BANCO DE ESPAÑA

Dynamics of the price distribution in a general model of state-dependent pricing

James Costain
Anton Nakov

División de Investigación, Servicio de Estudios
Banco de España

November 2008

Motivation

Sticky prices: crucial but controversial

- **Calvo (1983):** constant probability of adjustment
  - easy aggregation: central to DSGE models
  - no microfoundations / many costly mistakes
  - Lucas critique: Calvo parameter should change with inflation

- **Golosov-Lucas (2007):** menu costs as microfoundation for price stickiness
  - calibrated to match moments in microdata on price adjustments
  - money shocks have **much less persistent effects** than in Calvo
  - higher inflation: **more frequent adjustment**

Motivation

Sticky prices: crucial but controversial

- **Calvo (1983):** constant probability of adjustment
  - easy aggregation: central to DSGE models
  - no microfoundations / many costly mistakes
  - Lucas critique: Calvo parameter should change with inflation

- **Golosov-Lucas (2007):** menu costs as microfoundation for price stickiness
  - calibrated to match moments in microdata on price adjustments
  - money shocks have **much less persistent effects** than in Calvo
  - higher inflation: **more frequent adjustment**

- But **GL07 fits poorly** when nested in more general model

This paper

Study **dynamics** of simple **state-dependent** pricing model that **nests** Calvo-Yun, Golosov-Lucas and others

This paper

Study **dynamics** of simple **state-dependent** pricing model that **nests** Calvo-Yun, Golosov-Lucas and others

- Estimate to **match price adjustments** in US microdata
- **Simulate distributional dynamics** using method of Reiter (2008)
- Report **impulse response functions** for all three models

This paper: results

Study **dynamics** of simple **state-dependent** pricing model that **nests** Calvo-Yun, Golosov-Lucas and others

- Estimate to **match price adjustments** in US microdata
  - **GL07 model is rejected: no small price changes**
  - **Preferred model is closer to Calvo**, but mildly state-dependent

- **Simulate distributional dynamics** using method of Reiter (2008)
  - **Nonlinear** in idiosyncratic states / **linear** in aggregate state

- Report **impulse response functions** for all three models
  - **Much larger real effects** of money shocks than GL07 found
  - Almost like Calvo
  - Difference due to (counterfactually) **strong selection effect** in GL07
  - Effects of **autocorrelated shocks similar, but stronger**

Related literature: dynamics of state-dependent pricing

- **Partial equilibrium**
  - –Caballero-Engel (1993, 2007), Klenow-Kryvstov (2008)

- **General equilibrium without idiosyncratic shocks**
  - –Dotsey-King-Wolman (1999)
- **Strong restrictions on idiosyncratic processes**
  - –Caplin-Spulber (1987), Gertler-Leahy (2006)

Related literature: dynamics of state-dependent pricing

- **Partial equilibrium**
  - Caballero-Engel (1993, 2007), Klenow-Kryvstov (2008)

- **General equilibrium without idiosyncratic shocks**
  - Dotsey-King-Wolman (1999)
- **Strong restrictions on idiosyncratic processes**
  - Caplin-Spulber (1987), Gertler-Leahy (2006)

- **Distributional dynamics**
  - **Golosov-Lucas (2007):**
    - *assumed money shock i.i.d.*
    - *assumed* c *constant*
  - **Midrigan (2008):**
    - *different model: multiproduct firms*
    - *assume mean* p *is sufficient statistic (Krusell-Smith method)*
  - **Dotsey-King-Wolman (2008)**
    - *flexible and sticky-price firms; normal and extreme shocks*
    - *we fit finer histogram with fewer free parameters*

OUTLINE

(1) Introduction

(2) Monopolistic competitors in partial equilibrium

- Nesting various models of state dependence
- Finite grid approximation

(3) General equilibrium: steady state

(4) How to compute distributional dynamics

(5) Results: dynamics

- Impulse responses
- Inflation decomposition
- Transition dynamics

Model

Our model

- **Probability of adjustment** increases with the **value of adjustment:**

$$\lambda(L) \text{ where } \lambda' \geq 0$$

- **Firm-level** price adjustments due to **idiosyncratic shocks**
- Rest of model: standard “New Keynesian” DSGE

Our model

- **Probability of adjustment** increases with the **value of adjustment:**

$$\lambda(L) \text{ where } \lambda' \geq 0$$

  - –Interpretation: **stochastic menu costs**
  - –Interpretation: **axiom for boundedly rational choice**

- **Firm-level** price adjustments due to **idiosyncratic shocks**
- Rest of model: standard “New Keynesian” DSGE
  - –Focus on **dynamics, including distributional effects**

Our model

- **Probability of adjustment** increases with the **value of adjustment:**

$$\lambda(L) \text{ where } \lambda' \geq 0$$

  - **Stochastic menu costs** (DKW99 and Caballero-Engel)
  - **Axiom for boundedly rational choice** (Akerlof-Yellen 1985)

  - Nested: **Calvo (1983)**
  - Nested: **Dotsey-King-Wolman (1999)**
  - Nested: **Golosov-Lucas (2007)**
  - Nested: **Woodford (2008)**

- **Firm-level** price adjustments due to **idiosyncratic shocks**
- Rest of model: standard “New Keynesian” DSGE
  - Focus on **dynamics, including distributional effects**

Partial equilibrium,
steady state

Monopolistic competitor

Profits of firm *i* are: $P_{it}Y_{it} - WN_{it}$

Output is: $Y_{it} = A_{it}N_{it}$

Demand is: $Y_{it} = \xi P_{it}^{-\varepsilon}$

Monopolistic competitor

Profits of firm *i* are: $P_{it}Y_{it} - WN_{it}$

Output is: $Y_{it} = A_{it}N_{it}$

Demand is: $Y_{it} = \xi P_{it}^{-\varepsilon}$

A_{it} is exogenous shock

P_{it} is “sticky” decision

Monopolistic competitor

Profits of firm *i* are: $P_{it}Y_{it} - WN_{it}$

Output is: $Y_{it} = A_{it}N_{it}$

Demand is: $Y_{it} = \xi P_{it}^{-\varepsilon}$

With **sticky prices**, the value function is:

$$V(P_{it}, A_{it})$$

If the firm can **adjust its price**, value increases to:

$$V^{*}(A_{it}) \equiv \max_{P} V(P, A_{it})$$

Probability of adjustment

- Nominal **loss from failing to adjust:**

$$D \equiv D(P_{it}, A_{it}) \equiv V^*(A_{it}) - V(P_{it}, A_{it})$$

- **Probability of adjustment:**

$$\lambda(L) = \frac{\bar{\lambda}}{\bar{\lambda} + (1-\bar{\lambda})(\alpha / L)^{\xi}}, \quad \text{where} \quad L \equiv D / W.$$

  – We divide by the wage to express the loss **in units of labor time.**

Probability of adjustment

- Nominal **loss from failing to adjust:**

$$D \equiv D(P_{it}, A_{it}) \equiv V^*(A_{it}) - V(P_{it}, A_{it})$$

- **Probability of adjustment:**

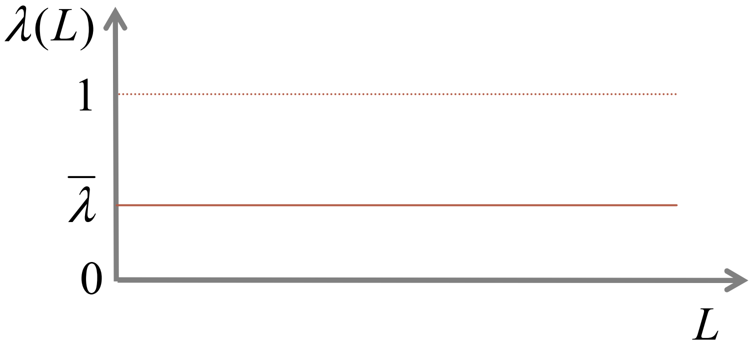
$$\lambda(L) = \frac{\bar{\lambda}}{\bar{\lambda} + (1-\bar{\lambda})(\alpha / L)^{\xi}}, \quad \text{where} \quad L \equiv D / W.$$

- **Special cases:**
  - – **ξ→0: equivalent to Calvo model**
  - – **ξ→∞: equivalent to fixed menu cost model**

Nesting alternative models

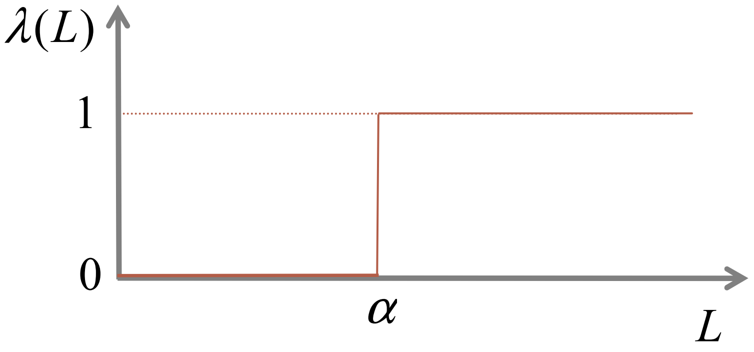
- **Calvo-Yun**

$$\lambda(L) = \xrightarrow{\xi\to 0} \bar{\lambda}$$



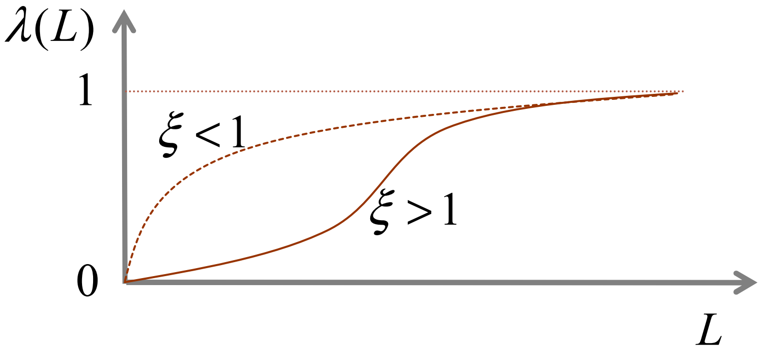
- **Golosov-Lucas**

$$\lambda(L) \xrightarrow{\xi\to\infty} \begin{cases} 0, & L < \alpha \\ 1, & L \geq \alpha \end{cases}$$



- **Costain-Nakov**

$$\lambda(L) = \frac{\bar{\lambda}}{\bar{\lambda} + (1-\bar{\lambda})(\alpha / L)^{\xi}}$$



Bellman equation at time of production

Firm that **produces at price *P_i* and productivity *A_i***
will adjust price with probability λ next period:

$$V(P_i, A_i) = \left(P_i - {}^{W}\!/\!_{A_i} \right) \xi P_i^{-\varepsilon} + \frac{1}{1+r} E\left\{ \left(1 - \lambda\left({}^{D}\!/\!_{W}\right)\right) V(P_i, A_i') + \lambda\left({}^{D}\!/\!_{W}\right) V^*(A_i') \mid A_i \right\}$$

This can be **simplified** using D= V^*(A)-V(P,A).

