CREDIT AND LIQUIDITY RISK IN SOVEREIGN BONDS

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Abstract

The recent financial crisis has shown huge increases in the sovereign yields of some countries. However, it is not clear a priori whether those increases come from fundamental changes in default risk or whether they are due to other determinants. We follow Ejsing et al. (2012) and Dubecq et al. (2013) to shed light on the credit and liquidity risk components of sovereign bond yields. We obtain sovereign and agency historical bond yields from France, Netherlands, Germany and Spain. Then, we consider several state-space models that exploit the differences in the yields between agency and sovereign bonds to identify the credit and liquidity factors. The estimated latent factors capture two distress periods, coinciding with the financial and the sovereign debt crises. In general, the credit effect dominates. We also identify a common European credit effect that allows us to distinguish idiosyncratic (credit) patterns.

1 Introduction

The term structure of sovereign bonds is a key piece of information in economics and finance. It not only reports how agents are discounting future events, but it also reflects the market's views about the financial shape of a country. The recent financial crisis has shown huge increases in the sovereign yields of some countries. However, it is not clear a priori whether those increases come from fundamental changes in default risk or whether they are due to other determinants. In this sense, it is of utmost importance to disentangle the main components of the term structure. A common approach followed by different researchers is to decompose sovereign bond yields into credit and liquidity effects. Credit risk accounts for the possible losses that a bond holder would suffer if the issuer defaults. Liquidity risk accounts for the impact on the price of the easiness or difficulty to trade the bond. Intuitively, the bond yield of a country should increase with the probability of default. Analogously, highly traded bonds are expected to offer a lower yield than less traded ones.

The goal of this paper is to empirically disentangle the credit and liquidity components in bond yields. We assume that the yield differences between two bonds with the same credit risk and time to expiration are due to liquidity reasons. Interestingly, this is a condition met by sovereign and agency bonds if the latter enjoy an explicit guarantee from the government. In other words, they are ex-ante equally credit risky. We consider sovereign and agency bond prices from four countries: France, Germany, the Netherlands and Spain, whose main agencies are, respectively, CADES *(Caisse d'Amortissement de la Dette Sociale;* social security debt redemption fund), KfW *(Kreditanstalt für Wiederaufbau;* Reconstruction Credit Institute), BNG *(Bank Nederlandse Gemeenten;* Dutch Municipal Bank) and ICO *(Instituto de Crédito Oficial;* Official Credit Institute).

We use the Nelson and Siegel (1987) formula to compute from the original bond raw data weekly constant maturity yields for the two, five and ten year maturities. Then, we employ state-space models to explain the spread between the yields and the risk-free rate as functions of credit and liquidity latent factors. This methodology is well suited for this application, because it can filter out the relevant factors underlying the data under a minimal set of assumptions. In this case, we can easily introduce reasonable and relatively innocuous identifying restrictions for the credit and liquidity factors. In all cases, we consider maximum likelihood estimation using the Kalman filter, which is the standard approach for the estimation of state-space models. We proxy for the risk-free rate using the Overnight Indexed Swap rate (OIS). We start initially with a simple benchmark affine model that only requires as inputs the agency and sovereign yields plus the OIS rate, for a given time to maturity, to identify the credit and liquidity factors. We carry out different estimations for each maturity. We continue by including Credit Default Swap rates (CDS), which basically reflect the insurance price of hedging against sovereign default. CDS rates are helpful to identify the credit risk factor more accurately, since their movements should not be driven by bond liquidity effects. Then, we study the presence of a common credit risk effect for the Netherlands, Germany and France. Finally we consider a consistent framework for the whole term structure of a given country by modelling the three different maturities in a joint setting. In particular, we follow Dubecq et al. (2013) by specifying a quadratic form for the stochastic discount factor, and estimate the factor dynamics under the physical and the risk-neutral measures.

A fundamental feature of all the specifications that we consider is that they allow us to quantify the impact of credit and liquidity risk on the yields. In this sense, we find that the credit factor has in general more influence than the liquidity factor. We capture two distress periods corresponding to the banking and the financial crisis. Interestingly, at those moments the credit factor displayed peaks but its effect on the yield was disguised by the liquidity factor, which accounted for the safe haven flows phenomena typical for highly rated sovereign bonds under tense financial episodes. Furthermore, when we account for a common (credit) factor, we can observe substantial differences in the country specific credit risks. Under this approach, the Netherlands' idiosyncratic credit factor remains quite stable around zero while France and Germany's exhibit a mirrored pattern, displaying the widest gap in 2012 and taking positive and negative values respectively.

The rest of the paper continues as follows. Section 2 reviews the previous literature on the topic. Section 3 discusses the data used and the computational methodology for the yield curves. The three affine models comprise section 4 and the fourth model is developed in section 5. Section 6 concludes. The appendix contains auxiliar results.

2 Related Literature

The discrete-time finance literature on the decomposition of term structures into credit and liquidity factors can be classified into two main categories. On the one hand, we have reduced-form approaches that model interest rates as linear approximations of some factors. On the other hand, other papers follow a more structural approach by directly modelling the stochastic discount factor and the default process to obtain the implied interest rates.

We integrate these two strands of the literature by building on the contributions of two previous papers: section 4 elaborates on the parametric framework designed by Ejsing et al. (2012), while section 5 adapts to our setting the framework of Dubecq et al. (2013), which was originally devised to model EURIBOR rates. Ejsing et al. (2012) try to disentangle the credit and liquidity risk contributions to French and German sovereign yields using a reduced-form affine setting. For the two countries, they exploit the assumption that sovereign and agency bonds are equally credit worthy. If this assumption holds, the difference between their yields should be due to liquidity reasons. Hence, they can use this feature to identify both factors. In contrast, Dubecq et al. (2013) propose a more structural approach to model the EURIBOR rates as the expected discounted values of the risk-free rate plus an intensity parameter. This intensity parameter is a quadratic function of credit and liquidity latent factors and takes different values under the physical and the risk neutral measure.

The paper by Ejsing et al. (2012) is a prominent example among the papers that identify liquidity by exploiting the fact that the agency and the sovereign bonds have the same credit risk. Another interesting example is provided by Monfort and Renne (2012), who analyze the yields of 8 euro-area countries. They assume perfect co-movement among countries in the liquidity factor and proxy it by the spread between the German Bund and the KfW (German agency) emissions. Similarly, Schwarz (2013) proposes a multi-country European model in which liquidity is proxied by the spread between the sovereign German bond and the KfW bond. However, she also proposes a credit measure "defined as the daily spread between actual unsecured interbank borrowing rates paid by banks that are relatively good credit risks versus those that are relatively bad credit risks". Another interesting example is Liu et al. (2006), who jointly model sovereign, repo and swap term structures as an affine five-factor model, two of them accounting for credit and liquidity risk. Favero et al. (2008) use the bid-ask spread to capture liquidity risk and the spread between the US sovereign yield and the US corporate swap rate for credit risk. Lastly, Dubecq et al. (2013) proxy liquidity premia by the first principal component of the KfW Bund spread, Tbill-repo spread and a factor based on a ECB survey, and credit premia by the first principal component of 36 Eurozone banks' CDS rates.

Ejsing et al. (2012) and Liu et al. (2006) model yields as linear functions of some factors. In contrast, Dubecq et al. (2013) make use of the quadratic Kalman filter designed by Monfort et al. (2013). Other researchers using a quadratic approach are Ahn et al. (2002), who also compare the performance of affine and quadratic term structure models and claim that the second outperform the first; and Constantinides (1992), who presents a model that "allows the term premium to change sign as a function of the state (variables) and the term to maturity and also allows for shapes of the yield curve that are observed (...) but are disallowed in the Cox, Ingersoll, and Ross (1985) model."

3 Data and Yield Curve Computation

We have obtained from Datastream daily yield data of fixed coupon sovereign and agency bonds for France, Germany, the Netherlands and Spain between January 2, 2007, and February 27, 2014. We consider the bonds issued by the main agencies in these countries (respectively CADES, KfW, BNE and ICO). These agencies are not fully comparable in terms of their activity. For example, the Spanish agency ICO works mainly providing loans to targeted companies, but the German KfW has a much broader scope, including mortgages, environmental and developing projects and also business financing. In any case, for our purposes the relevant common feature is that all these agencies have their debt backed by their respective governments. This implies that ex-ante their bonds have the same creditworthiness as the bonds issued by the sovereign.

Chart 1 shows the historical times to maturity in years for the agency bonds that are closer than 15 years to their maturity. We can observe a scarcer number of bonds for BNE and especially ICO. Unfortunately, this forces us to postpone the starting date of the sample for Spain to January 2 2010.

