Unique Monetary Equilibria with Interest Rate Rules*

Bernardino Adão  
Banco de Portugal  

Isabel Correia  
Banco de Portugal, Universidade Catolica Portuguesa and CEPR  

Pedro Teles  
Banco de Portugal, Universidade Catolica Portuguesa, and CEPR  


Abstract

In contrast to previous literature, we show that there are interest rate rules that implement unique global equilibria in standard monetary models. This is a contribution to a literature that either concentrates on conditions for local determinacy, or criticizes that approach showing that local determinacy might be associated with global indeterminacy. The interest rate

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Question

• How can monetary policy implement unique equilibria?
  (Can interest rate policy provide a nominal anchor?)

• Apparent success with inflation targeting.
  
  – Success is attributed to a Taylor rule.
  
  – In monetary models, Taylor rules do not pin down unique equilibria.
    
    * Equilibria may be locally unique, but not globally so.
A simple endowment economy

• Euler equation for the representative household:

\[
\frac{u_c(Y_t)}{P_t} = R_t E_t \beta u_c(Y_{t+1}) \frac{P_{t+1}}{P_t}
\]

\{Y_t\} is the endowment process.

• In log deviations from a deterministic steady state with constant inflation \(\pi^*\):

\[
\hat{R}_t = \hat{r}_t + E_t \hat{P}_{t+1} - \hat{P}_t,
\]

where \(r_t = \frac{u_c(Y_t)}{\beta E_t u_c(Y_{t+1})}\).
Monetary policy in the endowment economy

- Interest rate target:
  \[ \hat{R}_t = \hat{R}_t^* \]
  \[ \hat{R}_t = \hat{r}_t + E_t \hat{\pi}_{t+1} \]

  - Unique path for the conditional expectation of inflation \( E_t \hat{\pi}_{t+1} \),
  - but not for the initial price level, nor the distribution of realized inflation across states.
• Current feedback rule:

\[ \hat{R}_t = \hat{r}_t + \tau \hat{\pi}_t \]

\[ \hat{R}_t = \hat{r}_t + E_t \hat{\pi}_{t+1} \]

• Equilibria:

\[ \tau \hat{\pi}_t - E_t (\hat{\pi}_{t+1}) = 0. \]

  – Equilibrium with \( \hat{\pi}_t = 0 \) and \( \hat{R}_t = \hat{r}_t \).

  – Multiple other solutions:

    * If \( \tau > 1 \) (Taylor principle): Continuum of divergent solutions. The equilibrium with \( \hat{\pi}_t = 0 \) is locally unique.

    * If \( \tau < 1 \): Continuum of solutions converging to \( \hat{\pi}_t = 0 \).
• Forward looking rules:

\[ \hat{R}_t = \hat{r}_t + \tau E_t \hat{\pi}_{t+1} \]

\[ \hat{R}_t = \hat{r}_t + E_t \hat{\pi}_{t+1} \]

• Equilibria:

\[ (\tau - 1) E_t (\hat{\pi}_{t+1}) = 0 \]

• For \( \tau \neq 1 \), only expected inflation is pinned down, not the distribution of prices across states.
• Wicksellian interest rate rules (Woodford, 2003) have the interest rate respond to the price level rather than inflation.

• Policy rule:

\[ \hat{R}_t = \hat{r}_t + \phi \hat{P}_t, \]

with \( \phi > 0 \).

\[ \hat{R}_t = \hat{r}_t + E_t \hat{P}_{t+1} - \hat{P}_t, \]

• Equilibria:

\[ (1 + \phi) \hat{P}_t - E_t \hat{P}_{t+1} = 0 \]

– Equilibrium with \( \hat{P}_t = 0 \) and \( \hat{R}_t = \hat{r}_t \).

– The equilibrium with \( \hat{P}_t = 0 \) is locally unique if \( \phi > 0 \).

– Continuum of divergent solutions.
Unique solution for the dynamic equation?

- Price level targeting rule:

\[ \hat{R}_t = \hat{r}_t + E_t \hat{P}_{t+1} + \hat{\xi}_t \]

where \( \hat{\xi}_t \) is an exogenous random variable.

\[ \hat{R}_t = \hat{r}_t + E_t \hat{P}_{t+1} - \hat{P}_t, \]

- Equilibria

\[ \hat{P}_t = \hat{\xi}_t. \]
Massive literature starting with Sargent and Wallace (1975) and McCallum (1981), including recent literature on local and global determinacy in models with nominal rigidities:

- Conditions for a unique local equilibrium.

