad (46)$$

where

$$a_t \equiv \frac{\tau_{t-1}^p}{\tau_{t-1}^p + (t+1)\gamma_e}.$$

Since $\tilde{\pi}_t = \bar{\pi} + e_t$, (46) can be expressed as

$$\tilde{\pi}_t = a_t \tilde{\pi}_{t-1} + (1 - a_t) \bar{\pi} + (1 - a_t) \left(\frac{1}{t+1} \sum_{s=0}^t e_s \right). \quad (47)$$

Notice that $a_t \rightarrow 0$ as $t \rightarrow \infty$. Also, $\frac{1}{t+1} \sum_{s=1}^t e_s \rightarrow 0$ by the law of large numbers. Therefore, as the agents observe more information over time, they will eventually learn $\bar{\pi}$. Alternatively, we can write (47) in a recursive form:

$$\tilde{\pi}_t - \bar{\pi} = b_t (\tilde{\pi}_{t-1} - \bar{\pi}) + (1 - b_t) e_t, \quad (48)$$

where

$$\begin{aligned} b_t &\equiv \frac{\tau_{t-1}^p}{\tau_{t-1}^p + \gamma_e}, \\ \tau_t^p &\equiv \tau_{t-1}^p + \gamma_e. \end{aligned}$$

Equation (48) gives the evolution of the perceived inflation target.

Secondly, we consider the central bank. While equilibrium is given by both (20) and (22), all the variables in (22) are conditional on the bank's information set. Therefore, we take the

Euler equation (20) as the bank's observation equation. The bank's filtering problem is to estimate the perceived inflation target $\tilde{\pi}_t$ and aggregate output y_t . Notice that the right hand side of (20), $E_t^i \Delta q_{t+1}$, is determined endogenously in equilibrium as a function of $\tilde{\pi}_t$, y_t , and the bank's estimate of the economy. Equilibrium depends on the bank's policy and the bank's policy depends on its estimate of the state of the economy, and the bank's estimate in turn depends on the statistical relationship between the bank's observables and unobservable variables in equilibrium. Therefore it is necessary to solve the filtering problem and equilibrium simultaneously.¹² We solve the equilibrium and filtering by the method of undetermined coefficients. In Appendix B.2, it is shown that the evolution of the bank's estimation error of the perceived inflation target $\tilde{\pi}_t$ is given by

$$E_t^c \tilde{\pi}_t - \tilde{\pi}_t = d_t b_t (E_{t-1}^c \tilde{\pi}_{t-1} - \tilde{\pi}_{t-1}) + (1 - d_t) \frac{a_t}{B_t} (y_t^o - y_t), \quad (49)$$

where B_t and d_t are time-varying deterministic parameters defined in Appendix. The reason why B_t and d_t are time-varying is because the agents' learning about inflation target is transitory and the bank knows this fact. The evolution of the bank's estimation error of the aggregate output is given by

$$E_t^c y_t - y_t = d_t \frac{B_t}{B_{t-1}} (E_{t-1}^c y_{t-1} - y_{t-1}) + (1 - d_t) (y_t^o - y_t). \quad (50)$$

There is a close relationship between those estimates. Indeed, Appendix B.2 shows that the relationship is given by

$$E_t^c \tilde{\pi}_t - \tilde{\pi}_t = \frac{a_t}{B_t} (E_t^c y_t - y_t). \quad (51)$$

Equation (51) implies that imprecise estimate of the perceived inflation target results in imprecise estimate of aggregate output.

¹²See, Aoki (2003) and Svensson and Woodford (2003) for optimal filtering in forward looking models.

5 Inflation dynamics under imperfect two-way communication

5.1 Inflation dynamics

Now we are able to complete our analysis of inflation dynamics under imperfect two-way communication. Using (44) and (51), equilibrium inflation can be written as

$$\begin{aligned}\pi_t &= \bar{\pi} + (1 - \phi^{-1})e_t + \phi^{-1}(\tilde{\pi}_t - \bar{\pi}) + \phi^{-1}(E_t^c \tilde{\pi}_t - \tilde{\pi}_t) + (E_t^c \Delta y_t - \Delta y_t) \\ &= \bar{\pi} + (1 - \phi^{-1})e_t + \phi^{-1}(\tilde{\pi}_t - \bar{\pi}) + \phi^{-1} \frac{a_t}{B_t} (y_t - E_t^c y_t) + (E_t^c \Delta y_t - \Delta y_t),\end{aligned}\quad (52)$$

where $\tilde{\pi}_t - \bar{\pi}$ and $y_t - E_t^c y_t$ respectively evolve according to (48) and (50), and $E_t^c \Delta y_t - \Delta y_t$ evolves according to equation (B.36) that is shown in Appendix B. The first two terms ($\bar{\pi} + (1 - \phi^{-1})e_t$) of equation (52) are identical to (38), that is, the equilibrium inflation when the inflation target is observable and credible. The third term $\phi^{-1}(\tilde{\pi}_t - \bar{\pi})$ represents fluctuations that are caused by agents' uncertainty about the inflation target which affects their inflation expectations. The fourth term $\phi^{-1} a_t / B_t (y_t - E_t^c y_t)$ represents fluctuations caused by the bank's mismeasurement of the natural rate. Since we assume that output is *i.i.d.*, the natural rate is given by

$$E_t^c \bar{r}_t = E_t^c \Delta y_{t+1} = -E_t^c y_t.$$

The last term of equation (52), $(E_t^c \Delta y_t - \Delta y_t)$, represents the fluctuations caused by the bank's mismeasurement of aggregate inflation that it reacts to. Note that $\pi_t = \Delta q_t - \Delta y_t$. Therefore the mismeasurement of Δy_t results in mismeasurement of π_t . Equation (48) implies that $\tilde{\pi}_t - \bar{\pi} \rightarrow 0$ as $t \rightarrow \infty$, therefore the agents will eventually learn about $\bar{\pi}$. This in turn implies that the bank's uncertainty about perceived inflation target also diminishes over time. Therefore, $\pi_t \rightarrow \bar{\pi} + (1 - \phi^{-1})e_t$ as $t \rightarrow \infty$.

