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Preliminary

Abstract

This paper develops a dynamic stochastic general equilibrium model with rational inattention. Decisionmakers have limited attention and choose the optimal allocation of their attention. We study the implications of rational inattention for business cycle dynamics. For example, we study how rational inattention affects the impulse responses of prices and quantities to monetary policy, aggregate technology and micro-level shocks.

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1 Introduction

This paper develops a dynamic stochastic general equilibrium model with rational inattention. We model the idea that decisionmakers cannot attend perfectly to all available information. Therefore, the mapping between economic conditions and the price and quantity decisions taken by decisionmakers is not perfect. Decisionmakers make mistakes. However, decisionmakers try to minimize these mistakes.

The economy consists of households, firms and a government. Households supply differentiated types of labor, consume a variety of goods, and hold nominal government bonds. Firms supply differentiated goods that are produced with the different varieties of labor. The central bank sets the nominal interest rate according to a Taylor rule. Households take wage setting and consumption decisions. Firms take price setting and input decisions. There are no adjustment costs. We compute the impulse responses of prices and quantities to monetary policy shocks, aggregate technology shocks and micro-level shocks under perfect information and under rational inattention.

Rational inattention means that decisionmakers have limited attention and choose the optimal allocation of their attention. Following Sims (2003), we model decisionmakers' limited attention as a constraint on information flow. Decisionmakers choose how to use the available information flow. In other words, decisionmakers decide what to focus on. Decisionmakers decide how to allocate their attention across their different decision problems. Furthermore, for each decision problem decisionmakers decide how to attend to the different factors that may affect the optimal decision.

In Section 2 we present the assumptions that are the same under perfect information and under rational inattention. In Section 3 we solve the model under perfect information. When households and firms have perfect information, fluctuations in quantities and in relative prices are driven by aggregate technology shocks and by micro-level shocks. Monetary policy has no real effects. In Section 4 we solve the model under rational inattention on the side of firms, that is, we assume that decisionmakers in firms have limited attention and choose optimally the allocation of their attention. In that section we continue to assume that households have perfect information to isolate the role of rational inattention on the side of firms. To this end, our main findings are as follows. First, for our parameter values rational inattention on the side of firms implies that the impulse responses of inflation and output to monetary policy shocks resemble the impulse responses in a Calvo model with an average price duration of 7.5 months. At the same time, prices respond fairly quickly to aggregate technology shocks and almost perfectly to micro-level shocks. The reason is the optimal allocation of attention. Decisionmakers in firms decide to pay little attention to monetary policy, twice as much attention to aggregate technology, and a lot of attention to firm-specific conditions. Therefore, prices respond slowly to monetary policy shocks, fairly quickly to aggregate technology shocks, and almost perfectly to micro-level shocks. Second, losses in profits due to deviations of the actual price from the profit-maximizing price are an order of magnitude smaller than in a Calvo model that generates the same real effects. Specifically, losses in profits due to suboptimal price responses to aggregate conditions are 12 times smaller than in the Calvo model; and losses in profits due to suboptimal price responses to firm-specific conditions are 25 times smaller than in the Calvo model. One reason is the optimal allocation of attention. Under rational inattention prices respond slowly to monetary policy shocks, fairly quickly to aggregate technology shocks, and almost perfectly to micro-level shocks. In contrast in the Calvo model prices respond slowly to all these shocks. Another reason is that under rational inattention deviations of the actual price from the profit-maximizing price are less likely to be extreme than in the Calvo model. Third, rational inattention on the side of decisionmakers in firms implies that firms produce with a suboptimal input mix. Section 5 provides a description of how to solve the model with rational inattention on the side of households, and concludes.

2 Model

2.1 Households

There are J households. Households supply differentiated types of labor, consume a variety of goods, and can save by holding nominal government bonds.

Each household seeks to maximize the expected discounted sum of period utility. The discount factor is $\beta \in (0,1)$. The period utility function is given by

$$U\left(C_{jt}, L_{j1t}, \ldots, L_{jNt}\right) = \frac{C_{jt}^{1-\gamma} - 1}{1-\gamma} - \varphi \sum_{n=1}^{N} e^{-\chi_{jnt}} \frac{L_{jnt}^{1+\psi}}{1+\psi}, \tag{1}$$

with[1]

$$C_{jt} = \left(\sum_{i=1}^{I} C_{ijt}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}. \tag{2}$$

Here C_{jt} is composite consumption by household j in period t and C_{ijt} is consumption of good i by household j in period t. The parameter $\gamma > 0$ is the coefficient of relative risk aversion and the parameter $\theta > 1$ is the elasticity of substitution between different goods. Each household supplies N types of labor, where L_{jnt} is supply of household j's nth type of labor and χ_{jnt} is a preference shock affecting the disutility of supplying this type of labor. We introduce labor-specific preference shocks to generate variation in relative wage rates. We assume that each household supplies N types of labor to allow for a certain degree of risk sharing within the household. The parameter $\varphi > 0$ affects the disutility of supplying labor and the parameter $\psi > 0$ is the inverse of the Frisch elasticity of labor supply.

Households can save by holding nominal government bonds. The flow budget constraint of household j in period t reads

$$\sum_{i=1}^{I} P_{it} C_{ijt} + B_{jt} = R_{t-1} B_{jt-1} + (1+\tau_w) \sum_{n=1}^{N} W_{jnt} L_{jnt} + \frac{D_t}{J} - \frac{T_t}{J}. \tag{3}$$

Here P_{it} is the price of good i in period t, B_{jt} are bond holdings by household j between period t and period $t+1$, R_{t-1} is the nominal interest rate on bond holdings between period $t-1$ and period t, τ_w is a wage subsidy, W_{jnt} is the nominal wage rate for household j's nth type of labor in period t, (D_t/J) is a pro-rata share of nominal aggregate profits, and (T_t/J) is a pro-rata share of nominal lump-sum taxes. We assume that all J households have the same initial bond holdings, and we assume a natural debt limit.

Every period each household chooses a consumption vector, $(C_{1jt}, \ldots, C_{Ijt})$, and a vector of nominal wage rates, $(W_{j1t}, \ldots, W_{jNt})$. Each household commits to supply any quantity of labor at the chosen nominal wage rates.[2]

[1] Only for ease of exposition, we assume a constant elasticity of substitution between goods. We have also done the derivations using a general constant returns-to-scale aggregator.

[2] Bond holdings, B_{jt}, then follow from the flow budget constraint (3) and the labor demand function derived below.

2.2 Firms

There are I firms in the economy. Firms supply differentiated goods that are produced with the different varieties of labor.

The technology of firm i is given by

$$Y_{it} = e^{a_t} e^{a_{it}} L_{it}^{\alpha}, \tag{4}$$

with

$$L_{it} = \left(\sum_{j=1}^{J} \sum_{n=1}^{N} L_{ijnt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \tag{5}$$

Here Y_{it} is output, $(e^{a_t} e^{a_{it}})$ is total factor productivity and L_{it} is composite labor input of firm i in period t. Furthermore, L_{ijnt} is the input of type jn labor at firm i in period t. Recall that type jn labor is household j's nth type of labor. There are JN types of labor. Total factor productivity has an aggregate component, e^{a_t}, and a firm-specific component, $e^{a_{it}}$. The parameter $\alpha \in (0, 1]$ is the elasticity of output with respect to composite labor. The parameter $\eta > 1$ is the elasticity of substitution between different types of labor.[3]

Nominal profits of firm i in period t equal

$$(1 + \tau_p) P_{it} Y_{it} - \sum_{j=1}^{J} \sum_{n=1}^{N} W_{jnt} L_{ijnt}, \tag{6}$$

where τ_p is a production subsidy.

Every period each firm chooses a price and an input mix. Each firm commits to supply any quantity of the good at the chosen price.

2.3 Government

There is a monetary authority and a fiscal authority. Let $\Pi_t = (P_t / P_{t-1})$ denote inflation where P_t denotes a price index that will be defined later. Let $Y_t = \sum_{i=1}^{I} Y_{it}$ denote aggregate output. The central bank sets the nominal interest rate according to the rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} \right]^{1-\rho_R} e^{\varepsilon_t^R}, \tag{7}$$

[3] For ease of exposition all types of labor appear in the labor aggregator (5). One can also use a firm-specific labor aggregator in which only a firm-specific subset of types of labor appears.

where R, Π and Y are the values of the nominal interest rate, inflation and aggregate output in the non-stochastic steady state, and ε_t^R is a monetary policy shock. The policy parameters satisfy $\rho_R \in [0,1)$, $\phi_\pi > 1$ and $\phi_y \geq 0$.

The government budget constraint in period t reads

$$T_t + (B_t - B_{t-1}) = (R_{t-1} - 1) B_{t-1} + \tau_w \left(\sum_{j=1}^{J} \sum_{n=1}^{N} W_{jnt} L_{jnt} \right) + \tau_p \left(\sum_{i=1}^{I} P_{it} Y_{it} \right). \quad (8)$$

The government has to finance interest on nominal government bonds, the wage subsidy and the production subsidy. The government can collect lump-sum taxes or issue new government bonds.

2.4 Shocks

There are four types of shocks in the economy: aggregate technology shocks, monetary policy shocks, firm-specific productivity shocks and labor-specific preference shocks. We assume that, for all i and jn, the processes $\{a_t\}$, $\{\varepsilon_t^R\}$, $\{a_{it}\}$ and $\{\chi_{jnt}\}$ are independent. Furthermore, we assume that the firm-specific productivity processes, $\{a_{it}\}$, are independent across firms and that the labor-specific preference shocks, $\{\chi_{jnt}\}$, are independent across types of labor. Finally, we assume that all these processes are stationary Gaussian processes with mean zero. In the following, we denote the period t innovation to a_t, a_{it} and χ_{jnt} by ε_t^A, ε_{it}^I and ε_{jnt}^χ, respectively.

3 Solution under perfect information

In this section we derive the equilibrium under perfect information, that is, we assume that in period t all households and firms know all variables up to and including period t. We show that under perfect information fluctuations in quantities and in relative prices are driven by aggregate technology shocks and by micro-level shocks. Monetary policy has no real effects.

