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# **Search Frictions, Real Rigidities and Inflation Dynamics**

Carlos Thomas

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# Search frictions, real rigidities and inflation dynamics\*

Carlos Thomas<sup>†</sup>

Bank of Spain

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## Abstract

I analyze the effect of search frictions on inflation dynamics, in a New Keynesian model where the firms making the pricing decisions *are* subject to such frictions. I find that search frictions create real rigidities in price setting. This mechanism flattens the New Keynesian Phillips curve, relative both to the standard model with a frictionless labor market and a search model where price-setters are not subject to search frictions. This has a number of consequences for the economy's cyclical behavior. First, inflation becomes more persistent. Second, output responses to monetary shocks become larger and more persistent. Third, unemployment becomes more volatile conditional on monetary shocks. Search frictions thus reduce the need for the kind of ad-hoc persistence and amplification mechanisms commonly used in the literature.

*JEL classification:* E32, J40

*Keywords:* search and matching, real rigidities, New Keynesian Phillips curve

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<sup>†</sup>Research Department, Bank of Spain, Alcalá 48, 28014 Madrid, Spain. E-mail: carlos.thomas@bde.es. Tel.: +34 91 338 6280.

# 1 Introduction

The search and matching model has become a popular treatment of labor market dynamics in New Keynesian models of the monetary transmission mechanism.<sup>1</sup> Among other things, this kind of framework allows to analyze the joint dynamics of unemployment and inflation. Due to search frictions in the labor market, it takes time for unemployed workers to find jobs. This, together with recurrent job destruction, gives rise to unemployment in equilibrium. On the other side of the labor market, search frictions imply that it takes time for firms to find suitable workers. In principle, this latter feature implies that firms cannot immediately increase their number of workers in order to meet an increase in demand. By shutting down the employment margin as a means of increasing output in the short run, search frictions ought to have an important effect on the cyclical behavior of marginal costs. As a result, search frictions should also have important consequences for individual pricing decisions, and hence for inflation dynamics.

In fact, the existing literature has paid very little attention to this mechanism. The reason is an assumption commonly made in previous studies, namely that the firms making the pricing decisions are different from the firms that are subject to search frictions.<sup>2</sup> These two subsets of firms are usually called 'retailers' and 'producers', respectively. This assumption is very convenient, as it allows one to disentangle forward-looking vacancy posting and pricing decisions and thus simplify the analysis considerably. However, it eliminates from the outset the possibility of analyzing the effect of search frictions on the pricing decisions of individual firms. The aim of this paper is to analyze how search frictions affect inflation dynamics, in a model where price-setters face such frictions. In particular, I consider a framework in which firms reset their prices at random intervals. In order to meet a sudden change in demand for its product, each firm can adjust the number of hours worked by its employees, but not the size of its workforce. In order to adjust employment, the firm must first post vacancies and then wait for the latter to be filled.

I find that search frictions flatten the New Keynesian Phillips curve, relative both to the standard New Keynesian model with a frictionless labor market and the model with a producer-retailer structure. The reason is the following. Due to search frictions, firms' short-run marginal costs depend on the cost of increasing production along the intensive margin of labor. Wage bargaining between the firm and its workers implies that the latter must be compensated for

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<sup>1</sup>For a simple exposition of the search and matching model, see Pissarides (2000, Ch. 1).

<sup>2</sup>See Walsh (2003b, 2005), Trigari (2004, 2006), Christoffel and Linzert (2005), Andres et al. (2006), Blanchard and Gali (2008) and Thomas (2008).

the disutility of work. If the latter is realistically convex in hours worked, firms' marginal cost curves become upward-sloping. This gives price-setting firms an incentive to keep their prices in line with the overall price level. That is, search frictions give rise to *real rigidities* in prices, using Ball and Romer's (1990) terminology. Following for instance an aggregate shock that reduces marginal costs, each price-setter will consider a certain reduction in its nominal price. Given the prices of other firms, the reduction in the firm's nominal price represents a reduction in its real price. This leads the firm to anticipate greater sales and therefore higher marginal costs for the duration of the price contract, and the firm ends up choosing a smaller price cut than the one initially considered. Because all price-setters follow the same logic, real rigidities slow the adjustment of the overall price level in response to the same fluctuations in average real marginal costs. This mechanism is absent in the standard model, where firms can hire as much labor as they want at the market hourly wage. The same holds for the model with a producer-retailer structure, where retailers can buy as much intermediate input from producers as they need at a perfectly competitive price.

The flattening of the New Keynesian Phillips curve has important consequences for inflation dynamics, and more generally for the behavior of the economy as a whole. Inflation becomes more persistent conditional on both monetary and productivity shocks. Therefore, search frictions help the New Keynesian model account for the levels of inflation persistence observed in the data.<sup>3</sup> In addition, a slower adjustment of the price level implies a larger and more persistent response of output to monetary shocks. This way, search frictions also help the New Keynesian model explain the size and persistence of output responses to monetary shocks typically found in VAR studies.<sup>4</sup> Finally, unemployment becomes more volatile conditional on monetary shocks, due to the larger output fluctuations. Therefore, the well-known inability of the canonical search and matching model to produce empirically plausible levels of unemployment volatility can be partially addressed with the addition of staggered-price setting, once we let search frictions affect the price-formation mechanism.<sup>5</sup> By generating these effects endogenously, search frictions reduce the need for other kinds of persistence and amplification mechanisms typically used in the literature, such as backward-looking indexation or rule-of-thumb price setters in the case of inflation persistence, and wage stickiness or implausibly high

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<sup>3</sup>See Fuhrer and Moore (1995) and Pivetta and Reis (2007) for US evidence on inflation persistence and O'Reilly and Whelan (2005) for euro area evidence.

<sup>4</sup>See e.g. Christiano et al. (2005) for US evidence on the output response to monetary shocks.

<sup>5</sup>Shimer (2005) first emphasized the implausibly low unemployment volatility produced by the standard search and matching model. Andres et al. (2006) have analyzed the ability of price staggering to increase the volatility of unemployment and vacancies in the search and matching model, using a framework that separates pricing and vacancy posting decisions.

outside options for workers in the case of unemployment volatility.

Other papers have departed from the producer-retailer assumption in the context of New Keynesian models with search and matching frictions.<sup>6</sup> Closely related is the independent work of Kuester (2007), who identifies a real rigidity mechanism that is similar to the one presented here. His framework features firm-worker pairs where nominal wages as well as prices are bargained in a staggered fashion. This gives rise to real rigidities both in wage bargaining and in price bargaining, which increases the sluggishness of wages and prices in response to shocks. In contrast, I assume firms to be large, which realistically allows them to adjust both margins of labor over time. In addition, I assume flexible wage bargaining, which allows me to isolate the effect of search frictions and the resulting real price rigidities on inflation dynamics.

Finally, this paper is related to previous analyses of how the specificity of the labor input can give rise to real rigidities in New Keynesian models. In particular, Woodford (2003) considers a setup of industry-specific labor markets where firms in each industry hire labor at that industry's perfectly competitive wage. This generates upward-sloping marginal cost curves at the industry level and hence real rigidities. This paper uses instead a framework in which the search frictions that characterize the labor market give rise endogenously to long-run employment relationships, thus making labor specific to each firm.

The remainder of the chapter is organized as follows. Section 2 presents the model. Section 3 shows how to solve for individual pricing decisions in this framework, which is complicated by the fact that hiring and pricing decisions interact with each other. It then derives the New Keynesian Phillips curve and analyzes the effect of real price rigidities. Section 4 calibrates the model and quantifies the effect of real rigidities on equilibrium dynamics. Section 5 concludes.

## 2 The model

I now present a New Keynesian model with search and matching frictions in the labor market. The model therefore brings together two frameworks that have become the standard for analyzing the monetary transmission mechanism and the cyclical behavior of the labor market, respectively. The main difference with respect to previous models of this type is that I do not separate the firms making the pricing decisions from the firms that face search frictions. Instead, I consider a single set of firms which set prices and post vacancies in a labor market characterized by search frictions.

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<sup>6</sup>An example is Krause and Lubik (2007). These authors however do not discuss the existence of real price rigidities, focusing instead on relevance of real wage rigidity for inflation dynamics.

## 2.1 The matching function

The search frictions in the labor market are represented by a matching function,  $m(v_t, u_t)$ , where  $v_t$  is the total number of vacancies and  $u_t$  is the total number of unemployed workers. Normalizing the labor force to 1,  $u_t$  also represents the unemployment rate. The function  $m$  is strictly increasing and strictly concave in both arguments. Assuming constant returns to scale in the matching function,<sup>7</sup> the matching probabilities for unemployed workers,  $m(v_t, u_t)/u_t = m(v_t/u_t, 1)$ , and for vacancies,  $m(v_t, u_t)/v_t = m(1, u_t/v_t)$ , are functions of the ratio of vacancies to unemployment, also known as *labor market tightness*. Denote the latter by  $\theta_t \equiv v_t/u_t$ . In what follows, I let  $p(\theta_t) \equiv m(\theta_t, 1)$  denote the matching probability for unemployed workers; the latter is an increasing function of  $\theta_t$ : in a tighter labor market, job-seekers are more likely to find jobs. Similarly, I let  $q(\theta_t) \equiv m(1, 1/\theta_t)$  denote the matching probability for vacancies; the latter is decreasing in  $\theta_t$ : firms are less likely to fill their vacancies in a tighter labor market.

## 2.2 Households

In the presence of unemployment risk, we may observe differences in consumption levels between employed and unemployed consumers. However, under the assumption of perfect insurance markets, consumption is equalized across consumers. This is equivalent to assuming the existence of a large representative household, as in Merz (1995). In this household, a fraction  $n_t$  of its members are employed in a measure-one continuum of firms. The remaining fraction  $u_t = 1 - n_t$  search for jobs. All members pool their income so as to ensure equal consumption across members. Household welfare is given by

$$H_t = u(c_t) - n_t b - \int_0^1 n_{it} \frac{h_{it}^{1+\eta}}{1+\eta} di + \beta E_t H_{t+1}, \quad (1)$$

where  $n_{it}$  and  $h_{it}$  represent the number of workers and hours per worker respectively in firm  $i \in [0, 1]$ ,  $b$  is labor disutility unrelated to  $h_{it}$  (forgone utility from home production, commuting time, etc.), and  $c_t$  is the Dixit-Stiglitz consumption basket,

$$c_t \equiv \left( \int_0^1 c_{it}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}},$$

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<sup>7</sup>See Petrongolo and Pissarides (2001) for empirical evidence of constant returns to scale in the matching function for several industrialized economies.

where  $\gamma > 1$  measures the elasticity of substitution across differentiated goods. Cost minimization implies that the nominal cost of consumption is given by  $P_t c_t$ , where

$$P_t \equiv \left( \int_0^1 P_{it}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$$

is the corresponding price index. The household's budget constraint is given by

$$\frac{M_{t-1} + (1 + i_{t-1})B_{t-1} + T_t}{P_t} + \int_0^1 n_{it} w_t(h_{it}) di + \Pi_t \geq c_t + \frac{B_t + M_t}{P_t}, \quad (2)$$

where  $M_{t-1}$  and  $B_{t-1}$  are holdings of money and one-period nominal bonds, respectively,  $i_{t-1}$  is the nominal interest rate,  $T_t$  is a cash transfer from the government (which may be negative),  $w_t(h_{it})$  is the wage schedule negotiated in firm  $i$  and  $\Pi_t$  are aggregate real profits, which are reverted to households in a lump-sum manner.

