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Monetary Policy and the Great Moderation

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Abstract

Increased focus on price stability by a discretionary central bank reduces output *and* price volatility in a model with rationally inattentive firms. The volatility reduction can be arbitrarily large, e.g., imply a 'Great Moderation', and is particularly pronounced when firms can process information almost perfectly. The model-implied vector auto-regressive (VAR) dynamics are consistent with the empirical observation that the dynamics before and after the Great Moderation differ mainly with respect to the variance of the VAR residuals. These results emerge in the model because increased focus on price stability by the central bank facilitates firms' information processing problem, thereby aligns expectations better with policy decisions. This reduces aggregate real and nominal volatility.

Keywords: optimal monetary policy, information frictions, output and price volatility.

JEL-Class.No.: E31, E52, D82

1 Introduction

The aim of this paper is to present a simple monetary policy model in which increased emphasis on price stabilization by the central bank is associated with a significant reduction in the variance of aggregate output and inflation.

The model thus suggests the existence of a causal link between two major macroeconomic events that are widely believed to have taken place around 1980 in a number of developed economies: (1) following the inflation experience of the 1970's many central banks seem to have increasingly focused on insuring price stability.[1] In the U.S. this policy shift is typically associated with the appointment of Paul Volcker as chairman of the Federal Reserve and the subsequently

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[1] Prominent proponents of this view include Clarida, Galì, and Gertler (2000), Cogley and Sargent (2001), and Orphanides and Williams (2005).

implemented disinflation program; (2) the volatility of aggregate output and inflation has fallen significantly around the beginning of the 1980's, a fact generally referred to as the 'Great Moderation' and first documented in McConnell and Quiros (2000) and Blanchard and Simon (2001).

The monetary model presented is a standard rational expectations model with maximizing firms and consumers. A relatively new feature of the model is that firms are assumed to face constraints on the amount of information they can process about aggregate shocks and policy decisions, as introduced by Sims (2003). This follows a recent line of research stressing the scarcity of information in decision making based on the observation that processing and incorporating information into decisions is not a costless process.[2] The paper emphasizes the information processing problems of price setting firms as these appear of particular relevance for the conduct of monetary policy.[3]

The presence of information processing frictions implies that the quality of firms' information about their profit maximizing price is endogenous and depends, amongst other things, on the conduct of monetary policy. Specifically, monetary policies that give rise to large volatility of firms' profit maximizing price also make it harder for firms to process information about their truly optimal price, i.e., result in larger information processing errors. Processing errors increase the variability of firms' information sets and lead to a misalignment between the private sector information and the actual policy stance. These misalignments generate unpredictable elements in firms' price setting decisions and thereby amplify the nominal and real volatility in the economy.

In the present setting discretionary maximization of social welfare by the monetary authority is shown to generate excessively volatile monetary policy decisions. Through the channels just described this leads to excessive real and nominal volatility of the aggregate economy. Excess volatility is thereby particularly high in economies in which firms can process information rather well and emerges because discretionary policy fails to incorporate the amount of information noise it generates. This is so because the variance of information noise is a function of the *average* volatility of policy decisions in response to shocks, while the discretionary policy problem consists of determining the strength of the policy reaction to a *specific* shock realization. Since the latter contributes little (nothing with continuous shock distributions) to the overall variance of policy, it is rational to ignore it under discretionary maximization.

Assigning a price stabilization goal to monetary policy reduces the volatility of policy decisions and allows (for an appropriate weight on this policy goal) to replicate optimal commitment policy via discretionary maximization, as is the case in Vestin (2006) in a sticky price economy. With commitment the monetary policy response to shocks is less activist which reduces the variance of firms' optimal price and thereby their processing errors. This is shown to unambiguously lower aggregate price and output volatility. Indeed, the volatility

[2] See Moscarini (2004), Reis (2003, 2006), or Adam (2007) for recent applications.

[3] Mackowiak and Wiederholt (2005) have shown such frictions to be important for explaining firms' pricing behavior.

reduction associated with a shift from discretionary maximization to optimal commitment policy turns out largest if firms can process information very well.

The paper also analyzes the model-implied vector autoregressive (VAR) dynamics for output, prices and the monetary policy instrument. Interestingly, a marginal improvement in monetary policy (less activist policy) can result in no change of the auto-regressive coefficients - including those coefficients describing the VAR's 'policy equation' - but manifest itself via reduced variance of the VAR residuals. This suggests that it is well possible that the findings of the empirical VAR literature, e.g., Canova and Gambetti (2005), Primiceri (2005), or Sims and Zha (2006), are caused by improvements in monetary policy. More generally, the model suggests that the variability of private sector information sets induced by information processing errors is one of the 'fundamental' shocks entering the residuals of empirical VARs and that the volatility of information sets may be crucially influenced by the conduct of policy.

Obviously, the model (trivially) predicts lower price and output volatility following reduced variance of standard shocks, i.e., of those shocks which are not processing errors. Therefore, the model is equally consistent with the notion that the findings of the empirical VAR literature are simply the result of reduced shock variance.

In the literature only a few mechanisms have been suggested through which increased focus on price stability by monetary policy can cause reduced output and price volatility.

Orphanides and Williams (2003, 2005) show that the usual trade-off can disappear in a setting in which the private sector is perpetually learning about the dynamics of the economy by extrapolating from past economic behavior. More stable prices then reduce the volatility of the private sector's price expectations and thereby the overall volatility in the economy.

Mechanisms relying on rational expectations tend to be based on the presence of multiple equilibria. Clarida, Galì, and Gertler (2000), for example, interpret the 1970's as a period in which the monetary policy rule allowed for sunspot fluctuations, while from start of the 1980's policy behavior created a determinate equilibrium. Surico and Benati (2007) explore the implications of this idea for the Great Moderation. Branch et al. (2007) provide an explanation of the Great Moderation using a model in which the private sector can choose the level of attention and show how the model can give rise to a high and low attention equilibrium with high and low aggregate volatility, respectively.

The present model differs from the existing literature by assuming fully rational agents and by giving rise to a unique equilibrium prediction for all policy parameterizations. Due to the latter, the present model can rationalize policy-based explanations of the Great Moderation in a strong sense, as it unambiguously predicts a fall in nominal and real volatility in response to an improvement in the conduct of monetary policy.

The paper is structured as follows. Section 2 starts out by presenting a simple static version of the model with imperfectly informed firms and derives a linear-quadratic approximation to the monetary policy problem. After introducing firms' information processing constraints in section 3, I illustrate the monetary policy implications in section 4. In particular, it is shown how the presence of information processing constraints causes discretionary monetary policy to generate excessive aggregate volatility and how increased focus on price stability reduces volatility. Section 6 extends the static model to an infinite horizon economy. It derives and discusses the model-implied VAR dynamics for output, prices, and monetary policy and compares it to findings of the empirical literature. A conclusion briefly summarizes the main findings.

2 The Basic Model

This section introduces a stylized monetary policy model and derives a linear-quadratic approximation to the optimal monetary policy problem. To simplify the exposition, I consider a static model and assume firms' information to be exogenous. Endogenous information sets will be introduced in section 3 when studying firms that optimally process information subject to processing constraints. Section 6 extends the setup to a fully dynamic model.

Households The household sector is described by a representative consumer choosing aggregate consumption Y and labor supply L to maximize

$$\begin{aligned} & U(Y) - \nu V(L) \\ & s.t. \\ 0 = {} & WL + \Pi - T - PY \end{aligned} \tag{1}$$

where W denotes a competitive wage rate, Π monopoly profits from firms, T lump sum taxes, and P the price index of the aggregate consumption good. The parameter $\nu > 0$ is a stochastic labor supply shifter with $E[v] = 1$ and induces variations in the efficient level of output. Households are fully informed about all relevant aspects in the economy.[4] Furthermore, $U' > 0$, $U'' < 0$, $\lim_{Y\to\infty} U'(Y) = 0$, $V' > 0$, $V'' > 0$ and $V'(0) < U'(0)$.

Firms The supply side of the economy is characterized by a continuum of monopolistically competitive firms $i \in [0,1]$ that can freely adjust prices but possess imperfect information about the aggregate shocks hitting the economy. Firm i produces an intermediate good Y^i with labor input L^i according to a linear production function of the form

$$Y^i = L^i$$

Intermediate goods enter into aggregate output Y according to a Dixit-Stiglitz aggregator

$$Y = \left(\int_{[0,1]} (Y^i)^{\frac{\theta-1}{\theta}} \mathrm{di} \right)^{\frac{\theta}{\theta-1}} \tag{2}$$

where the demand elasticity $\theta > 1$ is stochastic with mean $E[\theta] = \overline{\theta}$.

[4] Households, however, would only need to know the wage rate, the prices charged by firms, and their income.

Let $Y^i(P^i/P)$ denote household's utility-maximizing demand for product Y^i induced by (2) when firm i charges price P^i and the price for the aggregate good is P. The profit maximization problem of firm i is then given by

$$\max_{P^i} E\left[(1+\tau)P^iY^i(P^i/P) - WY^i(P^i/P)|I\right] \tag{3}$$

where τ denotes an output subsidy and I the firm's information set, which contains information about the labor supply shock ν, the demand shock θ, and monetary policy decisions. For simplicity it is assumed in equation (3) that all firms possess the same information set I.[5] It is shown in section 5.3 that this assumption is not essential for the results that follow. All that is required is that firms share *some* common (noisy) piece of information about fundamentals.

Monetary Policy The monetary policymaker is supposed to choose nominal demand and maximize the utility of the representative agent (1). To simplify the analysis, I consider a linear-quadratic approximation to the optimal policy problem, i.e., a quadratic approximation to social welfare and a linear approximation to the implementability conditions characterizing optimal private sector behavior.

Appendix A.1 shows that for an appropriate level of the output subsidy τ a quadratic approximation of the representative agent's utility function (1) is given by

$$-(y-y_n)^2 \tag{4}$$

where $(y-y_n)$ denotes the output gap. In particular, y is the average output across firms, i.e., $y = \int y(j)\mathrm{d}j$, and y_n the efficient output level.[6] The latter depends on the labor supply shock ν only and is assumed normal, i.e., $y_n \sim N(0,\sigma_y^2)$. In the remaining part of the paper I will refer to y_n as a *natural rate shock*.