3.1 Equations characterizing equilibrium

Cost minimization implies that the demand for type jn labor in period t is given by

$$L_{jnt} = \left(\frac{W_{jnt}}{W_t}\right)^{-\eta} L_t, \tag{9}$$

where W_t is the following wage index

$$W_t = \left(\sum_{j=1}^{J}\sum_{n=1}^{N} W_{jnt}^{1-\eta}\right)^{\frac{1}{1-\eta}}, \tag{10}$$

and L_t is the aggregate labor input

$$L_t = \sum_{i=1}^{I} L_{it}. \tag{11}$$

The problem of household j is to choose a contingent plan for the consumption vector and for the vector of nominal wage rates so as to maximize

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_{jt}^{1-\gamma}-1}{1-\gamma} - \varphi\sum_{n=1}^{N} e^{-\chi_{jnt}}\frac{L_{jnt}^{1+\psi}}{1+\psi}\right)\right], \tag{12}$$

subject to the consumption aggregator (2), the flow budget constraint (3), the natural debt limit and the labor demand function (9). The first-order conditions for the household problem are

$$C_{jt}^{-\gamma} = E_t\left[\beta\frac{R_t}{\Pi_{t+1}}C_{jt+1}^{-\gamma}\right], \tag{13}$$

for all i

$$\frac{C_{ijt}}{C_{jt}} = \left(\frac{P_{it}}{P_t}\right)^{-\theta}, \tag{14}$$

and for all n

$$\frac{W_{jnt}}{P_t} = \frac{1}{1+\tau_w}\frac{\eta}{\eta-1}\varphi e^{-\chi_{jnt}}\left[\left(\frac{W_{jnt}}{W_t}\right)^{-\eta}L_t\right]^{\psi}C_{jt}^{\gamma}. \tag{15}$$

Here E_t is the expectation operator conditioned on information in period t, and P_t is the following price index

$$P_t = \left(\sum_{i=1}^{I} P_{it}^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{16}$$

Equation (13) is the consumption Euler equation, equation (14) characterizes the optimal consumption mix, and equation (15) characterizes the optimal nominal wage rates.

Throughout the paper we assume that the government sets the wage subsidy τ_w so as to correct the distortion arising from households' market power on the labor market. Here this implies that

$$1+\tau_w = \frac{\eta}{\eta-1}. \tag{17}$$

Multiplying equation (14) by C_{jt} and summing over all households yields the demand for good i in period t

$$C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\theta} C_t, \tag{18}$$

where C_t is aggregate consumption

$$C_t = \sum_{j=1}^{J} C_{jt}. \tag{19}$$

The problem of firm i under perfect information is to choose a price and an input mix so as to maximize profits (6) subject to the technology (4)-(5), the demand function (18) and the requirement that output has to equal demand. The firm problem is a static decision problem, because there are no adjustment costs and the demand function (18) is static. The first-order conditions for the firm problem are

$$P_{it} = \frac{1}{1+\tau_p}\frac{\theta}{\theta-1} W_t \frac{1}{\alpha} \frac{\left[\left(\frac{P_{it}}{P_t}\right)^{-\theta} C_t\right]^{\frac{1}{\alpha}-1}}{\left(e^{a_t} e^{a_{it}}\right)^{\frac{1}{\alpha}}}, \tag{20}$$

and for all jn

$$\frac{L_{ijnt}}{L_{it}} = \left(\frac{W_{jnt}}{W_t}\right)^{-\eta}. \tag{21}$$

Here W_t is the wage index (10). Equation (20) characterizes the profit-maximizing price. Throughout the paper we assume that the government sets the production subsidy τ_p so as to correct the distortion arising from firms' market power on the goods market. Here this implies that

$$1+\tau_p = \frac{\theta}{\theta-1}. \tag{22}$$

Equation (21) characterizes the profit-maximizing input mix. Multiplying equation (21) by L_{it} and summing over all firms yields the labor demand function (9).

3.2 Non-stochastic steady state

In the non-stochastic steady state, there are no shocks; all equations characterizing equilibrium are satisfied; and quantities, relative prices, the nominal interest rate and inflation are constant over time. We now report some relationships that hold in the non-stochastic steady state and that we will use below.

Equation (20) implies that in the non-stochastic steady state all firms set the same price. Therefore, households choose a consumption basket with equal weights. Thus all firms produce the same amount and have the same composite labor input. It follows from the price index (16), the consumption aggregator (2), the definition of aggregate output and the definition of the aggregate labor input (11) that

$$\left(\frac{P_i}{P}\right)^{1-\theta} = \left(\frac{C_{ij}}{C_j}\right)^{\frac{\theta-1}{\theta}} = \frac{Y_i}{Y} = \frac{L_i}{L} = \frac{1}{I}. \tag{23}$$

Here (P_i/P) denotes the value of (P_{it}/P_t) in the non-stochastic steady state, etc.

Since all households face the same decision problem, all households choose the same consumption level. It follows from the definition of aggregate consumption (19) that

$$\frac{C_j}{C} = \frac{1}{J}. \tag{24}$$

Furthermore, equation (15) implies that in the non-stochastic steady state all households set the same nominal wage rate for all different types of labor. Therefore, firms choose a labor mix with equal weights. It follows from the wage index (10) and the labor aggregator (5) that

$$\left(\frac{W_{jn}}{W}\right)^{1-\eta} = \left(\frac{L_{ijn}}{L_i}\right)^{\frac{\eta-1}{\eta}} = \frac{1}{JN}. \tag{25}$$

Here (W_{jn}/W) denotes the value of (W_{jnt}/W_t) in the non-stochastic steady state, etc.

3.3 Log-linearization

In this subsection, we log-linearize the equations characterizing equilibrium. In the next subsection, we report the log-linear equilibrium dynamics under perfect information. In the following, $\tilde{P}_{it} = (P_{it}/P_t)$ denotes the relative price of good i, $\tilde{W}_{jnt} = (W_{jnt}/P_t)$ denotes the real wage rate for type jn labor, and $\tilde{W}_t = (W_t/P_t)$ denotes the real wage index. Furthermore, small letters denote log-deviations from the non-stochastic steady state.

Log-linearizing the households' first-order conditions yields

$$c_{jt} = E_t \left[-\frac{1}{\gamma} \left(r_t - \pi_{t+1} \right) + c_{jt+1} \right], \tag{26}$$

$$c_{ijt} - c_{jt} = -\theta \tilde{p}_{it}, \tag{27}$$

and

$$\tilde{w}_{jnt} = -\frac{1}{1+\eta\psi} \chi_{jnt} + \frac{\eta\psi}{1+\eta\psi} \tilde{w}_t + \frac{\psi}{1+\eta\psi} l_t + \frac{\gamma}{1+\eta\psi} c_{jt}. \tag{28}$$

Dividing the definition of the price index (16) by P_t, log-linearizing and using (23) yields

$$\sum_{i=1}^{I} \tilde{p}_{it} = 0. \tag{29}$$

Log-linearizing the demand function (18) yields

$$c_{it} = -\theta \tilde{p}_{it} + c_t, \tag{30}$$

and log-linearizing the definition of aggregate consumption (19) and using (24) yields

$$c_t = \frac{1}{J} \sum_{j=1}^{J} c_{jt}. \tag{31}$$

Log-linearizing the firms' first-order conditions yields

$$\tilde{p}_{it} = \frac{1}{1+\theta\frac{1-\alpha}{\alpha}} \tilde{w}_t + \frac{\frac{1-\alpha}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}} c_t - \frac{\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}} \left(a_t + a_{it} \right), \tag{32}$$

and

$$l_{ijnt} - l_{it} = -\eta \left(\tilde{w}_{jnt} - \tilde{w}_t \right). \tag{33}$$

Dividing the definition of the wage index (10) by W_t, log-linearizing and using (25) yields

$$\tilde{w}_t = \frac{1}{JN} \sum_{j=1}^{J} \sum_{n=1}^{N} \tilde{w}_{jnt}. \tag{34}$$

Log-linearizing the production function (4) yields

$$y_{it} = a_t + a_{it} + \alpha l_{it}, \tag{35}$$

and log-linearizing the labor aggregator (5) and using (25) yields

$$l_{it} = \frac{1}{JN} \sum_{j=1}^{J} \sum_{n=1}^{N} l_{ijnt}. \tag{36}$$

Log-linearizing the labor demand function (9) yields

$$l_{jnt} = -\eta\left(\tilde{w}_{jnt} - \tilde{w}_t\right) + l_t, \tag{37}$$

and log-linearizing the definition of aggregate labor input (11) and using (23) yields

$$l_t = \frac{1}{I}\sum_{i=1}^{I} l_{it}. \tag{38}$$

Log-linearizing the monetary policy rule (7) yields

$$r_t = \rho_R r_{t-1} + (1-\rho_R)\left(\phi_\pi \pi_t + \phi_y y_t\right) + \varepsilon_t^R, \tag{39}$$

and log-linearizing the definition of aggregate output and using (23) yields

$$y_t = \frac{1}{I}\sum_{i=1}^{I} y_{it}. \tag{40}$$

3.4 Log-linear solution

Assume that I and N are sufficiently large so that[4]

$$\frac{1}{I}\sum_{i=1}^{I} a_{it} = 0, \tag{41}$$

and

$$\frac{1}{N}\sum_{n=1}^{N} \chi_{jnt} = 0. \tag{42}$$

The aggregate dynamics under perfect information are then given by

$$y_t = c_t = \frac{1+\psi}{1-\alpha+\alpha\gamma+\psi} a_t, \tag{43}$$

$$l_t = \frac{1-\gamma}{1-\alpha+\alpha\gamma+\psi} a_t, \tag{44}$$

$$\tilde{w}_t = \frac{\gamma+\psi}{1-\alpha+\alpha\gamma+\psi} a_t, \tag{45}$$

$$r_t - E_t\left[\pi_{t+1}\right] = \gamma\frac{1+\psi}{1-\alpha+\alpha\gamma+\psi} E_t\left[a_{t+1} - a_t\right]. \tag{46}$$

[4] Up to this point $N = 1$ is a special case of the model. Now we are making the assumption that N is sufficiently large so that households can insure against labor-specific preference shocks within the household. This assumption implies that all households have the same consumption level.

The proof is in Appendix A. Under perfect information the aggregate output, aggregate labor input, real wage index and real interest rate depend only on aggregate technology. Monetary policy has no real effects. The nominal interest rate and inflation follow from the monetary policy rule (39) and the real interest rate (46). Since $(1-\rho_R)\,\phi_\pi > 0$ and $(1-\rho_R)\,\phi_\pi + \rho_R > 1$, the equilibrium paths of the nominal interest rate and inflation are locally determinate.[5]

Substituting the solution (43) and (45) into the price setting equation (32) yields

$$\tilde{p}_{it} = -\frac{\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}} a_{it}. \tag{47}$$

It follows from the equation for the optimal consumption mix (27) that

$$c_{ijt} - c_{jt} = \frac{\theta\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}} a_{it}. \tag{48}$$

Thus, the relative price of good i and relative consumption of good i depend only on firm-specific productivity of firm i.

