Fiscal Solvency and Macroeconomic Uncertainty in Emerging Markets: The Tale of the Tormented Insurer

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June 2004

Abstract

Governments in emerging markets often behave like a "tormented insurer" who tries to keep government outlays smooth despite the randomness of public revenues and frictions in financial markets. They have access to a limited set of financial instruments and "liability dollarization" forces them to issue debt denominated in hard currencies, or indexed to tradable goods prices. How can a fiscal authority tell if the stock of public debt is consistent with fiscal solvency in this environment? This paper proposes a quantitative framework to answer this question by solving for the equilibrium dynamics of public debt of a two-sector small open economy subject to random income shocks, given tax and expenditure policies. This framework emphasizes macroeconomic uncertainty and the transmission mechanism by which this uncertainty affects debt dynamics when asset markets are incomplete and public debt is a "dollarized liability." In our model, a government trying to smooth outlays and make a credible commitment to repay cannot borrow above a "natural debt limit" set by the annuity value of the "catastrophic" level of the primary balance. This limit plays an important role in debt dynamics but it differs, in general, from the equilibrium debt levels along the stochastic equilibrium path. Liability dollarization implies that real-exchange-rate fluctuations affect the variability of revenues and the government's ability to service debt, and thus affect public debt dynamics. An application to Mexican data shows that the short- and long-run distributions of debt-output ratios deviate sharply from conventional estimates of "sustainable" debt ratios.

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We are grateful to Guillermo Calvo, Alejandro Izquierdo, Alejandro Werner and seminar participants at Iowa State University, North Carolina State University, Mexico's Ministry of Finance, and Université du Québec à Montreal for helpful comments and suggestions. The contents of this paper reflect work we did as consultants for the Research Department of the Inter-American Development Banks. The paper does not reflect the official views of the IADB.

1 Introduction

A central question in fiscal policy debates is whether the observed stock of public debt is consistent with fiscal solvency considerations - that is, consistent with the requirement to equate the present values of total government revenues and outlays. If it is, the observed debt-output ratio is commonly referred to as "sustainable." [1] If it is not, the fiscal position is judged to be unsustainable and in need of policy correction. In short, the goal of public debt sustainability analysis is to determine whether the government is living "within its means" and to facilitate the assessment of corrective policy measures when this is not the case.

The methodologies for evaluating fiscal sustainability that are most favored in policy-making institutions are based on (a) steady-state debt-output ratios implied by the stationary, growth-adjusted government budget constraint, or (b) econometric tests of the intertemporal government budget constraint. Interest in the latter is partly motivated by the fact that, while the steady-state analysis illustrates the level of debt that can be supported in the long-run equilibrium of a world without uncertainty, in practice the key issue is to assess public debt sustainability at a particular point in time (possibly far from steady state) and in a world where a variety of shocks can affect the government's ability to place and service debt.

Unfortunately, tests of the intertemporal government budget constraint fall somewhat short from delivering an effective methodology to make these assessments. Their main objective is to test the hypothesis that the fiscal solvency condition holds in a country's historical time-series data. These tests are not designed to connect the observed underlying sources of macroeconomic uncertainty with the dynamics of public debt in order to provide short- and long-run, forward-looking measures of sustainable public debt ratios.

The importance of incorporating uncertainty considerations into public debt sustainability analysis is clearly reflected in two striking empirical observations. First, as Figure 1 shows, countries with lower coefficients of variation in the ratios of public revenues to GDP support higher debt-output ratios on average. An unconditional panel regression suggests that an increase of 1 percent in the volatility of revenues reduces the mean debt-output ratio by 3 percentage points. Second, as Figure 2 shows, countries with lower GDP volatility support higher average debt-output ratios. Countries with a standard deviation of GDP growth in excess of 3 percent cannot support debt-output ratios higher than 50 percent. The samples in these figures are small because of serious limitations of cross-country databases

[1] This criterion of sustainability is different from the requirement that public debt plans formulated by the government in their strategic interaction with the private sector be free from time inconsistency. The literature examining this issue from the perspective of the theory of dynamic games also refers to public debt plans that satisfy this requirement as "sustainable."

on fiscal data. Yet, these observations clearly suggest that the stochastic nature of the environment in which governments operate must be taken into account in estimating sustainable debt ratios.

These considerations are particularly important for emerging markets. As the report by the International Monetary Fund (2003a) shows, emerging economies display significantly higher coefficients of variation in public revenues and larger cyclical fluctuations in economic activity than industrial countries. Moreover, the response of emerging economies to macroeconomic shocks also differs from that of industrial countries because of the financial frictions that emerging economies confront in world capital markets. The possibility of "Sudden Stops" to capital inflows and the syndrome of "liability dollarization" that affect these economies influence public debt sustainability analysis. Because of liability dollarization, emerging markets' public debt instruments are typically issued in hard currencies but largely leveraged on public revenues generated in the non-tradable-goods sector. In this situation, as Calvo, Izquierdo and Talvi (2003) showed, a foreign or domestic shock that triggers a Sudden Stop can force a large reversal of the current account and a collapse of the relative price of nontradable goods (or the real exchange rate), and the latter can compromise the ability to service public debt and result in sharp declines in sustainable debt-output ratios.

The aim of this paper is to propose a quantitative framework for assessing public debt sustainability that takes into account these elements of uncertainty and financial market imperfections. The starting point is the same from which both the long-run approach to fiscal sustainability and the intertemporal tests start: the budget constraint of the government. The framework proposed here differs in that it models explicitly the mechanism by which macroeconomic shocks affect the behavior of the government and the private sector. The framework is based on a model of a two-sector small open economy with stochastic endowments of tradable and nontradable goods. The government sets a time-invariant income tax rate and chooses optimal expenditure policies that are characterized by "expenditure smoothing." However, smoothing is difficult to accomplish because markets of contingent claims are incomplete (i.e., the government can only issue non-state-contingent debt) and because debt can only be issued in units of tradable goods (i.e., the government suffers of the syndrome of "liability dollarization"). Tax rates and government outlays are policy choices, but tax revenues and the financing needs of the public sector are endogenous outcomes that depend on variables beyond the control of the fiscal authority (such as the tax bases, the realizations of the shocks, and the equilibrium relative price of nontradables).

This stochastic framework makes explicit the operational implications of the government's desire to smooth its outlays. In particular, high aversion to very low expenditure levels leads the government to impose on itself a "natural debt limit" (NDL) analogous to

those that households adopt in models of incomplete markets and income uncertainty (see Aiyagari (1994) and Hugget (1993) and the analysis of optimal taxation with non-state-contingent public debt by Aiyagari et al. (2001)). This debt limit is given by the annuity value of the difference between the worst realization of public revenue and the minimum levels (or "basic needs levels) of outlays that the government can commit to adjust to in a state of "fiscal crisis" (defined as a long sequence of realizations of the lowest level of public revenue, which by definition can occur with non-zero probability). If the government borrows above the NDL, it exposes itself to the risk of lowering expenditures to extremely low, highly suboptimal levels. Since this NDL also represents the largest debt that allows the government to remain able to repay in all states of nature, the NDL can also be viewed as a credible commitment to repay (credible only from the perspective of an "ability to pay" criterion). If public debt exceeds the NDL, the government cannot credibly committ to be able to repay – since it would not be able to repay in the state of fiscal crisis.

The NDL is a key part of the debt sustainability framework proposed in this paper, but in general it is *not* the same as the equilibrium or sustainable level of public debt. The latter is determined by the government budget constraint taking into account the endogenous behavior of tax bases and the price of nontradables along a stochastic equilibrium path, and the tax and expenditure policies. Thus, the computation of the stochastic equilibrium dynamics of the economy is also central to the analysis.

One option to model taxes, debt and expenditures would be to consider optimal government policy in the traditional sense of Ramsey optimal taxation problems (by choosing optimal state-contingent rules for debt and tax rates for a given random process of government purchases). In contrast, the principle followed here is to adapt the model to the reality of emerging economies where government outlays tend not to be flexible and public revenues have important components exogenous to the government's actions (commodity export revenues, for example), or where tax policy deviates sharply from the predictions of optimal taxation theory (as illustrated by the procyclical nature of fiscal policy in developing countries documented by Talvi and Végh (2001)). Hence, the framework proposed here assumes that the government fixes an income tax rate and choose an optimal smooth path of government outlays in "normal" times, in which it may need to issue debt but the NDL does not bind. On the other hand, in times of fiscal crisis when the NDL binds the government adjusts outlays to a fixed minimum level.

The NDL and the dynamics of sustainable public debt depend on how the tax and expenditures policies are set. For example, if the government has no flexibility to reduce outlays during a fiscal crisis, the commitment to repay requires setting the "smoothed" level of government outlays equal to the worst realization of public revenues, so as to equate the annuity values of the inelastic outlays and the worst realization of revenues. In this case,

no positive amount of public debt is sustainable because the NDL is zero. At the other extreme, if outlays could be cut to zero in a fiscal crisis, the debt limit would be equal to the annuity value of the worst realization of tax revenue. This would yield the highest natural debt ceiling that the government could reach for a given stochastic process of tax revenue (although the incompleteness of asset markets would lead the government to engage in precautionary savings and avoid choosing the NDL as the equilibrium level of debt). Thus, governments that can complement a pledge to commit to repay with a commitment to undertake significant expenditure cuts during a fiscal crisis face higher NDLs and hence are allowed to borrow more.

The exogenous macroeconomic uncertainty coming from shocks to domestic income and the world-interest rate also plays a crucial role. Countries that have more volatile tax revenues face lower debt ceilings and are able to borrow less because their worst realization of public revenues is lower, for given tax and expenditure policies. The effects of "liability dollarization" are also at play. In particular, if the relative price of nontradable goods falls when the government hits its debt limit, the debt limit itself can feature an endogenous magnifying effect that tightens the debt limit further (since the value of tax revenues in units of tradable goods can fall with the relative price of nontradable goods). Through this mechanism, fluctuations of the real exchange rate can have important effects on the model's predictions for sustainable public debt ratios.

In summary, this paper develops an approach to study public debt dynamics and fiscal solvency that views the government as a "tormented insurer" operating in an imperfect and uncertain world. This insurer seeks to provide insurance to society by keeping government outlays smooth given the uncertainty of public revenues, but this is a challenging task because financial markets are incomplete and public debt is a "dollarized liability". As a result, the government cannot diversify away idiosyncratic risk and its ability to service debt fluctuates with swings in the real exchange rate. Faced with this situation, the insurer practices self-insurance and seeks to determine its optimum liability position in non-state-contingent debt so as to smooth its outlays as much as possible while not exposing it to the risk of becoming insolvent (i.e., while respecting the NDL that ensures that it can repay its obligations).

The paper first documents the results of implementing this approach to assess debt sustainability for the case of a representative emerging-market economy. This starting example is based on a basic one-sector model with ad-hoc government expenditure rules and a single, exogenous source of public revenue. In this basic model, the government keeps its expenditures fixed at an ad-hoc level unless its debt approaches the NDL, in which case expenditures are cut to an ad-hoc minimum level. It is shown that if the government is assumed to cut outlays by 4 percentage points of GDP in times of fiscal crisis, the model

yields a natural debt limit that is equal to 50% of the economy's GDP. This natural debt limit is very sensitive to small variations in the volatility of tax revenues, the world interest rate and the size of the cut in government outlays in a fiscal crisis. For example, a mean-preserving spread that increases the variability of tax revenues by 1/2 of a percentage points cuts the natural debt limit to 18 percent of output.

Stochastic simulations of this basic model show that, starting from debt ratios below 40 percent of GDP, the economy takes more than 20 quarters to hit the natural debt limit *on average*. However, this average is a misleading concept because this basic model has the unappealing feature that the long-run distribution of public debt is not unique and invariant to initial conditions. For sufficiently low initial debt ratios, the economy can follow paths in which public debt ends up vanishing in the long run. For sufficiently high initial debt ratios, the economy can follow paths in which it hits the NDL in public debt and it and falls into a fiscal crisis in six quarters or less. This result is reminiscent of the Barro's (1979) finding in his classic analysis of tax smoothing in a deterministic environment in which the dynamics of debt are indeterminate and depend entirely on initial conditions.

The two-sector dynamic stochastic general equilibrium model with endogenous expenditure policies features a unique and invariant long-run distribution of public debt. The quantitative implementation of the model is based on a baseline calibration designed so that the model's deterministic stationary equilibrium matches various features of the Mexican national accounts and fiscal data (particularly the average debt ratio of 46 percent of GDP). We examine the statistical moments that characterize the equilibrium stochastic processes of the model's endogenous variables in this baseline scenario, and study the short- and long-run features of debt dynamics. Then we examine the results of several alternative scenarios designed to illustrate the role played by the different factors that are key to drive debt dynamics in the model (such as the levels of "basic needs" of government outlays, the variability and persistence of GDP, the relative size of revenues collected from tradables and nontradables sectors, etc.).

The rest of the paper proceeds as follows. The next section surveys the existing methods to evaluate public debt sustainability and compares them with the framework proposed in this paper. This section also summarizes key stylized facts of public debt and revenue ratios that motivate the use of stochastic methods to study debt dynamics and fiscal solvency. Section 3 develops the basic one-sector variant of the model with ad-hoc expenditure rules and public revenues entirely driven by exogenous shocks. Section 4 presents the two-sector dynamic stochastic general equilibrium model. Section 5 discusses the calibration and the quantitative predictions of the model. Section 6 contains final remarks.

2 A Review of Public Debt Sustainability Analysis

This section provides a short review of the different methods that have been proposed for studying public debt sustainability. The aim is not to survey the literature thoroughly but to highlight the central differences between the existing methods and the framework that this paper develops.[2] The section ends documenting the major differences across industrial and developing countries regarding public revenue and public debt to serve as a motivation for the models of sections 3 and 4.

The starting point of all the methods for calculating sustainable public debt-output ratios is the period budget constraint of the government. In an economy in which output grows at an exogenous gross rate γ in the long run, the government budget constraint can be written as follows:[3]

$$\gamma b_{t+1}^{g} = b_t^g R_t - (t_t - g_t) \tag{1}$$

In this expression, b_{t+1}^{g} is the ratio of public debt issued by the end of period t and maturing at $t+1$ as a share of date $t+1$ output, b_t^g is the ratio of maturing public debt to output at date t, R_t is the gross real interest rate on public debt, t_t is the ratio of total government revenue to output, and g_t represents the ratio of total government outlays (current purchases plus transfers) to output. Thus $t_t - g_t$ is the primary fiscal balance as a share of output.

The methodologies for computing sustainable debt ratios differ in the manner in which they use the above constraint to assess whether observed debt ratios are consistent with the fiscal solvency condition that follows from solving the constraint forward. The solvency condition states that the present value of the primary balance must be equal to the interest and principal on the outstanding debt as of the initial date in which solvency is being evaluated.

2.1 Long-Run Methods.

The long-run methods for assessing public debt sustainability are based on long-run, perfect-foresight results that transform the government's budget constraint from an accounting identity into an equation that maps the steady-state primary balance into a sustainable debt-output ratio (see Buiter (1985)). Thus, this method defines the sustainable debt-output ratio as the value that it attains at steady state, when the primary balance has also attained its long-run equilibrium (see Buiter (1985), Blanchard (1990) and Blanchard et al. (1990)). Given the budget constraint (1), the steady-state debt output ratio satisfies the

[2] For comprhensive surveys, see Chalk and Hamming (2000) or IMF (2002)and (2003).