Appendix A.1 also derives the linear approximation to firms' optimal price setting behavior implied by problem (3)

$$p(i) = E\left[p + \xi(y-y_n) + \varepsilon|I\right] \tag{5}$$

The previous equation describes profit maximizing price setting behavior by firm i conditional on available information I.[7] The variable $p(i)$ thereby denotes firm i's profit maximizing price and p the average price charged by firms. The parameter $\xi > 0$ in equation (5) indicates the sensitivity of firms' prices to the output gap and is given by

$$\xi = -\frac{U''(\overline{Y})\overline{Y}}{U'(\overline{Y})} + \frac{V''(\overline{Y})\overline{Y}}{V'(\overline{Y})}$$

where $\overline{Y}$ denotes the steady state output level. The profit-maximizing price also depends on the expected value of ε, which is a function of the price elasticity θ:

$$\varepsilon \sim -(\theta - \overline{\theta}).$$

[5] With heterogeneous information sets, firms have to form so-called higher-order beliefs, i.e., beliefs about other firms' information sets. This setting is discussed in Adam (2007).

[6] Lower case letters indicate that variables are expressed as percentage deviations from steady state values.

[7] It also incorporates the implementability conditions implied by the optimal labor-leisure choice of households, see appendix A.1 for details.

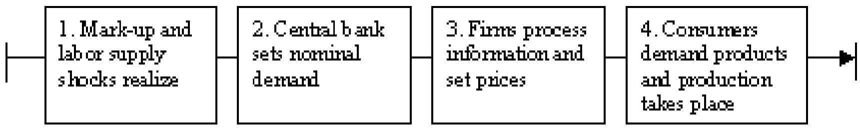

Figure 1: Sequence of events

Firms wish to charge a higher mark-up ($\varepsilon > 0$) whenever the price elasticity of demand θ falls below its mean $\overline{\theta}$. A positive value of ε thus reflects the fact that product demand has become less price-sensitive. We refer to ε as a *mark-up shock*.[8] It is assumed independent of the natural rate shock y_n and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$.

The central bank controls nominal spending q, which is defined as

$$q = y + p. \tag{6}$$

Combining this with equation (5), one obtains that in a symmetric price setting equilibrium

$$p = E\left[p^* | I\right] \tag{7}$$

where p^* denotes the optimal price under perfect information

$$p^* = q - y_n + \frac{1}{\xi}\varepsilon \tag{8}$$

The price p^* depends on the fundamental shocks and on monetary policy. As one would expect the optimal price increases (decreases) one-for-one with nominal demand (the efficient output level) and increases in response to mark-up shocks. The response to mark-up shocks is more pronounced the smaller is ξ. Low values of ξ indicate that marginal costs react only little to output. Therefore, to obtain the desired increase in mark-ups following a positive shock to ε, a larger real contraction is required, i.e., a larger increase in nominal price ceteris paribus.

Summarizing the previous results, the monetary policy problem consists of choosing q so as to maximize (4) subject to (6)-(8).

Timing of Events Figure 1 illustrates the sequence of events taking place in the economy. Namely, after the stochastic disturbances (y_n, ε) have realized the central bank sets the desired level of nominal demand. Firms then process information about the shocks and the central bank's policy choice, as is explained in the next section, then simultaneously determine prices. Finally, consumers demand products for consumption and production takes place.

[8] This follows Woodford (2003). Galí et al. (1999) refer to it as a 'cost-push shock'.

3 Optimal Information Processing

This section consider firms that in addition to choosing prices also determine what information to process. Firms choose prices and information structures to maximize their profits but face a constraint on the total amount of information that can be processed each period, as in Sims (2003). The section first derives results formally and then offers a more intuitive interpretation of the findings. The aim of this section is to show how in a setting with information processing constraints monetary policy gives rise to a 'coordination effect'.

A quadratic approximation of the firm's profit is given by

$$-E\left[(p-p^{*})^{2}\,|I\right] \tag{9}$$

The firm chooses p and I so as to maximize (9) subject to an information processing constraint

$$H\left(p^{*}\right)-H\left(p^{*}|I\right)<K \tag{10}$$

where $H(p^{*})$ denotes the entropy about p^{*} before processing information and $H\left(p^{*}|I\right)$ the entropy after information processing.[9] Intuitively, entropy is a measure of the uncertainty about a random variable. Stated in these terms, the processing constraint (10) provides a bound $K\in[0,\infty]$ on the maximum uncertainty reduction about p^{*} that can be achieved by processing information. The bound K is thereby measured in 'bits', i.e., number of zeros and ones, per unit of time. For $K=0$ no uncertainty reduction is possible, i.e., firms cannot process information at all, while for $K\to\infty$ firms process information perfectly.

For a given information structure I, the optimal price choice is $p=E\left[p^{*}|I\right]$ so that the expected loss associated with information structure I is equal to $Var(p^{*}|I)$. Choosing an optimal information structure thus amounts to minimizing $Var(p^{*}|I)$ subject to the constraint that the conditional entropy $H(p^{*}|I)$ cannot fall below the threshold defined by the processing constraint (10). Shannon (1948) shows that Gaussian variables minimize the conditional variance for a given entropy level.[10] It is therefore optimal to choose $p^{*}|I$ to be Gaussian so as to achieve an infimum value for $Var(p^{*}|I)$. Since p^{*} is Gaussian (due to the assumption of Gaussian fundamental shocks), the posterior $p^{*}|I$ is also Gaussian, provided the information I available to firms takes the form a signal s with representation

$$s=p^{*}+\eta, \tag{11}$$

where $\eta\sim N(0,\sigma_{\eta}^{2})$ denotes the firm's information processing noise, which is independent of all other random variables in the model. The variance of the processing noise η is thereby given by[11]

$$\sigma_{\eta}^{2}=\frac{1}{e^{2K}-1}Var(p^{*}). \tag{12}$$

[9] The entropy $H(X)$ of a continous random variable X is defined as $H(X)=-\int {}_2\log(x)p(x)dx$ where $p(x)$ is the probability density function of X and where the convention is to take ${}_2\log(x)p(x)=0$ when $p(x)=0$.

[10] Shannon solves the dual problem of maximizing entropy for a given variance.

[11] This follows from equation (10), the fact that the entropy of a Gaussian random variable is equal to one half its log variance plus a constant, and the updating formula for the variance of normal variables, i.e., $Var(p^{*}|s)=Var(p^{*})-Var(p^{*})^{2}/(Var(p^{*})+\sigma_{\eta}^{2})$.

This is the infimum variance such that the information structure $I = \{s\}$ still satisfies the constraint (10). In particular, choosing a lower variance would imply that firms process more than K bits of information. As one would expect, the processing noise falls and the information structure becomes more informative, if firms' processing capacity K increases.

Given the optimal information structure, the firm's optimal price is then

$$\begin{aligned} p &= E[p^*|I] \\ &= E[p^*|s] \\ &= k \cdot s, \end{aligned} \tag{13}$$

where the Kalman gain $k \in [0, 1]$ is

$$k = \frac{Var(p^*)}{Var(p^*) + \sigma_\eta^2} = (1 - e^{-2K}). \tag{14}$$

The Kalman gain k is a useful summary statistic indicating how well agents can process information about their environment. For $k = 0$ firms receive no information since $\sigma_\eta^2 = \infty$. Conversely, for $k = 1$ firms observe perfectly since $\sigma_\eta^2 = 0$. At intermediate values of k the variance of the observation noise η is positive and decreases with k.

Coordination Effect The expression for the variance of the processing noise in equation (12) shows that a more variable full information price p^* causes firms' to make larger processing errors. Intuitively, this occurs because information about a more variable environment is harder to track for any given capacity to process information. Since the noisy signal s enters with proportionality factor k into firms' prices, see equation (12), a more variable full information price causes firms' optimal price choice p to increasingly deviate from the full information price. This is formally summarized in the subsequent result:

Lemma 1 *A more variable optimal price p^* increases firms' pricing errors:*

$$Var(p - p^*) = (1 - k)\, Var(p^*)$$

Clearly, the variability of the full information price p^* depends on the conduct of monetary policy, see equation (8). Therefore, if monetary policy causes the full information price p^* to be very volatile, firms' will make larger processing errors, implying that firms' price choices will be less predictable for the monetary policymaker.[12] The endogeneity of the information structure thus suggests that monetary policy influences the economy along a new margin: by making p^* less variable, policy can make the private sectors' price choices more predictable, i.e., better coordinated around p^* defined in equation (8). I call this attenuation the **'coordination effect'**.

Importantly, the coordination effect not only reduces unpredictable movements in prices, but also unpredictable movements in the output gap. From the

[12] The processing error η is unpredictable for policymakers as it realizes after monetary policy has been set, see section 2.

definition $y = q - p$ and the results for firms' optimal price and information choice derived above one obtains

$$y - y_n = \left[(1-k)\, q - (1-k) y_n - \frac{k}{\xi} \varepsilon \right] - k\eta \tag{15}$$

which shows that, ceteris paribus, smaller information processing errors η also reduce the variance of unpredictable output gap movements. Yet, whether or not a policy change that reduces processing errors also reduces the volatility of the output gap depends also on how the policy shift affects the volatility of terms in the square bracket in equation (15). This issue is investigated in the next section.

4 Policy Implications of Processing Limitations

This section shows how the presence of the coordination effect influences the incentives of monetary policy to react to mark-up and natural rate shocks.

Under discretionary policymaking it turns out optimal to ignore the coordination effect. This is shown to result in suboptimally high real and nominal volatility, when compared to the outcome under commitment. Increased focus on price stability is shown to reduce aggregate volatility (real and nominal) and even allows to implement the commitment allocation through discretionary policymaking.

Throughout this section it is assumed that policymakers observe fundamental shocks without noise and that firms' processing capacity is exogenously given, i.e., independent of the way policy is conducted. Both assumptions will be relaxed in section 5, which shows that the main results extent to the case in which the central bank itself has limited capacity to process information and to the case with endogenous choice of firms' processing capacity.

4.1 Monetary Policy with Commitment

This section determines optimal monetary policy under commitment, which will serve as a benchmark for evaluating the distortions generated by discretionary policymaking. Under commitment the policymaker determines contingent policy taking into account that the variability of the full information price implied by policy decisions affects the information errors of the private sector.

