Unique Monetary Equilibria with Interest Rate Rules*

Bernardino Adão
Banco de Portugal

Isabel Correia
Banco de Portugal, Universidade Catolica Portuguesa and CEPR

Pedro Teles
Banco de Portugal, Universidade Catolica Portuguesa, and CEPR

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Abstract

In contrast to previous literature, we show that there are interest rate rules that implement unique global equilibria in standard monetary models. This is a contribution to a literature that either concentrates on conditions for local determinacy, or criticizes that approach showing that local determinacy might be associated with global indeterminacy. The interest rate

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Question

- How can monetary policy implement unique equilibria?

  (Can interest rate policy provide a nominal anchor?)

- Apparent success with inflation targeting.
  - Success is attributed to a Taylor rule.
  - In monetary models, Taylor rules do not pin down unique equilibria.
    - Equilibria may be locally unique, but not globally so.

A simple endowment economy

- Euler equation for the representative household:

$$\frac{u_c\left(Y_t\right)}{P_t} = R_t E_t \frac{\beta u_c\left(Y_{t+1}\right)}{P_{t+1}}$$

  $\{Y_t\}$ is the endowment process.

- In log deviations from a deterministic steady state with constant inflation π^*:

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{P}_{t+1} - \widehat{P}_t,$$

  where $r_t = \frac{u_c(Y_t)}{\beta E_t u_c(Y_{t+1})}$.

Monetary policy in the endowment economy

- Interest rate target:

$$\widehat{R}_t = \widehat{R}_t^*$$

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{\pi}_{t+1}$$

  - Unique path for the conditional expectation of inflation $E_t \widehat{\pi}_{t+1}$,
  - but not for the initial price level, nor the distribution of realized inflation across states.

- Current feedback rule:

$$\widehat{R}_t = \widehat{r}_t + \tau \widehat{\pi}_t$$

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{\pi}_{t+1}$$

- Equilibria:

$$\tau \widehat{\pi}_t - E_t \left(\widehat{\pi}_{t+1} \right) = 0.$$

  - Equilibrium with $\widehat{\pi}_t = 0$ and $\widehat{R}_t = \widehat{r}_t$.
  - Multiple other solutions:
    * If $\tau > 1$ (Taylor principle): Continuum of divergent solutions. The equilibrium with $\widehat{\pi}_t = 0$ is locally unique.
    * If $\tau < 1$: Continuum of solutions converging to $\widehat{\pi}_t = 0$.

- Forward looking rules:

$$\widehat{R}_t = \widehat{r}_t + \tau E_t \widehat{\pi}_{t+1}$$

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{\pi}_{t+1}$$

- Equilibria:

$$(\tau - 1) E_t (\widehat{\pi}_{t+1}) = 0$$

- For $\tau \neq 1$, only expected inflation is pinned down, not the distribution of prices across states.

- Wicksellian interest rate rules (Woodford, 2003) have the interest rate respond to the price level rather than inflation.

- Policy rule:

$$\widehat{R}_t = \widehat{r}_t + \phi \widehat{P}_t,$$

with $\phi > 0$.

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{P}_{t+1} - \widehat{P}_t,$$

- Equilibria:

$$(1 + \phi) \widehat{P}_t - E_t \widehat{P}_{t+1} = 0$$

  - Equilibrium with $\widehat{P}_t = 0$ and $\widehat{R}_t = \widehat{r}_t$.
  - The equilibrium with $\widehat{P}_t = 0$ is locally unique if $\phi > 0$.
  - Continuum of divergent solutions.

Unique solution for the dynamic equation?

- Price level targeting rule:

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{P}_{t+1} + \widehat{\xi}_t$$

where $\widehat{\xi}_t$ is an exogenous random variable.

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{P}_{t+1} - \widehat{P}_t,$$

- Equilibria

$$\widehat{P}_t = \widehat{\xi}_t.$$

- Massive literature starting with Sargent and Wallace (1975) and McCallum (1981), including recent literature on local and global determinacy in models with nominal rigidities:

  – Conditions for a unique local equilibrium.

    McCallum (1981), Woodford (2003), Clarida, Gali and Gertler (1999, 2000), Carlstrom and Fuerst (2001, 2002), Benhabib, Schmitt-Grohe and Uribe (2001a), among others.