[3] At the highest level of generality, this constraint is merely an accounting identity that relates all the flows of government receipts and payments to the change in public debt

following condition:

$$b^g = \frac{t-g}{R-\gamma} \tag{2}$$

where variables without time subscripts correspond to steady-state values. In policy applications, condition (2) is interpreted either as an indicator of the "permanent" value (or growth-adjusted annuity value) of the primary balance-output ratio that is needed to stabilize the debt-output ratio at a target level, or as an indicator of the "sustainable" debt-output ratio consistent with the permanent primary balance-output ratio.

An important shortcoming of this long-run approach is that it fails to recognize that the "long run" is a theoretical construct. In the short run, governments face a budget constraint that does not reduce to the simple formula of the long-run analysis. In a world without uncertainty in which the economy grows gradually to a stationary state, there can be temporarily high debt ratios, or temporarily large primary deficits, that are consistent with government solvency. Furthermore, incurring in such temporarily high debt or deficits could be optimal from a tax-smoothing perspective (see for example the quantitative simulations of the effects of tax reforms in Mendoza and Tesar (1998)). In a world with uncertainty, there can be sufficiently adverse realizations of the primary balance such that a public debt ratio allowed for in a deterministic long-run environment can be too large to be repayable in the short run. Thus, a country that keeps its public debt-output ratio at the level that corresponds to the stationary state of a deterministic model can make serious mistakes (by, for example, not borrowing enough to fully exploit the benefits of economic reforms or borrowing too much relative to is ability to repay).

2.2 Intertemporal Methods.

The realization of the flaws affecting the long-run calculations of sustainable debt ratios led to the development of methods that test whether the intertemporal government budget constraint holds in the data. These methods shifted the focus from analyzing stationary debt-output ratios to studying the time-series properties of the fiscal balance. The aim was to test whether these properties are consistent with the conditions required to satisfy the government's solvency condition. This condition serves as a means to link the short-run dynamics of debt and the primary balance with the long-run solvency constraint of the government.

In their original form (see Hamilton and Flavin (1986)), the intertemporal methods aimed to test whether the data can reject the hypothesis that the condition ruling out Ponzi games on public debt holds. This condition states that at any date t, the discounted value of the stock of public debt $t+j$ periods into the future should vanish as j goes to

infinity:

$$\lim_{j\to\infty}\prod_{k=0}^{j}\left[\gamma_{t+k}/R_{t+k}\right]\gamma b^{g}_{t+1+j}=0.$$

In other words, the debt-output ratio cannot grow faster than the growth-adjusted gross interest rate in the long run. When this no-Ponzi-game (NPG) condition holds, the forward solution of eq. (1) implies that the present value of the primary fiscal balance (as a share of output) is equal to the interest and principal on the outstanding debt-output ratio. Thus, the existing public debt-output ratio is deemed "sustainable" because the government is able to honor it overtime. The survey by Chalk and Hemming (2000) provides a detailed review of the literature on empirical tests of this hypothesis.

By their nature, these intertemporal-budget-constraint methods introduced elements of uncertainty into public debt sustainability analysis, but mostly in an indirect manner. Uncertainty was introduced mainly as a source of statistical error in hypothesis testing. Some of the tests focused on the above NPG condition in expected value while others considered intertemporal optimality conditions to reformulate the test as an orthogonality condition. The orthogonality condition states that the sequence of expected growth-adjusted real interest rates used to discount the "terminal" debt stock must match the intertemporal marginal rate of substitution in private consumption at equilibrium:

$$\lim_{j\to\infty}\prod_{k=0}^{j}E_t\left[\frac{\beta^{t+1+j}u'(c_{t+1+j})}{u'(c_t)}b^{g}_{t+1+j}\right]=0$$

where β is the growth-adjusted discount factor and $u'(c_t)$ is the marginal utility of consumption as a share of output.

Bohn (1998) provides a very useful alternative interpretation of the intertemporal methods that reduces to testing whether or not the primary balance responds positively to increases in public debt. Under his approach, if the primary balance-output ratio and the debt-output ratio are stationary time-series processes, the following regression can be used to test for sustainability:

$$s_t=\rho\, b^{g}_{t}+\alpha\, Z_t+\epsilon_t$$

where s_t is the ratio of the primary fiscal balance over GDP, ϵ_t is a well-behaved error term, and Z_t is a vector of determinants of the primary balance other than the initial stock of public debt. Bohn estimates the equation above including the cyclical variations in U.S. GDP and a measure of "abnormal" government expenditures as elements of Z_t. A positive coefficient ρ indicates that the primary balance displays a linear response that is both positive and systematic to increases in debt. By imposing this property on the budget constraint (1), one can show that $\rho>0$ is sufficient to ensure that the intertemporal government budget

constraint holds. Hence, $\rho > 0$ is Bohn's measure of fiscal sustainability.

Bohn found strong evidence in favor of $\rho > 0$ in U.S. data. In addition,Chapter 3 of the publication by the IMF (2003) shows results of the application of this test for a sample of industrial and developing countries. The results indicate that the sustainability condition holds for industrial countries and for developing countries with low debt ratios, and it fails for developing countries with high debt ratios.

2.3 Stochastic Methods and Methods with Financial Frictions.

Recent developments in public debt sustainability analysis follow two strands. One strand incorporates elements of the financial frictions that have played an important role in recent emerging-markets crises. In particular, public debt in many emerging markets displays "liability dollarization"(i.e., debt is denominated in foreign currency or indexed to the price level but leveraged on public revenues that depend to a large extent on prices, incomes and expenditures of the nontradables sector). As a result, abrupt changes in domestic relative prices that are common in the aftermath of a large devaluation, or a "Sudden Stop"to net capital inflows, can alter dramatically standard long-run calculations of sustainable debt ratios and render levels of debt that looked sustainable in one situation unsustainable in another. Calvo et al. (2003) evaluate these effects for the Argentine case and show that large changes in the relative price of nontradables alter significantly the assessments obtained with standard steady-state sustainability analysis.

The second strand emphasizes the fact that governments, particularly in emerging markets, face significant sources of aggregate uncertainty as they try to assess the patterns of government revenue and expenditures, and hence the level of debt that they can afford to maintain. From the perspective of these stochastic methods, measures of sustainability derived from the long-run approach or the intertemporal analysis are seen as being of limited use for governments that hold large stocks of debt and face large shocks to their revenues. The key question for these governments is not whether their debt is sustainable at a deterministic steady state, or whether in the sample of their recent or historical past the NPG condition holds. The key question is whether their current debt-output ratio is sustainable given the current domestic and international economic environment and its immediate future prospects.

Most of the existing stochastic methods for assessing fiscal sustainability propose alternative strategies for dealing with macroeconomic uncertainty, although these strategies follow non-structural or reduced form representations of the process that drives the dynamics of public debt. For instance, a method proposed at the IMF by Barnhill and Kopits (2003) incorporates uncertainty by adapting the value-at-risk (VaR) principles of the finance indus-

try to public debt instruments in order to estimate the probability of a negative net worth position for the government. A second method recently considered for country surveillance at the IMF (see International Monetary Fund (2003b)) modifies the long-run method to incorporate variations to the determinants of sustainable public debt in the right-hand-side of equation (2). This method is also used to examine the short-term debt dynamics that result from different assumptions about the short-run path of the variables that enter the government budget constraint (1) in deterministic form.[4] The same IMF document proposes a stochastic simulation approach that computes the probability density function of possible debt-output ratios. The IMF's stochastic simulation model, like the VaR approach, is based on a non-structural time-series analysis of the macroeconomic variables that drive the dynamics of public debt (particularly output growth, interest rates, and the primary balance). The difference is that the stochastic simulation model produces simulated probability distributions based on forward simulations of a vector-autoregression model that combines the determinants of debt dynamics as endogenous variables with a vector of exogenous variables. The distributions are then used to make assessments of sustainable debt in terms of the probability that the simulated debt ratios are greater or equal than a critical value.

Xu and Ghezzi (2002)developed a third stochastic method to evaluate sustainable public debt. Their method computes "fair spreads"on public debt that reflect the default probabilities implied by a continuous-time stochastic model of the dynamics of treasury reserves in which exchange rates, interest rates, and the primary fiscal balance follow Brownian motion processes (so that they capture drift and volatility observed in the data). Default occurs when treasury reserves are depleted, and thus debt is deemed unsustainable when the properties of the underlying Brownian motion is such that the expected value of treasury reserves declines to zero (which occurs at an exponential rate).

Although the stochastic methods described above make significant progress in incorporating macroeconomic uncertainty into debt sustainability analysis, they are not robust to the Lucas critique since they follow from a non-structural representation of the determinants of the public debt dynamics. This is not a serious limitation when these methods are used for an ex-post evaluation of fiscal solvency conditions, but it can be a shortcoming for a forward-looking analysis that requires a framework for describing how equilibrium prices and allocations, and hence the ability of the government to raise revenue and service debt, adjust to alternative tax and expenditure policies or other changes in the environment.

The framework proposed in this paper provides an explicit dynamic general equilibrium model of the mechanism by which macroeconomic shocks affect government finances and yields estimates of sustainable public debt that are robust to the Lucas critique. The

[4]For example, deterministic debt dynamics up to 10 periods into the future are computed for variations of the growth rate of output of two standard deviations relative to its mean.

framework determines sustainable debt ratios consistent with the goal to smooth government outlays and respect the commitment to repay implied by this goal, rather than with the exposure to negative net worth or depletion of treasury reserves. This framework also takes into account elements of the financial frictions strand of fiscal sustainability models by incorporating the real-exchange-rate effects identified by Calvo et al. (2002).

2.4 Cross-Country Empirical Regularities of Public Debt and Revenues.

A comparison of the average ratios of public revenue to GDP using data for the period 1990-2002 for 47 industrial and developing countries shows that industrial countries generate significantly larger revenue-GDP ratios in general (see Figure 3).[5] In addition, coefficients of variation show that revenue-output ratios are significantly more stable in industrial countries than in developing countries (see Figure 4). As illustrated in Figure 1 in the Introduction, an unconditional scattered diagram shows that countries with lower coefficients of variation in revenue-output ratios generally support higher mean debt-output ratios. The report by IMF (2003a) shows that the same is true for countries with higher mean revenue-GDP ratios.

The IMF (2003a) report went deeper into a review of the characteristics of the tax structures across countries and found major differences in the averages and coefficients of variation of effective tax rates. The report shows estimates of the averages and the coefficients of variation of effective direct and indirect tax rates for a subset of industrial and developing countries for the period 1970-2000, computed using a simplified version of the methodology proposed by Mendoza et. al (1994). Mean effective tax rates in industrial countries exceed those of developing countries by large margins. The differences in mean effective income tax rates are particularly striking. Industrial countries collect on average more than 30 percent of the total annual flow of payments to factors of production in taxes, while developing countries outside Eastern Europe collect less than 15 percent. From an accounting perspective, this wide gap in mean effective tax rates could reflect smaller statutory tax rates in developing countries, but it also reflects the lower "yields"of the tax systems in developing countries because the effective tax rates are measured in terms of what is actually paid in each tax relative to the relevant tax base.

The differences in the volatility of effective tax rates across industrial and developing countries are also staggering. Coefficients of variation of effective direct and indirect tax rates in large industrial countries are below 4 percent, whereas those for developing countries

[5]This review is largely a summary of the facts documented in Chapter 3 of the report by the IMF (2003a).

are in a similar range only in the case of Chile. In general, developing countries display coefficients of variation in excess of 7 percent and 6 percent in direct and indirect tax rates respectively, and they can be as high as 22 percent for direct tax rates and 17 percent for indirect tax rates.

In summary, developing countries seem to be severely handicapped in their ability to raise government revenues on average and they also face much higher volatility in their revenue base. The stochastic model of public debt sustainability proposed in this paper predicts that these two characteristics of developing countries, combined with structural rigidities in their ability to adjust public expenditures, play a key role in explaining why emerging economies should be expected to sustain lower ratios of public debt to GDP than industrial countries.

3 The Basic Model of a Tormented Insurer's Public Debt

The starting point of the methodology proposed in this paper is the same as in the traditonal methods of debt sustainability: the assumption of a government aiming to make a credible commitment to repay its debt. For example, the stationary condition obtained in the long-run approach makes this assumption implicitly because that conditions implies that both the period government budget constraint and the government's NPG condition hold. Before proceeding to study the two-sector general equilibrium model, it is useful to illustrate some of the key implications that follow from this commitment to repay under uncertainty in a basic one-sector model in which the government follows ad-hoc rules to smooth its outlays and public revenue is an exogenous random process.

A sustainable public debt policy under uncertainty is defined as one that is consistent with the government's solvency condition for given paths of government revenues and outlays. In the case of a government that, as a "tormented insurer," tries to smooth outlays because it has extreme aversion to outcomes that could force it into very low levels of outlays (or, alternatively, because large cuts in government outlays are too costly to undertake), a sustainable debt policy has a key additional feature: The sustainable debt must be consistent with the government's goal not to experience an excessive, sudden collapse in outlays. It is straightforward to see that, if the government wants to rule out non-positive levels of outlays, the government budget constraint implies that public debt must not exceed the debt that can be serviced if public revenues remain "almost surely" at their lowest level for a long period of time. In other words, the aim to avoid non-negative consumption requires the government to credibly commit to be able to repay in all states of nature.

The commitment is credible only in the sense of this "ability to pay" criterion because the government is assumed to be committed to repay from a strategic or "willingness to pay" perspective. However, as argued later, the basic model presented below can be made compatible with a willingness to pay criterion based on credit-market participation constraints for non-contingent debt.

Generalizing the above concept of repayment commitment, it follows that a government credibly committed to service its debt in all states of nature must take into account the probabilistic processes and policy variables that determine the dynamics of the primary balance. In particular, the commitment requires the government to impose on itself a "natural debt limit" (NDL) by which it cannot borrow more than the amount of debt it could service in the worst-case scenario that we label a state of "fiscal crisis." The state of fiscal crisis is the one at which the fiscal authority arrives after experiencing a long sequence of the worst realization of public revenues (that is, if public revenues were to remain "almost surely" at their lowest possible level). In addition, in a fiscal crisis the government can be assumed to have the flexibility to adjust its outlays to some target minimum level. This state of "fiscal crisis" has non-zero probability of occurring even in the long run as long as there are non-zero transition probabilities of moving across all realizations of public revenues. In this environment, the government knows that from today's perspective, there is a chance that it can end up in a fiscal crisis at some future date (after a long sequence of draws of the worst realization of revenues and with expenditures adjusted down to their minimum level). Therefore, to credibly commit to repay (i.e., to be able to do its best to keep government outlays smooth) it must not hold more debt than it could service in a fiscal crisis.

Formalizing the above notion of repayment commitment requires a explicit setup describing the probabilistic dynamics of the components of the primary balance. On the revenue side, the probabilistic process driving public revenue reflects the uncertainty affecting tax rates and tax bases. In emerging markets, this process has two components. One component is the combined result of domestic tax policy and the endogenous response of the tax bases to this policy and the underlying shocks driving business cycles. The second component is largely exogenous to tax policy and reflects the nontrivial effects of fluctuations in commodity prices and exports on public revenues. In Mexico, for example, although oil exports are less than 15 percent of total exports, oil-related revenues still represent more than half of public revenues. On the expenditure side, government outlays adjust largely in response to policy decisions, but the manner in which they respond varies widely across countries. In emerging markets in particular, there is a tendency for fiscal policy to be procyclical, so that expenditures tend to contract during downturns.

