Dynamics of the price distribution in a general model of state-dependent pricing

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Motivation

Sticky prices: crucial but controversial

- **Calvo (1983)**: constant probability of adjustment
  - easy aggregation: central to DSGE models
  - no microfoundations / many costly mistakes
  - Lucas critique: Calvo parameter should change with inflation

- **Golosov-Lucas (2007)**: menu costs as microfoundation for price stickiness
  - calibrated to match moments in microdata on price adjustments
  - money shocks have **much less persistent effects** than in Calvo
  - higher inflation: **more frequent adjustment**
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  - calibrated to match moments in microdata on price adjustments
  - money shocks have **much less persistent effects** than in Calvo
  - higher inflation: **more frequent adjustment**

- **But GL07 fits poorly** when nested in more general model
This paper

Study **dynamics** of simple **state-dependent** pricing model that **nests** Calvo-Yun, Golosov-Lucas and others
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- Estimate to match price adjustments in US microdata

- Simulate distributional dynamics using method of Reiter (2008)

- Report impulse response functions for all three models
This paper: results

Study **dynamics** of simple **state-dependent** pricing model that **nests** Calvo-Yun, Golosov-Lucas and others

- Estimate to **match price adjustments** in US microdata
  - GL07 model is rejected: no small price changes
  - Preferred model is closer to Calvo, but mildly state-dependent

- Simulate **distributional dynamics** using method of Reiter (2008)
  - **Nonlinear** in idiosyncratic states / **linear** in aggregate state

- Report **impulse response functions** for all three models
  - **Much larger real effects** of money shocks than GL07 found
  - Almost like Calvo
  - Difference due to (counterfactually) **strong selection effect** in GL07
  - Effects of **autocorrelated shocks** similar, but stronger
Related literature: dynamics of state-dependent pricing

- **Partial equilibrium**

- **General equilibrium without idiosyncratic shocks**
  - Dotsey-King-Wolman (1999)

- **Strong restrictions on idiosyncratic processes**
Related literature: dynamics of state-dependent pricing

- **Partial equilibrium**

- **General equilibrium without idiosyncratic shocks**
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- **Strong restrictions on idiosyncratic processes**

- **Distributional dynamics**
  - Golosov-Lucas (2007):
    - assumed money shock i.i.d.
    - assumed \( c \) constant
  - Midrigan (2008):
    - different model: multiproduct firms
    - assume mean \( p \) is sufficient statistic (Krusell-Smith method)
  - Dotsey-King-Wolman (2008)
    - flexible and sticky-price firms; normal and extreme shocks
    - we fit finer histogram with fewer free parameters
OUTLINE

(1) Introduction

(2) Monopolistic competitors in partial equilibrium
   - Nesting various models of state dependence
   - Finite grid approximation

(3) General equilibrium: steady state

(4) How to compute distributional dynamics

(5) Results: dynamics
   - Impulse responses
   - Inflation decomposition
   - Transition dynamics
Model
Our model

- **Probability of adjustment** increases with the **value of adjustment**:

\[ \lambda(L) \text{ where } \lambda' \geq 0 \]

- **Firm-level** price adjustments due to **idiosyncratic shocks**
- Rest of model: standard “New Keynesian” DSGE
Our model

- **Probability of adjustment** increases with the **value of adjustment**: 
  $$\lambda(L) \text{ where } \lambda' \geq 0$$
  - Interpretation: **stochastic menu costs**
  - Interpretation: **axiom for boundedly rational choice**

- **Firm-level** price adjustments due to **idiosyncratic shocks**
  - Rest of model: standard “New Keynesian” DSGE
    - Focus on **dynamics, including distributional effects**
Our model

- **Probability of adjustment** increases with the **value of adjustment**: 
  \[ \lambda(L) \text{ where } \lambda' \geq 0 \]

  - **Stochastic menu costs** (DKW99 and Caballero-Engel)
  - **Axiom for boundedly rational choice** (Akerlof-Yellen 1985)
    - Nested: Calvo (1983)
    - Nested: Dotsey-King-Wolman (1999)
    - Nested: Golosov-Lucas (2007)
    - Nested: Woodford (2008)

- **Firm-level** price adjustments due to **idiosyncratic shocks**

- Rest of model: standard “New Keynesian” DSGE
  - Focus on **dynamics, including distributional effects**
Partial equilibrium, steady state
Monopolistic competitor