In order to investigate the properties of inflation under imperfect two-way communication, we conduct a simple stochastic simulation of our model. We generate artificial normally-distributed shocks and obtain the stochastic process of inflation under the learning process of

the agents and the central bank. Each simulation generates inflation dynamics of 50 periods.¹³ Then, for each simulation we compute the first order autocorrelation and standard deviation of inflation. In order to examine how the stochastic properties of inflation are affected by learning, the statistics are estimated over the first 25 periods and the second 25 periods separately. The above process is repeated 1000 times, and the statistics are averaged over 1000 sets. Our data frequency should be interpreted as annual rather than quarterly, since our model is a flexible price model.

In the simulation, several parameters must be specified. What we have in mind in the analysis is the disinflation process that occurred in the 1980's in several developed countries such as the US. We choose the central bank's long-run inflation target, $\bar{\pi}$, of 2%, and the agent's prior in (40), $\tilde{\pi}_{-1}$, of 10%. According to the analysis of [Kozicki and Tinsley \(2001, 2005\)](#), these values are roughly in line with the US economy at the beginning of 1980s. We set $E_{-1}^c \tilde{\pi}_{-1} = 12\%$, which means that the bank overestimated the initial perceived target by 2 percentage points.¹⁴ The volatility of e_t is taken from empirical volatility of monetary policy shock. (See equation (25)). The standard deviation of monetary policy shock is set to 1%. According to [Roberts \(2004\)](#), this is in line with the FED policy between 1960 and 1983. This implies that the standard deviation of e_t is $(\phi - 1)^{-1}$, which in turn implies that $\gamma_e = (1 - \phi)^2$. The standard deviation of measurement error, ε_t^o , is cited from [Orphanides \(2001\)](#). We calculate the standard deviation of the cumulative revisions of the output measures from his estimates. The resulting standard deviation is 0.89, implying $\gamma_{\varepsilon^o} = 1.26$.¹⁵ The policy coefficient, ϕ , is set to 1.5, like Taylor rule. Finally, we set the precision of initial prior, both τ_{-1}^p and τ_{-1}^c , to one. We also examine robustness against changing the parameter values.

Since our model is highly stylized, the simulation exercise should not be interpreted as trying to match the data. In particular, we have assumed for tractability that all the structural shocks are white noise processes. Therefore, the persistence reported below are purely driven

¹³More precisely, we simulate the economy for 55 periods and discard the first 5 periods to remove the effects of the initial values of shocks (all the shocks at $t = -1$ are set equal to zero).

¹⁴This range is within the difference in the estimate of perceived target between [Bekaert et al. \(2006\)](#) and [Kozicki and Tinsley \(2005\)](#).

¹⁵This may underestimate the degree of uncertainty facing the bank because it implicitly assumes that the final output data corresponds to true output.

by learning by central bank and agents. In reality, shocks can be persistent process. This implies that the persistence reported below may be interpreted as the lower bound that our theoretical model can generate.

Table 1. Time-varying persistence and volatility of inflation under imperfect two-way communication

| | A | B | C | D | E | F | G | H |
|-----------------------------|--------|-----------|------------|----------|-------------|-----------|-----------|-----------|
| $\bar{\pi}$ | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\tilde{\pi}_{-1}$ | 10 | 20 | 10 | 10 | 10 | 10 | 10 | 10 |
| γ_u | 1 | 1 | 1 | 4 | 1 | 1 | 1 | 1 |
| γ_{ε^o} | 1.26 | 1.26 | 1.26 | 1.26 | 0.32 | 1.26 | 1.26 | 1.26 |
| ϕ | 1.5 | 1.5 | 1.3 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 |
| τ_{-1}^p | 1 | 1 | 1 | 1 | 1 | 10 | 1 | 1 |
| τ_{-1}^c | 1 | 1 | 1 | 1 | 1 | 1 | 10 | 1 |
| $E_{-1}^c \tilde{\pi}_{-1}$ | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 14 |
| $\rho_1(\pi_t)$ | 0.30 | 0.67 | 0.48 | 0.21 | 0.34 | 0.38 | 0.35 | 0.36 |
| $\rho_2(\pi_t)$ | -0.020 | 0.030 | 0.026 | -0.028 | -0.020 | 0.08 | -0.021 | -0.012 |
| $\sigma_1(\pi_t)$ | 0.89 | 1.36 | 1.20 | 0.42 | 0.91 | 0.91 | 0.92 | 0.94 |
| $\sigma_2(\pi_t)$ | 0.68 | 0.70 | 0.82 | 0.34 | 0.68 | 0.71 | 0.68 | 0.69 |

Note: Inflation levels and standard deviations are measured in percentage points (for example, $\bar{\pi} = 2$ means inflation target of 2%). $\rho_1(\pi_t)$ and $\rho_2(\pi_t)$ respectively denote the first-order serial correlation of inflation in the first and second subperiod. $\sigma_1(\pi_t)$ and $\sigma_2(\pi_t)$ denote the standard deviation in the first and second subperiod. Cases A-H are explained below.

Case A: benchmark.

Case B: higher perceived target. Initial perceived inflation target is set to 20%. This may correspond to the case of the introduction of inflation targeting in some emerging countries. When Chile, Israel and Hungary adopted inflation targeting, the inflation rates were about 20%.

Case C: less aggressive monetary policy. ϕ is set to 1.3, which is lower than the original Taylor rule.

Case D: smaller monetary policy shock. The standard deviation of monetary policy shock is set to 0.5% and hence $\gamma_u=4$. According to Roberts (2004), this is almost in line with the Fed policy after 1984.

Case E: large measurement error. The standard deviation of measurement error is twice larger: $\gamma_{\varepsilon^o} = 1.26/4$

Case F: stubborn agent's belief. The initial value of the agent's precision parameter, τ_{-1}^p , is 10, which means that the public is more convinced by their own belief.

Case G: stubborn central-bank belief. The initial value of the central-bank precision parameter, τ_{-1}^c , is 10.

Case H: imprecise bank's estimate of perceived target. Bank's initial estimate of $\tilde{\pi}_{-1}$ is increased to 14.

