# Central bank's two-way communication with the public and inflation dynamics

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26th February 2008

# Motivation: Two-way communication between central bank and markets

- CB communicates to markets the future course of monetary policy
- Markets provide CB with information about aggregate state of the economy.
- Prices depend on market participants' view on future monetary policy — perceived inflation target.
- How does observability/credibility of IT affect this two-way communication?

# What we do

- Island economy where inflation target may not be observable to agents
- When IT observable, information about the state of the economy revealed to CB.
- When not observable, there are two equilibria
  - The same equilibrium as above
  - Equilibrium in which information revelation becomes less perfect.
    - Inflation becomes persistent.
    - Inflation becomes volatile because CB fails to estimate economic shocks accurately
- Information revealed to CB endogenously determined by policy
- Two-way communication is complementary

# Related Literature

- Literature on transparency (Morris and Shin ('02), Svensson, Eusepi and Preston)
  - Imperfect common knowledge and strategic interaction among agents
- MP under data uncertainty (Orphanides '01, '02, '03, Svensson-Woodford '03, Aoki '03)
  - Data uncertainty is taken as given
- Imperfect credibility (Erceg-Levin, '03)
  - Consider private-sector uncertainty. CB has perfect information
- Excess sensitivity of long rates in non-IT countries (Gurkaynak, Sack, Swanson, '05, Gurkaynak, Levin, Swanson, '06)

# Inflation targeting and information revelation
## intuition

Suppose nominal interest rate $r_t$ increases

- Two possibilities ($r_t = \bar{r}_t + E_t\pi_{t+1}$)
  - inflation expectations increased
  - natural rate increased
- When CB uncertain about perceived IT, CB cannot distinguish those two.

# Structural Equations

# Outline of the model

A simple model of inflation determination in an island economy

- stochastic endowment and flexible prices
- each island produces island-specific goods
- agents consume only a subset of goods
- assets: Lucas tree and nominal bond
- Monetary policy follows a simple rule
(No optimisation. Focus on filtering and equilibrium)
- Information dispersed

# Model

- Continuum of island $i \in [0, 1]$
- In island $i$,
  - Lucas tree with measure 1. Produces $y_t(i)$.
  - $y_t(i)$ is stochastic (supply shock).
- nominal bond (zero net supply)

# Preferences

- Agent in island $i$ (agent $i$) maximises

$$E_0^i \sum_{t=0}^{\infty} \beta^t \log C_t^i, \tag{1}$$

- with the Cobb-Douglas aggregator

$$C_t^i = \frac{1}{n} \prod_{j \in J_i} C^i(j)^{1/n}. \tag{2}$$

- $E_t^i$: conditional on agent $i$'s info.
- $J_i$: consumption basket of agent $i$. Contains $n$ goods.

# Optimisation

- Agent $i$ chooses consumption $C_t^i(j)$, holdings of trees $S_t^i(j), j \in J_i$, and holdings of nominal bond $B_t^i$ to maximise utility subject to:
- budget constraint

$$\sum_{j \in J_i} p(j) c^i(j) + \sum_{j \in J_i} \underbrace{S_{t+1}^i(j)}_{\text{tree}} q_t(j) + \underbrace{B_{t+1}^i}_{\text{bond}}$$

$$= \sum_{j \in J_i} S_t^i(j) \left[ q_t(j) + p_t(j) y_t(j) \right] + R_t B_t^i \equiv \underbrace{W_t^i}_{\text{total wealth}} \tag{3}$$

# Linearised model

- Linearised IS

$$r_t = E_t^i \pi_{t+1}(i) + \bar{r}_t(i), \tag{4}$$

where

$$\bar{r}_t(i) \equiv E_t^i \left[ y_{t+1}(i) - y_t(i) \right] \equiv E_t^i \Delta y_{t+1}(i) \tag{5}$$

represents the natural interest rate for island $i$.

- Asset price

$$\Delta q_t = \pi_t(i) + \Delta y_t(i), \quad \forall i, \tag{6}$$

where $\Delta q_t \equiv \log(Q_{t+1}/Q_t)$.