Profits of firm $i$ are: $P_{it}Y_{it} - WN_{it}$

Output is: $Y_{it} = A_{it}N_{it}$

Demand is: $Y_{it} = \xi P_{it}^{-\epsilon}$
**Monopolistic competitor**

Profits of firm $i$ are: $$P_{it} Y_{it} - W N_{it}$$

Output is: $$Y_{it} = A_{it} N_{it}$$

Demand is: $$Y_{it} = \xi P_{it}^{-\varepsilon}$$

$A_{it}$ is exogenous shock

$P_{it}$ is “sticky” decision
Monopolistic competitor

Profits of firm $i$ are: $P_{it} Y_{it} - WN_{it}$

Output is: $Y_{it} = A_{it} N_{it}$

Demand is: $Y_{it} = \xi P_{it}^{-\epsilon}$

With sticky prices, the value function is:

$$V(P_{it}, A_{it})$$

If the firm can adjust its price, value increases to:

$$V^*(A_{it}) \equiv \max_P V(P, A_{it})$$
Probability of adjustment

- Nominal loss from failing to adjust:
  \[ D \equiv D(P_{it}, A_{it}) \equiv V^*(A_{it}) - V(P_{it}, A_{it}) \]

- Probability of adjustment:
  \[ \lambda(L) = \frac{\lambda}{\lambda + (1 - \hat{\lambda})(\alpha / L)^\xi}, \text{ where } L \equiv D / W. \]

- We divide by the wage to express the loss in units of labor time.
Probability of adjustment

- **Nominal loss from failing to adjust:**

  \[ D \equiv D(P_{it}, A_{it}) \equiv V^*(A_{it}) - V(P_{it}, A_{it}) \]

- **Probability of adjustment:**

  \[
  \lambda(L) = \frac{\overline{\lambda}}{\overline{\lambda} + (1 - \overline{\lambda})(\alpha / L)^{\xi}}, \quad \text{where} \quad L \equiv D/W.
  \]

- **Special cases:**
  - \( \xi \to 0 \): equivalent to Calvo model
  - \( \xi \to \infty \): equivalent to fixed menu cost model
Nesting alternative models

- **Calvo-Yun**
  \[
  \lambda(L) = \lim_{\xi \to 0} \lambda
  \]

- **Golosov-Lucas**
  \[
  \lambda(L) = \begin{cases} 
    0, & L < \alpha \\
    1, & L \geq \alpha
  \end{cases}
  \]

- **Costain-Nakov**
  \[
  \lambda(L) = \frac{\overline{\lambda}}{\lambda + (1 - \overline{\lambda})(\alpha / L)^\xi}
  \]
Firm that produces at price $P_i$ and productivity $A_i$ will adjust price with probability $\lambda$ next period:

$$V(P_i, A_i) = \left( P_i - \frac{W}{A_i} \right) \xi P_i^{-\varepsilon} + \frac{1}{1+r} E\left\{\left(1 - \lambda \left( \frac{D}{W} \right) \right) V(P_i, A_i') + \lambda \left( \frac{D}{W} \right) V^*(A_i') \mid A_i \right\}$$

This can be simplified using $D = V^*(A) - V(P, A)$. 
Simplifying Bellman

Steady state Bellman equation:

\[ V(P_i, A_i) = \left( P_i - \frac{W}{A_i} \right) \xi P_i^{-\epsilon} + \frac{1}{1+r} E\{V(P_i, A_i') + G(P_i, A_i') \mid A_i\} \]

where

\[ G(P_i, A_i') \equiv \lambda^{D(P_i, A_i')/W} D(P_i, A_i') \]

\[ D(P_i, A_i') \equiv V^*(A_i') - V(P_i, A_i'). \]
Alternative models

1. Calvo model:
   \[ G(P_i, A_i) = \lambda D(P_i, A_i) \]

2. Bounded rationality:
   \[ G(P_i, A_i) = \lambda \left( D(P_i, A_i) / W \right) D(P_i, A_i) \]

3. Fixed menu costs:
   \[ G(P_i, A_i) = 1(D(P_i, A_i) \geq W\kappa)(D(P_i, A_i) - W\kappa) \]

4. Stochastic menu costs:
   \[ G(P_i, A_i) = \lambda \left( D(P_i, A_i) / W \right) \left( D(P_i, A_i) - W\kappa \right) \]