Employed members separate from their jobs at the exogenous rate  $\lambda$ , whereas unemployed members find jobs at the rate  $p(\theta_t)$ . Therefore, the household's employment rate evolves according to the following law of motion,

$$n_{t+1} = (1 - \lambda)n_t + p(\theta_t)(1 - n_t). \quad (3)$$

It is useful at this point to find the utility that the marginal worker in firm  $i$  contributes to the household. Equations (1), (2) and (3) imply that

$$\frac{\partial H_t}{\partial n_{it}} = u'(c_t)w_t(h_{it}) - b - \frac{h_{it}^{1+\eta}}{1+\eta} - p(\theta_t)\beta E_t \int_0^1 \frac{v_{jt}}{v_t} \frac{\partial H_{t+1}}{\partial n_{jt+1}} dj + (1 - \lambda)\beta E_t \frac{\partial H_{t+1}}{\partial n_{it+1}}, \quad (4)$$

where  $p(\theta_t)\frac{v_{jt}}{v_t}$  is the probability of being matched to firm  $j \in [0, 1]$ . The right hand side of equation (4) consists of the real wage (in utility units), minus labor disutility and outside opportunities, plus the continuation value of the job.

I assume the existence of a standard cash-in-advance (CIA) constraint on the purchase of consumption goods. Assuming that goods markets open after the closing of financial markets, the household's nominal expenditure in consumption cannot exceed the amount of cash left after bond transactions have taken place,

$$P_t c_t \leq M_{t-1} + T_t - B_t. \quad (5)$$

Cash transfers are given by  $T_t = M_t^s - M_{t-1}^s$ , where  $M_t^s$  is exogenous money supply. The growth

rate in money supply,  $\zeta_t \equiv \log(M_t^s/M_{t-1}^s)$ , follows an autoregressive process,  $\zeta_t = \rho_m \zeta_{t-1} + \varepsilon_t^m$ , where  $\varepsilon_t^m$  is an iid shock. Assuming that the nominal interest rate (i.e. the opportunity cost of holding money) is always positive, equation (5) holds with equality. In equilibrium, money demand equals money supply,  $M_t = M_t^s$ , which implies  $M_{t-1} + T_t = M_t$ . Combining this with (5) and the fact that bonds are in zero net supply ( $B_t = 0$ ), I obtain

$$P_t c_t = M_t. \quad (6)$$

### 2.3 Firms

Profits in firm  $i \in [0, 1]$  are given by

$$\Pi_{it} = \frac{P_{it}}{P_t} y_{it}^d - w_t(h_{it})n_{it} - \frac{\chi}{u'(c_t)} v_{it} + E_t \beta_{t,t+1} \Pi_{it+1}, \quad (7)$$

where  $P_{it}$  and  $y_{it}^d$  are the firm's nominal price and real sales, respectively,  $v_{it}$  are vacancies posted in period  $t$ ,  $\chi$  is the utility cost of posting a vacancy and  $\beta_{t,T} \equiv \beta^{T-t} \frac{u'(c_T)}{u'(c_t)}$  is the stochastic discount factor between periods  $t$  and  $T$ . Due to imperfect substitutability among individual goods, the firm faces the following demand curve for its product,

$$y_{it}^d = \left( \frac{P_{it}}{P_t} \right)^{-\gamma} y_t, \quad (8)$$

where aggregate demand is given by  $y_t = c_t$ . Once the firm has chosen a price, it commits to supplying whichever amount is demanded at that price,  $y_{it}^s = y_{it}^d$ . The firm's production technology is given by

$$y_{it}^s = A_t n_{it} h_{it},$$

where  $A_t$  is an exogenous aggregate technology shock. The log of latter,  $a_t \equiv \log A_t$ , evolves according to  $a_t = \rho_a a_{t-1} + \varepsilon_t^a$ , where  $\varepsilon_t^a$  is an iid shock.

In each period, the individual firm posts a number  $v_{it}$  of vacancies. Assuming that firms are large,  $\lambda$  and  $q(\theta_t)$  are the fraction of workers that separate from the firm and the fraction of vacancies that the firm fills, respectively. Due to the time involved in searching for suitable workers, new hires become productive in the following period. Therefore, the firm's workforce,  $n_{it}$ , is given at the start of the period. The law of motion of the firm's employment stock is given by

$$n_{it+1} = (1 - \lambda)n_{it} + q(\theta_t)v_{it}. \quad (9)$$



Since  $n_{it}$  is predetermined and the firm is demand-constrained, in the short run the firm has to adjust hours per worker so as to provide the required amount of output. Using the firm's production function, hours per worker are given by

$$h_{it} = \frac{y_{it}^d}{A_t n_{it}}. \quad (10)$$

Workers must be compensated for the incurred labor disutility according to a wage schedule agreed by firm and workers. The derivation of this wage schedule is presented next.

### 2.3.1 Wage bargaining

I assume that the firm bargains individually with each worker. Both the firm and the worker enjoy a surplus value from their employment relationship. The worker's surplus in consumption units, which I denote by  $S_{it}^w \equiv (\partial H_t / \partial n_{it}) / u'(c_t)$ , is given by equation (4) divided by  $u'(c_t)$ , that is,

$$S_{it}^w = w_t(h_{it}) - \frac{b + h_{it}^{1+\eta} / (1 + \eta)}{u'(c_t)} - p(\theta_t) E_t \beta_{t,t+1} S_{t+1}^w + (1 - \lambda) E_t \beta_{t,t+1} S_{it+1}^w, \quad (11)$$

where  $S_{t+1}^w \equiv \int_0^1 (v_{jt} / v_t) S_{jt+1}^w dj$ . On the firm's side, the surplus obtained from the marginal worker equals her marginal contribution to profits, which from equations (7), (9) and (10) is given by

$$\frac{\partial \Pi_{it}}{\partial n_{it}} = w'_t(h_{it}) \left( -\frac{\partial h_{it}}{\partial n_{it}} \right) n_{it} - w_t(h_{it}) + (1 - \lambda) E_t \beta_{t,t+1} \frac{\partial \Pi_{it+1}}{\partial n_{it+1}}. \quad (12)$$

Notice in particular that, in a context of monopolistic competition and infrequent price adjustment, once the firm has set a price its revenue is independent of  $n_{it}$ . Therefore, the contribution of the marginal worker to flow profits is given, not by the marginal revenue product of the worker (as in standard RBC models), but by the marginal reduction in the wage bill.<sup>8</sup> Indeed, if the worker walked away from the job, and given the impossibility of hiring a replacement immediately, the firm would need to increase the number of hours of (and therefore the wage payments to) all other workers in order to meet its demand.

Letting  $\xi$  denote the firm's bargaining power, the real wage is chosen to maximize

$$\xi \log \frac{\partial \Pi_{it}}{\partial n_{it}} + (1 - \xi) \log S_{it}^w,$$

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<sup>8</sup>This result is analogous to the one in Woodford's (2005) model of firm-specific capital, where the marginal contribution of capital to flow profits is given by the marginal reduction in the wage bill, rather than the marginal revenue product of capital.

subject to (11) and (12). The first order condition is given by  $(1 - \xi)\partial\Pi_{it}/\partial n_{it} = \xi S_{it}^w$ . This, combined with (11) and (12), yields the following real wage equation,

$$w_t(h_{it}) = (1 - \xi)w'_t(h_{it})h_{it} + \xi \left[ \frac{b + h_{it}^{1+\eta}/(1 + \eta)}{u'(c_t)} + p(\theta_t)E_t\beta_{t,t+1}S_{t+1}^w \right], \quad (13)$$

where I have also used the fact that, from equation (10),  $\partial h_{it}/\partial n_{it} = -h_{it}/n_{it}$ . The worker therefore receives a weighted average of the marginal reduction in the firm's wage bill and her opportunity cost of holding the job. Equation (13) is a differential equation, the solution to which is given by

$$w_t(h_{it}) = \tilde{\xi} \frac{h_{it}^{1+\eta}/(1 + \eta)}{u'(c_t)} + \xi \left[ \frac{b}{u'(c_t)} + p(\theta_t)E_t\beta_{t,t+1}S_{t+1}^w \right], \quad (14)$$

where  $\tilde{\xi} \equiv \xi/[1 - (1 - \xi)(1 + \eta)]$ . I restrict the parameter  $\tilde{\xi}$  to be positive, which requires  $\xi > \eta/(1 + \eta)$ . Otherwise, wage payments would be less than workers' opportunity cost and no worker would be willing to work in this economy.<sup>9</sup> According to equation (14), under the realistic assumption that labor disutility is convex in hours worked ( $\eta > 0$ ), the wage payment received by the worker is also a convex function of  $h_{it}$ . This implies that the *marginal real wage*,

$$w'_t(h_{it}) = \tilde{\xi} \frac{h_{it}^\eta}{u'(c_t)}, \quad (15)$$

is an increasing function of  $h_{it}$ . Therefore, the firm finds it more and more expensive to increase output by increasing hours per worker.

In order to gain more intuition about the wage schedule, I use equation (15) in order to rewrite equation (13) as

$$w_t(h_{it}) = (1 - \xi)\tilde{\xi}(1 + \eta) \frac{h_{it}^{1+\eta}/(1 + \eta)}{u'(c_t)} + \xi \left[ \frac{b + h_{it}^{1+\eta}/(1 + \eta)}{u'(c_t)} + p(\theta_t)E_t\beta_{t,t+1}S_{t+1}^w \right].$$

Therefore, the disutility of working hours,  $h_{it}^{1+\eta}/(1 + \eta)$ , improves the worker's compensation in two ways. First, by raising her opportunity cost. Second, by increasing the value that the

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<sup>9</sup>Notice that  $\lim_{\xi \rightarrow 1} \tilde{\xi} = 1$ , which implies that

$$\lim_{\xi \rightarrow 1} w_t(h_{it}) = \frac{h_{it}^{1+\eta}/(1 + \eta) + b}{u'(c_t)} + p(\theta_t)E_t\beta_{t,t+1}S_{t+1}^w.$$

That is, in the limiting case in which the firm has all the bargaining power, the worker is simply paid its opportunity cost.

firm attaches to the job, via the marginal reduction in its wage bill.