Simplifying Bellman

Steady state Bellman equation:

current profits

$$V(P_i, A_i) = \left(P_i - {W}/{A_i}\right)\xi P_i^{-\varepsilon} +$$

$$\tfrac{1}{1+r} E\left\{V(P_i, A_i') + G(P_i, A_i') \mid A_i\right\}$$

value of not adjusting expected gains from adjusting

where

$$G(P_i, A_i') \equiv \lambda\left({D(P_i, A_i')}/{W}\right) D(P_i, A_i')$$

$$D(P_i, A_i') \equiv V^*(A_i') - V(P_i, A_i').$$

Alternative models

1. Calvo model:

$$G(P_i, A_i) = \lambda D(P_i, A_i)$$

2. Bounded rationality:

$$G(P_i, A_i) = \lambda\left(D(P_i, A_i)/W\right)D(P_i, A_i)$$

3. Fixed menu costs:

$$G(P_i, A_i) = 1\left(D(P_i, A_i) \geq W\kappa\right)\left(D(P_i, A_i) - W\kappa\right)$$

4. Stochastic menu costs:

$$G(P_i, A_i) = \lambda\left(D(P_i, A_i)/W\right)\left(D(P_i, A_i) - WE(\kappa \mid {\scriptstyle D(P_i, A_i) \geq W\kappa})\right)$$

5. Woodford's model:

$$\lambda\left(D(P_i, A_i)/W\right) = \frac{\bar{\lambda}\exp((D/W - \kappa)/\theta)}{(1-\bar{\lambda}) + \bar{\lambda}\exp((D/W - \kappa)/\theta)}$$

Distributional dynamics

and

finite grid approximation

Finite grid of real states

- Define **grids:** $\Gamma^{A} \equiv \left\{ a^{1}, a^{2}, \ldots a^{\#^{A}} \right\}, \quad \Gamma^{P} \equiv \left\{ p^{1}, p^{2}, \ldots p^{\#^{P}} \right\}$

- Grid represents **real prices, deflated by money supply:** $M_{t+1} = \mu M_{t}$

  Beginning-of-period prices: $\widetilde{p}_{it} \equiv \widetilde{P}_{it} / M_{t} \in \Gamma^{P}$

  Prices at time of production: $p_{it} \equiv P_{it} / M_{t} \in \Gamma^{P}$

- **Real value function:** $V(P_{it}, A_{it}) = M_{t} v(p_{it}, A_{it})$

Finite grid of real states

- Define **grids:** $\Gamma^A \equiv \{a^1, a^2, \ldots a^{\#^A}\}, \quad \Gamma^P \equiv \{p^1, p^2, \ldots p^{\#^P}\}$

- Grid represents **real prices, deflated by money supply:** $M_{t+1} = \mu M_t$

  Beginning-of-period prices: $\tilde{p}_{it} \equiv \tilde{P}_{it} / M_t \in \Gamma^P$

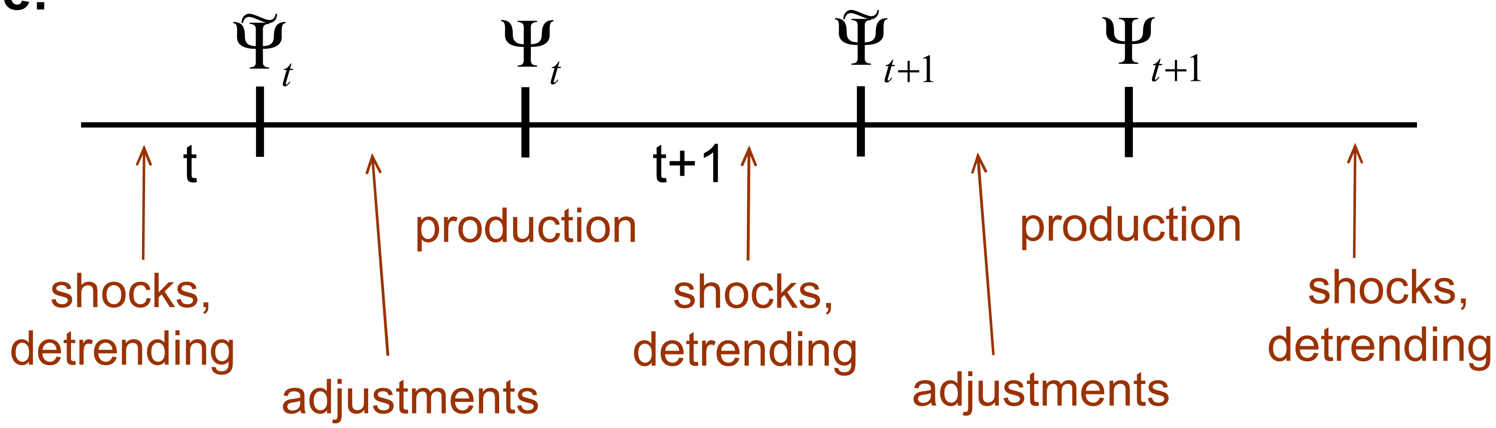
  Prices at time of production: $p_{it} \equiv P_{it} / M_t \in \Gamma^P$

- **Real value function:** $V(P_{it}, A_{it}) = M_t v(p_{it}, A_{it})$

  Next period: $V(P_{it}, A_{i,t+1}) = M_{t+1} v(\mu^{-1} p_{it}, A_{i,t+1})$

Distributional dynamics

Time line:



$\tilde{\Psi}_t$ Ψ_t $\tilde{\Psi}_{t+1}$ Ψ_{t+1}

t

t+1

shocks, detrending

adjustments

production

shocks, detrending

adjustments

production

shocks, detrending

Beginning of period:

$$\tilde{\Psi}_t : \ \tilde{\psi}_t^{jk} = prob\left(\tilde{p}_{it} = p^j, A_{it} = a^k\right)$$

N^P x N^A

Time of production:

$$\Psi_t : \ \psi_t^{jk} = prob\left(p_{it} = p^j, A_{it} = a^k\right)$$

N^P x N^A

Matrix notation

- Define **grids:** $\Gamma^A \equiv \left\{a^1, a^2, \ldots a^{\#^A}\right\}, \quad \Gamma^P \equiv \left\{p^1, p^2, \ldots p^{\#^P}\right\}$

- **Productivity shocks** $\mathbf{S}$**:** $s^{jk} = prob(a^j \mid a^k)$

- **Adjust real prices:**

  Define $\mathbf{R}$ with ones in column *j*, row *j-#μ*, zeros elsewhere

- **Current profits** $\mathbf{U}$**:** $u^{jk} = \left(p^j - w/a^k\right)\xi\left(p^j\right)^{-\varepsilon}$

Now rewrite Bellman...

Matrix notation

- Define **grids:** $\Gamma^A \equiv \{a^1, a^2, \dots a^{\#^A}\},\quad \Gamma^P \equiv \{p^1, p^2, \dots p^{\#^P}\}$

- **Productivity shocks $\mathbf{S}$:** $s^{jk} = prob(a^j \mid a^k)$

  N^A x N^A

- **Adjust real prices:**

  Define $\mathbf{R}$ with ones in column *j*, row *j-#μ*, zeros elsewhere

  N^P x N^P

  → Deflates real price from p^j to $p^j/\mu = p^{j-\#\mu}$

- **Current profits $\mathbf{U}$:** $u^{jk} = \left(p^j - w/a^k\right)\xi\left(p^j\right)^{-\varepsilon}$

  N^P x N^A

Now rewrite Bellman...

Backwards induction in matrix notation

Guess $\mathbf{V}$ with elements $v^{jk} \equiv v(p^j, a^k): \quad p^j \in \Gamma^P, a^k \in \Gamma^A$

1. **Optimal value:** $\mathbf{v}^* = \max \mathbf{V}$
2. **Loss from not adjusting:** $\mathbf{D} = \mathbf{1} * \mathbf{v}^* - \mathbf{V}$
3. **Expected gains:** $\mathbf{G} = \lambda(\mathbf{D}/w) .* \mathbf{D}$
4. **Work backwards:** $\mathbf{V} = \mathbf{U} + \beta \mathbf{R}' * (\mathbf{V} + \mathbf{G}) * \mathbf{S}$

and return to 1.

Backwards induction in matrix notation

Guess $\mathbf{V}$ with elements $v^{jk} \equiv v(p^j, a^k): \quad p^j \in \Gamma^P, a^k \in \Gamma^A$

N^P x N^A

1. **Optimal value:** $$\mathbf{v}^* = \max \mathbf{V}$$ 1 x N^A

2. **Loss from not adjusting:** $$\mathbf{D} = \mathbf{1} * \mathbf{v}^* - \mathbf{V}$$ N^P x N^A

3. **Expected gains:** $$\mathbf{G} = \lambda(\mathbf{D}/w).*\mathbf{D}$$ N^P x N^A

4. **Work backwards:** $$\mathbf{V} = \mathbf{U} + \beta \mathbf{R}' * (\mathbf{V} + \mathbf{G}) * \mathbf{S}$$

and return to 1.

$u^{jk} = (p^j - w/a^k)\xi(p^j)^{-\varepsilon}$

Relates p^j to $p^{j-\#\mu}$

Markov process for A shocks

Distributional dynamics in matrix notation

Shocks and detrending:

Productivity *A* follows Markov process $\mathbf{S}$,

price p^j deflated to p^j/μ :

$$\widetilde{\mathbf{\Psi}}_t = \mathbf{R} * \mathbf{\Psi}_{t-1} * \mathbf{S}'$$

Price adjustments:

Change to optimal price with probability $\mathbf{\Lambda} \equiv \lambda(\mathbf{D}/w)$:

$$\mathbf{\Psi}_t = (\mathbf{1} - \mathbf{\Lambda}) .* \widetilde{\mathbf{\Psi}}_t + \mathbf{P} .* (\mathbf{1} * (\mathbf{\Lambda} .* \widetilde{\mathbf{\Psi}}_t))$$

Distributional dynamics in matrix notation

Shocks and detrending:

Productivity *A* follows Markov process $\mathbf{S}$,

price p^j deflated to p^j/μ :

$$\widetilde{\mathbf{\Psi}}_t = \mathbf{R} * \mathbf{\Psi}_{t-1} * \mathbf{S}'$$

R deflates and rounds
up or down to grid

Price adjustments:

Change to optimal price with probability $\mathbf{\Lambda} \equiv \lambda(\mathbf{D} / w)$:

$$\mathbf{\Psi}_t = (\mathbf{1} - \mathbf{\Lambda}) .* \widetilde{\mathbf{\Psi}}_t + \mathbf{P} .* (\mathbf{1} * (\mathbf{\Lambda} .* \widetilde{\mathbf{\Psi}}_t))$$

P selects optimal price
and rounds to grid

General equilibrium:

steady state results

Households, firms, central bank

Utility of households:

$$\frac{C_t^{1-\gamma}}{1-\gamma} - \chi N_t + \nu \log(M_t / P_t),\ \text{discount factor}\ \beta$$

Period budget constraint:

$$P_t C_t + M_t + \tfrac{1}{1+r_t} B_t = W_t N_t + M_{t-1} + T_t + B_{t-1} + \Pi_t$$

Consumption aggregation:

$$C_t = \left\{ \int_0^1 C_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right\}^{\frac{\varepsilon}{\varepsilon-1}} \quad \rightarrow \quad C_{it} = (P_{it} / P_t)^{-\varepsilon} C_t$$

Money supply:

$$M_{t+1}\mu_{t+1} = M_t, \quad (\mu_{t+1} - \mu) = \phi(\mu_t - \mu) + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim i.i.d.$$

General equilibrium: Steady state fixed point

- **Guess:** p

- **Euler, FOC:**

$$C^{\gamma} = (1-\beta\mu)/p, \;\; w = \chi C^{\gamma} \quad \rightarrow \quad u^{jk} = \left(p^{j} - \frac{w}{a^{k}}\right) C \left(\frac{p^{j}}{p}\right)^{-\varepsilon}$$

- **Solve Bellman:**

$$\mathbf{V} = \mathbf{U} + \beta \mathbf{R}' * (\mathbf{V} + \mathbf{G}) * \mathbf{S}$$

- **Calculate distributions:** $\widetilde{\mathbf{\Psi}} = \mathbf{R} * \mathbf{\Psi} * \mathbf{S}'$

$$\mathbf{\Psi} = (\mathbf{1} - \mathbf{\Lambda}).* \widetilde{\mathbf{\Psi}} + \mathbf{P}.*(\mathbf{1}*(\mathbf{\Lambda}.*\widetilde{\mathbf{\Psi}})$$

- **Calculate** $p = \left\{ \sum_{j} \sum_{k} \psi^{jk} (p^{j})^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}}$ **and return.**

Parameters

As in Golosov and Lucas (2007):

- Discounting: β=0.99 quarterly
- CRRA: γ=2
- Labor supply coefficient: χ=6
- Money demand coefficient: ν=1
- Elasticity of substitution: ε=7
- Money growth: 0% (as in AC Nielsen data)

Adjustment probability:

$$\lambda(L) = \frac{\bar{\lambda}}{\bar{\lambda} + (1-\bar{\lambda})(\alpha / L)^{\xi}}$$

- Productivity process: $\log A_{t+1} = \rho \log A_t + \varepsilon^A_{t+1},$

  with std(ε^A)=σ^A

Simulate at **monthly** frequency.