5. Woodford’s model:
   \[ \lambda(D(P_i, A_i) / W) = \frac{\overline{\lambda} \exp((D/W - \kappa)/\theta)}{(1 - \overline{\lambda}) + \overline{\lambda} \exp((D/W - \kappa)/\theta)} \]
Distributional dynamics
and
finite grid approximation
Finite grid of real states

- Define grids: \( \Gamma^A \equiv \{a^1, a^2, \ldots, a^{A^\#}\} \), \( \Gamma^P \equiv \{p^1, p^2, \ldots, p^{P^\#}\} \)

- Grid represents real prices, deflated by money supply: \( M_{t+1} = \mu M_t \)

  Beginning-of-period prices: \( \tilde{p}_{it} \equiv \tilde{P}_{it} / M_t \in \Gamma^P \)

  Prices at time of production: \( p_{it} \equiv P_{it} / M_t \in \Gamma^P \)

- Real value function: \( V(P_{it}, A_{it}) = M_t \nu(p_{it}, A_{it}) \)
Finite grid of real states

- Define grids: \( \Gamma^A \equiv \{a^1, a^2, \ldots, a^{#A}\} \), \( \Gamma^P \equiv \{p^1, p^2, \ldots, p^{#P}\} \)

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  Prices at time of production: \( p_{it} \equiv P_{it} / M_t \in \Gamma^P \)

- Real value function: \( V(P_{it}, A_{it}) = M_t v(p_{it}, A_{it}) \)

  Next period: \( V(P_{it}, A_{i,t+1}) = M_{t+1} v(\mu^{-1} p_{it}, A_{i,t+1}) \)
Distributional dynamics

Time line:

\[ \Psi_t: \tilde{\psi}_{jk}^{\Psi_t} = \text{prob}(\tilde{p}_{it} = p^j, A_{it} = a^k) \]

Beginning of period:

Production shocks, detrending

\[ \tilde{\Psi}_{t}: \tilde{\psi}_{jk}^{\tilde{\Psi}_t} = \text{prob}(\tilde{p}_{it} = p^j, A_{it} = a^k) \]

\[ N^P \times N^A \]

Time of production:

Production shocks, detrending

\[ \Psi_{t+1}: \psi_{jk}^{\Psi_{t+1}} = \text{prob}(p_{it} = p^j, A_{it} = a^k) \]

\[ N^P \times N^A \]
Matrix notation

- Define grids: \( \Gamma^A \equiv \{a^1, a^2, \ldots, a^\#^A\} \quad \Gamma^P \equiv \{p^1, p^2, \ldots, p^\#^P\} \)

- Productivity shocks \( S : \quad s^{jk} = \text{prob}(a^j \mid a^k) \)

- Adjust real prices:
  Define \( R \) with ones in column \( j \), row \( j-\#\mu \), zeros elsewhere

- Current profits \( U : \quad u^{jk} = \left(p^j - w/a^k\right)^\xi \left(p^j\right)^{-\varepsilon} \)

Now rewrite Bellman...
Matrix notation

- Define grids: \[ \Gamma_A \equiv \{a^1, a^2, \ldots a^{#A}\} \quad \Gamma_P \equiv \{p^1, p^2, \ldots p^{#P}\} \]

- Productivity shocks \( S \): \[ s^{jk} = \text{prob}(a^j | a^k) \quad N^A \times N^A \]

- Adjust real prices:
  Define \( R \) with ones in column \( j \), row \( j-\#\mu \), zeros elsewhere
  \[ N^P \times N^P \]
  → Deflates real price from \( p^i \) to \( p^i / \mu = p^{i-\#\mu} \)

- Current profits \( U \):
  \[ u^{jk} = (p^j - w/a^k)\xi(p^j)^{-\varepsilon} \quad N^P \times N^A \]

Now rewrite Bellman...
Backwards induction in matrix notation

Guess $V$ with elements $v^{jk} \equiv v(p^j, a^k) : p^j \in \Gamma^P, a^k \in \Gamma^A$

1. Optimal value: $v^* = \max V$

2. Loss from not adjusting: $D = 1 * v^* - V$

3. Expected gains: $G = \lambda(D / w) * D$

4. Work backwards: $V = U + \beta R'(V + G) * S$

and return to 1.
Backwards induction in matrix notation

Guess \( V \) with elements \( v^{jk} \equiv v(p^j, a^k) : p^j \in \Gamma^P, a^k \in \Gamma^A \)

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2. Loss from not adjusting: \( D = 1^* v^* - V \)

3. Expected gains: \( G = \lambda(D / w)^* D \)

4. Work backwards: \( V = U + \beta R'^*(V + G) * S \)

and return to 1.