### 2.3.2 Vacancy posting

The firm chooses the number of vacancies  $v_{it}$  that maximize profits subject to equation (9). This yields the following first order condition,

$$\frac{\chi}{u'(c_t)} = q(\theta_t) E_t \beta_{t,t+1} \frac{\partial \Pi_{it+1}}{\partial n_{it+1}}. \quad (16)$$

Using the fact that equation (16) and the bargaining rule,  $S_{jt}^w = \frac{1-\xi}{\xi} \partial \Pi_{jt} / \partial n_{jt}$ , hold in every firm, I can write the term  $E_t \beta_{t,t+1} S_{t+1}^w$  in equation (14) as  $\frac{1-\xi}{\xi} \frac{\chi}{u'(c_t) q(\theta_t)}$ . This, together with  $p(\theta_t)/q(\theta_t) = \theta_t$ , allows me to write the real wage schedule as

$$w_t(h_{it}) = \tilde{\xi} \frac{h_{it}^{1+\eta}/(1+\eta)}{u'(c_t)} + \xi \frac{b}{u'(c_t)} + (1-\xi) \frac{\chi}{u'(c_t)} \theta_t. \quad (17)$$

Combining equations (12), (15), (16), (17) and  $\partial h_{it} / \partial n_{it} = -h_{it}/n_{it}$ , I can express the firm's vacancy posting decision as

$$\frac{\chi}{q(\theta_t)} = \beta E_t \left\{ \tilde{\xi} \eta \frac{h_{it+1}^{1+\eta}}{1+\eta} - \xi b - (1-\xi) \chi \theta_{t+1} + (1-\lambda) \frac{\chi}{q(\theta_{t+1})} \right\}. \quad (18)$$

According to equation (18), the firm's incentives to hire are driven by fluctuations in hours per employee. Intuitively, if the firm expects hours to be higher in the future, it also expects larger reductions in its wage bill from having additional workers. This leads the firm to post more vacancies today, up to the point in which the expected marginal benefit of hiring equals its marginal cost,  $\chi/q(\theta_t)$ .

### 2.3.3 Pricing decision

As is standard in the New Keynesian literature, I use the Calvo (1983) model of staggered price setting. Each period, a randomly selected fraction  $\delta$  of firms cannot change their price. Therefore,  $\delta$  also represents the probability that a firm is not able to change its price in the following period. The part of the firm's profits that depends on its current price is given by

$$E_t \sum_{T=t}^{\infty} \delta^{T-t} \beta_{t,T} \left[ \left( \frac{P_{it}}{P_T} \right)^{1-\gamma} y_T - w_T \left( \left( \frac{P_{it}}{P_T} \right)^{-\gamma} \frac{y_T}{A_T n_{iT|t}} \right) n_{iT|t} \right], \quad (19)$$

where I have used equations (8) and (10) to write the firm's demand and hours per worker in terms of the current price,  $P_{it}$ . The subscript  $iT|t$  indicates that the firm has not reset its price since period  $t$ . When a firm has the chance to reset its price, it chooses  $P_{it}$  so as to maximize (19). The first order condition is given by

$$E_t \sum_{T=t}^{\infty} \delta^{T-t} \beta_{t,T} P_T^\gamma y_T \left( \frac{P_{it}^*}{P_T} - \frac{\gamma}{\gamma-1} mc_{iT|t} \right) = 0, \quad (20)$$

where  $P_{it}^*$  is the pricing decision and

$$mc_{iT|t} \equiv w'_T \left( \left( \frac{P_{it}^*}{P_T} \right)^{-\gamma} \frac{y_T}{A_T n_{iT|t}} \right) \frac{1}{A_T}$$

is the real marginal cost in period  $T$ , conditional on the firm not having changed its price since period  $t$ . Therefore, real marginal costs are given the marginal real wage divided by the marginal product of labor,  $A_T$ . Using equation (15), I can alternatively write real marginal costs in terms of the marginal rate of substitution between consumption and leisure,

$$mc_{iT|t} = \tilde{\xi} \left( \frac{y_{iT|t}}{A_T n_{iT|t}} \right)^\eta \frac{1}{u'(c_T) A_T}, \quad (21)$$

where  $y_{iT|t} = (P_{it}^*/P_T)^{-\gamma} y_T$ . Equation (21) implies that, under the assumption of convexity in labor disutility ( $\eta > 0$ ), the firm's marginal cost curve is an increasing function of its output level.

### 3 Log-linear equilibrium dynamics

Following standard practice in the New Keynesian literature, I now perform a log-linear approximation of the equilibrium conditions around a zero-inflation steady state. This will allow me obtain the law of motion of inflation, also known as the 'New Keynesian Phillips curve'. At this point, I assume the following functional forms for the utility function and the matching function,

$$u(c) = \frac{c^{1-\sigma^{-1}}}{1-\sigma^{-1}},$$

$$m(v, u) = v^\epsilon u^{1-\epsilon},$$

where  $\sigma > 0$  and  $\epsilon \in (0, 1)$ . In terms of notation, I will use 'hats' to denote percentage deviations of a certain variable from its steady-state value, and 'tildes' to denote percentage deviations of that variable from its cross-sectional average.

### 3.1 Relative dynamics of the firm

Log-linearization of the firm's pricing decision, equation (20), yields

$$\log P_{it}^* = (1 - \delta\beta)E_t \sum_{T=t}^{\infty} (\delta\beta)^{T-t} [\widehat{m}c_{iT|t} + \log P_T]. \quad (22)$$

Equation (21) implies that the real marginal cost in period  $T \geq t$  of a firm that has not changed its price since period  $t$  can be expressed as

$$\widehat{m}c_{iT|t} = \widehat{m}c_T + \eta(\hat{y}_{iT|t} - \hat{y}_T) - \eta\tilde{n}_{iT|t}, \quad (23)$$

where

$$\hat{y}_{iT|t} = \hat{y}_T - \gamma(\log P_{it}^* - \log P_T) \quad (24)$$

and  $\widehat{m}c_T$  is the average real marginal cost. Notice that a firm's relative marginal cost is decreasing in its relative stock of workers,  $\tilde{n}_{iT|t}$ . Having more workers allows the firm to produce a certain amount of output with a smaller number of hours per worker, which reduces the marginal labor disutility of its workers and hence the marginal real wage. I now combine (22), (23) and 24) to obtain

$$(1 + \eta\gamma) \log P_{it}^* = (1 - \delta\beta)E_t \sum_{T=t}^{\infty} (\delta\beta)^{T-t} [\widehat{m}c_T + (1 + \eta\gamma) \log P_T - \eta\tilde{n}_{iT|t}]. \quad (25)$$

This expression for a firm's pricing decision is very similar to the one produced by a standard New Keynesian model.<sup>10</sup> The only difference is the presence of the  $E_t\tilde{n}_{iT|t}$  terms, which reflect the fact that a firm's marginal cost is decreasing in its stock of workers. These additional terms complicate the analysis in the following way. In order to determine  $\log P_{it}^*$ , we need to compute the expected path of  $\tilde{n}_{iT|t}$ . The latter however depends on the firm's current and future expected vacancy posting decisions, which in turn depend on the price chosen today. Solving for the firm's pricing decision therefore requires that one considers the effect of a firm's relative price on the evolution of its relative employment stock.

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<sup>10</sup>See e.g. Walsh (2003a, chap. 3).

With this purpose, I now follow Woodford's (2005) method to solve for the firm's relative dynamics.<sup>11</sup> I start by noticing that, in a log-linear approximation, the firm's pricing decision is a linear function of the state of the economy and its individual state,  $\hat{n}_{it}$ . On the other hand, since price-setters are randomly chosen, their average employment stock coincides with the economy-wide average employment stock. Therefore, it is plausible to guess that a firm's pricing decision, relative to the average pricing decision, is proportional to its relative employment stock,

$$\log P_{it}^* = \log P_t^* - \tau^* \tilde{n}_{iT|t}. \quad (26)$$

I now log-linearize the vacancy posting decision, equation (18), and rescale the resulting expression by  $\frac{y u'(c)}{n}$  to obtain

$$\frac{s_v}{\lambda} (1 - \epsilon) \hat{\theta}_t = \beta E_t \left\{ \frac{\eta}{\mu} \hat{h}_{it+1} + [(1 - \lambda)(1 - \epsilon) - (1 - \xi)p(\theta)] \frac{s_v}{\lambda} \hat{\theta}_{t+1} \right\}, \quad (27)$$

where  $s_v \equiv \frac{\chi v}{y u'(c)}$  is vacancy posting costs over GDP in the steady state and  $\mu \equiv \gamma/(\gamma - 1)$  is the monopolistic mark-up. Notice that the coefficient  $\tilde{\xi}$  no longer appears in the log-linearized equation. This is due to the fact that steady-state labor disutility,  $h^{1+\eta}/(1 + \eta)$ , is inversely proportional to  $\tilde{\xi}$ .<sup>12</sup> Notice also that the only idiosyncratic term in equation (27) is  $E_t \hat{h}_{it+1}$ . The latter depends on  $P_{it}$  (by affecting the firm's demand in  $t + 1$  should it not reset its price) as well as on its stock of workers at the beginning of  $t + 1$ . It is now possible to obtain the following result.<sup>13</sup>

**Proposition 1** *Let relative pricing decisions be given by equation (26), up to a log-linear approximation. Then the relative employment stock of any firm evolves according to*

$$\tilde{n}_{it+1} = -\tau^n (\log P_{it} - \log P_t), \quad (28)$$

where

$$\tau^n = \frac{\gamma \delta}{1 - \gamma(1 - \delta)\tau^*}. \quad (29)$$

<sup>11</sup>Woodford (2005) develops his method in the context of a model with where capital, rather than labor, is specific to each individual firm.

<sup>12</sup>From equations (10), (20) and (21), steady-state labor disutility is given by

$$\frac{h^{1+\eta}}{1 + \eta} = \frac{u'(c)}{1 + \eta} \frac{h^\eta}{u'(c)} h = \frac{u'(c)}{1 + \eta} \frac{mcA}{\tilde{\xi}} \left( \frac{y}{An} \right) = \frac{u'(c)y}{n(1 + \eta)} \frac{1}{\tilde{\xi}\mu}.$$

In the derivation of equation (27), I have also used the fact that, in the steady state,  $q(\theta)v = \lambda n$ .

<sup>13</sup>The proofs of all propositions are in the Appendix.

Intuitively, firms with a higher price in the current period also expect to have a higher price in the next period, which means that they also expect lower demand. Anticipating this, such firms post a number of vacancies that leaves them with a smaller workforce than the average firm in the following period.