  – Conditions for local determinacy may be conditions for global indeterminacy.

    Benhabib, Schmitt-Grohe and Uribe (2001b, 2002, 2003), Schmitt-Grohe and Uribe (2001).

  – Christiano and Rostagno (2002) and Atkeson, Chari and Kehoe (2007).

  – Loisel (2006). Similar mechanism to ours in a linear dynamic model.

- We show that it is possible to implement a unique equilibrium globally with an interest rate feedback rule.

A model with flexible prices

- Identical households, competitive firms, and a government.
- Preferences over consumption and leisure.
- The production uses labor only with a linear technology.
- There are shocks to productivity and government expenditures.
- Cash-in-advance constraint with the timing structure as in Lucas (1980).
- Lump-sum taxes.

Households

- Preferences:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(C_t, L_t\right) \right\}$$

- Budget constraints

$$M_t + B_t + E_t Q_{t,t+1} Z_{t+1} \leq \mathbb{W}_t,$$

$$\mathbb{W}_{t+1} = M_t + R_t B_t + Z_t - P_t C_t + W_t N_t - P_t T_t.$$

together with a terminal condition.

- Cash-in-advance constraint

$$P_t C_t \leq M_t$$

- Marginal conditions:

$$\frac{u_L(t)}{u_C(t)} = \frac{W_t}{R_t P_t}$$

$$\frac{u_C(t)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}} \right]$$

$$Q_{t,t+1} = \beta \frac{u_C(t+1)}{u_C(t)} \frac{P_t}{P_{t+1}}$$

Firms

- The firms are competitive and prices are flexible.
- Production function of the representative firm is linear

$$Y_t = A_t N_t$$

- The equilibrium real wage is

$$\frac{W_t}{P_t} = A_t$$

Government

- The policy variables are lump-sum taxes T_t, state-noncontingent interest rates R_t, state-contingent nominal returns $Q_{t,t+1}^{-1}$, money supplies M_t, state-noncontingent public debt B_t, state-contingent debt $Z_{t+1} = 0$.

- Policy: Maps sequences of quantities, prices and policy variables into sets of sequences of the policy variables.

Market clearing

$$C_t + G_t = A_t N_t,$$

$$1 - L_t = N_t.$$

Equilibrium

- Equilibrium conditions for the variables $\{C_t, L_t, R_t, M_t, P_t\}$ are:

$$C_t + G_t = A_t(1 - L_t),$$

$$\frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t},$$

(from these get $C_t = C(R_t)$, $L_t = L(R_t)$),

$$\frac{u_C\,(t)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}} \right],$$

$$P_t C_t \leq M_t.$$

- The equilibrium conditions for the variables $\{R_t, M_t, P_t\}$ can be written as:

$$\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[\frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}}\right], t \geq 0$$

$$P_t \leq \frac{M_t}{C(R_t)}, t \geq 0.$$

- Interest rate policy:

- Need to set prices (money supply) in every state at date T and, after that, in $\Phi_t - \Phi_{t-1}$ states for every $t \geq T+1$.

$$\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[\frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}} \right], \; t \geq 0$$

$$P_t \leq \frac{M_t}{C(R_t)}, \; t \geq 0$$

Feedback Rules

- Interest rate rules such that there is a unique equilibrium:

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}},$$

ξ_t is an exogenous variable.

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}}$$

- From

$$\frac{u_C\,(t)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}} \right]$$

get

$$\frac{u_C(t)}{P_t} = \xi_t,\ t \geq 0$$

and

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}},\ t \geq 0$$

- From

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}},\ t \geq 0,$$

$$\frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t},\ t \geq 0,$$

$$C_t + G_t = A_t(1 - L_t),\ t \geq 0,$$

and

$$\frac{u_C(t)}{P_t} = \xi_t,\ t \geq 0$$

determine the four variables R_t, C_t, L_t, P_t uniquely.

- The cash-in-advance conditions, if with equality, determine M_t.

- Does the policy rule resemble the rules followed by central banks?

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}},$$

- Depending on the exogenous process ξ_t, can implement each allocation in a set of implementable allocations, including the (Friedman rule) optimal allocation.