Table 1 shows the simulation results. The benchmark case is Case A. First-order autocorrelation and standard deviation of inflation become smaller in the second half period than the first half period. In the first half, the first-order autocorrelation is about 0.3, implying that inflation

can be persistent even if all the shocks are purely transitory. The standard deviation is also high (0.89). On the contrary, both inflation persistence and the volatility of inflation decline in the second half period. Inflation is almost white noise and its standard deviation drops below 0.68. This is because the agents and central bank eventually learn about the inflation target and the perceived inflation target respectively. Cases B-H examine robustness. In all these cases, the persistence and volatility of inflation are higher in the first subperiod than the second. Therefore the result that learning by central bank and agents add persistence and volatility is robust to variations in parameter values. Case B shows that, compared with Case A, a higher initial perceived target results in larger persistence and volatility in the first subperiod. Case C shows that a weaker monetary policy response to inflation can make inflation more persistent and volatile. This is consistent with the previous literature that finds that inflation was more volatile and persistent in the 1970s in the US when the Fed's response to inflation was weaker.¹⁶ Case D shows that smaller monetary policy shock results in lower persistence and volatility. This is because agents' learning becomes easier. Also smaller monetary policy shock decreases directly inflation volatility. Case E shows that larger measurement error results in higher persistence and volatility. This is because central bank's learning about output becomes more difficult. In Case F, the agents are more convinced about their initial perceived target.¹⁷ In this case, the agents would put less weight on new information when they update their perceived target. As a result, inflation becomes more persistent and volatile. Cases G and H examine the effects of the bank's uncertainty on inflation dynamics. In Case G, the bank's initial precision is high, implying that the bank is more convinced of its initial prior about perceived target. Similar to Case F, this results in higher persistence and volatility in the first subperiod. Finally, in Case H, the bank's initial estimate of the perceived inflation target is further away from the perceived target. This results in higher persistence and volatility than the benchmark case in the first subperiod.

¹⁶Clarida et al. (2000) shows that a smaller monetary policy response to inflation can result in indeterminacy of rational expectations equilibrium, resulting in high inflation volatility. Benati (2008) finds that inflation is more persistent in the 70s than 90s.

¹⁷This case can be interpreted as the situation in which the bank's announcement of the target is less credible.

5.2 Communication and measurement of economic activity

Under imperfect communication, inflation is persistent and volatile in the early phase of learning. Equation (48) shows that monetary policy shock e_t has persistent effects on the perceived target. This agrees with [Erceg and Levin \(2003\)](#), who consider an economy in which the agents' learning about inflation target can make inflation process persistent.¹⁸ In our model, there is another channel. The bank's uncertainty about perceived inflation target causes mismeasurement of the natural rate and aggregate inflation, and this fact de-stabilizes inflation (equation (37)). Equation (49) shows that the bank's learning can a persistent process, adding persistence to inflation.

Our model shows that the two-way communication between the public and the bank is complementary. When the bank fails to communicate its inflation target, financial markets fail to reveal information about the aggregate state of the economy to the bank. The literature on monetary policy under data uncertainty assumes that the degree of uncertainty facing the bank is exogenously given.¹⁹ In this paper we show that the degree of uncertainty is endogenously determined by the communication problem. Improving communication from the bank can make the measurement problem less serious. This has important policy implications. If we take the measurement problem as exogenously given, a policy prescription may be not to respond actively to those economic variables subject to measurement errors. For example, [Orphanides and Williams \(2005\)](#) argue that it is desirable for monetary policy not to respond actively to the unemployment gap because it can be subject to large measurement errors. [Orphanides \(2003b\)](#) argues that a version of nominal income targeting performs well under uncertainty because it is less sensitive to measurement errors.²⁰ On the other hand, our model implies that imper-

¹⁸[Erceg and Levin \(2003\)](#) assume that the central bank has perfect information, which in our model corresponds to the case of $E_t^c \tilde{\pi}_t = \tilde{\pi}_t$.

¹⁹For example, [Orphanides \(2001, 2002, 2003a\)](#) show that the mismeasurement of economic activity, such as the output gap and natural unemployment rate, was responsible for the 'Great Inflation' of the 1970s-1980s in the United States.

²⁰In our model, if the monetary policy rule (22) is replaced by

$$r_t = \phi(\pi_t^o - \tilde{\pi}_t) + \tilde{\pi}_t + E_t^c \bar{r}_t,$$

then the bank reacts directly to the measurement error of inflation. Therefore this rule may destabilize inflation through its response to measurement error, as is argued by [Orphanides and Williams \(2005\)](#). Nominal income targeting can avoid responding to measurement errors in our model because nominal income (q_t) is directly ob-

fect communication can endogenously amplify the measurement problem and that improving communication and gaining credibility help the bank reduce the measurement problem.

5.3 Learning and time-varying stochastic process of inflation

Our model shows that the persistence and volatility of inflation decline as both of the agents and central bank learn. It is interesting to compare this observation with some empirical studies. [Benati \(2008\)](#) found that under inflation targeting inflation persistence declined significantly and exhibits almost no persistence in UK, Canada, Sweden and New Zealand, while it was highly persistent between the breakdown of Bretton Woods and the introduction of inflation targeting. In [Benati \(2004\)](#), he found that the volatility of GDP and inflation in the UK has decreased since the introduction of inflation targeting in 1992. Our model can offer an explanation of his findings. As inflation targeting becomes credible, the agents' uncertainty about the long-run inflation rate has decreased over time. A decrease in the agents' uncertainty has reduced the uncertainty facing the bank. As a result, the bank's stabilization policy has improved, making inflation process less volatile and less persistent.