Proposition 1 allows me to write

$$\begin{aligned} E_t \sum_{T=t}^{\infty} (\delta\beta)^{T-t} \tilde{n}_{iT|t} &= \tilde{n}_{it} + \delta\beta E_t \sum_{T=t}^{\infty} (\delta\beta)^{T-t} \tilde{n}_{iT+1|t} \\ &= \tilde{n}_{it} - \delta\beta\tau^n E_t \sum_{T=t}^{\infty} (\delta\beta)^{T-t} (\log P_{it}^* - \log P_T). \end{aligned} \quad (30)$$

Using (30) in equation (25), I can write the firm's pricing decision as

$$(1 + \phi) \log P_{it}^* = (1 - \delta\beta) E_t \sum_{T=t}^{\infty} (\delta\beta)^{T-t} [\widehat{m}c_T + (1 + \phi) \log P_T] - (1 - \delta\beta) \eta \tilde{n}_{it}, \quad (31)$$

where  $\phi \equiv \eta\gamma - \delta\beta\eta\tau^n$ . Averaging (31) across price-setters, and using the fact that the latter are randomly chosen, I obtain

$$(1 + \phi) \log P_t^* = (1 - \delta\beta) E_t \sum_{T=t}^{\infty} (\delta\beta)^{T-t} [\widehat{m}c_T + (1 + \phi) \log P_T]. \quad (32)$$

Subtracting (32) from (31) yields

$$(1 + \phi)(\log P_{it}^* - \log P_t^*) = -(1 - \delta\beta) \eta \tilde{n}_{it}.$$

This is consistent with my initial guess, equation (26), only if

$$\tau^* = \frac{(1 - \delta\beta)\eta}{1 + \eta\gamma - \delta\beta\eta\tau^n}. \quad (33)$$

Therefore, if relative pricing decisions and relative employment stocks are to have a solution, the latter is given by equations (26) and (28), respectively, where the parameters  $\tau^*$  and  $\tau^n$  must satisfy equations (29) and (33). I now analyze whether such a solution exists.

### 3.1.1 Existence of solution

Using (29) to substitute for  $\tau^n$  in (33), I obtain the following equation for  $\tau^*$ ,

$$\tau^* = \frac{(1 - \delta\beta)\eta}{1 + \eta\gamma - \delta\beta\eta \left( \frac{\gamma\delta}{1 - \gamma(1 - \delta)\tau^*} \right)}.$$

This can be written as

$$a(\tau^*)^2 + b\tau^* + c = 0, \quad (34)$$

where

$$a \equiv (1 + \eta\gamma)\gamma(1 - \delta) > 0, \quad (35)$$

$$b \equiv -[1 + \gamma(2 - \delta - \delta\beta)\eta] < 0, \quad (36)$$

$$c \equiv (1 - \delta\beta)\eta > 0. \quad (37)$$

The quadratic equation (34) has two solutions. The latter are real numbers if and only if  $b^2 - 4ac > 0$ . The following result establishes that this is indeed the case.

**Proposition 2** *Let the parameters  $a$ ,  $b$  and  $c$  in equation (34) be given by equations (35), (36) and (37), respectively, where  $\eta > 0$ ,  $\gamma > 1$  and  $0 < \beta, \delta < 1$ . Then the two solutions of equation (34) are real numbers.*

Equation (34) has therefore two real solutions, given by

$$(\tau_1^*, \tau_2^*) = \left( \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \frac{-b + \sqrt{b^2 - 4ac}}{2a} \right).$$

It is also possible to show that the solutions for both  $\tau^*$  and  $\tau^n$  have to be positive. To see this, define

$$\tau_1^n(\tau^*) \equiv \frac{1 + \eta\gamma - \frac{(1 - \delta\beta)\eta}{\tau^*}}{\delta\beta\eta},$$

$$\tau_2^n(\tau^*) \equiv \frac{\gamma\delta}{1 - \gamma(1 - \delta)\tau^*}.$$

The function  $\tau_1^n(\tau^*)$  is obtained by solving for  $\tau_1^n$  in equation (33). The solutions for  $\tau^n$  and  $\tau^*$  are given by the two points of intersection of both functions in  $(\tau^*, \tau^n)$  space. Both functions are increasing in  $\tau^*$ . For  $\tau^* < 0$ ,  $\tau_1^n(\tau^*) > \frac{1 + \eta\gamma}{\delta\beta\eta}$  and  $\tau_2^n(\tau^*) < \gamma\delta$ . Since  $\frac{1 + \eta\gamma}{\delta\beta\eta} > \gamma\delta$ , there can be no solution for  $\tau^* < 0$ . But if  $\tau^* > 0$ , then  $\tau_2^n(\tau^*) > \gamma\delta > 0$ , which implies that  $\tau^n$  must be positive too.



### 3.1.2 Convergence

Equation (34) has two real solutions,  $\tau_1^*$  and  $\tau_2^*$ , where  $\tau_1^* < \tau_2^*$ . However,  $\tau_2^*$  implies explosive dynamics. To see this, notice that a firm's relative price and employment stock evolve according to

$$\begin{bmatrix} E_t \tilde{P}_{it+1} \\ \tilde{n}_{it+1} \end{bmatrix} = \begin{bmatrix} \delta + (1 - \delta)\tau^*\tau^n & 0 \\ -\tau^n & 0 \end{bmatrix} \begin{bmatrix} \tilde{P}_{it} \\ \tilde{n}_{it} \end{bmatrix}.$$

This system implies convergent dynamics only if the eigenvalues of the 2x2 matrix are inside the unit circle. These eigenvalues are 0 and  $\delta + (1 - \delta)\tau^*\tau^n$ . Since  $\delta + (1 - \delta)\tau^*\tau^n > 0$  (as a result of both  $\tau^*$  and  $\tau^n$  being positive), a non-explosive solution must satisfy  $\delta + (1 - \delta)\tau^*\tau^n < 1$ , or simply  $\tau^*\tau^n < 1$ . Using equation (29), this requires in turn

$$\tau^* < \frac{1}{\gamma}. \quad (38)$$

I now define  $F(\tau^*) \equiv a(\tau^*)^2 + b\tau^* + c$ , where  $a$ ,  $b$  and  $c$  are given by equations (35), (36) and (37), respectively. Since  $F(\tau^*)$  is a convex function, it follows that  $F(\tau^*) < 0 \Leftrightarrow \tau^* \in (\tau_1^*, \tau_2^*)$ , where  $\tau_1^*, \tau_2^*$  are the two roots of  $F(\tau^*)$ . Evaluating  $F(\cdot)$  at  $\frac{1}{\gamma}$ , I obtain

$$\begin{aligned} F\left(\frac{1}{\gamma}\right) &= (1 + \eta\gamma)\frac{1}{\gamma}(1 - \delta) - \left[\frac{1}{\gamma} + (2 - \delta - \delta\beta)\eta\right] + (1 - \delta\beta)\eta \\ &= -\frac{\delta}{\gamma} < 0. \end{aligned}$$

It follows that  $\tau_1^* < \frac{1}{\gamma} < \tau_2^*$ , which means that  $\tau_2^*$  violates (38) and therefore implies explosive dynamics. As emphasized by Woodford (2005), in order for a log-linear approximation around the steady state to be an accurate approximation of the model's exact equilibrium conditions, the dynamics of firms' relative prices and employment stocks must remain forever near enough to the steady state. Since  $\tau_2^*$  violates this condition, from now onwards I set  $\tau^* = \tau_1^*$ .

## 3.2 Real rigidities and inflation dynamics

I am now ready to discuss the presence of real rigidities in this framework and how they affect inflation dynamics. The average pricing decision, equation (32), can be written as

$$(1 + \phi) \log P_t^* = (1 - \delta\beta) [\widehat{mc}_t + (1 + \phi) \log P_t] + \delta\beta E_t (1 + \phi) \log P_{t+1}^*. \quad (39)$$

In the Calvo model of staggered price-setting, the law of motion for the price level is given by  $P_t^{1-\gamma} = \delta P_{t-1}^{1-\gamma} + (1-\delta)(P_t^*)^{1-\gamma}$ . The latter admits the following log-linear approximation,

$$\log P_t^* - \log P_t = \frac{\delta}{1-\delta} \pi_t, \quad (40)$$

where  $\pi_t \equiv \log(P_t/P_{t-1})$  is the inflation rate. Combining (39) and (40), I obtain the familiar *New Keynesian Phillips curve*,

$$\pi_t = \kappa \widehat{m}c_t + \beta E_t \pi_{t+1}, \quad (41)$$

where

$$\kappa \equiv \frac{(1-\delta\beta)(1-\delta)}{\delta} \frac{1}{1+\phi}, \quad (42)$$

$$\phi \equiv \eta\gamma - \delta\beta\eta\tau^n. \quad (43)$$

The parameter  $\phi$  has two components,  $\eta\gamma$  and  $\delta\beta\eta\tau^n$ . The term  $\eta\gamma$  reflects the existence in this framework of *strategic complementarities* in price-setting, also known as *real rigidities* after Ball and Romer (1990). This mechanism has the effect of slowing the adjustment of the overall price level in response to fluctuations in average real marginal costs. To see this, take a price-setter that is considering a reduction in its price. Given the pricing decisions of the other firms, a reduction in the firm's nominal price represents also a reduction in its real price. This increases its sales (with elasticity  $\gamma$ ) and therefore, given its employment stock, the required amount of hours per worker. This increases the firm's marginal costs through the increase in workers' marginal disutility of labor (with elasticity  $\eta$ ). The anticipated rise in its current and future expected marginal costs leads the firm to choose a smaller price cut than the one initially considered. Therefore, the fact that some firms keep their prices unchanged leads price-setters to change theirs by little, hence the 'strategic complementarity' in price-setting. Equivalently, price-setters have an incentive to keep their prices in line with the overall price level, hence the 'real rigidity' in prices. Because all price-setters follow the same logic, the price level and therefore inflation become less sensitive to changes in average real marginal costs.

The term  $\delta\beta\eta\tau^n$  reflects the fact that the position of a firm's marginal cost curve depends on its stock of workers, by affecting how many hours per worker are needed to produce a certain amount of output. This has the effect of accelerating price adjustment. To see this, take the same firm considering a price cut. From Proposition 1, today's price cut leads the firm to expect a larger relative employment stock and, by equation (23) a lower marginal cost in future periods. Holding everything else constant, this would lead the firm to choose an even larger price cut than initially considered. It is possible to show however that this latter effect

is dominated by the real rigidity effect. Using the definition of  $\tau^n$ , equation (29), I can write

$$\begin{aligned}\eta\gamma - \delta\beta\eta\tau^n &= \eta\gamma - \delta\beta\eta\left(\frac{\gamma\delta}{1 - \gamma(1 - \delta)\tau^*}\right) \\ &= \eta\gamma\left(1 - \frac{\delta^2\beta}{1 - \gamma(1 - \delta)\tau^*}\right).\end{aligned}$$

The latter expression is positive only if the expression in brackets is. Given that  $\tau^*$  must be smaller than  $\frac{1}{\gamma}$  in order for the model to have convergent dynamics, we have that

$$1 - \frac{\delta^2\beta}{1 - \gamma(1 - \delta)\tau^*} > 1 - \frac{\delta^2\beta}{1 - \gamma(1 - \delta)\frac{1}{\gamma}} = 1 - \delta\beta > 0.$$

It follows that  $\phi > 0$ . Therefore, the real rigidities in price-setting that arise under search frictions unambiguously flatten the New Keynesian Phillips curve.