- Define a set of implementable equilibria where the sequences of policy variables can be any sequences that satisfy the government budget constraint.

- The interest rate rule can be used to implement uniquely each implementable equilibrium in that set.

- With

$$\xi_t = \frac{1}{k\beta^t}, t \geq 0,$$

where k is a positive constant, and

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}$$

get

$$R_t = \frac{\frac{1}{k\beta^t}}{\beta E_t \frac{1}{k\beta^{t+1}}} = 1.$$

- Let $C_t = C^*(A_t, G_t)$, $L_t = L^*(A_t, G_t)$ be the first best allocation:

$$\frac{u_C(t)}{u_L(t)} = \frac{1}{A_t}, \ t \geq 0,$$

$$C_t + G_t = A_t(1 - L_t), \ t \geq 0,$$

- The price level is given by

$$\frac{u_C(t)}{P_t} = \xi_t, \ t \geq 0$$

or

$$\frac{u_C(C^*(A_t, G_t), L^*(A_t, G_t))}{P_t} = \frac{1}{k\beta^t}, \ t \geq 0.$$

- The equilibrium money stock is obtained using the cash-in-advance constraint if it holds with equality.

- There are other possible equilibrium processes for the path of the price level associated with the Friedman rule. The rule with

$$\xi_t = \frac{\mu_t}{k\left(\rho\beta\right)^t},$$

where

$$\mu_t = \rho\mu_{t-1} + \varepsilon_t$$

and ε_t is a white noise, also has

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}} = \frac{\frac{\mu_t}{k(\rho\beta)^t}}{\beta E_t \frac{\rho\mu_t}{k(\rho\beta)^{t+1}}} = 1$$

and achieves the first best allocation with different processes for the price level depending on the choice of k, ρ and ε_t.

Robustness:
Capital

- Intertemporal condition

$$\frac{u_C(t)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}} \right]$$

- Interest rate rule

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}},$$

- Get

$$\frac{u_C(t)}{P_t} = \xi_t,\ t \geq 0,$$

and

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}.$$

- Once the sequence of nominal interest rates R_t is determined, the allocations in the model with capital are also uniquely determined and then the price level is also determined uniquely.

Sticky prices: Prices set in advance

- Continuum of firms, indexed by $i \in [0,1]$, each producing a differentiated good also indexed by i. The firms are monopolistic competitive and set prices in advance with different lags.

- C_t is now the composite good

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \theta > 1,$$

  $c_t(i)$ is consumption of good i.

- The demand function for each good i is

$$c_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} C_t,$$

  $p_t(i)$ is the price of good i and P_t is the price level,

$$P_t = \left[\int p_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}.$$

- The households' intertemporal and intratemporal conditions are as before.

- The government must finance an exogenous path of government purchases $\{G_t\}_{t=0}^{\infty}$, such that

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 0.$$

and minimizes expenditures on G_t, so that

$$\frac{g_t(i)}{G_t} = \left(\frac{p_t(i)}{P_t} \right)^{-\theta}.$$

- A fraction α_j firms set prices j periods in advance with $j = 0, ...J-1$. Firms decide the price for period t with the information up to period $t-j$ to maximize profits:

$$E_{t-j}\left\{Q_{t-j,t+1}\left[p_t(i)y_t(i) - W_t n_t(i)\right]\right\},$$

subject to

$$y_t(i) \le A_t n_t(i)$$

and

$$y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t,$$

where $y_t(i) = c_t(i) + g_t(i)$ and $Y_t = C_t + G_t$.

- The optimal price for a firm setting the price for period t, j periods in advance, is

$$p_t(i) \equiv p_{t,j} = \frac{\theta}{(\theta-1)} E_{t-j}\left[\eta_{t,j}\frac{W_t}{A_t}\right],$$

where

$$\eta_{t,j} = \frac{Q_{t-j,t+1}P_t^{\theta}Y_t}{E_{t-j}\left[Q_{t-j,t+1}P_t^{\theta}Y_t\right]}.$$