- IS equation can be written as

$$r_t = E_t^i \Delta q_{t+1}, \quad \forall i. \tag{7}$$

# Assumptions

- Output in each island consists of aggregate and idiosyncratic components:

$$y_t(i) = y_t + \varepsilon_t(i). \tag{8}$$

- $y_t$: aggregate supply shock; *i.i.d.* normal
- $\varepsilon_t(i)$: idiosyncratic supply shock in island $i$.
- *i.i.d.* normal across islands and across time, so that $\int_0^1 \varepsilon_t(i)di = 0$.
- Then we have

$$y_t = \int_0^1 y_t(i)di, \quad \pi_t = \int_0^1 \pi_t(i)di.$$

# Monetary policy

- CB follows a simple rule:

$$r_t = \phi(E_t^C \pi_t - \bar{\pi}_t) + \bar{\pi}_t + E_t^C \bar{r}_t, \quad \phi > 1, \qquad (9)$$

- $\bar{\pi}_t$: inflation target
  - $\bar{\pi}_t \equiv \bar{\pi} + e_t$
  - $\bar{\pi}$: long-run inflation target
  - $e_t$: transitory deviation from $\bar{\pi}$; *i.i.d* normal
- $E_t^C \pi_t$: estimate of aggregate inflation
- $E_t^C \bar{r}_t$: estimate of the aggregate natural rate

# Information set of agent $i$

- Agent observes
  - prices and quantities of goods he consumes
  - interest rate $r_t$ and asset price $q_t$
  - $E_t^c \pi_t$ (this implies $E_t^c \bar{r}_t$ observable)
- Agent may not observe $\bar{\pi}$ and $e_t$ separately

# Information set of CB

- Bank visits $m$ islands and constructs noisy data on aggregate output and prices

$$y_t^o \equiv \frac{1}{m} \sum_{i \in J_c} y_t(i) = y_t + \varepsilon_t^o$$

- $r_t$ and $q_t$: observable

# Benchmark: When $\bar{\pi}$ is observable

## Information fully revealed to CB and agents

- From IS and monetary policy rule, equilibrium is given by
$$\pi_t = \bar{\pi} + (1 - \phi^{-1})e_t.$$
- CB fully offsets the effects of $\bar{r}_t$ on $\pi_t$
- Inflation expectations anchored by $\bar{\pi}$
- By looking at $\Delta q_t$, CB can identify $y_t$ even if $y_t$ not directly observable.
- This is because
$$\Delta q_t = \pi_t + \Delta y_t$$
$$\Delta q_t - \bar{\pi} - (1 - \phi^{-1})e_t = \Delta y_t$$
- Two-way communication works well

# Equilibrium under unobservable $\bar{\pi}$

# Fully-revealing is one of the equilibria

- Equilibrium is given by

$$\pi_t = \bar{\pi} + (1 - \phi^{-1})e_t.$$

- By looking at $\Delta q_t$, CB can identify $y_t$ even if $y_t$ not directly observable.
- Now agents also can identify $\bar{\pi}$ even if it is not directly observable

$$\Delta q_t - \Delta y_t = -\bar{\pi} + (1 - \phi^{-1})e_t$$

$$\bar{\pi}_t = \bar{\pi} + e_t$$

- But this is not the unique equilibrium even if $\phi > 1$

# Partially revealing equilibrium

- There is an equilibrium in which:
  - $\bar{\pi}$ is not revealed to agents immediately
  - $y_t$ is not revealed to CB immediately.
- Multiple equilibria even if the Taylor principle is satisfied
- Two-way communication does not work well

# Equilibrium given belief

From IS and monetary policy rule,

$$\pi_t = \bar{\pi} + (1 - \phi^{-1})e_t + \phi^{-1}(\tilde{\pi}_t - \bar{\pi}) + \phi^{-1}(E_t^c \tilde{\pi}_t - \tilde{\pi}_t) + (E_t^c \Delta y_t - \Delta y_t). \quad (10)$$

- $\tilde{\pi}_t$: perceived target
- $\tilde{\pi}_t - \bar{\pi}$: PA estimation error of IT
- $E_t^c \tilde{\pi}_t - \tilde{\pi}_t$: CB estimation error of perceived IT
- $E_t^c \Delta y_t - \Delta y_t$: CB estimation error of $\Delta y_t$
- 2nd order belief matters