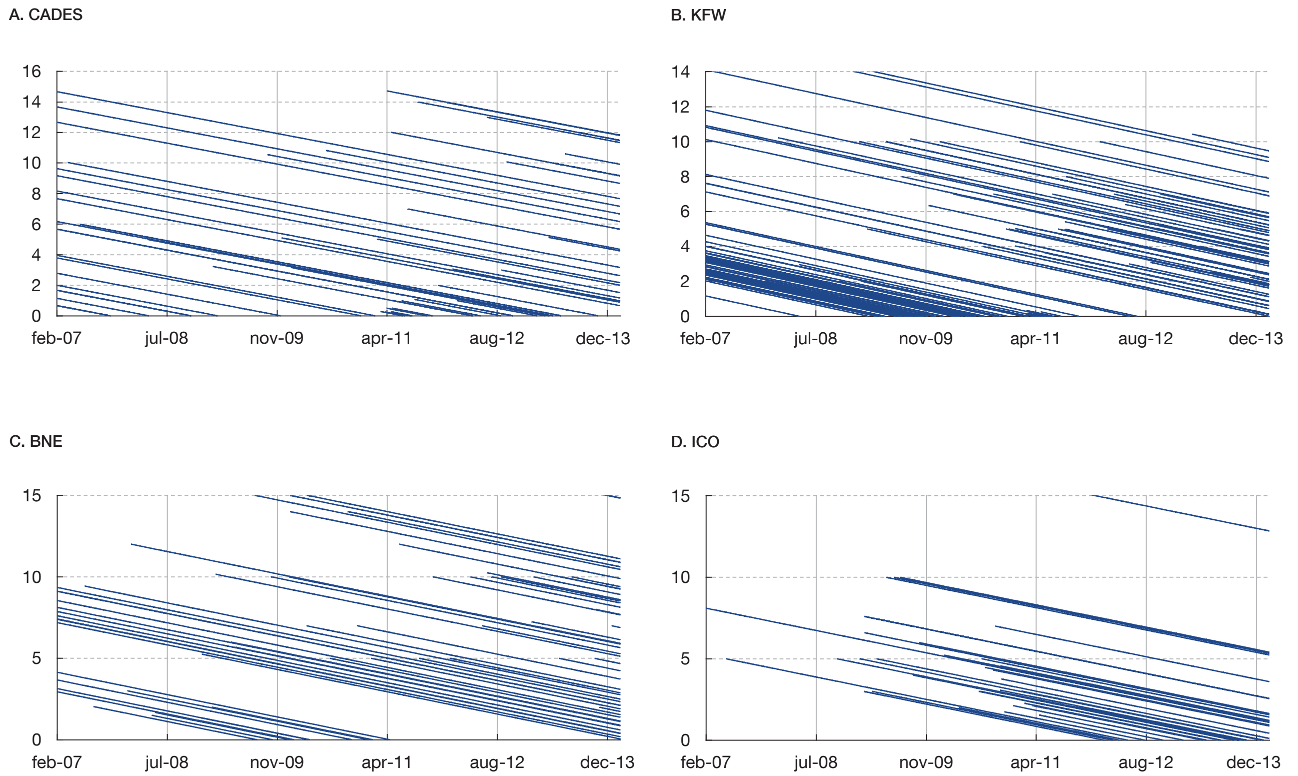
For each day and issuer, we have computed a yield curve using the popular Nelson and Siegel (1987) formula

$$y(\tau) = \beta_0 + \beta_1 \frac{1 - e^{-\tau\lambda}}{\tau\lambda} + \beta_2 \left[\frac{1 - e^{-\tau\lambda}}{\tau\lambda} - e^{-\tau\lambda} \right], \tag{1}$$

where $y(\tau)$ denotes the yield to maturity. The three additive terms can be interpreted as level, slope and curvature factors as described by Litterman and Sheinckman (1991). Alternatively, β_0 can be interpreted as a long term factor (constant loading), β_1 as a short term factor (the loading decreases fast to 0 with time to maturity τ) and β_2 a medium term factor (hump shaped loading in τ whose right tail tends to 0). Furthermore, to transform a

TIME TO MATURITY OF THE AGENCY BONDS IN THE DATABASE

CHART 1



SOURCES: Author's elaboration using data from Datastream.
NOTES: The time to maturity is expressed in years. The graphs have been truncated at the 15 year maturity, since our analysis is focused on the 2, 5 and 10 year maturities.

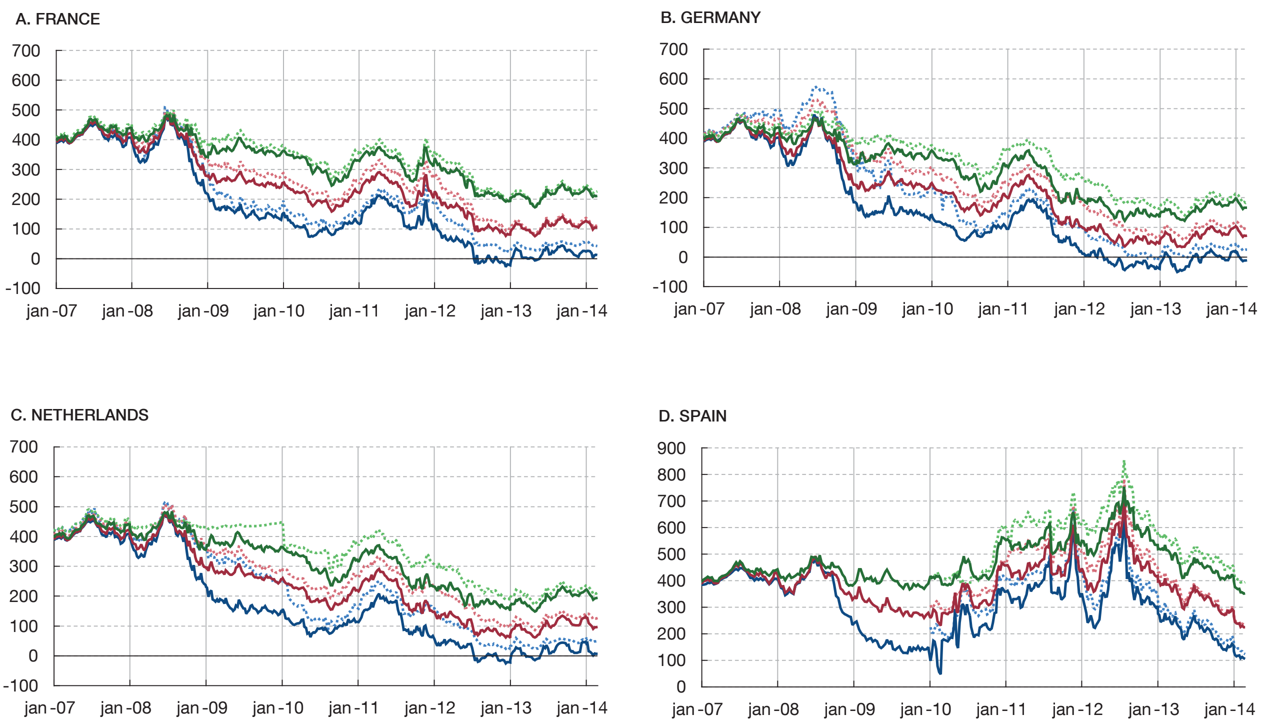
computationally intensive numerical optimisation problem into simple least squares, we set $\lambda = 0.0609$ as suggested by Diebold and Li (2006), which maximizes the loading of β_2 for $\tau = 29.45$, or roughly speaking, 30 months, in consonance with its medium term interpretation.

We have disregarded all bonds maturing before one year to avoid the distortions caused by abrupt variations of the yields close to expiration. Besides, yields that are two standard deviations away from their daily means are eliminated. For the case of the agencies, we also reject yields that are 200 basis points above or below the yield curve of the corresponding sovereign. Once all the daily yield curves are computed, we collect the yields to maturity for the 2, 5 and 10 year time-horizons and obtain the weekly median to further eliminate possible outliers. Chart 2 shows the evolution of these series for all issuers. We can observe that, prior to the financial crisis beginning in the autumn of 2008, the differential between sovereign and agency bonds was very small and the term structure was relatively flat. In addition, the yields from the different countries were very similar. As the crisis unfolded, we can observe growing differentials between sovereign and agency bonds and a steepening of the term structure within each country. As expected, the agency bond yields generally lie above their sovereign yields counterparts. In addition, we can also notice widening differentials across countries. In particular, the European sovereign crisis starting in 2010 barely affected France, Germany and Netherlands compared to Spain, which suffered a substantial increase in both sovereign and agency yields.[1]

1 See Castro and Mencía (2014), for a thorough discussion of the differentials between Euro-area sovereign bonds.

SOVEREIGN AND AGENCY YIELDS FOR CONSTANT MATURITIES

CHART 2



SOURCE: Author's elaboration using data from Datastream.
NOTES: The constant maturity yields have been obtained by applying the Nelson and Siegel (1987) formula to the raw data. The two, five and ten year maturities are represented with blue, red and green lines, respectively. Sovereign yields are plotted using dark coloured lines, while the analogous agency yields are plotted with light coloured dotted lines. Agency yields for Spain are not plotted prior to January 2, 2010.

As mentioned in the introduction, we take the OIS rate (at 2, 5 and 10 years) as a proxy for the risk-free rate. The OIS is a fixed for floating interest rate swap that places in the floating leg the Euro overnight index average.[2] Before the crisis, it was common to consider interbank offered rates (e. g. LIBOR, EURIBOR...) for this purpose. However, the great recession has heavily questioned the validity of that assertion [see Dubecq et al. (2013)]. Chart 3 illustrates this fact, displaying the overlapping between the OIS and EURIBOR 3-month rates before 2007 and a gap after. OIS is generally considered a better proxy of the risk-free rate because it does not imply large transactions of capital as no principal is exchanged and also because it enjoys credit and netting enhancement mechanisms, such as margin accounts.[3] The reason as for why CDSs are taken as a proxy for credit risk is straightforward as they basically constitute insurance against sovereign default. The data for the OIS and the CDS rates have also been downloaded from Datastream.