\( u^{jk} = (p^j - w/a^k)\xi(p^j)^{-\epsilon} \)

\( \text{Relates } p^i \text{ to } p^{i-\#\mu} \)

Markov process for A shocks
Distributional dynamics in matrix notation

Shocks and detrending:

Productivity \( A \) follows Markov process \( S \),

price \( p^i \) deflated to \( p^i/\mu \):

\[
\tilde{\Psi}_t = R \ast \Psi_{t-1} \ast S'
\]

Price adjustments:

Change to optimal price with probability \( \Lambda \equiv \lambda(D/w) \):

\[
\Psi_t = (1 - \Lambda) \ast \tilde{\Psi}_t + P \ast (1 \ast (\Lambda \ast \tilde{\Psi}_t))
\]
Distributional dynamics in matrix notation

Shocks and detrending:

Productivity $A$ follows Markov process $S$,
price $p_i$ deflated to $p_i/\mu$:

$$\tilde{\Psi}_t = R \cdot \Psi_{t-1} \cdot S'$$

$R$ deflates and rounds up or down to grid

Price adjustments:

Change to optimal price with probability $\Lambda \equiv \lambda(D/\nu)$:

$$\Psi_t = (1 - \Lambda) \cdot \tilde{\Psi}_t + P \cdot (1 \cdot (\Lambda \cdot \tilde{\Psi}_t))$$

$P$ selects optimal price and rounds to grid
General equilibrium:
steady state results
Utility of households:

\[
\frac{C_t^{1-\gamma}}{1-\gamma} - \chi N_t + \nu \log(M_t / P_t), \text{ discount factor } \beta
\]

Period budget constraint:

\[
P_tC_t + M_t + \frac{1}{1+r_t} B_t = W_t N_t + M_{t-1} + T_t + B_{t-1} + \Pi_t
\]

Consumption aggregation:

\[
C_t = \left\{ \int_0^1 C_{it}^{\frac{\varepsilon}{\varepsilon-1}} \, d\varepsilon \right\}^{\frac{\varepsilon}{\varepsilon-1}} \rightarrow C_{it} = (P_{it} / P_t)^{-\varepsilon} C_t
\]

Money supply:

\[
M_{t+1} \mu_{t+1} = M_t, \quad (\mu_{t+1} - \mu) = \phi(\mu_t - \mu) + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim i.i.d.
\]
General equilibrium:
Steady state fixed point

- **Guess:** \( p \)

- **Euler, FOC:**
  \[
  C^r = (1 - \beta \mu) / p, \quad w = \chi C^r \quad \rightarrow \quad u^{jk} = \left( p_j - \frac{w}{a_k} \right) C \left( \frac{p_j}{p} \right)^{-\epsilon}
  \]

- **Solve Bellman:**
  \[
  V = U + \beta R^*(V + G) * S
  \]

- **Calculate distributions:**
  \[
  \tilde{\Psi} = R * \Psi * S'
  \]
  \[
  \Psi = (1 - \Lambda) * \tilde{\Psi} + P * (1 * (\Lambda * \tilde{\Psi})
  \]

- **Calculate**
  \[
  p = \left\{ \sum_j \sum_k \psi^{jk} (p_j)^{1-\epsilon} \right\}^{1/(1-\epsilon)} \quad \text{and return.}
  \]
Parameters

As in Golosov and Lucas (2007):
- Discounting: $\beta=0.99$ quarterly
- CRRA: $\gamma=2$
- Labor supply coefficient: $\chi=6$
- Money demand coefficient: $\nu=1$
- Elasticity of substitution: $\varepsilon=7$
- Money growth: 0% (as in AC Nielsen data)

- Productivity process: \[ \log A_{t+1} = \rho \log A_t + \varepsilon^A_{t+1}, \]
  with $\text{std}(\varepsilon^A)=\sigma^A$

Simulate at **monthly** frequency.

Adjustment probability:
\[ \lambda(L) = \frac{\bar{\lambda}}{\bar{\lambda} + (1 - \bar{\lambda})(\alpha / L)^{\xi}} \]
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Simulate at **monthly** frequency.