Parameters

As in Golosov and Lucas (2007):

- Discounting: β=0.99 quarterly
- CRRA: γ=2
- Labor supply coefficient: χ=6
- Money demand coefficient: ν=1
- Elasticity of substitution: ε=7
- Money growth: 0% (as in AC Nielsen data)

Adjustment probability:

$$\lambda(L) = \frac{\bar{\lambda}}{\bar{\lambda} + (1-\bar{\lambda})(\alpha / L)^{\xi}}$$

- Productivity process: $\log A_{t+1} = \rho \log A_t + \varepsilon^A_{t+1},$

  with $\mathrm{std}(\varepsilon^A)=\sigma^A$

Simulate at **monthly** frequency.

Estimate free parameters

Estimation

- **Estimated parameters**:

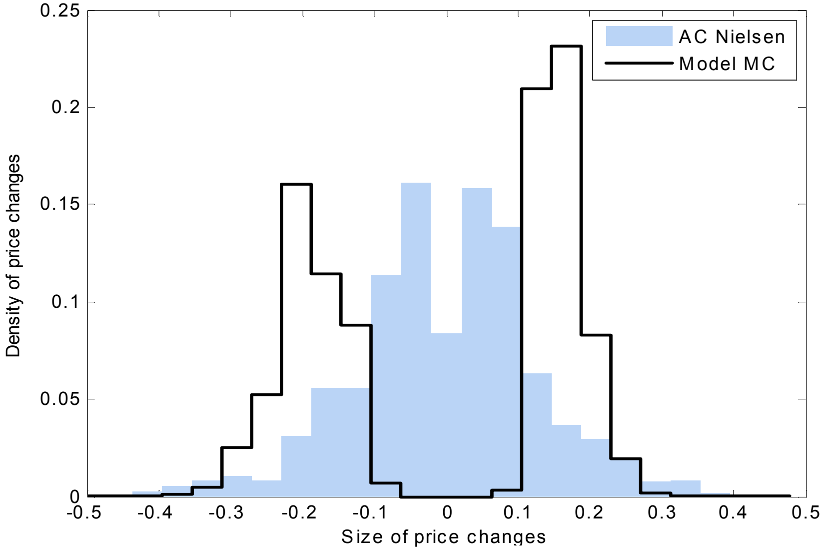
$$\log A_{t+1} = \rho \log A_t + \varepsilon^A_{t+1}, \quad std(\varepsilon^A_{t+1}) = \sigma^A$$

$$\lambda(L) = \frac{\bar{\lambda}}{\bar{\lambda} + (1-\bar{\lambda})(\alpha / L)^{\xi}}$$

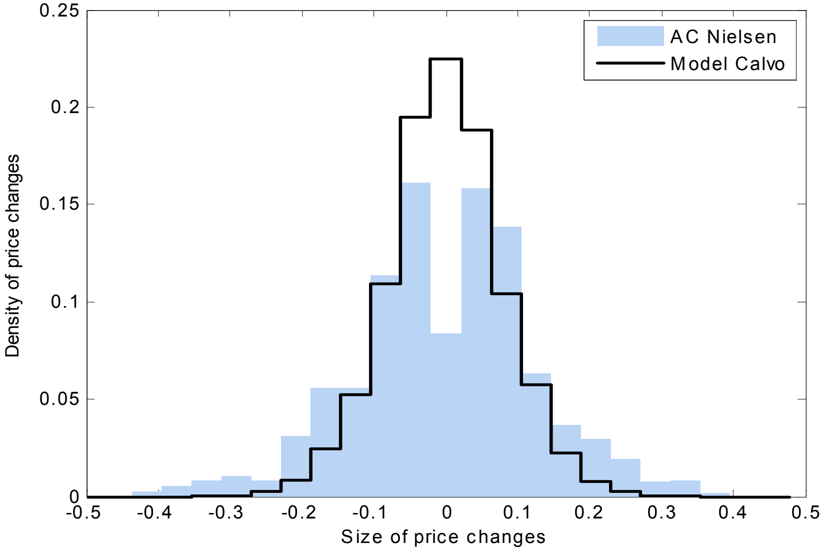
- Microdata on **price adjustments:**
  - –AC Nielsen supermarket data (Midrigan 2008)

- Minimize **objective function** with two terms:
  - –Mean adjustment **frequency** in model vs. data
  - –**Histogram** of price adjustments in model vs. data

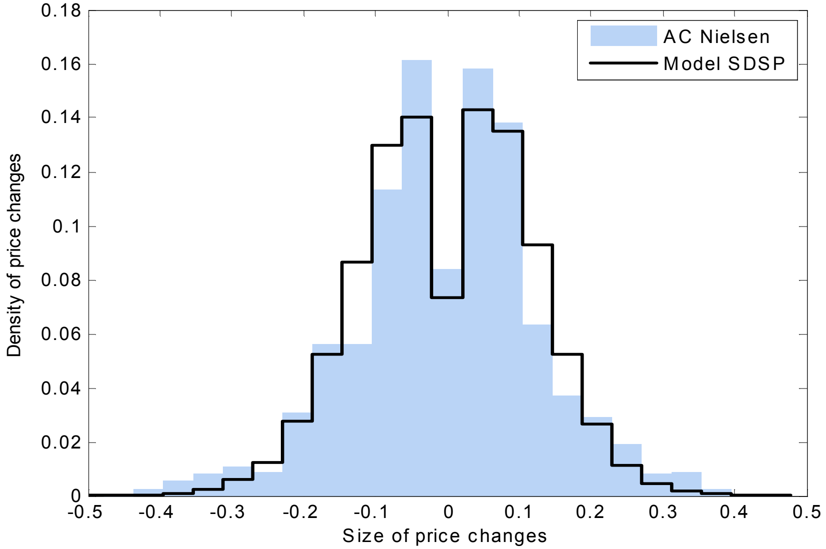
Price changes: models vs. evidence



Actual and simulated distribution of price changes

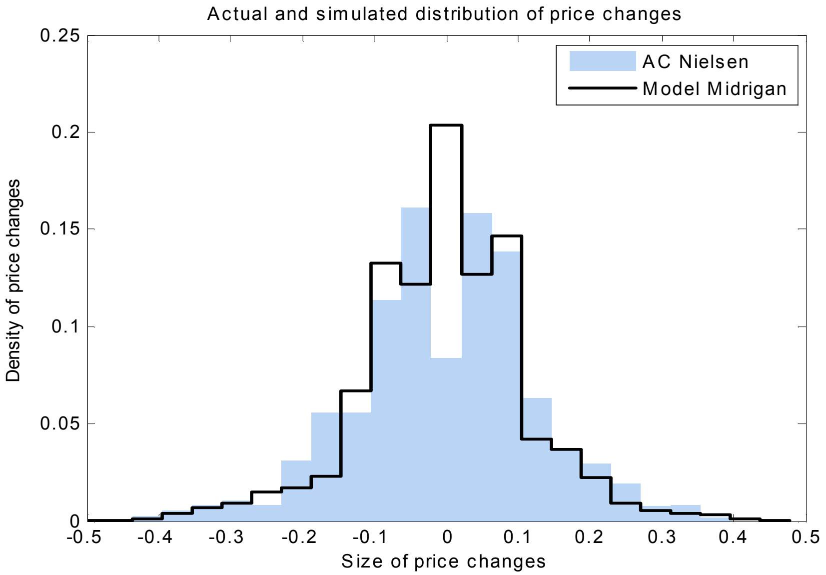


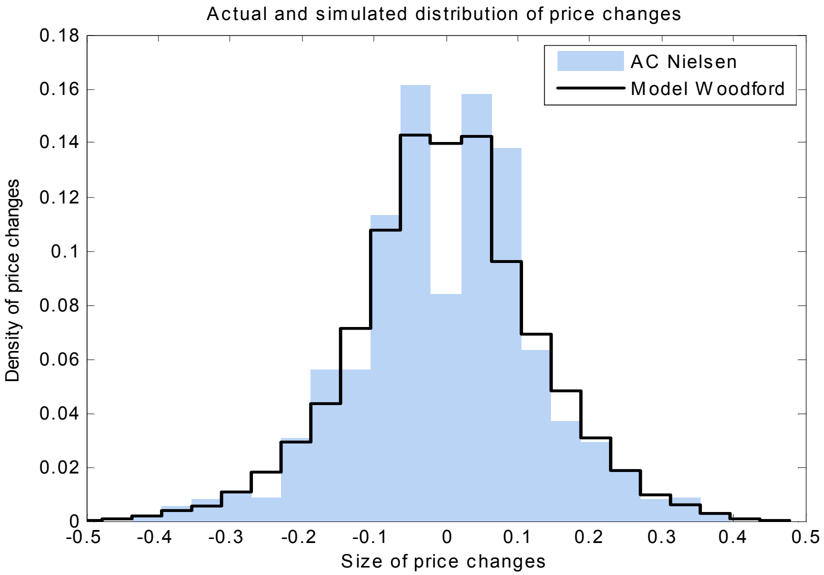
Actual and simulated distribution of price changes