### 3.3 Aggregate equilibrium

Equilibrium in the search model with real rigidities is characterized by the following 5 equations,

$$\pi_t = \kappa\widehat{m}c_t + \beta E_t\pi_{t+1}, \quad (44)$$

$$\widehat{m}c_t = (\eta + \sigma^{-1})\hat{y}_t - (1 + \eta)a_t - \eta\hat{n}_t, \quad (45)$$

$$\hat{y}_t = \hat{y}_{t-1} + \zeta_t - \pi_t, \quad (46)$$

$$\frac{s_v}{\lambda}(1 - \epsilon)\hat{\theta}_t = \beta E_t\left\{\frac{\eta}{\mu}(\hat{y}_{t+1} - a_{t+1} - \hat{n}_{t+1}) + [(1 - \lambda)(1 - \epsilon) - (1 - \xi)p(\theta)]\frac{s_v}{\lambda}\hat{\theta}_{t+1}\right\}, \quad (47)$$

$$\hat{n}_{t+1} = (1 - \lambda - p(\theta))\hat{n}_t + \lambda\epsilon\hat{\theta}_t. \quad (48)$$

Equation (45) is obtained by log-linearizing (21), averaging across all firms and using the fact that  $\hat{c}_t = \hat{y}_t$ . Equation (46) is obtained by log-linearizing (6), taking first differences and using again  $\hat{c}_t = \hat{y}_t$ . Equation (47) is obtained by averaging equation (27) across firms and using  $\hat{h}_t = \hat{y}_t - a_t - \hat{n}_t$  to substitute for average hours per employee.<sup>14</sup> Finally, equation (48) is the log-linear approximation of equation (3), where I also use the steady-state condition  $\lambda n = p(\theta)(1 - n)$ . This log-linear representation allows to understand easily the effect of shocks on the economy. In response to a positive monetary shock (an increase in  $\zeta_t$ ), aggregate demand

<sup>14</sup>As shown in the Appendix,  $E_t\hat{h}_{it+1}$  can be written as  $E_t\hat{h}_{t+1} - \gamma\delta\tilde{P}_{it} - [1 - \gamma(1 - \delta)\tau^*]\tilde{n}_{it+1}$ , which averages to  $E_t\hat{h}_{t+1}$ .

increases, which puts upward pressure on real marginal costs, and therefore on inflation. The expected expansion in aggregate demand leads firms to anticipate longer hours per employee in the future, which leads them to post more vacancies. This results in a tightening of the labor market, with the subsequent increase in employment. In response to a positive productivity shock (an increase in  $a_t$ ), real marginal costs fall and so does inflation. For a constant level of nominal GDP, the fall in prices produces an expansion in aggregate demand. The effect on employment is however ambiguous. If the expansion in output is strong enough relative to the increase in  $a_t$ , firms expect hours per employee to be higher, which leads them to post more vacancies and thus increase employment. If the increase in output is weak enough, the opposite will be true.

### 3.3.1 Comparison to the standard New Keynesian model

In its standard formulation, the New Keynesian model assumes the existence of a frictionless labor market, where labor hours are sold at a perfectly competitive hourly wage. In this framework, variations in labor input are due solely to changes in the intensive margin,  $h_{it}$ . Therefore, households' utility flow and the production function are given by

$$\frac{c_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \int_0^1 \frac{h_{it}^{1+\eta}}{1+\eta} di,$$

$$y_{it} = A_t h_{it},$$

respectively. As is well-known, this framework produces the following inflation equation,<sup>15</sup>

$$\pi_t = \kappa_{nk} \widehat{m}c_t + \beta E_t \pi_{t+1}, \quad (49)$$

where

$$\kappa_{nk} \equiv \frac{(1-\delta\beta)(1-\delta)}{\delta}. \quad (50)$$

Therefore, the inflation equation is the same in both models, except for the slope. This can be seen by comparing expressions (42) and (50). In the model with search frictions, the presence of real rigidities ( $\phi > 0$ ) flattens the New Keynesian Phillips curve. This effect is absent in the standard model, because firms can hire as much labor as they want at the market hourly wage. As a result, a firm's marginal costs is independent of its own output and therefore of its pricing

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<sup>15</sup>See e.g. Walsh (2003a).

decisions. Real marginal costs in the standard model are given by

$$\widehat{mc}_t = (\eta + \sigma^{-1}) \hat{y}_t - (1 + \eta)a_t. \quad (51)$$

The latter differs from (45) in that employment has no effect on real marginal costs, due to the fact that the standard model abstracts from the extensive margin of labor. The equilibrium conditions in the standard model are given by equations (46), (49) and (51).

Relative to the standard model, under search frictions the presence of real rigidities slows the adjustment of prices to shocks, which makes inflation more persistent. By increasing inflation persistence in an endogenous way, search frictions help the New Keynesian framework account for the observed levels of inflation persistence. Search frictions thus reduce the need for ad-hoc persistence mechanisms such as backward-looking indexation of rule-of-thumb price-setters. On the other hand, conditional on monetary shocks the slower price adjustment makes output responses larger and more persistent. Again, search frictions reduce the need for ad-hoc persistence mechanisms, such as habit persistence.

Notice that differences in inflation and output dynamics between the search model and the standard model are not due *only* to the flatness of the New Keynesian Phillips. The reason is that real marginal costs under search frictions are affected by the evolution of the employment stock. In order to isolate the pure effect of real rigidities on output and inflation dynamics, it is useful to consider a search model in which such rigidities are absent.

### 3.3.2 Comparison to a search model with a producer-retailer structure

Most of the literature on New Keynesian models with search and matching frictions separates vacancy-posting and pricing decisions by assuming a producer-retailer structure, in which the former are subject to search frictions and the latter to staggered price-setting. While simplifying the analysis considerably, this assumption eliminates from the outset the possibility of analyzing the effect of search frictions on pricing decisions. In such models, producers produce a homogenous intermediate good that is sold to retailers at a perfectly competitive real price. We may denote the latter by  $mc_t$ . Each retailer then transforms the intermediate good into a differentiated final good using a linear technology. Therefore,  $mc_t$  represents the real marginal cost common to all retailers. It is relatively straightforward to construct a model with this kind of producer-retailer structure that is otherwise equivalent to the search model with real rigidities presented in this paper. In particular, I may assume that household preferences and the production function of producers are the same as in the model with real rigidities. The Appendix derives the equilibrium conditions in such a model. Once the producer-retailer model

is log-linearized, the inflation equation is exactly the same as in the standard New Keynesian model, equation (49). In particular, the slope coefficient is given again by (50). Because retailers can buy as much intermediate input as they need at the real price  $mc_t$ , their pricing decisions have no effect on their own marginal costs, just as in the case of a frictionless labor market. Therefore, the simplicity bought by the producer-retailer structure comes at the cost of ignoring how search frictions affect the pricing behavior of individual firms.

As shown in the appendix, the log-linear job creation condition in the producer-retailer model is given by

$$\frac{s_v}{\lambda}(1 - \epsilon)\hat{\theta}_t = \beta E_t \left\{ \xi \frac{\eta}{\mu} (\hat{y}_{t+1} - a_{t+1} - \hat{n}_{t+1}) + [(1 - \lambda)(1 - \epsilon) - (1 - \xi)p(\theta)] \frac{s_v}{\lambda} \hat{\theta}_{t+1} \right\}.$$

The latter differs from the corresponding expression in the model with real rigidities, equation (47), only in the term multiplying hours per employee on the right hand side, which is greater in the model with real rigidities ( $\eta/\mu > \xi\eta/\mu$ ). This reflects the fact that, in the latter model, hours per employee have a stronger positive effect on the firm's surplus via the marginal reduction in its wage bill. This effect is absent in the producer-retailer model, because the firms making the hiring decisions (the producers) are not demand-constrained and their surplus depends simply on marginal revenue product; as a result, the same fluctuations in average hours per employee produce smaller fluctuations in labor market tightness and employment. I may assume however that retail sales in the producer-retailer model are subsidized at the rate  $s$ , such that the *effective* retailer mark-up is given by  $\mu_s \equiv \frac{\mu}{1+s}$ . I now set  $s = \xi^{-1} - 1$  so as to guarantee that  $\xi\eta/\mu_s = \eta/\mu$ . With this modification of the producer-retailer model, both models are observationally equivalent. Indeed, the log-linear equilibrium conditions of both models are exactly the same, except for the flatter slope of the New Keynesian Phillips curve in the model with real rigidities ( $\kappa < \kappa_{nk}$ ). The producer-retailer model thus serves as a 'control' that allows me to isolate the effect of real rigidities in models with search frictions and staggered price-setting. In particular, the improvements relative to the standard model (higher inflation persistence, and larger and more persistent output responses to monetary shocks) continue to take place relative to the producer-retailer model. However, whereas the comparison of inflation and output dynamics in the standard model and the search model with real rigidities is partly contaminated by the effect of employment fluctuations on marginal costs in the latter model, such contamination is completely absent when we compare the search model with and without real rigidities.

Interestingly, real rigidities also have the property of amplifying unemployment fluctuations in response to monetary shocks. Conditional on such shocks, average hours per employee in

the economy are given by  $\hat{h}_t = \hat{y}_t - \hat{n}_t$ . Given the average employment stock, the stronger output response in the model with real rigidities produces larger fluctuations in hours per employee. Because the latter are the driving force of job creation, unemployment also fluctuates more. This way, staggered price setting together with real rigidities help tackle the so-called ‘unemployment volatility puzzle’, i.e. the failure of the canonical search and matching model to produce empirically plausible levels of volatility in unemployment.<sup>16</sup> I turn now to the quantitative analysis of these mechanisms.

## 4 Quantitative analysis

### 4.1 Calibration

I calibrate the model to monthly US data. Following most of the RBC literature, I set the discount factor to 4% per quarter, or  $\beta = 0.99^{1/3}$ . I also choose standard values for the intertemporal elasticity of substitution,  $\sigma = 1$ , and for the autocorrelation of the technology shock process,  $\rho_a = 0.95^{1/3}$ .

Calibrating the convexity of labor disutility,  $\eta$ , requires more attention. In a standard RBC framework, this parameter represents the inverse of the elasticity of labor supply. This interpretation does not carry over to this context, where hours per worker are demand-determined rather than chosen by the workers. It is however possible to derive an alternative interpretation. From equation (15),  $\eta$  represents the elasticity of the marginal wage with respect to hours per worker. Using US manufacturing data, Bils (1987) constructs a measure of marginal wages by assuming an overtime premium of 50% (as required by US labor regulations) and estimating by how much average overtime hours rise following an increase in average hours per worker. He then finds an elasticity of marginal wages with respect to hours per worker of 1.39. Therefore, I set  $\eta$  to 1.4.<sup>17</sup>

Regarding the New Keynesian side of the model, following the evidence in Bils and Klenow (2004) I assume that firms change prices every 1.5 quarters, or 4.5 months, which implies  $\delta = \frac{4.5-1}{4.5} = 0.78$ . As in Woodford (2005), I choose a monopolistic mark-up of  $\mu = 1.15$ , which implies  $\gamma = \frac{\mu}{1-\mu} = 7.67$ . Following Cooley and Quadrini (1999), the autocorrelation of the monetary shock process is set to  $\rho_m = 0.49^{1/3}$ .