- Conditions for local determinacy may be conditions for global indeterminacy.


• We show that it is possible to implement a unique equilibrium globally with an interest rate feedback rule.
A model with flexible prices

- Identical households, competitive firms, and a government.
- Preferences over consumption and leisure.
- The production uses labor only with a linear technology.
- There are shocks to productivity and government expenditures.
- Cash-in-advance constraint with the timing structure as in Lucas (1980).
- Lump-sum taxes.
Households

- Preferences:

\[ U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right\} \]

- Budget constraints

\[ M_t + B_t + E_t Q_{t,t+1} Z_{t+1} \leq W_t, \]

\[ W_{t+1} = M_t + R_t B_t + Z_t - P_t C_t + W_t N_t - P_t T_t. \]

together with a terminal condition.

- Cash-in-advance constraint

\[ P_t C_t \leq M_t \]
- Marginal conditions:

\[
\frac{u_L(t)}{u_C(t)} = \frac{W_t}{R_t P_t}
\]

\[
\frac{u_C(t)}{P_t} = R_t E_t \left[ \frac{\beta u_C(t + 1)}{P_{t+1}} \right]
\]

\[
Q_{t,t+1} = \beta \frac{u_C(t + 1)}{u_C(t)} \frac{P_t}{P_{t+1}}
\]
Firms

- The firms are competitive and prices are flexible.
- Production function of the representative firm is linear
  \[ Y_t = A_t N_t \]
- The equilibrium real wage is
  \[ \frac{W_t}{P_t} = A_t \]
Government

- The policy variables are lump-sum taxes $T_t$, state-noncontingent interest rates $R_t$, state-contingent nominal returns $Q_{t,t+1}^{-1}$, money supplies $M_t$, state-noncontingent public debt $B_t$, state-contingent debt $Z_{t+1} = 0$.

- Policy: Maps sequences of quantities, prices and policy variables into sets of sequences of the policy variables.
Market clearing

\[ C_t + G_t = A_t N_t, \]

\[ 1 - L_t = N_t. \]
Equilibrium

- Equilibrium conditions for the variables \( \{C_t, L_t, R_t, M_t, P_t\} \) are:

\[
C_t + G_t = A_t (1 - L_t),
\]

\[
\frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t},
\]

(from these get \( C_t = C(R_t), \ L_t = L(R_t) \)),

\[
\frac{u_C(t)}{P_t} = R_t E_t \left[ \frac{\beta u_C(t + 1)}{P_{t+1}} \right],
\]

\[
P_t C_t \leq M_t.
\]
• The equilibrium conditions for the variables \( \{R_t, M_t, P_t\} \) can be written as:

\[
\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[ \frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}} \right], \quad t \geq 0
\]

\[
P_t \leq \frac{M_t}{C(R_t)}, \quad t \geq 0.
\]
• Interest rate policy:

• Need to set prices (money supply) in every state at date $T$ and, after that, in $\Phi_t - \Phi_{t-1}$ states for every $t \geq T + 1$.

$$
\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[ \frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}} \right], \quad t \geq 0
$$

$$
P_t \leq \frac{M_t}{C(R_t)}, \quad t \geq 0
$$
Feedback Rules

- Interest rate rules such that there is a unique equilibrium:

\[ R_t = \frac{\xi_t}{E_t^{\beta u_C(t+1)}}, \]

\( \xi_t \) is an exogenous variable.
\[ R_t = \frac{\xi_t}{E_t \beta u_C(t+1)} \]

- From

\[ \frac{u_C(t)}{P_t} = R_t E_t \left[ \frac{\beta u_C(t+1)}{P_{t+1}} \right] \]

get

\[ \frac{u_C(t)}{P_t} = \xi_t, \ t \geq 0 \]

and

\[ R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}, \ t \geq 0 \]
• From

\[ R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}, \quad t \geq 0, \]

\[ \frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t}, \quad t \geq 0, \]

\[ C_t + G_t = A_t(1 - L_t), \quad t \geq 0, \]

and

\[ \frac{u_C(t)}{P_t} = \xi_t, \quad t \geq 0 \]

determine the four variables \( R_t, C_t, L_t, P_t \) uniquely.