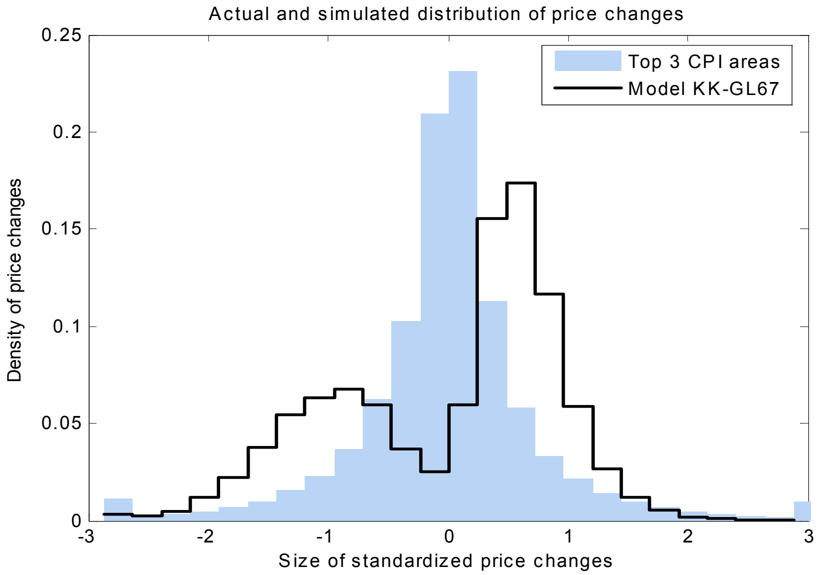


Actual and simulated distribution of price changes

Price changes:
models vs. evidence







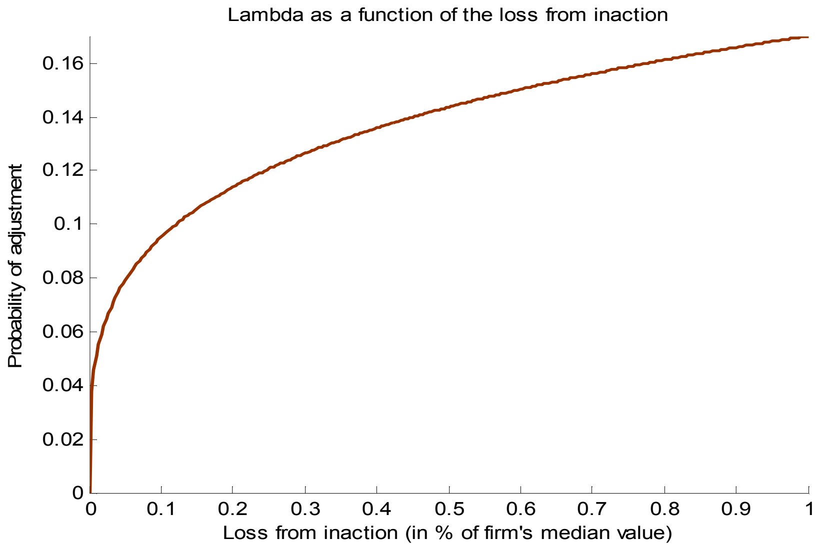
Price changes: models vs. evidence

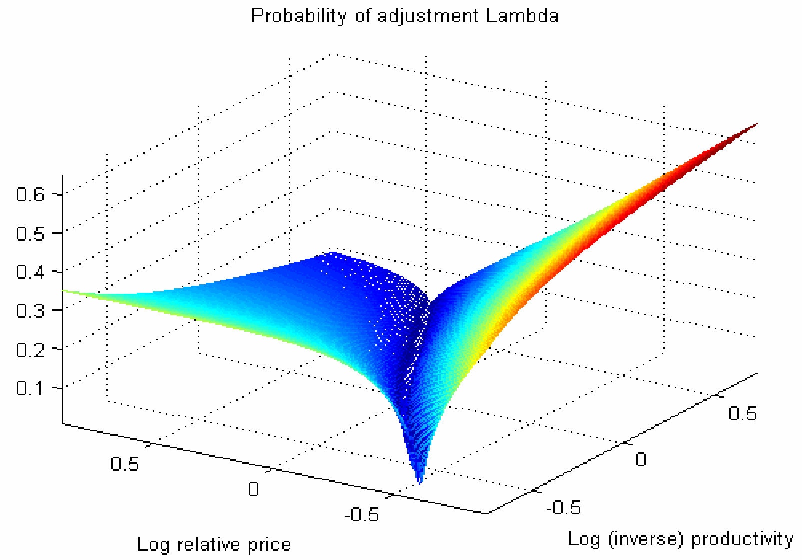
	GL07	Calvo	CN	Data
Monthly frequency of changes	10%	10%	10%	10% (NS07)
Mean absolute price change	18.3	6.4	10.1	10.5 (VM08)
Standard deviation	18.8	8.2	12.1	13.2 (VM08)
Changes less than 5%	0%	49.7%	25.2%	25% (VM08)
Variance of λ relative to GL07	1	0	0.025	

Benchmark calibration: probability of adjustment

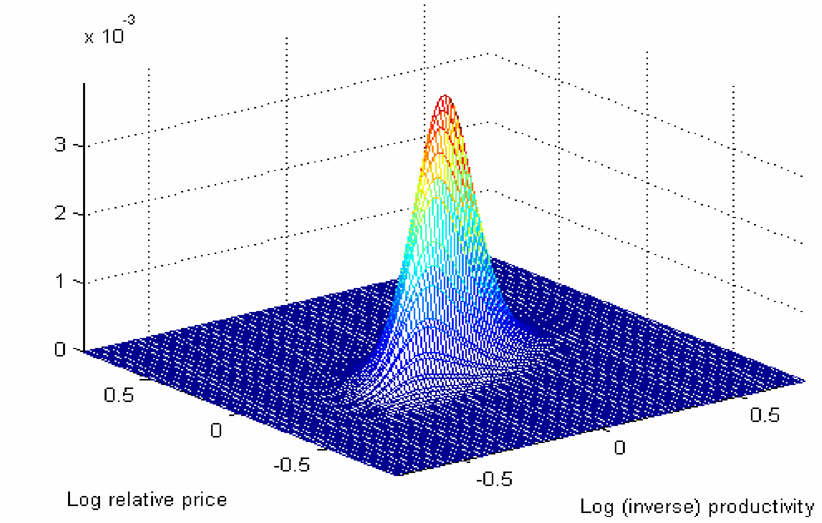
$$\lambda(L) = \frac{\bar{\lambda}}{(1-\bar{\lambda})(\alpha / L)^{\xi} + \bar{\lambda}}$$

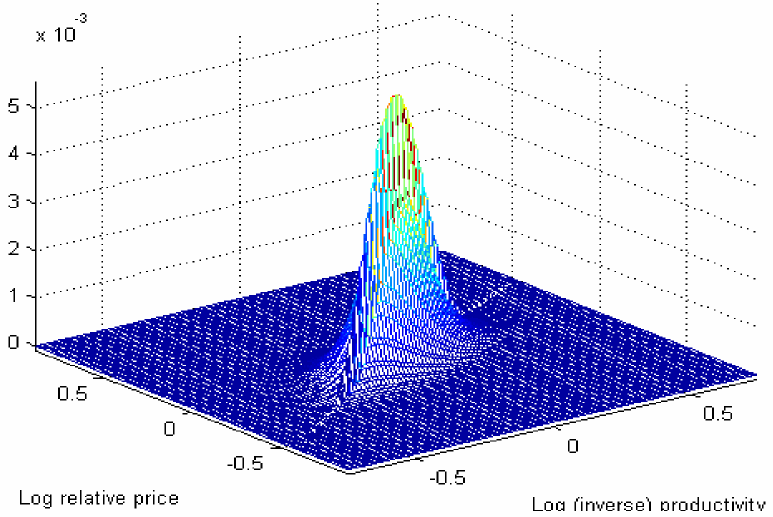
$$(\sigma_{\varepsilon}^{2}, \rho, \bar{\lambda}, \alpha, \boxed{\xi}) = (0.005,\ 0.881,\ 0.109,\ 0.031,\ \boxed{0.290})$$

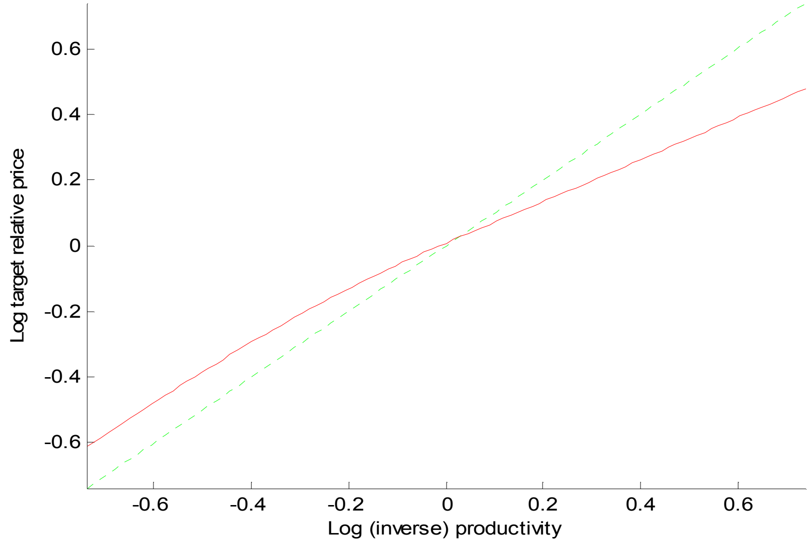


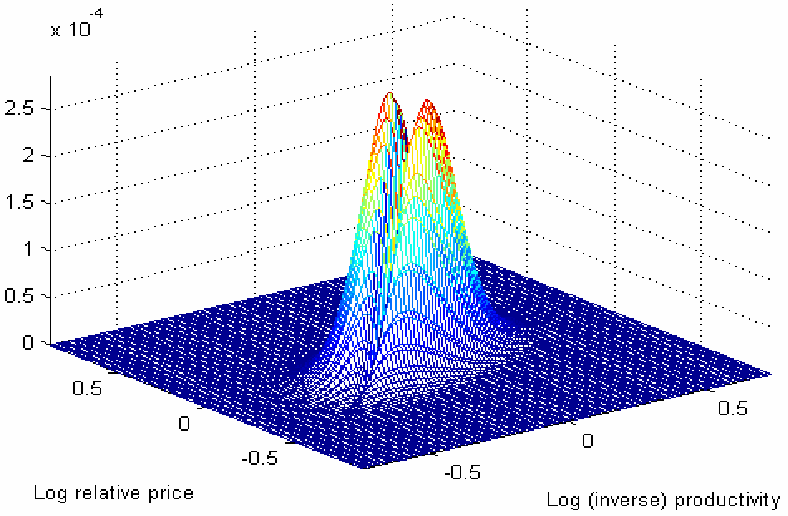


Benchmark calibration: distributions and price policy


Density of firms after mc shock and inflation


Stationary density of firms


Optimal price policy


Density of adjusting firms

Computing general equilibrium:

dynamics

Is Krusell-Smith method suitable for menu cost model?

Consider Golosov-Lucas (2007) menu cost model...

- Suppose firm adjusts if it deviates from optimum by 5%.

Two possible initial conditions:

- All firms deviate by 1% from optimal price
  - Result: 0% of firms adjust

- 10% of firms deviate by 10% from optimal price
  - Result: 10% of firms adjust

- Krusell-Smith method assumes **tomorrow's mean varies smoothly with today's mean**
- Fails to predict tomorrow's mean in menu cost model because individual choices are **highly nonlinear**

Method of Reiter (2008)

Reiter (2008, forthcoming *JEDC*):

- Often, individual shocks bigger than aggregate shocks.
- Therefore: **individual choice** needs **nonlinear solution**, but **linear solution** suffices for **aggregate dynamics.**

- Step 1: detailed **nonlinear** solution of **steady state** on **grid**
  - Solve individual choices by **backwards induction** on grid

- Step 2: **Linearize dynamics at every grid point**
  - Viewed **point by point**, the Bellman equation is just a system of first-order expectational difference equations
  - Many equations, but standard toolkits applicable (Sims, Klein, etc)

- Surprise: **It's easy!!!**

Is Reiter's method suitable for menu cost model?

Consider Golosov-Lucas (2007) menu cost model...

- –Suppose firm adjusts if it deviates from optimum by 5%.

Two possible initial conditions:

- All firms deviate by 1% from optimal price
  - –Result: 0% of firms adjust

- 10% of firms deviate by 10% from optimal price
  - –Result: 10% of firms adjust

- Reiter's method can capture this difference: 1% and 10% from optimal price are **different grid points**
- Each grid point treated by a **different equation**... coefficients of these equations **not** linearly related

Step 1: steady state (already done)

- **Guess:** p

- **Euler, FOC:**

$$C^{\gamma} = (1-\beta\mu)/p,\ \ w = \chi C^{\gamma} \quad \rightarrow \quad u^{jk} = \left(p^{j} - \frac{w}{a^{k}}\right) C \left(\frac{p^{j}}{p}\right)^{-\varepsilon}$$

- **Solve Bellman:**

$$\mathbf{V} = \mathbf{U} + \beta \mathbf{R}' * (\mathbf{V} + \mathbf{G}) * \mathbf{S}$$

- **Calculate distributions:** $\widetilde{\mathbf{\Psi}} = \mathbf{R} * \mathbf{\Psi} * \mathbf{S}'$

$$\mathbf{\Psi} = (\mathbf{1} - \mathbf{\Lambda}).*\widetilde{\mathbf{\Psi}} + \mathbf{P}.*(\mathbf{1}*(\mathbf{\Lambda}.*\widetilde{\mathbf{\Psi}})$$

- **Calculate** $p = \left\{\sum_{j}\sum_{k}\psi^{jk}(p^{j})^{1-\varepsilon}\right\}^{\frac{1}{1-\varepsilon}}$ **and return.**

Dynamics

1. F.O.C. labor: $C_t^{-\gamma} = \chi / w_t$
2. Euler equation: $1 - p_t C_t^{\gamma} = \beta E_t \left(\mu_{t+1} \frac{p_t C_t^{\gamma}}{p_{t+1} C_{t+1}^{\gamma}} \right)$
3. Labor demand: $N_t = \sum_j \sum_k \Psi_t^{jk} (p^j / p_t)^{-\varepsilon} \, {}^{C_t}\!\big/_{a^k} \equiv \Delta_t C_t$
4. Price index: $p_t = \left\{ \sum_j \sum_k \Psi_t^{jk} (p^j)^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}}$

Dynamics

Matrices $\mathbf{U}_t, \mathbf{V}_t, \mathbf{D}_t, \mathbf{G}_t, \mathbf{\Psi}_t, \widetilde{\mathbf{\Psi}}_t, \mathbf{1}, \mathbf{P}_t$ have size N^p x N^A.