Summarizing results derived in sections 2 and 3, the policy problem under commitment can be expressed as

$$\max_{q} -E[(y - y_n)^2] \tag{16}$$

$$s.t.:$$

$$y - y_n = \underbrace{(1-k)\, q}_{\text{standard effect}} - (1-k) y_n - \frac{k}{\xi}\varepsilon - \underbrace{k\eta}_{\text{coordination effect}} \tag{17}$$

$$\sigma_\eta^2 = \frac{1-k}{k} Var(p^*) \tag{18}$$

$$p^* = q - y_n + \frac{1}{\xi}\varepsilon \tag{19}$$

with equation (17) describing the behavior of the output gap as a function of the policy choice, the fundamental shocks, and the realization of the processing error, and equation (18) determining the variance of the processing errors.

To gain insights into the policy problem, it is useful to distinguish in equation (17) between the 'standard effects' of monetary policy and the 'coordination effect', as discussed before in section 3. The 'standard effect' of monetary policy is that arising in traditional imperfect information models, e.g., Lucas (1972, 1973). It predicts that an increase in nominal demand by one unit moves output (and the output gap) by $(1-k)$ units because the presence of processing constraints implies that firms react only with strength $k \in [0,1]$ to nominal demand movements. The standard effect of monetary policy can be used to amplify or dampen the natural rate and mark-up shocks entering on the r.h.s. of equation (17), but cannot be used to eliminate the effects of the processing error η, which only realizes after monetary policy has been determined. Monetary policy influences the behavior of the output gap through the standard effect and the coordination effect and has to trade-off the effects on the output gap arising from both policy channels. Below I discuss the nature of this trade-off in response to natural rate and mark-up shocks.

Consider the case of a natural rate shock first. In response to these shocks the output gap can be fully closed simply by moving nominal demand one for one with the natural rate shock. This stabilizes the full information price level, thereby eliminates all processing errors (achieves maximum coordination) but also fully offsets the effects of natural rate shocks in equation (17) through the standard effects of monetary policy. Natural rate shocks, thus, do not generate a trade-off between the standard and the coordination effects of monetary policy. Note that closing the output gap is equivalent to stabilizing the price level in response to natural rate shocks.

The situation differs radically for mark-up shocks. Ignoring the coordination effect in equation (15) for a moment, policy can stabilize the output gap through the standard effects of policy by setting

$$q = \frac{k}{(1-k)}\frac{1}{\xi}\varepsilon \tag{20}$$

i.e., by appropriately nominally accommodating in response to positive mark-up shocks. The coordination effect in isolation, however, suggests that closing the output gap requires nominally contracting in response to positive mark-up shocks, i.e., to set

$$q = -\frac{1}{\xi}\varepsilon$$

Mark-up shocks thus generate a trade-off implied, which is illustrated in figure 2. The figure depicts the required amount of contraction/accommodation to a positive mark-up shock ($\varepsilon = 1$) as a function of the processing index k. The optimal reaction to mark-up shocks, which takes into account standard and coordination effects, is a convex combination of the two policies shown in the figure. The following proposition shows that the two policy incentives balance each other precisely at the point where it is optimal not to react to mark-up shocks at all.

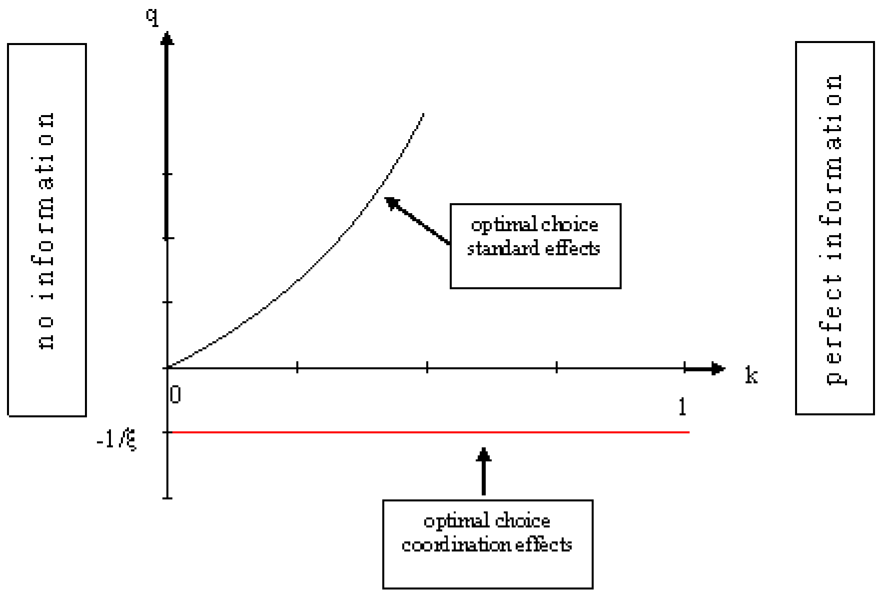

Figure 2: Policy trade-off for mark-up shocks

Proposition 2 *In a rational expectations equilibrium with optimal information processing by firms, optimal monetary policy under commitment is to set*

$$q = y_n \tag{21}$$

The implied variability of the output gap and prices are

$$E[(y - y_n)^2] = \frac{k}{\xi^2}\sigma_\varepsilon^2 \tag{22}$$
$$E[p^2] = \frac{k}{\xi^2}\sigma_\varepsilon^2$$

Note that the previous proposition has the slightly non-intuitive implication that output gap variability increases, i.e., welfare decreases, when firms can process information better (larger values of k). Increased processing capacity has two opposing effects in the model. On the one hand, it reduces the size of processing errors, which enhances welfare; on the other hand, it allows firms to observe aggregate mark-up shocks better. As a result, firms' react more strongly to these welfare reducing shocks. The latter effect dominates the first, causing output and price variability to linearly increase with firms' ability to process information (k).

4.2 Discretionary Policy Activism

This section considers a discretionary policymaker determining monetary policy at the time of implementation, i.e., only after the mark-up and natural rate shocks have realized, see figure 1. A policymaker that determines policy after economic disturbances have materialized can safely ignore the coordination effects of policy decisions, i.e., can treat σ_η^2 in the policy problem (16) as given. This is rational because the policy reaction to a particular shock realization contributes little (nil with continuous shock distribution) to the ex-ante variability of the full information price p^*. As the following proposition shows, ignoring the coordination effects of policy can have stark policy implications:

Proposition 3 *In a rational expectations equilibrium with optimal information processing by firms, optimal discretionary policy is*

$$q = \frac{k}{(1-k)}\frac{1}{\xi}\varepsilon + y_n \tag{23}$$

The variance of the output gap and the price level implied by discretionary policy is

$$E[(y-y_n)^2] = \frac{k}{(1-k)\xi^2}\sigma_\varepsilon^2 \tag{24}$$
$$E[p^2] = \frac{k}{(1-k)^2\xi^2}\sigma_\varepsilon^2$$

Ignoring the coordination effect, discretionary monetary policy nominally accommodates mark-up shocks, as suggested by figure 2. Accommodative policy causes q and ε to move in the same direction, thereby increasing the overall variability of the full information price p^*, see equation (8). Increased variability of the full information price increases firms' processing errors and thereby the variability of prices *and* output compared to the case with commitment. This result holds independently of the model parameterization.

The increase in aggregate volatility is particularly pronounced when firms can process information almost perfectly (k close to 1). While an increase in the processing index reduces firms processing errors (ceteris paribus), it also results in stronger nominal accommodation of mark-up shocks by policy, as suggested by the upper curve in figure 2. The overall effect is an increase in processing errors resulting in higher nominal and real volatility. In the limit, as $k \to 1$, real and nominal volatility increase without bound compared to the case with policy commitment.[13]

From propositions 2 and 3 it is clear that discretionary policy approaches the commitment solution as $k \to 0$. For low levels of firms' processing capacity, lack of monetary commitment would thus not entail large utility costs due to increased aggregate volatility. Yet, if firms' processing capacity increases over time, a possible monetary commitment problem would become increasingly apparent. The increase in aggregate volatility from the 1960's to the 1970's experienced in a number of developed countries might thus be interpreted as the result of a monetary commitment problem and of an underlying trend that would allow firms to increasingly process aggregate information. By the beginning of the 1980's the monetary commitment problem has become so apparent that policymakers were forced to actively look for a solution to the commitment problem. This issue is discussed in the next section.

[13] Note that the discretionary policy reaction to natural rate shocks remains is optimal because such shocks do not generate a trade-off between the coordination effects and standard effects of policy.

4.3 Price Stability as a Policy Objective

The inefficiently high nominal and real variability resulting from discretionary policy in the previous section is the result of inefficiently high volatility of the full information price p^* that emerges ex-post when treating the degree of coordination of private sector expectations as given. This suggests that a discretionary policymaker might endogenize the coordination effects generated by policy, if assigned also a price stabilization objective. This motivates consideration of the following policy problem

$$\max -E\left[(y-y_n)^2+\omega p^2\right] \tag{25}$$

$$s.t.$$

$$p = k\left(q - y_n + \frac{1}{\xi}\varepsilon + \eta\right) \tag{26}$$

$$y - y_n = (1-k)\,q - (1-k)y_n - \frac{k}{\xi}\varepsilon - k\eta$$

$$\sigma_\eta^2 \text{ given}$$

where $\omega \geq 0$ is the weight attached to price stability in the objective function. Note that the policymaker continues to act under discretion, i.e., treats the variance of the processing error as exogenous. The lemma below summarizes the main results.

Lemma 4 *Consider discretionary monetary policy with weight $\omega \geq 0$ on stabilizing prices. The variance of the output gap and prices is decreasing in ω for $0 \leq \omega < \frac{1-k}{k}$. For $\omega = \frac{1-k}{k}$ discretionary monetary policy is identical to the policy under commitment.*

The model thus predicts that, independently of the precise model parameterization, increased focus on price stability reduces aggregate price and output gap volatility under discretionary policymaking. Moreover, for a sufficiently high weight on price stability, discretionary monetary policy becomes identical to optimal policy under policy commitment. The latter suggests that a policy regime shift from discretionary maximization of social welfare towards discretionary maximization of an optimally weighted sum of social welfare and price stability can result in a large reduction of aggregate nominal and real volatility, i.e., give rise to a policy-induced 'Great Moderation' effect. It follows from propositions 2 and 3 that the volatility reduction associated with such a shift is particularly pronounced if firms can process information very well and becomes arbitrarily large as firms' processing capacity $k \to 1$.