Since all households face the same decision problem and labor-specific preference shocks average out within the household, all households choose the same consumption level. Therefore, $c_{jt} = c_t$. Substituting $c_{jt} = c_t$ and the solution (43)-(45) into the wage setting equation (28) yields

$$\tilde{w}_{jnt} - \tilde{w}_t = -\frac{1}{1+\eta\psi} \chi_{jnt}. \tag{49}$$

It follows from the equation for the profit-maximizing input mix (33) that

$$l_{ijnt} - l_{it} = \frac{\eta}{1+\eta\psi} \chi_{jnt}. \tag{50}$$

Hence, the relative wage rate for type jn labor and the relative input of type jn labor depend only on the labor-specific preference shock.

In summary, under perfect information fluctuations in quantities and in relative prices are driven by aggregate technology shocks and by micro-level shocks. Monetary policy has no real effects. Next we solve the model assuming that decisionmakers in firms have limited attention.

[5] See Woodford (2003), chapter 2, Proposition 2.8.

4 Case 1: Firms rational inattention, households perfect information

In this section we solve the model under rational inattention on the side of firms, that is, we assume that decisionmakers in firms have limited attention and choose optimally the allocation of their attention. For the moment, we continue to assume that households have perfect information to isolate the role of rational inattention on the side of firms.

4.1 Firms' objective

We assume that the decisionmaker in firm i chooses the allocation of his/her attention so as to maximize the expected discounted sum of profits. Nominal profits are given by (6), technology is given by (4)-(5), and the demand function is given by (18) because households have perfect information. Substituting technology and demand function into the expression for nominal profits and dividing by P_t yields the real profit function. Computing a log-quadratic approximation of the real profit function around the non-stochastic steady state yields the following expression for the expected discounted sum of losses in profits due to suboptimal decisions[6]

$$E\left[\sum_{t=0}^{\infty}\beta^t\frac{1}{2}\left(x_t-x_t^*\right)'H\left(x_t-x_t^*\right)\right]. \tag{51}$$

Here x_t is the following vector of dimension JN

$$x_t=\begin{pmatrix}\tilde{p}_{it}\\ \hat{l}_{i11t}\\ \vdots\\ \hat{l}_{iJ(N-1)t}\end{pmatrix}, \tag{52}$$

where $\tilde{p}_{it}=p_{it}-p_t$ is the relative price of good i in period t and $\hat{l}_{ijnt}=l_{ijnt}-l_{it}$ is the relative input of type jn labor at firm i in period t. The profit-maximizing decisions in period t, denoted x_t^*, are given by equations (32)-(34). The matrix H is of dimension

[6] In this expression losses in profits have a negative sign.

$(JN) \times (JN)$ and is given by

$$H = Y_i \alpha \frac{\theta - 1}{\theta} \begin{bmatrix} -\frac{\theta}{\alpha}\left(1 + \frac{1-\alpha}{\alpha}\theta\right) & 0 & \cdots & \cdots & 0 \\ 0 & -\frac{2}{\eta JN} & -\frac{1}{\eta JN} & \cdots & -\frac{1}{\eta JN} \\ \vdots & -\frac{1}{\eta JN} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & -\frac{1}{\eta JN} \\ 0 & -\frac{1}{\eta JN} & \cdots & -\frac{1}{\eta JN} & -\frac{2}{\eta JN} \end{bmatrix}. \tag{53}$$

The derivation of expression (51) is in Appendix B. After the log-quadratic approximation of the real profit function, losses in profits due to suboptimal decisions depend only on the deviations of the decisions from the profit-maximizing decisions, and the H matrix contains all the information (up to second order) about how costly different types of mistakes are. See expression (51). Furthermore, the profit-maximizing decisions are given by the usual log-linear first-order conditions. See equations (32)-(34). The last result will imply that under unlimited attention the equilibrium will equal the log-linear solution under perfect information presented in Section 3.

The matrix H is the matrix of second derivatives of the real profit function with respect to x_t evaluated at the non-stochastic steady state. The diagonal elements of the matrix H contain information about the cost of a mistake in a single variable. The off-diagonal elements of the matrix H contain information about how a mistake in one variable affects the cost of a mistake in another variable. Recall that Y_i is the firm's steady state output, α is the elasticity of output with respect to composite labor, θ is the price elasticity of demand, η is the elasticity of substitution between different types of labor, and JN is the number of different types of labor. The cost of a mistake in the price setting decision is increasing in the price elasticity of demand and decreasing in the elasticity of output with respect to composite labor. The parameter α matters because it determines the concavity of the production function. The cost of a mistake in the relative input of one type of labor is decreasing in the elasticity of substitution between different types of labor and decreasing in the number of different types of labor.

4.2 Firms' attention problem

We now formalize the idea that decisionmakers cannot attend perfectly to all available information. Following Sims (2003), we model limited attention as a constraint on information flow. Decisionmakers choose how to use the available information flow. Decisionmakers decide what to focus on. Specifically, decisionmakers decide how to allocate their attention across their different decision problems, and for each decision problem decisionmakers decide how to attend to the different factors that may affect the optimal decision. Formally, the attention problem of the decisionmaker in firm i reads

$$\max_{B(L),C(L)} E\left[\sum_{t=0}^{\infty}\beta^t \frac{1}{2}\left(x_t - x_t^*\right)' H \left(x_t - x_t^*\right)\right], \tag{54}$$

subject to an equation linking an argument of the objective and a decision variable

$$\tilde{p}_{it} - \tilde{p}_{it}^* = p_{it} - p_{it}^*, \tag{55}$$

the equations characterizing the profit-maximizing decisions

$$p_{it}^* = \underbrace{A_1(L)\,\varepsilon_t^A}_{p_{it}^{A*}} + \underbrace{A_2(L)\,\varepsilon_t^R}_{p_{it}^{R*}} + \underbrace{A_3(L)\,\varepsilon_{it}^I}_{p_{it}^{I*}} \tag{56}$$

$$\hat{l}_{ijnt}^* = A_4(L)\,\varepsilon_{jnt}^{\chi}, \tag{57}$$

the equations specifying the actual decisions

$$p_{it} = \underbrace{B_1(L)\,\varepsilon_t^A + C_1(L)\,\nu_{it}^A}_{p_{it}^A} + \underbrace{B_2(L)\,\varepsilon_t^R + C_2(L)\,\nu_{it}^R}_{p_{it}^R} + \underbrace{B_3(L)\,\varepsilon_{it}^I + C_3(L)\,\nu_{it}^I}_{p_{it}^I} \tag{58}$$

$$\hat{l}_{ijnt} = B_4(L)\,\varepsilon_{jnt}^{\chi} + C_4(L)\,\nu_{ijnt}^{\chi}, \tag{59}$$

and the information flow constraint

$$\mathcal{I}\left(\left\{p_{it}^{A*}, p_{it}^{R*}, p_{it}^{I*}, \hat{l}_{i11t}^*, \ldots, \hat{l}_{iJ(N-1)t}^*\right\}; \left\{p_{it}^A, p_{it}^R, p_{it}^I, \hat{l}_{i11t}, \ldots, \hat{l}_{iJ(N-1)t}\right\}\right) \leq \kappa. \tag{60}$$

Here ν_{it}^A, ν_{it}^R, ν_{it}^I and ν_{ijnt}^{χ} follow idiosyncratic Gaussian white noise processes with unit variance that are mutually independent and independent of all other shocks in the economy.

Expression (54) is the expected loss in profits due to suboptimal decisions. A more negative value means a larger expected loss in profits. Equation (55) states that the mistake in the relative price of good i equals the mistake in the dollar price of good i. This equation is important because the objective depends on the relative price of good i, while the decisionmaker sets the dollar price of good i. The equation follows from $\tilde{P}_{it} = (P_{it}/P_t)$ and the fact that small variables denote log-deviations from the non-stochastic steady state. Equations (56)-(57) characterize the profit-maximizing decisions. Here $A_1(L)$, $A_2(L)$, $A_3(L)$ and $A_4(L)$ are infinite-order lag polynomials. These two equations follow from equations (32)-(34) and the stochastic processes for p_t, c_t, a_t, a_{it} and $\tilde{w}_{jnt}$. Equations (58)-(59) specify the actual decisions. Choosing the lag polynomials $B_1(L)$ and $C_1(L)$ to $B_4(L)$ and $C_4(L)$ amounts to choosing the stochastic processes for the actual decisions. These lag polynomials imply a mapping between shocks and price setting and input mix decisions. For example, if $B_1(L) = A_1(L)$ and $C_1(L) = 0$, the price set by the decisionmaker responds perfectly to aggregate technology shocks. The information flow constraint (60) introduces limited attention on the side of the decisionmaker. Limited attention is modeled as a bound on information flow. The operator $\mathcal{I}$ measures information flow between stochastic processes.[7] In particular, here the operator $\mathcal{I}$ measures information flow between the profit-maximizing behavior and the actual behavior. In other words, the left-hand side of (60) measures how much information the actual behavior contains about the profit-maximizing behavior, and vice versa because information flow is symmetric. The parameter κ is the bound on information flow. The parameter κ indexes the decisionmaker's total attention. We will choose the parameter κ such that the private marginal value of information flow is small.

Note that we assume that the noise shocks ν_{it}^{A}, ν_{it}^{R}, ν_{it}^{I} and ν_{ijnt}^{χ} follow Gaussian processes. It turns out that Gaussianity is optimal because the objective is quadratic and the profit-maximizing decisions follow Gaussian processes.[8] Furthermore, note that we assume that ν_{it}^{A}, ν_{it}^{R}, ν_{it}^{I} and ν_{ijnt}^{χ} are mutually independent. In the future, we also plan to study the case where these noise shocks can be correlated. Finally, note that we assume that these noise shocks are idiosyncratic. This assumption accords well with the idea that the friction is the decisionmaker's limited attention rather than the availability of information.

[7] For a definition of the operator $\mathcal{I}$, see equations (1)-(4) in Maćkowiak and Wiederholt (2007).

[8] See Sims (2006) or Maćkowiak and Wiederholt (2007).