5. **Bellman is:**

$$\mathbf{V}_t = \mathbf{U}_t + \beta E_t \left\{ \frac{p_t C_{t+1}^{\gamma}}{p_{t+1} C_t^{\gamma}} \mathbf{R}_{t+1}' * (\mathbf{V}_{t+1} + \mathbf{G}_{t+1}) * \mathbf{S} \right\}$$

6. **Distributions satisfy:**
   - Beginning of period: $$\widetilde{\mathbf{\Psi}}_{t+1} = \mathbf{R}_{t+1} * \mathbf{\Psi}_t * \mathbf{S}'$$
   - Time of production: $$\mathbf{\Psi}_t = (\mathbf{1} - \mathbf{\Lambda}_t) .* \widetilde{\mathbf{\Psi}}_t + \mathbf{P}_t .* (\mathbf{1} * (\mathbf{\Lambda}_t .* \widetilde{\mathbf{\Psi}}_t))$$
   - JUST A HUGE SYSTEM OF 1ST-ORDER DIFFERENCE EQS... **CAN BE LINEARIZED!** (Reiter 2008)

Step 2: Linearized dynamics

Suppose $\hat{\mu}_{t+1} = \phi\hat{\mu}_t + \varepsilon_{t+1}$ with ε_{t+1} *iid*, mean zero.

Define row vector: $\mathbf{X}_t \equiv (vec(\mathbf{V}_t)', C_t, p_t, vec(\mathbf{\Psi}_{t-1})')$

length: $2N^P N^\pi +2$

Equilibrium dynamics can be summarized by:

$$E_t \boldsymbol{F}\left(\mathbf{X}_{t+1}, \mathbf{X}_t, \hat{\mu}_{t+1}, \hat{\mu}_t\right) = 0$$

$2N^P N^\pi +3$ first-order expectational difference equations in $2NPN\pi +3$ series.

Step 2: Linearized dynamics

Suppose $\hat{\mu}_{t+1} = \phi\hat{\mu}_t + \varepsilon_{t+1}$ with ε_{t+1} *iid*, mean zero.

Define row vector: $\mathbf{X}_t \equiv (vec(\mathbf{V}_t)', C_t, p_t, vec(\mathbf{\Psi}_{t-1})')$

length: $2N^PN^\pi +2$

Equilibrium dynamics can be summarized by:

$$E_t \boldsymbol{F}\left(\mathbf{X}_{t+1}, \mathbf{X}_t, \hat{\mu}_{t+1}, \hat{\mu}_t\right) = 0$$

$2N^PN^\pi +3$ first-order expectational difference equations in 2NPNπ +3 series.

(We've eliminated w_t , N_t , and $\tilde{\Psi}_t$.)

Step 2: Linearized dynamics

Suppose $\hat{\mu}_{t+1} = \phi\hat{\mu}_t + \varepsilon_{t+1}$ with ε_{t+1} *iid*, mean zero.

Define row vector: $\mathbf{X}_t \equiv (vec(\mathbf{V}_t)', C_t, p_t, vec(\mathbf{\Psi}_{t-1})')$

length: $2N^P N^\pi + 2$

Steady state satisfies:

$$\boldsymbol{F}\left(\mathbf{X}^*, \mathbf{X}^*, 0, 0\right) = 0$$

We’ve already solved for the steady state (“step 1”).

Step 2: Linearized dynamics

Suppose $\hat{\mu}_{t+1} = \phi\hat{\mu}_t + \varepsilon_{t+1}$ with ε_{t+1} *iid*, mean zero.

Define row vector: $\mathbf{X}_t \equiv (vec(\mathbf{V}_t)', C_t, p_t, vec(\mathbf{\Psi}_{t-1})')$

length: $2N^P N^\pi + 2$

Linearization around steady state:

$$AE_t\Delta\mathbf{X}_{t+1} + B\Delta\mathbf{X}_t + CE_t\hat{\mu}_{t+1} + D\hat{\mu}_t = 0$$

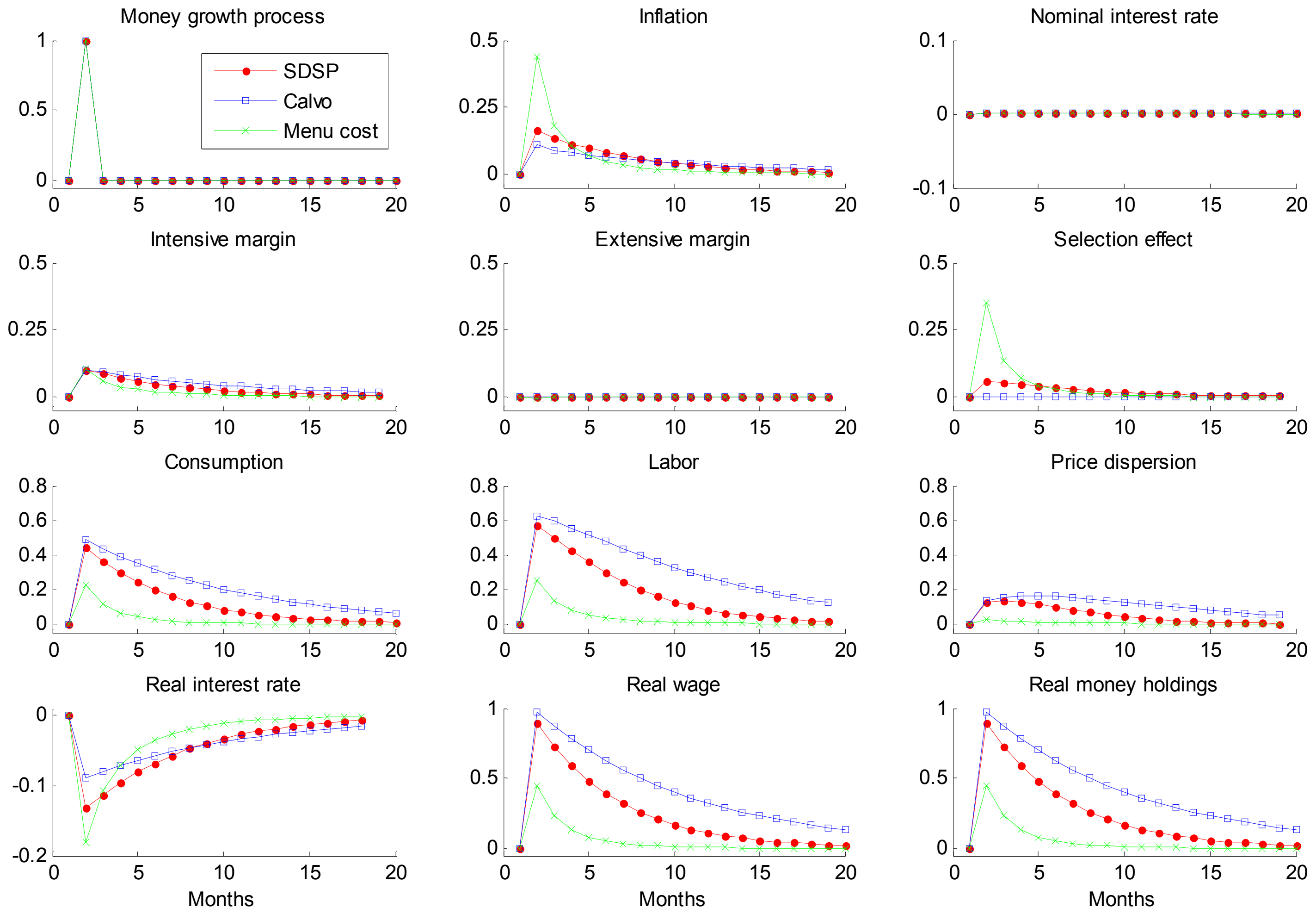
Can be solved by Paul Klein’s “QZ” algorithm.

Alternatives: Chris Sims’ algorithm, etc.

General equilibrium:

dynamic results

Impulse responses: *iid* money growth shock



Inflation decompositions

- **Inflation identity**

$$\pi_t = \sum_{j,k} x_t^{jk} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}$$

where $x_t^{jk} = \log\left(\frac{p_t^*(a^k)}{p^j}\right)$ is the **desired price adjustment**, in logs

Inflation decompositions

- **Inflation identity**

$$\pi_t = \sum_{j,k} \overset{\substack{\text{Desired}\\ \text{price change}\\ \downarrow}}{x_t^{jk}} \underset{\substack{\uparrow\\ \text{Probability of}\\ \text{adjustment}}}{\lambda_t^{jk}} \tilde{\Psi}_t^{jk} \leftarrow \substack{\text{Distribution}\\ \text{after shock}}$$

Inflation decompositions

- **Inflation identity**

$$\pi_t = \sum_{j,k} \overset{\substack{\text{Desired}\\ \text{price change}\\ \downarrow}}{x_t^{jk}} \underset{\substack{\uparrow\\ \text{Probability of}\\ \text{adjustment}}}{\lambda_t^{jk}} \widetilde{\Psi}_t^{jk} \leftarrow \substack{\text{Distribution}\\ \text{after shock}}$$

- **Klenow and Kryvtsov**
  - Intensive and extensive margin

$$\pi_t = \frac{\sum_{j,k} x_t^{jk} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}}{\sum_{j,k} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}} \sum_{j,k} \lambda_t^{jk} \widetilde{\Psi}_t^{jk} = av_t \, fr_t$$

$$\Delta\pi_t \approx \overline{fr}\,\Delta av_t + \overline{av}\,\Delta fr_t$$

Inflation decompositions

- **Inflation identity**

Desired
price change

$$\pi_t = \sum_{j,k} x_t^{jk} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}$$

Distribution
after shock

Probability of
adjustment

- **Klenow and Kryvtsov**
  - Intensive and extensive margin

$$\pi_t = \frac{\sum_{j,k} x_t^{jk} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}}{\sum_{j,k} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}} \sum_{j,k} \lambda_t^{jk} \widetilde{\Psi}_t^{jk} = av_t\, fr_t$$

$$\Delta\pi_t \approx \underbrace{\overline{fr}\,\Delta av_t}_{\text{Intensive margin}} + \underbrace{\overline{av}\,\Delta fr_t}_{\text{Extensive margin}}$$

Inflation decompositions

- **Inflation identity**

Desired
price change

$$\pi_t = \sum_{j,k} x_t^{jk} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}$$

Distribution
after shock

Probability of
adjustment

Inflation decompositions

- **Inflation identity**

Desired
price change

$$\pi_t = \sum_{j,k} x_t^{jk} \lambda_t^{jk} \widetilde{\Psi}_t^{jk} \leftarrow$$

Distribution
after shock

Probability of
adjustment

- **Costain and Nakov**
  - Intensive margin, extensive margin, and selection effect

$$\pi_t = \sum_{j,k} x_t^{jk} \widetilde{\Psi}_t^{jk} \sum_{j,k} \lambda_t^{jk} \widetilde{\Psi}_t^{jk} + \sum_{j,k} x_t^{jk} (\lambda_t^{jk} - \sum \lambda_t^{jk} \widetilde{\Psi}_t^{jk}) \widetilde{\Psi}_t^{jk} = av_t^* \, fr_t + sel_t$$

$$\Delta\pi_t \approx \overline{fr}\, \Delta av_t^* + \overline{av}^* \, \Delta fr_t + \Delta sel_t$$

Inflation decompositions

- **Inflation identity**

$$\pi_t = \sum_{j,k} \overset{\substack{\text{Desired}\\ \text{price change}\\ \downarrow}}{x_t^{jk}} \underset{\substack{\uparrow\\ \text{Probability of}\\ \text{adjustment}}}{\lambda_t^{jk}} \widetilde{\Psi}_t^{jk} \leftarrow \substack{\text{Distribution}\\ \text{after shock}}$$