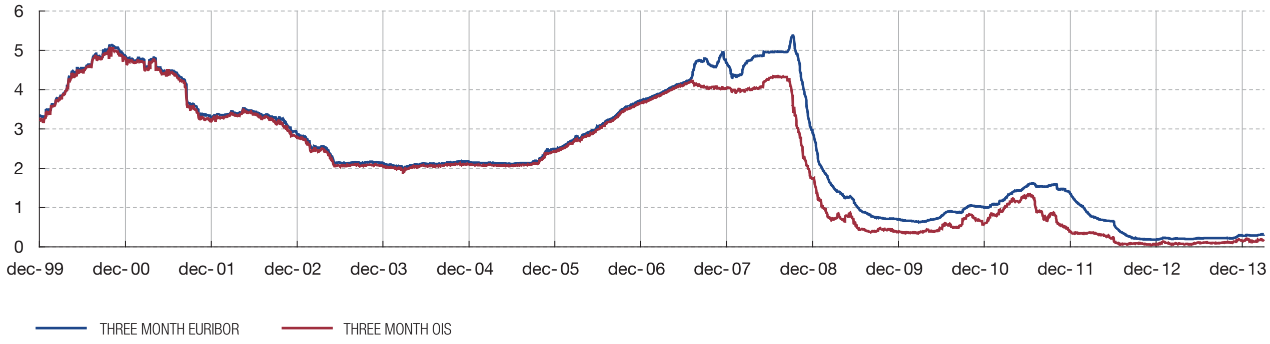
4 Affine Models

In this section, we initially apply, with some minor changes, the model by Ejsing et al. (2012) to the four countries in our database. Then, we continue with two extensions. First, we include CDS sovereign rate data in the measurement equation to enhance the identification of the factors. And secondly, we pool all the countries in a joint framework to estimate a common European factor.

2 The European Banking Federation defines the EONIA as "... the effective overnight reference rate for the euro. It is computed as a weighted average of all overnight unsecured lending transactions in the interbank market, undertaken in the European Union and European Free Trade Association (EFTA) countries".

3 Bomfim (2002) mentions three main credit enhancement mechanisms: "(i) credit triggers clauses, which give the higher-quality counterparty the right to terminate the swap if its counterparty's credit rating falls below, say, BBB, (ii) the posting of collateral against the market value of the swap, and (iii) requirements to obtain insurance or guarantees from highly-rated third parties".

HISTORICAL EVOLUTION OF THE THREE-MONTH EURIBOR AND OIS RATES CHART 3



SOURCE: Datastream.

4.2 BENCHMARK MODEL

We define the spread of bond i at time t as $s_{i,t} = y_{i,t} - r_t$, where $y_{i,t}$ is the yield to maturity of the bond i at time t, and r_t is the risk-free rate at time t (proxied by the OIS).

Ejsing et al. (2012) propose a model in which the spreads between the yield to maturity of the bonds and the risk-free rate are linear functions of credit and liquidity risk. Instead, we consider affine functions of the factors, which do not impose zero intercepts. In particular, we consider the following state space model:

$$\text{Measurement}: \mathbf{s}_t = \begin{pmatrix} \delta_{sov} \\ \delta_{agn} \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & \theta \end{pmatrix} \mathbf{x}_t + \epsilon_t, \tag{2}$$

$$\text{Transition}: \mathbf{x}_t = \begin{pmatrix} \alpha_c & 0 \\ 0 & \alpha_l \end{pmatrix} \mathbf{x}_{t-1} + \mathbf{v}_t, \tag{3}$$

where $\mathbf{s}_t = (s_{sov,t}, s_{agn,t})'$ is the vector containing the sovereign and agency spreads at time t, while $\mathbf{x}_t = (x_{c,t}, x_{l,t})'$ is the vector containing the credit and liquidity latent factors, respectively. We assume that the residuals are Gaussian and independent, so that $\mathbf{x}_t$ captures all the correlation between the bond yields. Specifically, $\epsilon_t \sim$ iid N $[\mathbf{0}, \sigma^2 \mathbf{I}_2]$ and $\mathbf{v}_t \sim$ iid N $[\mathbf{0}, \text{diag}(\boldsymbol{\omega}_v)]$, where $\boldsymbol{\omega}_v = (\sigma^2_{vc}, \sigma^2_{vl})'$, $\sigma^2_{vc} = 1 - \alpha^2_c$ and $\sigma^2_{vl} = 1 - \alpha^2_l$ to fix the otherwise undetermined scale of the latent vector $\mathbf{x}_t$.[4]

We assume that the autoregressive terms α_c and α_l are smaller than one in absolute value. Thus, we implicitly assume that the yields are stationary and mean-reverting. Notice that we assume that the credit and liquidity risk factors are conditionally and unconditionally independent. Ejsing et al. (2012) show that this is a necessary condition for identification in this framework. In any case, this orthogonality assumption is empirically sensible, as the correlation between the proxies for credit and liquidity risk tends to deliver values close to zero in practice.

Notice that all the loadings are normalised to 1 except for the one on liquidity in the agency equation. This is the key for the identification of the two factors. In order to identify the two components of $\mathbf{x}_t$, the liquidity loading for the agency spreads θ should be different from 1. In other words, the contribution of the credit factor to both agency and sovereign bonds yields is the same, while it is different on the liquidity side. For the sovereign it will just be $x_{l,t}$ and for the agency $\theta x_{l,t}$.

4 Ejsing et al. (2012) do allow for different (and constant) variances for $_t$. They do also estimate σ^2_{vc} and σ^2_{vl}. Our more parsimonious approach implies that $V(\mathbf{x}_t) = \mathbf{I}_2$.

To grasp the intuition, we can succinctly represent the spread as the sum of two components:

$$\text{spread}_i = \text{credit premium}_i + \text{liquidity premium}_i, \tag{4}$$

for i = {sov,agn}. In general, we should expect less credit-risky bonds to be pricier than more credit-risky bonds; a higher probability of default pushes the yield up and is translated in our model as a positive credit factor, especially under financial distress. On the contrary, more liquid bonds will experience an appreciation in their price or equivalently a lower yield. This phenomenon is intensified during market downturns, where agents generally want to hold in their portfolio a larger proportion of safe and liquid assets, such as sovereign bonds. This capital flows are known in the finance literature as safe haven flows [Longstaff (2004)]. In our model, the liquidity factor decreases, or even becomes negative, to reflect better liquidity conditions. For the agencies, what matters is the product $\theta x_{l,t}$. Since $s_{agn,t} > s_{sov,t}$ in general, we have that

$$s_{agn,t} - s_{sov,t} = \delta_{agn} - \delta_{sov} + (\theta - 1)\, x_{l,t} > 0. \tag{5}$$

In principle, we expect $x_{l,t}$ to be smaller than zero on average, so that a higher liquidity enters in our model through a more negative value of $x_{l,t}$ that reduces the yields. If that is the case, assuming that the impact of the intercepts in (5) is negligible, we should expect θ to be smaller than one.

Table 1.A shows the estimates of θ at the 2, 5 and 10 year horizons for the four countries. We have carried out different estimations for each country and maturity. As expected, all the estimates are smaller than 1. In fact, almost all of them are negative and significantly

PARAMETER ESTIMATES OF THE BENCHMARK MODEL — TABLE 1

A. LIQUIDITY LOADING ON THE AGENCY SPREADS (θ). ALL MATURITIES

Maturity	France	Germany	Netherlands	Spain
2 years	-0.290	-3.797	-1.094	0.150
	(0.075)	(0.189)	(0.105)	(0.073)
5 years	-0.022	-1.639	-0.599	-0.063
	(0.062)	(0.141)	(0.048)	(0.151)
10 years	-0.166	-0.648	-0.599	-1.366
	(0.039)	(0.068)	(0.048)	(0.111)

B. REMAINING PARAMETERS. FIVE YEARS TO MATURITY

	France	Germany	Netherlands	Spain
δ_{sov}	0.192	-0.010	0.117	2.281
	(1.298)	(1.405)	(1.344)	(0.911)
δ_{agn}	0.282	0.233	0.358	2.629
	(0.889)	(1.942)	(1.107)	(0.497)
α_c	0.999	1.000	0.999	0.972
	(0.000)	(0.034)	(0.034)	(0.003)
α_l	0.999	1.000	0.999	0.994
	(0.034)	(0.033)	(0.034)	(0.002)
σ	0.006	0.015	0.000	0.000
	(0.003)	(0.002)	(0.008)	(0.021)

SOURCE: Author's elaboration using data from Datastream.
NOTE: The parameters have been obtained by maximum likelihood from different estimations for each country and maturity. Standard errors are reported below the estimates in parenthesis.

different from 1. Except for Spain, the coefficients tend to be smaller for the shortest horizon, which might imply that liquidity risk differentials between sovereign and agency bonds diminish with the horizon. Table 1.B shows the remaining parameters of the model, only for the five-year horizon for the sake of brevity. We can observe that the intercept terms are not significant except for Spain, where they increase with the time to maturity and the relationship $\delta_{sov} < \delta_{agn}$ always holds. In contrast, the estimates show that the two latent factors are very persistent in all countries.