Adjustment probability:

$$\lambda(L) = \frac{\bar{\lambda}}{\bar{\lambda} + (1 - \bar{\lambda})(\alpha / L)^\xi}$$
### Estimation

- **Estimated parameters:**
  \[
  \log A_{t+1} = \rho \log A_t + \epsilon^A_{t+1}, \quad \text{std}(\epsilon^A_{t+1}) = \sigma^A
  \]

- **Microdata on price adjustments:**
  - AC Nielsen supermarket data (Midrigan 2008)

- **Minimize objective function with two terms:**
  - Mean adjustment **frequency** in model vs. data
  - **Histogram** of price adjustments in model vs. data
Price changes: models vs. evidence

Actual and simulated distribution of price changes

- AC Nielsen
- Model MC

Actual and simulated distribution of price changes

- AC Nielsen
- Model Calvo

Actual and simulated distribution of price changes

- AC Nielsen
- Model SDSP

Size of price changes
Density of price changes
Price changes: models vs. evidence
### Price changes: models vs. evidence

<table>
<thead>
<tr>
<th></th>
<th>GL07</th>
<th>Calvo</th>
<th>CN</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly frequency of changes</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>10% (NS07)</td>
</tr>
<tr>
<td>Mean absolute price change</td>
<td>18.3</td>
<td>6.4</td>
<td>10.1</td>
<td>10.5 (VM08)</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>18.8</td>
<td>8.2</td>
<td>12.1</td>
<td>13.2 (VM08)</td>
</tr>
<tr>
<td>Changes less than 5%</td>
<td>0%</td>
<td>49.7%</td>
<td>25.2</td>
<td>25% (VM08)</td>
</tr>
<tr>
<td>Variance of $\lambda$ relative to GL07</td>
<td>$1$</td>
<td>$0$</td>
<td>$0.025$</td>
<td></td>
</tr>
</tbody>
</table>
Benchmark calibration: probability of adjustment

\[ \lambda(L) = \frac{\bar{\lambda}}{(1 - \bar{\lambda})(\alpha / L)^{\xi} + \bar{\lambda}} \]

\((\sigma_{\epsilon}^2, \rho, \bar{\lambda}, \alpha, \xi) = (0.005, 0.881, 0.109, 0.031, 0.290)\)
Benchmark calibration: distributions and price policy

Density of firms after mc shock and inflation

Stationary density of firms

Optimal price policy

Density of adjusting firms
Computing general equilibrium: dynamics
Is Krusell-Smith method suitable for menu cost model?

Consider Golosov-Lucas (2007) menu cost model...
  –Suppose firm adjusts if it deviates from optimum by 5%.

Two possible initial conditions:

- All firms deviate by 1% from optimal price
  –Result: 0% of firms adjust

- 10% of firms deviate by 10% from optimal price
  –Result: 10% of firms adjust

- Krusell-Smith method assumes tomorrow’s mean varies smoothly with today’s mean
- Fails to predict tomorrow’s mean in menu cost model because individual choices are highly nonlinear
Method of Reiter (2008)

Reiter (2008, forthcoming *JEDC*):
- Often, individual shocks bigger than aggregate shocks.
- Therefore: **individual choice** needs **nonlinear solution**, but **linear solution** suffices for **aggregate dynamics**.

- **Step 1**: detailed **nonlinear** solution of **steady state** on grid
  - Solve individual choices by **backwards induction** on grid

- **Step 2**: **Linearize dynamics at every grid point**
  - Viewed **point by point**, the Bellman equation is just a system of first-order expectational difference equations
  - Many equations, but standard toolkits applicable (Sims, Klein, etc)

- **Surprise**: **It’s easy!!!**
Consider Golosov-Lucas (2007) menu cost model...

- Suppose firm adjusts if it deviates from optimum by 5%.

Two possible initial conditions:

- All firms deviate by 1% from optimal price
  - Result: 0% of firms adjust

- 10% of firms deviate by 10% from optimal price
  - Result: 10% of firms adjust

- Reiter’s method can capture this difference: 1% and 10% from optimal price are different grid points
- Each grid point treated by a different equation... coefficients of these equations not linearly related
Step 1: steady state
(already done)

- **Guess:** \( p \)

- **Euler, FOC:**
  \[
  C^r = (1 - \beta \mu) / p, \quad w = \chi C^r \quad \rightarrow \quad u^{jk} = \left( p^j - \frac{w}{a^k} \right) C \left( \frac{p^j}{p} \right)^{-\epsilon}
  \]

- **Solve Bellman:**
  \[
  V = U + \beta R' * (V + G) * S
  \]