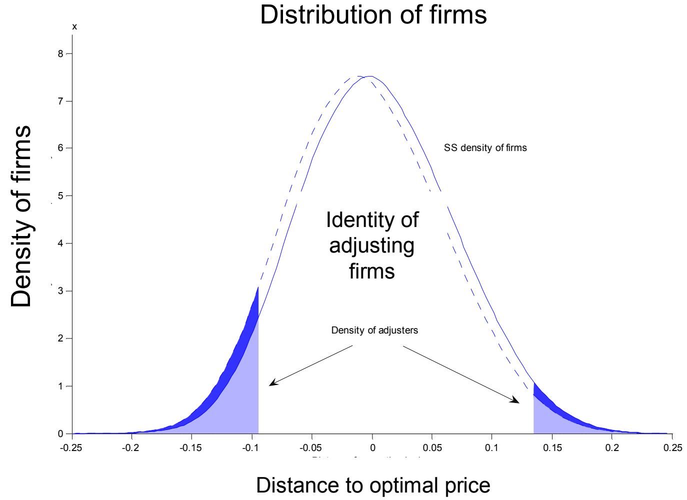
- **Costain and Nakov**
  - Intensive margin, extensive margin, and selection effect

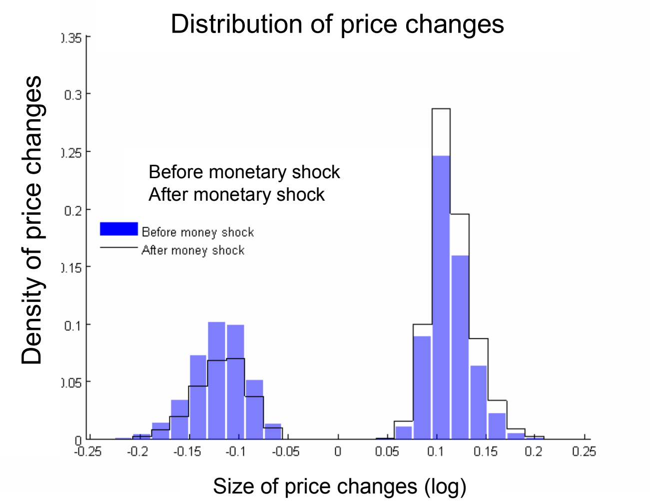
$$\pi_t = \sum_{j,k} x_t^{jk} \widetilde{\Psi}_t^{jk} \sum_{j,k} \lambda_t^{jk} \widetilde{\Psi}_t^{jk} + \sum_{j,k} x_t^{jk} (\lambda_t^{jk} - \sum \lambda_t^{jk} \widetilde{\Psi}_t^{jk}) \widetilde{\Psi}_t^{jk} = av_t^* \, fr_t + sel_t$$

$$\Delta\pi_t \approx \underbrace{\overline{fr}\,\Delta av_t^*}_{\substack{\text{Intensive}\\ \text{margin}}} + \underbrace{\overline{av}^*\ \Delta fr_t}_{\substack{\text{Extensive}\\ \text{margin}}} + \underbrace{\Delta sel_t}_{\substack{\text{Selection}\\ \text{effect}}}$$

Why the difference with GL07?

Golosov-
Lucas
(2007)

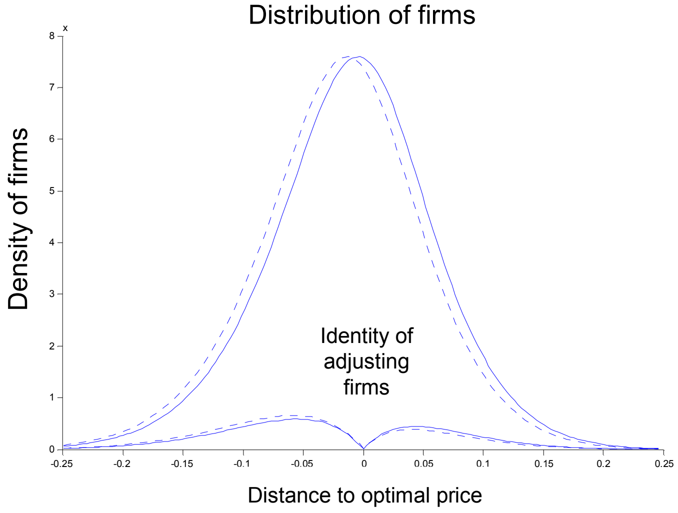


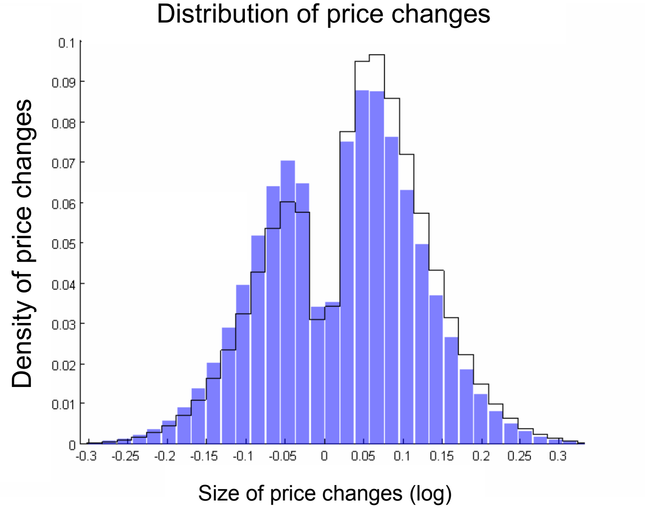


- Fixed menu costs imply strong **selection effect**:
  - –Firms that adjust are far from optimal price.
- Shock redistributes mass from price decreases to price increases
  - –Large change in average adjustment, Δav_t ...
  - –...even if small change in average desired adjustment, Δav_t^*
- Depends on firms jumping from $\lambda^{jk}=0$ to $\lambda^{jk}=1$
  - –Such **strong state dependence rejected** by estimate of λ(L)

Why the difference with GL07?

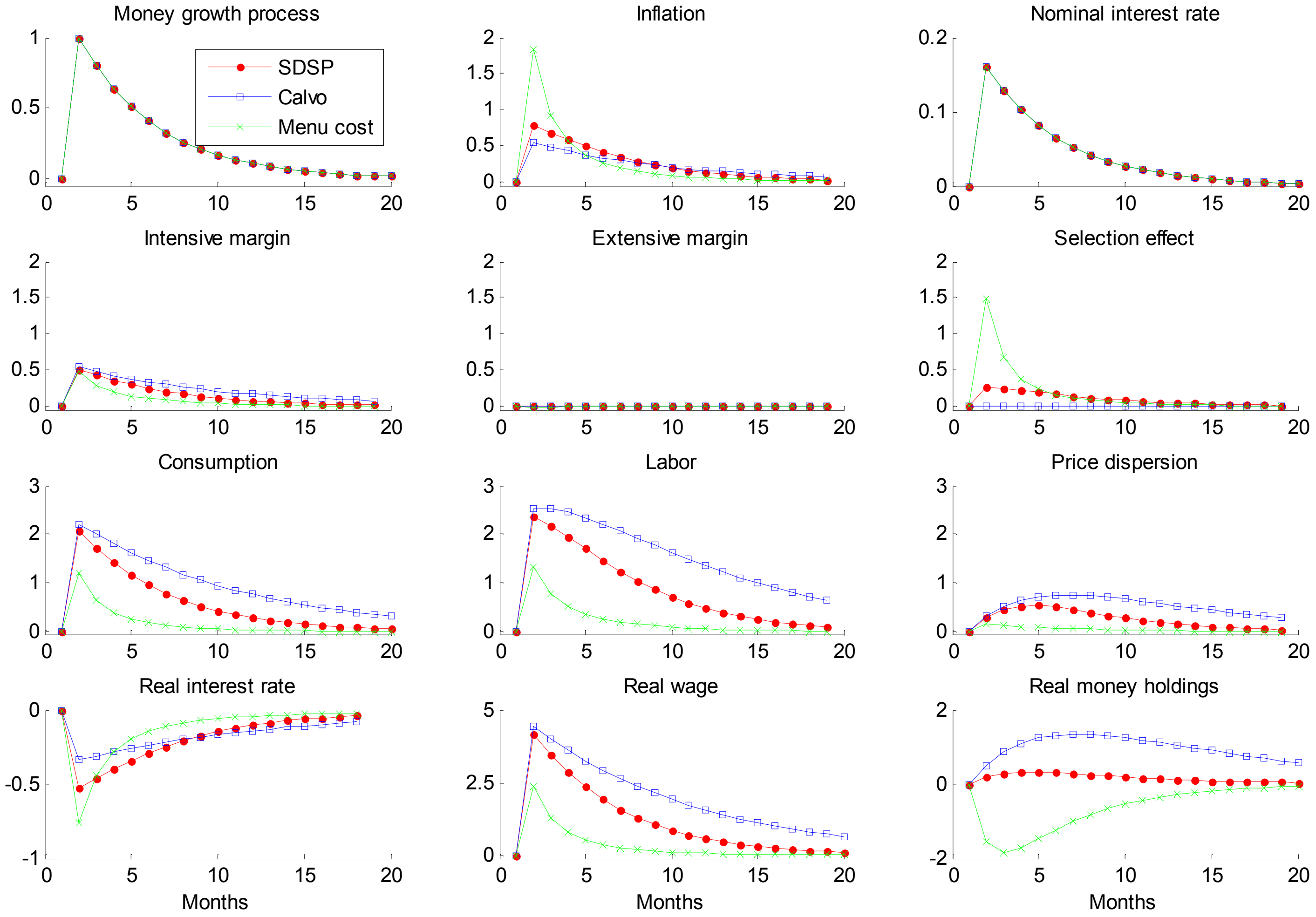
Costain-Nakov (2008)





- In estimated model, many adjusters are near optimal price
- Selection effect smaller (1/3 of change in inflation)
  - Average adjustment similar to average desired adjustment: $\Delta av_t \approx \Delta av_t^*$
- Shock falls less on inflation, **more on output**