4.3 Computing the equilibrium

We use an iterative procedure to solve for the equilibrium of the model. First, we make a guess concerning the processes for the profit-maximizing price and the profit-maximizing input mix. See equations (56)-(57). Second, we solve the firms' attention problem (54)-(60). Third, we aggregate the individual prices to obtain the aggregate price level:

$$p_t = \frac{1}{I}\sum_{i=1}^{I} p_{it}. \tag{61}$$

Fourth, we compute the aggregate dynamics implied by the price level dynamics. The following equations have to be satisfied in equilibrium:

$$r_t = \rho_R r_{t-1} + (1-\rho_R)\left[\phi_\pi\left(p_t - p_{t-1}\right) + \phi_y y_t\right] + \varepsilon_t^R, \tag{62}$$

$$c_t = E_t\left[-\frac{1}{\gamma}\left(r_t - p_{t+1} + p_t\right) + c_{t+1}\right], \tag{63}$$

$$y_t = c_t, \tag{64}$$

$$a_t = \rho_A a_{t-1} + \varepsilon_t^A, \tag{65}$$

$$y_t = a_t + \alpha l_t, \tag{66}$$

$$\tilde{w}_t = \psi l_t + \gamma c_t. \tag{67}$$

The first equation is the Taylor rule. The second equation is the consumption Euler equation. The third equation is the requirement that output equals demand. The fourth equation is the assumed process for aggregate technology. The fifth equation is the aggregate production function. The sixth equation follows from optimal wage setting by households. See Appendix C. We employ a standard solution method for linear rational expectations models to solve the system of equations containing the price level dynamics and those six equations. We obtain the law of motion for $(r_t, c_t, y_t, a_t, l_t, \tilde{w}_t)$ implied by the price level dynamics. Fifth, we compute the law of motion for the profit-maximizing price from equation (32) which we reproduce for convenience:

$$p_{it}^* = p_t + \frac{1}{1+\theta\frac{1-\alpha}{\alpha}}\tilde{w}_t + \frac{\frac{1-\alpha}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}c_t - \frac{\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}\left(a_t + a_{it}\right). \tag{68}$$

If the process for the profit-maximizing price differs from our guess, we update our guess. We iterate until we reach a fixed point. Finally, we compute the equilibrium relative wage rates and the equilibrium input mix. This is explained in Appendix C.

4.4 Benchmark parameter values and solution

In this section we report the numerical solution of the model for the following parameter values. We set $\beta = 0.99$, $\gamma = 1$, $\psi = 1$, $\theta = 4$, $\alpha = 2/3$ and $\eta = 4$.

To set the parameters governing the process for aggregate technology, equation (65), we consider quarterly U.S. data from 1960 Q1 to 2006 Q4. We first compute a time series for aggregate technology, a_t, using equation (66) and measures of y_t and l_t. We use the log of real output per person, detrended with a linear trend, as a measure of y_t. We use the log of hours worked per person, demeaned, as a measure of l_t.[9] We then fit equation (65) to the time series for a_t obtaining $\rho_A = 0.96$ and a standard deviation of the innovation equal to 0.0085. In the benchmark economy we set $\rho_A = 0.95$ and we set the standard deviation of ε_t^A equal to 0.0085.

To set the parameters of the Taylor rule we consider quarterly U.S. data on the Federal Funds rate, inflation and real GDP from 1960 Q1 to 2006 Q4.[10] We fit the Taylor rule (62) to the data obtaining $\rho_R = 0.89$, $\phi_\pi = 1.53$, $\phi_y = 0.33$, and a standard deviation of the innovation equal to 0.0021. In the benchmark economy we set $\rho_R = 0.9$, $\phi_\pi = 1.5$, $\phi_y = 0.33$, and the standard deviation of ε_t^R equal to 0.0021.

We assume that firm-specific productivity follows a first-order autoregressive process. Recent papers calibrate the autocorrelation of firm-specific productivity to be about two-thirds in monthly data, e.g., Klenow and Willis (2007) use 0.68, Midrigan (2006) uses 0.5, and Nakamura and Steinsson (2007) use 0.66. Since $(2/3)^3$ equals about 0.3, we set the autocorrelation of firm-specific productivity in our quarterly model equal to 0.3. We then choose the standard deviation of the innovation to firm-specific productivity such that the average absolute size of price changes in our model equals 9.7 percent under perfect information. The value 9.7 percent is the average absolute size of price changes excluding sales reported in Klenow and Kryvtsov (2007). This yields a standard deviation of the

[9] We use data for the non-farm business sector. The source of the data is the website of the Federal Reserve Bank of St.Louis.

[10] We compute a time series for four-quarter inflation rate from the price index for personal consumption expenditures excluding food and energy. We compute a time series for percentage deviations of real GDP from potential real GDP. The sources of the data are the websites of the Federal Reserve Bank of St.Louis and the Congressional Budget Office.

innovation to firm-specific productivity equal to 0.22.

We assume that labor-specific preference shocks follow a white noise process. This simplifying assumption implies that solving for relative wage rates and relative labor inputs is straightforward. See Appendix C. To set the standard deviation of labor-specific preference shocks we proceed as follows. Autor, Katz and Kearney (2005) report the variance of log hourly wages of men in the U.S. between 1975 and 2003. The average variance of log hourly wages of men in this period was 0.32. We choose the variance of χ_{jnt} such that the variance of $\tilde{w}_{jnt}$ in our model equals 0.32 under perfect information. This yields a standard deviation of labor-specific preference shocks equal to 2.83. See equation (49) and recall that $\psi = 1$ and $\eta = 4$. We set the number of different types of labor that a firm hires to $JN = 100$.

We compute the solution of the model by fixing the marginal value of information flow. The total information flow, κ, is then determined within the model. The idea is the following. When the marginal value of information flow is high, decisionmakers have a high incentive to increase information flow in order to take better decision. In contrast, when the marginal value of information flow is low, decisionmakers have little incentive to increase information flow. We set the marginal value of information flow equal to 0.25 percent of a firm's steady state output.

We first report the optimal allocation of attention at the rational inattention fixed point. The total information flow at the solution equals 133 bits. The decisionmaker in a firm allocates: 2.46 bits of information flow (his/her attention) to tracking firm-specific productivity, 1.31 bits of information flow to tracking each relative wage rate, 0.76 bits of information flow to tracking aggregate technology, and 0.41 bits of information flow to tracking monetary policy. The expected per period loss in profits due to imperfect tracking of firm-specific productivity equals 0.18 percent of the firm's steady state output. The expected per period loss in profits due to imperfect tracking of aggregate technology equals 0.12 percent of the firm's steady state output. The expected per period loss in profits due to imperfect tracking of monetary policy equals 0.07 percent of the firm's steady state output. Together these numbers imply that the total expected per period loss in profits due to suboptimal price setting equals 0.37 percent of the firm's steady state output. We think that this is a reasonable number.

Figures 1 and 2 show impulse responses of the price level, inflation, output, and the nominal interest rate at the rational inattention fixed point (green lines with circles). For comparison the figures also include impulse responses of the same variables at the perfect information equilibrium (blue lines with points). All impulse responses are to shocks of one standard deviation. All impulse responses are drawn such that an impulse response equal to one means "a one percent deviation from the non-stochastic steady state". Time is measured in quarters along horizontal axes.

Consider Figure 1. The price level shows a dampened and delayed response to a monetary policy shock compared with the case of perfect information. The response of inflation to a monetary policy shock is persistent. Output falls after a positive innovation in the Taylor rule and the decline in output is persistent. The nominal interest rate increases on impact and converges slowly to zero. The impulse responses to a monetary policy shock under rational inattention differ markedly from the impulse responses to a monetary policy shock under perfect information. Under perfect information the price level adjusts fully on impact to a monetary policy shock, there are no real effects, and the nominal interest rate fails to change.

Consider Figure 2. The price level and inflation show a dampened response to an aggregate technology shock compared with the case of perfect information. The output gap is negative for a few quarters after the shock. Output and the nominal interest rate show hump-shaped impulse responses to an aggregate technology shock. Note that under rational inattention the response of the price level to an aggregate technology shock is less dampened and less delayed than the response of the price level to a monetary policy shock. The reason is the optimal allocation of attention. Since decisionmakers in firms allocate about twice as much attention to aggregate technology than to monetary policy, prices respond faster to aggregate technology shocks than to monetary policy shocks. Therefore, the output gap is negative for only 5 quarters after an aggregate technology shock, while the output gap is negative for more than 10 quarters after a monetary policy shock.[11]

[11] See also Paciello (2007). Paciello solves the white noise case of a similar model analytically, where white noise case means that: (i) all exogenous processes are white noise processes, (ii) there is no lagged interest rate in the Taylor rule, and (iii) the price level instead of the inflation rate appears in the Taylor rule. The analytical solution in the white noise case helps to understand in more detail the differential response of

Figure 3 shows the impulse response of an individual price to a firm-specific productivity shock. Note that prices respond almost perfectly to firm-specific productivity shocks. The reason is the optimal allocation of attention.

4.5 Comparison to the Calvo model

For comparison, we solved the Calvo model for the same parameter values and assuming that prices change every 2.5 quarters on average.[12] Figures 4 and 5 show the impulse responses in the benchmark economy with rational inattention (green lines with circles) and the impulse responses in the perfect information, Calvo model (red lines with crosses). The impulse responses to a monetary policy shock are very similar in the two models. In contrast, the impulse responses to an aggregate technology shock are quite different in the two models. Inflation responds to a monetary policy shock by the same amount on impact in the benchmark economy and in the Calvo model, while inflation responds to an aggregate technology shock twice more strongly on impact in the benchmark economy than in the Calvo model. The reason is that decisionmakers in firms in the benchmark economy allocate about twice as much attention to aggregate technology than to monetary policy.

Firms in the benchmark economy and firms in the Calvo model experience profit losses due to deviations of the actual price from the profit-maximizing price. It turns out that profit losses due to deviations of the actual price from the profit-maximizing price are an order of magnitude smaller than in the Calvo model generating the same real effects. Specifically, the expected loss in profits due to suboptimal price responses to aggregate conditions is 12 times smaller than in the Calvo model; and the expected loss in profits due to suboptimal price responses to firm-specific conditions is 25 times smaller than in the Calvo model. One reason is that in the benchmark economy prices respond slowly to monetary policy shocks, faster to aggregate technology shocks, and almost perfectly to micro-level shocks. In contrast, in the Calvo model prices respond slowly to all those shocks. Another reason is that under rational inattention deviations of the actual price from the profit-maximizing price are less likely to be extreme than in the Calvo model.

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prices to aggregate technology shocks and to monetary policy shocks.

[12] Klenow and Kryvtsov (2007) find that the median price duration excluding sales is 7.2 months, or about 2.5 quarters.

4.6 Varying parameter values

Figure 6 compares the benchmark economy to an economy with a higher degree of real rigidity.[13] We set $\psi = 0$ implying that the coefficient on aggregate output in the equation for the profit-maximizing price falls from 1 to 0.5 (after substituting in the wage equation). Real effects of monetary policy shocks become larger and more persistent. After a monetary policy shock the output gap is now negative for about 20 quarters instead of 10 quarters. At the same time, profit losses due to imperfect tracking of aggregate conditions decrease. The expected loss in profits due to imperfect tracking of aggregate technology falls by 25 percent; and the expected loss in profits due to imperfect tracking of monetary policy falls by 60 percent.