The basic model assumes that public revenues follow a Markov chain with a known vector of discrete realizations and a known, non-degenerate transition probability matrix.

The lowest realization of revenues is denoted as $\underline{t}$. The government aims to keep its outlays constant at a positive level g as long as it has access to debt markets. Otherwise, if the NDL binds, government outlays are reduced to $\underline{g}$. The interest rate and the growth are kept constant for simplicity. In this environment, the NDL implies that the public debt ratio must satisfy this constraint:

$$b^g_{t+1} \leq \phi = \frac{\underline{t} - \underline{g}}{R - \gamma}$$

Hence, ϕ is the natural debt limit on the public debt-GDP ratio. The NDL will be lower for governments that have (a) higher variability in tax revenues (for example, if the Markov chain is symmetric so that the absolute value of $\underline{t}$ can be written as a multiple of the standard deviation of public revenues, lower values of $\underline{t}$ reduce ϕ), (b) less flexibility to adjust government outlays, and (c) lower growth rates or higher real interest rates.

By eq. (1) and the above NDL, if the government starts with sufficiently low debt at date 0 and the realization of the revenue-output ratio is $\underline{t}$, the government will keep its outlays at g by increasing b_{t+1}. In an example with zero initial debt, it is straightforward to show that if the government keeps drawing the minimum realization of revenue, it will take at most the T periods that satisfy the following equation for the government to hit the NDL:

$$(R/\gamma)^T = \frac{g - \underline{g}}{g - \underline{t}} \tag{4a}$$

In this example, the highest number of periods that the government can access the debt market (if revenues remain "almost surely" at their minimum) depends on the ratio of the excess of "normal" government spending over its minimum level relative to the excess of normal spending over the minimum level of revenues. At any date in which the debt ratio starts at ϕ and the realization of tax revenues is $\underline{t}$, the budget constraint and the NDL imply that debt remains at ϕ and $g = \underline{g}$. Hence, in this example the government uses debt to keep its outlays as smooth as possible (at the level g) given its capacity to service debt as determined by the volatility of its tax revenues reflected in the value of $\underline{t}$.

The credibility of the announcement setting $\underline{g}$ is an important part of the commitment to repay. The ability to issue debt and the credibility of the announcement that government outlays will be cut in a fiscal crisis depend on each other because a government with a credible commitment to major expenditure cuts can borrow more and hence, everything else the same, this government faces a lower probability to be called to act on its commitment.

The condition defining the natural debt limit has a similar form as the long-run sustainability condition (2). However, the implications of the two conditions for debt sustainability are very different. The long-run condition can easily identify as sustainable a debt-output ratio that is unsustainable once uncertainty of the determinants of the fiscal balance and the

NDL are taken into account. Consider two governments with identical long-run averages of tax revenue-output ratios at 20 percent. The tax revenue-output ratio of government A has a standard deviation of 1 percent relative to the mean, while that of Government B has a standard deviation of 5 percent relative to the mean. If the distributions of tax revenue-output ratios are Markov processes with $\underline{t}$ set two-standard-deviations below the mean, the basic stochastic model would compute the sustainable debt ratio for A using a value $\underline{t}$ =0.18, while for B it would $\underline{t}$ =0.1. The long-run method yields the same debt ratio for both governments at 20 percent, using their common 20 percent average tax revenue-output ratio. In contrast, the basic stochastic model would find that debt ratio unsustainable for both governments and would produce a sustainable debt ratio for B that is significantly lower than that for A.

Another key difference between the stochastic method proposed here and the long-run approach is how the two view sustainable debt ratios. In the long-run analysis, the debt ratio is viewed as either a target to which a government should be forced to move to, or as the anchor for a target primary balance-GDP ratio that should be achieved by means of a policy correction. In contrast, the NDL of the stochastic model only sets an upper bound for public debt. The NDL is not the equilibrium or sustainable debt policy consistent. It does plays a central role in determining both the equilibrium path and the sustainable debt, but it is not the model's measure of sustainable debt. Depending on the probabilistic and policy assumptions driving taxes and expenditures, a country can exhibit levels of debt lower than ϕ most of the time, and may take a very long time on average to enter a state of fiscal crisis or even never arrive at it.

Table 1 presents illustrative calculations of NDLs in the basic model under alternative assumptions about the variability of public revenue-output ratios, the level of $\underline{g}$, and the world interest rate. The table takes typical values of the growth rate, the mean public revenue-output ratio, and the mean ratio of total government outlays in emerging markets (3.7, 22.9 and 21.7 percent respectively). The data reported in International Monetary Fund (2003a) show that these figures are similar to the corresponding figures observed in Mexico in the last 20 years; this similarity will facilitate the comparison of the results in this section with the numerical simulations applied to the case of Mexico later in the paper. Case 1 in the Table shows natural debt limits for a "low risk" environment in which the real interest rate is 6.5 percent. Case 2 considers a "high risk" environment in which the interest rate is 10 percent. The public revenue-output ratio is assumed to follow a discrete, symmetric Markov process with a minimum realization $\underline{t}$ set two standard deviations below the mean. The Table shows natural debt limits for coefficients of variation in public revenue ranging from 4.4 to 13.1 percent and for commitments to expenditure cuts during fiscal crises of 2 to 8 percentage points of GDP. Scenarios that yield negative debt limits are reported as zeros,

since negative debt limits indicate that in those cases the government cannot borrow.

Coefficients of variation of public revenue-output ratios in excess of 4 percent are very common in emerging economies (see IMF (2003a)). For a "low-risk" emerging market with these characteristics, Table 1 indicates that the government would need to commit to fiscal cuts of at least 4 percentage points of GDP in order to attain debt limits that include observed average debt-GDP ratios. Moreover, natural debt limits are very sensitive to modest changes in the volatility of revenues and the commitment to expenditure cuts. Economies with coefficients of variation in public revenue in excess of 6.5 percent and commitments to expenditure cuts of 2 percent of GDP cannot sustain positive public-debt output ratios with a credible commitment to repay. On the other hand, if these economies can reduce the volatility of the public revenue-output ratio to 4.3 percent and/or make credible commitments to larger expenditure cuts of 4 percentage points of GDP or more, their natural debt limits would rise sharply.

The results in Table 1 can also be used to explain why the governments of industrial countries can in general sustain higher debt ratios than those of emerging economies. Large industrial countries exhibit coefficients of variation in public revenues ranging between 2 and 4 percent, whereas the coefficients of variation in developing countries exceed 5 percent in general and are above 8 percent for several middle-income emerging countries like Argentina, Brazil, Korea, Indonesia and Mexico (see IMF (2003a)). Moreover, the gap between interest rates and growth rates is smaller for industrial countries, as they pay negligible country risk premia. These factors imply that, from the perspective of the model, the governments of industrial countries are capable of making credible commitments to repay higher levels of debt-GDP ratios, and hence if macroeconomic conditions require it they are able to borrow more than the governments of emerging countries.

As explained earlier, the natural debt limit is an important part of the analysis but in general it does not correspond to the model's sustainable debt ratio. The model provides information on the short- and long-run dynamics of sustainable debt ratios that satisfy the government budget constraint above the NDL along an equilibrium path. This can be illustrated with a quantitative application calibrated to a representative emerging market economy. The simulations are conducted at a quarterly frequency. The growth rate is set to 3.7 percent. The world real interest rate is set to be consistent with the annual real interest rate of 6.5 percent widely used in Real-Business-Cycle models. Public revenues follow a Markov process with the following mean, standard deviation and first-order autocorrelation: 0.229, 0.185 and 0.601. The GDP share of total government outlays (current purchases plus transfer payments) in normal times is set to 0.217, and the mean public debt-GDP ratio, 0.459, over the same sample period. The minimum value of the ratio of government outlays to GDP is set to obtain a natural debt limit of 0.5. This implies setting the minimum

outlays-GDP ratio in a fiscal crisis at 83.5 percent of the same ratio in normal times.

The simulations consider a grid of initial public debt-GDP ratios that spans the interval from 0 to 0.5 (which is the natural debt limit). The short-run dynamics of sustainable debt can be traced from any initial public debt ratio in this interval. Figure 5 shows average and "extreme" estimates of the number of periods that it takes to hit a fiscal crisis (i.e., to hit the NDL) for different initial debt ratios. From each initial condition at present, there are different stochastic paths that public debt, revenues and outlays can follow in the future, and each of these paths features a different number of periods to hit a fiscal crisis. The figure reports the mean and the mean plus two standard deviations of this measure of time to a fiscal crisis (the latter is referred to as the "extreme" estimate). Depending on initial conditions, there are scenarios in which a fiscal crisis never occurs (particularly for low initial debt ratios). In these cases, the measure in the vertical axis goes to infinity and hence these cases are ignored in Figure 5.

Figure 5 shows that initial public debt ratios of 25 percent or less never lead to a fiscal crisis. The average time to a fiscal crisis is high (at 24 quarters or more) for initial debt ratios below 40 percent, but it declines rapidly to less than 10 quarters for initial debt ratios around 45 percent. Moreover, even though it can take long to hit a fiscal crisis on average with a low initial debt ratio, there are sequences of adverse realizations of public revenue within the two-standard-deviations boundary that lead to a fiscal crisis much sooner. This is illustrated by the extreme measure of the time to a fiscal crisis for an initial debt ratio of 30 percent. While the average time to a fiscal crisis is above 40 quarters, the two-standard-deviation scenario leads to a fiscal crisis before 8 quarters. These large differences between the mean and extreme number of periods to hit a fiscal crisis for *low* initial public debt ratios are an striking illustration of the importance of uncertainty in analyzing public debt sustainability. However, they also point to one important weakness of the basic model: this model can produce multiple long-run distributions of public debt or degenerate distributions with their mass concentrated in the extremes (i.e., public debt eventually moves to the upper or lower bounds of the debt grid in the long run). We produced a large set of 1000-quarters stochastic simulations all starting from a public debt ratio of 10 percent and found that about half of them converged to the debt limit and the other half converged to zero.

It is interesting to note that these properties of the long-run distribution of debt in this basic model are in line with the findings of Barro's (1979) classic work on public debt and tax smoothing. In Barro's setup, the long-run level of debt and its short therm dynamics depend entirely on initial conditions. Still, these unappealing features of the basic model raise questions about some of its assumptions. First, the government expenditure rule keeping outlays constant except when doing so threatens fiscal solvency is too rudimentary to become an effective insurance mechanism for the government, and could be dominated by

other rules capable of observing the commitment to fiscal solvency. Second, public revenues are entirely driven by an exogenous random process and hence there is no feedback between the government's debt and expenditure policies and its ability to raise revenue. Third, by assumption, the model rules out the possibility of default. The model examined in the next section makes progress in addressing the first two limitations and some of the implications of default risk are considered next.

Default has been ignored intentionally because the model is intended as a forward-looking policy tool aimed at determining levels of public debt consistent with fiscal solvency, rather than as a positive theory of public debt that tries to explain the past experience of defaulting countries. Still, the notion that default risk is a determinant of the cost of public debt that should be part of debt sustainability analysis is worth considering. A straightforward manner to introduce default risk into the basic model is to consider the case of a lender interested in designing a credit contract that enforces the government's "participation constraint." This constraint requires the government to always find it preferable (from the point of view of its strategic payoff function) to fulfill its financial obligations than to go on default. A risk-averse lender with a constant-relative-risk-aversion (CRRA) payoff function would want the participation constraint to hold at all times, since the lender would not want to take the risk of not being paid. If non-contingent public debt is the only financial instrument available, the lender could manage the risk of default by comparing the government's payoff under the credit relationship and under financial autarky and imposing a limit on government borrowing. This limit would be the smallest debt for which the government is indifferent between defaulting and repaying across all possible realizations of the random variables that determine the government's payoff. Given this debt limit, we could go back to the condition that defines the basic model's NDL and solve it for an "effective" real interest rate that would enforce the debt limit consistent with the participation constraint as the natural debt limit. The difference between this interest rate and the risk-free rate can be viewed as a default risk premium.

4 The Stochastic General Equilibrium Model

This section generalizes the basic model of the last section to the case of a two-sector dynamic, stochastic general equilibrium model of a small open economy. The general equilibrium model differs in three key respects. First, the government chooses optimally its expenditures policy (including the allocations of purchases of tradable and nontradable goods). Second, public debt is now a "dollarized liability" denominated in units of tradables but financed to a significant extent on public revenues collected from the nontradable sector. Third, public revenue is no longer an exogenous random process. Instead, tax

revenues depend in part on the endogenous dynamics of the relative price of nontradables (which is the unique determinant of the model's real exchange rate).

Non-state-contingent public and private bonds are the only financial assets available in this economy so markets for contingent claims are incomplete. The incompleteness of financial markets, coupled with the CRRA nature of preferences, induces public and private precautionary savings effects, as households and the government seek to self-insure against the non-diversifiable risk of very low consumption if a long series of adverse income shocks occurs. These effects lead them to impose on themselves natural debt limits which, as in the case of the basic model, represent implicit credible commitments to remain able to repay debts even if incomes were to remain "almost surely" at their lowest realizations. The government's NDL is now determined jointly by the properties of the equilibrium processes of tax revenues, government outlays, the relative price of nontradables and the interest rate.

The economy grows in the long run at a constant exogenous gross rate γ. This common trend is shared by consumption allocations, sectoral income endowments of tradables and nontradables, and net exports. In the short run, the economy displays fluctuations around this trend induced by stochastic shocks to the sectoral endowments. The analysis focuses on the deviations from trend and hence all variables are detrended by expressing them as ratios relative to aggregate output in units of tradable goods (the price of tradables is the model's numeraire). Detrended variables are denoted by lowercase letters.

4.1 The Private Sector

Households derive utility from consumption of private and public goods, with each represented by composites of tradables and nontradables. We assume for simplicity a period utility function with separable preferences across private and public goods. Households collect government transfers and draw stochastic endowment incomes of tradable and nontradable goods each period. Endowment incomes are taxed by the government at a pre-determined, time-invariant income tax rate. Households have access to a world credit market of one-period real bonds issued at a world-determined real interest rate in units of tradables. They can also buy bonds issued by the domestic government denominated in units of tradables.

The representative household chooses stochastic sequences of consumption and bond holdings so as to maximize Epstein's (1983) Stationary Cardinal Utility:

$$\max_{\{c_t^T, c_t^N, b_{t+1}\}_{t=0}^{\infty}} E\left[\sum_{t=0}^{\infty} \exp\left\{-\sum_{\tau=0}^{t-1} \beta \ln(1 + c_\tau + g_\tau)\right\} \left(\frac{c_t^{1-\sigma}}{1-\sigma} + \frac{g_t^{1-\sigma^g}}{1-\sigma^g}\right)\right] \tag{5}$$

where c_t and g_t are CES composites of private and public goods defined by the following

functions:

$$c_t = c(c_t^T, c_t^N) = \left[\omega_h \, (c_t^T)^{-\eta} + (1-\omega_h)(c_t^N)^{-\eta}\right]^{-1/\eta}$$

$$g_t = g(g_t^T, g_t^N) = \left[\omega_g \, (g_t^T - \underline{g}^T)^{-\eta} + (1-\omega_g)(g_t^N - \underline{g}^N)^{-\eta}\right]^{-1/\eta}$$

In these CES composite goods, ω_g and ω_h are the weighing factors and η determines the elasticity of substitution between consumption of tradable and nontradable goods, $(1/(1+\eta))$, which is assumed to be the same for private and public expenditures. $\underline{g}^T$ and $\underline{g}^N$ represent "basic-needs" levels of government consumption in tradables and nontradables respectively. The role of these basic needs is discussed later in this section.