# Private-sector filtering

- Observation equation identical to all agents:

$$\bar{\pi}_t = \bar{\pi} + e_t$$

- Consider the case in which the initial perceived target identical for all agents
- Perceived inflation target after $t$ observations:

$$\tilde{\pi}_t - \bar{\pi} = b_t(\tilde{\pi}_{t-1} - \bar{\pi}) + (1 - b_t)e_t, \quad (11)$$

  - $b_t \to 1$ as $t \to \infty$
  - Agents eventually learn $\bar{\pi}$

# CB-filtering about $y_t$ and $\tilde{\pi}_t$

- Endogenous variables are determined simultaneously with CB filtering. Solve by the method of undetermined coefficients.
- Estimated perceived inflation target

$$E_t^c \tilde{\pi}_t - \tilde{\pi}_t = d_t b_t \left( E_{t-1}^c \tilde{\pi}_{t-1} - \tilde{\pi}_{t-1} \right) + (1 - d_t) \frac{a_t}{B_t} (y_t^o - y_t). \quad (12)$$

- Estimated output

$$y_t - E_t^c y_t = d_t \frac{B_t}{B_{t-1}} \left( y_{t-1} - E_{t-1}^c y_{t-1} \right) - (1 - d_t)(y_t^o - y_t), \quad (13)$$

- $B_t$, $d_t$: time-varying deterministic coefficients

# Summary of Equilibrium

- Equilibrium is given by

$$\pi_t = \bar{\pi} + (1-\phi^{-1})e_t + \phi^{-1}(\tilde{\pi}_t - \bar{\pi}) + \phi^{-1}(E_t^c \tilde{\pi}_t - \tilde{\pi}_t) + (E_t^c \Delta y_t - \Delta y_t). \quad (14)$$

- Inflation fluctuations due to miscommunication
  - Private-sector uncertainty about IT
  - CB uncertainty about perceived IT and aggregate output (natural rate)
  - Both learning has recursive representation

# Implications: Endogenous data uncertainty and inflation dynamics

# Information revelation

- To central bank
  - $y_t$ and $\tilde{\pi}_t$ not revealed immediately
  - Estimation error of perceived IT and aggregate output closely related:

$$E_t^c \tilde{\pi}_t - \tilde{\pi}_t = -\frac{\alpha_t}{B_t}\left(E_t^c y_t - y_t\right). \qquad (15)$$

- To agents
  - $\bar{\pi}$ is not revealed immediately.

# Inflation dynamics

Inflation is persistent and volatile

- persistence: recursive nature of learning.
- volatility:
  - volatile inflation expectations due to uncertain IT
  - policy mistakes due to unobservability of the natural rate

# Unobservable $\bar{\pi}$ and data uncertainty

- Unobservable inflation target creates uncertainty about perceived inflation target
- → identification of shocks difficult. → source of natural rate mis-measurement
- this causes policy mistakes, generating inflation volatility and persistence.

# Policy implications

- Two-way communication is complementary
- If monetary policy becomes credible, CB uncertainty becomes smaller.

# Time-varying stochastic process of inflation

Our model implies

- $\pi_t \to \bar{\pi} + (1 - \phi^{-1})e_t$ as $t \to \infty$.
- $\pi_t$ becomes less persistent over time
- $\pi_t$ becomes less volatile over time
- Consistent with Benati ('08, QJE)
- Changes in stochastic process due to changes in expectations — Bernanke (2004)'s view on Great Moderation

# Summary

- Two-way communication is complementary
- Mis-measurement of the states of the economy endogenously determined
- Change in stochastic process of inflation driven by changes in expectations
- Future work
  - add real effects of monetary policy
  - heterogeneity in perceived inflation target
  - yield curve analysis

# APPENDIX

# Optimisation

- Consumption for each good

$$P_t(j)C_t^i(j) = \frac{1}{n}P_t^i C_t^i, \quad P_t^i \equiv \prod_{j \in J_i} P_t(j)^{1/n}, \tag{16}$$

- Euler equation for bond

$$\frac{1}{C_t^i} = \beta E_t^i \left[ \frac{1}{C_{t+1}^i} R_t \frac{P_t^i}{P_{t+1}^i} \right], \tag{17}$$

