A Discussion of Eusepi and Preston's *Expectations, Learning and Business Cycle Fluctuations*

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Can learning help the RBC model overcome some of its empirical short comings?

EP introduces learning into two simple RBC models

1. Standard linearized RBC model with endogenous labour supply
2. Non-separable utility and (slightly) increasing returns to scale

EP's answer: Sort of, and more so if (2) is used.

- Learning improves propagation and amplification properties in (1) + (2)
- Learning improves (2) in more dimensions

A productivity shock causes agents to revise their expectations about returns to capital upwards and wages downwards relatively more than under RE. Consumption (and other variables) respond stronger on impact.

Expectations, Learning and Business Cycle Fluctuations

Paper closely related to Williams (2002) who does similar exercise (with separable utility) and simpler model for "structural/infinite horizon" learning

My comments and suggestions are in order of increasing degree of *outside the scope of this paperness.*

- What drives the difference between the results from "Infinite Horizon" and "Euler equation" learning?
- Can we make the agents better econometricians and still have interesting results?
- Can we test whether learning is important empirically without horse racing models?

Two strategies for modelling boundedly rational consumption

"Infinite horizon learning" Preston (2005)

$$\begin{aligned} c_t &= \frac{1-\beta}{\epsilon_c}\left[\beta^{-1}k_t + \overline{R}R_t^K - \beta^{-1}\gamma_t + \epsilon_w w_t\right] \\ &+ \widehat{E}_t\left[\frac{1-\beta}{\epsilon_c} - \beta\right]\beta\sum_{T=t}^{\infty}\beta^{T-t}\overline{R}R_{T+1}^K \\ &+ \widehat{E}_t\frac{1-\beta}{\epsilon_c}\beta\epsilon_w\sum_{T=t}^{\infty}\beta^{T-t}w_{T+1} \end{aligned}$$

"Euler equation learning" (Everybody else)

$$c_t = \widehat{E}_t c_{t+1} - \widehat{E}_t\left[\beta\overline{R}R_{t+1}^K + \gamma_y\right]$$

Perceived Law of Motion

$$\begin{aligned} R_t^K &= \omega_0^r + \omega_1^r k_t + e_t^r \\ w_t &= \omega_0^w + \omega_1^w k_t + e_t^w \\ k_{t+1} &= \omega_0^k + \omega_1^k k_t + e_t^k \end{aligned}$$

Stack $\omega' s$ in the vector ω_t and the left hand sid variables in z_t and let $q_t = \begin{pmatrix} 1 & k_t \end{pmatrix}$

$$\begin{aligned} \omega_t &= \omega_{t-1} + g R_t^{-1} q_{t-1} \left(z_t - \omega_{t-1}' q_{t-1} \right) \\ R_t &= R_{t-1} + g \left(q_{t-1} q_{t-1'} - R_{t-1} \right) \end{aligned}$$

Rephrasing Infinite Horizon as Euler equation learning

We can rewrite the decision rule

$$c_t = a_{t-1} + b_{t-1} k_{t+1} + d_{t-1} \gamma_t$$

derived from the PLM of capital, wages and returns as an Euler equation

$$\begin{aligned} c_t &= \widehat{E}_t c_{t+1} - \widehat{E}_t \left[\beta \overline{R} R^K_{t+1} + \gamma_t \right] \\ &= a_{t-1} + b_{t-1} \left(\omega^k_{0,t-1} + \omega^k_{1,t-1} k_{t+1} \right) - \beta \overline{R} \omega^r_{1,t-1} k_{t+1} + \gamma_t \end{aligned}$$

where $a = a(\omega)$, $b = b(\omega)$ and $d = d(\omega)$

Why do we get different results from IH and EE?

$$\begin{aligned} c_t &= a_{t-1} + b_{t-1}\left(\omega^k_{0,t-1} + \omega^k_{1,t-1}k_{t+1}\right) - \beta\overline{R}\omega^r_{1,t-1}k_{t+1} + \gamma_t \\ &\neq \omega^c_{0,t-1} + \omega^c_{1,t-1}k_{t+1} - \beta\overline{R}\omega^r_{1,t-1}k_{t+1} + \gamma_t \end{aligned}$$

where $\omega^c_{0,t-1} + \omega^c_{1,t-1}$ are the time t estimates of the parameters in

$$c_t = \omega^c_0 + \omega^c_1 k_{t-1}$$

Under RE, either approach would give the same answer.