In eq. (5), σ is the private sector's coefficient of relative risk aversion and β is the elasticity of the rate of time preference with respect to $1 + c_t + g_t$. We use preferences with endogenous discounting for the standard reason: to ensure that the small open economy model can support a unique, invariant long-run distribution of assets (see Arellano and Mendoza (2003) for details). The endogenous discount factor introduces an impatience effect because the rate of time preference increases with past consumption levels. However, since in quantitative applications this effect has shown to be negligible (see Mendoza (1991)), we follow Schmitt-Grohe and Uribe (2001) in assuming that atomistic agents do not internalize the effects of their consumption plans on the rate of time preference.

Households maximize utility subject to the following budget constraint for $t = 0, ..., \infty$:

$$c_t^T + p_t^N c_t^N + \gamma b_{t+1} \leq (1-\tau)(y_t^T + p_t^N y_t^N) + R\, b_t + p_t^N\, w \tag{6}$$

The uses of income in the left-hand-side of the equation consist of purchases of tradable goods, nontradable goods and bonds all valued in units of tradables (p_t^N is the relative price of nontradables in units of tradables). The household chooses an aggregate bond position b_{t+1}which is composed of domestic government bonds, b_{t+1}^g, and international bonds, b_{t+1}^I, (i.e., $b_{t+1} \equiv b_{t+1}^g + b_{t+1}^I$). Without loss of generality, the government is assumed to issue its bonds at the same terms that domestic households face in the international bond market. Hence, domestic households are indifferent between both types of assets and the two pay the same world-determined gross real interest rate R.

The right-hand-side of (6) represents the household after-tax income which has three components. First, stochastic endowment income collected from the tradable and nontradable sectors, y_t^j for $j = T, N$, which is taxed by the government at the income tax rate τ. Second, payments of interest and principal on total bond holdings, $R\, b_t$. Third, transfer payments from the government $p_t^N w$, which are set at a fixed level in units of nontradables and thus have a value in units of tradables that moves together with the real exchange rate, p_t^N.

Endowment incomes follow random processes defined as perturbations of exponential support around mean values :

$$y_t^T = y^T \exp(e_t^T); \qquad y_t^N = y^N \exp(e_t^N) \tag{7}$$

The exponential shocks are Markov processes with known vectors of realizations (I values for e^T, J values for e^N, where I and J are positive integers). Define $\overline{e}$ as a discrete-valued random vector that includes the joint realizations of endowments shocks (i.e., the $I \times J$ pairs (e_h^T, e_i^N), for i=1,...,I, j=1,...,J, that represent all the combinations of the possible realizations of each shock). Hence, $\overline{e}$ is a column vector with $s = I \times J$ rows. The joint states included in $\overline{e}$ follow a Markov process defined by $(\overline{e}, P, \pi_0)$, where P is an $s \times s$ state-transition probability matrix of moving from each joint state represented by a pair in s to another joint state in s in one period and π_0 is the initial probability vector.

Households choose optimal plans for bond holdings, consumption of tradables and consumption of nontradables, taking as given endowment incomes, prices and fiscal policy variables (i.e., public consumption of tradables and nontradables, the income tax rate and the level of transfers). The first-order conditions of the households' maximization problem are the budget constraint in equation (6) and the following conditions

$$\left(\frac{1-\omega_h}{\omega_h}\right)\left(\frac{c_t^T}{c_t^N}\right)^{1+\eta} = p_t^N \tag{8a}$$

$$\left[c(c_t^T, c_t^N)\right]^{-\sigma} c_{c^T}(c_t^T, c_t^N) = (1 + c_t + g_t)^{\beta} E_t \left\{\left[c(c_{t+1}^T, c_{t+1}^N)\right]^{-\sigma} c_{c^T}(c_{t+1}^T, c_{t+1}^N) R\right\} \tag{8b}$$

where $c_{c^T}(c_t^T, c_t^N)$ is the derivative of the CES aggregator with respect to consumption of tradables. These two conditions have straightforward interpretation. Eq. (8a) equates the marginal rate of substitution in consumption of tradables and nontradables with the relative price of nontradables. Eq. (8b) is the consumption Euler equation for tradable goods that equates the marginal cost and benefit of an additional unit of savings in foreign bonds or domestic government bonds. The household is indifferent between the two financial instruments, but at equilibrium the composition of the portfolio is well-defined because the supply of government debt is limited.[6]

4.2 The Public Sector

The government chooses public expenditures so as to maximize its contribution to private utility, which is the CRRA component of private utility that depends on public expenditures.

[6]The domestic government sets its supply of debt so that it crosses the demand from households that is infinitely-elastic at the world interest rate at the level of debt it needs to sell.

This feature of the model produces the desire to smooth public outlays that the government had in the basic model as an endogenous outcome that will be reflected in the government's optimal plans. CRRA utility and the incompleteness of asset markets also provides the same incentives as in the basic model for the government to engage in precautionary savings and impose on itself a natural debt limit.The commitment to expenditure cuts in a fiscal crisis is also a feature of this model that is captured by the Stone-Geary formulation of the CES composite of public expenditures.

The government chooses levels of public expenditures and a debt policy so as to maximize its contribution to private utility,[7]

$$\max_{\{g_t^T, g_t^N, b_{t+1}^g\}_{t=0}^{\infty}} E\left[\sum_{t=0}^{\infty} \exp\left\{-\sum_{\tau=0}^{t-1} \beta \ln(1+c_\tau+g_\tau)\right\}\left(\frac{g_t^{1-\sigma^g}}{1-\sigma^g}\right)\right] \tag{9}$$

subject to the following government budget constraint for $t = 0, ..., \infty$,

$$\gamma b_{t+1}^g = b_t^g R + g_t^T + p_t^N(g_t^N + w) - \tau(y_t^T + p_t^N y_t^N) \tag{10}$$

As this budget constraint indicates, the government collects revenues from the nontradables sector and buys nontradable goods and services, and therefore the dynamics of the equilibrium relative price of nontradables affect the dynamics of total tax revenues and outlays valued in units of tradables. Since debt is a "dollarized liability" denominated in units of tradables, it follows that the dynamics of the price of nontradables will be key for determining the government's ability to issue and service debt.

Government transfers w are introduced to capture payments for welfare and entitlement programs that in most countries represent a large fraction of total government outlays and pertain mainly to the nontradables sector. Since welfare programs are also the most inflexible component of government outlays, they are modelled as a fixed quantity of nontradable goods (this assumption is also in line with the aim of the government to act as a "social insurer").

The government's optimality conditions for $t = 0, ..., \infty$ are the budget constraint (10) and the following conditions:

$$\left(\frac{1-\omega_g}{\omega_g}\right)\left(\frac{g_t^T - \underline{g}^T}{g_t^N - \underline{g}^N}\right)^{1+\eta} = p_t^N \tag{11a}$$

[7]The government is benevolent in the sense that it tries to maximize its contribution to private utility, but it does so ignoring the indirect effects of its actions on private consumption (i.e., it takes c_t as given). This is a reasonable assumption because in the model there are no direct distortions from fiscal policy on private actions. An alternative interpretation is that the government is not benevolent. Instead, the government and the private sector maximize their own payoffs. The private payoff depends on the governments' but both players act atomistically and take all prices as given.

$$\left[g(g_t^T, g_t^N)\right]^{-\sigma} g_{g^T}(g_t^T, g_t^N) = (1 + c_t + g_t)^{\beta} E_t \left\{ \left[g(g_{t+1}^T, g_{t+1}^N)\right]^{-\sigma} g_{g^T}(g_{t+1}^T, g_{t+1}^N) R \right\} \quad (11b)$$

Condition (11a) equates the marginal rate of substitution in public consumption of tradables and nontradables with the relative price of nontradables. The marginal rate of substitution includes the effect of the basic needs $\underline{g}^T$ and $\underline{g}^N$. For a given level of g_t^T (g_t^N), raising the value of $\underline{g}^T$ ($\underline{g}^N$) increases the marginal utility of tradables (nontradables) expenditures. The expenditure Euler equation (11b) is the efficiency condition that characterizes optimal public debt management. The government aims to use public debt so as to equate the marginal cost and benefit of issuing an extra unit of debt to reallocate expenditures from the current period to the next. The Euler equation is expressed in terms of tradables expenditures, but Dornbusch's (1983) classic treatment of a condition like this applies to this model, so that duality principles can be used to re-write the Euler equation in terms of aggregate consumption g and the "consumption-based real interest rate" that takes into account the rate of depreciation of the real exchange rate (i.e. the change in the CES relative price index of g, which is an increasing function of p^N).

A key feature of the optimal public debt plan is that, because the marginal utility of the government's contribution to private utility goes to infinity as public expenditures approach their basic needs levels, the government never want to be exposed to the risk of being unable to provide for at least its basic needs. The extreme aversion to this situation leads the government to impose on itself a Natural Debt Limit because, as in the case of the basic model, the government needs to make sure that it does not borrow more than it can service if the economy were to remain "almost surely" in the worst state of public revenues and public expenditures were to drop to their basic needs levels. Thus, in this economy with uncertainty and incomplete markets, respecting the NDL is the *only* mean the government has to ensure that it is not exposed to the risk of being unable to provide at least the basic needs levels of expenditures. Respecting the NDL implies the government has the incentive to engage in precautionary savings to build up a buffer stock of assets. In this way, the government can make the best use it can of the debt market to smooth expenditures by retiring debt when revenues are high and issuing debt when revenues are low.

As in the basic model, the NDL has the equivalent interpretation of representing a credible commitment to remain able to repay in all states of nature. The advantage is that modelling the decision-making problem of the government makes it clear that this commitment is not an ad-hoc assumption that trivially neglects the reality of sovereign defaults, but it is a strong implication of the "tormented insurer" behavior of the government implied by its aim to try to smooth its outlays and the frictions of financial markets. As argued earlier, the NDL represents only a commitment to remain *able* to repay given a probabilistic process for the exogenous shocks hitting the economy, it does not preclude

that the government may default because it is *unwilling* to repay for strategic reasons or because it can become unable to repay as a result of large, unexpected shocks.

The NDL of the government in this model economy can be written as follows:

$$\phi = \frac{1}{R-\gamma} \times \min\left[\tau\left(y_t^T + p_t^N y_t^N\right) - \left(\underline{g}^T + p_t^N(\underline{g}^N + tr)\right)\right] \tag{12}$$

In this expression, the lowest realizations of endowment incomes are determined by the characteristics of the exogenous Markov processes of income shocks and the levels of basic needs of public expenditures are an exogenous policy choice. However, because of liability dollarization, the NDL as a whole is an endogenous variable because it depends on the equilibrium relative price of nontradables. Thus, in this economy the natural debt limit is linked to the properties of equilibrium real exchange rates and as a result it cannot be computed separately from the solution of the model's competitive equilibrium.[8]

As with the basic model, the NDL and the sustainable or equilibrium level of public debt are not the same in general. The sustainable level of debt is represented by the government's optimal debt policy that satisfies its budget constraints and the optimality conditions examined above. As we illustrate in the quantitative application of the next Section, the sustainable level of debt depends, among other things, on the the volatility and persistence of exogenous shocks, the levels of the government's basic needs, the relative size of the tradables and nontradables sectors and equilibrium allocations of the private sector.

4.3 The Competitive Equilibrium

The economy's competitive equilibrium is defined by sequences of allocations $\{c_t^T, c_t^N, b_{t+1}^g, b_{t+1}^I, b_{t+1}, g_t^T, g_t^N\}_{t=0}^{\infty}$, and a sequence of prices $\{p_t^N\}_{t=0}^{\infty}$, such that: 1) Households choose $\{c_t^T, c_t^N, b_{t+1}\}_{t=0}^{\infty}$ to maximize utility subject to their budget constraint, taking as given an initial stock of assets b_0, the stochastic processes driving the sequence of endowments $\{y_t^T, y_t^N\}_{t=0}^{\infty}$, the sequence of relative prices $\{p_t^N\}_{t=0}^{\infty}$, and the fiscal policy characterized by w and the sequences of government expenditures $\{g_t^T, g_t^N\}_{t=0}^{\infty}$. 2) The government chooses sequences of expenditures $\{g_t^T, g_t^N\}_{t=0}^{\infty}$ and public debt $\{b_t^g\}_{t=1}^{\infty}$ so as to maximize its contribution to private utility subject to the government budget constraint, given the initial stock of public debt b_0^g, the stochastic processes driving the sequences of endowments $\{y_t^T, y_t^N\}_{t=0}^{\infty}$, and the sequence of prices $\{p_t^N\}_{t=0}^{\infty}$. 3) The following market-clearing conditions hold for $t = 0...\infty$:

$$y_t^T = -b_t^I R + \gamma b_{t+1}^I + c_t^T + g_t^T \tag{13a}$$

[8]Notice that, other things equal, ϕ raises with p^N if $(\tau \min(y_t^N)) > \underline{g}^N$, and ϕ falls with p^N if $(\tau \min(y_t^N)) < \underline{g}^N$.

$$y_t^T = c_t^N + g_t^N \tag{13b}$$

This competitive equilibrium can be represented as the solution to a social planner's problem that maximizes a weighed sum of the utility functions of private and public goods subject to the market-clearing conditions. We used this planner's problem representation of the equilibrium to design the solution method used to solve the model in the next section.

In the planner's problem, the planner's weighs have an analogous interpretation to the weighs typically used in two-country real-business-cycle models, in which the weighs are linked to the distribution of wealth across countries. The difference is that in this case the weighs are linked to the distribution of the small open economy's wealth across the private and public sectors of the economy. In a simple case of a one-good model with zero initial assets, a deterministic endowment, logarithmic utility, and a constant rate of time preference equal to the real interest rate, the government's weigh in the planner's problem needed to replicate the competitive equilibrium is equal to the present value of tax revenue. The weigh of the private sector is the present value of after-tax income. In the more general model we specified here, the present values are conditional expected present values that take uncertainty into account and the weighs are also a function of the coefficients of relative risk aversion.

5 A Quantitative Application: The Case of Mexico

This Section explores the quantitative implications of the model using a series of numerical simulations calibrated to the Mexican case. The model is set to a quarterly frequency and the calibration strategy follows closely the calibration to Mexico described in Mendoza (2002).

The calibration exercise is conducted in two stages. First, parameter values are set so that the deterministic, balanced-growth stationary equilibrium of the model matches key characteristics of the Mexican economy reflected in time-series averages from national accounts and fiscal policy data. Second, the properties of the Markov processes of endowment shocks are set so as to match the statistical moments of the observed fluctuations of Mexico's tradables and non-tradables output.

