The Dynamics of the U.S. Net Foreign Liabilities An Empirical Characterization*

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Abstract

This paper provides empirical evidence on the adjustment dynamics of the US net foreign liabilities, net output and consumption, in response to shocks. We use empirical techniques that allow us to quantify the relative importance of permanent and transitory innovations. We find that transitory shocks contribute considerably to the variation in all three variables for a horizon up to a year, and their contribution remain significant for a horizon up to five years. A permanent shock — that we interpret as a technological shock — dominates the variation of all variables at longer horizons. In response to this shock, net foreign liabilities, net output and consumption all increase — consistent with the effect of productivity gains raising domestic return to capital and thus generating an inflow of foreign capital. Conversely, shocks that cause net output and consumption to increase temporarily are accompanied by short-run accumulation of net foreign assets. This is in contrast with traditional model predicting procyclical current account deficits. Instead, our results are qualitatively consistent with predictions of the intertemporal approach to the current account.

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1 Introduction

Modern textbooks in international macroeconomics stress that, since the current account is the difference between national saving and investment, external deficits or surpluses result from intertemporal investment and consumption decisions by firms, households and the government. Thus, when international markets provide limited insurance opportunities, borrowing and lending enable economic agents to smooth consumption through intertemporal trade, enhancing economic efficiency.[1] Namely, in response to temporary shocks to net output, domestic households can increase both current and future consumption by lending internationally, either directly or through financial institutions. In response to permanent shocks to productivity, the domestic economy can sustain high rates of domestic investment without cutting current consumption, by borrowing in international markets.

For more than a decade, these basic propositions have been tested empirically using variants of the present-value model originally conceived by Campbell [1987] and Campbell and Shiller [1987]. Test results are mixed. The model is successful in some cases, unsuccessful in others. The extent to which the model performance is driven by the empirical failure of the auxiliary assumptions commonly adopted to make the model testable (without being necessary to the main theoretical proposition) is unclear. In addition, present-value tests do not distinguish between shocks that drive the dynamics of net foreign liabilities of a country, i.e. whether these shocks are temporary or permanent. Recent literature is addressing these issues theoretically and numerically using DSGE models (see Nason and Rogers [2003]).

In this paper we provide an empirical characterization of the link between net output, consumption and the accumulation of net foreign liabilities, with the goal of shedding light on the empirical foundations of present value tests as well as more recent DSGE models. Drawing on previous work by Campbell and Mankiw [1989] and Lettau and Ludvigson [2001, 2004], we focus our study on the dynamics of US net foreign liabilities. We find that in the short run (for an horizon of up to 4 quarters), the dynamics of US net foreign liabilities is strongly influenced by temporary disturbances that raise US output and consumption together with US foreign assets — as implied by the basic textbook model of the current account. This result is robust to different identification schemes and different methodologies to construct confidence intervals. Over longer horizons, permanent shocks take over. Although the impulse response for permanent shocks are less robust to different methodologies, we find a recurrent pattern: the dynamic of net liabilities is dominated by shocks that increase net output and consumption in the long run, associated with large accumulation of net foreign liabilities — consistent with capital inflows in response to permanent innovations to productivity.

Relative to the traditional model (e.g. Mundell-Fleming), the dynamic response of the system

[1] Depending on model specification and parameter values, the allocation when international trade in asset is restricted to a single non contingent bond can be quite close to efficient risk-sharing. International trade in bonds induce a positive ex-ante correlation between national consumption measured in PPP terms. Without full insurance, however, ex post such correlation can be low and negative (see the discussion in Corsetti *et al.* [2004]).

to temporary shocks does not provide support to the hypothesis of a pro-cyclical deterioration of the current account. While the methodology we employ does not allow us to identify temporary shocks in a structural sense, in our results temporary output expansions are associated with external surpluses (not with deficits) in a quite robust way.[2] Net external liabilities grow with output, instead, in response to permanent shocks to the system.

These results are qualitatively consistent with basic predictions of the intertemporal approach to the current account — for convenience, we briefly summarize the main predictions of models adopting this approach in an appendix. Looking at the pattern of the impulse responses to permanent and temporary shocks in our analysis, our results seem to corroborate the view that the rejection of the intertemporal model in empirical studies might well be due to auxiliary assumptions on preferences or asset returns.