- Under RE, capital is the only state variable relevant for making predictions
- Under learning, the state is seven dimensional (capital + parameters in ω) but agent condition only on capital (and with suboptimal coefficients)

In this model, there is only one way to behave optimally, but many ways to be boundedly rational.

Suggestions, Part I

It would be interesting to see:

- Impulse responses of parameters to a productivity shock for both approaches
  - Are the parameters from the wages and returns regressions more volatile?
- Differences in implied consumption Euler equation errors from Infinite Horizon learning vs Euler equation learning
  - Conjecture: Given that unconditional moments of RE and Euler Equation learning are very similar....

Hard to stick to intention of endowing agents only with knowledge of their preferences and constraints

- Composite parameters in their decision rule (e.g. ϵ_c) are functions of parameters (e.g. α) that also determine coefficients that they estimate.

Constant gain learning

Motivation for learning is often stated as a desire to put the agents inside the models on the same footing as the econometrician

- EP make their agents a little less clever than a good econometrician
  - Constant gain in stable environment means that agents throw away information
- However, decreasing gain leads to convergence to REE and no additional dynamics

Can we make the agent more rational while retaining the interesting dynamics?

Steady state learning and Bayesian agents

Steady State Learning and Nash Equilibria, Fudenberg and Levine (*Econometrica* 1993)
Basic idea: Let short lived agents learn about parameters like Bayesians

- Sensible if agents learn about local conditions so that long history of data does not exist. (Learning about own wages?)
- Delivers constant gain learning in steady state
- New and perhaps interesting implications: Dispersion of actions between generations of agents

Feasible?
I think so, but one would need to keep track of generations (no representative agent updating equation)

- The need for many generations in order to achieve appropriately low gain could be replaced with tighter priors

Convincing sceptics

EP chooses productivity innovation variance and learning gain to match variance of output and serial correlation of consumption

- Are we sure that the best fitting RE model is compared with the learning model?

Horse racing (calibrated/estimated) learning and RE models:

- Straight forward, but unlikely to convince people with different priors

What prediction from the model are unique to learning models?

- Correlation between structural innovations and parameters in ALM?
  - Evaluating response of $T_1(\omega)$ to productivity shock at PLM=ALM should give an idea of unconditional correlation
- Structure imposed by "monotonicity" of actions in expectations may give enough structure

Conclusions

More careful analysis of what drives the different results from Infinite Horizon learning and Euler equation learning.

- What does expectational errors look like in a simulated time series?

Convince the reader that you are comparing the best fitting RE model with the best fitting learning model

- Perhaps by estimating competing models using ML

More ambitious/speculative suggestions:
- Bayesian agents with finite lives
- Think about what predictions from learning that are unique to this class of models

Extras

$$a = \left[\epsilon_c^{-1} - \frac{\beta}{1-\beta}\right] \beta \overline{R} \omega_o^r + \epsilon_c^{-1} \beta \epsilon_w \omega_o^w$$

$$b = \left[\frac{1-\beta}{\epsilon_c} - \beta\right] \beta \overline{R} \frac{\omega_1^r}{1-\beta\omega_1^k} + \frac{1-\beta}{\epsilon_c} \beta \epsilon_w \frac{\omega_1^w}{1-\beta\omega_1^k}$$

$$\epsilon_c = \frac{c}{k} + \frac{1-H}{H}\left[\frac{\epsilon_L}{\sigma} - \psi \frac{\sigma-1}{\sigma}\right]^{-1} R \frac{1-\alpha}{\alpha}$$