Figure 7 compares the benchmark economy to an economy with larger monetary policy shocks. We increase the standard deviation of monetary policy shocks by roughly a factor of two, from 0.0021 to 0.004. This matches the standard deviation of monetary policy shocks estimated by Justiniano and Primiceri (2006) for the high inflation episode in the 1970s.[14] At the rational inattention fixed point with larger monetary policy shocks firms allocate 0.84 bits to monetary policy, an increase by 100 percent compared to the benchmark economy.[15] Since firms allocate more attention to monetary policy, a monetary policy shock of a given size has smaller real effects and larger inflationary effects.[16] However, the additional attention that decisionmakers pay to monetary policy is not sufficient to compensate fully for the fact that the average size of monetary policy shocks has increased. The firms' tracking problem has become more complicated. Therefore, the expected loss in profits due to imperfect tracking of monetary policy almost doubles and the variance of output due to monetary policy shocks increases.

[13] Ball and Romer (1990) refer to the elasticity of the profit-maximizing price with respect to aggregate output as the degree of real rigidity. A low elasticity corresponds to a high degree of real rigidity.

[14] Justiniano and Primiceri (2006) estimate a DSGE model that allows for time variation in the size of shocks.

[15] We are holding constant the marginal value of information flow. The total information flow is then determined within the model.

[16] In Figure 7 one must divide an impulse response in the economy with larger monetary policy shocks by roughly one-half to obtain an impulse response to a shock of the same size as in the benchmark economy.

5 Conclusion

We have introduced rational inattention on the side of firms into a dynamic stochastic general equilibrium model. The impulse responses under rational inattention have several properties of empirical impulse response functions, e.g., (i) prices respond slowly to monetary policy shocks, (ii) prices respond faster to aggregate technology shocks, and (iii) prices respond quickly to disaggregate shocks (see Boivin, Giannoni, and Mihov (2007)). In addition, profit losses due to deviations of the actual price from the profit-maximizing price are an order of magnitude smaller than in the perfect information, Calvo model generating the same real effects.

We are currently working on introducing rational inattention on the side of households. The approach is very similar. We first derive a quadratic objective, and we then solve a problem of the form (54)-(60) where the households' decision variables are the consumption vector and the nominal wage rates.

A Solution under perfect information

First, $y_{it} = c_{it}$ and equations (29), (30) and (40) imply that

$$y_t = c_t.$$

Second, computing the average of the production function (35) over all i and using (38), (40) and (41) yields

$$y_t = a_t + \alpha l_t.$$

Third, computing the average of the price setting equation (32) over all i and using (29), (41) and $l_t = \frac{1}{\alpha}(c_t - a_t)$ yields

$$\tilde{w}_t = c_t - l_t.$$

The real wage index equals output per labor input. Fourth, computing the average of the wage setting equation (28) over all jn and using (31), (34) and (42) yields

$$\tilde{w}_t = \psi l_t + \gamma c_t.$$

The real wage index equals the marginal rate of substitution of consumption for leisure. When we solve the last four equations for y_t, c_t, l_t and $\tilde{w}_t$, we arrive at equations (43)-(45). Finally, computing the average of the Euler equation (26) over all j and using (31) yields

$$c_t = E_t\left[-\frac{1}{\gamma}(r_t - \pi_{t+1}) + c_{t+1}\right].$$

Substituting the solution for c_t into the last equation yields equation (46).

B Derivation of the firms' objective

Nominal profits of firm i in period t equal

$$(1+\tau_p) P_{it} Y_{it} - L_{it}\left(\sum_{j=1}^{J}\sum_{n=1}^{N} W_{jnt}\hat{L}_{ijnt}\right),$$

where $\hat{L}_{ijnt} = (L_{ijnt}/L_{it})$. The term in brackets is the wage bill per unit of composite labor. The production function (4) implies that

$$L_{it} = \left(\frac{Y_{it}}{e^{a_t + a_{it}}}\right)^{\frac{1}{\alpha}}.$$

The labor aggregator (5) implies that

$$1=\sum_{j=1}^{J}\sum_{n=1}^{N}\hat{L}_{ijnt}^{\frac{\eta-1}{\eta}},$$

or equivalently

$$\hat{L}_{iJNt}=\left(1-\sum_{jn\neq JN}\hat{L}_{ijnt}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}.$$

Furthermore, since households have perfect information, the demand function equals

$$C_{it}=\left(\frac{P_{it}}{P_t}\right)^{-\theta}C_t.$$

Substituting the production function, the labor aggregator and the demand function into the expression for nominal profits yields the profit function

$$\left(1+\tau_p\right)P_{it}\left(\frac{P_{it}}{P_t}\right)^{-\theta}C_t-\left(\frac{\left(\frac{P_{it}}{P_t}\right)^{-\theta}C_t}{e^{a_t+a_{it}}}\right)^{\frac{1}{\alpha}}\left[\sum_{jn\neq JN}W_{jnt}\hat{L}_{ijnt}+W_{JNt}\left(1-\sum_{jn\neq JN}\hat{L}_{ijnt}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}\right].$$

Dividing by P_t yields the real profit function

$$\left(1+\tau_p\right)\tilde{P}_{it}^{1-\theta}C_t-\left(\frac{\tilde{P}_{it}^{-\theta}C_t}{e^{a_t+a_{it}}}\right)^{\frac{1}{\alpha}}\left[\sum_{jn\neq JN}\tilde{W}_{jnt}\hat{L}_{ijnt}+\tilde{W}_{JNt}\left(1-\sum_{jn\neq JN}\hat{L}_{ijnt}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}\right],\quad(69)$$

where $\tilde{P}_{it}=(P_{it}/P_t)$ and $\tilde{W}_{jnt}=(W_{jnt}/P_t)$. One can express the real profit function in terms of log-deviations from the non-stochastic steady state

$$\begin{aligned}&\left(1+\tau_p\right)\tilde{P}_iC_ie^{(1-\theta)\tilde{p}_{it}+c_t}\\&-L_ie^{\frac{1}{\alpha}(-\theta\tilde{p}_{it}+c_t-a_t-a_{it})}\tilde{W}\frac{1}{JN}\left[\sum_{jn\neq JN}e^{\tilde{w}_{jnt}+\hat{l}_{ijnt}}+e^{\tilde{w}_{JNt}}\left(JN-\sum_{jn\neq JN}e^{\frac{\eta-1}{\eta}\hat{l}_{ijnt}}\right)^{\frac{\eta}{\eta-1}}\right]\end{aligned}\quad(70)$$

Here we have used equation (25).

We assume that firm i chooses the allocation of attention so as to maximize the expected discounted sum of profits

$$E\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_{jt}}{C_{j0}}\right)^{-\gamma}F\left(\tilde{P}_{it},C_t,e^{a_t},e^{a_{it}},\tilde{W}_{11t},\ldots,\tilde{W}_{JNt},\hat{L}_{i11t},\ldots,\hat{L}_{iJ(N-1)t}\right)\right],\quad(71)$$

where $\beta^t \left(\frac{C_{jt}}{C_{j0}}\right)^{-\gamma}$ is the stochastic discount factor and F is the real profit function (69). Two remarks concerning firm i's objective may be helpful. First, C_{jt} is composite consumption by household j in period t. Since in equilibrium all households have the same composite consumption, the stochastic discount factor does not depend on j. Second, E is the unconditional expectation operator. Here we are using the assumption that firms choose the allocation of attention before receiving any information. This assumption slightly simplifies the computation of the equilibrium. The assumption can be relaxed.

One can express the objective (71) in terms of log-deviations from the non-stochastic steady state

$$E\left[\sum_{t=0}^{\infty} \beta^t e^{-\gamma(c_{jt}-c_{j0})} f\left(\tilde{p}_{it}, c_t, a_t, a_{it}, \tilde{w}_{11t}, \ldots, \tilde{w}_{JNt}, \hat{l}_{i11t}, \ldots, \hat{l}_{iJ(N-1)t}\right)\right], \tag{72}$$

where f is the real profit function (70).

Next we compute a second-order Taylor approximation around the non-stochastic steady state of the term inside the expectation operator of (72). Afterwards, we deduct from the quadratic objective the value of the quadratic objective at the profit-maximizing behavior $\left\{\tilde{p}_{it}^*, \hat{l}_{i11t}^*, \ldots, \hat{l}_{iJ(N-1)t}^*\right\}_{t=0}^{\infty}$. This yields the following expression for the expected discounted sum of losses in profits due to suboptimal behavior:

$$E\left[\sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left(x_t - x_t^*\right)' H \left(x_t - x_t^*\right)\right],$$

where

$$\underset{(JN\times 1)}{x_t} = \begin{pmatrix} \tilde{p}_{it} \\ \hat{l}_{i11t} \\ \vdots \\ \hat{l}_{iJ(N-1)t} \end{pmatrix},$$

$$\underset{(JN\times JN)}{H} = Y_i \alpha \frac{\theta - 1}{\theta} \begin{bmatrix} -\frac{\theta}{\alpha}\left(1 + \frac{1-\alpha}{\alpha}\theta\right) & 0 & \cdots & \cdots & 0 \\ 0 & -\frac{2}{\eta JN} & -\frac{1}{\eta JN} & \cdots & -\frac{1}{\eta JN} \\ \vdots & -\frac{1}{\eta JN} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & -\frac{1}{\eta JN} \\ 0 & -\frac{1}{\eta JN} & \cdots & -\frac{1}{\eta JN} & -\frac{2}{\eta JN} \end{bmatrix},$$

and x_t^* is given by the following two equations:

$$\tilde{p}_{it}^* = \frac{1}{1+\theta\frac{1-\alpha}{\alpha}}\tilde{w}_t + \frac{\frac{1-\alpha}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}c_t - \frac{\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}\left(a_t + a_{it}\right),$$

and

$$\hat{l}_{ijnt}^* = -\eta\left(\tilde{w}_{jnt} - \tilde{w}_t\right).$$

Here $\tilde{w}_t = \frac{1}{JN}\sum_{j=1}^{J}\sum_{n=1}^{N}\tilde{w}_{jnt}$. Note that deducting from the quadratic objective the value of the quadratic objective at the profit-maximizing behavior is simply a monotone transformation of the quadratic objective. This transformation simplifies the quadratic objective without affecting the solution to the optimization problem.

C Solving for relative wage rates

In this subsection, we solve for the relative wage rate for type jn labor and the relative input of type jn labor at firm i. Solving for relative wage rates is not trivial because limited attention on the side of decisionmakers in firms lowers the wage elasticity of labor demand, which affects households' wage setting behavior. To make the derivation as clear as possible, we assume that all the χ_{jnt} follow a common white noise process.