Relative to present value and DSGE models in the literature, we derive our empirical framework by imposing a smaller set of equilibrium restrictions, namely, we make use of transversality conditions but not of the Euler equations from the representative national consumer's problem. So, we do not need to impose restrictive assumptions on preferences, i.e. specific functional forms for the utility function of the national representative consumer. By the same token, we need not impose a constant interest rate — it is well understood that allowing for stochastic real interest rates improves the match between the model and the data (see for instance Bergin and Sheffrin [2000] and Nason and Rogers [2003]).

We proceed as follows. As a preliminary step, we test a (weak) implication of the intertemporal budget constraint for the US: under the plausible assumption that real rates of return are stationary, the budget constraint implies that consumption, net output and the stock of net foreign liabilities be cointegrated — a condition for which we find support in the data.

We then move to the heart of our analysis: we make use of the long-run restrictions implied by cointegration to identify empirically trend and cyclical components,[3] and relate these components to aggregate consumption, net output and foreign assets. As mentioned above, we find that in response to temporary shocks that raises US net output, consumption also increases temporarily, but less than output. Hence, the economy runs a surplus and accumulates net foreign assets. Instead, a permanent shock that raises long-run per-capita net output leads to permanently higher per-capita consumption, but also raises net foreign liabilities. This is qualitatively consistent with the effects of a permanent shock to productivity that raises US returns above world level, thus attracting capital from abroad.

For a horizon of four quarters, the transitory shock accounts for approximately 60 percent of the variance in net output, 43 percent of the variance in consumption and 15 percent of the variance in net foreign liabilities. At a horizon of twenty quarters ahead, the transitory shock keeps contributing a non negligible amount to the forecast error variance of these three variables

[2] Using a VAR, Kim and Roubini [2003] have recently noted that US government spending shocks are associated with current account surpluses.

[3] There are several studies that employ the restrictions implied by cointegration to identify specific innovations in a range of structural models (e.g. King et al. [1991], Melander et al. [1992], Warne [1993]).

(27, 10 and 6 percent, respectively). But at longer horizons the systems is dominated by the two permanent shocks we identify in our analysis. At a horizon of forty quarters, the first of these permanent shocks accounts for 79 percent of the variance in net output, 73 percent of the variance in consumption and 87 percent of the variance in net foreign liabilities.

The shocks that move the variables in our systems in either a permanent or a temporary fashion correspond to a variety of structural disturbances hitting the economy at either national or international level — the methodology employed in our paper does not allow identification of structural disturbances. But in some sense this strengthens the central message of our paper: shocks that result into temporary innovations in current output also raise current consumption and induce net foreign asset accumulation; permanent output innovations are associated with current account deficits. By the same token, our study does not distinguish between country-specific and global shocks, a distinction that would be crucial in the analysis of small open economies. This is a clear limitation of our analysis. But since our case study is the US, we claim that such limitation is not too consequential for our results. Because of the economic size of this country, most domestic shocks have global repercussions, but still have a clear asymmetric component relative to the rest of the world.[4]

Our approach is related to Gourinchas and Rey [2004], who also build an open-economy empirical model following Campbell and Mankiw [1989] and Lettau and Ludvigson [2001, 2004]. But the focus of our study is different. We examine the adjustment of the capital account (current account) in response to shocks that hit the economy and focus on quantity adjustments, whereas Gourinchas and Rey [2004] analyze the adjustment of the financial account towards the equilibrium through the adjustment of asset prices rather than quantities.

The rest of the paper is organized as follows. Section 2 provides a theoretical motivation for our work. Section 3 lays out our empirical methodology. Section 4 presents our empirical results. Section 5 concludes.

2 Intertemporal Budget Constraint and Common Trends in Consumption, Net Output and Foreign Debt

Consider an open-economy in which all goods are traded. Domestic agents can borrow and lend in the international bond market at a time-varying (stochastic) real interest rate. The sequence of current account surpluses can be written as

$$B_{t+i+1} - B_{t+i} = Z_{t+i} - C_{t+i} + r_{t+i}B_{t+i}, \qquad i = 0, 1, ...;\ B_t : given \tag{1}$$

where B_{t+i} is the stock of net foreign assets at the beginning of period $t+i$ and r_{t+i} is the world real interest rate, which may vary over time. Define $R_{t,s}$ as the market discount factor for date

[4] For an analysis of global vs. country-specific shocks see Glick and Rogoff [1995], and Iscan [2000]. Hoffmann [2001, 2003] is amongst the first contributions that exploits cointegration and the stationarity of the current account, in order to identify permanent and transitory global and country-specific shocks.