• The cash-in-advance conditions, if with equality, determine \( M_t \).
• Does the policy rule resemble the rules followed by central banks?

\[ R_t = \frac{\xi_t}{E_t \beta u_C(t+1)} \cdot \frac{1}{P_{t+1}} \]

• Depending on the exogenous process \( \xi_t \), can implement each allocation in a set of implementable allocations, including the (Friedman rule) optimal allocation.
• Define a set of implementable equilibria where the sequences of policy variables can be any sequences that satisfy the government budget constraint.

• The interest rate rule can be used to implement uniquely each implementable equilibrium in that set.
• With
\[ \xi_t = \frac{1}{k\beta^t}, \quad t \geq 0, \]
where \( k \) is a positive constant, and
\[ R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}} \]
get
\[ R_t = \frac{1}{\beta E_t \frac{1}{k\beta^{t+1}}} = 1. \]
• Let \( C_t = C^*(A_t, G_t) \), \( L_t = L^*(A_t, G_t) \) be the first best allocation:

\[
\frac{u_C(t)}{u_L(t)} = \frac{1}{A_t}, \ t \geq 0,
\]

\[
C_t + G_t = A_t(1 - L_t), \ t \geq 0,
\]

• The price level is given by

\[
\frac{u_C(t)}{P_t} = \xi_t, \ t \geq 0
\]

or

\[
\frac{u_C(C^*(A_t, G_t), L^*(A_t, G_t))}{P_t} = \frac{1}{k^{\beta t}}, \ t \geq 0.
\]

• The equilibrium money stock is obtained using the cash-in-advance constraint if it holds with equality.
• There are other possible equilibrium processes for the path of the price level associated with the Friedman rule. The rule with

\[ \xi_t = \frac{\mu_t}{k(\rho\beta)^t}, \]

where

\[ \mu_t = \rho\mu_{t-1} + \varepsilon_t \]

and \( \varepsilon_t \) is a white noise, also has

\[ R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}} = \frac{\mu_t}{k(\rho\beta)^t} = 1 \]

and achieves the first best allocation with different processes for the price level depending on the choice of \( k, \rho \) and \( \varepsilon_t \).
Robustness:
Capital

• Intertemporal condition

\[
\frac{u_C(t)}{P_t} = R_t E_t \left[ \beta u_C(t+1) \right] / P_{t+1}
\]

• Interest rate rule

\[
R_t = \frac{\xi_t}{E_t \beta u_C(t+1) / P_{t+1}}
\]

• Get

\[
\frac{u_C(t)}{P_t} = \xi_t, \quad t \geq 0,
\]

and

\[
R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}.
\]

• Once the sequence of nominal interest rates $R_t$ is determined, the allocations in the model with capital are also uniquely determined and then the price level is also determined uniquely.
Sticky prices: Prices set in advance

- Continuum of firms, indexed by $i \in [0, 1]$, each producing a differentiated good also indexed by $i$. The firms are monopolistic competitive and set prices in advance with different lags.

- $C_t$ is now the composite good

$$C_t = \left[ \int_0^1 c_t(i)^{\theta-1} \frac{\theta}{\theta-1} \, di \right]^{\frac{\theta}{\theta-1}}, \theta > 1,$$

$c_t(i)$ is consumption of good $i$.

- The demand function for each good $i$ is

$$c_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} C_t,$$

$p_t(i)$ is the price of good $i$ and $P_t$ is the price level,

$$P_t = \left[ \int p_t(i)^{1-\theta} \, di \right]^{\frac{1}{1-\theta}}.$$
• The households’ intertemporal and intratemporal conditions are as before.

• The government must finance an exogenous path of government purchases \( \{G_t\}_{t=0}^{\infty} \), such that

\[
G_t = \left[ \int_0^1 g_t(i) \frac{\theta-1}{\theta} di \right]^{\frac{\theta}{\theta-1}}, \theta > 0.
\]

and minimizes expenditures on \( G_t \), so that

\[
\frac{g_t(i)}{G_t} = \left( \frac{p_t(i)}{P_t} \right)^{-\theta}.
\]
• A fraction $\alpha_j$ firms set prices $j$ periods in advance with $j = 0, \ldots, J - 1$. Firms decide the price for period $t$ with the information up to period $t - j$ to maximize profits:

$$E_{t-j} \{ Q_{t-j,t+1} [p_t(i)y_t(i) - W_t n_t(i)] \} ,$$

subject to

$$y_t(i) \leq A_t n_t(i)$$

and

$$y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t,$$

where $y_t(i) = c_t(i) + g_t(i)$ and $Y_t = C_t + G_t$.