The averages from the data that the model is set to match in the first stage of the calibration are the following:

1. The public debt-GDP ratio is set to the quarterly equivalent of an annual ratio of 45.9 percent, which is Mexico's average public debt-GDP ratio over the period 1990-2002 in the data reported in IMF (2003a).

2. The ratio of net foreign assets to GDP is set to the quarterly equivalent of an annual

ratio of -35 percent, which is the average for Mexico in the data on net foreign assets constructed by Lane and Milesi-Ferretti (1999)).

3. The ratio of tradables GDP to nontradables GDP is set to 64.8 percent to match the average for Mexico over the 1988-1998 period. This sample is shorter than for the national aggregates because a consistent set of sectoral national accounts that is required to construct the accounts for the tradables and nontradables sectors is only available from 1988 (for details on the sectoral decomposition of the data see Mendoza (2002)).

4. The sectoral ratios of national accounts measures of consumption, government purchases and investment to GDP in the tradables and nontradables sectors are also set to match Mexican averages over the same period (the ratios are 66.5, 0.09 and 20.0 percent respectively for tradables and 70.8, 14.1 and 15.1 percent respectively for nontradables).

5. The average GDP growth rate from the same source and sample is 1.5 percent per year.

The calibration also sets value for the world real interest rate, the coefficient of relative risk aversion and the elasticity of substitution in consumption of tradables and nontradables. The risk aversion coefficient and the real interest rate are set to values commonly used in quantitative applications of equilibrium business cycle models (σ_i for $i = h, g$ is set at 2 and the net real interest rate is set at 6.5 percent per year). The elasticity of substitution between tradable and nontradable goods $1/(1+\eta)$ is set to the estimate for developing countries produced by Ostry and Reinhart (1992. They estimated the elasticity at 0.76 which implies η=0.316. In addition, the deterministic steady state values of total income in units of tradables (i.e., $y = y^T + p^N y^N$) and p^N are normalized to 1.

Given the constraints imposed by the information taken from Mexican data listed in (1)-(5) and the above values of σ_h, σ_g, η, R, y and p^N, the model's steady-state equilibrium conditions produce values for the CES weighing parameters ($\omega_h = 0.342$ and $\omega_g = 2431$), the income tax rate ($\tau = 0.239$), the ratio of transfer payments to GDP $w = 0.096$, the elasticity of the rate of time preference function $\beta = 0.135$.. In solving for this deterministic steady state equilibrium, investment expenditures are incorporated as lump-sum expenditures so that the model can match the observed consumption and government expenditures shares (even though the model abstracts from capital accumulation decisions). Finally, the basic needs of government expenditures are set to equal to zero in this baseline calibration.

The two endowment shocks are modelled as two-point, symmetric Markov processes that follow the rule of simple persistence as in Mendoza (2002). The shocks are required to

share the same first-order autocorrelation coefficient but they can have different standard deviations and their contemporaneous correlation can vary within a certain range inside the unit circle. The standard deviation of the tradable (nontradable) endowment shock is set to 3.68 (2.74) percent and the common autocorrelation of the shocks in both sectors is equal to 0.60; the correlation between the endowment shocks is equal to 0.685. These figures correspond to the standard deviation and first-order autocorrelation of the cyclical component of tradable GDP in Mexico as estimated by Mendoza (2002) using quarterly, seasonally-adjusted data for the period 1980:1-1997:4.

The model is solved by parameterizing the expectation function in the right-hand-side of the Euler equation of the planner's problem, following Marcet's parameterized expectation method. The expectations function is approximated using Chebychev polynomials that depend on the state variables of the model (i.e. the two endowment shocks and the stock of international assets). A detailed description of the solution method is provided in an Appendix available from the authors on request.

5.1 Results from the Baseline Calibration

The key features of the stochastic competitive equilibrium solved using the baseline calibration are illustrated in Figure 6 and Table 3. Table 3 shows the key statistical moments of the main macroeconomic aggregates computed with the long-run distribution of the model's state variables (b^I, e^T, e^N).[9] Figure 6 shows the long-run probability distribution of the ratio of public debt to GDP induced by the long-run distribution of the state variables via the intertemporal government budget constraint. The long-run average of b^g is 45.3 percent, which is close to Mexico's mean public debt ratio of 45.9 percent. The model was calibrated so that the *deterministic* steady state mimics this 45.9 percent debt ratio but the model with uncertainty will in general produce a mean debt ratio lower than this deterministic stationary state because of the precautionary savings effect (which is absent from a deterministic environment). The reduction of 0.6 of a percentage point in the mean debt-output ratio of the stochastic simulation is the result of precautionary savings. However, this small reduction suggests that the precautionary savings effect is not very strong in the baseline calibration.

The coefficient of variation of the public debt ratio is 7.3 percent, which is about twice as large as the coefficient of variation of total GDP (at tradables goods prices). This indicates that public debt ratios as low 0.32 or as high as 0.6 are within the two-standard-deviation

[9] All of the model's endogenous variables are functions of these three state variables in the recursive formulation of the planner's problem that represents the competitive equilibrium of the economy. Hence, the long-run distributions of all the endogenous macroeconomic aggregates and their corresponding moments are induced by the long-run distribution of (b^I, e^T, e^N).

thresholds of the public debt distribution (see also Figure 6). It follows from this result that, in the case of Mexico, levels of public debt above 60 percent of GDP are "unsustainable," in the sense that they are inconsistent with the realizations of debt ratios that are compatible with fiscal solvency and the stochastic competitive equilibrium of the economy. Fluctuations in public debt are highly persistent (the first-order autocorrelation of the debt ratio is 0.99). Public debt shares this near-unit-root behavior with total assets and net foreign assets, and this is a typical result in RBC models of the small open economy. Public debt is also highly countercyclical. The correlation between public debt (which is denominated in units of tradables) and total GDP valued at tradables prices is -0.95. The government issues debt when output is low and retires debt when output is high. Alternatively, taking into account that debt is a dollarized liability, we can say that the public debt ratio rises when the real exchange rate is low and drops when the real exchange rate is high (the relative price of nontradables is highly procyclical). This is a key finding in that is line with the conclusions of Calvo et al. (2003): a fall in the real exchange rate can increase the debt ratio (and bring it closer to the 60 percent two-standard-deviation threshold) even if there is no change in the fiscal policy stance.

The model also yields "sustainability" measures for external debt and the total asset position. The long-run average of the foreign asset-GDP ratio is -0.337. Hence, the long-run average of the foreign debt ratio that is consistent with external solvency is about 34 percent of GDP. This debt ratio is highly volatile, with a coefficient of variation of 21 percent, which is nearly 6 times larger than the coefficient of variation of output. Since fluctuations in foreign debt are also highly persistent, it follows that sequences of bad shocks can make this economy's foreign debt (assets) grow to levels as high (low) as 75 (8) percent of GDP, and that on average it will take a long time to reduce the debt (assets) from these high levels.

The moments of government expenditures in Table 3 show the outcome of the government's aim to smooth its outlays. The coefficient of variation of total government expenditures is 2.6 percent, which is significantly lower than the coefficient of variation of public revenues of 3.6 percent. The model captures the procyclical nature of fiscal policy: public expenditures are procyclical. The CES aggregate of expenditures and government purchases of tradables are highly correlated with GDP. The correlation between government purchases at tradables prices and GDP is about 0.56. Government expenditures in nontradables are uncorrelated with GDP, but since the real exchange rate (or the price of nontradables) and the expenditure in tradables are highly procyclical, total government purchases in tradables prices end up displaying a procyclical pattern. Thus, the procyclical pattern of the real exchange rate is an important factor driving the procyclial nature of total government expenditures.

The procyclical nature of fiscal policy is also reflected in the primary balance. The primary balance is negatively correlated with total output at tradables prices (the coefficient of correlation is -0.4). Thus, the primary fiscal balance tends to move into deficits when output is high and surpluses when output is low. Note that in this model tax revenues are perfectly correlated with GDP at tradables prices by construction, given the assumption that incomes are exogenous endowments.

Figure 6 and Table 3 focus on the long-run equilibrium. Figure 6, for example, shows the probabilities of reaching different sustainable debt ratios in the long run, regardless of the debt ratio observed today. In contrast, Figures 7-9 provide information about the short-run dynamics of public debt. Figure 7 shows the short-run transitional dynamics of the distribution of public debt 1, 5, 10 and 25 years ahead of the initial date, starting from a public debt ratio of 30 percent of GDP. One year into the future, the debt ratios consistent with fiscal solvency, once uncertainty and the equilibrium response to that uncertainty are taken into account, are only those in the narrow range of 32-35 percent of GDP. The distribution widens and shifts to the rigth as we increased the forecast period. Ten years into the future, the debt ratio can vary between 30 and 40 percent of GDP. Eventually, as the forecast period increases beyond 25 years, the debt distribution converges to the long-run distribution shown in Figure 6. The plots in Figure 7 show that this convergence process is very slow, which is consistent with the high first-order autocorrelation of the debt ratio. These findings suggest that one should raise serious doubts about the sustainability of public debt when year-on-year variations in debt ratios are larger than 1 or 2 percentage points of GDP.

Figure 8 shows forecasting functions of the equilibrium Markov process of public debt conditional on three initial debt ratios (the calibrated average of 45.9 percent and values 30 percent above and below this average). These forecasting functions can be interpreted as conditional impulse response functions that plot the "mean forecast" response of the public debt ratio to initial impulses equal to plus and minus one-standard deviation shocks to the endowments, conditional on the three different initial debt ratios. The data are plotted as deviations from the long-run average of the public debt ratio shown in Table 3 (0.453). The plots show that on impact the endowment shocks trigger deviations of about 4-5 percent in the debt ratio, relative to the initial condition. Because the shocks themselves have a certain degree of positive persistence and they induce high persistence in GDP, debt rises (falls) for a short period of time after a negative (positive) shock hits, and then debt converges monotonically to the long-run average (i.e., the deviations from the mean go to zero). The message is similar to the one derived from Figure 7: the mean forecast of public debt from any given initial conditions shows that, for typical (i.e., average) realizations of the endowment shocks, debt ratios vary very gradually over the short run. Thus large year-

on-year movements in debt ratios should not be regarded as consistent with fiscal solvency.

Figure 8 emphasizes the predicted *mean* public debt ratios at each point in time conditional on an initial debt ratio and an initial impulse in the form endowment shocks. Like impulse response functions, the curves in the Figure show how debt responds over time (on average) to endowment shocks that hit the economy at date zero and then vanish at the rate captured by their first order autocorrelation coefficients. In contrast, Figure 9 plots a series of hypothetical stochastic time-series simulations of the public debt ratios predicted by the model drawing realizations of endowment shocks each period from their Markov process using a random number generator. All of these stochastic simulations start from the debt ratio in the deterministic steady state of 46 percent of GDP. Thus, Figure 9 shows debt ratios consistent with fiscal solvency and the model's equilibrium at any date t in which the economy is being hit by new shocks (instead of just an initial impulse as in Figure 8). Some of these simulations show the possibility that long sequences of adverse shocks can drive the debt ratio into levels around 50 percent. Given that Figure 6 shows that debt ratios as high as 60 percent have non-trivial probability of occurring in the long run, there can be even longer sequences of adverse shocks that could yield stochastic simulations in which the debt ratio reaches as high as 60 percent. However, these sequences have very low probability since, by construction, generating a large set of stochastic simulations at random will yield long-run moments of public debt identical to those shown in Table 3. Figure 9 also shows that allowing for new shocks to hit the economy each period does not change the prediction that sustainable debt ratios cannot vary widely from one period to the next (the largest year-on-year changes are about 2 percentage points of GDP).

Tables 4A-4B present the results of a sensitivity analysis that shows how the predictions of the baseline calibration regarding the long-run moments of key macroeconomic aggregates vary as we introduce different changes in parameter values. This exercise considers five alternative scenarios: (1) basic needs of government expenditures of tradables and nontradables set at 10 percent of the levels in the deterministic steady state, (2) increased variance in the endowment shocks of tradables and nontradables, (3) an increase in the risk aversion coefficient of the government, (4) increasing the average size of tradables GDP relative to nontradables GDP, (5) lowering the average size of tradables GDP relative to nontradables GDP.

Introducing basic needs results in a significant cut in the long-run average of the sustainable public debt ratio, from 45.2 percent in the baseline calibration to 15.5 percent with basic needs of government expenditures (see the second column of Table 4A). This change has no effects on the sectoral measures of government revenue, but since it induces an increase of 7 percent in the average relative price of nontradables, it increases the average total revenue in units of tradables by one percentage point of GDP. On the expenditure side, the

means of the sectoral allocations of expenditures on tradables and nontradables increase but their coefficients of variation decline. Thus, basic needs result in higher and smoother levels of government purchases. The mean of total expenditures in units of tradables increases as a result of the combined effect of these higher average sectoral allocations of expenditures and the real exchange rate appreciation. While the increases in total revenue and total expenditures nearly offset each other, the real appreciation increases the cost of paying for the fixed level of transfer payments W (which are fixed at 0.096), and hence the primary balance shrinks by almost 2 percentage points of GDP.

The sharp reduction in the mean public debt-output ratio is caused by the reduction of the primary balance *and* by the stronger precautionary savings effect that results from the tigthening of the government's NDL implied by the existence of basic needs. It is worth noting that in this experiment liability dollarization works to reduce sustainable debt ratios in response to real appreciation (i.e., a rise in the price of nontradables). Real appreciation makes total outlays more expensive given the higher cost of providing the inflexible amount of non-tradables transfer payments. This "flow" effect is different from the "stock" effect emphasized by Calvo et al. (2003), according to which a real depreciation reduces sustainable debt ratios because of its effect on the ability of the government to service a stock debt denominated in units of tradables with revenues from the nontradables sector. This effect is shown in other alternative scenarios shown in Table 4B.

The third column of Table 4A shows results for a scenario in which the standard deviation of the two endowment shocks is tripled. This increase in the exogenous uncertainty of the economy lowers the mean sustainable debt ratio by 5 percentage points of GDP. The tripling of the standard deviation of the shocks induces a tripling of the coefficients of variation of total output and total tax revenues in units of tradables. It also results in a 6 percent increase in the average relative price of nontradables, and a sharp increase in its coefficient of variation by a factor of almost 3.4. As in the basic needs experiment, the appreciation of the mean real exchange rate contributes to worsen the average primary balance. The drop in the sustainable debt ratio results from this "flow effect" of the real appreciation and the stronger precautionary savings effect induced by the increase in the NDL on public debt (which in turn results from the increased volatility of revenues and the primary balance).

The last column of Table 4A reports results for a simulation in which the coefficient of relative risk aversion increases slightly from 2 to 2.1. The outcome is a large drop in the long-run average of the public debt ratio of nearly 28 percentage points of GDP. The increase in σ^g increases the curvature of the government's payoff function (i.e., it strengthens the precautionary savings motive) and at the same time it redistributes the economy's wealth across the public and private sectors. The planner's representation of the competitive equilibrium features a condition keen to a "risk pooling" condition that equates the

public and private marginal utilities of consumption of tradables at all dates and states. An increase in the curvature of the government's payoff function requires higher (lower) sectoral allocations of public (private) expenditures in order to satisfy the "risk pooling" condition. Since the economy's stochastic equilibrium paths of the endowments of tradables and nontradables are the same as in the baseline, this reallocation of expenditures from the private to the public sector reflects a wealth reallocation. This wealth effect can also be observed in the fact that large drop in public debt is accompanied by a small change in the mean level of foreign assets (it falls by 2 percentage points of GDP), so the drop in public debt reflects a net reduction of about 26 percentage points of GDP in the total asset position of the private sector. Table 4B shows the results of two scenarios changing the relative size of the tradables and nontradables endowments. The second column increases the ratio of tradables to nontradables GDP from 0.65 in the baseline calibration to 0.8. The last column reduces the ratio to 0.5. Interestingly, in both cases the long-run average of the public debt ratio falls relative to the baseline calibration (by about 15 percentage points of GDP in the first case and 5 in the second). These results show how the tension between the "stock" and "flow" effects of the real exchange rate on the fiscal position affects debt dynamics.