s consumption, so that

$$R_{t,s} = \left[\prod_{j=t+1}^{s} (1 + r_j) \right]^{-1} \tag{2}$$

with $R_{t,t} = 1$. With optimizing national consumers, consumption plans will obey the following transversality condition:

$$\lim_{k \to \infty} E_t \left(R_{t,t+k} B_{t+k+1} \right) = 0. \tag{3}$$

Summing up the current account over all periods and using this optimality condition we then write an intertemporal budget constraint as:

$$\sum_{i=0}^{\infty} R_{t,t+i} C_{t+i} = B_t + \sum_{i=0}^{\infty} R_{t,t+i} Z_{t+i} \tag{4}$$

It is convenient to denote the present discounted value of consumption and net output as

$$\begin{aligned} \Phi_t &\equiv \textstyle\sum_{i=0}^{\infty} R_{t,t+i} C_{t+i} \\ \Psi_t &= \textstyle\sum_{i=0}^{\infty} R_{t,t+i} Z_{t+i} \end{aligned}$$

Since the current account and the capital account satisfy the identity

$$CA_t + KA_t \equiv 0,$$

we can define $D_t = -B_t$ as the stock of net foreign liabilities, and write the intertemporal budget constraint as:

$$\Psi_t = D_t + \Phi_t. \tag{5}$$

Following Campbell and Mankiw [1989] and Bergin and Sheffrin [2000], we derive an approximate expression for the above intertemporal budget constraint, by taking a first-order Taylor approximation of (5), imposing two transversality conditions and taking expectations.[5] Throughout this paper we use lower case letters to denote log variables (e.g., $c_t \equiv \ln(C_t)$ and $\phi_t = \ln \Phi_t$) and define $r_t \approx \ln(1 + r_t)$. We obtain:

$$\begin{gathered} c_t - \frac{1}{\rho_{D\Psi}} z_t + \left(\frac{1}{\rho_{D\Psi}} - 1 \right) d_t \approx \\ E_t \left\{ -\sum_{i=1}^{\infty} \rho_{C\Phi}^{i} \Delta c_{t+i} + \frac{1}{\rho_{D\Psi}} \sum_{i=1}^{\infty} \rho_{Z\Psi}^{i} \Delta z_{t+i} + \sum_{i=1}^{\infty} \rho_{C\Phi}^{i} r_{t+i} - \frac{1}{\rho_{D\Psi}} \sum_{i=1}^{\infty} \rho_{Z\Psi}^{i} r_{t+i} \right\}, \end{gathered} \tag{6}$$

[5] The two transversality conditions are:

$$\begin{gathered} \lim_{T \to \infty} \rho_{C\Phi}^{T} \left(c_{t+T} - \phi_{t+T} \right) \to 0 \\ \lim_{T \to \infty} \rho_{Z\Psi}^{T} \left(z_{t+T} - \psi_{t+T} \right) \to 0 \end{gathered}$$

where $\phi_t \equiv \ln \Phi_t$ and $\psi_t = \ln \Psi_t$. Details of the derivation and discussion are provided in the appendix.

where $\rho_{D\Psi} \equiv 1 - \exp\left(\overline{d_t - \psi_t}\right)$, $\rho_{C\Phi} \equiv 1 - \exp\left(\overline{c_t - \phi_t}\right)$ and $\rho_{Z\Psi} \equiv 1 - \exp\left(\overline{z_t - \psi_t}\right)$. Note that the left-hand side of the above expression can be interpreted as a log-linear approximation of the capital account. Thus, we define KA_t^* as the following approximation for the capital account:

$$KA_t^* \equiv c_t - \varphi_z z_t + (\varphi_z - 1)\, d_t, \tag{7}$$

where $\varphi_z \equiv \frac{1}{\rho_{D\Psi}}$, so we can write

$$KA_t^* = E_t \left\{ -\sum_{i=1}^{\infty} \rho_{C\Phi}^i \Delta c_{t+i} + \frac{1}{\rho_{D\Psi}} \sum_{i=1}^{\infty} \rho_{Z\Psi}^i \Delta z_{t+i} + \left(\sum_{i=1}^{\infty} \rho_{C\Phi}^i r_{t+i} - \frac{1}{\rho_{D\Psi}} \sum_{i=1}^{\infty} \rho_{Z\Psi}^i r_{t+i} \right) \right\}. \tag{8}$$

This expression shows that KA_t^* embodies rational forecasts of interest rates (returns), consumption growth and net output growth. This is intuitively appealing, and expressions similar to (8) have been extensively examined in the literature, especially for testing present value relations of the current account (Bergin and Sheffrin, [2000]; Sheffrin and Woo [1990a], [1990b] - *inter alia*). A few points about (8) deserve emphasis.