- **Calculate distributions:**
  \[
  \Psi = R * \Psi * S'
  \]
  \[
  \Psi = (1 - \Lambda) * \Psi + P * (1 * (\Lambda * \Psi)
  \]

- **Calculate**
  \[ p = \left\{ \sum_j \sum_k \psi^{jk} (p^j)^{1 - \epsilon} \right\}^{1 - \epsilon} \]
  and return.
Dynamics

1. F.O.C. labor: \( C_t^{-\gamma} = \chi / w_t \)

2. Euler equation: \( 1 - p_tC_t^\gamma = \beta E_t \left( \mu_{t+1} \frac{p_tC_t^\gamma}{p_{t+1}C_{t+1}^\gamma} \right) \)

3. Labor demand: \( N_t = \sum_j \sum_k \Psi_t^{jk} (p_j / p_t)^{-\varepsilon} C_t / \alpha_k \equiv \Delta_t C_t \)

4. Price index: \( p_t = \left\{ \sum_j \sum_k \Psi_t^{jk} (p_j)^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}} \)
Matrices $U_t, V_t, D_t, G_t, \Psi_t, \tilde{\Psi}_t, 1, P_t$ have size $N^p \times N^A$.

5. **Bellman is:**

$$V_t = U_t + \beta E_t \left\{ \frac{p_t C_{t+1}^r}{p_{t+1} C_t^r} R_{t+1}' \ast (V_{t+1} + G_{t+1}) \ast S \right\}$$

6. **Distributions satisfy:**

- **Beginning of period:**
  $$\tilde{\Psi}_{t+1} = R_{t+1} \ast \Psi_t \ast S'$$

- **Time of production:**
  $$\Psi_t = (1 - \Lambda_t) \ast \tilde{\Psi}_t + P_t \ast (1 \ast (\Lambda_t \ast \tilde{\Psi}_t))$$

- **JUST A HUGE SYSTEM OF 1ST-ORDER DIFFERENCE EQS... CAN BE LINEARIZED!** (Reiter 2008)
Step 2: Linearized dynamics

Suppose \( \mu_{t+1} = \phi \mu_t + \varepsilon_{t+1} \) with \( \varepsilon_{t+1} \) iid, mean zero.

Define row vector: 
\[
X_t \equiv (\text{vec}(V_t)', C_t, p_t, \text{vec}({\Psi_{t-1}})')
\]
length: \( 2N^pN^n + 2 \)

Equilibrium dynamics can be summarized by:
\[
E_t F(X_{t+1}, X_t, \mu_{t+1}, \mu_t) = 0
\]
\( 2N^pN^n + 3 \) first-order expectational difference equations in \( 2NPN^n + 3 \) series.
Step 2: Linearized dynamics

Suppose $\mu_{t+1} = \phi \mu_t + \varepsilon_{t+1}$ with $\varepsilon_{t+1}$ iid, mean zero.

Define row vector: $X_t \equiv (\text{vec}(V_t)', C_t, p_t, \text{vec}(\Psi_{t-1})')$

length: $2NP\pi + 2$

Equilibrium dynamics can be summarized by:

$$E_t F(X_{t+1}, X_t, \mu_{t+1}, \mu_t) = 0$$

$2NP\pi + 3$ first-order expectational difference equations in $2NP\pi + 3$ series.

(We’ve eliminated $w_t, N_t,$ and $\Psi_t$.)
Suppose $\bar{\mu}_{t+1} = \phi \bar{\mu}_t + \varepsilon_{t+1}$ with $\varepsilon_{t+1} iid$, mean zero.

Define row vector: $X_t \equiv (\text{vec}(V_t)', C_t, \ p_t, \text{vec}(\Psi_{t-1}')$)

length: $2NP^TN^T + 2$

Steady state satisfies:

$$F(X^*, X^*, 0, 0) = 0$$

We’ve already solved for the steady state (“step 1”).
Step 2: Linearized dynamics

Suppose $\mu_{t+1} = \phi \mu_t + \epsilon_{t+1}$ with $\epsilon_{t+1}$ iid, mean zero.

Define row vector: $X_t \equiv (\text{vec}(V_t)', C_t, p_t, \text{vec}(\Psi_{t-1})')$

length: $2NP^N + 2$

Linearization around steady state:

$$AE_t \Delta X_{t+1} + B \Delta X_t + CE_t \hat{\mu}_{t+1} + D \hat{\mu}_t = 0$$

Can be solved by Paul Klein’s “QZ” algorithm.