In the case in which the tradables output grows larger than the nontradables output, the long-run average of p^N grows by 25 percent. The government reallocates its expenditures from the nontradables sector into the tradables sector in response to this real appreciation but still the mean value of total expenditures in units of tradables increases by 1.4 percentage points. The value of tax revenues grows with the real appreciation by 3.2 percentage points, but once the effect of the real appreciation on the cost of the fixed level of transfers is taken into account, the net result is a reduction in the primary balance. Hence, in this case the "flow" effect of the real appreciation lowers the sustainable debt because it increases proportionally more the cost of total government outlays than the total revenues.

Consider next the case in which the nontradables GDP grows relative to the tradables GDP (the last column of Table 4B). The "flow" effect is pushing for an increase in the debt ratio, since in this case the average of p^N falls to by about 13 percent, leading to a reallocation of expenditures from tradable goods into nontradables goods that results in a cut in the value of total expenditures in units of tradables of about 0.4 percentage points. The cut in the total value of outlays is even larger because the real depreciation lowers the cost of the fixed transfer payments. Yet, the long-run average of total revenues falls by about 2.2 percentage points of GDP because of the combined effect of the smaller size of the tradables sector and the endogenous collapse in the price of nontradables. The drop in revenues is larger than the drop in total outlays and hence the averages of the primary balance in units of tradables and the public debt ratio fall.

The above findings, showing that the long-run sustainable debt ratio can fall in response to either an increase or a decrease in the relative size of the tradables sector vis-a-vis the nontradables sector, is likely to depend on the elasticity of substitution in consumption across tradables and nontradables. This is important because, as can be seen in the third column of Table 3, the increase in total expenditures when the tradables sector grows relatively larger than the nontradables sector is driven largely by the fact that the drop in non-tradables expenditures is more than offset by the real appreciation of the currency. The elasticity of substitution in consumption is likely to affect this outcome, and hence it is likely to affect the strength of the fiscal "flow" effect of movements in the real exchange rate. We intend to study this conjecture in a future version of this paper.

6 Concluding Remarks

Emerging economies seem less able to sustain high ratios of public debt to GDP than industrial countries. At the same time, emerging countries display more volatility in their public revenues and less flexibility in their ability to adjust government outlays, and they have only limited access to a narrow set of non-state-contingent debt instruments. In addition, emerging countries suffer of the syndrome of "liability dollarization" in public debt. That is, they leverage public debt denominated in units of tradable goods on the large fraction of revenues they collect from the nontradables sector of their economies, and they are exposed to large fluctuations in the domestic relative prices of nontradables.

This environment presents a challenge for a fiscal authority trying to assess whether its tax, expenditures and public debt policies are consistent with a sustainable debt position (defined as the goal of maintaining fiscal solvency). This paper examines public debt sustainability from this perspective using a dynamic stochastic general equilibrium model that features aggregate, non-insurable shocks to the output of the tradables and nontradables sector as well as liability dollarization. This model provides an analytical, structural framework for mapping the underlying exogenous shocks hitting the economy into endogenous fluctuations in public revenues and relative prices and endogenous dynamics of public debt. The paper implements the model for a case calibrated to the Mexican economy and produces the model's estimates of short- and long-run distributions of public debt ratios consistent with fiscal solvency requirements.

The starting point of the method proposed in this paper is the same assumption of a government that is aiming to keep a credible commitment to repay its debt implicit in conventional methods for assessing fiscal solvency (such as the long-run or Blanchard ratios relating public debt to the primary balance and the methods based on econometric tests of the intertemporal government budget constraint). This commitment to repay is not imposed

as an ad-hoc assumption but is derived from optimal plans made by a government that, acting like a "tormented insurer," tries to smooth its outlays in the face of the exogenous volatility of public revenues. If the government displays a standard CRRA payoff function, it will never want to face a situation in which a sequence of negative revenue shocks compromises its ability to provide for a positive amount of government outlays (of for at least arbitrary "basic needs" levels of government outlays). To avoid this outcomes, the government imposes on itself a "natural debt limit" analogous to the endogenous debt limits that households facing non-diversifiable income uncertainty need to satisfy in the literature on savings under incomplete markets.

The NDL on public debt is jointly determined by the variance of the exogenous random shocks hitting the economy, the minimum levels (or basic needs) levels of outlays the government can commit to undertake, and the endogenous equilibrium relative price of nontradable goods that determines the value of revenues and outlays in the units in which public debt is issued. This NDL is an important piece of our methodology for assessing fiscal sustainability under uncertainty but it is generally not the same as the equilibrium or "sustainable" debt ratio. The latter is obtained from the law of motion of public debt implied by the government budget constraint along a stochastic equilibrium path, and is thus influenced by the processes driving exogenous shocks, the endogenous optimal policies setting government outlays, and the equilibrium stochastic process of the relative price of nontradable goods.

We show that in a basic one-sector model in which public revenues are an exogenous Markov process and public expenditures follows ad-hoc smoothing rules, the model reproduces some of the features of debt dynamics in Barro's (1979) classic model of debt under tax smoothing. That is, the long-run distribution of debt is not determined within the model. The long-run behavior of debt is degenerate, featuring outcomes in which for some set of initial debt ratios public debt always ends up hitting the NDL in the long run, and for others public debt always ends up vanishing.

Quantitative experiments based on this basic model calibrated to an "average" emerging economy show that, for public debt-GDP ratios under 30 percent, there are large differences in the number of periods that it can take for the economy to hit its NDL depending on future realizations of income shocks. On the "average" path implied by mean forecast functions, the average time to a fiscal crisis can be above 40 quarters. In a path in which income remains two-standard-deviations below trend, however, the government is predicted to hit its NDL in less than 8 quarters. This suggests that on average one could expect a typical emerging country to go for as long as 10 years without hitting a fiscal crisis, but if there are two "sufficiently bad" years of output shocks in a row, the government can find its day of reckoning much sooner.

The above results are in sharp contrast with the predictions of conventional methods for calculating sustainable public debt. For the same "average" emerging economy, the sustainable debt ratio predicted by the long-run deterministic approach would be 45 percent of GDP (this is the ratio of the difference between average tax revenue minus average total outlays divided by the difference between the real interest rate minus the average growth rate). As calibrated, the stochastic model yields a natural debt limit of 50 percent, which is above this estimate of sustainable debt from the long-run approach. In this benign case, sticking to the 45 percent estimate would prevent the government from fulfilling its insurer role efficiently (i.e., there are states of the economy in which it could exploit the extra 5 percentage points of public debt before hitting the debt limit to keep outlays smooth while fulfilling its commitment to repay). However, slight modifications to consider larger shocks to income or weaker commitments to adjust outlays reduce the NDL below 45 percent, and in these cases aiming to practice a fiscal policy that assumes a debt ratio that can be as large as 45 percent of GDP would be a major mistake serious mistake.

Moving from the basic model to the two-sector general equilibrium model, in which debt is a dollarized liability and the government chooses optimally its debt policy to conform to its desire to smooth government purchases, results in significant changes. In this case, the distribution of public debt is no longer degenerate. The model yields a unique, invariant long-run distribution of the state variable of the model (which are the stock of external debt and the output shocks), and this distribution and the government budget constraint induce a unique, invariant long-run distribution of the public debt-GDP ratio. We use these distributions to study the long-run statistical moments that characterize the model's key macroeconomic aggregates, including the sustainable public debt ratio, and to study the transitional dynamics of the debt distribution, conditional impulse response functions of public debt to output shocks, and time series simulations of public debt paths for given initial conditions.

The results of a baseline calibration to the case of Mexico and several alternative scenarios illustrate the importance of uncertainty, asset-market incompleteness and liability dollarization in assessing sustainable public debt ratios. The long-run average debt ratio (i.e., the model's key measure of sustainable debt in the long run) falls when (a) the minimum or basic needs levels of government expenditures increase, (b) the variability of the exogenous output shocks increase, (c) the curvature of the government's payoff function increases, or (d) the mean of tradables GDP increases or falls relative to the mean of nontradables GDP. These results are explained by the changes in the strength of precautionary savings effects under each scenario, and by the combination of "flow" and "stock" effects of endogenous changes in the relative price of nontradables. The flow effect refers to changes in the primary balance that result from the fact that the cost of expenditures varies relative

to the value of revenues. The stock effect refers to how, because of liability dollarization, changes in the value of revenues in units of tradables in response to endogenous movements in the real exchange rate affect the ability of the government to sustain public debt denominated in units of tradables.

The results of the baseline calibration support a long-run average debt ratio of about 45 percent, and predict that debt ratios of 60 percent or more are not consistent with fiscal solvency in the short run for all possible sequences of output shocks within a two-standard deviation threshold. Fluctuations in public debt are also predicted to be highly persistent. This suggests that a good short-run measure of public debt sustainability is the year-on-year variation of the public debt ratio. Countries that exhibit large *variations* on their public debt ratios from one year to the next are unlikely to be following a path of public debt consistent with fiscal solvency and macroeconomic equilibrium, even if the *levels* of debt ratios are relatively low.

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Table 1: Estimates of the Natural Limit on the Public Debt

	Standard deviation of t	Coefficient of variation of t	Adjustment in g (percentage points of GDP)			
			0.02	0.04	0.06	0.08
Case 1			$\gamma = 1.037$	$\mathbf{R = 1.065}$	$E[t] = 0.229$	g=0.224
	0.01	4.37	17.86	89.29	160.71	232.14
	0.01	6.55	0.00	53.57	125.00	196.43
	0.02	8.73	0.00	17.86	89.29	160.71
	0.03	10.92	0.00	0.00	53.57	125.00
	0.03	13.10	0.00	0.00	17.86	89.29
Case 2			$\gamma = 1.037$	$\mathbf{R = 1.100}$	$E[t] = 0.229$	g=0.224
	0.01	4.37	7.94	39.68	71.43	103.17
	0.01	6.55	0.00	23.81	55.56	87.30
	0.02	8.73	0.00	7.94	39.68	71.43
	0.03	10.92	0.00	0.00	23.81	55.56
	0.03	13.10	0.00	0.00	7.94	39.68

Note: The lowest realization of the public revenue-GDP ratio is set two standard deviations below $E[t]$.

Table 2: Parameter Values for the Calibrated Deterministic Stationary State

Aggregate and sectorial macroeconomic ratios					
Mexican Data		Implied by the Deterministic Steady State			
$C^T/Y^T =$	0.665	$C/Y =$	0.691	$G^T/Y^T =$	0.090
$I^T/Y^T =$	0.200	$I/Y =$	0.170	$G^N/Y^N =$	0.141
$C^N/Y^N =$	0.708	$G/Y =$	0.121	$W/Y =$	0.096
$I^N/Y^N =$	0.151	$TB/Y =$	0.018		
$B^I/Y =$	-0.350	$CA/Y =$	-0.005		
$B^g/Y =$	0.459	$B/Y =$	0.109		
$Y^T/Y^N =$	0.648				
Preferences					
Mexican Data		Implied by the Deterministic Steady State			
$\sigma_h =$	2.000	$\omega_h =$	0.342	$\sigma_h =$	2.000
$\sigma_g =$	2.000	$\omega_g =$	0.240	$\sigma_g =$	2.000
$\eta =$	0.316	$\eta =$	0.316	$\beta =$	0.135
		$\mu =$	0.968		
Growth, interest, and tax rates					
Mexican Data		Implied by the Deterministic Steady State			
$\gamma =$	1.015	$\tau =$	0.240		
$R =$	1.065				
Endowment shocks					
$\sigma^T =$	0.037	$\sigma^N =$	0.027		
$\rho^T =$	0.600	$\rho^N =$	0.600	$\rho^{T,N} =$	0.685
Basic Needs and Normalizations					
$y =$	1.000	$\underline{G}^T/G^T =$	0.000	$\underline{G}^N/G^N =$	0.000
$p^N =$	1.000				

Table 3: Business Cycle Statistics in the Limiting Distribution

Variable (x)	$E[x]$	Std. Dev. % of $E[x]$	Std. Dev. $\frac{\text{s.d.}[x]}{\text{s.d.}[y]}$	$\rho(x_t, x_{t-1})$	$\rho(x_t, y_t)$
GDP in units of tradables (y)	1.004	3.57	1.000	0.940	1.000
Tradable GDP (y^T)	0.393	3.68	1.029	0.600	0.356
Non-tradable GDP (y^N)	0.607	2.74	0.767	0.600	0.034
Public debt (b^g)	0.453	7.28	2.037	0.991	-0.952
Total government expenditures (g)	0.122	5.09	1.423	0.971	0.942
CES government expenditures (CES_g)	0.093	2.55	0.714	0.757	0.556
Gov. expend. on tradables (g^T)	0.036	4.13	1.157	0.990	0.956
Gov. expend. on nontradables (g^N)	0.086	3.15	0.882	0.603	0.063
Total tax revenue (tr)	0.241	3.57	1.000	0.940	1.000
Tax revenue from tradables (tr^T)	0.094	3.68	1.029	0.600	0.356
Tax revenue from nontradables (tr^N)	0.146	2.74	0.767	0.600	0.034
Primary fiscal balance (pfb)	0.023	0.68	0.189	0.697	-0.413
Relative price of nontradables (p^N)	1.007	6.75	1.890	0.802	0.732
Total consumption (c)	0.695	4.84	1.354	0.976	0.947
CES consumption (CES_c)	0.360	2.54	0.712	0.776	0.594
Consumption of tradables (c^T)	0.263	4.02	1.124	0.992	0.957
Consumption of nontradables (c^N)	0.430	3.24	0.907	0.600	0.028
Total private assets (b)	0.116	13.84	3.874	0.997	0.905
International assets (b^I)	-0.337	21.05	5.891	0.998	0.924
Trade balance (tb)	0.017	1.76	0.493	0.737	-0.393
Current account (ca)	-0.005	1.42	0.398	0.613	0.402

Notes: $\rho(x_t, x_{t-1})$ represents the autocorrelation of the variable x and $\rho(x_t, y_t)$ represents the contemporaneous correlation between the variable x and the GDP in terms of tradable goods, y

Table 4: Alternative Scenarios (Part A)