Let $\hat{w}_{jnt} = \tilde{w}_{jnt} - \tilde{w}_t$ denote the relative wage rate for type jn labor. We guess that in equilibrium

$$\hat{w}_{jnt} = A\chi_{jnt}, \tag{73}$$

where A is an unknown coefficient. Let $\hat{l}_{ijnt} = l_{ijnt} - l_{it}$ denote the relative input of type jn labor at firm i. The profit-maximizing relative input of type jn labor at firm i is given by equation (33):

$$\hat{l}_{ijnt}^* = -\eta\hat{w}_{jnt}. \tag{74}$$

Since χ_{jnt} follows a Gaussian white noise process, both $\hat{w}_{jnt}$ and $\hat{l}_{ijnt}^*$ follow Gaussian white noise processes. Furthermore, tracking an optimal decision that follows a Gaussian white noise process with an information flow equal to κ_χ yields the following decision under rational inattention when the aim is to minimize the mean squared error

$$\hat{l}_{ijnt}^{RI} = \left(1 - \frac{1}{2^{2\kappa_\chi}}\right)\hat{l}_{ijnt}^* + \sqrt{\frac{1}{2^{2\kappa_\chi}} - \frac{1}{2^{4\kappa_\chi}}}\sqrt{Var\left(\hat{l}_{ijnt}^*\right)}\nu_{ijnt}^{\chi}, \tag{75}$$

where ν^{χ}_{ijnt} follows an independent Gaussian white noise process with unit variance. See, for example, Proposition 3 in Maćkowiak and Wiederholt (2007). Using equation (74) to substitute for $\hat{l}^{*}_{ijnt}$ in equation (75), we arrive at

$$\hat{l}^{RI}_{ijnt} = -\eta \left(1 - \frac{1}{2^{2\kappa_{\chi}}}\right) \left(\hat{w}_{jnt} - \sqrt{\frac{1}{2^{2\kappa_{\chi}} - 1}} \sqrt{Var\left(\hat{w}_{jnt}\right)} \nu^{\chi}_{ijnt}\right). \quad (76)$$

Now it is easy to verify that the signal

$$s_{ijnt} = \hat{w}_{jnt} - \sqrt{\frac{1}{2^{2\kappa_{\chi}} - 1}} \sqrt{Var\left(\hat{w}_{jnt}\right)} \nu^{\chi}_{ijnt} \quad (77)$$

has the property

$$\hat{l}^{RI}_{ijnt} = E\left[\hat{l}^{*}_{ijnt} | s_{ijnt}, s_{ijnt-1}, \ldots\right].$$

Thus, one can interpret the decision under rational inattention as being due to the fact that firms pay limited attention to the relative wage rate for type jn labor. Furthermore, comparing (74) to (76) one can see that firms' limited attention lowers the wage elasticity of labor demand from η to $\eta\left(1 - \frac{1}{2^{2\kappa_{\chi}}}\right)$.

Computing the average of (76) over all firms and using the fact that noise is idiosyncratic yields

$$l_{jnt} - l_t = -\eta \left(1 - \frac{1}{2^{2\kappa_{\chi}}}\right) \hat{w}_{jnt}, \quad (78)$$

where $l_{jnt} = \frac{1}{I}\sum_{i=1}^{I} l_{ijnt}$ and $l_t = \frac{1}{I}\sum_{i=1}^{I} l_{it}$. Exponentiating both sides of equation (78), multiplying by L_{jn} and using the fact that in the non-stochastic steady state $L_{jn} = \left(\frac{W_{jn}}{W}\right)^{-\eta} L$ yields

$$L_{jnt} = \left(\frac{W_{jn}}{W}\right)^{-\frac{\eta}{2^{2\kappa_{\chi}}}} \left(\frac{W_{jnt}}{W_t}\right)^{-\eta\left(1 - \frac{1}{2^{2\kappa_{\chi}}}\right)} L_t. \quad (79)$$

The first term on the RHS is due to the fact that in the non-stochastic steady state the wage elasticity of labor demand equals η rather than $\eta\left(1 - \frac{1}{2^{2\kappa_{\chi}}}\right)$.

When firms have limited attention, the household problem is to maximize (12) subject to (2), (3) and the new labor demand function (79). The optimality conditions for consumption are still equations (13) and (14) and, as long as $\eta\left(1 - \frac{1}{2^{2\kappa_{\chi}}}\right) > 1$, the optimal wage rate for type jn labor is given by

$$\frac{W_{jnt}}{P_t} = \frac{1}{1 + \tau_w} \frac{\eta\left(1 - \frac{1}{2^{2\kappa_{\chi}}}\right)}{\eta\left(1 - \frac{1}{2^{2\kappa_{\chi}}}\right) - 1} \varphi e^{-\chi_{jnt}} L^{\psi}_{jnt} C^{\gamma}_{jt}.$$

We continue to assume that the government sets the wage subsidy τ_w so as to correct the distortion arising from households' market power on the labor market. This now implies that

$$1+\tau_w = \frac{\eta\left(1-\frac{1}{2^{2\kappa_\chi}}\right)}{\eta\left(1-\frac{1}{2^{2\kappa_\chi}}\right)-1}.$$

Therefore the wage setting equation reduces to

$$\frac{W_{jnt}}{P_t} = \varphi e^{-\chi_{jnt}} L_{jnt}^{\psi} C_{jt}^{\gamma}. \tag{80}$$

Taking logs on both sides of (80) and using the fact that (80) with $\chi_{jnt} = 0$ also holds in the non-stochastic steady state yields

$$\tilde{w}_{jnt} = -\chi_{jnt} + \psi l_{jnt} + \gamma c_{jt}.$$

Using the labor demand function (78) to substitute for l_{jnt} in the last equation yields

$$\tilde{w}_{jnt} = -\chi_{jnt} + \psi\left[-\eta\left(1-\frac{1}{2^{2\kappa_\chi}}\right)(\tilde{w}_{jnt}-\tilde{w}_t) + l_t\right] + \gamma c_{jt}. \tag{81}$$

Computing the average of (81) over all types of labor and using (34), (42) and $c_{jt} = c_t$ yields the following expression for the real wage index

$$\tilde{w}_t = \psi l_t + \gamma c_t. \tag{82}$$

Computing the difference between (81) and (82) and using again $c_{jt} = c_t$ yields the following expression for the relative wage rate for type jn labor

$$\tilde{w}_{jnt} - \tilde{w}_t = -\frac{1}{1+\eta\left(1-\frac{1}{2^{2\kappa_\chi}}\right)\psi}\chi_{jnt}. \tag{83}$$

Comparing (73) and (83) shows that the guess (73) was correct. Firm i's profit-maximizing input mix then follows from equation (74) and the actual input mix under rational inattention follows from equation (76).

We still need to solve for the equilibrium attention allocated to the relative wage rate for type jn labor. Equations (74), (76) and (83) imply that the mean squared error in the relative input of type jn labor equals

$$E\left[\left(\hat{l}_{ijnt}^{*} - \hat{l}_{ijnt}^{RI}\right)^2\right] = \frac{1}{2^{2\kappa_\chi}} \frac{\eta^2}{\left[1+\left(1-\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi\right]^2}\sigma_\chi^2.$$

The derivative of the mean squared error with respect to κ_χ equals

$$\frac{\partial E\left[\left(\hat{l}^{*}_{ijnt}-\hat{l}^{RI}_{ijnt}\right)^{2}\right]}{\partial \kappa_\chi} = -2\ln\left(2\right)\frac{1}{2^{2\kappa_\chi}}\frac{1+\left(1+\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi}{\left[1+\left(1-\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi\right]^{3}}\eta^{2}\sigma_\chi^{2}.$$

It follows from objective (54) that the marginal value of paying attention to the relative wage rate for type jn labor equals

$$\lambda_\chi = \frac{1}{1-\beta}\frac{1}{\eta JN}2\ln\left(2\right)\frac{1}{2^{2\kappa_\chi}}\frac{1+\left(1+\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi}{\left[1+\left(1-\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi\right]^{3}}\eta^{2}\sigma_\chi^{2}. \tag{84}$$

By equating the marginal value of attention across different activities we obtain the equilibrium κ_χ.

D Solving the Calvo model

If we assume that firms and households have perfect information but firms face a Calvo friction, we obtain the following version of the New Keynesian Phillips curve

$$\pi_t = \frac{\left(1-\lambda\right)\left(1-\lambda\beta\right)}{\lambda}\frac{\frac{\psi}{\alpha}+\gamma+\frac{1-\alpha}{\alpha}}{1+\frac{1-\alpha}{\alpha}\theta}\left(c_t-c_t^{f}\right)+\beta E_t\left[\pi_{t+1}\right], \tag{85}$$

where $(1-\lambda)$ is the fraction of goods prices that change every period and c_t^f is the flexible price solution given by equation (43). The aggregate dynamics are obtained by solving the system containing equations (62)-(67) and equation (85). The solution of the Calvo model reported in Figures 4-5 sets $\lambda = 0.6$.

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Figure 1: Impulse responses, benchmark economy