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<sup>16</sup>The unemployment volatility puzzle, originally emphasized by Shimer (2005), has received a considerable degree of attention in the recent literature. See Pissarides (2007) for an overview.

<sup>17</sup>Notice that my choice of  $\eta$  is actually conservative. If I was to calibrate  $\eta^{-1}$  to match labor supply elasticity, and given that the latter is no larger than 0.5 according to most micro studies (see e.g. Card, 1994),  $\eta$  would be no smaller than 2. This would make real rigidities even stronger and the Phillips curve even flatter.

Given the values of  $\eta, \beta, \gamma$  and  $\delta$ , equations (33) and (29) jointly imply  $\tau^* = 0.072$  and  $\tau^n = 6.80$ . From (43), the parameter measuring (net) real rigidity equals  $\phi = 3.35$ . From (42), the slope of the New Keynesian Phillips curve equals  $\kappa = 0.015$ . This compares with a slope of  $\kappa_{nk} = 0.064$  in the standard New Keynesian model and the producer-retailer model.

The parameters that describe the labor market  $(\lambda, p(\theta), \epsilon)$  are calibrated as in Thomas (2008), based also on US data (see Table 1 below for details). The choice of firms' bargaining power,  $\xi$ , is constrained by the requirement that the parameter  $\tilde{\xi}$  in equation (14) is positive, which requires  $\xi > \eta/(1 + \eta)$ . Given my choice of  $\eta$ , any  $\xi$  greater than 0.58 is consistent with  $\tilde{\xi} > 0$ . Given the absence of direct evidence on  $\xi$ , I set it equal to  $\epsilon$ . It is however important to emphasize that  $\xi$  has no effect on real rigidities,  $\phi$ , and therefore on the slope of the New Keynesian Phillips curve,  $\kappa$ . Finally, the steady-state ratio of vacancy-posting costs to GDP,  $s_v$ , is set to 1% following most of the literature.<sup>18</sup>

Table 1. Parameter values

|             | Value               | Description                   |               | Value               | Description                   |
|-------------|---------------------|-------------------------------|---------------|---------------------|-------------------------------|
| $\beta$     | 0.99 <sup>1/3</sup> | discount factor               | $\xi$         | 0.6                 | firm's bargaining power       |
| $\eta$      | 1.4                 | convexity of labor disutility | $\rho_a$      | 0.95 <sup>1/3</sup> | AC of technology shock        |
| $\delta$    | 0.78                | fraction of sticky prices     | $\rho_m$      | 0.49 <sup>1/3</sup> | AC of monetary shock          |
| $\gamma$    | 7.67                | elasticity of demand curves   | $\phi$        | 3.35                | net real rigidity             |
| $\sigma$    | 1                   | intertemporal elast. of subs. | $\kappa$      | 0.015               | slope of NKPC                 |
| $\lambda$   | 0.035               | job separation rate           | $\kappa_{nk}$ | 0.064               | slope of NKPC, standard model |
| $p(\theta)$ | 0.30                | job finding rate              | $s_v$         | 0.01                | hiring costs/GDP              |
| $\epsilon$  | 0.6                 | elasticity of matching fct.   |               |                     |                               |

## 4.2 Simulation

I now simulate the economy's response to monetary and productivity shocks, and calculate a set of relevant statistics for inflation, output and unemployment.

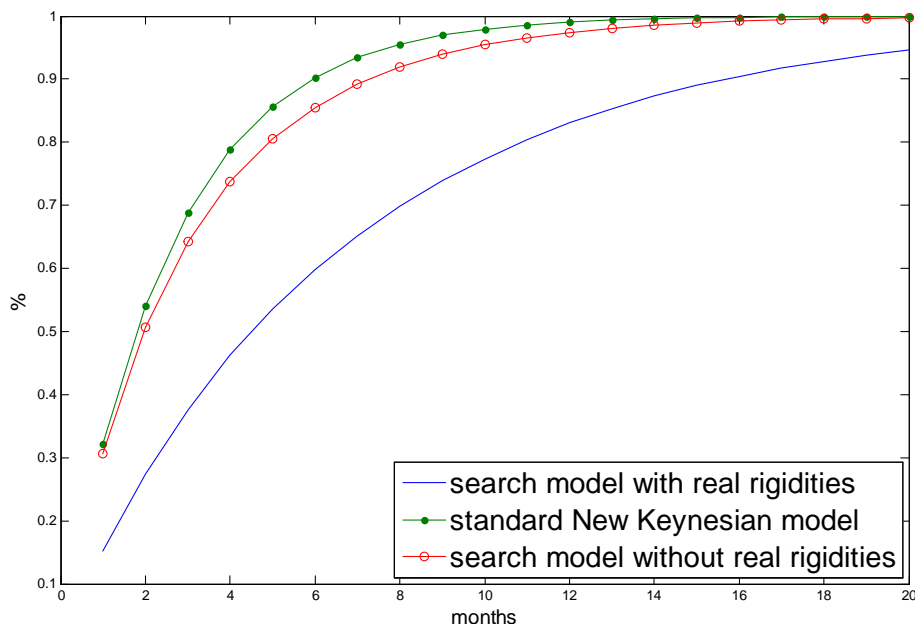
### 4.2.1 Transitory monetary shocks

For the purpose of illustration, I first analyze the economy's response to transitory monetary shocks ( $\rho_m = 0$ ). This exercise allows to quantify the amount of persistence due solely to the

<sup>18</sup>See e.g. Andolfatto (1996), Gertler and Trigari (2006) and Blanchard and Gali (2008).



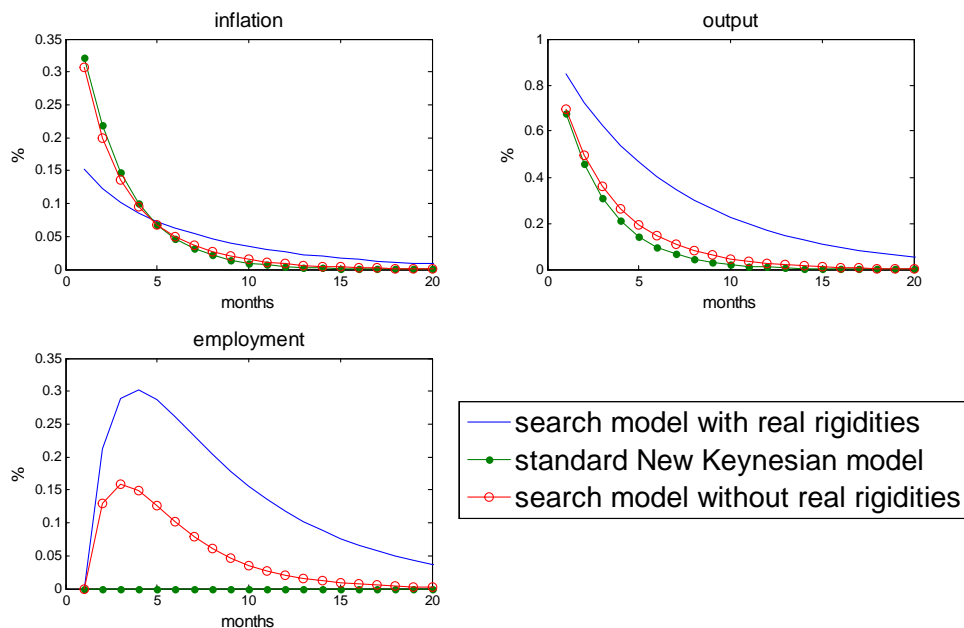
Figure 1: Price level response to a transitory monetary shock



endogenous propagation mechanism in the model. Figure 1 displays the adjustment of the overall price level to a 1% positive shock to money growth under each of the three models considered. In the search model with real rigidities (solid line), the adjustment of the price level is much slower. As shown by the upper left panel of Figure 2, the slower price adjustment leads to an inflation response that is both weaker and more persistent. The aggregate demand relation, equation (6), can alternatively be written as  $\log y_t = \log M_t - \log P_t$ , where  $\log M_t$  jumps immediately from 0 (its normalized initial value) to 1 in the simulation. Given the exogenous expansion in nominal GDP, the slower response of  $\log P_t$  leads to a larger and more persistent output response, as shown in the upper right panel of Figure 2. This produces a similar behavior of average hours per employee (not shown in the figures); since the latter drive job creation, the employment response is larger too, as shown in the lower left panel. Finally, given that  $u_t = 1 - n_t$ , the stronger employment increase translates into a stronger fall in unemployment.

Table 2 displays the effect of search frictions and real rigidities on the persistence of inflation and output, as measured by the coefficients of first-order autocorrelation. Conditional on transitory monetary shocks, inflation persistence increases from 68% in the standard model and 67% in the search model without real rigidities (the 'producer-retailer' model), to 84%

Figure 2: Impulse-responses to a transitory monetary shock



in the search model with real rigidities. Output persistence increases from 68% and 72%, respectively, to 86%.

Table 2. Conditional first-order autocorrelations

|                            | Search, real rigidities | Standard NK model | Search, no real rigidities |
|----------------------------|-------------------------|-------------------|----------------------------|
| Transitory monetary shocks |                         |                   |                            |
| inflation                  | 0.84                    | 0.68              | 0.67                       |
| output                     | 0.86                    | 0.68              | 0.72                       |
| Persistent monetary shocks |                         |                   |                            |
| inflation                  | 0.92                    | 0.87              | 0.88                       |
| output                     | 0.98                    | 0.96              | 0.97                       |
| Productivity shocks        |                         |                   |                            |
| inflation                  | 0.81                    | 0.66              | 0.65                       |
| output                     | 0.999                   | 0.997             | 0.997                      |

Finally, Table 3 compares the two models with search frictions in terms of unemployment

volatility. Conditional on transitory monetary shocks, the standard deviation of unemployment in the model with real rigidities is 2.4 times as large as in the producer-retailer model.