	Baseline Calibration				Basic Needs $\underline{g}^k/g^k$ =0.10; $k = T, N$				High Endowment Volatility σ^T=11%; σ^N=8.2%				High Government Risk Aversion σ_g=2.1			
	$E[x]$	$s.d.[x]$	$\rho[x]$	$\rho[x,y]$	$E[x]$	$s.d.[x]$	$\rho[x]$	$\rho[x,y]$	$E[x]$	$s.d.[x]$	$\rho[x]$	$\rho[x,y]$	$E[x]$	$s.d.[x]$	$\rho[x]$	$\rho[x,y]$
b^g	0.452	7.28	0.99	-0.95	0.155	6.87	0.99	-0.94	0.401	21.73	0.99	-0.94	0.173	7.43	0.99	-0.94
g	0.122	5.09	0.97	0.94	0.140	4.93	0.96	0.94	0.123	10.08	0.97	0.94	0.136	5.08	0.97	0.94
g^T	0.036	4.13	0.99	0.96	0.040	3.70	0.99	0.96	0.037	12.15	0.99	0.96	0.040	4.11	0.99	0.96
g^N	0.086	3.15	0.60	0.06	0.093	2.91	0.60	0.03	0.086	9.43	0.60	0.05	0.095	3.07	0.60	0.04
tr	0.241	3.57	0.94	1.00	0.250	3.56	0.94	1.00	0.248	10.61	0.94	1.00	0.241	3.59	0.94	1.00
tr^T	0.094	3.68	0.60	0.36	0.094	3.68	0.60	0.32	0.095	10.99	0.60	0.34	0.094	3.68	0.60	0.35
tr^N	0.146	2.74	0.60	0.03	0.146	2.74	0.60	-0.00	0.146	8.20	0.60	0.02	0.146	2.74	0.60	0.03
pfb	0.023	0.68	0.70	-0.41	0.008	0.70	0.68	-0.41	0.020	2.02	0.70	-0.42	0.009	0.71	0.70	-0.42
p^N	1.007	6.75	0.80	0.73	1.070	6.75	0.79	0.74	1.061	20.13	0.79	0.74	1.010	6.80	0.80	0.74
c	0.695	4.84	0.98	0.95	0.722	4.76	0.97	0.95	0.723	14.23	0.97	0.95	0.681	4.88	0.98	0.95
c^T	0.263	4.02	0.99	0.96	0.270	3.94	0.99	0.96	0.271	11.81	0.99	0.96	0.257	4.05	0.99	0.96
c^N	0.430	3.24	0.60	0.03	0.423	3.30	0.60	-0.01	0.431	9.70	0.60	0.01	0.420	3.26	0.60	0.03
b^I	-0.337	21.06	1.00	0.92	-0.087	19.71	1.00	0.92	-0.237	62.76	1.00	0.91	-0.357	21.18	1.00	0.93
ca	-0.005	1.42	0.61	0.40	-0.001	1.37	0.61	0.36	-0.004	4.22	0.61	0.39	-0.005	1.42	0.61	0.40

Notes: See the variable nomenclature on Table 3; $E[x]$ stands for the mean of variable x; $s.d.[x]$ for the standard deviation in percentages of the mean; $\rho[x]$ for the autocorrelation of x; and $\rho[x,y]$ for the correlation between variable x and the GDP in units of tradables, y.

Continued on next page

Table 4: -Continued from previous page- Alternative Scenarios (Part B)

	Baseline Calibration				$y^T/y^N = 0.810$				$y^T/y^N = 0.486$			
	$E[x]$	$s.d.[x]$	$\rho[x]$	$\rho[x,y]$	$E[x]$	$s.d.[x]$	$\rho[x]$	$\rho[x,y]$	$E[x]$	$s.d.[x]$	$\rho[x]$	$\rho[x,y]$
b^g	0.452	7.28	0.99	-0.95	0.295	8.03	0.99	-0.95	0.399	5.63	0.99	-0.95
g	0.122	5.09	0.97	0.94	0.136	5.42	0.97	0.94	0.118	4.36	0.96	0.94
g^T	0.036	4.13	0.99	0.96	0.039	4.40	0.99	0.96	0.035	3.52	0.99	0.96
g^N	0.086	3.15	0.60	0.06	0.078	3.15	0.60	0.07	0.095	3.15	0.60	-0.00
tr	0.241	3.57	0.94	1.00	0.273	3.77	0.94	1.00	0.219	3.21	0.93	1.00
tr^T	0.094	3.68	0.60	0.36	0.107	3.68	0.60	0.35	0.079	3.68	0.60	0.31
tr^N	0.146	2.74	0.60	0.03	0.133	2.74	0.60	0.04	0.162	2.74	0.60	-0.03
pfb	0.023	0.68	0.70	-0.41	0.015	0.74	0.71	-0.44	0.020	0.60	0.67	-0.38
p^N	1.007	6.75	0.80	0.73	1.251	7.01	0.82	0.75	0.872	6.24	0.76	0.71
c	0.695	4.84	0.98	0.95	0.770	5.17	0.98	0.95	0.676	4.13	0.97	0.95
c^T	0.263	4.02	0.99	0.96	0.282	4.28	0.99	0.96	0.261	3.42	0.99	0.96
c^N	0.430	3.24	0.60	0.03	0.391	3.24	0.60	0.03	0.477	3.24	0.60	-0.04
b^I	-0.337	21.06	1.00	0.92	-0.677	22.58	1.00	0.94	0.754	16.32	1.00	0.88
ca	-0.005	1.42	0.61	0.40	-0.010	1.43	0.62	0.41	0.011	1.31	0.61	0.31

Figure 1. Mean Debt-GDP Ratios and Coefficients of Variation of Public Revenue-GDP Ratios (IMF 1990-2002)

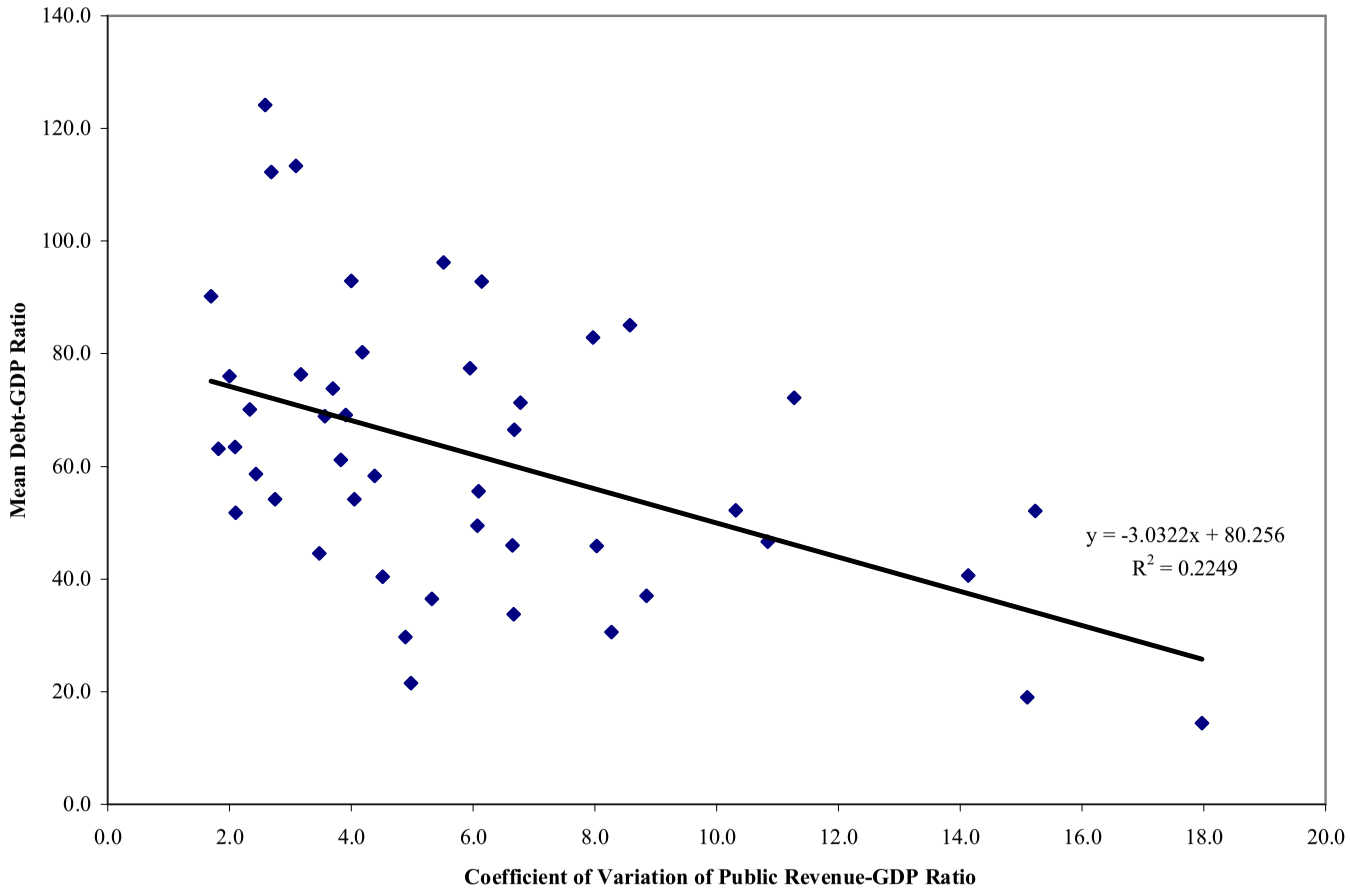


Figure 2. Mean Debt-GDP Ratios and Standard Deviation of GDP Growth (IMF-GFS 1991-2001)