Also, several articles have documented that macroeconomic volatility in several OECD countries have declined over the past twenty years — the so called 'Great Moderation.' See, for example, [Ahmed et al. \(2004\)](#), [Stock and Watson \(2003\)](#), [Cogley and Sargent \(2005\)](#). Our model, even though it is stylized, has an interesting implication for the econometric analysis of the Great Moderation. In the literature, two competing explanations for the Great Moderation are considered very likely. One is "good policy", i.e. improvements in monetary policy. The other is "good luck", i.e. a fortuitous reduction in exogenous shocks. Several prominent studies have provided support for the good-luck hypothesis. However, [Bernanke \(2004a\)](#) and [Benati and Surico \(2008\)](#) argue that the existing studies may incorrectly identify the effect of good policy as good luck. Econometricians typically do not measure exogenous shocks directly but instead infer them from movements in macroeconomic variables that they cannot otherwise explain. When the central bank's inflation target is not made observable to the pub-

servable. However, in order to respond to the natural interest rate the bank still needs to estimate y_t .

lic, the change in de-anchored inflation expectation may result in what appear to be change in exogenous shocks. Shocks in this sense may certainly depend on monetary policy regime. Accordingly, as the inflation expectation becomes to be anchored gradually, the standard deviation of innovation in the reduced form regression may become smaller even when the magnitude of exogenous shock is constant. This makes an econometric analysis based on reduced-form regression incorrectly lead to good-luck bias. Their argument is closely related to the prediction of our model. In our model, the volatility and persistence of inflation changes over time as the bank and agents learn, even though the monetary policy rule (22) and variances of shocks (σ_e , σ_y) are kept constant. The communication problem creates fluctuations in inflation expectations through the agents' learning. This diminishes over time as the agents learn. In addition to this, this problem creates additional uncertainty facing the bank, which makes policy erratic. This also contributes to variances that diminish over time.

6 Conclusion

The main message of this paper is that the two-way communication between the central bank and the public is complementary. We reached this conclusion by considering an island economy in which the degree of information aggregation by markets is an equilibrium outcome. While the previous literature on communication has focused on the effects of central-bank communication on expectations of the public, we showed that the degree of the bank's uncertainty regarding the aggregate state of the economy can crucially depend on its communication strategy. Our model implies that, by communicating well with markets, the bank can reduce its measurement problem. When the communication is not well functioning, the model predicts that inflation can be persistent and volatile even when there is no intrinsic persistence and structural shocks are white noise. Central bankers often regard financial markets as the mirror that reflects macroeconomic activities. The mirror can get clouded if the bank's communication is imperfect.

There is a number of directions to future research. Firstly, it is important to introduce a source of the real effects of monetary policy, such as price stickiness or information stickiness,

to analyze the implications of both inflation and output. Secondly, in reality there are many other ways of central bank communication other than announcing inflation target. Some central banks publish their forecasts of the future path of inflation and/or policy rates. In our stylized current setting, it is straightforward to show that, if those forecasts are credible, announcing the predictions and making the inflation target credible are equivalent. This prediction might be too extreme compared with reality. As Dale et al. (2008) argues, a fruitful analysis of what to communicate (communication strategy) would need to consider imperfections in agents' ability to interpret the bank's announcement. Thirdly, it would be interesting to model heterogeneity in perceived inflation target. In our model, since the agents have the same observation equation, their perceived inflation target is identical. This assumption significantly simplified the analysis because in the current model the agents do not need to infer the perceived inflation target of the others. If we allow heterogeneity, we conjecture that the information would be even less likely to reveal than our current analysis because there would be one more layer of filtering. Fourthly, it would be interesting to analyze the model's implications for the yield curve.²¹ Finally, it would be interesting to examine some other monetary policy regimes with stronger nominal anchor. In the present paper, it is assumed that the bank changes the nominal interest rate in response to deviations of inflation from its target value. It would be interesting to analyze price level targeting in the context of interest-rate rules, or monetary-aggregate control instead of interest-rate control. Recently, the Bank of Canada is investigating the potential benefits of price level targeting. It would be important to analyze how different policy regimes perform under different degree of communication.

²¹Gurkaynak et al. (2005) shows that long-term nominal interest rates tend to be sensitive to changes in current monetary policy actions when there is uncertainty about nominal anchor. This is because current monetary policy actions bring some news about long-run inflation target.

Appendix

A Equilibrium conditions in Section 2.1

Here we show the equilibrium conditions of the model presented in Section 2.1. Suppose agent i maximizes (1) subject to (2) and (3) and the initial condition $S_0^i(i) = 1$. Then, the following conditions must hold in equilibrium:

- (a) the relative price of two goods i, j is given by (12) and asset prices are all equal;
- (b) for all agents i , the optimal holdings of nominal bond is $B_t^i = 0$;
- (c) for all agents i , the optimal portfolio of trees is indeterminate but satisfies (9);
- (d) the price of tree j is given by (13);
- (e) consumption function of agent i is given by (8).

What we need to show is that (a)-(e) satisfy the first order conditions (4)-(7), budget constraint (3) and the market equilibrium conditions (9)-(11). Under (a)-(c), we can express the consumption function (8) as

$$P_t^i C_t^i = (1 - \beta) [Q_t(i) + P_t(i) Y_t(i)]. \quad (\text{A.1})$$

Under (a)-(c) and the initial condition $S_0^i(i) = 1$, the budget constraint (3) becomes (after using (4))

$$P_t^i C_t^i = P_t(i) Y_t(i). \quad (\text{A.2})$$

Substitute (A.2) into (A.1), we obtain

$$Q_t(i) = \frac{\beta}{1 - \beta} P_t(i) Y_t(i), \quad (\text{A.3})$$

which is equation (13).

Now we show that (a)-(e) satisfy the first order conditions and the market clearing conditions. It is obvious that under (b) the market clearing for the nominal bond is satisfied. That is, $\int_0^1 B_t^i di = 0$. When $Q_t(i) = Q_t(j)$ for all i, j , all trees become perfect substitutes, so $S_t^i(j)$ becomes indeterminate as long as it satisfies $\sum_{j \in J_i} S_t^i(j) = 1$.²² It is possible to construct $S_t^i(j)$ such that $\sum_{j \in J_i} S_t^i(j) = 1$ and the market clearing condition $\sum_{j \in I_i} S_t^j(i) = 1$. One such example is $S_t^i(j) = 1/n$ for all i, j . This clearly satisfies (9).

Next we consider the goods-market clearing. The market clearing condition for good i is given by equation (11):

$$P_t(i)Y_t(i) = \sum_{j \in I_i} \frac{1}{n} P_t^j C_t^j. \quad (11)$$

When consumption is given by (A.2) for all agents, equation (11) becomes

$$P_t(i)Y_t(i) = \sum_{j \in I_i} \frac{1}{n} P_t(j)Y_t(j). \quad (A.4)$$

Now it is clear that (a) satisfies (A.4).