Impulse responses: autocorrelated money growth



Real impact of money growth shocks

"Phillips curve": $\log(C_t) = \alpha + \beta \log(\mu_t)$

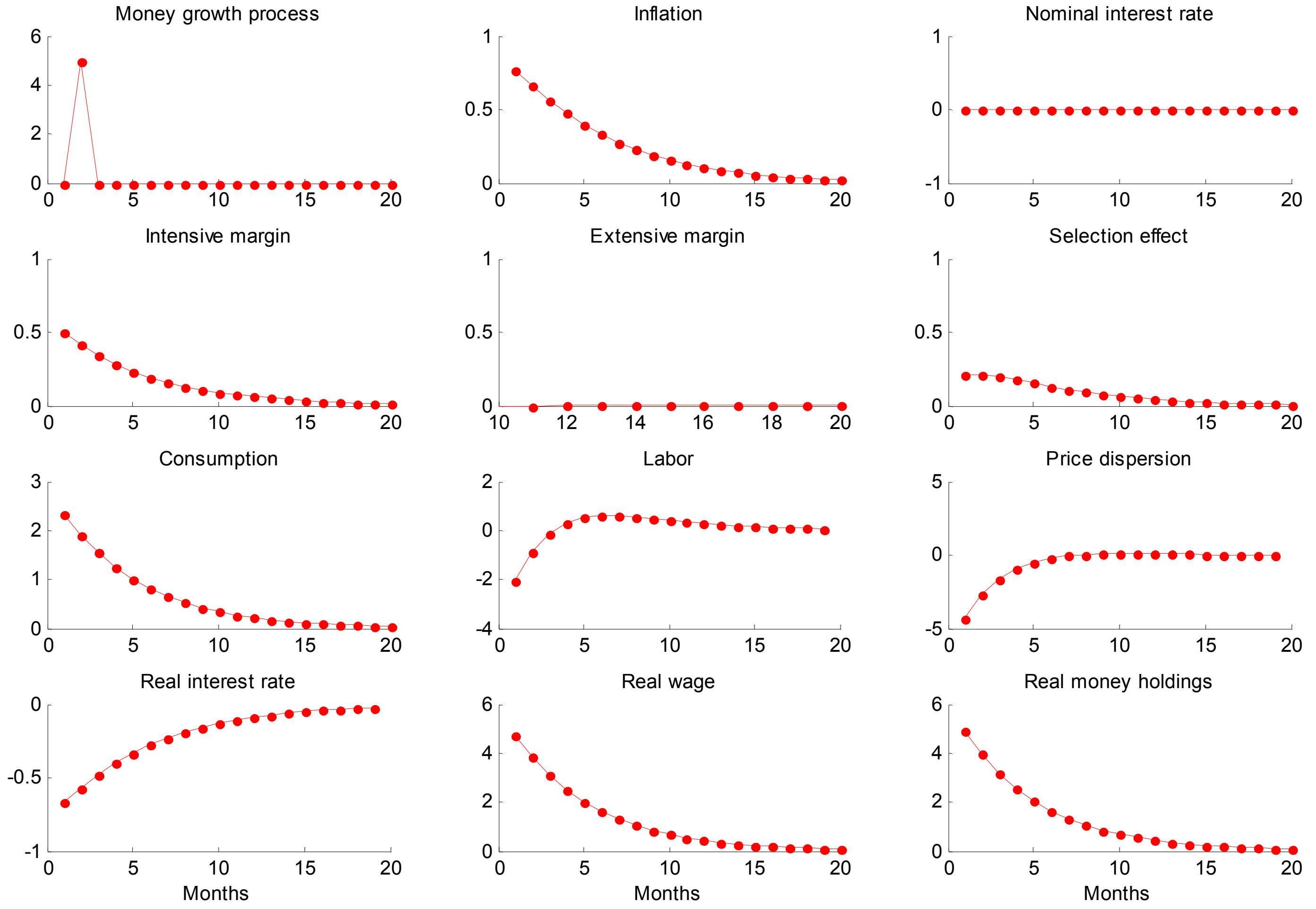
Uncorrelated shocks $(\phi_\mu = 0)$	GL07	Calvo	CN
Std dev money growth shock (x100)	0.52	1.05	0.81
Std dev quarterly inflation (x100)	0.25	0.25	0.25
% explained by nominal shock	100%	100%	100%
Std dev quarterly output growth (x100)	0.13	0.72	0.49
% explained by nominal shock	26%	142%	96%
Slope of the Phillips curve (β)	0.23	0.52	0.46
Standard error	0.00	0.03	0.02
R^2	0.71	0.20	0.33

Real impact of money growth shocks

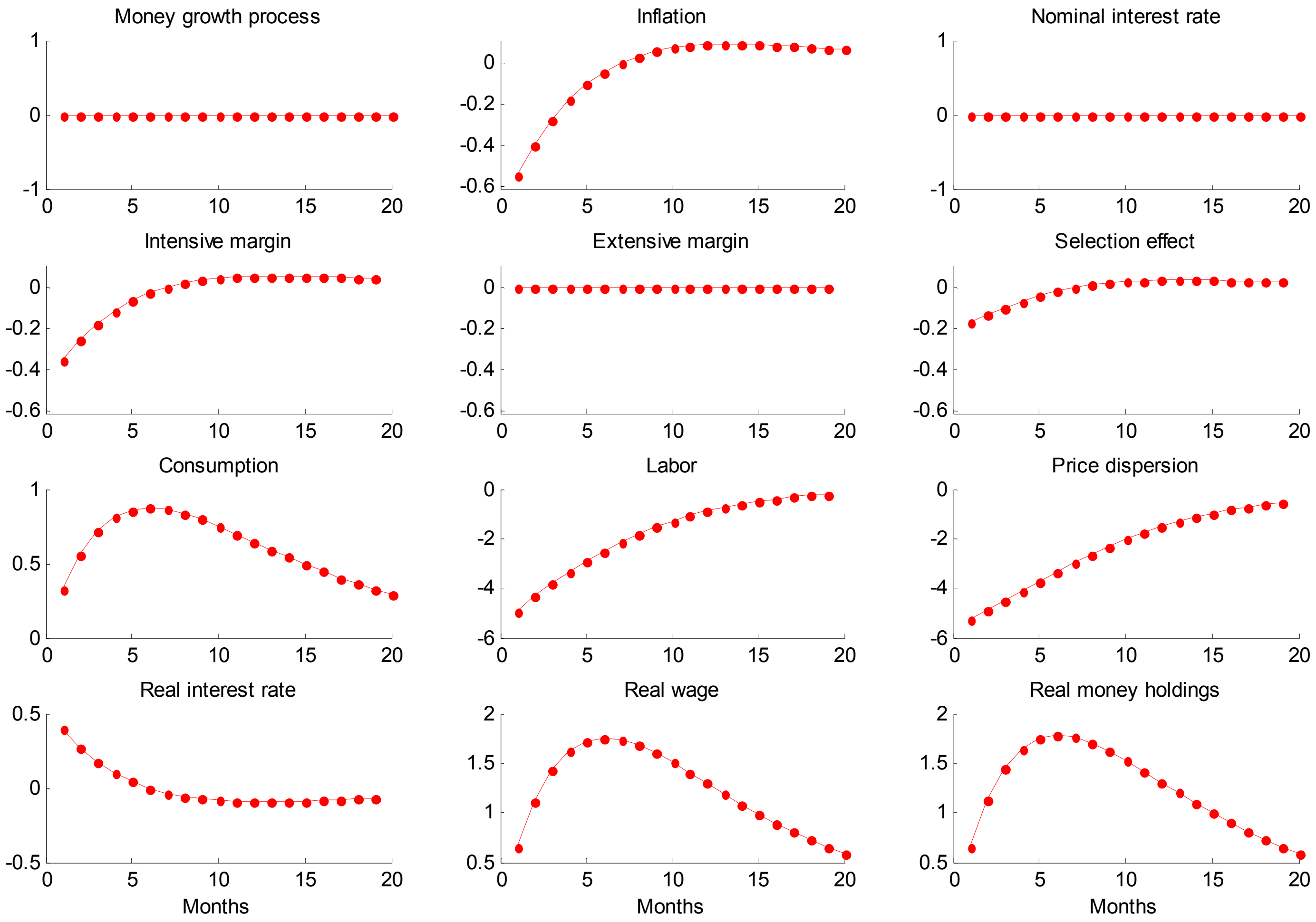
"Phillips curve": $\log(C_t) = \alpha + \beta \log(\mu_t)$

Correlated shocks $(\phi_\mu = 0.8)$	GL07	Calvo	CN
Std dev money growth shock (x100)	0.11	0.21	0.16
Std dev quarterly inflation (x100)	0.25	0.25	0.25
% explained by nominal shock	100%	100%	100%
Std dev quarterly output growth (x100)	0.15	0.67	0.47
% explained by nominal shock	29%	131%	91%
Slope of the Phillips curve (β)	0.82	2.88	2.20
Standard error	0.01	0.03	0.00
R^2	0.89	0.88	0.99