5 Robustness

This section relaxes the assumption that the central bank can process information about mark-up and natural rate shocks perfectly. In addition, it studies the effects of endogenizing firms' choice of processing capacity and of introducing idiosyncratic elements in firms' processing errors.

5.1 Central Bank Processing Limitations

This section considers a central bank facing information processing limitations of the same kind as previously introduced for firms. In particular, the central bank's choice for nominal demand q is now assumed to be subject to an information flow constraint

$$H(q^*) - H(q^*|q) < K^{CB}$$

where $K^{CB} \geq 0$ denotes the central bank's capacity to process information (expressed in bits per unit of time) and

$$q^* = a\varepsilon_t + by_{n,t} \quad (27)$$

the optimal decision under perfect central bank information. The coefficients a and b remain to be determined. As with firms, the central bank's optimal choice of information structure is a signal of the form

$$s^{CB} = q^* + \eta^{CB}$$

where $\eta^{CB} \sim N(0, \sigma^2_{\eta,CB})$ is a processing error limiting the information about q^* contained in s^{CB}. The processing noise η^{CB} is independent of all other random variables and has (infimum) variance

$$\sigma^2_{\eta,CB} = \frac{1-k^{CB}}{k^{CB}} Var(q^*) \quad (28)$$

where $k^{CB} = 1 - e^{-2K^{CB}} \in [0,1]$ denotes the central bank's processing index. To describe central bank behavior I consider linear policies of the form

$$q = E\left[q^*|s^{CB}\right] = k^{CB} s^{CB} \quad (29)$$

Since policy can freely choose the reaction coefficients a and b in equation (27), choosing the reaction coefficient with respect to the signal s^{CB} in equation (29) is without loss of generality.[14]

Firms' behavior remains described by the equations derived in section 4, which allows to express the central bank's maximization problem under commitment as

$$\max_{a,b} -E[(y-y_n)^2] \quad (30)$$

$$\begin{aligned} s.t. &: \\ y - y_n &= (1-k)\, q - (1-k) y_n - \frac{k}{\xi}\varepsilon - k\eta \\ \sigma^2_\eta &= \frac{1-k}{k} Var\left(p^*\right) \\ p^* &= q - y_n + \frac{1}{\xi}\varepsilon \\ q &= k^{CB}\left(a\varepsilon + by_n + \eta^{CB}\right) \\ \sigma^2_{\eta,CB} &= \frac{1-k^{CB}}{k^{CB}} Var(a\varepsilon + by_n) \end{aligned}$$

[14] The optimality of a linear reaction function follows from the linear quadratic nature of the policy problem, see problem (30) below.

Substituting the constraints into the objective function and taking first order conditions delivers that optimal policy is given by

$$a = 0$$
$$b = 1$$

This together with equation (27) shows that optimal policy displays certainty equivalence. The implied volatility of the output gap and the price level are

$$E[(y - y_n)^2] = \frac{k}{\xi^2}\sigma_\varepsilon^2 + (1-k)\,(1-k^{CB})\sigma_{y_n}^2$$
$$E\left[p^2\right] = \frac{k}{\xi^2}\sigma_\varepsilon^2 + k(1-k^{CB})\sigma_{y_n}^2$$

Unlike in the case with perfect central bank information, variations in potential output y_n now have an effect on the output gap to the extent these variations are neither observed by the central bank nor the private sector. Moreover, variations in the output gap perceived by firms but not by the central bank now lead to movements in the price level, as firms try to respond to variations in the natural rate of output that the central bank fails to nominally accommodate due to processing limitations.

With discretionary policy, the policy problem is identical to (30), except that the policymaker treats σ_η^2 as independent of policy decisions. Discretionary optimal policy is to set

$$a = \frac{k}{1-k}\frac{1}{\xi}$$
$$b = 1$$

and displays again certainty equivalence. The implied variance of the output gap and inflation are

$$E[(y - y_n)^2] = \frac{\left(1 - k(1-k^{CB})\right)}{1-k}\frac{k}{\xi^2}\sigma_\varepsilon^2 + (1-k)(1-k^{CB})\sigma_{y_n}^2$$
$$E\left[p^2\right] = \frac{\left(k^{CB} + (1-k^{CB})(1-k)^2\right)}{(1-k)^2}\frac{k}{\xi^2}\sigma_\varepsilon^2 + k(1-k^{CB})\sigma_{y_n}^2$$

and highlight that, as before, the volatility increase due to discretionary policy is again particularly pronounced when firms can process information almost perfectly.

5.2 Endogenizing Firms' Capacity Choice

This section shows that with discretionary policy increased focus on price stability continues to imply lower price and output gap volatility, even if firms choose their information processing capacity optimally.

To model firms' choice of processing capacity, a game with the following sequence of events is considered. First, monetary policy is assigned the weight

$\omega \geq 0$ on price stabilization in its objective function (25). Second, firms simultaneously choose their processing index k_i ($i \in [0, 1]$) taking as given the choice of other firms. The costs of acquiring capacity k_i are thereby described by a cost function $c(k_i)$ with strictly positive first and second derivatives. Third shocks realize, thereafter the policymaker (discretionarily) determines and implements monetary policy. Fourth, firms process information about shocks and policy decisions and set their prices. Finally production and consumption takes place.

I am interested in the effects of increased emphasis on price stabilization by the central bank (larger ω) on aggregate volatility. The following proposition summarizes the main result.

Proposition 5 *Suppose firms' capacity choice problem has an interior solution and for each policy weight ω there exists a unique symmetric Nash equilibrium $k^*(\omega)$ for firms' capacity choice. A marginal increase in ω then results in a new symmetric Nash equilibrium with lower output gap and lower price volatility, for all ω sufficiently small.*

As is the case with exogenous processing capacity, increased focus on price stabilization initially lowers aggregate nominal and real volatility. The assumptions stated at the beginning of the proposition thereby simply insure that meaningful comparative statics are associated with a change in the policy weight ω. The proof is provided in Appendix A.2. The crucial step in the proof consists of showing that firms' equilibrium choice for processing capacity is decreasing in the policy weight ω, i.e., that firms choose to observe mark-up shocks less precisely, if the policymaker focuses more on price stability. Reduced processing capacity dampens firms' price response to (their imperfect signal about) mark-up shocks and this contributes further to reducing price and output gap volatility. The reduction in aggregate volatility is thus even larger once one allows firms to optimally choose their information processing capacity.

5.3 Idiosyncratic Processing Errors

This section relaxes the common knowledge assumption, i.e, the assumption that the information processing errors in equation (11) are identical across firms. While it appears reasonable to assume that there is some common component in firms' processing errors, e.g., because firms all read the same (noisy) newspaper reports, it is equally reasonable to postulate the presence of idiosyncratic processing errors. This section shows that the previous analysis readily extends to a setting with such idiosyncratic information sets.

Suppose that equation (11) is replaced by

$$s^i = p^* + \eta^c + \eta^i$$

where η^c denotes a processing error common to all firms, while η^i is firm i's idiosyncratic processing error that is assumed independent of η^c and p^*. Let a share $\alpha \in [0, 1]$ of the total noise be common to all firms with the remaining share being idiosyncratic, i.e.,

$$\sigma^2_{\eta^c} = \alpha \sigma^2_\eta$$
$$\sigma^2_{\eta^i} = (1 - \alpha) \sigma^2_\eta$$

where the total observation noise σ_η^2 is given by (12). For $\alpha = 1$ one recovers the common knowledge setup studied thus far in the paper, while for $\alpha = 0$ one obtains a setting with purely idiosyncratic noise. The latter has been studied in Adam (2007) where it is shown that a firm's optimal price in the presence of idiosyncratic information noise is given by[15]

$$p(i) = \xi E\left[\sum_{m=0}^{\infty}(1-\xi)^m \left(p^{*(m)}\right)|s^i\right] \tag{31}$$

with $p^{*(m)}$ denoting the so-called m-th order expectation of the optimal price p^*, which can be defined recursively as

$$\begin{aligned} p^{*(0)} &= p^* \\ p^{*(m)} &= \int_{i\in[0,1]} E\left[p^{*(m-1)}|s^i\right] \text{di} \end{aligned}$$

As shown in appendix A.3 the higher-order expectations are given by

$$E^i\left[p^{*(m)}\right] = k\left(k(1-\alpha)+\alpha\right)^m s^i$$

so that equation (31) implies the aggregate price level to be given by

$$\begin{aligned} p &= \int_{i\in[0,1]} p(i)\text{di} \\ &= \frac{\xi k}{1-(1-\xi)\left(k(1-\alpha)+\alpha\right)}\left(q - y_n + \frac{1}{\xi}\varepsilon + \eta^c\right) \end{aligned} \tag{32}$$

For $\alpha = 1$ this expression simplifies to the price level expression derived for the common knowledge setting studied thus far, see for example equation (26). For $\alpha > 0$ it differs from the common knowledge expression only by a proportionality factor and the fact that η^c instead of η enters as the noise term. Since the variance of η^c is proportional to the variance of η, the qualitative behavior of the aggregate price level does not change when introducing idiosyncratic processing errors. In particular, overly activist discretionary policy will still lead to excessive (real and nominal) aggregate volatility, with the volatility increasing without bound as $k \to 1$.

6 Infinite Horizon Economy

This section extends the static setting considered thus far to an infinite horizon economy with persistent shocks, showing how the previous findings naturally extend to an intertemporal setup. A major objective of this section is to demonstrate that the dynamic model is consistent with a number of stylized facts about aggregate volatility that have been documented in the empirical literature on the Great Moderation. This literature seems to largely agree on the following set of facts:

[15] This follows from equation (24) in Adam (2007), which is valid independently of the assumed information structure.

Fact 1: Aggregate real and nominal volatility pre 1984 was significantly higher than thereafter. This is the case for the United States but also for a number of other industrialized economies (McConnell and Quiros (2000), Blanchard and Simon (2001)).