Finally, we need to show that (A.1) satisfies the first-order condition (7). Since $Q_t(i) = Q_t(j)$ and $P_t(i)Y_t(i) = P_t(j)Y_t(j)$ for all i, j we only need to check that

$$\frac{1}{P_t^i C_t^i} = \beta E_t^i \left[\frac{1}{P_{t+1}^i C_{t+1}^i} \frac{Q_{t+1}(i) + P_{t+1}(i)Y_{t+1}(i)}{Q_t(i)} \right] \quad (A.5)$$

is satisfied. Under (A.1) and the portfolio decisions (b) and (c), $P_{t+1}^i C_{t+1}^i$ is given by

$$\begin{aligned} P_{t+1}^i C_{t+1}^i &= (1 - \beta) W_{t+1}^i \\ &= (1 - \beta) \beta W_t^i \frac{Q_{t+1}(i) + P_{t+1}(i)Y_{t+1}(i)}{Q_t(i)}. \end{aligned} \quad (A.6)$$

When $P_{t+1}^i C_{t+1}^i$ is given by (A.6) it is easily shown that the first order condition (A.5) is satisfied.

²²Later, we will show that the consumption function with $\sum_{j \in J_i} S_t^i(j) = 1$ and $B_t^i = 0$ satisfies the first order condition.

Now we have verified that (a)-(e) are all consistent with the first order conditions and the market clearing conditions. Lastly, the aggregate price level is determined to satisfy the other first order condition (6), which is analyzed in the main text.

B Bank's filtering and equilibrium in Section 4.2

B.1 Constructing the observation equation of the bank

Key equations for the bank's filtering is (20) and (41). In addition to those, the bank has noisy measures of aggregate state of the economy (27).²³ Equations (20) and (41) imply²⁴

$$\begin{aligned} r_t &= E_t^i \Delta q_{t+1} \\ &= (1 - \phi^{-1}) \tilde{\pi}_t + \phi^{-1} E_t^i E_{t+1}^c \tilde{\pi}_{t+1} + E_t^i E_{t+1}^c \Delta y_{t+1}. \end{aligned} \quad (\text{B.7})$$

In order to solve the bank's filtering problem, it is convenient to rewrite (B.7) in terms of $\tilde{\pi}_{-1}$. By taking conditional expectation E_t^c of equation (47) and subtracting the resulting equation from (47), we have

$$E_t^c \tilde{\pi}_t - \tilde{\pi}_t = a_t (E_t^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1}), \quad (\text{B.8})$$

and therefore

$$E_t^i E_{t+1}^c \tilde{\pi}_{t+1} - \tilde{\pi}_t = a_{t+1} (E_t^i E_{t+1}^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1}). \quad (\text{B.9})$$

Here we use the fact that $E_t^i \tilde{\pi}_{t+1} = \tilde{\pi}_t$. Equation (B.8) shows that what matters to the bank's estimation error of the perceived inflation target is its estimation error of the initial perceived target. Substituting (B.9) into (B.7), one obtains

$$r_t = \tilde{\pi}_t + \phi^{-1} a_{t+1} (E_t^i E_{t+1}^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1}) + E_t^i E_{t+1}^c \Delta y_{t+1}.$$

²³We also have (29) as noisy inflation measure, but it is redundant once we have (27) since $q_t = y_t + p_t$.

²⁴Recall that $\tilde{\pi}_t(i) = \tilde{\pi}_t$ for all i because of assumption (40).

Substituting (46) into the above equation, and collecting the variables that are observable to the bank to the left hand side and the unobservables to the right hand side, one obtains

$$r_t - (1 - a_t) \frac{1}{t+1} \sum_{s=0}^t \bar{\pi}_s = (a_t - \phi^{-1} a_{t+1}) \tilde{\pi}_{-1} + \phi^{-1} a_{t+1} E_t^i E_{t+1}^c \tilde{\pi}_{-1} + E_t^i E_{t+1}^c \Delta y_{t+1}. \quad (\text{B.10})$$

Define X_t by

$$X \equiv r_t - (1 - a_t) \frac{1}{t+1} \sum_{s=0}^t \bar{\pi}_s. \quad (\text{B.11})$$

Note that X_t is directly observable both to the central bank and agents. Then equation (B.10) can be written as

$$X_t = (a_t - \phi^{-1} a_{t+1}) \tilde{\pi}_{-1} + \phi^{-1} a_{t+1} E_t^i E_{t+1}^c \tilde{\pi}_{-1} + E_t^i E_{t+1}^c \Delta y_{t+1}. \quad (\text{B.12})$$

Equation (B.12) involves the agents' expectations about the bank's future estimate of $\tilde{\pi}_{-1}$. It comes from the second term in equation (B.7), which in turn comes from agents' expectations about the future monetary policy. The agents' expectations about future monetary policy depends on their expectations about the bank's filtering in the subsequent periods.

To summarize, the bank's observation equations are (B.12) and (27). Now the remaining task is to compute the equilibrium and the bank's filtering. Equation (B.12) still contains endogenous variables, namely, r_t , $E_t^i E_{t+1}^c \tilde{\pi}_{-1}$ and $E_t^i E_{t+1}^c \Delta y_{t+1}$. Those terms should be determined jointly with the bank's filtering. In the next section, we will compute the equilibrium and the filtering by the method of undetermined coefficients.

B.2 Deriving the equilibrium and the bank's filtering

Notice that the economy can be represented by X_t , y_t , y_t^o , $\tilde{\pi}_{-1}$, and $E_{t-1}^c \tilde{\pi}_{-1}$. Noisy output measure y_t^o affects the bank's policy through filtering, and policy in turn affects equilibrium inflation. Past estimate of the initial perceived target, $E_{t-1}^c \tilde{\pi}_{-1}$, affects equilibrium at time t through bank's filtering. Finally, we include $\tilde{\pi}_{-1}$, not $\tilde{\pi}_t$, because equation (47) shows that the bank's uncertainty about $\tilde{\pi}_t$ is due to its uncertainty about $\tilde{\pi}_{-1}$. Therefore, we guess that in

equilibrium equation (B.12) takes the following form:

$$A_t X_t = -y_t + B_t \tilde{\pi}_{-1} + C_t E_{t-1}^c \tilde{\pi}_{-1} + D_t y_t^o, \quad (\text{B.13})$$

where A_t , B_t , C_t , D_t are time-varying coefficients to be determined. The coefficient on y_t is normalized to one. While $E_{t-1}^c \tilde{\pi}_{-1}$ and y_t^o are directly observable to the bank, y_t and $\tilde{\pi}_{-1}$ are not. Equation (B.13) shows that the bank cannot identify y_t and $\tilde{\pi}_{-1}$ separately.