Alternatives: Chris Sims’ algorithm, etc.
General equilibrium:
dynamic results
Impulse responses: \textit{iid} money growth shock

Money growth process

Inflation

Nominal interest rate

Intensive margin

Extensive margin

Selection effect

Consumption

Labor

Price dispersion

Real interest rate

Real wage

Real money holdings

SDSP
Calvo
Menu cost

Impulse responses: \textit{iid} money growth shock
Inflation decompositions

- Inflation identity

\[ \pi_t = \sum_{j,k} x_{t}^{jk} \lambda_{t}^{jk} \Psi_{t}^{jk} \]

where \( x_{t}^{jk} = \log \left( \frac{p_{t}^{*}(a_{k})}{p_{j}} \right) \) is the desired price adjustment, in logs
Inflation decompositions

- Inflation identity

\[ \pi_t = \sum_{j,k} x_t^{jk} \lambda_t^{jk} \Psi_t^{jk} \]

- Desired price change

- Distribution after shock

- Probability of adjustment
### Inflation decompositions

- **Inflation identity**

  Desired price change

  \[ \pi_t = \sum_{j,k} x_t^{jk} \lambda_t^{jk} \tilde{\Psi}_t^{jk} \]

  Probability of adjustment

- **Klenow and Kryvtsov**
  - Intensive and extensive margin

  \[
  \pi_t = \frac{\sum_{j,k} x_t^{jk} \lambda_t^{jk} \tilde{\Psi}_t^{jk}}{\sum_{j,k} \lambda_t^{jk} \tilde{\Psi}_t^{jk}} \sum_{j,k} \lambda_t^{jk} \tilde{\Psi}_t^{jk} = av_t fr_t
  \]

  \[
  \Delta\pi_t \approx fr \Delta av_t + av \Delta fr_t
  \]
Inflation decompositions

- **Inflation identity**
  
  Desired price change
  
  \[ \pi_t = \sum_{j,k} x_t^{jk} \lambda_t^{jk} \tilde{\Psi}_t^{jk} \]
  
  Distribution after shock
  
  Probability of adjustment

- **Klenow and Kryvtsov**
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  \[ \Delta \pi_t \approx \bar{fr} \Delta av_t + av \Delta \bar{fr}_t \]

  Intensive margin Extensive margin
Inflation decompositions

- Inflation identity

\[ \pi_t = \sum_{j,k} x_t^{jk} \lambda_t^{jk} \Psi_t^{jk} \]

- Desired price change
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Inflation decompositions

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\[ \pi_t = \sum_{j,k} x_t^{jk} \lambda_t^{jk} \Psi_t^{jk} \]

Desired price change

Distribution after shock

Probability of adjustment

- Costain and Nakov
  - Intensive margin, extensive margin, and selection effect

\[ \pi_t = \sum_{j,k} x_t^{jk} \Psi_t^{jk} \sum_{j,k} \lambda_t^{jk} \Psi_t^{jk} + \sum_{j,k} x_t^{jk} (\lambda_t^{jk} - \sum_{j,k} \lambda_t^{jk} \Psi_t^{jk}) \Psi_t^{jk} = av^*_t \Delta r_t + sel_t \]

\[ \Delta \pi_t \approx fr \Delta av^*_t + av \Delta fr_t + \Delta sel_t \]
**Inflation decompositions**

- **Inflation identity**
  
  Desired price change
  
  \[ \pi_t = \sum_{j,k} x_{ik} \lambda_i \psi_{i} \]

  Distribution after shock
  
  Probability of adjustment

- **Costain and Nakov**
  - Intensive margin, extensive margin, and selection effect
  
  \[ \pi_t = \sum_{j,k} x_{ik} \psi_{i} \sum_{j,k} \lambda_i \psi_{i} + \sum_{j,k} x_{ik} (\lambda_i \psi_{i} - \sum_{j,k} \lambda_i \psi_{i} \hat{\psi}_{i}) \hat{\psi}_{i} = av^* fr_t + sel_t \]

  \[ \Delta \pi_t \approx fr \Delta av^* + av^* \Delta fr_t + \Delta sel_t \]

  Intensive margin  Extensive margin  Selection effect
Why the difference with GL07?

- Fixed menu costs imply strong **selection effect**:  
  - Firms that adjust are far from optimal price.