First, under the weak maintained hypothesis that the real rate of return r_t, Δz_t and Δc_t are covariance stationary, then the budget constraint implies that the logs of consumption, net output and net foreign liabilities must be cointegrated. Even if net foreign wealth is non-stationary in levels (as predicted by standard infinite-horizon intertemporal model), the transversality condition (3) prevents it to wander away from net output and consumption.

Using a well known result by Campbell and Shiller [1987] in the framework of present-value models, if Z_t is well characterized as an integrated process, then current account as the discounted sum of expected changes in Z_t will be stationary — this follows from the Wiener-Kolmogorov formula (see Sargent [1987]).[6] Essentially, we make use of this property of the current account in deriving empirical implications of the intertemporal budget constraint via a log-linear approximation of the present value relation in terms of KA_t^*. In line with the literature, KA_t^* is stationary.

Second, the cointegrating residual is $c_t + \beta_d z_t + \beta_d d_t$, and the cointegrating parameters β_z and β_d are equal to the theoretical parameters $-\varphi_z$ and $(\varphi_z - 1)$. By definition, φ_z is the a function of the expected ratio of net foreign debt to domestic *private* wealth, defined as the present discounted value of output net of (private and public) gross investment and government consumption. In short, φ_z can be interpreted as a function of the average portfolio share of net foreign assets in domestic private wealth. In order to log-linearize the intertemporal budget constraint of the country, we need assume that φ_z (essentially $\exp\left(\overline{d_t - \psi_t}\right)$) is constant. This is consistent with recent work by Kraay and Ventura [2000, 2002] and Ventura [2003], who advocate models of the current account allowing for international portfolio diversification in which the portfolio share of foreign wealth is constant. In these models, a constant φ_z follows from a time-invariant D_t/Ψ_t. But we note that our methodology is valid under a much weaker condition:

[6] The Wiener-Kolomogov formula essentially states that for a covariance stationary process Y_t, with a Wold MA representation, $Y_t = \Upsilon(L)\, v_t$ and $\delta \in (0,1)$, then the MA representation for $X_t = \sum_{i=1}^{\infty} \delta^i \mathrm{E}_t Y_{t+i}$ is given by $X_t = \delta\left[\left(\Upsilon(L) - \Upsilon(\delta)\right)/(L-\delta)\right] v_t$.

all we need is a well defined expected value of the ratio of net foreign debt to domestic *private* wealth. For instance, φ_z is also constant when D_t/Ψ_t varies over time following a stationary distribution.

In our econometric study below we are unable to reject the hypothesis that φ_z is constant — which may correspond to a portfolio share of foreign assets in wealth that is either time-invariant, or (more plausibly) follows some stationary distribution. With a time varying D_t/Ψ_t, however, it is possible that a country switches its international net position during the sample period. Since in deriving our log-linear approximation we have assumed that no variable switches sign, a problem in applying our methodology to the US is that this country is a net creditor in the first part of our sample, and becomes a net debtor during the 1980s.

Third, if the cointegrating relation on the left-hand-side of (8) is not constant, it must forecast either changes in rates of return, net output or consumption growth, or some combination of the three. In particular, (8) implies that the cointegrating residual $c_t + \beta_z z_t + \beta_d d_t$ should summarize expectations of future rates of return, net output and consumption growth, and provide a rational forecast of them. Very strong versions of such predictive ability have been widely tested in the literature by means of the present value test of the current account, the dual of the capital account we focus on here.

The empirical approach we describe below simply exploits the above cointegrating relation — without imposing additional structure. As long as budget constraints are not violated, a country's net output, consumption and net foreign debt should commove in the long-run and therefore be cointegrated. In fact, as we discuss shortly, our empirical findings support this hypothesis.

3 Econometric Framework

This section describes our approach to isolating the permanent and transitory shocks of a n-dimensional cointegrated vector $\mathbf{x}_t$. In our application, $\mathbf{x}_t = (c_t, z_t, d_t)'$.