Fact 2: VAR evidence for the pre and post 1984 periods indicates that the different levels of volatility are mainly the result of larger VAR residuals and only to a minor extent the result of a change in the autoregressive coefficients of the VAR (Sims and Zha (2006), Primiceri (2005)).[16]

Fact 3: When exchanging the monetary policy equation of VAR estimates in the pre and post 1984 period while keeping unchanged the remaining equations of the VAR equations as well as the VAR residuals, then this leads to virtually unchanged economic outcomes (Canova and Gambetti (2005), Primiceri (2005)).

Fact 4: It appears that only a negligible share of the variance reduction following 1984 is due to reduced variance of monetary policy shocks (Sims and Zha (2006)).

The next section introduces the dynamic model and derives optimal policy under discretion and commitment. Section 6.2 shows that the dynamic model predicts all four facts mentioned above to be consistent with an improvement in monetary policy, i.e., with a monetary policy regime shift away from purely discretionary conduct of policy (some way) towards the commitment solution.

6.1 Dynamic Model

This section introduce the infinite horizon economy and derives optimal monetary policy. As in most of the earlier setup, I abstract from monetary policy shocks, i.e., monetary policy information processing errors. This implies that the model is consistent from the outset with Fact 4 mentioned in the previous section.

Infinite Horizon Economy Consider an infinite horizon economy without capital in which the firm side is identical to the one described in section 2. The representative consumer now maximizes

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(U(Y_t)-\nu_t V(L_t)+\chi_t D(M_t/P_t)\right)\right] \tag{33}$$

$$s.t.$$

$$P_tY_t+B_t+M_t=W_tL_t+\Pi_t-T_t+R_{t-1}B_{t-1}+M_{t-1}$$

where M_t denotes nominal money balances, B_t nominal bonds, R_t the nominal interest rate, and χ_t a money demand shock with $E[\chi_t]=\chi$. Previously used variables retain their definition from section 2. To simplify the analysis I will consider a 'cashless' limit economy ($\chi\to 0$), which allows to abstract from the utility implications of monetary policy that operate through level of real cash balances.

The government flow budget constraints is

$$B_t+M_t=M_{t-1}+B_{t-1}R_{t-1}-T_t+\tau Y_t$$

[16] Cogley and Sargent (2005) argue that changes in the autoregressive coefficients may be also relevant but are statistically harder to detect.

The government chooses an efficient output subsidy τ to eliminate the steady state distortions from monopolistic competition. In addition, it chooses a sequence of conditional debt and lump sum tax plans (B_t, T_t), which are assumed to give rise to a bounded path for the real value of outstanding government claims. The latter implies that Ricardian equivalence holds so that the fiscal choices for (B_t, T_t) do not affect the equilibrium outcome. One can thus abstract from fiscal policy when analyzing the conduct of monetary policy.

LQ-Approximation of the Monetary Policy Problem I now derive a linear-quadratic approximation to the monetary policy problem for the dynamic economy. As in the static model, monetary policy maximizes social welfare, where the policy choices are subject to a number of implementability constraints. The constraints consist of firms' price setting equation and the laws governing firms' beliefs under optimal information processing. I continue to assume that the monetary policy 'instrument' is nominal demand rather than the nominal interest rate or some monetary aggregate. This is motivated by analytical convenience and to increase comparability with the findings from the static model. The equilibrium outcomes remain unaffected by this assumption.[17]

A quadratic approximation of the utility of the representative household (33) is given by

$$-E_0[\sum_{t=0}^{\infty} \beta^t (y_t - y_{n,t})^2]$$

where $y_{n,t}$ denotes again the efficient output level.[18] For simplicity, I consider the limiting case $\beta \to 1$, which allows to express household utility and thus the monetary policy objective as

$$-E\left[(y_t - y_{n,t})^2\right] \tag{34}$$

where $E[\cdot]$ denotes the unconditional expectations operator.

For a given information set, the linear approximation to profit maximizing price setting behavior by firms continues to be described by equations (7) and (8), i.e.:

$$p_t = E\left[q_t - y_{n,t} + \frac{1}{\xi}\varepsilon_t | I_t\right] \tag{35}$$

where I_t denotes firms' information set at time t. The stochastic disturbances $x_t' = (\varepsilon_t, y_{n,t})'$ thereby evolve according to[19]

$$x_t = \rho x_{t-1} + v_t \tag{36}$$

[17] This is so because the paper effectively considers Ramsey allocation problems with and without commitment and does not address issues of equilibrium implementation. As can be easily shown, the equilibrium path for prices, output gap and nominal demand derived below imply a corresponding path for the nominal interest rate and money demand. The choice of policy instrument can matter, however, for implementing a desirable allocation as the unique equilibrium outcome. This issue is beyond the scope of this paper.

[18] See equation (55) in appendix A.1 for the definition of y_n.

[19] At the cost of additional notational complexity one could easily allow for shocks with different persistence.

where $\rho \in (-1, 1)$ and $v_t \sim iiN(0, \Sigma_v)$ with

$$\Sigma_v = \begin{pmatrix} \sigma_{v_1}^2 & 0 \\ 0 & \sigma_{v_2}^2 \end{pmatrix}.$$

At the time when monetary policy is implemented, the economy is characterized by the state variables x_t and $x_{t|t-1}$ where the latter denote firms' $t-1$ expectations of x_t, see the timeline in figure 1. In general, optimal policy should condition its choices on x_t as well as on $x_{t|t-1}$. Yet, nominal demand variations that depend on private sector beliefs are fully perceived by firms, therefore do not generate real effects and can be safely ignored. This allows considering policy of the form

$$q = a \cdot \varepsilon_t + b \cdot y_{n,t} \tag{37}$$

as in the static setup. Given policy of this form, firms' optimal signal (11) can be written as

$$s_t = H' x_t + \eta_t \tag{38}$$

where

$$H' = (\frac{1}{\xi} + a, b - 1)$$

The state equations (36) and the observation equation (38) together define firms' Kalman filtering problem. Letting $x_{t|t}$ denote agents' time t estimate of x_t, appendix A.5 shows that the Kalman filter updating equations imply the following evolution of firms' beliefs under optimal information processing:

$$\begin{aligned} H' x_{t|t} &= (1-k) H' x_{t|t-1} + k s_t \\ &= \rho (1-k) H' x_{t-1|t-1} + k s_t \end{aligned} \tag{39}$$

This filtering problem has two non-standard features. First, the observation equation H depends on the policy parameters a and b. This is due to firms being able to *choose* which variables to observe through the information channel. Firms' choice thereby depends on the policy pursued by the central bank. Second, the variance of the observation noise η_t in equation (38) is endogenous since the information flow generated by the signal s_t is constrained by firms' information processing capacity.[20]

Defining the state of the economy as $\Xi_t = \left(x_t', H' x_{t|t} \right)$ and using the previous results, one can write the evolution of the state as

$$\Xi_t = A \cdot \Xi_{t-1} + B \omega_t \tag{40}$$

where the state innovation vector

$$\omega_t' = (v_{1t}, v_{2t}, \eta_t)' \tag{41}$$

consists of the innovations to fundamentals (price mark-up, natural output level) *and* of innovations to firms information sets (η_t) that are the result of processing errors.[21] The latter will be crucial for interpreting the findings that follow. Explicit expressions for the matrices A and B are given in appendix A.4.

[20]In particular, $VAR(\eta_t) = \frac{1-k}{k} H' P_{t|t-1} H$ where $P_{t|t-1}$ denotes agents' uncertainty about x_t prior to observing s_t.

[21]For the moment, I abstract from the money demand shocks χ_t in the state vector, as these do not affect the equilibrium dynamics of the variables under consideration.

The equilibrium price, the output gap, and monetary policy are functions of the state vector

$$\begin{pmatrix} p_t \\ y_t - y_{n,t} \\ q_t \end{pmatrix} = C \cdot \Xi_t \tag{42}$$

where the explicit expression for C can be found in appendix A.4.

The linear quadratic policy problem now consists of choosing the policy reaction coefficients (a, b) so as to maximize objective (34) subject to the equations (40) and (42), which summarize optimal price setting behavior and optimal information processing by firms. Under commitment, the policymaker thereby recognizes that the variance of the observation error η_t entering the state innovation vector ω_t in equation (40) is a function of its policy. Under discretionary policy, the policymaker treats the variance of observation errors as given, as was the case in the static model.

Optimal Policy The following proposition shows that the results from the static model carry over in a natural way to the infinite horizon setting. In particular, discretionary monetary policy suboptimally accommodates markup shocks with the degree of accommodation increasing in firms' processing capacity and becoming unbounded in the limit as firms process information perfectly:

Proposition 6 *Optimal policy under commitment sets* $a = 0$ *and* $b = 1$. *Under discretion the policymaker chooses* $a = \frac{k}{\xi(1-k)(1-\rho^2(1-k))} > 0$ *and* $b = 1$.

The proof is given in appendix A.5.

6.2 Model Implied VAR Dynamics

This section shows that the dynamics of prices, output gap, and monetary policy follow a first-order vector-autoregression (VAR). Moreover, following improvements in monetary policy, as defined below, the changes in the VAR dynamics are shown to reproduce Facts 1-3 mentioned at the beginning of section 6.[22]

Throughout this section, monetary policy in the VAR is identified with nominal demand to keep in line with the previous convention. Section 6.3 shows that the main conclusions extend to a VAR involving interest rates instead of nominal demand and to an augmented VAR involving also a monetary aggregate.