• The optimal price for a firm setting the price for period $t$, $j$ periods in advance, is

$$p_t(i) \equiv p_{t,j} = \frac{\theta}{(\theta - 1)} E_{t-j} \left[ \eta_{t,j} \frac{W_t}{A_t} \right] ,$$

where

$$\eta_{t,j} = \frac{Q_{t-j,t+1} P^\theta_t Y_t}{E_{t-j} \left[ Q_{t-j,t+1} P^\theta_t Y_t \right]} .$$
• Substituting the state contingent prices $Q_{t-j,t+1}$ in the price setting conditions, and using the intertemporal condition as well as the households’ intratemporal condition,

$$E_{t-j} \left[ \frac{u_C(t)}{R_t} P_t^{\theta-1} A_t (1 - L_t) - \frac{\theta}{(\theta - 1)} u_L(t) P_t^{\theta-1} (1 - L_t) \frac{P_t}{p_{t,j}} \right] = 0, \; j = 0, \ldots, J-1.$$  

• Under flexible prices, this condition is

$$\frac{u_C(t)}{u_L(t)} = \frac{1}{A_t}.$$
• Market clearing

\[ c_t(i) + g_t(i) = A_t n_t(i), \]
\[ \int_0^1 n_t(i) di = N_t. \]

• Aggregate resource constraints

\[ (C_t + G_t) \int_0^1 \left( \frac{p_t(i)}{P_t} \right)^{-\theta} di = A_t N_t. \]
• When prices are set in advance, if policy is conducted with the interest rate feedback rule

\[ R_t = \frac{\xi_t}{E_t\beta u_C(t+1)} \frac{1}{P_{t+1}} \]

where \( \xi_t \) is an exogenous variable, there is a unique equilibrium.
\[
\frac{u_C(t)}{P_t} = \xi_t, \quad t \geq 0
\]
\[
R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}, \quad t \geq 0
\]

These conditions together with
\[
(C_t + G_t) \sum_{j=0}^{J-1} \alpha_j \left( \frac{p_{t,j}}{P_t} \right)^{-\theta} = A_t N_t,
\]
\[
E_{t-j} \left[ \frac{u_C(t)}{R_t} P_t^{\theta-1} A_t (1 - L_t) - \frac{\theta}{(\theta - 1)} u_L(t) P_t^{\theta-1} (1 - L_t) \frac{P_t}{p_{t,j}} \right] = 0, \quad j = 0, \ldots, J-1,
\]
\[
P_t = \left[ \sum_{j=0}^{J-1} \alpha_j (p_{t,j})^{1-\theta} \right]^{\frac{1}{1-\theta}}.
\]

and the cash-in-advance constraints (if with equality), determine uniquely the variables \( R_t, C_t, L_t, P_t, p_{t,j}, j = 0, \ldots, J-1, \) and \( M_t. \) \( p_{0,j}, j = 1, \ldots, J-1 \) are exogenous.
Calvo (1983) staggered prices

- Standard new Keynesian model.
- Exogenous velocity made arbitrarily large, so that it is a cashless economy.
- Log-linearized model.
• Cashless economies

\[
\frac{P_tC_t}{v_t} \leq M_t,
\]

where \(v_t \to \infty\).

• In the limit case, the households conditions are

\[
u_C(t) \over u_L(t) = \frac{P_t}{W_t}, \quad t \geq 0,
\]

and

\[
\frac{u_C(t)}{P_t} = E_t \left[ R_{t+1} \frac{\beta u_C(t+1)}{P_{t+1}} \right].
\]
• Calvo pricing

• In each period, a fraction $1 - \alpha$ of firms can choose optimally their prices, $p_t^*$.

• Optimal price

$$\frac{p_t^*}{P_t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{j=0}^{\infty} (\alpha/\beta)^j u_c(t+j+1)P_t}{E_t \sum_{j=0}^{\infty} (\alpha/\beta)^j u_c(t+j+1)P_t} \left( \frac{P_{t+j}}{P_t} \right)^{1+\theta} Y_{t+j}.$$

$s_{t+j} = \frac{W_{t+j}}{A_{t+j}P_{t+j}}$ is the real marginal cost.

• The expression for the price level is

$$P_t^{1-\theta} = (1 - \alpha) p_t^{1-\theta} + \alpha (P_{t-1})^{1-\theta}$$
Loglinearize around a steady-state with zero inflation. Let $\pi_t = \frac{P_t}{P_{t-1}}$.