3.1 Data and Preliminary Analysis

In our empirical analysis, c_t is (the log of) real per-capita expenditure on nondurables and services.[7] The log of the net output, z_t, is gross domestic product net of investment, durable goods and government expenditure, expressed in real, per-capita terms. The log of the stock of net foreign liabilities, d_t, is also expressed in real, per-capita terms. We stress here that d_t records more than bonds — as it includes the whole array of assets and liabilities traded internationally. The variable d_t is derived by cumulating the current account deficits over the sample period, and rescaling the original series so that it becomes positive throughout the sample. Data limitations

[7]Since we are mainly interested in the current account dynamics generated by consumption smoothing, we prefer to exclude expenditure on durables expenditure — as they replace (or add to) capital stock rather than buying a service flow from the existing capital stock. We include durable expenditure in investment.

do not allow us to use series of net foreign liabilities allowing for capital gains and losses on a wide array of assets — as proposed by Lane and Milesi-Ferretti [2001]. Namely, the series built in this study is at a lower frequency (annual), and for a smaller sample than the one we adopt in our work. Yet we should note here that our series and the Lane & Milesi-Ferretti series are quite correlated.[8] Clearly, the series d_t is not an ideal measure, but, as discussed below, its deficiencies are arguably not too consequential as regards the main results of our analysis. A full description of the data is provided by the appendix.

Table 1 reports the summary statistics of the data. The standard deviation of the quarterly net foreign liabilities growth is over ten times as high as that of consumption growth, and over four times as high as that of net output growth. The correlation between consumption and net output growth rate is roughly 0.4, while the growth rate of net liabilities is positively correlated with consumption growth and negatively correlated with net output growth — the correlation coefficient being equal to 0.05 and -0.08, respectively. Figure 1 plots the series used in the analysis, in level and growth rates.

`[Insert Table 1 and Figure 1 about here]`

3.2 The Econometric Model

In the econometric analysis we employ a vector autoregression (VAR) with k lags, which in its vector equilibrium correction (VEqCM henceforth) is given by

$$\Delta \mathbf{x}_t = \Pi \mathbf{x}_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta \mathbf{x}_{t-i} + \boldsymbol{\delta} + \mathbf{u}_t; \quad \mathbf{u}_t \sim NIID\left(\mathbf{0}, \Omega\right), \tag{9}$$

where $\mathbf{x}_t$ is a $n \times 1$ vector of variables in the system, and $\boldsymbol{\delta}$ is a vector of constants.[9]

The hypothesis that $\mathbf{x}_t$ is $I(1)$ is formulated as the reduced rank hypothesis of the matrix Π (see Johansen, [1995]). Here one assumes that the matrix Π has rank $r < n$. In this case Π can be decomposed as the product of two matrices $\boldsymbol{\alpha}\boldsymbol{\beta}'$ where $\boldsymbol{\alpha}, \boldsymbol{\beta}$ are each $n \times r$ and have full rank $r < n$,

$$\Pi = \boldsymbol{\alpha}\boldsymbol{\beta}'. \tag{10}$$

Furthermore, the full rank of

$$\boldsymbol{\alpha}_{\perp}' \Gamma \boldsymbol{\beta}_{\perp}, \tag{11}$$

is required, where $\boldsymbol{\alpha}_{\perp}$ and $\boldsymbol{\beta}_{\perp}$ are $n \times (n-r)$ matrices orthogonal to $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ respectively. Following this parameterization, there are r linearly-independent stationary relations given by

[8] The correlation between our measure of net foreign liabilities and that reported in Lane and Milesi-Ferretti [2001] (based on adjusted cumulative current account) is 0.987.

[9] Notice that if we allow the parameters $\Theta = \left\{\Pi, \Gamma_1, ..., \Gamma_{k-1}, \boldsymbol{\delta}, \Omega\right\}$ to vary unrestrictedly, then the model (9) corresponds to the *I(0)* model, i.e., a reparameterization of a VAR in levels. The *I(1)* model is obtained as sub-model of (9) if certain restrictions are satisfied. Thus, the higher order model is nested within the more general *I(0)* model.

the cointegrating vectors $\boldsymbol{\beta}$; the matrix $\boldsymbol{\alpha}$ gives the speed of adjustment of the endogenous variables to their 'equilibrium' values (the cointegrating relations), while there are also $n-r$ linearly-independent non-stationary relations. These last relations define the common stochastic trends of the system. In this case, the moving average representation (or solution) of $\mathbf{x}_t$ as a function of the innovations $\mathbf{u}_t$, the initial conditions $\mathbf{x}_0$, and the deterministic variables $\boldsymbol{\delta}$ is given by

$$\mathbf{x}_t = \mathbf{C}\left(1\right)\sum_{i=1}^{t}\mathbf{u}_i + \mathbf{C}\left(1\right)\boldsymbol{\delta}t + \mathbf{C}^*\left(\mathbf{L}\right)\left(\mathbf{u}_t + \boldsymbol{\delta}\right) + \mathbf{A}, \tag{12}$$

where $\mathbf{C}\left(1\right) = \boldsymbol{\beta}_\perp\left(\boldsymbol{\alpha}'_\perp\Gamma\boldsymbol{\beta}_\perp\right)^{-1}\boldsymbol{\alpha}'_\perp$, $\mathbf{C}^*\left(\mathbf{L}\right)$ is a polynomial in the lag operator, and $\mathbf{A}$ is a function of the initial conditions, such that $\boldsymbol{\beta}'\mathbf{A} = \mathbf{0}$.