- Substituting the state contingent prices $Q_{t-j,t+1}$ in the price setting conditions, and using the intertemporal condition as well as the households' intratemporal condition,

$$E_{t-j}\left[\frac{u_C(t)}{R_t}P_t^{\theta-1}A_t\left(1-L_t\right)-\frac{\theta}{(\theta-1)}u_L(t)P_t^{\theta-1}\left(1-L_t\right)\frac{P_t}{p_{t,j}}\right]=0,\ j=0,...J-1.$$

- Under flexible prices, this condition is

$$\frac{u_C(t)}{u_L(t)}=\frac{1}{A_t}.$$

- Market clearing

$$c_t(i) + g_t(i) = A_t n_t(i),$$

$$\int_0^1 n_t(i) di = N_t.$$

- Aggregate resource constraints

$$(C_t + G_t) \int_0^1 \left(\frac{p_t\,(i)}{P_t} \right)^{-\theta} di = A_t N_t.$$

- When prices are set in advance, if policy is conducted with the interest rate feedback rule

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}},$$

where ξ_t is an exogenous variable, there is a unique equilibrium.

$$\frac{u_C(t)}{P_t} = \xi_t,\ t \geq 0$$

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}},\ t \geq 0$$

These conditions together with

$$(C_t + G_t) \sum_{j=0}^{J-1} \alpha_j \left(\frac{p_{t,j}}{P_t} \right)^{-\theta} = A_t N_t,$$

$$E_{t-j} \left[\frac{u_C(t)}{R_t} P_t^{\theta-1} A_t \left(1 - L_t\right) - \frac{\theta}{(\theta-1)} u_L(t) P_t^{\theta-1} \left(1 - L_t\right) \frac{P_t}{p_{t,j}} \right] = 0,\ j = 0, ...J{-}1,$$

$$P_t = \left[\sum_{j=0}^{J-1} \alpha_j \left(p_{t,j}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

and the cash-in-advance constraints (if with equality), determine uniquely the variables R_t, C_t, L_t, P_t, $p_{t,j}$, $j = 0, ...J-1$, and M_t. $p_{0,j}$, $j = 1, ...J-1$ are exogenous.

Calvo (1983) staggered prices

- Standard newkeynesian model.
- Exogenous velocity made arbitrarily large, so that it is a cashless economy.
- Log-linearized model.
- Loisel (2006).

- Cashless economies

$$\frac{P_t C_t}{v_t} \leq M_t,$$

where $v_t \to \infty$.

- In the limit case, the households conditions are

$$\frac{u_C(t)}{u_L(t)} = \frac{P_t}{W_t},\ t \geq 0,$$

and

$$\frac{u_C\left(t\right)}{P_t} = E_t\left[R_{t+1}\frac{\beta u_C(t+1)}{P_{t+1}}\right].$$

- Calvo pricing
- In each period, a fraction $1-\alpha$ of firms can choose optimally their prices, p_t^*.
- Optimal price

$$\frac{p_t^*}{P_t} = \frac{\theta}{\theta-1}\frac{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{u_c(t+j+1)P_t}{P_{t+j+1}} s_{t+j} \left(\frac{P_{t+j}}{P_t}\right)^{1+\theta} Y_{t+j}}{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{u_c(t+j+1)P_t}{P_{t+j+1}} \left(\frac{P_{t+j}}{P_t}\right)^{\theta} Y_{t+j}}.$$

$s_{t+j} = \frac{W_{t+j}}{A_{t+j}P_{t+j}}$ is the real marginal cost.

- The expression for the price level is

$$P_t^{1-\theta} = (1-\alpha)\, p_t^{*1-\theta} + \alpha \left(P_{t-1}\right)^{1-\theta}$$

- Loglinearize around a steady-state with zero inflation. Let $\pi_t = \frac{P_t}{P_{t-1}}$.
- Price setting condition:

$$\widehat{\pi}_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\widehat{s}_t + \beta E_t \widehat{\pi}_{t+1}$$

where

$$\widehat{s}_t = \widehat{\varpi}_t - \widehat{A}_t$$

- The loglinearization of the intratemporal and intertemporal conditions gives

$$\phi_c \widehat{C}_t + \phi_L \widehat{L}_t = \widehat{\varpi}_t$$

where $\phi_x = \frac{\partial \frac{u_L(t)}{u_C(t)}}{\partial x} \frac{u_C(t)}{u_L(t)} x$, $x = C, L$, and

$$E_t \widehat{R}_{t+1} - E_t (\widehat{\pi}_{t+1}) = \widehat{r}_t$$

where

$$\widehat{r}_t \equiv -\left(\frac{u_{cc} C}{u_c}\right) E_t \left(\widehat{C}_{t+1} - \widehat{C}_t\right) - \left(\frac{u_{cl} L}{u_c}\right) E_t \left(\widehat{L}_{t+1} - \widehat{L}_t\right).$$