Now the bank's observation equations are equations (27) and (B.13). By substituting (27) into (B.13) to eliminate y_t , and moving all the variables that are observable to the bank to the left hand side, we obtain

$$A_t X_t - C_t E_{t-1}^c \tilde{\pi}_{-1} + (1 - D_t) y_t^o = \varepsilon_t^o + B_t \tilde{\pi}_{-1}. \quad (\text{B.14})$$

Equation (B.14) shows that the bank's filtering problem reduces to the sequential updating of a constant, $\tilde{\pi}_{-1}$. A slight complication is that it involves a time-varying coefficient B_t . Define the new observable variable by

$$V_t \equiv A_t X_t - C_t E_{t-1}^c \tilde{\pi}_{-1} + (1 - D_t) y_t^o.$$

From (27) and (28), we obtain

$$\varepsilon_t^o \sim N(0, \gamma_{\varepsilon^o}^{-1}), \quad \gamma_{\varepsilon^o} \equiv m^2 \gamma_{\varepsilon}. \quad (\text{B.15})$$

From equations (B.14) and (B.15) V_t is normally distributed

$$V_t \sim N\left(B_t \tilde{\pi}_{-1}, \gamma_{\varepsilon^o}^{-1}\right).$$

Let the prior distribution at time t be

$$B_{t-1} \tilde{\pi}_{-1} \sim N\left(B_{t-1} E_{t-1}^c \tilde{\pi}_{-1}, (\tau_{t-1}^c)^{-1}\right),$$

where τ_{t-1}^c is the bank's precision at the end of time $t-1$ (i.e., before the bank observes time- t variables). Then the prior for $B_t \tilde{\pi}_{-1}$ is given by

$$B_t \tilde{\pi}_{-1} \sim N \left(B_t E_{t-1}^c \tilde{\pi}_{-1}, \frac{B_t^2}{B_{t-1}^2} (\tau_{t-1}^c)^{-1} \right). \quad (\text{B.16})$$

The posterior mean of $B_t \tilde{\pi}_{-1}$ is given by (see DeGroot (1970))

$$B_t E_t^c \tilde{\pi}_{-1} = d_t B_t E_{t-1}^c \tilde{\pi}_{-1} + (1 - d_t) V_t, \quad (\text{B.17})$$

where

$$d_t \equiv \frac{\frac{B_{t-1}^2}{B_t^2} \tau_{t-1}^c}{\frac{B_{t-1}^2}{B_t^2} \tau_{t-1}^c + \gamma \varepsilon^o}. \quad (\text{B.18})$$

The law of motion of τ_t^c can be written as

$$\tau_t^c = \frac{B_{t-1}^2}{B_t^2} \tau_{t-1}^c + \gamma \varepsilon^o. \quad (\text{B.19})$$

Using the definition of V_t , one can rewrite (B.17) as

$$E_t^c \tilde{\pi}_{-1} = \left\{ d_t - (1 - d_t) \frac{C_t}{B_t} \right\} E_{t-1}^c \tilde{\pi}_{-1} + (1 - d_t) \frac{A_t}{B_t} X_t + \frac{1 - d_t}{B_t} (1 - D_t) y_t^o. \quad (\text{B.20})$$

Equation (B.20) gives the bank's estimate of the initial perceived target. How about the bank's estimate of output? Notice that (B.13) implies

$$E_t^c y_t = -A_t X_t + B_t E_t^c \tilde{\pi}_{-1} + C_t E_{t-1}^c \tilde{\pi}_{-1} + D_t y_t^o.$$

By substituting equation (B.20) into the above equation, we obtain the bank's estimate of output, $E_t^c y_t$, as

$$E_t^c y_t = d_t (B_t + C_t) E_{t-1}^c \tilde{\pi}_{-1} - d_t A_t X_t + (1 - d_t + d_t D_t) y_t^o. \quad (\text{B.21})$$

Finally, by substituting (B.13) into (B.20), one obtains the recursive formula for the bank's

estimate of $\tilde{\pi}_{-1}$ as

$$E_t^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1} = d_t (E_{t-1}^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1}) + \frac{1-d_t}{B_t} (y_t^o - y_t). \quad (\text{B.22})$$

Having derived the bank's filtering, next we substitute the results of the bank's filtering into the equilibrium relation, namely, equation (B.12). For this purpose, let us first discuss what information is revealed to the agents when the equilibrium takes the form given by equation (B.13). Equation (B.21) implies that the agents can identify $E_{t-1}^c \tilde{\pi}_{-1}$. This is because X_t , y_t^o and $E_t^c y_t$ are directly observable.²⁵ Then, when equilibrium is given by (B.13), output y_t is revealed to the agents. Therefore $E_t^i y_t = y_t$. This is because X_t , y_t^o and $E_{t-1}^c \tilde{\pi}_{-1}$ are observable to the agents and we assume that the initial perceived inflation target $\tilde{\pi}_{-1}$ is identical and this fact is common knowledge (thus $\tilde{\pi}_{-1}$ is observable to the agents). However, $\tilde{\pi}$ is not revealed because X_t (equation (B.11)), $E_{t-1}^c \tilde{\pi}_{-1}$ (whose evolution is given by equation (B.20)), y_t and y_t^o do not contain information that allows the agents to identify $\tilde{\pi}$. Given the information revealed to agents, we are now ready to rewrite equation (B.12). Regarding the second term of equation (B.12), $E_t^i E_{t+1}^c \tilde{\pi}_{-1}$, equation (B.22) implies that

$$E_t^i E_{t+1}^c \tilde{\pi}_{-1} = d_{t+1} E_t^c \tilde{\pi}_{-1} + (1-d_{t+1}) \tilde{\pi}_{-1}. \quad (\text{B.23})$$

Here we used the fact that y_t and y_t^o are *i.i.d.* with zero mean. Next we compute the third term of equation (B.12), $E_t^i E_{t+1}^c \Delta y_{t+1}$. Since V_{t+1} is observable to the bank at time $t+1$, we have $V_{t+1} = E_{t+1}^c V_{t+1}$. Then, equation (B.14) implies

$$\begin{aligned} V_{t+1} &= B_{t+1} \tilde{\pi}_{-1} + y_{t+1}^o - y_{t+1} \\ &= B_{t+1} E_{t+1}^c \tilde{\pi}_{-1} + y_{t+1}^o - E_{t+1}^c y_{t+1}. \end{aligned}$$

This in turn implies

$$B_{t+1} (E_{t+1}^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1}) = E_{t+1}^c y_{t+1} - y_{t+1}.$$

²⁵It is assumed that $E_t^c y_t$ is announced by the bank (Section 2.3).