The following proposition summarizes the model implications for the dynamics of prices, output gap and monetary policy:

Proposition 7 *Suppose* $a \neq 0$ *and let* $z_t' = (p_t, y_t - y_{n,t}, q_t)'$. *Then*

$$z_t = D z_{t-1} + u_t \tag{43}$$

with

$$D = \begin{pmatrix} \frac{(bk+a\xi)\rho}{\xi a} & \frac{(b+a\xi)k\rho}{\xi a} & \frac{(1-b)k\rho}{\xi a} \\ -\frac{\rho kb}{\xi a} & -\frac{(bk-a\xi+ak\xi)\rho}{\xi a} & \frac{(b-1)k\rho}{\xi a} \\ 0 & 0 & \rho \end{pmatrix}$$

[22] As discussed at the beginning of section 6.1, Fact 4 is also replicated because the model abstracts from monetary policy shocks.

and

$$u_t = CB\omega_t$$

with $\omega_t \in R^3$ *denoting the vector of fundamental shocks defined in (41).*

Proof. If $a \neq 0$, one can invert the matrix C in equation (42) and use it to substitute Ξ_t and Ξ_{t-1} in (40), which delivers the stated result. ■

The expression for the autoregressive matrix D in proposition 7 shows that a change in the monetary policy reaction coefficients (a, b) does not affect the last row of this matrix. Therefore, monetary policy affects only the impact matrix CB pre-multiplying the structural economic shocks and the rows in the autoregressive matrix D governing the evolution of prices and output-gaps. This implies that any in change monetary policy is consistent with Fact 3 mentioned at the beginning of section 6. Specifically, if a researcher estimates VARs for two different policy regimes and exchanges the VAR's 'policy equation' across regimes (while leaving untouched the residuals of each regime), the conclusion will be that such a change makes no difference for economic outcomes.

Note that the variables z_t entering the VAR in equation (43) fully reveal the state vector Ξ_t, provided $a \neq 0$. This implies that the previous and subsequent results cannot be overturned by adding additional information (observables) to the VAR: observing the variables entering z_t is already the best situation an econometrician might hope for.

I now turn consideration to the effects of a marginal improvement in monetary policy on the variance of prices, output gap and monetary policy, and on the variance of the VAR residuals u_t. As suggested by proposition 6, I define an improvement in monetary policy over the discretionary outcome as a reduction in the policy reaction coefficient a towards zero, i.e., as a policy change resulting in less accommodation of mark-up shocks. The following lemma summarizes a first finding. The proof is in appendix A.6.

Lemma 8 *Suppose* $a > 0$ *and* $b = 1$*. The variance of all VAR residuals is strictly increasing in* a*.*

The lemma shows that the innovation variances of the VAR in equation (43) are all decreasing as monetary policy shifts from the discretionary solution towards the commitment outcome. Improvements in monetary policy around the 1980's are thus consistent with the empirical finding that around the same time VAR residuals start to display smaller variances (Fact 2).

The next lemma establishes that improvements in monetary policy also reduce the overall variance of prices, output gap and monetary policy, i.e., the dynamic model delivers Fact 1. This is not immediate from the previous lemma because changes in a also affect the autoregressive matrix D in equation (43), with some of the autoregressive coefficients possibly strongly increasing as a falls to levels close to zero.

Lemma 9 *Suppose* $a > 0$ *and* $b = 1$*. The unconditional variance of prices, output gap, and monetary policy are strictly increasing in* a*.*

The proof can be found in appendix A.7. As in the static model, the variance reduction resulting from less activist monetary policy is due to reduced private sector observation errors. The next result shows that arbitrarily large variance reductions can result from improvements in monetary policy:

Lemma 10 *Consider a regime shift from discretionary optimal policy towards fully optimal policy, see proposition 6. This generates an arbitrarily large (relative and absolute) reduction in the volatility of prices and the output gap, provided firms can process information sufficiently well (k sufficiently close to 1).*

The proof is given in appendix A.8. As in the static case, the volatility reduction can be arbitrarily large because the volatility generated by discretionary policy increases without bound as firms start to process information almost perfectly ($k \to 1$).

Finally, the subsequent lemma shows that, consistent with Fact 2 mentioned at the beginning of section 6, the model implies policy improvements to mainly operate through the VAR residuals and only to a lesser extend through changes in the VAR's autoregressive coefficients:

Lemma 11 *Consider discretionary conduct of monetary policy and a marginal change in the policy response to mark-up shocks a. For k close to 1*

$$\frac{\partial D}{\partial a} \approx 0 \tag{44}$$

$$\frac{\partial diag(VAR(u_t))}{\partial a} > 0 \tag{45}$$

The proof is found in A.9.

Provided firms can process information sufficiently well (k sufficiently close to 1), the results of Lemma 8 - 11 show that the model reproduces all empirical facts listed at the beginning of section 6 following an improvement in the conduct of monetary policy.

6.3 VAR with Interest Rates and Monetary Aggregates

The previous section identified monetary policy in the VAR using nominal demand. Clearly, this is at odds with much of the empirical literature that - following the monetary policy practice of recent decades - uses nominal interest rates instead. This section discusses how the previous results extend when monetary policy in the VAR is identified using nominal interest rates instead of nominal demand. Moreover, it shows that all previous results remain unaffected when augmenting the VAR with a monetary aggregate.

Linearizing the consumption Euler equation implied by the first order conditions of (33) delivers

$$\begin{aligned} i_t &= -\left(\frac{U''\overline{Y}}{U'}\right)(E_t y_{t+1} - y_t) + E_t p_{t+1} - p_t \\ &= (E_t q_{t+1} - q_t) - \left(\frac{U''\overline{Y}}{U'} + 1\right)(E_t y_{t+1} - y_t) \end{aligned} \tag{46}$$

Assuming log utility in consumption, the short-term interest rate consistent with the path for nominal demand implied by equation (43) is given by

$$i_t = (\rho - 1)q_t$$

With log consumption utility all previous results, therefore, fully extend to a VAR including short-term nominal interest rates instead of nominal demand. If consumption utility deviates from the log case, then current and expected future output affect nominal interest rates, see equation (46). Since the evolution of output depends on the policy coefficients a and b, see equation (43), the autoregressive coefficients of the VAR for the interest rate equation will generally not be independent of policy anymore. This may potentially cause a policy shift to show up in the autoregressive coefficients. This could cause the model to be at odds with Fact 3 mentioned in section 6. To what extend the AR coefficients of the interest rate equation do indeed change depends on the degree to which consumption utility deviates from log utility. It also appears that the empirical literature is not at odds with the notion that these coefficients have indeed changed somewhat, although not by a statistically significant amount.

Finally, I discuss the effects of including monetary aggregates into the VAR. The first order conditions of problem (33) deliver a money demand equation of the form

$$D'_t = \frac{1}{\chi_t}\frac{(R_t - 1)}{R_t}U'_t$$

Linearizing this equation and using (46) to substitute nominal interest rates, delivers a linearized money demand equation of the form

$$m_t = \alpha_0 q_t + \alpha_1 p_t + \alpha_2 \xi_t \tag{47}$$

where m_t denotes nominal balances[23], $\xi_t = (\chi_t - \chi)/\chi$ money demand shocks, and α_i linearization coefficients ($i = 0, 1, 2$). One can can easily augment the VAR in equation (43) with the evolution of monetary aggregates implied by equation (47). Since the dynamics of output gap, prices, and policy instrument remain unaffected, all previously derived results carry over to such an augmented VAR.

7 Conclusions

Taking into account the endogeneity of decision makers' information structures appears to have important implications for the conduct of monetary policy and stabilization policy more generally. In particular, discretionary policy decisions tend to be overly activist and this may considerably complicate the information processing problems faced by private agents. As a result, private agents' decisions may become contaminated by large and unpredictable elements (information processing errors), potentially causing a significant increase in aggregate volatility. Overall, it appears that information processing constraints have the potential to vindicate Milton Friedman's conviction that overly ambitious use of monetary policy for stabilization purposes may simply end up amplifying economic fluctuations.

[23]Expressed in percent deviation form its deterministic level.

A Appendix

A.1 Price setting equation and welfare objective

I first derive the linearized price setting equation (5). The product demand functions associated with the Dixit-Stiglitz aggregator (2) are

$$Y^i(P^i) = \left(P^i/P\right)^{-\theta} Y \tag{48}$$

where

$$P = \left(\int \left(P^j\right)^{1-\theta} dj\right)^{\frac{1}{1-\theta}} \tag{49}$$

Using (48) the first order condition of the firms' profit maximization problem (3) delivers

$$P^i = E\left[\frac{1}{1+\tau}\frac{\theta}{\theta-1}W|I\right] \tag{50}$$

In a symmetric equilibrium $P^i = P$. Equation (48) then implies $Y^j = Y$ and the household's first order condition can be written as

$$W = \frac{\nu V'(Y)}{U'(Y)}P \tag{51}$$

Combining (50) and (51) delivers

$$P^i = E\left[\frac{1}{1+\tau}\frac{\theta}{\theta-1}\frac{\nu V'(Y)}{U'(Y)}P|I\right] \tag{52}$$

In the symmetric deterministic steady state $P^i = P = \overline{P}$, $Y^i = Y = \overline{Y}$, $\theta = \overline{\theta}$, and $\nu = 1$ where $\overline{Y}$ solves

$$\frac{1}{1+\tau}\frac{\overline{\theta}}{\overline{\theta}-1}\frac{V'(\overline{Y})}{U'(\overline{Y})} = 1 \tag{53}$$

and $\overline{P}$ is any value chosen by the central bank. Steady state output $\overline{Y}$ is first best for

$$\tau = \frac{1}{\overline{\theta}-1}$$

which implies

$$V'(\overline{Y}) = U'(\overline{Y}) \tag{54}$$

For a given the labor supply shock ν, the first best output level Y_n solves

$$\frac{\nu V'(Y_n)}{U'(Y_n)} = 1$$

Linearizing this equation around the steady state delivers

$$\nu - 1 = -\frac{V''(\overline{Y})U'(\overline{Y}) - V'(\overline{Y})U''(\overline{Y})}{\left(U'(\overline{Y})\right)^2}\overline{Y}\left(\frac{Y_n - \overline{Y}}{\overline{Y}}\right) \tag{55}$$

Linearizing (52) around the deterministic steady state and using (55) delivers (5) where:

$$\begin{aligned}
\varepsilon_t &= -\frac{1}{\overline{\theta}-1}\frac{(\theta-\overline{\theta})}{\overline{\theta}} \\
\xi &= \frac{V''(\overline{Y})U'(\overline{Y}) - V'(\overline{Y})U''(\overline{Y})}{\left(U'(\overline{Y})\right)^2}\overline{Y} \\
&= \frac{V''(\overline{Y})\overline{Y}}{V'(\overline{Y})} - \frac{U''(\overline{Y})\overline{Y}}{U'(\overline{Y})}
\end{aligned}$$

Next, I derive the welfare approximation (4). Consider a symmetric equilibrium where $P^i = P$ and $Y^i = Y$. A second order approximation of the utility Ω of the representative agent around the steady state level $\overline{\Omega}$ is then given by

$$\begin{aligned}
\Omega - \overline{\Omega} = {} & U'(\overline{Y})(Y-\overline{Y}) - V'(\overline{Y})(Y-\overline{Y}) \\
& + \frac{1}{2}U''(\overline{Y})(Y-\overline{Y})^2 - \frac{1}{2}V''(\overline{Y})(Y-\overline{Y})^2 \\
& - V'(\overline{Y})(Y-\overline{Y})(\nu-1) + O(2) + t.i.p
\end{aligned} \tag{56}$$

where $t.i.p.$ denotes (first and higher order) terms that are independent of policy and $O(2)$ summarizes endogenous terms of order larger than two. Using equations (54) and (55) and adding $\frac{1}{2}\left[U''(\overline{Y}) - V''(\overline{Y})\right]\left(Y^n - \overline{Y}\right)^2$, which is a term independent of policy, the welfare approximation (56) can be written as

$$\begin{aligned}
\Omega - \overline{\Omega} = {} & \frac{1}{2}\left[U''(\overline{Y}) - V''(\overline{Y})\right]\overline{Y}^2\left(\frac{(Y-\overline{Y})}{\overline{Y}} - \frac{(Y^n-\overline{Y})}{\overline{Y}}\right)^2 \\
& + O(2) + t.i.p
\end{aligned} \tag{57}$$

which is of the form postulated in (4).