Price setting condition:

$$\hat{\pi}_t = \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \hat{s}_t + \beta E_t \hat{\pi}_{t+1}$$

where

$$\hat{s}_t = \hat{\omega}_t - \hat{A}_t$$

The loglinearization of the intratemporal and intertemporal conditions gives

$$\phi_c \hat{C}_t + \phi_L \hat{L}_t = \hat{\omega}_t$$

where $\phi_x = \frac{\partial u_L(t)}{\partial x} u_L(t) x$, $x = C, L$, and

$$E_t \hat{R}_{t+1} - E_t (\hat{\pi}_{t+1}) = \hat{r}_t$$

where

$$\hat{r}_t \equiv - \left( \frac{u_{cc}^C}{u_c} \right) E_t \left( \hat{C}_{t+1} - \hat{C}_t \right) - \left( \frac{u_{cl}L}{u_c} \right) E_t \left( \hat{L}_{t+1} - \hat{L}_t \right).$$
• The loglinearization of the resource constraints

\[(C_t + G_t) \int_0^1 \left( \frac{p_t(i)}{P_t} \right)^{-\theta} \, di = A_t N_t \]

is

\[\frac{C}{Y} \hat{C}_t + \frac{G}{Y} \hat{G}_t = \hat{A}_t + \hat{N}_t,\]

with

\[\hat{L}_t = -\kappa \hat{N}_t\]

where \(\kappa = \frac{N}{L}\).
• Consider the following interest rate rule

\[ \hat{R}_{t+1} = \hat{r}_t + \hat{P}_{t+1} - \xi_t \]

where \( \xi_t \) is an exogenous process.

• Together with the Euler condition

\[ E_t \hat{R}_{t+1} - E_t (\hat{\pi}_{t+1}) = \hat{r}_t, \]

gives

\[ \hat{P}_t = \xi_t, \]

so that the price level is uniquely pinned down.

• Since \( \hat{\pi}_{t+1} = \hat{P}_{t+1} - \hat{P}_t = \xi_{t+1} - \xi_t \), from the Phillips curve

\[ \hat{\pi}_t = \lambda \left( \hat{\omega}_t - \hat{A}_t \right) + \beta E_t \hat{\pi}_{t+1} \]

determine \( \hat{\omega}_t \) uniquely.
• From the intratemporal and resource constraints, get \( \hat{L}_t, \hat{N}_t \) and \( \hat{C}_t \):

\[
\phi_c \hat{C}_t - \phi_L \kappa \hat{N}_t = \hat{\omega}_t
\]

\[
\frac{C}{Y} \hat{C}_t + \frac{G}{Y} \hat{G}_t = \hat{A}_t + \hat{N}_t,
\]

\[
\hat{L}_t = -\kappa \hat{N}_t
\]

• This pins down \( \hat{r}_t \)

\[
\hat{r}_t \equiv -\left( \frac{u_{cc} C'}{u_c} \right) E_t \left( \hat{C}_{t+1} - \hat{C}_t \right) - \left( \frac{u_c L}{u_c} \right) E_t \left( \hat{L}_{t+1} - \hat{L}_t \right).
\]

and, therefore, \( \hat{R}_{t+1} \)

\[
E_t \hat{R}_{t+1} - E_t (\hat{\pi}_{t+1}) = \hat{r}_t.
\]
In order to implement the equilibrium with zero inflation, need $\xi_t = 0$. Then $\hat{P}_t = 0$, so that inflation is the steady state zero inflation. From

$$\hat{\pi}_t = \lambda (\hat{\omega}_t - \hat{A}_t) + \beta E_t \hat{\pi}_{t+1},$$

have $\hat{\omega}_t = \hat{A}_t$, as under flexible prices.
• The rule is the same independently of the price setting restrictions.

• \( \hat{P}_t = \xi_t \). Under flexible prices we have, instead of the Phillips curve,

\[
\hat{\omega}_t = \hat{A}_t
\]

The allocations, \( \hat{L}_t, \hat{N}_t \) and \( \hat{C}_t \), are determined uniquely.
Concluding remarks.

- Interest rate rules can implement unique local equilibria with stable prices. These are normally associated with multiple global equilibria.