Table 3. Conditional standard deviation of unemployment (%)

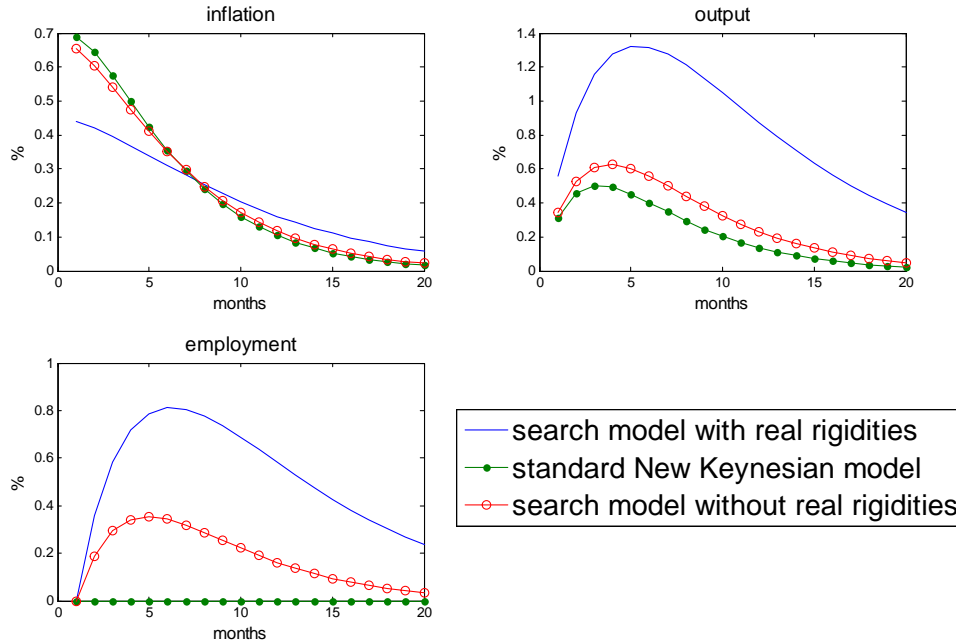
|                            | Search, real rigidities | Search, no real rigidities |
|----------------------------|-------------------------|----------------------------|
| Transitory monetary shocks | 4.12                    | 1.74                       |
| Persistent monetary shocks | 13.75                   | 5.06                       |
| Productivity shocks        | 4.59                    | 1.96                       |

A striking feature of the preceding analysis is that the behavior of inflation and output in the search model with a producer-retailer structure is virtually the same as in the standard New Keynesian model with a frictionless labor market. It is easy to see why. As far as inflation and output dynamics are concerned, the only difference between both models is the fact that in the producer-retailer model real marginal costs depend on employment (compare equations 51 and 45). However, the employment response in the producer-retailer model is not strong enough to make much of a difference to real marginal costs, and hence to inflation and output dynamics.

#### 4.2.2 Persistent monetary shocks

I now return to the more realistic case of persistent monetary shocks and simulate again the economy's response to a 1% shock to money growth. The results, displayed in Figure 3, confirm those obtained in the case of a transitory shock. The inflation response is weaker and more persistent in the search model with real rigidities, and the output response is larger and more persistent. The differences in persistence are now smaller than in the transitory-shock case, due to the persistence in the shock itself (see the autocorrelations in Table 2). The employment response is larger than in the producer-retailer model. Once again, inflation and output dynamics in the standard and the producer-retailer models are very similar, due to the relatively weak employment response in the latter. Finally, the values displayed in Table 3 imply that the standard deviation of unemployment in the model with real rigidities is now 2.7 times as large as in the producer-retailer model.

Figure 3: Impulse-responses to a persistent monetary shock



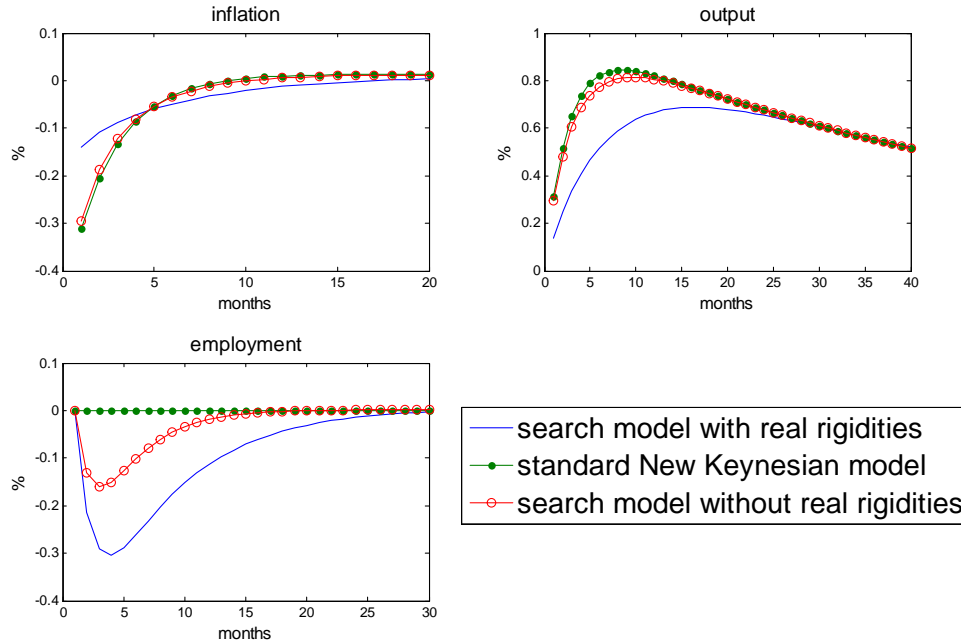
### 4.2.3 Productivity shocks

To conclude the quantitative analysis, I simulate the economy’s response to a 1% positive productivity shock. The results are displayed in Figure 4.

Once again, the inflation response is smaller and more persistent in the search model with real rigidities. As can be seen in Table 2, the first-order autocorrelation of inflation increases from 66% and 65% in the standard and producer-retailer models, respectively, to 81% in the search model with search frictions. Given the results obtained in the case of monetary shocks, it follows that the *unconditional* persistence of inflation (i.e. taking into account both productivity and monetary shocks) increases as a result of real rigidities. Search frictions and the resulting real rigidities therefore help the New Keynesian framework account for the levels of inflation persistence observed in the data. Since nominal GDP remains constant in this exercise, the smaller drop in prices in the search model with real rigidities produces a weaker rise in output.

It is interesting to notice that the employment response is stronger in the search model with real rigidities than in the producer-retailer model. In both models, the rise in exogenous productivity is stronger than the rise in aggregate demand. This leads to a fall in hours per employee (not shown in the figure), which in turn produces a fall in job creation and

Figure 4: Impulse responses to a positive productivity shock



employment. Since aggregate demand increases by less in the model with real rigidities, hours per employee fall by more, which explains the larger drop in employment. According to Table 3, conditional on productivity shocks unemployment volatility increases by a factor of 2.3 relative to the producer-retailer model. Therefore, under my baseline calibration, real rigidities amplify unemployment fluctuations conditional also on productivity shocks. Contrary to the case of monetary shocks however, the effect of real rigidities on unemployment volatility depends on the response of output relative to  $a_t$ , which in turn hinges on the assumed calibration.

## 5 Conclusion

I have analyzed the effect of search frictions in the labor market on inflation dynamics, in a model where the firms making the pricing decisions are also subject to search frictions. In doing so, I depart from most of the earlier literature on New Keynesian models with search and matching frictions, which separates the firms making the pricing decisions from the firms that face search frictions. The framework presented here allows to understand how search frictions affect the pricing policies of individual firms.

I have found that search frictions give rise to real rigidities in price-setting. This mechanism

leads each individual price-setter to make smaller price changes in response to the same macroeconomic fluctuations. On the aggregate, real rigidities slow the adjustment of the overall price level. This is reflected in a smaller sensitivity of inflation to average real marginal costs, i.e. in a flatter New Keynesian Phillips curve. This effect helps the New Keynesian model improve its empirical performance along a number of dimensions. Inflation becomes more persistent. Output responses to monetary shocks become larger and more persistent. Finally, unemployment becomes more volatile conditional on monetary shocks.

It would be interesting to analyze how the real rigidity mechanism generated by search frictions would affect optimal monetary policy. As emphasized by Levin, Lopez-Salido and Yun (2007), real rigidities have important effects on the nature of monetary policy in the context of New Keynesian models. I leave this topic for future research.

## 6 Appendix

### 6.1 Proof of Proposition 1

From equation (27) in the text, I can write the firm's vacancy posting decision as

$$\frac{s_v}{\lambda}(1 - \epsilon)\hat{\theta}_t = \beta E_t \left\{ \frac{\eta}{\mu} \left( \tilde{h}_{it+1} + \hat{h}_{t+1} \right) + [(1 - \lambda)(1 - \epsilon) - (1 - \xi)p(\theta)] \frac{s_v}{\lambda} \hat{\theta}_{t+1} \right\}, \quad (\text{A1})$$

where  $\tilde{h}_{it+1} = \hat{h}_{it+1} - \hat{h}_{t+1}$  is the firm's relative number of hours per worker. Hours per worker admit the following exact log-linear representation,

$$\hat{h}_{it} = \hat{y}_{it}^d - a_t - \hat{n}_{it}.$$

Therefore, I can write  $\tilde{h}_{it} = \tilde{y}_{it}^d - \tilde{n}_{it}$ . This becomes

$$\tilde{h}_{it} = -\gamma \tilde{P}_{it} - \tilde{n}_{it}$$

once I use the fact that  $\tilde{y}_{it}^d = -\gamma \tilde{P}_{it}$ . The firm's expected relative price is given by

$$\begin{aligned} E_t \tilde{P}_{it+1} &= \delta E_t (\log P_{it} - \log P_{t+1}) + (1 - \delta) E_t (\log P_{it+1}^* - \log P_{t+1}) \\ &= \delta E_t (\tilde{P}_{it} - \pi_{t+1}) + (1 - \delta) E_t \left( \log P_{it+1}^* - \log P_{t+1}^* + \frac{\delta}{1 - \delta} \pi_{t+1} \right) \\ &= \delta \tilde{P}_{it} - (1 - \delta) \tau^* \tilde{n}_{it+1}. \end{aligned}$$

In the second equality I have used the fact that, in the Calvo model,  $\log P_{it+1}^* - \log P_{t+1} = \frac{\delta}{1 - \delta} \pi_{t+1}$ , where  $\pi_t \equiv \log(P_t/P_{t+1})$  is the inflation rate. In the third equality I have used  $\log P_{it+1}^* - \log P_{t+1}^* = -\tau^* \tilde{n}_{it+1}$ . Expected relative hours are then given by

$$\begin{aligned} E_t \tilde{h}_{it+1} &= -\gamma E_t \tilde{P}_{it+1} - \tilde{n}_{it+1} \\ &= -\gamma \delta \tilde{P}_{it} - [1 - \gamma(1 - \delta)\tau^*] \tilde{n}_{it+1}. \end{aligned} \quad (\text{A2})$$

Averaging (A1) across all firms and subtracting the resulting expression from (A1) yields  $E_t \tilde{h}_{it+1} = 0$ . Combining this with (A2), I finally obtain

$$\tilde{n}_{it+1} = -\frac{\gamma \delta}{1 - \gamma(1 - \delta)\tau^*} \tilde{P}_{it}.$$

## 6.2 Proof of Proposition 2

Using equations (35), (36) and (37), the inequality  $b^2 > 4ac$  can be written as

$$[1 + \gamma(2 - \delta - \delta\beta)\eta]^2 > 4(1 + \eta\gamma)\gamma(1 - \delta)(1 - \delta\beta)\eta.$$