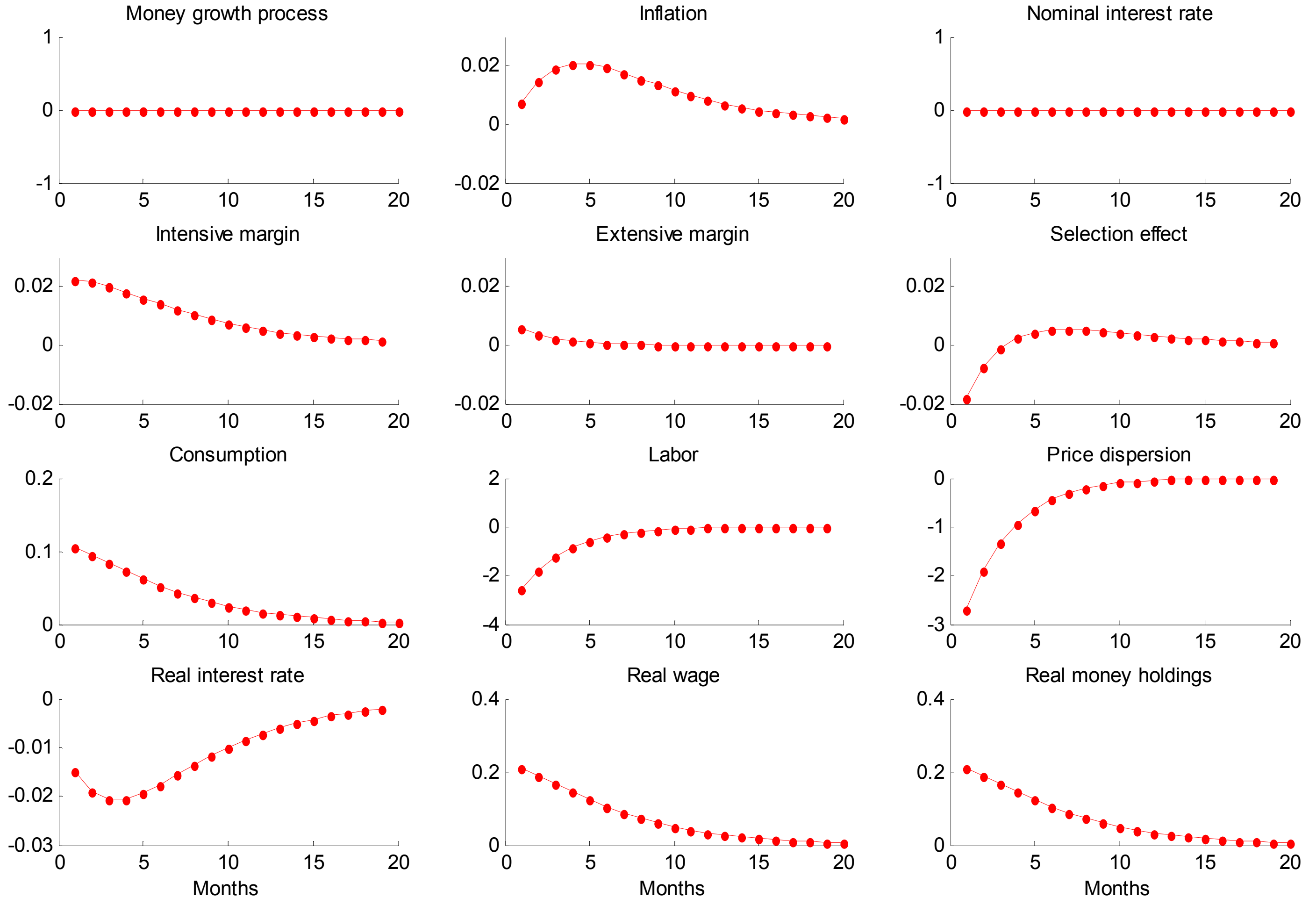
Transitional dynamics: shift in p



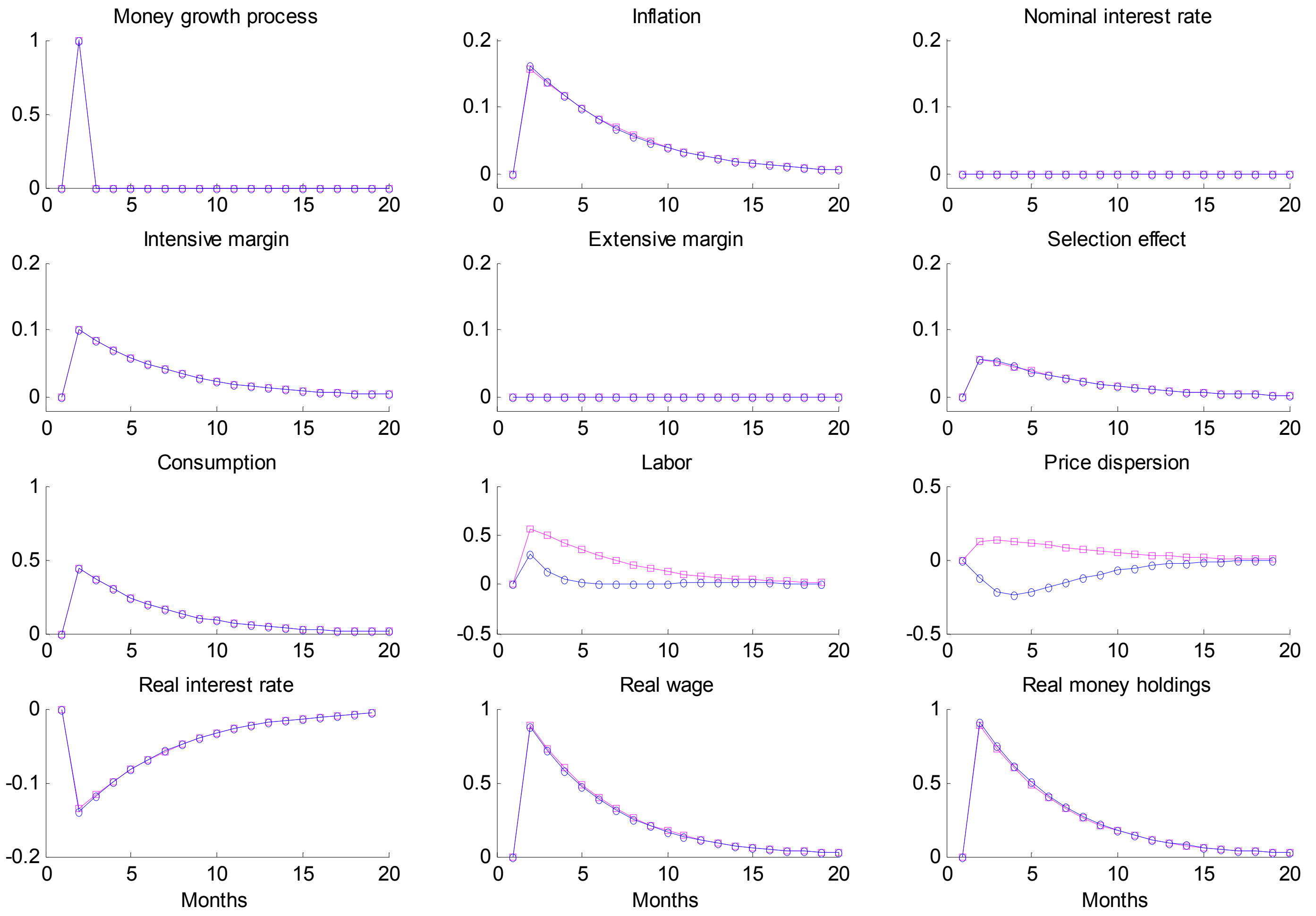
Transitional dynamics: shift in A



Transitional dynamics: “post-euro”



Effect of starting conditions: IRFs after productivity shock



Conclusions

- Now feasible to **compute DSGE distributional dynamics with state-dependent prices**
  - Reiter (2008)
- **Benchmark calibration: money growth shocks not neutral**
- **Autocorrelated shocks: effects similar, but stronger**
- Near-neutrality in GL07 calibration due to strong **selection effect**
  - strong selection effect and lack of small price changes both due to counterfactual degree of state dependence
- Additional findings:
  - **Price dispersion** quantitatively important (unlike standard NK DSGE)
  - Increased **aggregate productivity** causes labor to fall (like NK DSGE)

Conclusions

- Now feasible to **compute DSGE distributional dynamics with state-dependent prices**
  - Reiter (2008)

- **Benchmark calibration: money growth shocks not neutral**

- **Autocorrelated shocks: effects similar, but stronger**

- Near-neutrality in GL07 calibration due to strong **selection effect**
  - strong selection effect and lack of small price changes both due to counterfactual degree of state dependence

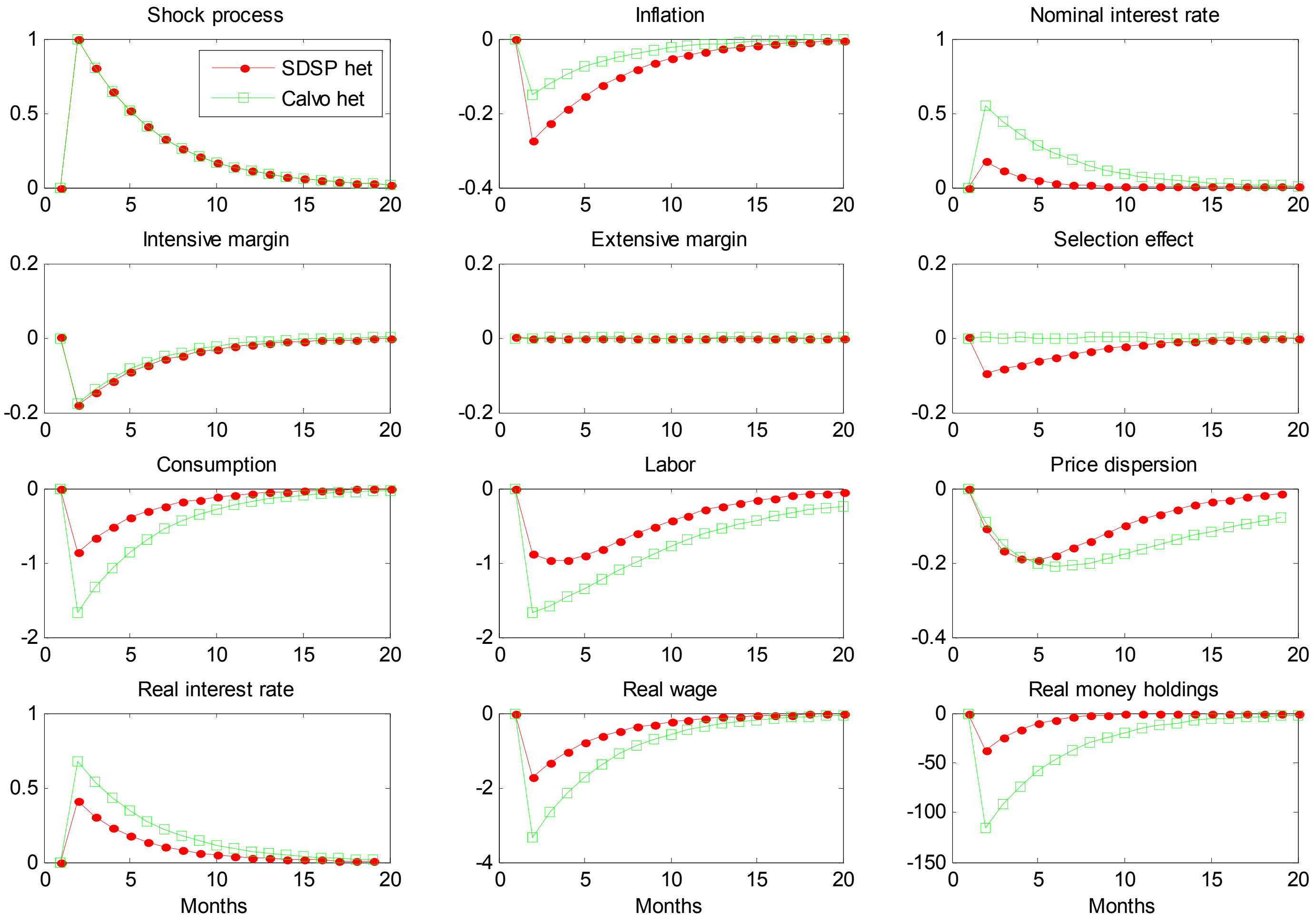
- Additional findings:
  - **Price dispersion** quantitatively important (unlike standard NK DSGE)
  - Increased **aggregate productivity** causes labor to fall (like NK DSGE)

- **Phillips curve is alive and well!**

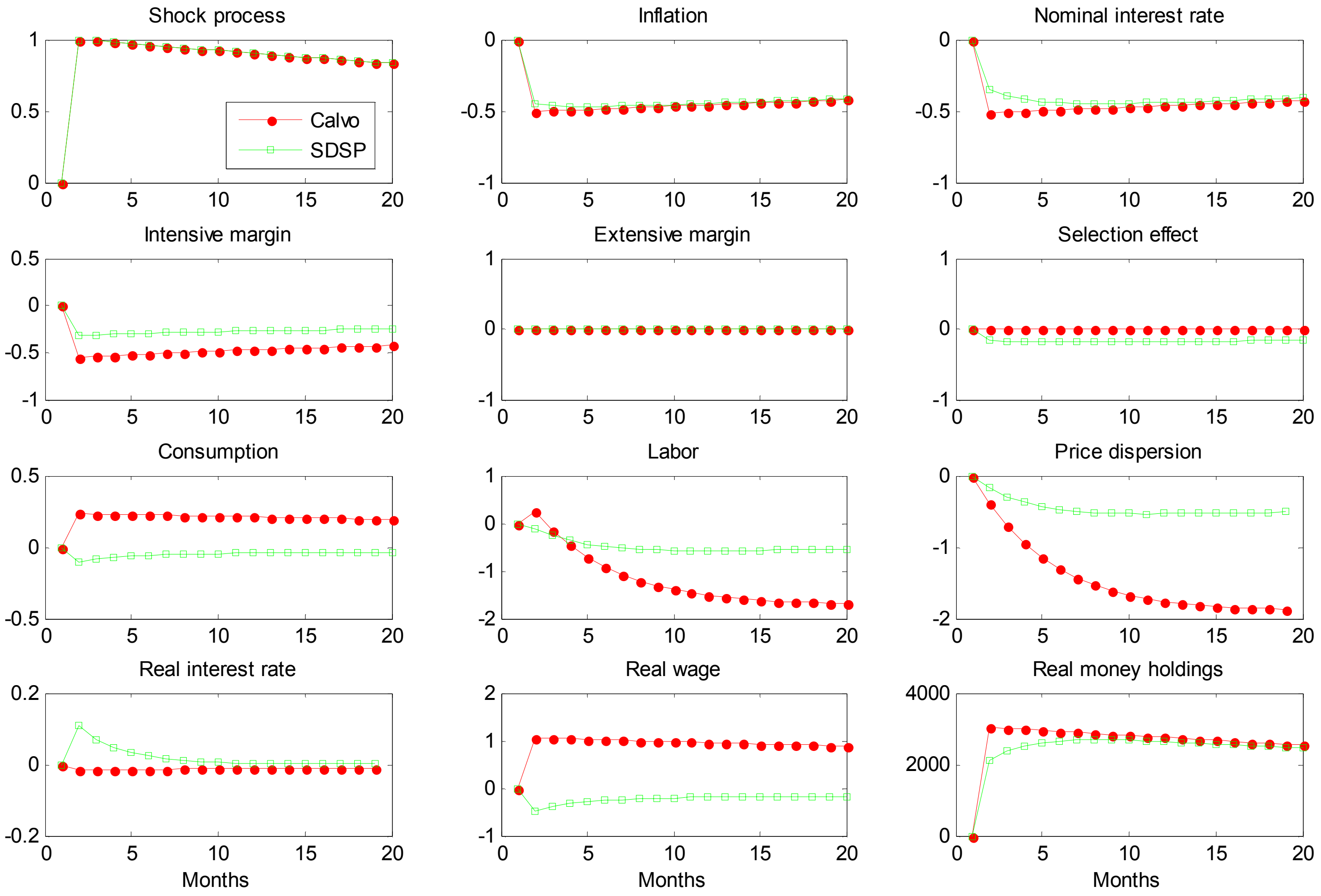
Some extensions:

Taylor rules

Shock in Taylor rule



Taylor rule: “Permanent” disinflation



James Costain

THANK YOU FOR YOUR ATTENTION

BANCO DE ESPAÑA

150 AÑOS DE HISTORIA
1856-2006

SERVICIO DE ESTUDIOS