- Shock redistributes mass from price decreases to price increases  
  - Large change in average adjustment, \( \Delta a_{t} \) ...  
  - ...even if small change in average desired adjustment, \( \Delta a_{t}^{*} \)

- Depends on firms jumping from \( \lambda^{jk}=0 \) to \( \lambda^{jk}=1 \)  
  - Such **strong state dependence rejected** by estimate of \( \lambda(L) \)
Why the difference with GL07?

- In estimated model, many adjusters are near optimal price

- Selection effect smaller (1/3 of change in inflation)
  - Average adjustment similar to average desired adjustment: $\Delta v_t \approx \Delta v_t^*$

- Shock falls less on inflation, more on output
Impulse responses: autocorrelated money growth
Real impact of money growth shocks

“Phillips curve”:  \( \log(C_t) = \alpha + \beta \log(\mu_t) \)

<table>
<thead>
<tr>
<th>Uncorrelated shocks ( (\phi_\mu = 0) )</th>
<th>GL07</th>
<th>Calvo</th>
<th>CN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std dev money growth shock ( \times 100 )</td>
<td>0.52</td>
<td>1.05</td>
<td>0.81</td>
</tr>
<tr>
<td>Std dev quarterly inflation ( \times 100 )</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>% explained by nominal shock</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Std dev quarterly output growth ( \times 100 )</td>
<td>0.13</td>
<td>0.72</td>
<td>0.49</td>
</tr>
<tr>
<td>% explained by nominal shock</td>
<td>26%</td>
<td>142%</td>
<td>96%</td>
</tr>
<tr>
<td>Slope of the Phillips curve ( \beta )</td>
<td>0.23</td>
<td>0.52</td>
<td>0.46</td>
</tr>
<tr>
<td>Standard error</td>
<td>0.00</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.71</td>
<td>0.20</td>
<td>0.33</td>
</tr>
</tbody>
</table>
"Phillips curve": \( \log(C_t) = \alpha + \beta \log(\mu_t) \)

<table>
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<tr>
<th>Correlated shocks ( (\phi_\mu = 0.8) )</th>
<th>GL07</th>
<th>Calvo</th>
<th>CN</th>
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</thead>
<tbody>
<tr>
<td>Std dev money growth shock (x100)</td>
<td>0.11</td>
<td>0.21</td>
<td>0.16</td>
</tr>
<tr>
<td>Std dev quarterly inflation (x100)</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
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<tr>
<td>% explained by nominal shock</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Std dev quarterly output growth (x100)</td>
<td>0.15</td>
<td>0.67</td>
<td>0.47</td>
</tr>
<tr>
<td>% explained by nominal shock</td>
<td>29%</td>
<td>131%</td>
<td>91%</td>
</tr>
<tr>
<td>Slope of the Phillips curve ( (\beta) )</td>
<td>0.82</td>
<td>2.88</td>
<td>2.20</td>
</tr>
<tr>
<td>Standard error</td>
<td>0.01</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.89</td>
<td>0.88</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Transitional dynamics: shift in $p$
Transitional dynamics: shift in A
Transitional dynamics: "post-euro"
Effect of starting conditions: IRFs after productivity shock
Conclusions

- Now feasible to compute DSGE distributional dynamics with state-dependent prices
  – Reiter (2008)

- Benchmark calibration: money growth shocks not neutral

- Autocorrelated shocks: effects similar, but stronger

- Near-neutrality in GL07 calibration due to strong selection effect
  – Strong selection effect and lack of small price changes both due to counterfactual degree of state dependence

- Additional findings:
  – Price dispersion quantitatively important (unlike standard NK DSGE)
  – Increased aggregate productivity causes labor to fall (like NK DSGE)
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- Additional findings:
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- Phillips curve is alive and well!
Some extensions:

Taylor rules
Shock in Taylor rule

- Shock process
- Inflation
- Nominal interest rate
- Intensive margin
- Extensive margin
- Selection effect
- Consumption
- Labor
- Price dispersion
- Real interest rate
- Real wage
- Real money holdings

Graphs showing the impact of a shock on various economic variables over time, including inflation, the nominal interest rate, consumption, labor, price dispersion, real interest rate, real wage, and real money holdings.
Taylor rule: “Permanent” disinflation

- Shock process
- Inflation
- Nominal interest rate
- Intensive margin
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- Labor
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Calvo
SDSP
James Costain

THANK YOU FOR YOUR ATTENTION