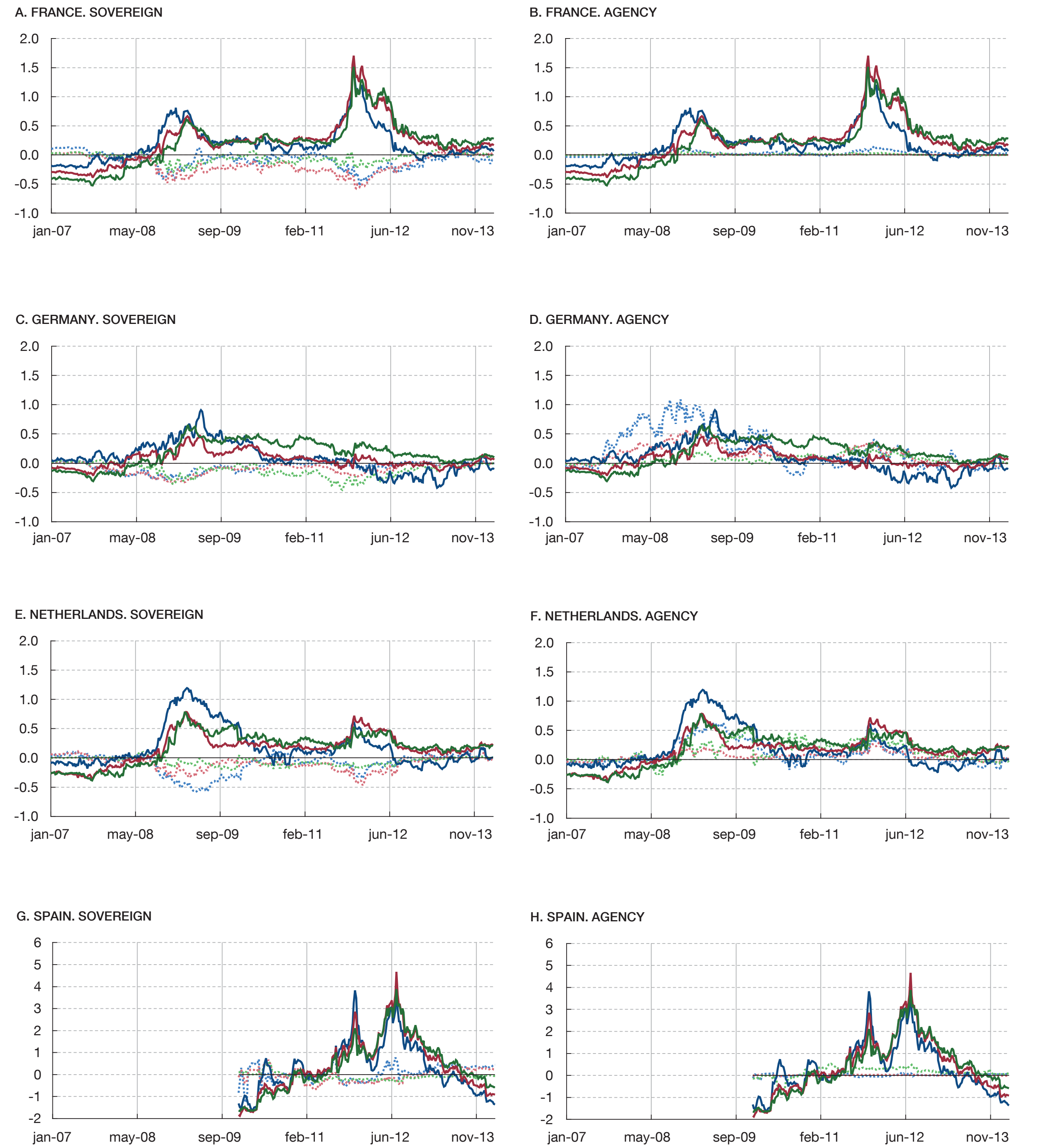
Chart 4 shows the filtered credit and liquidity factors. The credit risk factors reflect two crises: the 2008-2009 financial crisis and the 2011-2012 European sovereign crisis. According to this specification, Germany and the Netherlands were more severely hit by the first crisis, while the second crisis was relatively more important for France and especially Spain. The liquidity factor, which is generally negative over the sample, tends to reduce sovereign yields, but it leaves agency yields almost equal to the credit risk factor. Thus, the credit factor generally dominates over the liquidity factor, with the only exception of the German agency KfW. In this case, Chart 4.C shows that the liquidity crisis affected the liquidity of KfW bonds more than credit risk at the short term, and it had a non-negligible impact over the long maturities as well. For the sovereign bonds and the remaining agency bonds, the credit risk factor is much larger in magnitude than the liquidity factor. Therefore, we can conclude that the financial crisis was driven by both credit and liquidity factors, while the sovereign crisis was mainly driven by credit risk concerns. From a term structure perspective, we can observe that the credit risk factors are increasing with the time to maturity after 2012 for all countries. Before 2012 there is not a clear differentiation between different maturities. In contrast, there is not a stable pattern on the term structure of the liquidity factors.

The nature of the financial crisis differs across countries, due in part to the fact that different EU member countries have undergone different situations in their financial systems and their real economies. For example, Dutch banks were among the most exposed in Europe to American financial markets and after the Lehman bankruptcy in 2008 and the consequent spread of the storm to both sides of the Atlantic, the government had to partially nationalize some institutions, such as Fortis, and took measures to strengthen the deposit guarantee scheme [Masselink and Den Noord (2009)]. German banks, however, suffered from their originate-to-distribute business model, which basically consisted in expanding the lending capacity through collateralised debt obligations and other securities of the sort. By 2009 several German institutions had to be recapitalised by the government and a “bad-bank” was created to transfer all non-performing securitised assets. In all cases, the assistance provided by national governments contributed to reduce the financial tensions, though it was translated into increasing sovereign risk. This, added to the possible understanding of a tacit and implicit guarantee by the government to banks, lead to the sovereign debt crisis three years later. At this stage, the unconventional monetary policy measures taken by the ECB (e. g. Long Term Refinancing Operations, Outright Monetary Transactions...) as well as the measures taken in coordination with financial authorities (e. g. Stress tests in Spain) eventually brought back calm to the financial arena.

As a validity check of the model, we compare the CDS rates on the sovereign bonds with the estimated credit factors in Chart 5. We focus on the 5 year time to maturity due to its higher liquidity. CDS rates are a very popular proxy for the credit risk of bonds as they essentially provide insurance against default. As we can see in the picture, the lines move closely, which supports our findings. We can also observe that CDS rates are generally

CREDIT AND LIQUIDITY FILTERED FACTORS FROM THE BENCHMARK MODEL

CHART 4



SOURCE: Author's elaboration using data from Datastream.
NOTES: The two, five and ten year maturities are represented with blue, red and green lines, respectively. The credit factors are plotted using dark coloured lines, while the analogous liquidity factors are plotted with ligth coloured dotted lines. The factors for Spain cannot be reliably computed prior to January 2, 2010.

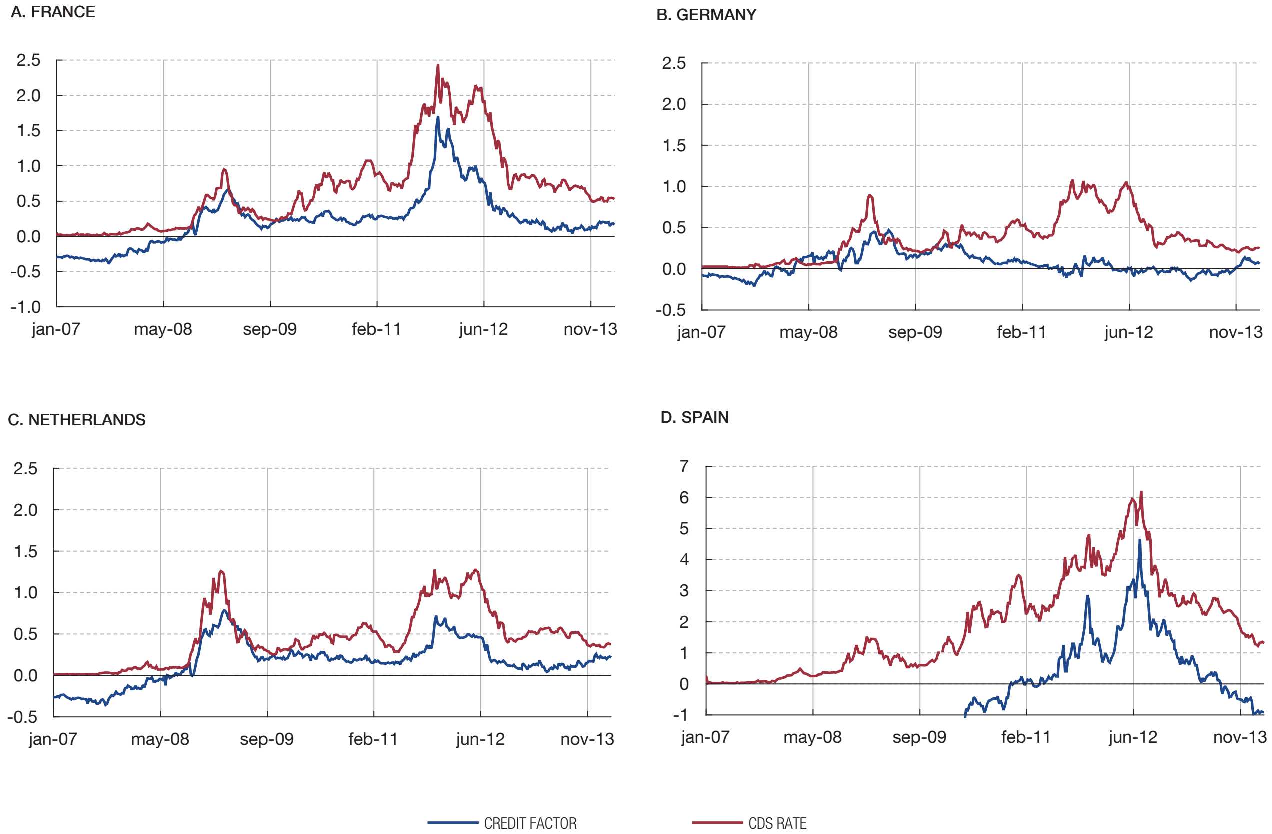
above the credit factors. This might reflect the higher illiquidity of these assets, though this assertion must be considered as a tentative explanation; a possible line of research could be started in this direction.