A.2 Proof of proposition 5

Firm i chooses its capacity index k_i so as to

$$\max_{k_i} \Pi(k_i, k_{-i}, \omega) - c(k_i)$$

where $\Pi(k_i, k_{-i}, \omega)$ denotes the firm's expected profits from selling goods and $c(k_i)$ the costs of acquiring processing capacity. The firm takes as given the policy weight ω placed on price stabilization as well as the capacity choice k_{-i} of other firms. Under the maintained assumptions, the firm's optimal capacity choice k_i^* solves

$$\Pi_{k_i}(k_i^*, k_{-i}, \omega) - c_{k_i}(k_i) = 0 \tag{58}$$

and there is a unique symmetric Nash equilibrium k^* solving

$$\Pi_{k_i}(k^*, k^*, \omega) - c_{k_i}(k^*) = 0 \tag{59}$$

The price and output gap variances implied by an (arbitrary) capacity choice k and a policy weight ω are

$$Var(p) = \frac{(1-k)^2 k}{\left((1-k)^2 + \omega k^2\right)^2 \xi^2} \sigma_\varepsilon^2$$

$$Var(y - y_n) = \frac{\left(3k^2 - 3k - k^3 + k^3\omega^2 + 1\right)}{\left((1-k)^2 + \omega k^2\right)^2} \frac{k}{\xi^2} \sigma_\varepsilon^2$$

The marginal effects of ω on the Nash equilibrium are thus

$$\frac{dVar(p)}{d\omega} = \frac{\partial Var(p)}{\partial \omega} + \frac{\partial Var(p)}{\partial k}\frac{\partial k^*}{\partial \omega}$$

$$\frac{dVar(y-y_n)}{d\omega} = \frac{\partial Var(y-y_n)}{\partial \omega} + \frac{\partial Var(y-y_n)}{\partial k}\frac{\partial k^*}{\partial \omega}$$

As is easily verified $\partial Var(p)/\partial\omega < 0$, $\partial Var(y-y_n)/\partial\omega < 0$, $\partial Var(p)/\partial k > 0$, and $\partial Var(y-y_n)/\partial k > 0$ for ω sufficiently small. One thus has $dVar(p)/d\omega < 0$ and $dVar(y-y_n)/d\omega < 0$, provided $\frac{\partial k^*}{\partial \omega} < 0$. From the implicit function theorem and (59)

$$\frac{\partial k^*}{\partial \omega} = -\frac{\partial \left(\Pi_{k_i}(k^*, k^*, \omega) - c_{k_i}(k^*)\right)/\partial \omega}{\partial \left(\Pi_{k_i}(k^*, k^*, \omega) - c_{k_i}(k^*)\right)/\partial k_i}$$

$$= -\frac{\Pi_{k_i\omega}(k^*, k^*, \omega)}{\Pi_{k_ik_i}(k^*, k^*, \omega) - c_{k_ik_i}}$$

We know $c_{k_ik_i} > 0$ and below we show that $\Pi_{k_ik_i}(k^*, k^*, \omega) = 0$ and $\Pi_{k_i\omega}(k^*, k^*, \omega) < 0$. This together establishes $\frac{\partial k^*}{\partial \omega} < 0$. The sign of the derivatives $\Pi_{k_ik_i}$ and $\Pi_{k_i\omega}$ can be determined using the second order approximation to firms' profits (9):

$$E\left[-E\left[\left(p^i - p^*\right)^2 |I\right]\right] = -E\left[\left(k_i s^i - p^*\right)^2\right]$$

$$= -E\left[\left(k_i p^* + k_i \eta - p^*\right)^2\right]$$

$$= -(1-k_i)^2 E\left[(p^*)^2\right] - (k_i)^2 E\left[\left(\eta^i\right)^2\right]$$

Optimal discretionary policy for given ω and k is

$$q = \frac{\left((1-k)k - \omega k^2\right)}{\left((1-k)^2 + \omega k^2\right)} \frac{1}{\xi}\varepsilon + y_n$$

and implies

$$E[(p^*)^2] = \left(\frac{(1-k)}{\left((1-k)^2 + \omega k^2\right)} \frac{1}{\xi}\right)^2 \sigma_\varepsilon^2$$

and

$$E\left[\left(\eta^i\right)^2\right] = \frac{1-k_i}{k_i} E[(p^*)^2]$$

The quadratic approximation of firms' profits is thus

$$E\left[-E\left[\left(p^i - p^*\right)^2 |s^i\right]\right] = \frac{(k_i - 1)(k-1)^2}{\left((1-k)^2 + k^2\omega\right)^2 \xi^2}\sigma_\varepsilon^2$$

and implies

$$\Pi_{k_i k_i}(k^*, k^*, \omega) = 0$$

$$\Pi_{k_i \omega}(k^*, k^*, \omega) = -2\frac{(1-k^*)^2 (k^*)^2}{\left((1-k^*)^2 + (k^*)^2\omega\right)^3 \xi^2}\sigma_\varepsilon^2 < 0$$

which establishes the claim.■

A.3 Higher-Order Expectations

We know that

$$\sigma_\eta^2 = \frac{1-k}{k}Var(p^*)$$

$$var(s^i) = \frac{1}{k}Var(p^*)$$

$$cov(p^*, s^i) = var(p^*)$$

$$cov(p^* + \eta^c, s^i) = \frac{\alpha(1-k)+k}{k}Var(p^*)$$

Using the updating formulae for the conditional mean of jointly normally distributed random variables one gets

$$E\left[p^*|s^i\right] = ks^i$$

and

$$p^{*(1)} = \int_{i\in[0,1]} E\left[p^*|s^i\right] = k\left(p^* + \eta^c\right)$$

Furthermore,

$$\begin{aligned} E\left[p^{*(1)}|s^i\right] &= kE\left[\left(p^* + \eta^c\right)|s^i\right] \\ &= k\left(\frac{cov(p^* + \eta^c, s^i)}{var(s^i)}s^i\right) \\ &= k\left(k(1-\alpha)+\alpha\right)s^i \end{aligned}$$

so that

$$p^{*(2)} = k\left(k(1-\alpha)+\alpha\right)\left(p^* + \eta^c\right)$$

and

$$\begin{aligned} E\left[p^{*(2)}|s^i\right] &= k\left(k(1-\alpha)+\alpha\right)E\left[\left(p^* + \eta^c\right)|s^i\right] \\ &= k\left(k(1-\alpha)+\alpha\right)^2 s^i \end{aligned}$$

Repeatedly integrating over i and taking the expectations $E\left[\cdot|s^i\right]$ delivers:

$$E\left[p^{*(m)}|s^i\right] = k\left(k(1-\alpha)+\alpha\right)^m s^i$$

A.4 Details on the State Dynamics

The matrices in equation (40) are defined as follows:

$$A = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho & 0 \\ k\rho\left(a + \frac{1}{\xi}\right) & k\rho\,(b-1) & \rho(1-k) \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ k\left(a + \frac{1}{\xi}\right) & k\,(b-1) & k \end{pmatrix}$$

The matrix in equation (42) is given by

$$C = \begin{pmatrix} 0 & 0 & 1 \\ a & (b-1) & -1 \\ a & b & 0 \end{pmatrix}$$

A.5 Optimal Policy in the Dynamic Economy

We start by deriving the commitment solution. Using (42) the policy objective can be expressed as

$$\begin{aligned} -E\left[(y - y_{n,t})^2\right] &= -E\left[\left(a\varepsilon_t + (b-1)y_{n,t} - H'x_{t|t}\right)^2\right] \\ &= -\left[a^2\sigma_\varepsilon^2 + (b-1)^2\sigma_y^2 + E\left[(H'x_{t|t})^2\right]\right. \\ &\left. -2a\cdot cov(\varepsilon_t, H'x_{t|t}) - 2(b-1)\cdot cov(y_{n,t}, H'x_{t|t})\right] \end{aligned} \tag{60}$$

I first derive an explicit expression for $H'x_{t|t}$ that allows to compute the variance and the covariance terms appearing in (60). The Kalman filter updating equations are

$$x_{t|t} = x_{t|t-1} + P_{t|t-1}H(H'P_{t|t-1}H + \sigma_\eta^2)^{-1}(s_t - H'x_{t|t-1}) \tag{61}$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H + \sigma_\eta^2)^{-1}H'P_{t|t-1} \tag{62}$$

where $x_{t|t}$ is the posterior mean of x_t and $x_{t|t-1}$ the prior mean. Likewise, $P_{t|t}$ denotes the posterior covariance matrix of x_t and $P_{t|t-1}$ the prior covariance matrix. Equation (12) implies that the variance of the channel noise is given by

$$\sigma_\eta^2 = \frac{1-k}{k}H'P_{t|t-1}H \tag{63}$$

From equations (62) and (63)

$$\begin{aligned} H'P_{t|t}H &= (1-k)H'P_{t|t-1}H \\ &= (1-k)H'\left(\rho^2 P_{t-1|t-1} + \Sigma_v\right)H \end{aligned} \tag{64}$$