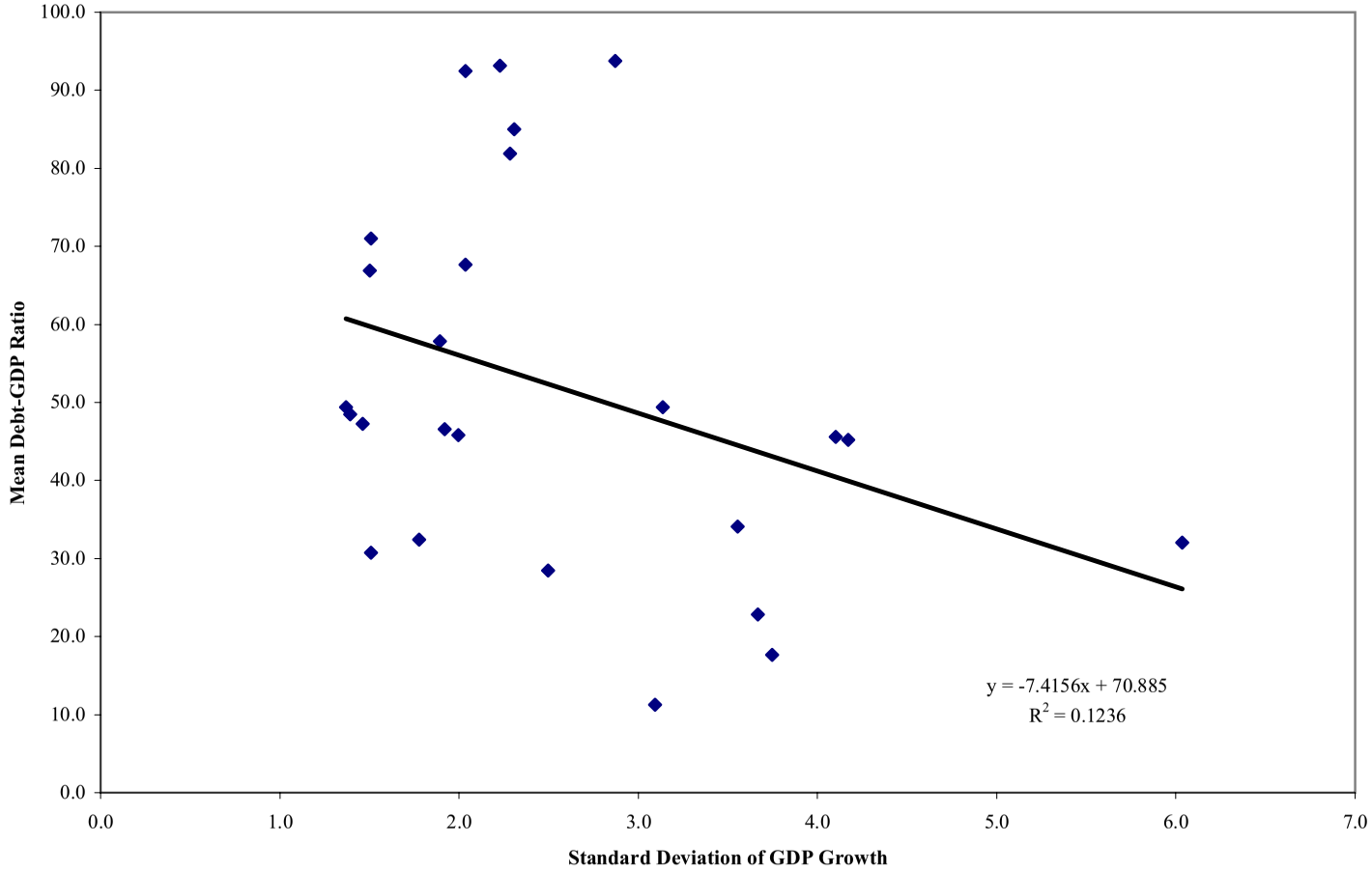


Figure 3. Average Public Revenue-GDP Ratios: 1990-2002

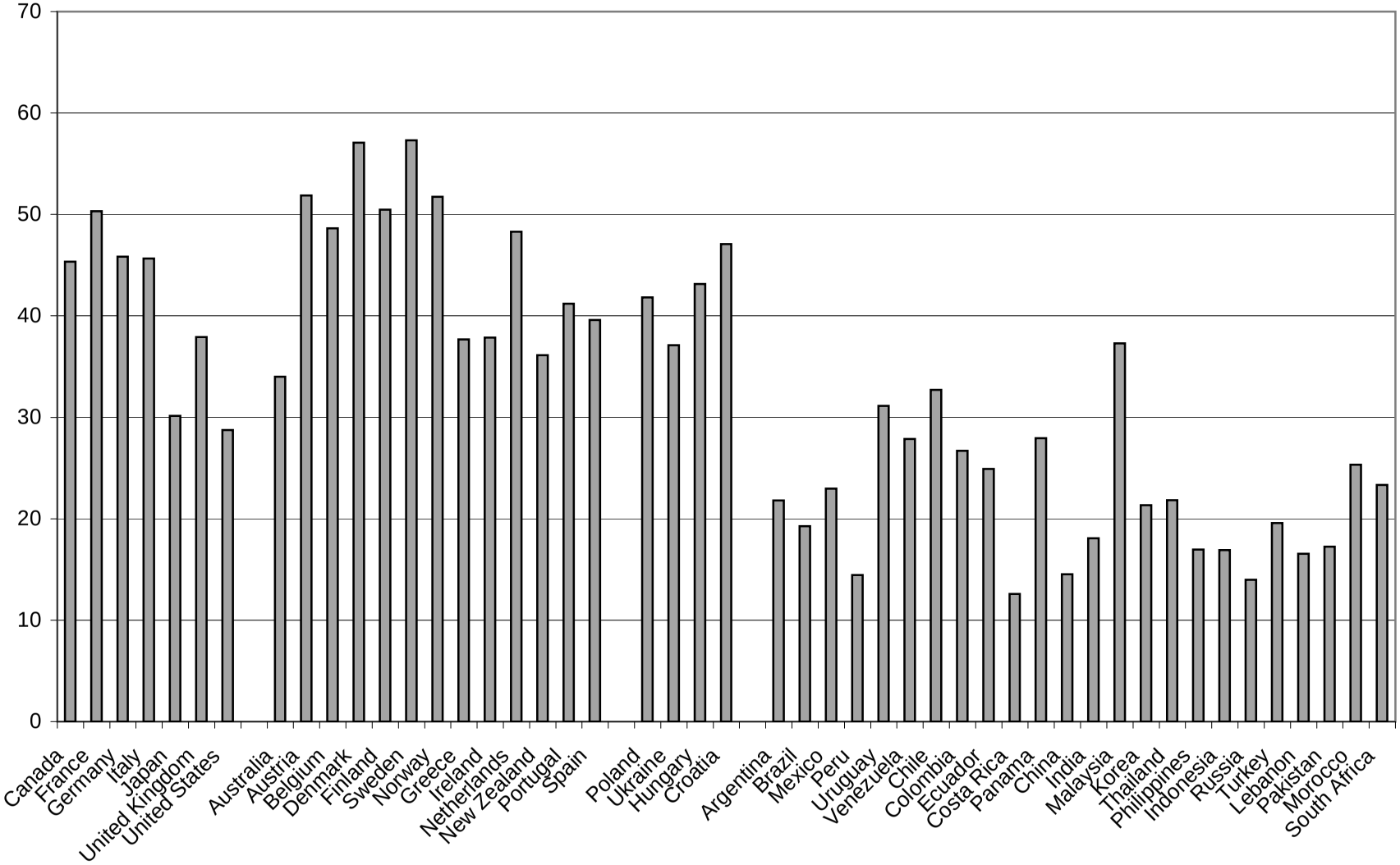


Figure 4. Coefficients of Variation of Public Revenue-GDP Ratios: 1990-2002

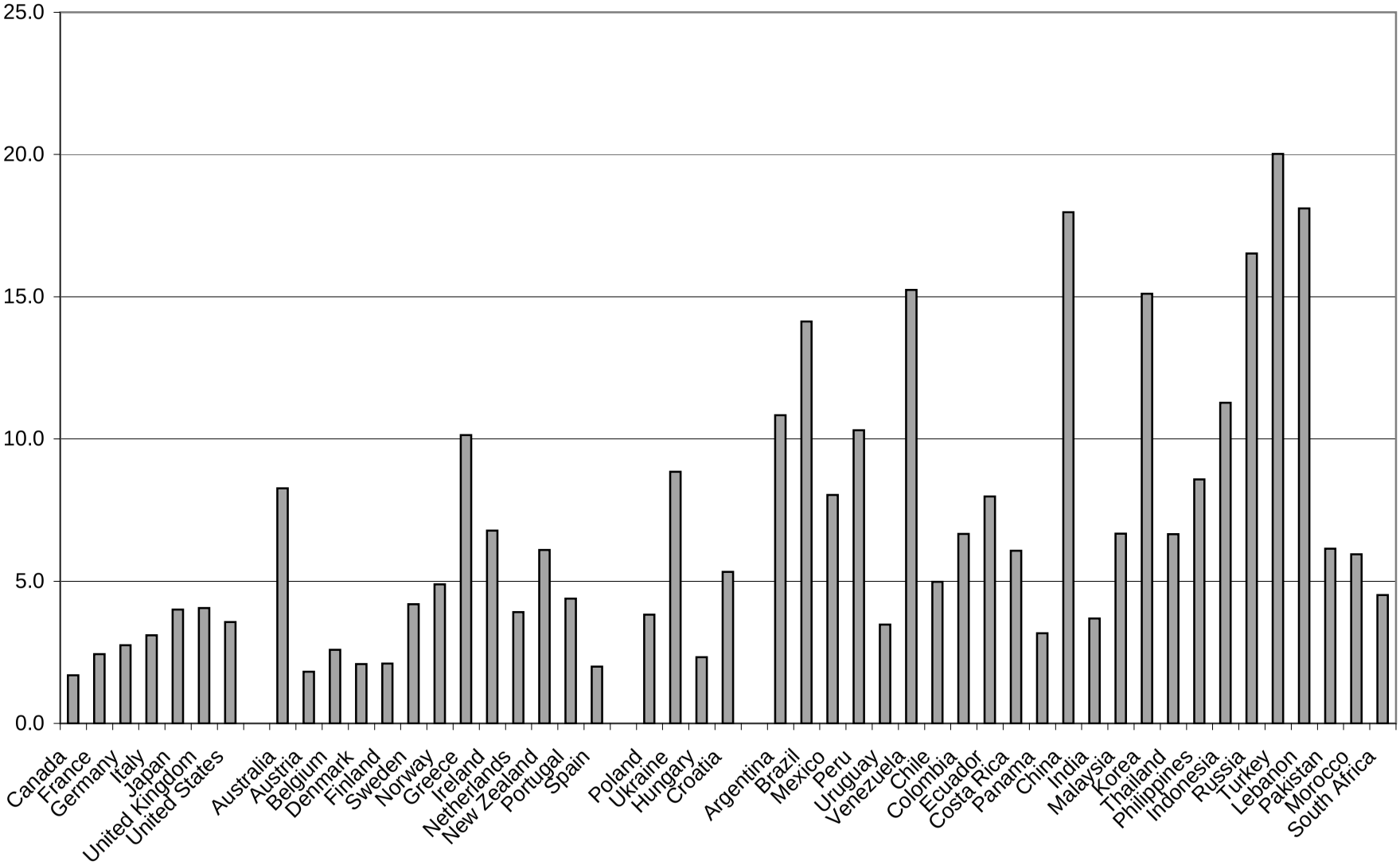
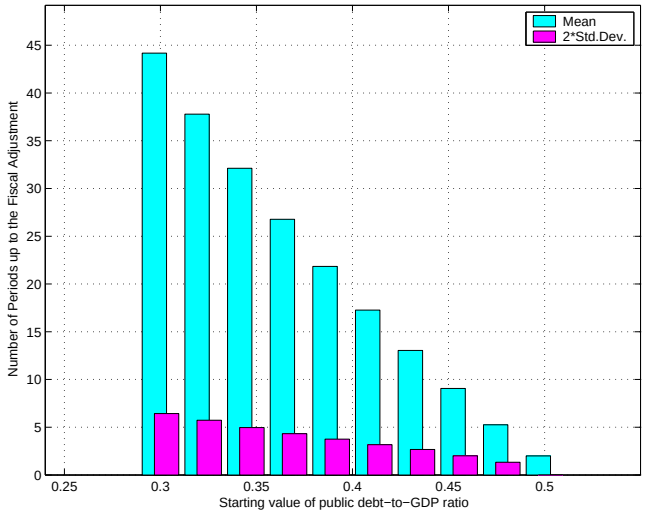


Figure 5. Number of Periods Before the Fiscal Adjustment in the Simplified Model.



Notes: For each starting value of the debt-GDP ratio, the bars show the average numbers of quarters it takes the debt ratio to hit the maximum debt limit.

Figure 6. Long-Run Distribution of Public Debt

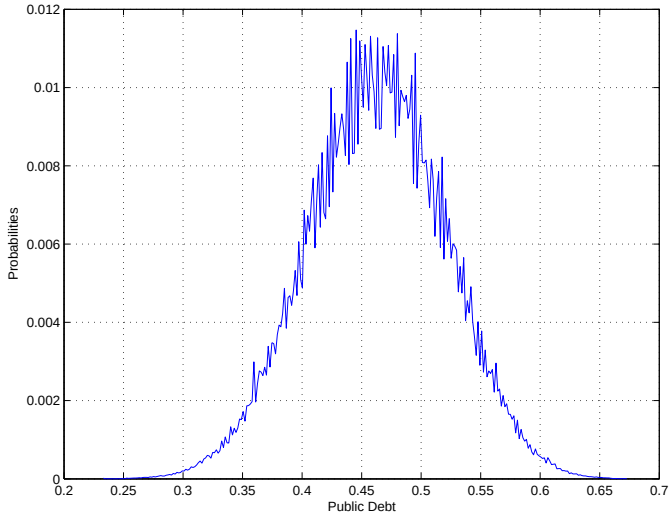
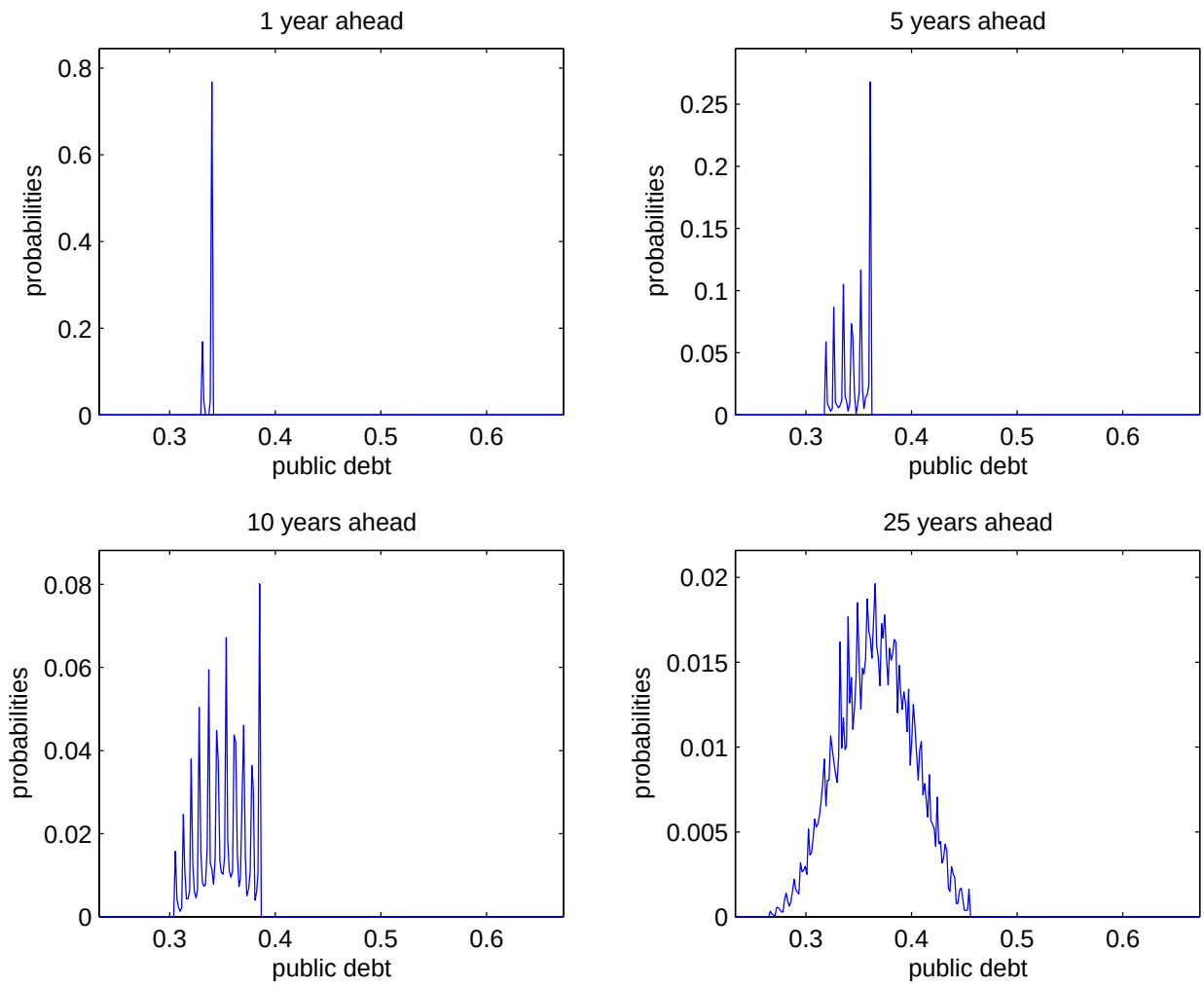
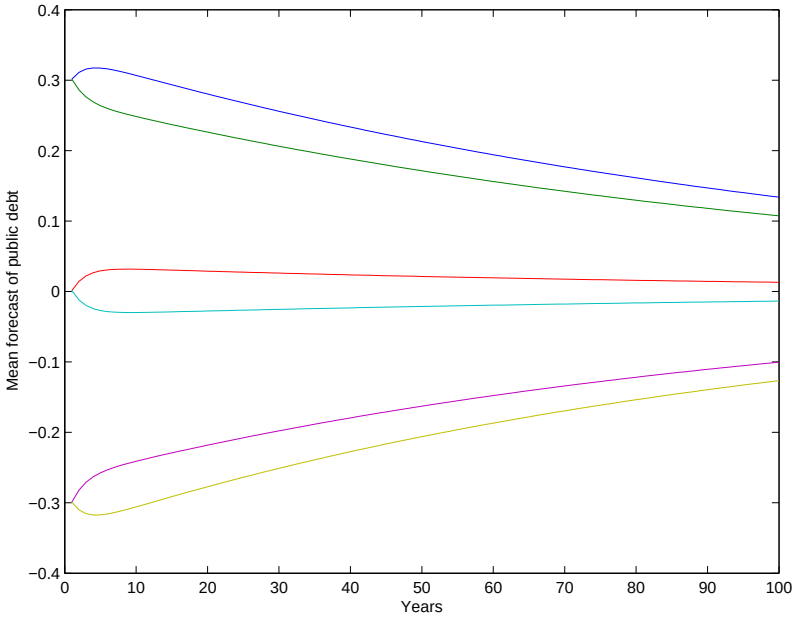


Figure 7. Forecast of Public-Debt-to-GDP Ratio



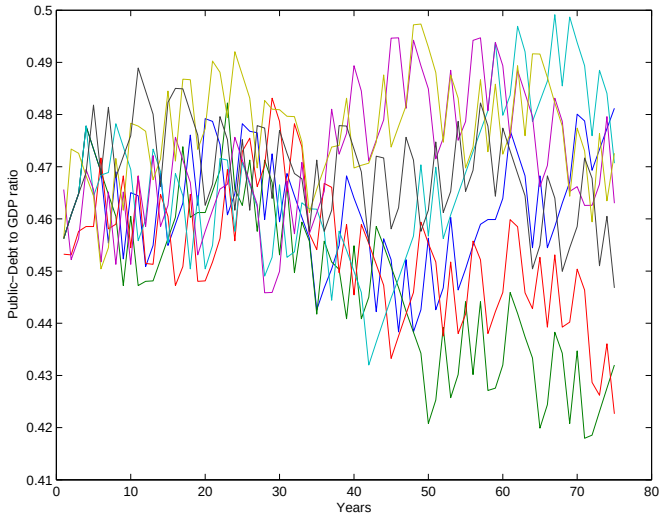
Note: The initial ratio of public debt to GDP is set equal to 30% of the value of that ratio in the non stochastic steady state.

Figure 8. Forecast of Public-Debt-to-GDP Ratio



Note: The graph shows the expected path of the deviations of the debt-to-GDP ratio with respect to the steady-state value of that ratio for three starting values; the starting values are chosen to be equal to 30% higher than the nonstochastic ratio of public debt to GDP; equal to that ratio; and 30% below that ratio; For each starting value, there exist two possible values of the productivity shock.

Figure 9. Stochastic Simulations of the Debt-to-GDP Ratio.



The starting value of the debt-to-GDP ratio is equal to 0.459, which corresponds to the steady-state value of that ratio.