This in turn can be written as

$$(1 + \eta\gamma)^2 + [(1 - \delta - \delta\beta)\eta\gamma]^2 > 2(1 + \eta\gamma) [(1 - \delta)(1 - \delta\beta) + \delta^2\beta] \eta\gamma.$$

This can be written as

$$\begin{aligned} & (1 + \eta\gamma) \{1 + \eta\gamma - [(1 - \delta)(1 - \delta\beta) + \delta^2\beta] \eta\gamma\} \\ & + \eta\gamma \{(1 - \delta - \delta\beta)^2 \eta\gamma - [(1 - \delta)(1 - \delta\beta) + \delta^2\beta] (1 + \eta\gamma)\} > 0. \end{aligned} \tag{B1}$$

I can now write (B1) as

$$\begin{aligned} & (1 + \eta\gamma) \{1 + \delta [1 - \delta\beta + \beta(1 - \delta)] \eta\gamma\} \\ & - \eta\gamma \{ \delta [1 - \delta + \beta(1 - \delta\beta)] \eta\gamma + [(1 - \delta)(1 - \delta\beta) + \delta^2\beta] \} > 0. \end{aligned}$$

This can be expressed as

$$\begin{aligned} & 1 + (\eta\gamma)^2 \delta \{1 - \delta\beta + \beta(1 - \delta) - [1 - \delta + \beta(1 - \delta\beta)]\} \\ & + \eta\gamma \{ \delta [1 - \delta\beta + \beta(1 - \delta)] + 1 - [(1 - \delta)(1 - \delta\beta) + \delta^2\beta] \} > 0. \end{aligned}$$

Cancelling terms, I can finally write

$$1 + (\eta\gamma)^2 \delta^2 (1 - \beta)^2 + 2\eta\gamma\delta [1 - \delta\beta + \beta(1 - \delta)] > 0,$$

which holds for any  $\delta, \beta \in [0, 1]$ , as required by the theory.

## 6.3 A search model with a producer-retailer structure

Consider an economy where technology and preferences are the same as in the model presented in this paper, but with a different goods-market structure. In particular, a continuum of identical producers produce a homogenous intermediate good that is sold to retailers at the



perfectly competitive price  $mc_t$ . Profits of an individual producer are given by

$$\Pi_{it} = mc_t A_t n_{it} h_{it} - w_t n_{it} - \frac{\chi}{u'(c_t)} v_{it} + E_t \beta_{t,t+1} \Pi_{it+1}.$$

The surplus of worker and firm are given respectively by

$$S_{it}^w = w_t - \frac{b + h_{it}^{1+\eta}/(1+\eta)}{u'(c_t)} - p(\theta_t) E_t \beta_{t,t+1} S_{t+1}^w + (1-\lambda) E_t \beta_{t,t+1} S_{it+1}^w,$$

$$\frac{\partial \Pi_{it}}{\partial n_{it}} = mc_t A_t h_{it} - w_t + (1-\lambda) E_t \beta_{t,t+1} \frac{\partial \Pi_{it+1}}{\partial n_{it+1}}. \quad (\text{C1})$$

Hours per employee are chosen in a privately efficient way, that is, so as to maximize the joint match surplus,  $S_{it}^w + \frac{\partial \Pi_{it}}{\partial n_{it}}$ . The resulting first order condition is given by

$$mc_t A_t = \frac{h_{it}^\eta}{u'(c_t)}, \quad (\text{C2})$$

which implies that hours are equalized across firms,  $h_{it} = h_t$ . The real wage is chosen so as to maximize  $\xi \log S_{it}^w + (1-\xi) \log \frac{\partial \Pi_{it}}{\partial n_{it}}$ . The solution for the real wage is given by

$$w_t = (1-\xi) mc_t A_t h_t + \xi \left[ \frac{b + h_t^{1+\eta}/(1+\eta)}{u'(c_t)} + p(\theta_t) E_t \beta_{t,t+1} S_{t+1}^w \right]$$

$$= (1-\xi) \left[ mc_t A_t h_t + \frac{\chi}{u'(c_t)} \theta_t \right] + \xi \frac{b + h_t^{1+\eta}/(1+\eta)}{u'(c_t)}. \quad (\text{C3})$$

The firm's vacancy posting decision is given by equation (16) in the text. Combining this with (C1) and (C3) yields the following job creation condition,

$$\frac{\chi}{q(\theta_t)} = \beta E_t \left\{ \xi \left[ u'(c_{t+1}) mc_{t+1} A_{t+1} h_{t+1} - b - \frac{h_{t+1}^{1+\eta}}{1+\eta} \right] - (1-\xi) \chi \theta_{t+1} + (1-\lambda) \frac{\chi}{q(\theta_{t+1})} \right\}. \quad (\text{C4})$$

Notice that, from equation (C2), the marginal revenue product of a job (in utils) can be written as  $u'(c_{t+1}) mc_{t+1} A_{t+1} h_{t+1} = h_{t+1}^{1+\eta}$ . This allows me to obtain the following log-linearization of equation (C4),

$$\frac{s_v}{\lambda} (1-\epsilon) \hat{\theta}_t = \beta E_t \left\{ \xi \frac{\eta}{\mu} \hat{h}_{t+1} + [(1-\lambda)(1-\epsilon) - (1-\xi)p(\theta)] \frac{s_v}{\lambda} \hat{\theta}_{t+1} \right\}.$$

Retailers buy the intermediate input at the real price  $mc_t$  and transform it into differentiated final goods with a linear technology. Therefore,  $mc_t$  is also the real marginal cost of retailers and is independent of their pricing decisions. The optimal pricing decision common to all price-setting retailers is given by

$$E_t \sum_{T=t}^{\infty} \delta^{T-t} \beta_{t,T} P_T^\gamma y_T \left( \frac{P_t^*}{P_T} - \frac{\gamma}{\gamma-1} mc_T \right) = 0.$$

Log-linearizing the previous equation and combining it with  $\pi_t = \frac{1-\delta}{\delta} (\log P_t^* - \log P_t)$ , I obtain equation (49) in the text.

## References

- [1] Andolfatto, David, 1996, Business Cycles and Labor-Market Search, *American Economic Review*, 86(1), 112-132.
- [2] Andres, Javier, Rafael Doménech and Javier Ferri, 2006, Price Rigidity and the Volatility of Vacancies and Unemployment, working paper.
- [3] Ball, Laurence and David Romer, 1990, Real Rigidities and the Non-Neutrality of Money, *Review of Economic Studies*, 57(2), 183-203.
- [4] Bills, Mark, 1987, The Cyclical Behavior of Marginal Cost and Price, *American Economic Review*, 77(5), 838-855.
- [5] Bills, Mark and Peter J. Klenow, 2004, Some Evidence on the Importance of Sticky Prices, *Journal of Political Economy*, 112(5), 947-985.
- [6] Blanchard, Olivier and Jordi Gali, 2008, A New Keynesian Model with Unemployment, working paper.
- [7] Calvo, Guillermo, 1983, Staggered Prices in a Utility-Maximizing Framework, *Journal of Monetary Economics*, 12(3), 383-398.
- [8] Card, David, 1994, Intertemporal Labor Supply: An Assessment, in: *Advances in Econometrics: Sixth World Congress*, Christopher Sims (ed.), Cambridge University Press.
- [9] Christiano, Lawrence J., Martin Eichenbaum and Charles L. Evans, 2005, Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy*, 113(1), 1-45.
- [10] Christoffel, Kai and Tobias Linzert, 2005, The Role of Real Wage Rigidities and Labor Market Frictions for Unemployment and Inflation Dynamics, ECB Discussion Paper 556.
- [11] Cooley, Thomas F. and Vincenzo Quadrini, 1999, A neoclassical model of the Phillips curve relation, *Journal of Monetary Economics*, 44(2), 165-193.
- [12] Dotsey, Michael and Robert G. King, 2005, Pricing, Production and Persistence, FRB of Philadelphia working paper 05-4.
- [13] Fuhrer, Jeff and George Moore, 1995, Inflation Persistence, *Quarterly Journal of Economics*, 110(1), 127-159 .

- [14] Gertler, Mark and Antonella Trigari, 2006, Unemployment Fluctuations with Staggered Nash Wage Bargaining, working paper.
- [15] Hall, Robert E., 1980, Employment Fluctuations and Wage Rigidity, *Brookings Papers on Economic Activity*, 1, 91-123.
- [16] Krause, Michael and Thomas Lubik, 2007, The (Ir)relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions, in press, *Journal of Monetary Economics*.
- [17] Kuester, Keith, 2007, Real Price and Wage Rigidities in a Model with Matching Frictions, ECB Working Paper No. 720.
- [18] Levin, Andrew, David Lopez-Salido and Tack Yun, 2007, Strategic Complementarities and Optimal Monetary Policy, Kiel Working Papers 1355.
- [19] Merz, Monika, 1995, Search in the labor market and the real business cycle, *Journal of Monetary Economics*, 36(2), 269-300.
- [20] O'Reilly, Gerard and Karl Whelan, 2005, Has euro area inflation persistence changed over time?, *Review of Economics and Statistics*, 87(4), 709–720.
- [21] Petrongolo, Barbara and Christopher A. Pissarides, 2001, Looking into the Black Box: A Survey of the Matching Function. *Journal of Economic Literature*, 39(2), 390-431.
- [22] Pissarides, Christopher A., 2000, *Equilibrium Unemployment Theory*. MIT Press.
- [23] Pissarides, Christopher A., 2007, The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?, The Walras-Bowley lecture, North American Summer Meetings of the Econometric Society.
- [24] Pivetta, Frederic and Ricardo Reis, 2007, The Persistence of Inflation in the United States, *Journal of Economic Dynamics and Control*, 31 (4), 1326-1358.
- [25] Shimer, Robert, 2005, The Cyclical Behavior of Equilibrium Unemployment and Vacancies, *American Economic Review*, 95(1), 25-49.
- [26] Thomas, Carlos, 2008, Search and matching frictions and optimal monetary policy, forthcoming, *Journal of Monetary Economics*.
- [27] Trigari, Antonella, 2004, Equilibrium Unemployment, Job Flows and Inflation Dynamics, ECB working paper 304.

- [28] Trigari, Antonella, 2006, The Role of Search Frictions and Bargaining for Inflation Dynamics, IGIER Working Paper No. 304.
- [29] Walsh, Carl E., 2003a, *Monetary Theory and Policy*, MIT Press.
- [30] Walsh, Carl E., 2003b, Labor Market Search and Monetary Shocks, in: *Elements of Dynamic Macroeconomic Analysis*, S. Altug, J. Chadha and C. Nolan (eds.), Cambridge University Press, 451-486.
- [31] Walsh, Carl E., 2005, Labor Market Search, Sticky Prices, and Interest Rate Policies, *Review of Economic Dynamics*, 8(4), 829-849.
- [32] Woodford, Michael, 2003, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.
- [33] Woodford, Michael, 2005, Firm-Specific Capital and the New-Keynesian Phillips Curve, *International Journal of Central Banking*, 1(2), 1-46.