4.2 ADDING CDSs

Given the implications of Chart 5 and the arguments provided in the last paragraph, a natural extension of the previous model would include CDS rates into the measurement equation. In this way, we enhance the identification of the factors by considering an

CREDIT FACTORS AND CDS RATES. FIVE YEARS TO MATURITY

CHART 5



SOURCE: Author's elaboration using data from Datastream.
NOTES: The plotted credit factors have been obtained using the benchmark model. The credit factor for Spain cannot be reliably computed prior to January 2, 2010.

additional source of information. The resulting state-space model is characterised by the following enlarged measurement equation:

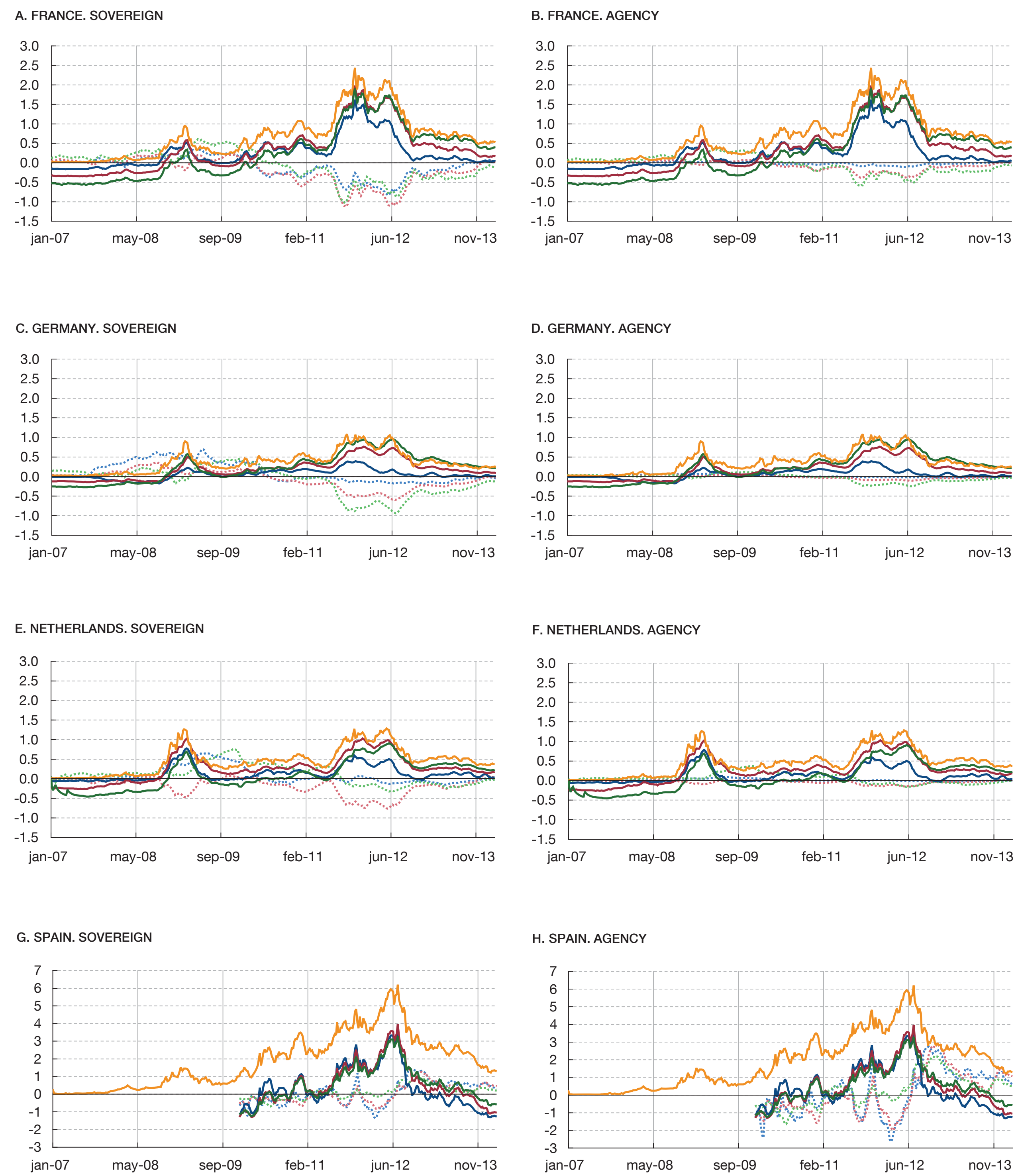
$$\begin{pmatrix} s_t \\ cds_t \end{pmatrix} = \begin{pmatrix} \delta_{sov} \\ \delta_{agn} \\ \delta_{cds} \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & \theta \\ 1 & 0 \end{pmatrix} \mathbf{x}_t + \epsilon_t, \qquad (6)$$

where cds_t denotes the CDS rate for the corresponding sovereign bond, δ_{cds} denotes the intercept in the CDS equation, $\epsilon_t \sim$ iid $N(\mathbf{0}, \sigma^2 \mathbf{I}_3)$ and (3) would still be the transition equation in this case. Notice that the credit factor loading in the CDS equation is one, the same as in the sovereign bond equation, while the liquidity factor loading is 0. The same considerations about θ and the liquidity factor explained in subsection 4.1 apply here as well.

Chart 6 shows the filtered credit and liquidity factors resulting from the estimation of (6), together with the 5 year CDS rates. The dynamics of the factors are similar to those observed for the benchmark model. Once again, we capture the two distress periods corresponding to years 2009 and 2012. The credit factor still shows a higher impact on the yields than the liquidity factor. However, now the effect of the sovereign crisis on German bonds is slightly different. Cahrt 6.C shows that there was indeed a deterioration of credit risk in Germany during the sovereign crisis, but it was compensated by the negative liquidity factor that reduced its sovereign yield to almost zero. Thus, German sovereign bonds were actually perceived as riskier, but their yields could stay low thanks to their increasing demand (here captured by the liquidity factor). In contrast, the German agency

CREDIT AND LIQUIDITY FILTERED FACTORS FROM THE EXTENDED MODEL WITH CDS DATA

CHART 6



SOURCE: Author's elaboration using data from Datastream.
NOTES: The two, five and ten year maturities are represented with blue, red and green lines, respectively. The credit factors are plotted using dark coloured lines, while the analogous liquidity factors are plotted with light coloured dotted lines. The 5 year CDS are plotted with yellow lines. The factors for Spain cannot be reliably computed prior to January 2, 2010.

bond yields did not benefit from such a great liquidity effect. Interestingly, the difference between Spanish CDS rates and the credit factors is more marked than for the other countries. Furthermore, the differential between the CDS rates and the credit factor are comparatively larger than for other countries. This is perhaps due to a larger risk aversion implicit in the Spanish CDS rates during the sovereign crisis. On the light of these new

PARAMETER ESTIMATES OF THE MODEL THAT INCLUDES CDS SPREADS

TABLE 2

A. LIQUIDITY LOADING ON THE AGENCY SPREADS (θ). ALL MATURITIES

	France	Germany	Netherlands	Spain
2 years	0.741	—	—	0.801
	(0.026)			(0.022)
5 years	0.792	-1.253	-0.403	0.848
	(0.016)	(0.014)	(0.028)	(0.032)
10 years	0.998	0.740	1.239	1.447
	(0.011)	(0.016)	(0.045)	(0.071)

B. REMAINING PARAMETERS. FIVE YEARS TO MATURITY

	France	Germany	Netherlands	Spain
δ_{sov}	0.134	-0.156	0.110	2.074
	(1.284)	(1.402)	(1.353)	(0.969)
δ_{agn}	0.348	0.350	0.393	2.491
	(1.155)	(1.597)	(1.032)	(0.875)
δ_{cds}	0.344	0.182	0.199	2.338
	(0.900)	(0.971)	(0.957)	(0.572)
α_c	0.999	1.000	0.999	0.981
	(0.000)	(0.001)	(0.036)	(0.002)
α_l	0.999	1.000	0.999	0.994
	(0.000)	(0.038)	(0.038)	(0.001)
σ	0.076	0.112	0.076	0.165
	(0.002)	(0.002)	(0.002)	(0.006)

SOURCE: Author's elaboration using data from Datastream.
NOTE: The parameters have been obtained by maximum likelihood from different estimations for each country and maturity. Standard errors are reported below the estimates in parenthesis.

estimates, the sovereign crisis was much more severe than the financial crisis, both in terms of credit and liquidity risk.