- The loglinearization of the resource constraints

$$(C_t + G_t) \int_0^1 \left(\frac{p_t(i)}{P_t} \right)^{-\theta} di = A_t N_t$$

is

$$\frac{C}{Y}\widehat{C}_t + \frac{G}{Y}\widehat{G}_t = \widehat{A}_t + \widehat{N}_t,$$

with

$$\widehat{L}_t = -\kappa \widehat{N}_t$$

where $\kappa = \frac{N}{L}$.

- Consider the following interest rate rule

$$\widehat{R}_{t+1} = \widehat{r}_t + \widehat{P}_{t+1} - \xi_t$$

where ξ_t is an exogenous process.

- Together with the Euler condition

$$E_t \widehat{R}_{t+1} - E_t \left(\widehat{\pi}_{t+1} \right) = \widehat{r}_t,$$

gives

$$\widehat{P}_t = \xi_t,$$

so that the price level is uniquely pinned down.

- Since $\widehat{\pi}_{t+1} = \widehat{P}_{t+1} - \widehat{P}_t = \xi_{t+1} - \xi_t$, from the Phillips curve

$$\widehat{\pi}_t = \lambda \left(\widehat{\varpi}_t - \widehat{A}_t \right) + \beta E_t \widehat{\pi}_{t+1}$$

determine $\widehat{\varpi}_t$ uniquely.

- From the intratemporal and resource constraints, get $\widehat{L}_t$, $\widehat{N}_t$ and $\widehat{C}_t$:

$$\phi_c \widehat{C}_t - \phi_L \kappa \widehat{N}_t = \widehat{\varpi}_t$$

$$\frac{C}{Y}\widehat{C}_t + \frac{G}{Y}\widehat{G}_t = \widehat{A}_t + \widehat{N}_t,$$

$$\widehat{L}_t = -\kappa \widehat{N}_t$$

- This pins down $\widehat{r}_t$

$$\widehat{r}_t \equiv -\left(\frac{u_{cc}C}{u_c}\right) E_t\left(\widehat{C}_{t+1} - \widehat{C}_t\right) - \left(\frac{u_{cl}L}{u_c}\right) E_t\left(\widehat{L}_{t+1} - \widehat{L}_t\right).$$

and, therefore, $\widehat{R}_{t+1}$

$$E_t \widehat{R}_{t+1} - E_t\left(\widehat{\pi}_{t+1}\right) = \widehat{r}_t.$$

- In order to implement the equilibrium with zero inflation, need $\xi_t = 0$. Then $\widehat{P}_t = 0$, so that inflation is the steady state zero inflation. From

$$\widehat{\pi}_t = \lambda \left(\widehat{\varpi}_t - \widehat{A}_t \right) + \beta E_t \widehat{\pi}_{t+1},$$

have $\widehat{\varpi}_t = \widehat{A}_t$, as under flexible prices.

- The rule is the same independently of the price setting restrictions.
- $\widehat{P}_t = \xi_t$. Under flexible prices we have, instead of the Phillips curve,

$$\widehat{\varpi}_t = \widehat{A}_t$$

The allocations, $\widehat{L}_t$, $\widehat{N}_t$ and $\widehat{C}_t$, are determined uniquely.

Concluding remarks.

- Interest rate rules can implement unique local equilibria with stable prices. These are normally associated with multiple global equilibria.
- One way out is the rule proposed in this paper.