- One way out is the rule proposed in this paper.
Extra-credit

• Backward interest rate feedback rule:

\[ \hat{R}_t = \hat{r}_t + \tau \hat{\pi}_{t-1}, \]

the dynamic equation is

\[ \tau \hat{\pi}_{t-1} - E_t (\hat{\pi}_{t+1}) = 0. \]

− If \( \hat{\pi}_{-1} = 0 \), there is a solution with \( \hat{\pi}_t = 0 \) all \( t \).

− There are again multiple solutions and a locally determinate solution, \( \hat{\pi}_t = 0 \), with \( \tau > 1 \), provided \( \hat{\pi}_{-1} = 0 \).
Money supply rules

Proposition 1 Suppose the cash-in-advance constraint holds exactly. Every equilibrium in Definition 1 can be implemented (uniquely) with the money supply feedback rule,

\[ M_t = \frac{C_t u_C(t)}{\xi_t}, \]

where \( \xi_t \) is an exogenous variable.

- Using the cash in advance conditions with equality,

\[ \frac{u_C(t)}{P_t} = \xi_t \]

- Using the intertemporal conditions,

\[ R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}. \]

This, together with the intratemporal conditions and the resource constraints, determine \( C_t, L_t, P_t, R_t \), all \( t \geq 0 \) and \( s^t \).
Finite vs infinite horizon

- Equilibrium in a finite horizon economy
- Equilibrium conditions for the variables \( \{C_t, L_t, R_t, M_t, P_t\} \) are:

\[
C_t + G_t = A_t(1 - L_t),
\]

\[
\frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t},
\]

for \( 0 \leq t \leq T \) (from these get \( C_t = C(R_t), L_t = L(R_t) \));

\[
P_tC_t \leq M_t,
\]

for \( 0 \leq t \leq T \) (which gives \( P_t \leq \frac{M_t}{C(R_t)} \));

\[
\frac{u_C(t)}{P_t} = R_tE_t \left[ \frac{\beta u_C(t + 1)}{P_{t+1}} \right],
\]

for \( 0 \leq t \leq T - 1 \).
The equilibrium conditions for the variables \( \{R_t, M_t, P_t\} \) are:

\[
\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[ \frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}} \right],
\]

\[
0 \leq t \leq T - 1
\]

\[
P_t \leq \frac{M_t}{C(R_t)}, 0 \leq t \leq T
\]
Interest Rate Policy.

- Interest rates are set in every date and state.
  
  There is a unique equilibrium if prices (money supply?) are set in every state at date $T$

\[
\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[ \frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}} \right],
\]

\[
0 \leq t \leq T - 1
\]

\[
P_T \leq \frac{M_T}{C(R_T)}
\]

- Deterministic economy.
- Uncertainty. Need a nominal anchor for every history.
• Arbitrarily large time horizon.

• Does it matter whether policy is conducted with an interest rate or a money supply rule?

• Does it matter which particular feedback rule is used?

• Does it matter whether prices are flexible or sticky?

• Preferences and technology?
Infinite horizon

- Need to set prices (money supply) in every state at date $T$ and, after that, in $\Phi_t - \Phi_{t-1}$ states for every $t \geq T + 1$.

$$\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[ \frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}} \right], \quad t \geq 0$$

$$P_t \leq \frac{M_t}{C(R_t)}, \quad t \geq 0$$

- If policy is a money supply rule and

$$P_t = \frac{M_t}{C(R_t)}, \quad t \geq 0,$$

then

$$\frac{u_C(C(R_t), L(R_t))}{\frac{M_t}{C(R_t)}} = \beta R_t E_t \left[ \frac{u_C(C(R_{t+1}), L(R_{t+1}))}{\frac{M_{t+1}}{C(R_{t+1})}} \right], \quad t \geq 0$$
• In the infinite horizon, preferences are relevant:

• The utility function is additively separable and logarithmic in consumption.

\[ U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\ln(C_t) + v(L_t)] \right\} \]
• Equilibrium conditions:

$$\frac{1}{M_t} = \beta R_t E_t \frac{1}{M_{t+1}}$$

$$P_t = \frac{M_t}{C(R_t)}$$

$$C_t = C(R_t)$$

$$L_t = L(R_t)$$
• In the finite horizon economy:

\[
\frac{1}{M_t} = \beta R_t E_t \left[ \frac{1}{M_{t+1}} \right], \quad t = 0, ..., T - 1
\]

When the money supply is set exogenously in every state the nominal interest rates, \( R_t \), are determined for \( t = 0, ..., T - 1 \). It is still necessary to set exogenously the interest rates, \( R_T \), in every terminal state.