Table 2 shows the parameters obtained in these estimations. The most remarkable difference with respect to Table 1 lies on the signs of the θ parameters. Now, they are generally positive and increasing with the time to maturity. They are smaller than one in most estimations, though. The two cases in which they are larger than one are characterised by a positive liquidity factor. The intercepts are once again not statistically different from 0 for all countries but Spain, where still the relationship $\delta_{agn} > \delta_{sov}$ holds.

In fact, the ordering this time is $\delta_{agn} > \delta_{cds} > \delta_{sov}$ for all maturities. Lastly, the factors also exhibit a very persistent autoregressive pattern.

In conclusion, including an extra source of information not only helps to improve the identification of the credit factor, it contributes to the liquidity side of the model as well.

4.3 JOINT MODEL, LOOKING FOR A COMMON FACTOR

The observed co-movement of the credit and liquidity factors across different countries in Charts 4 to 6 suggests the presence of a common European driver. Given the panel dimensionality of our data, we can identify the factor that generates this effect. In particular, we introduce a common European credit factor in our framework. We exclude Spanish data to use the longest possible complete panel.

Let us denote as $x_{eur,t}$ the factor accounting for the common European credit shock at time t. Then we have:

$$s_{i,sov,t} = \delta_{i,sov} + x_{eur,t} + x_{i,c,t} + x_{i,l,t} + \epsilon_{i,sov,t} \tag{7}$$

$$s_{i,agn,t} = \delta_{i,agn} + x_{eur,t} + x_{i,c,t} + \theta_i \, x_{i,l,t} + \epsilon_{i,agn,t} \tag{8}$$

where i is the country indicator (France, Germany or the Netherlands).

As we have done in the previous models, we normalise all the loadings to 1 except for the liquidity loading of the agencies. Thus, the liquidity factor is still the only source of differentiation between agency and sovereign yields.

Table 3.A shows that all the agency liquidity loadings estimates are negative and statistically significant in this model. In this sense, this model reflects the same features as the previous model. In addition, Table 3.B, which reports the remaining parameters for the five-year maturity case, shows that once again we do not obtain statistically significant intercepts for the spreads but we still estimate a very persistent autoregressive process. However, it is more interesting to look at Chart 7, which shows the evolution of the national credit and liquidity factors. Interestingly, we can notice that the credit factors no longer include the common European component. This is why now the Dutch credit factors remain almost flat at zero, while the German ones even take negative values. In contrast, the French

PARAMETER ESTIMATES OF THE THREE-COUNTRY JOINT-MODEL **TABLE 3**

A. LIQUIDITY LOADING ON THE AGENCY SPREADS (θ). ALL MATURITIES

	France	Germany	Netherlands
2 years	-2.055	-2.831	-2.524
	(0.437)	(0.558)	(0.558)
5 years	-0.499	-0.864	-0.991
	(0.230)	(0.181)	(0.095)
10 years	-0.409	-1.208	-3.914
	(0.217)	(0.141)	(0.532)

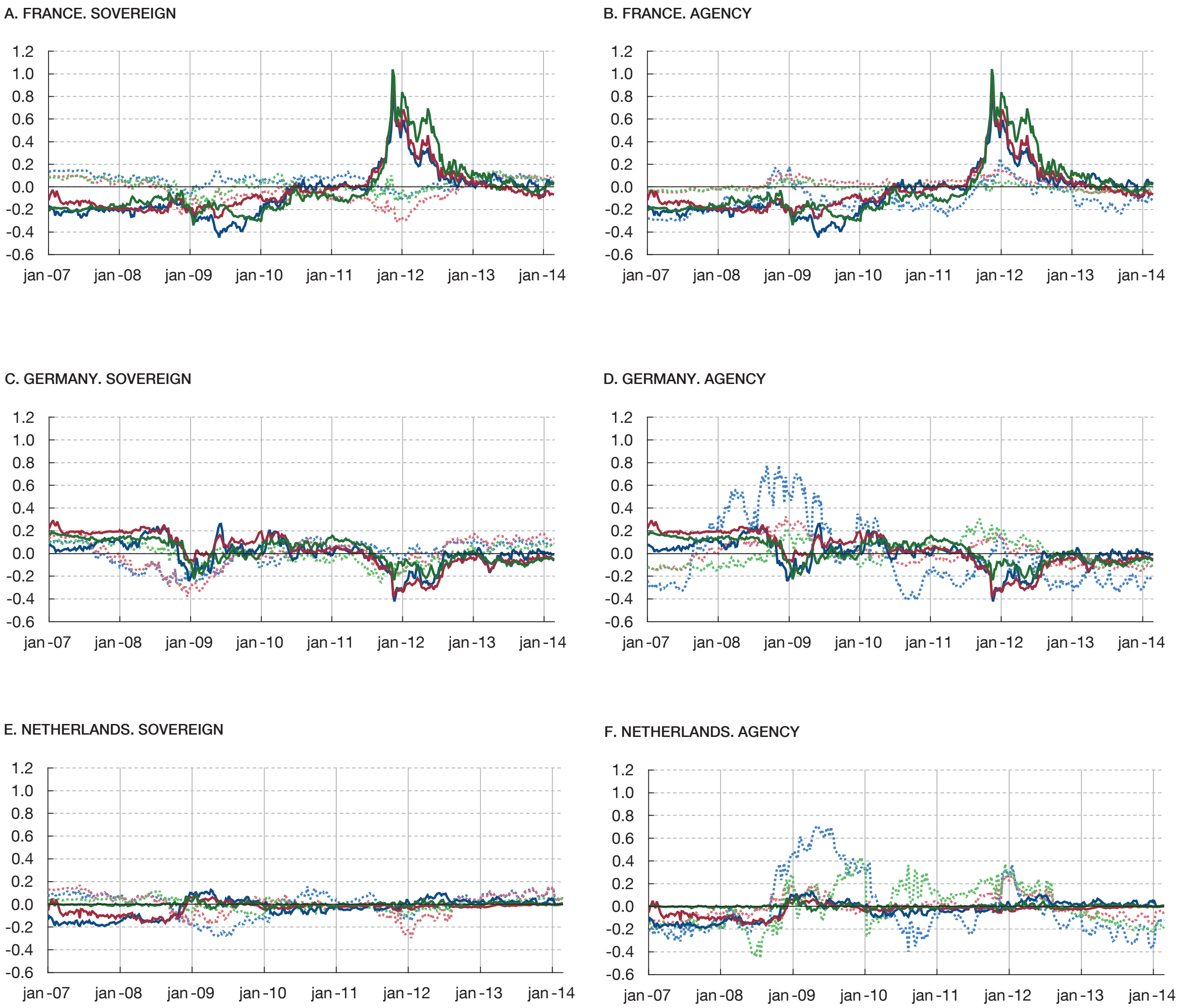
B. REMAINING PARAMETERS. FIVE YEARS TO MATURITY

	France	Germany	Netherlands
Intercept δ			
Sovereign yield	-0.090	-0.479	-0.203
	(1.149)	(1.017)	(1.416)
Agency yield	0.108	0.006	0.201
	(1.045)	(1.015)	(1.416)
Autocorrelation α			
Sovereign factor	0.978	0.985	1.000
	(0.011)	(0.008)	(0.066)
Agency factor	0.998	0.991	0.954
	(0.002)	(0.006)	(0.015)
Common factor		1.000	
		(0.037)	

SOURCE: Author’s elaboration using data from Datastream.
NOTE: The parameters have been obtained by maximum likelihood from a joint estimation for the three countries. Standard errors are reported below the estimates in parenthesis.

NATIONAL CREDIT AND LIQUIDITY FILTERED FACTORS FROM THE JOINT MODEL

CHART 7



SOURCE: Author's elaboration using data from Datastream.
NOTES: The two, five and ten year maturities are represented with blue, red and green lines, respectively. The credit factors are plotted using dark colured lines, while the analogous liquidity factors are plotted with ligth coloured dotted lines.