Equation (64) implies that in steady state

$$P = \frac{1-k}{1-(1-k)\rho^2}\Sigma_v \tag{65}$$

Using equations (40) and (63) equation (61) implies

$$\begin{aligned} H'x_{t|t} &= (1-k)H'x_{t|t-1} + ks_t \\ &= \rho(1-k)H'x_{t-1|t-1} + ks_t \end{aligned}$$

and since $|\rho(1-k)| < 1$

$$H'x_{t|t} = \sum_{j=0}^{\infty} ((1-k)\rho)^j \, k(H'x_{t-j} + \eta_{t-j}) \tag{66}$$

Equation (66) implies that

$$cov(\varepsilon_t, H'x_{t|t}) = \frac{k}{1-(1-k)\rho^2}\left(a + \frac{1}{\xi}\right)\sigma_\varepsilon^2 \tag{67}$$

$$cov(y_{n,t}, H'x_{t|t}) = \frac{k}{1-(1-k)\rho^2}\,(b-1)\,\sigma_y^2 \tag{68}$$

Equation (66) also implies that

$$var(H'x_{t|t}) = k^2\left(E\left[\left(\sum_{j=0}^{\infty}((1-k)\rho)^j\, H'x_{t-j}\right)^2\right] + \left[\frac{1}{1-((1-k)\rho)^2}\sigma_\eta^2\right]\right) \tag{69}$$

Some tedious but straightforward calculations show that

$$E\left[\left(\sum_{j=0}^{\infty}((1-k)\rho)^j\, H'x_{t-j}\right)^2\right] = \frac{1}{\left(1-((1-k)\rho)^2\right)}\left(\frac{2}{(1-(1-k)\rho^2)} - 1\right)H'Var(x_t)H$$

Furthermore, from equation (63)

$$\begin{aligned} \frac{1}{1-((1-k)\rho)^2}\sigma_\eta^2 &= \frac{1}{1-((1-k)\rho)^2}\frac{1-k}{k}H'P_{t|t-1}H \\ &= \frac{1}{1-((1-k)\rho)^2}\frac{1-k}{k}H'(\rho^2 P_{t-1|t-1} + \Sigma_v)H \end{aligned} \tag{70}$$

Using the steady state expression (65) the steady state version of equation (70) is

$$\begin{aligned} \frac{1}{1-((1-k)\rho)^2}\sigma_\eta^2 &= \frac{1}{1-((1-k)\rho)^2}\frac{1}{1-(1-k)\rho^2}\frac{1-k}{k}H'\Sigma_v H \\ &= \frac{(1-\rho^2)}{1-((1-k)\rho)^2}\frac{1}{1-(1-k)\rho^2}\frac{1-k}{k}H'Var(x_t)H \end{aligned} \tag{71}$$

Combining equations (69), (70), and (71) and simplifying delivers

$$E\left[(H'x_{t|t})^2\right] = \frac{k}{1-(1-k)\rho^2}H'Var(x_t)H \tag{72}$$

Substituting (67), (68), and (72) into (60) delivers

$$\begin{aligned}-E\left[(y-y_{n,t})^2\right] &= -a^2\sigma_\varepsilon^2-(b-1)^2\sigma_y^2\\ &\quad -\frac{k}{1-(1-k)\rho^2}\left(\left(a+\frac{1}{\xi}\right)^2\sigma_\varepsilon^2+(b-1)^2\sigma_y^2\right)\\ &\quad +2a\frac{k}{1-(1-k)\rho^2}\left(a+\frac{1}{\xi}\right)\sigma_\varepsilon^2\\ &\quad +2\frac{k}{1-(1-k)\rho^2}(b-1)^2\sigma_y^2 \end{aligned} \tag{73}$$

The first order conditions for maximizing (73) with respect to a and b deliver result in the proposition for the commitment case.

Under discretion, the policymaker takes the observation noise σ_η^2 in equation (69) as exogenous. Under discretion the policy objective (73) thus has to be modified to:[24]

$$\begin{aligned}-E\left[(y-y_{n,t})^2\right] &= -a^2\sigma_\varepsilon^2-(b-1)^2\sigma_y^2\\ &\quad -\frac{k^2}{\left(1-((1-k)\rho)^2\right)}\left(\frac{2}{(1-(1-k)\rho^2)}-1\right)\left(\left(a+\frac{1}{\xi}\right)^2\sigma_\varepsilon^2+(b-1)^2\sigma_y^2\right)\\ &\quad -k^2\left[\frac{1}{1-((1-k)\rho)^2}\sigma_\eta^2\right]\\ &\quad +2a\frac{k}{1-(1-k)\rho^2}\left(a+\frac{1}{\xi}\right)\sigma_\varepsilon^2\\ &\quad +2\frac{k}{1-(1-k)\rho^2}(b-1)^2\sigma_y^2 \end{aligned} \tag{74}$$

The first order conditions with respect to a and b deliver the result stated in the proposition for the case without commitment.

A.6 Proof of lemma 8

From proposition 7 we have $u_t = CB\omega_t$. For $b=1$ we get (see appendix A.4):

$$CB = \begin{pmatrix} \left(a+\frac{1}{\xi}\right)k & 0 & k\\ (1-k)\left(a-\frac{k}{(1-k)}\frac{1}{\xi}\right) & 0 & -k\\ a & 1 & 0 \end{pmatrix}$$

From equation (63) follows that the variance of the observation noise is

$$\sigma_\eta^2 = \frac{1-k}{k}\frac{1}{1-(1-k)\rho^2}\left(\frac{1}{\xi}+a\right)^2\sigma_1^2 \tag{75}$$

Therefore, letting $u_t{}' = (u_t^1, u_t^2, u_t^3)'$, the variances of the VAR residuals are

[24] Note that the covariance terms (67) and (68) remain unaffected by treating σ_η^2 as exogenous.

$$Var(u_t^1) = \left(k^2 + k(1-k)\frac{1}{1-(1-k)\rho^2}\right)\left(a + \frac{1}{\xi}\right)^2 \sigma_{v_1}^2$$
$$Var(u_t^2) = \left((1-k)^2\left(a - \frac{k}{(1-k)}\frac{1}{\xi}\right)^2 + (1-k)\frac{1}{1-(1-k)\rho^2}\left(\frac{1}{\xi}+a\right)^2\right)\sigma_{v_1}^2$$
$$Var(u_t^3) = a^2\sigma_{v_1}^2 + \sigma_{v_2}^2$$

From the previous expressions it is obvious that $\frac{\partial Var(u_t^1)}{\partial a} > 0$, $\frac{\partial Var(u_t^3)}{\partial a} > 0$, and straightforward to establish $\frac{\partial Var(u_t^2)}{\partial a} > 0$, provided $a > 0$.

A.7 Proof of lemma 9

Given the policy rule (37), the result is immediate for the variance of the policy instrument. From proposition 7 we have

$$z_t = Dz_{t-1} + CB\omega_t$$

Taking variances on both sides and applying the columnwise vectorization operator $vec(\cdot)$ one gets

$$vec\,(var(z_t)) = (I_{9x9} - D \otimes D)^{-1} vec(CB\Sigma C'B')$$

where $\Sigma = var(\omega_t)$ and I_{9x9} is a 9-dimensional identity matrix. Deriving the explicit expressions for $vec\,(var(z_t))$ and computing the derivatives shows that $\frac{\partial \sigma_p^2}{\partial a} > 0$ and $\frac{\partial \sigma_{y-y_n}^2}{\partial a} > 0$, provided $a > 0$.

A.8 Proof of lemma 10

From the proof of lemma 9 in appendix A.7 one obtains explicit expression for the variance of prices and output gap. Evaluating these at discretionary and at fully optimal policy shows that

$$\frac{\sigma_{p,d}^2}{\sigma_{p,c}^2} = \frac{(1-\rho(1-k))^2\,(1+\rho(1-k))^2}{(1-\rho^2(1-k))^2}\frac{1}{(1-k)^2}$$
$$\frac{\sigma_{y-y_n,d}^2}{\sigma_{y-y_n,c}^2} = \frac{\left(1+\rho^4-2\rho^2+3k\rho^2-3k\rho^4-2k^2\rho^2+3k^2\rho^4-k^3\rho^4\right)}{(1-\rho^2(1-k))^2}\frac{1}{(1-k)}$$

where $\sigma_{p,d}^2$ and $\sigma_{p,c}^2$ denote the variance of prices under discretionary and commitment policy, respectively, and $\sigma_{y-y_n,d}^2$ and $\sigma_{y-y_n,c}^2$ the corresponding variances of the output gap. As is easily seen from the previous equations, the variance ratios become unbounded as $k \to 1$.

A.9 Proof of lemma 11

Evaluating $\frac{\partial D}{\partial a}$ at the discretionary monetary policy solution delivers

$$\frac{\partial D}{\partial a} = \begin{pmatrix} -\frac{1}{k}\xi\rho\,(-k+1)^2 & -\frac{1}{k}\xi\rho\,(-k+1)^2 & 0 \\ \frac{1}{k}\xi\rho\,(-k+1)^2 & \frac{1}{k}\xi\rho\,(-k+1)^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and shows that for $k \approx 1$ one obtains (44). Moreover, taking the derivative of

$$diag(Var(u_t)) = diag(Var(CB\omega_t))$$

with respect to a delivers

$$\frac{\partial diag(Var(u_t))}{\partial a} = \begin{pmatrix} 2\frac{\left(k^2\rho^2 - k\rho^2 + 1\right)(a\xi+1)\sigma_1^2}{(k\rho^2-\rho^2+1)\xi}k \\ 2\frac{\left(a\xi\rho^2 - k\rho^2 - a\xi - 2ak\xi\rho^2 + k^2\rho^2 + ak^2\xi\rho^2\right)}{(k\rho^2-\rho^2+1)\xi}(k-1)\sigma_1^2 \\ 2\sigma_1^2 a \end{pmatrix}$$

Evaluating at the discretionary monetary policy solution and taking the limit $k \to \infty$ delivers (45).

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