- Euler equation for tree: for all $j \in J_i$,

$$\frac{1}{C_t^i} = \beta E_t^i \left[ \frac{1}{C_{t+1}^i} \frac{Q_{t+1}(j) + P_{t+1}(j)Y_{t+1}(j)}{Q_t(j)} \right] \tag{18}$$

# Market equilibrium

- Market for each tree

$$\sum_{j \in I_i} S_t^j(i) = 1 \quad \text{for each } i, t \tag{19}$$

- Market for bond

$$\int_0^1 B_t^i di = 0 \tag{20}$$

- Market for each good

$$P_t(i) Y_t(i) = \sum_{j \in I_i} \frac{1}{n} P_t^j C_t^j \quad \forall i, t. \tag{21}$$

$I_i$: set of islands that consume good produced in $i$.

# Equilibrium prices

- relative price of good $i$ and $j$

$$\frac{P_t(i)}{P_t(j)} = \frac{Y_t(j)}{Y_t(i)}, \tag{22}$$

- price of tree $i$

$$Q_t(i) = \frac{\beta}{1-\beta} P_t(i) Y_t(i) \tag{23}$$

- (22) and (23) imply $Q_t(i) = Q_t(j), \forall i, j$. Portfolio decision is indeterminate as long as it satisfies

$$\sum_{j \in I_i} S_t^j(i) = 1 \tag{24}$$

# Expectational IS equation

- BC, (22) and (24) imply that

$$P_t^i C_t^i = P_t(i) Y_t(i). \tag{25}$$

- Substituting (25) into the Euler equation (17), we obtain

$$E_t^i \beta \left[ \frac{P_t(i) Y_t(i)}{P_{t+1}(i) Y_{t+1}(i)} \right] R_t = 1. \tag{IS}$$

- Equation (IS) represents the expectational IS equation for island $i$.

# Stochastic simulation

- Each simulation generates inflation of 50 periods
- Split to the first and second 25 periods
- Compute 1st order serial correlation and standard deviation
- 1000 replication

# Calibration

- $\bar{\pi} = 2\%$
- $\tilde{\pi}_{-1} = 10\%$: Kozicki and Tinsley ('01)
- $E^c_{-1}\tilde{\pi}_{-1} = 12\%$: within the difference in the empirical estimates of perceived target

# Time-varying persistence and volatility of inflation under imperfect two-way communication

| | A | B | C | D |
|---|---|---|---|---|
| $\bar{\pi}$ | 2 | 2 | 2 | 2 |
| $\tilde{\pi}_{-1}$ | 10 | **20** | 10 | 10 |
| $\gamma_u$ | 1 | 1 | 1 | **4** |
| $\gamma_{\varepsilon^o}$ | 1.26 | 1.26 | 1.26 | 1.26 |
| $\phi$ | 1.5 | 1.5 | **1.3** | 1.5 |
| $\tau^p_{-1}$ | 1 | 1 | 1 | 1 |
| $\tau^c_{-1}$ | 1 | 1 | 1 | 1 |
| $E^c_{-1}\tilde{\pi}_{-1}$ | 12 | 12 | 12 | 12 |
| $\rho_1(\pi_t)$ | 0.30 | 0.67 | 0.48 | 0.21 |
| $\rho_2(\pi_t)$ | -0.020 | 0.030 | 0.026 | -0.028 |
| $\sigma_1(\pi_t)$ | 0.89 | 1.36 | 1.20 | 0.42 |
| $\sigma_2(\pi_t)$ | 0.68 | 0.70 | 0.82 | 0.34 |

A: Benchmark; B: higher perceived target;
C: less aggressive MP; D: smaller MP shock