Extra-credit

- Backward interest rate feedback rule:

$$\widehat{R}_t = \widehat{r}_t + \tau \widehat{\pi}_{t-1},$$

  the dynamic equation is

$$\tau \widehat{\pi}_{t-1} - E_t \left(\widehat{\pi}_{t+1} \right) = 0.$$

  - If $\widehat{\pi}_{-1} = 0$, there is a solution with $\widehat{\pi}_t = 0$ all t.
  - There are again multiple solutions and a locally determinate solution, $\widehat{\pi}_t = 0$, with $\tau > 1$, provided $\widehat{\pi}_{-1} = 0$.

Money supply rules

Proposition 1 *Suppose the cash-in-advance constraint holds exactly. Every equilibrium in Definition 1 can be implemented (uniquely) with the money supply feedback rule,*

$$M_t = \frac{C_t u_C(t)}{\xi_t},$$

where ξ_t is an exogenous variable.

- Using the cash in advance conditions with equality,

$$\frac{u_C(t)}{P_t} = \xi_t$$

- Using the intertemporal conditions,

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}.$$

This, together with the intratemporal conditions and the resource constraints, determine C_t, L_t, P_t, R_t, all $t \geq 0$ and s^t.

Finite vs infinite horizon

- Equilibrium in a finite horizon economy
- Equilibrium conditions for the variables $\{C_t, L_t, R_t, M_t, P_t\}$ are:

$$C_t + G_t = A_t(1 - L_t),$$
$$\frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t},$$

for $0 \leq t \leq T$ (from these get $C_t = C(R_t)$, $L_t = L(R_t)$);

$$P_t C_t \leq M_t,$$

for $0 \leq t \leq T$ (which gives $P_t \leq \frac{M_t}{C(R_t)}$);

$$\frac{u_C\left(t\right)}{P_t} = R_t E_t\left[\frac{\beta u_C(t+1)}{P_{t+1}}\right],$$

for $0 \leq t \leq T - 1$.

The equilibrium conditions for the variables $\{R_t, M_t, P_t\}$ are:

$$\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[\frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}}\right],$$
$$0 \leq t \leq T-1$$

$$P_t \leq \frac{M_t}{C(R_t)}, \; 0 \leq t \leq T$$

Interest Rate Policy.

- Interest rates are set in every date and state.

  There is a unique equilibrium if prices (money supply?) are set in every state at date T

$$\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[\frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}} \right],$$
$$0 \leq t \leq T - 1$$

$$P_T \leq \frac{M_T}{C(R_T)}$$

- Deterministic economy.
- Uncertainty. Need a nominal anchor for every history.

- Arbitrarily large time horizon.
- Does it matter whether policy is conducted with an interest rate or a money supply rule?
- Does it matter which particular feedback rule is used?
- Does it matter whether prices are flexible or sticky?
- Preferences and technology?

Infinite horizon

- Need to set prices (money supply) in every state at date T and, after that, in $\Phi_t - \Phi_{t-1}$ states for every $t \geq T+1$.

$$\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[\frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}}\right], \, t \geq 0$$

$$P_t \leq \frac{M_t}{C(R_t)}, \, t \geq 0$$

- If policy is a money supply rule and

$$P_t = \frac{M_t}{C(R_t)}, \, t \geq 0,$$

then

$$\frac{u_C(C(R_t), L(R_t))}{\frac{M_t}{C(R_t)}} = \beta R_t E_t \left[\frac{u_C(C(R_{t+1}), L(R_{t+1}))}{\frac{M_{t+1}}{C(R_{t+1})}}\right], \, t \geq 0$$

- In the infinite horizon, preferences are relevant:
- The utility function is additively separable and logarithmic in consumption.

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln \left(C_t \right) + v(L_t) \right] \right\}$$

- Equilibrium conditions:

$$\frac{1}{M_t} = \beta R_t E_t \frac{1}{M_{t+1}}$$

$$P_t = \frac{M_t}{C(R_t)}$$

$$C_t = C(R_t)$$

$$L_t = L(R_t)$$

- In the finite horizon economy:

$$\frac{1}{M_t} = \beta R_t E_t \left[\frac{1}{M_{t+1}} \right], \, t = 0, ..., T-1$$

When the money supply is set exogenously in every state the nominal interest rates, R_t, are determined for $t = 0, ..., T-1$. It is still necessary to set exogenously the interest rates, R_T, in every terminal state.