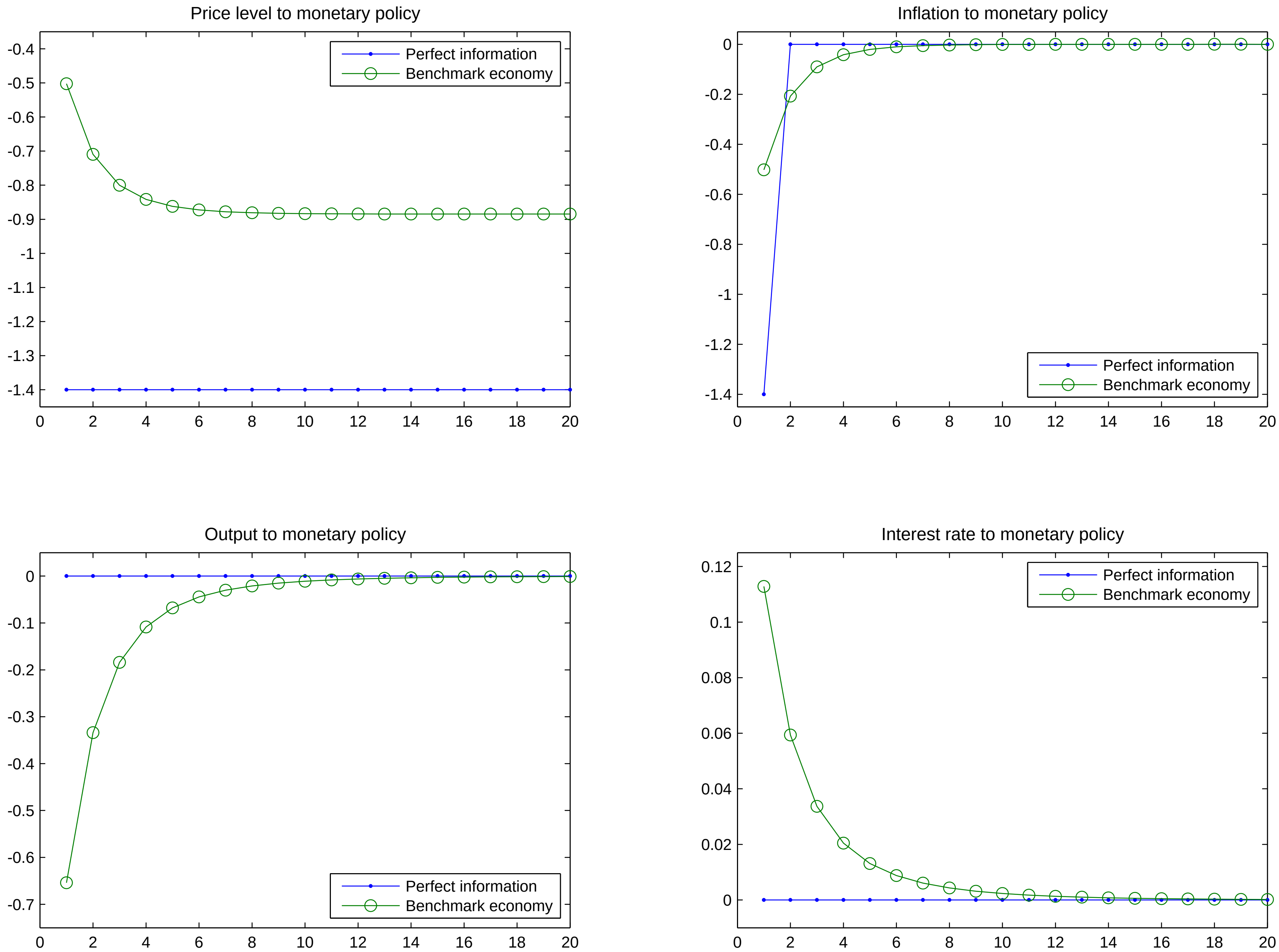

Figure 2: Impulse responses, benchmark economy

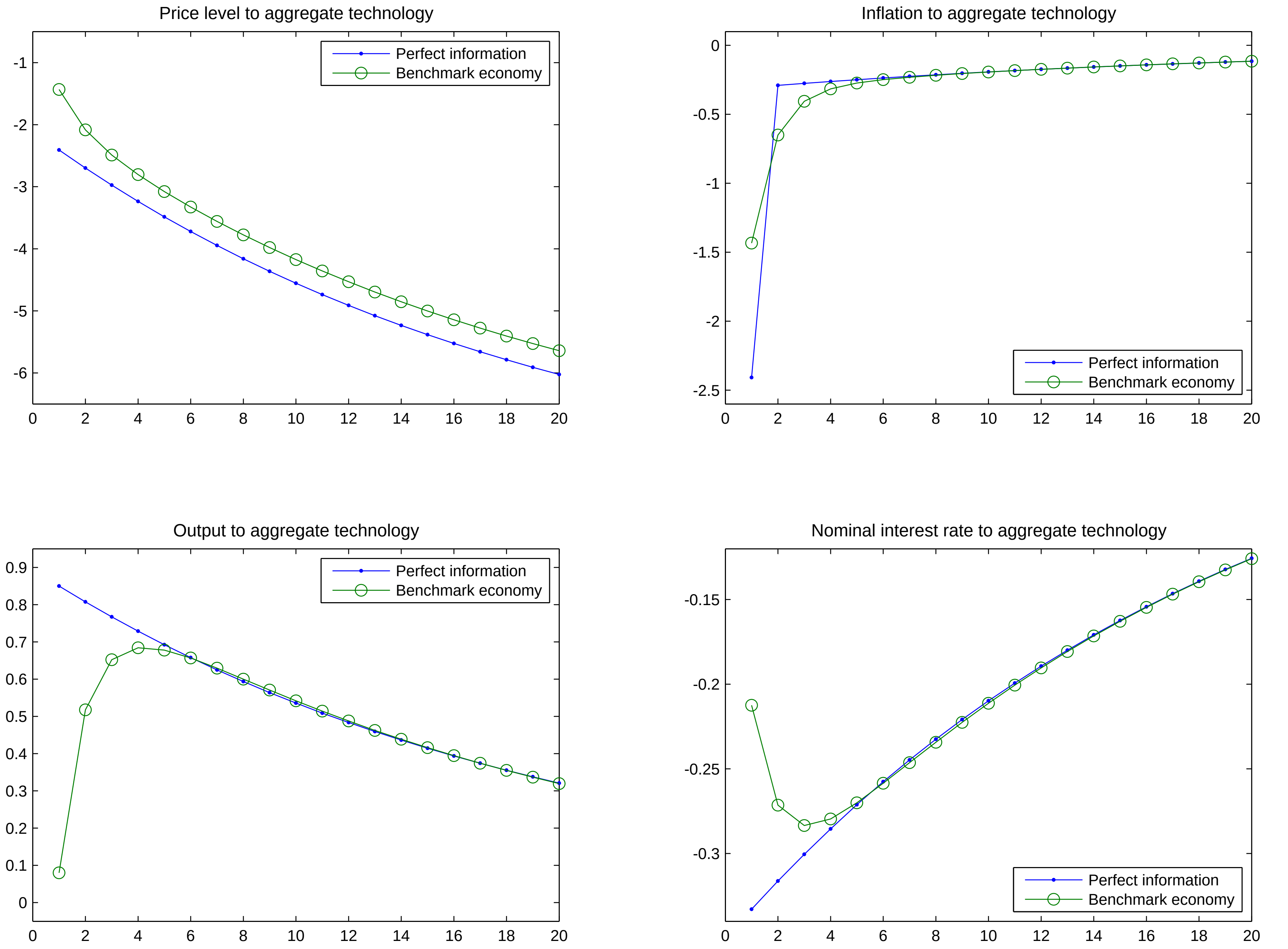

Figure 3: Impulse response of an individual price to a firm-specific productivity shock