Similarly, since $E_{t+1}^c V_t = V_t$, we have

$$B_t (E_{t+1}^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1}) = E_{t+1}^c y_t - y_t.$$

From those equations we obtain²⁶

$$(B_{t+1} - B_t) (E_{t+1}^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1}) = E_{t+1}^c \Delta y_{t+1} - \Delta y_{t+1}.$$

By taking E_t^i of both sides, $E_t^i E_{t+1}^c \Delta y_{t+1}$ is given by

$$E_t^i E_{t+1}^c \Delta y_{t+1} = (B_{t+1} - B_t) (E_t^i E_{t+1}^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1}) - y_t. \quad (\text{B.24})$$

Finally, substitute (B.23) and (B.24) into (B.12). Then equilibrium X_t is indeed given by

$$\begin{aligned} X_t = & \left[(B_{t+1} - B_t + \phi^{-1} a_{t+1}) d_{t+1} \frac{1-d_t}{B_t} A_t \right] X_t \\ & + [a_t - (B_{t+1} - B_t + \phi^{-1} a_{t+1}) d_{t+1}] \tilde{\pi}_{-1} \\ & + (B_{t+1} - B_t + \phi^{-1} a_{t+1}) d_{t+1} \left[d_t - (1-d_t) \frac{C_t}{B_t} \right] E_{t-1}^c \tilde{\pi}_{-1} \\ & + \left[(B_{t+1} - B_t + \phi^{-1} a_{t+1}) d_{t+1} \frac{1-d_t}{B_t} (1-D_t) \right] y_t^o \\ & - y_t. \end{aligned} \quad (\text{B.25})$$

By comparing our guess (B.13) and (B.25), we have the following four identities:

$$A_t = 1 - (B_{t+1} - B_t + \phi^{-1} a_{t+1}) d_{t+1} \frac{1-d_t}{B_t} A_t, \quad (\text{B.26})$$

$$B_t = a_t - (B_{t+1} - B_t + \phi^{-1} a_{t+1}) d_{t+1}, \quad (\text{B.27})$$

$$C_t = (B_{t+1} - B_t + \phi^{-1} a_{t+1}) d_{t+1} \left[d_t - (1-d_t) \frac{C_t}{B_t} \right], \quad (\text{B.28})$$

$$D_t = (B_{t+1} - B_t + \phi^{-1} a_{t+1}) d_{t+1} \frac{1-d_t}{B_t} (1-D_t). \quad (\text{B.29})$$

By substituting (B.18), equations (B.28) and (B.19) represent a system of deterministic differ-

²⁶Notice that $E_t^i \Delta y_{t+1} = -y_t$ since y_t is revealed to agents.

ence equation with respect to B_t and τ_t^c

$$B_t = a_t - (B_{t+1} - B_t + \phi^{-1} a_{t+1}) \frac{\frac{B_t^2}{B_{t+1}^2} \tau_t^c}{\frac{B_t^2}{B_{t+1}^2} \tau_t^c + \gamma \varepsilon^o}, \quad (\text{B.30})$$

where τ_t^c is given by equation (B.19). Once B_t and τ_t^c are solved, d_t is solved by (B.18). Then equations (B.27), (B.29) and (B.29) respectively determine A_t , C_t and D_t . For simulation in Section 5, we solve for B_t numerically.

Equation (B.19) and (B.30) show that B_t depends both on B_{t-1} and B_{t+1} . Dependence on B_{t-1} results from the recursive nature of filtering. Dependence on B_{t+1} results from the interaction between forward-looking nature of inflation and the bank's filtering. In our model, the current equilibrium variables depend on the agents' expectations about the future monetary policy. The future monetary policy in turn depends on how the central bank will estimate the future state of the economy. This is represented in equation (B.24). Therefore, the way the bank will estimate the state of the economy in the next period will affect the current equilibrium. As a result, the bank's filtering in the current period is affected by its filtering in the future periods. The existing literature such as Aoki (2003) and Svensson and Woodford (2003) focuses on stationary filtering by central bank, so that the Kalman gain is constant over time. In our model, since the bank's learning is about a constant $\tilde{\pi}_{-1}$, filtering is not stationary. That is the main reason why d_t and B_t , which are related to the Bank's Kalman gain, is not constant over time. In forward looking models like ours, the current equilibrium depends on agents' expectations about the future Kalman gain of the bank. Therefore current Kalman gain also depends on expectations about the future Kalman gain.

Let us finish this section by characterizing the stochastic properties of the central-bank uncertainty. From equations (B.13), (B.20) and (B.28), the evolution of $E_t^c \tilde{\pi}_{-1}$ is given by

$$E_t^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1} = d_t (E_{t-1}^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1}) + \frac{1 - d_t}{B_t} \varepsilon_t^o. \quad (\text{B.31})$$

By using (B.8) and (B.31), and noticing that $a_t/a_{t-1} = b_t$, one obtains

$$E_t^c \tilde{\pi}_t - \tilde{\pi}_t = b_t d_t (E_{t-1}^c \tilde{\pi}_{t-1} - \tilde{\pi}_{t-1}) + \frac{a_t(1-d_t)}{B_t} \varepsilon_t^o. \quad (\text{B.32})$$

This is equation (49). As discussed in Section 4.2, there is a close relationship between the bank's estimates of the perceived inflation target and the estimates of the natural rate. Taking the conditional expectation E_t^c of equation (B.14) and subtracting that conditional expectation from (B.14), we have

$$E_t^c \varepsilon_t^o - \varepsilon_t^o = -B_t (E_t^c \tilde{\pi}_{t-1} - \tilde{\pi}_{t-1}). \quad (\text{B.33})$$

Using (B.33) and (B.8), and by noticing that $\varepsilon_t^o = y_t^o - y_t$, we can see that

$$E_t^c \tilde{\pi}_t - \tilde{\pi}_t = \frac{a_t}{B_t} (E_t^c y_t - y_t), \quad (\text{B.34})$$

which is equation (51). By substituting (51) into (B.32), we obtain the evolution of the bank's estimate of y_t , which is (50).