- Persistence and volatility become smaller over time

# Time-varying persistence and volatility of inflation under imperfect two-way communication

| | E | F | G | H |
|---|---|---|---|---|
| $\bar{\pi}$ | 2 | 2 | 2 | 2 |
| $\tilde{\pi}_{-1}$ | 10 | 10 | 10 | 10 |
| $\gamma_u$ | 1 | 1 | 1 | 1 |
| $\gamma_{\varepsilon^o}$ | **0.32** | 1.26 | 1.26 | 1.26 |
| $\phi$ | 1.5 | 1.5 | 1.5 | 1.5 |
| $\tau^p_{-1}$ | 1 | **10** | 1 | 1 |
| $\tau^c_{-1}$ | 1 | 1 | **10** | 1 |
| $E^c_{-1}\tilde{\pi}_{-1}$ | 12 | 12 | 12 | **14** |
| $\rho_1(\pi_t)$ | 0.34 | 0.38 | 0.35 | 0.36 |
| $\rho_2(\pi_t)$ | -0.020 | 0.08 | -0.021 | -0.012 |
| $\sigma_1(\pi_t)$ | 0.91 | 0.91 | 0.92 | 0.94 |
| $\sigma_2(\pi_t)$ | 0.68 | 0.71 | 0.68 | 0.69 |

E: large measurement error; F: stubborn agents' belief;
G: stubborn CB belief; H: imprecise CB estimate of perceived target

# CB-filtering about perceived IT

Constructing observation equations

- Evolution of perceived IT

$$\tilde{\pi}_t = a_t \tilde{\pi}_{-1} + (1 - a_t)\left(\frac{1}{t}\sum_{s=1}^{t} \bar{\pi}_s\right),$$

- equilibrium $\Delta q_t$

$$\Delta q_t = (1 - \phi^{-1})\bar{\pi}_t + \phi^{-1} E_t^c \tilde{\pi}_t + E_t^c \Delta y_t.$$

- IS

$$r_t = E_t^i \Delta q_{t+1}$$

- Noisy output data

$$y_t^o = y_t + \varepsilon_t^o$$

# Observation equation

- From those equation, we obtain

$$X_t = \left(a_t - \phi^{-1} a_{t+1}\right) \tilde{\pi}_{-1} + \phi^{-1} a_{t+1} E_t^i E_{t+1}^c \tilde{\pi}_{-1} + E_t^i E_{t+1}^c \Delta y_{t+1},$$

- where $X_t$ is observable to CB:

$$X \equiv r_t - (1 - a_t) \frac{1}{t} \sum_{s=0}^{t} \bar{\pi}_s$$

- $E_t^i E_{t+1}^c \tilde{\pi}_{-1}$ and $E_t^i E_{t+1}^c \Delta y_{t+1}$ are determined simultaneously with filtering
- Solve by the method of undetermined coefficients

# Equilibrium and CB filtering (1)

- Guess:

$$A_t X_t = -y_t + B_t \tilde{\pi}_{-1} + C_t E_{t-1|c} \tilde{\pi}_{-1} + D_t y_t^o, \quad \text{(guess)}$$

$A_t$, $B_t$, $C_t$, $D_t$ to be determined jointly with Kalman filtering about $y_t$ and $\tilde{\pi}_{-1}$.

- $B_t$ represents the effects of initial perceived target ($\tilde{\pi}_{-1}$) on current equilibrium

# Equilibrium and CB filtering (2)

- Derive Kalman filter based on (guess), and substitute it back to (guess).
- Then solve for $A_t$, $B_t$, $C_t$, $D_t$.
- Then $B_t$ satisfies

$$B_t = -(B_{t+1} - B_t + \phi^{-1} a_{t+1}) d_{t+1} + a_t$$

where

$$d_t \equiv \frac{\frac{B_{t-1}^2}{B_t^2} \tau_{t-1}^c}{\frac{B_{t-1}^2}{B_t^2} \tau_{t-1}^c + \gamma_{\varepsilon^o}}$$

$$\tau_t^c = \frac{B_{t-1}^2}{B_t^2} \tau_{t-1}^c + \gamma_{\varepsilon^o}.$$

- Once $B_t$ is determined, $A_t$, $C_t$, $D_t$ are determined.

# Equilibrium properties (1)

- Simultaneity of equilibrium and CB filtering
- PS expectations about future CB filtering matters to $\pi_t$
- Current CB filtering depends on PS expectations about future CB filtering
- Intuition:
  - Forward-looking nature of inflation
  - Inflation determined by expectations about future MP
  - Future MP depends on future CB filtering

# Equilibrium property (2)

$B_t$ depends on:

- $B_{t-1}$: recursive nature of filtering
- $B_{t+1}$: forward-looking nature of inflation
  - $\pi_t$ depends on PS expectations about future MP
  - future MP depends on filtering $d_{t+1}$
  - current filtering depends on PS expectations