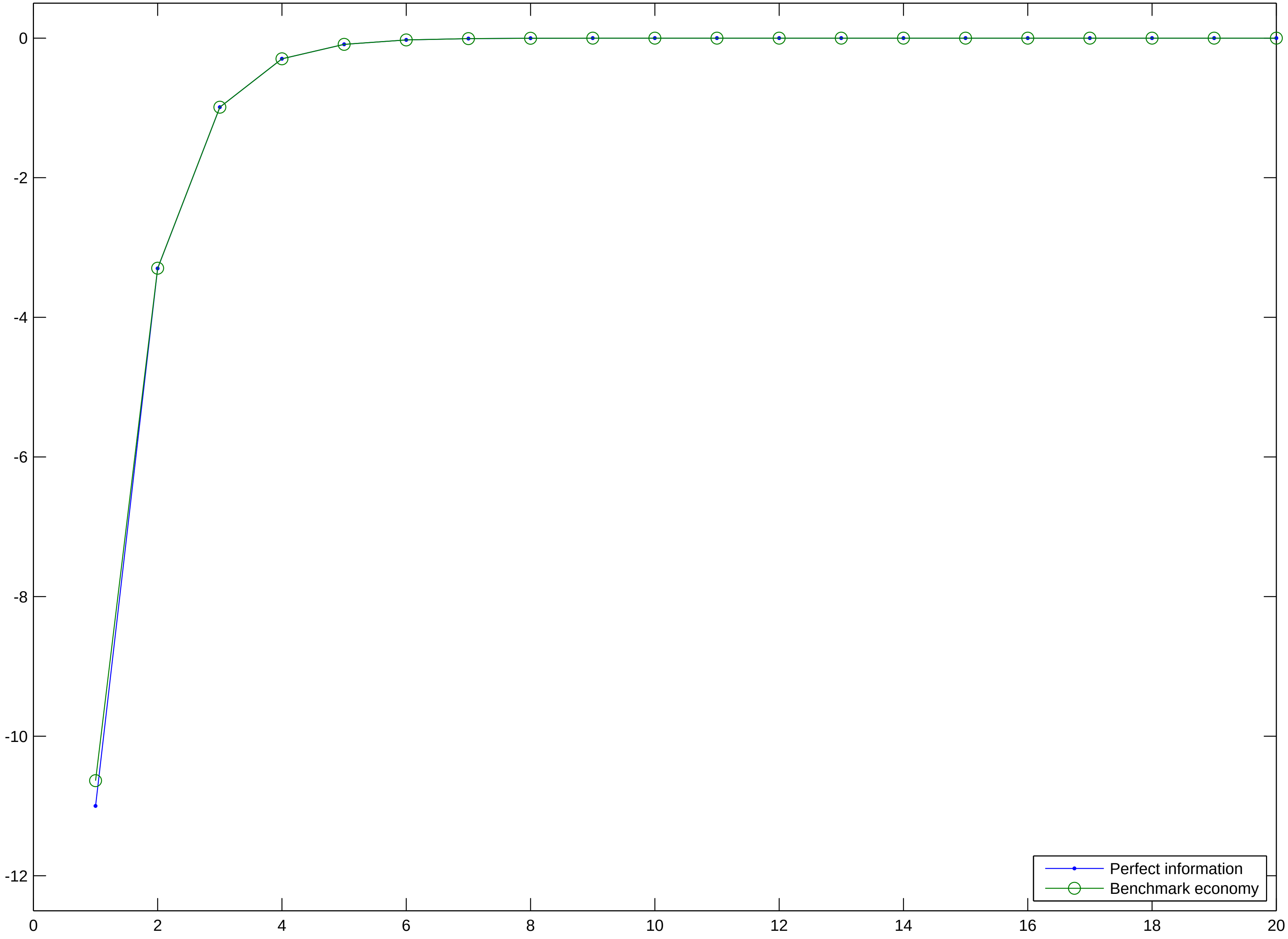

Figure 4: Impulse responses, benchmark economy and the Calvo model

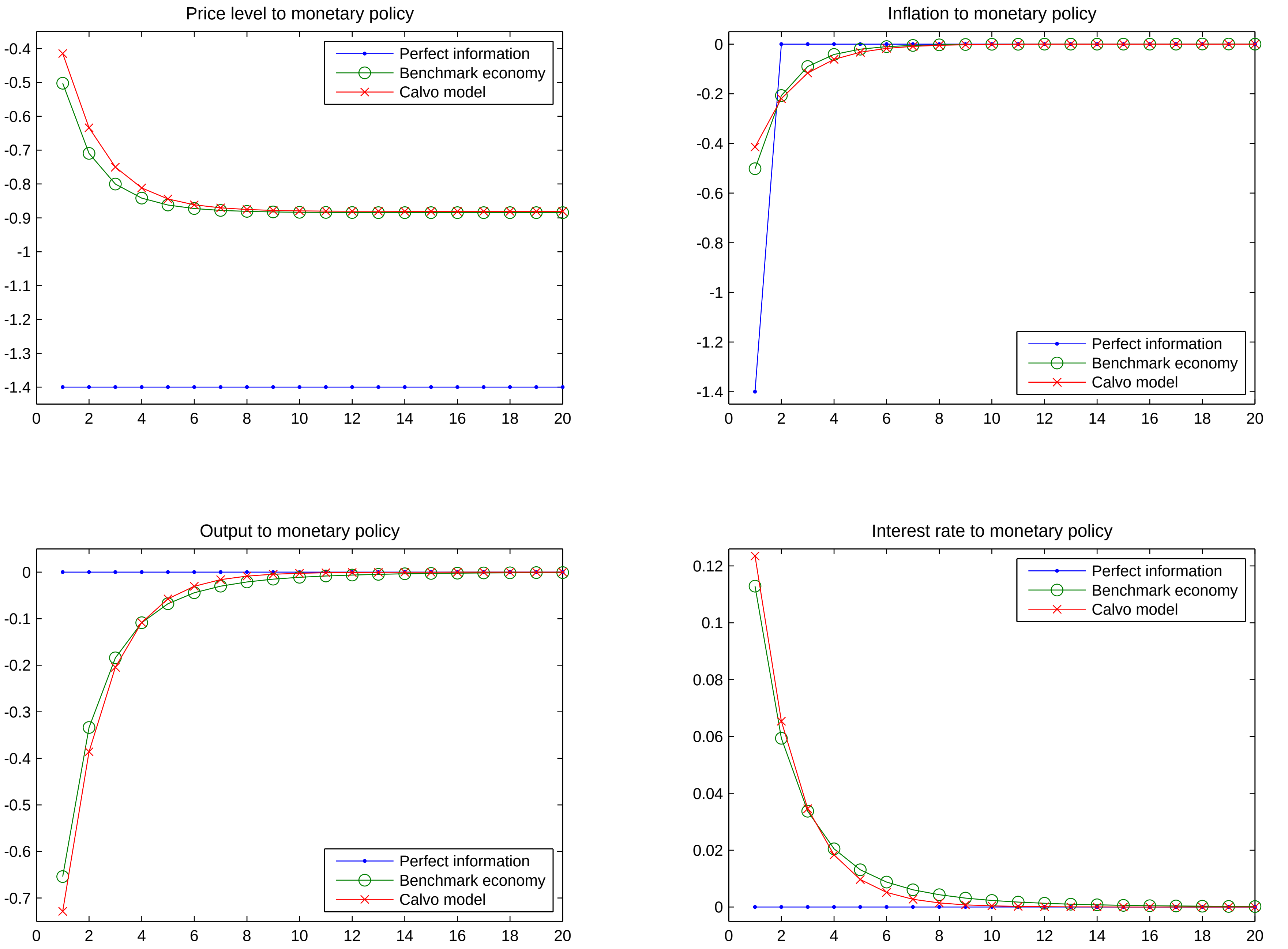

Figure 5: Impulse responses, benchmark economy and the Calvo model

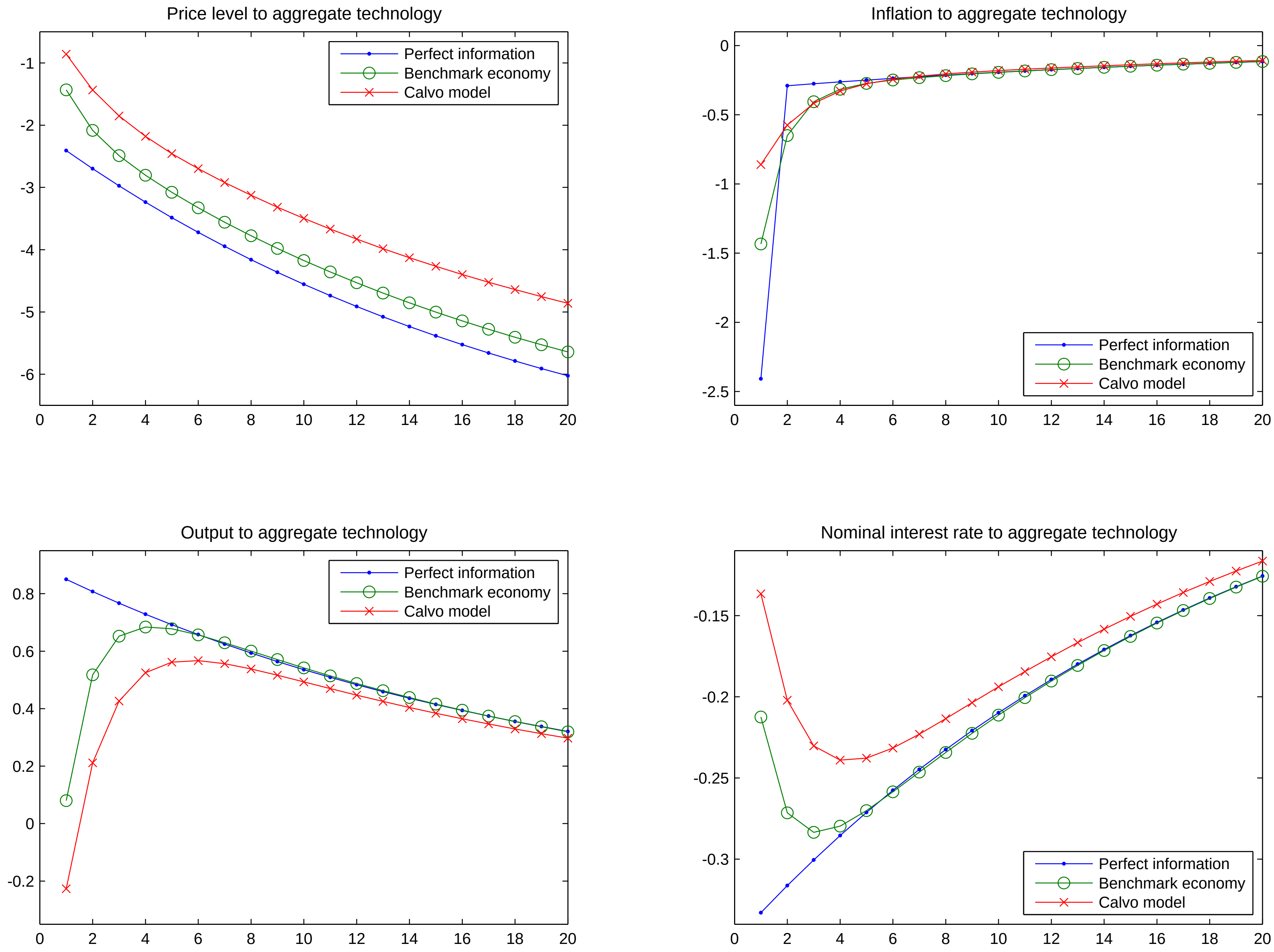

Figure 6: Impulse responses, benchmark economy and an economy with more real rigidity

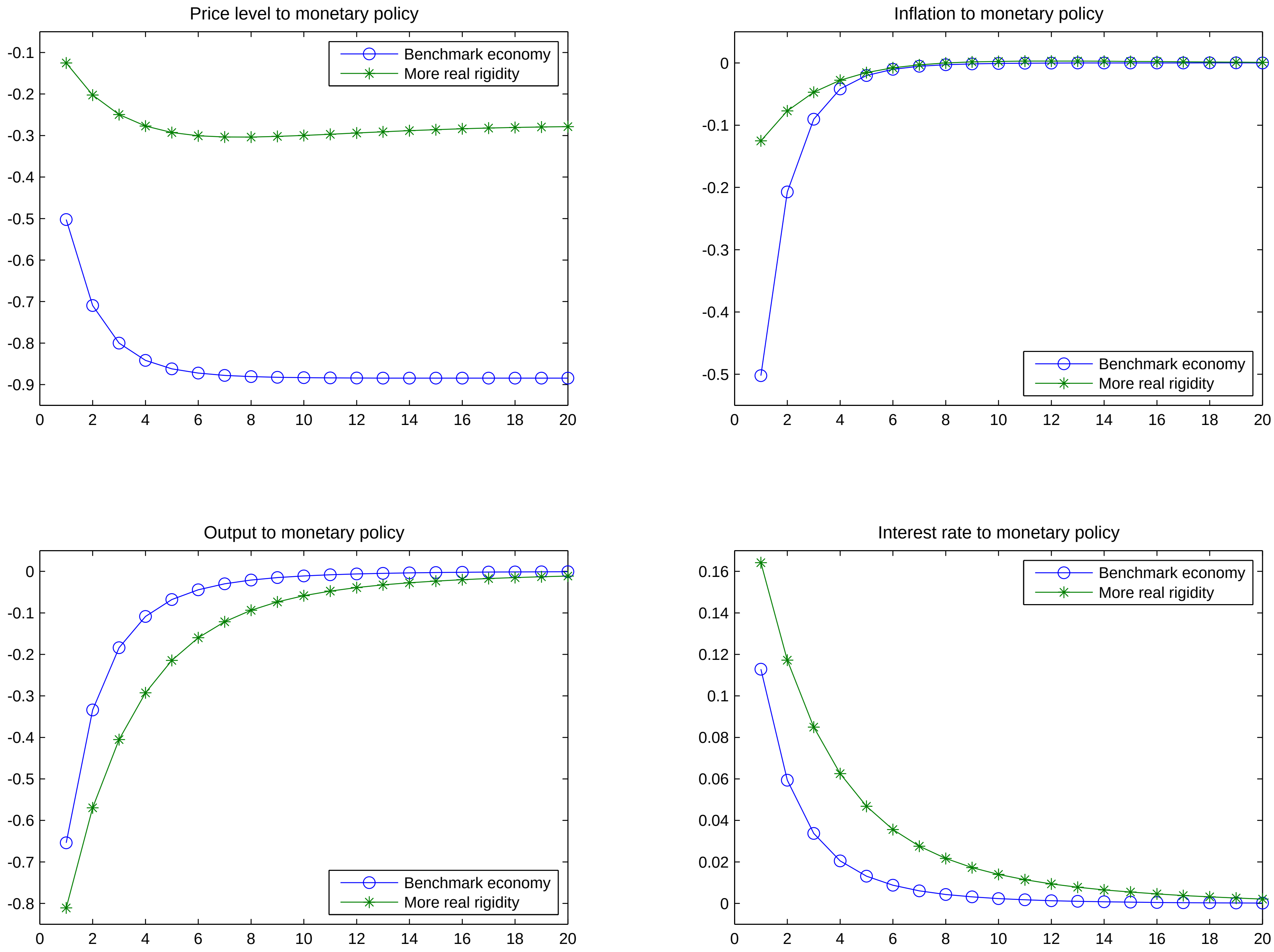

Figure 7: Impulse responses, benchmark economy and an economy with larger monetary policy shocks