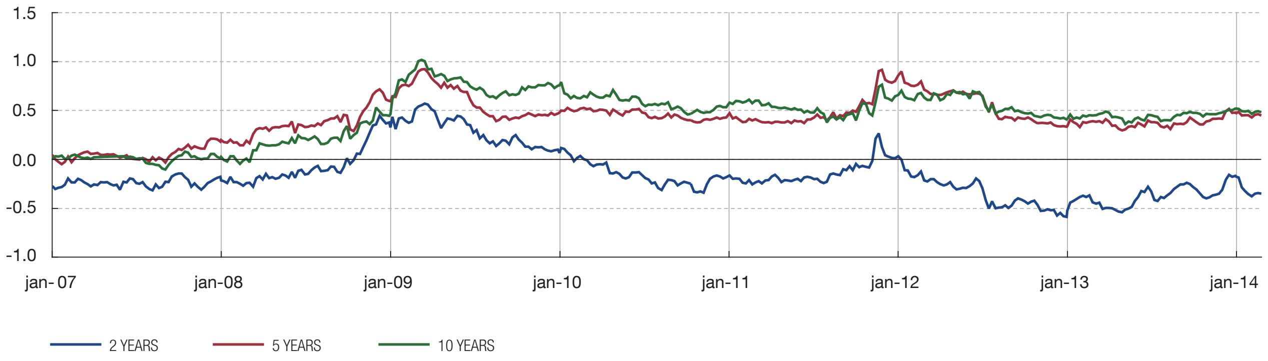
credit factors still suffer substantial increases during the Sovereign crisis. Hence, the impact of the crisis in Germany and the Netherlands is greatly explained by the pan-European effects shown in Chart 8, whereas this common factor is not able to explain all the rise in French sovereign yields. This result suggests the presence of a second factor that introduced contagion between only a group of countries that were more vulnerable to the sovereign crisis. The liquidity factors present a similar pattern as in the previous models. Another striking result from Chart 7 is that it is no longer easy to observe the effect of the Financial crisis. In this sense, Chart 8 shows that this crisis was fully captured by the common European credit factor, in contrast to the Sovereign crisis.

5 Quadratic Model

In this section, we adapt the EURIBOR model proposed by Dubecq et al. (2013) to a sovereign bond yield application. The models that we have analysed in the previous sections feature a constant stochastic discount factor. In contrast, this new model provides a more ambitious structure that explicitly provides estimates in the physical and in the risk neutral measures and allows the modelisation of different maturities in a joint setting. From

COMMON CREDIT FACTORS IN THE JOINT MODEL

CHART 8



SOURCE: Author's elaboration using data from Datastream.

a practical point of view, there are two relevant differences with respect to the previous sections. First, now we no longer have maturity dependent factors but maturity dependent loadings. And secondly, we consider quadratic functions of the factors to model yield-OIS spreads to ensure that they do not take negative values.

Let the yield to maturity of a risk-free bond at time t with maturity (in years) n be

$$r_{t,n} = \frac{1}{n} \log E^{Q} \left[\exp \left(\sum_{j=1}^{n} r_{t+j} \right) \right],$$

where r_t is the one year risk-free rate. In addition, let us denote as $y_{i,t,n}$ the yield to maturity of a risky bond i at time t with time to maturity n. This yield satisfies the following relationship

$$y_{i,t,n} = \frac{1}{n} \log E^{Q} \left[\exp \left(\sum_{j=1}^{n} r_{t+j} + \lambda_{t+j} \right) \right],$$

where λ_t is the intensity parameter. We assume that r_t and λ_t are independent under the risk-neutral measure (Q for short), which implies that we can write the spread of bond i at time t with maturity n as

$$\begin{aligned} s_{i,t,n} &= y_{i,t,n} - r_{t,n}, \\ &= \frac{1}{n} \log E^{Q} \left[\exp \left(\textstyle\sum_{j=1}^{n} \lambda_{t+j} \right) \right] \end{aligned} \quad (9)$$

As we did in the previous section, we proxy $r_{t,n}$ with the corresponding OIS rate. However, we will model the intensity parameter as a quadratic function of the factor x_t which will be the sum of a credit and a liquidity latent factor. Specifically,

$$\lambda_t = \lambda_0 + \lambda_1 x_t + \lambda_2 x_t^2, \quad (10)$$

where $x_t = x_{c,t} + x_{l,t}$ follows a first order autoregressive process under Q

$$x_t = \mu^* + \phi^* x_{t-1} + \epsilon_t^*, \quad (11)$$

with $\epsilon_t^* \sim$ iid N(0,1). The asterisk superindex denotes that we refer to the Q measure. To ensure the identification of the parameters, we normalise the variance of the factor x_t to 1. And again, we assume that $|\phi^*| < 1$.

As Dubecq et al. (2013) show, the intensity parameter λ_t is related to the probability of default.[5] It must be non-negative so that the probability of default lies between 0 and 1. We can ensure that λ_t is non-negative by setting $\lambda_2 \geq 0$ and $\lambda_0 \geq 0.25\,\lambda_1^2 \;/\, \lambda_2$, which in practice makes (10) a convex parabola with a non-negative minimum. This is the main reason why it is convenient to model the intensity as a quadratic function of the factors instead of an affine one. In addition, in this way we make the intensities more flexible as the quadratic function embeds the affine specification as a particular case.

We have to define the joint dynamics of $x_{c,t}$ and $x_{l,t}$ under Q so that (11) holds. We assume that they follow the diagonal VAR(1) process

$$\begin{pmatrix} x_{c,t} \\ x_{l,t} \end{pmatrix} = \begin{pmatrix} \mu_c^* \\ \mu_l^* \end{pmatrix} + \begin{pmatrix} \phi_c^* & 0 \\ 0 & \phi_l^* \end{pmatrix} \begin{pmatrix} x_{c,t-1} \\ x_{l,t-1} \end{pmatrix} + \boldsymbol{\epsilon}_t^* \tag{12}$$

where $\boldsymbol{\epsilon}_t^* \sim$ iid N $(\mathbf{0}, \text{diag}\,(\omega))$ and $\omega = (\sigma_c^2, \sigma_l^2)'$[6] Similarly, we also propose a diagonal VAR(1) process under the real measure P,

$$\begin{pmatrix} x_{c,t} \\ x_{l,t} \end{pmatrix} = \begin{pmatrix} \mu_c \\ \mu_l \end{pmatrix} + \begin{pmatrix} \phi_c & 0 \\ 0 & \phi_l \end{pmatrix} \begin{pmatrix} x_{c,t-1} \\ x_{l,t-1} \end{pmatrix} + \boldsymbol{\epsilon}_t, \tag{13}$$

where $\boldsymbol{\epsilon}_t \sim$ iid N $(\mathbf{0}, \text{diag}\,(\omega))$.

It can be shown that $\mu^* = \mu_c^* + \mu_l^*$, $\phi^* = \phi_c^* = \phi_l^*$ and $1 = \sigma_c^2 + \sigma_l^2$ must hold so that (10) and (12) are consistent. Then, we can use an exponentially affine stochastic discount factor to connect the real and risk-neutral measures. In fact, it can be shown that there is a one to one between these two measures [see Dubecq et al. (2013)].

Furthermore, the spreads in this framework can be expressed in closed form. In particular, we can write

$$s_{i,t,n} = \theta_{0,n} + \theta_{1,n}\, x_t + \theta_{2,n}\, x_t^2, \tag{14}$$

where the explicit formulas for the coefficients can be found in the appendix.

In sum, we use as our dependent variables the 2, 5 and 10 year sovereign spreads, which we model by adding an error term to (15) for n = 2,5 and 10:

$$s_{sov,t,n} = \theta_{0,n} + \theta_{1,n}\, x_t + \theta_{2,n}\, x_t^2 + \eta_{sov,t,n},$$

In addition, we identify the credit factor using the 5 year CDS rate against the sovereign bond, our proxy for credit risk:

$$cds_{t,5} = \pi_{c,0} + \pi_{c,1}\, x_{c,t} + \pi_{c,2}\, x_{c,t}^2 + \eta_{c,t},$$

where $\pi_{c,0}$, $\pi_{c,1}$ and $\pi_{c,2}$ are constant scalars. It is a quadratic extension of the credit factor identifying relationship (6). Finally, we use the difference between the agency and sovereign 5 year yields to identify the liquidity factor:

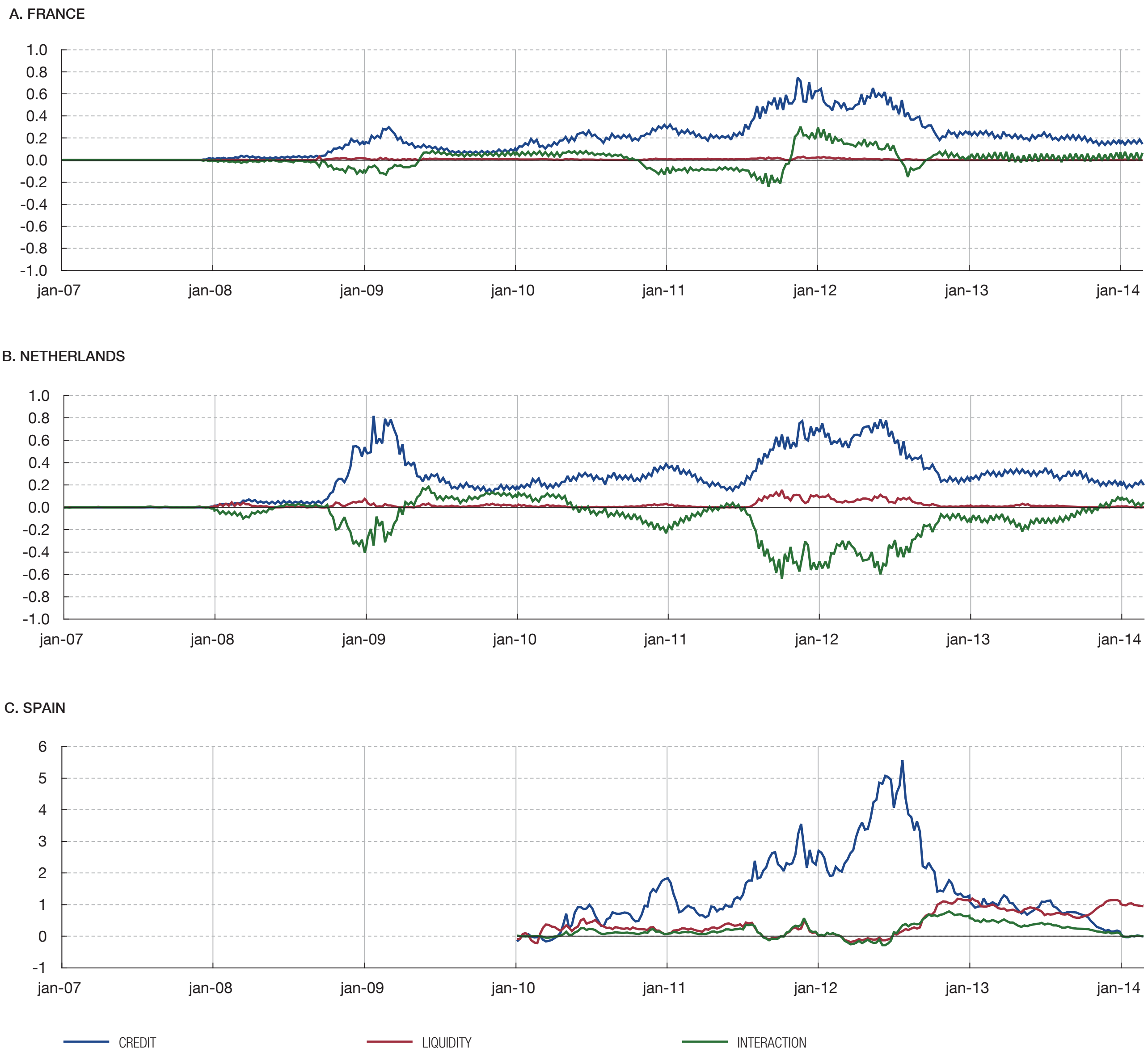
$$y_{agn,t,5} - y_{sov,t,5} = \pi_{l,0} + \pi_{l,1}\, x_{l,t} + \pi_{l,2}\, x_{l,t}^2 + \eta_{l,t},$$

5 Specifically, the probability of default at time t given information available at t–1 would be 1–exp $(-\lambda_t)$.

6 Notice the notational distinction between $\boldsymbol{\epsilon}_t^*$ and ϵ_t^*. The first is a vector and the second a scalar.

FIVE-YEAR FACTOR CONTRIBUTIONS IN THE QUADRATIC MODEL

CHART 9



SOURCE: Author's elaboration using data from Datastream.

where $\pi_{l,0}$, $\pi_{l,1}$ and $\pi_{l,2}$ are also constant scalars. We assume that the error terms are independent jointly Gaussian variables with zero means.

Although we jointly consider the 2, 5 and 10 year spreads, we only use the 5 year credit and liquidity proxies. We do this because it is enough to consider one maturity to identify these factors. In addition, it has the advantage of significantly reducing the number of parameters to be estimated. Lastly, the 5 year maturity is the most liquid one for the case of CDS spreads.[7]

This problem is not linear, so we can no longer use the standard Kalman filter. Instead, we follow Trolle and Schwartz (2009) and others by considering the extended Kalman filter. In practice, this involves computing a first-order Taylor approximation of (15) around $x_{t|t-1}$, which denotes the Kalman filter prediction of x_t given information known at t – 1.

7 Also we impose, as in section 4, that $V(x_{c,t}) = V(x_{l,t}) = 1$.

FIVE-YEAR SOVEREIGN FACTOR LOADINGS IN THE QUADRATIC MODEL | TABLE 4

	France	Netherlands	Spain
$\theta_{0,5}$	0.041	0.003	0.639
$\theta_{1,5}$	0.002	-0.000	-0.124
$\theta_{2,5}$	0.003	0.002	0.007

SOURCE: Author's elaboration using data from Datastream.

Contrary to what occurred to the models in section 4, the factors themselves are not very informative about the impact they have on the spreads. For that reason it is convenient to rewrite the spreads in a way that the partial contributions of each factor show off:

$$s_{i,t,n} = \theta_{0,n} + \underbrace{\theta_{1,n}\, x_{c,t} + \theta_{2,n}\, x^2_{c,t}} + \underbrace{\theta_{1,n}\, x_{l,t} + \theta_{2,n}\, x^2_{l,t}} + \underbrace{2\,\theta_{2,n}\, x_{c,t}\, x_{l,t}}.$$

The first bracket denotes the credit contribution, the second bracket the liquidity contribution and the third represents the interaction between both factors.

Chart 9 plots the 5 year contributions for France, Netherlands and Spain. The corresponding $\theta_{k,5}$ loadings can be found in Table 4. For these three countries, we obtain comparable results to the previous affine models, though the safe-haven liquidity premia is not captured by the liquidity contribution, but by the interaction term. Spain's sovereign bonds do not experience the downward force on the yields that the other bonds enjoy under stress periods. Unfortunately, the model is not able to fit the German data well,[8] probably because the 2 and 5 year German yields are below the OIS rate through most of the period considered (the respective spread means are –0.1281 and –0.0391).[9] This fact is at odds with the assumption of positive intensities λ_t. Therefore, we cannot expect plausible results in this situation.

6 Conclusions

In this paper, we analyse the relevance of the credit and liquidity components in explaining the movements of sovereign yields. In particular, we shed light on the impact that credit and liquidity risk have on sovereign bond yields for France, Germany, the Netherlands and Spain. This analysis is fundamental to understand the determinants of the developments in the sovereign markets during the recent European sovereign crisis. We disentangle the credit and liquidity components exploiting the fact that agency bonds in these countries are subject to the same credit risk than sovereign bonds, but they are generally less liquid. We start by considering a benchmark model that only uses sovereign and agency country yields for a given maturity. Then, we show that we can improve the identification of the components by considering CDS rates in the estimation. In addition, we observe a clear co-movement of the credit factors that motivates the introduction of a joint model for France, Germany and the Netherlands.

In all these specifications, we clearly observe two distress periods, which correspond to the financial and the European sovereign crises. In general, the credit component is the dominant force, pushing up sovereign yields at times of stress. However, this effect is alleviated by the liquidity premia in some countries, originated by the safe haven flows typical of hectic financial episodes. Interestingly, the joint multi-country model shows that the idiosyncratic credit components significantly differ among sovereigns. In particular, the

8 This results are not reported for the sake of brevity..

9 For the 10 year case the mean is positive, taking a value of 0.0532.

sovereign crisis mainly affected Germany and the Netherlands through the common European factor, whereas the French yields suffered an additional idiosyncratic deterioration.

Finally, we consider a more structural approach in which we directly model the default process. Specifically, we follow Dubecq et al. (2013) by modelling the default intensity as a quadratic discrete-time process. The results that we obtain from this model confirm our previous findings. Nevertheless, they show that part of the liquidity effect is due to the interaction of credit and liquidity, and not to liquidity alone.

An interesting avenue for future research would be to investigate impact of the Quantitative Easing program introduced by the ECB in 2015 on sovereign yields. To the extent that this program does not affect fundamentals, it should reduce sovereign yields through the liquidity factor. However, if it indirectly affects debt sustainability as well (e. g. through a positive impact on the real economy), it might also have an impact on the credit component. It would also be interesting to analyse the presence of credit factors that affected only a subset of countries. In addition, it would be interesting to extend the quadratic model into this multi-country setting. Lastly, it might be very useful to include other European countries in the database, although data limitations would require a careful treatment of incomplete panels.

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APPENDIX
Parameters of the quadratic pricing model

We can write

$$E^Q\left[\exp\left(\sum_{j=1}^{n}\lambda_{t+j}\right)\right] = \exp\left(a_n + b_n x_t + c_n x_t^2\right)$$

where

$$a_n = a_{n-1} - \lambda_0 - \log\sqrt{1-2\left(c_{n-1}-\lambda_2\right)} + \frac{1}{2}\,\frac{\left(b_{n-1}-\lambda_1\right)^2}{1-2\left(c_{n-1}-\lambda_2\right)}$$

$$+ \frac{\mu^*\left(b_{n-1}-\lambda_1\right)}{1-2\left(c_{n-1}-\lambda_2\right)} + \frac{\mu^{*2}\left(c_{n-1}-\lambda_2\right)}{1-2\left(c_{n-1}-\lambda_2\right)}$$

$$b_n = \frac{\phi^*\left(b_{n-1}-\lambda_1\right)}{1-2\left(c_{n-1}-\lambda_2\right)} + \frac{2\mu^*\phi^*\left(c_{n-1}-\lambda_2\right)}{1-2\left(c_{n-1}-\lambda_2\right)}$$

$$c_n = \frac{\phi^{*2}\left(c_{n-1}-\lambda_2\right)}{1-2\left(c_{n-1}-\lambda_2\right)}$$

and $a_n = b_n = c_n = 0$. Finally, the coefficients in (14) can be expressed as $\theta_{0,n} = -a_n / n$, $\theta_{1,n} = -b_n / n$ and $\theta_{2,n} = -c_n / n$.