Finally, we can compute the evolution of $E_t^c \Delta y_t - \Delta y_t$ in equation (52). Again, from equation (B.14) at time t and $t-1$ we can obtain

$$\begin{aligned} E_t^c y_t - y_t &= B_t (E_t^c \tilde{\pi}_{t-1} - \tilde{\pi}_{t-1}), \\ E_t^c y_{t-1} - y_{t-1} &= B_{t-1} (E_t^c \tilde{\pi}_{t-1} - \tilde{\pi}_{t-1}). \end{aligned}$$

From these equations and equation (B.8),

$$\begin{aligned} E_t^c \Delta y_t - \Delta y_t &= (B_t - B_{t-1}) (E_t^c \tilde{\pi}_{t-1} - \tilde{\pi}_{t-1}) \\ &= \frac{B_t - B_{t-1}}{a_t} (E_t^c \tilde{\pi}_t - \tilde{\pi}_t). \end{aligned} \quad (\text{B.35})$$

Substituting (B.35) into (B.32), we obtain

$$E_t^c \Delta y_t - \Delta y_t = d_t \frac{B_t - B_{t-1}}{B_{t-1} - B_{t-2}} (E_{t-1}^c \Delta y_{t-1} - \Delta y_{t-1}) + (1-d_t) \frac{B_t - B_{t-1}}{B_t} \varepsilon_t^o. \quad (\text{B.36})$$

B.3 Proof of equations (42) and (43)

We show that, conditional on the information revealed in equilibrium under the imperfect two-way communication, equation (42) and (43) holds.

Proof of equation (42) Note that equilibrium satisfies (B.22) and (B.24). For $s = 1$, (B.22) and (B.24) imply that

$$E_t^i E_{t+1}^c \Delta y_{t+1} = (B_{t+1} - B_t) d_{t+1} (E_t^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1}) - y_t.$$

(Note that $E_t^c \tilde{\pi}_{-1}$ and y_t are revealed to agents in equilibrium.) By taking E_t^c of the above equation, we obtain

$$E_t^c E_t^i E_{t+1}^c \Delta y_{t+1} = -E_t^c y_t = E_t^c \Delta y_{t+1}. \quad (\text{B.37})$$

For $s \geq 2$, (B.22) and (B.24) imply that

$$E_{t+s-1}^i E_{t+s}^c \Delta y_{t+s} = (B_{t+s} - B_{t+s-1}) d_{t+s} (E_{t+s-1}^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1}) - y_{t+s-1}.$$

By taking E_t^i of the above equation, we obtain

$$E_t^i E_{t+s}^c \Delta y_{t+s} = (B_{t+s} - B_{t+s-1}) d_{t+s} (E_t^i E_{t+s-1}^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1}).$$

Then, by taking E_t^c of the above equation, we obtain

$$E_t^c E_t^i E_{t+s}^c \Delta y_{t+s} = (B_{t+s} - B_{t+s-1}) d_{t+s} (E_t^c E_t^i E_{t+s-1}^c \tilde{\pi}_{-1} - E_t^c \tilde{\pi}_{-1}). \quad (\text{B.38})$$

Now we show that

$$E_t^c E_t^i E_{t+s-1}^c \tilde{\pi}_{-1} - E_t^c \tilde{\pi}_{-1} = 0. \quad (\text{B.39})$$

Since we assume that y_t is *i.i.d.* with mean zero, equation (B.22) implies

$$E_t^i E_{t+s-1}^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1} = d_{t+s-1} d_{t+s-2} \dots d_{t+1} (E_t^i E_t^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1}). \quad (\text{B.40})$$

Notice that in equation (B.40), $E_t^i E_t^c \tilde{\pi}_{-1} = E_t^c \tilde{\pi}_{-1}$. This is because $E_t^c \tilde{\pi}_{-1}$ is revealed in equilibrium. By taking E_t^c of equation (B.40), we obtain (B.39). By substituting (B.39) into (B.38), we finally obtain

$$E_t^c E_t^i E_{t+s}^c \Delta y_{t+s} = 0 = E_t^c \Delta y_{t+s}, \quad s \geq 2.$$

Proof of equation (43) Since $E_{t+s}^c \Delta y_{t+s+1} = -y_{t+s}$ for $s \geq 0$, equation (43) reduces to

$$E_t^c E_t^i E_{t+s}^c y_{t+s} = E_t^c y_{t+s}, \quad s \geq 0. \quad (\text{B.41})$$

For $s = 0$, equation (B.41) holds because $E_t^i E_t^c y_t = E_t^c y_t$ (see Section 2.3). Notice that $E_t^c y_{t+s} = 0$ for $s \geq 1$. Therefore, in order to prove (B.41) we need to show that $E_t^c E_t^i E_{t+s}^c y_{t+s} = 0$ for $s \geq 1$. Equation (B.21) implies

$$E_t^i E_{t+s}^c y_{t+s} = d_{t+s} (B_{t+s} + C_{t+s}) E_t^i E_{t+s-1}^c \tilde{\pi}_{-1} - d_{t+s} A_{t+s} E_t^i X_{t+s}, \quad s \geq 1. \quad (\text{B.42})$$

Substituting (B.13) into (B.42) to eliminate $A_{t+s} X_{t+s}$, we obtain

$$E_t^i E_{t+s}^c y_{t+s} = d_{t+s} B_{t+s} (E_t^i E_{t+s-1}^c \tilde{\pi}_{-1} - \tilde{\pi}_{-1}), \quad s \geq 1. \quad (\text{B.43})$$

Since equation (B.39) holds, by taking E_t^c of (B.43) we obtain

$$E_t^c E_t^i E_{t+s}^c y_{t+s} = 0.$$

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