Under the cointegrating restrictions one can estimate a VEqCM representation for $\mathbf{x}_t$ which takes the form

$$\Gamma\left(L\right)\Delta\mathbf{x}_t = \boldsymbol{\delta} + \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{x}_{t-1} + \mathbf{u}_t. \tag{13}$$

The term $\boldsymbol{\beta}'\mathbf{x}_{t-1}$ gives last period's equilibrium error; $\boldsymbol{\alpha}$ is the vector of "adjustment" coefficients (or loadings) that tells us which of the variables react to last periods equilibrium error (cointegrating residual); that is which of the variables, and by how much, adjust to restore the equilibrium relation $\boldsymbol{\beta}'\mathbf{x}_{t-1}$ back to its mean when a deviation occurs. By virtue of the Granger Representation Theorem (GRT, Engle and Granger [1987]), if a vector of variables $\mathbf{x}_t$ is cointegrated, then at least one of the adjustment parameters in the $n \times r$ matrix $\boldsymbol{\alpha}$ must be non-zero in the VEqCM representation (13). Thus if x_i does at least some of the adjusting needed to restore the long-run equilibrium subsequent to a shock that distorts this equilibrium, then some of the parameters in the $1 \times r$ vector $\boldsymbol{\alpha}_i$ should be different from zero in the equation for Δx_i in the VEqCM representation (13).

3.3 Permanent and Transitory Decomposition

Our empirical approach is based on using the restrictions implied by cointegration to identify the permanent and transitory components of the three variable system, $\mathbf{x}_t$. Identification is possible because cointegration places restrictions on the long-run multipliers of the shocks in a model where innovations are distinguished by their degree of persistence, as shown, for example, in Gonzalo and Granger [1995], Johansen [1995], King *et al.* [1991], Mellander *et al.* [1992] and Warne [1993]. While this approach does not identify shocks that are structural in any sense,[10] as we will argue below, it will yield results that have some natural structural interpretation — potentially useful as a guide to further analysis.

[10] Strictly speaking, cointegration allows us to find a suitable rotation that maps reduced form shocks $\mathbf{u}_t$ into shocks $\mathbf{e}_t$, such that $n-r$ of them have permanent effects on $\mathbf{x}_t$ and the rest r have only a transitory effect on $\mathbf{x}_t$. But as explained in King *et al.* [1991] and Warne [1993], if some structural shocks are assumed to be permanent and some transitory, then cointegration can considerably reduce the number of restrictions that need to be imposed to identify the shocks.

This procedure takes several steps. We first estimate the VEqCM, then use the estimated parameters to back out the long-run restrictions. More specifically, cointegration restricts the matrix of long-run multipliers of shocks in the system, which identifies the permanent components. The transitory components are identified in a 'residual' manner. In order to study the dynamic impact of the transitory innovations, it is assumed that they are orthogonal to the permanent innovations.

It is useful to review our methodology in some detail, and explain how it is related to our application. From the GRT it follows that, under the maintained hypothesis that the growth rates in $\mathbf{x}_t$ are covariance stationary, there exists a multivariate Wold representation of the form

$$\Delta\mathbf{x}_t = \boldsymbol{\kappa} + \mathbf{C}\left(L\right)\mathbf{u}_t, \tag{14}$$

where $\mathbf{C}\left(L\right)$ is a $n \times n$ matrix polynomial in the lag operator. We want to map these reduced form innovations into transformed innovations $\mathbf{e}_t$ that are distinguished by whether they have permanent or transitory effects. Without loss of generality the shocks $\mathbf{e}_t$ are ordered so that the first $n-r$ of them have permanent effects; and the last r of them have transitory effects. Following Gonzalo and Granger [1995], we define a shock $\mathbf{e}_t^P$, as permanent if $\lim_{h\to\infty} \partial E\left(\mathbf{x}_{t+h}\right)/\partial\mathbf{e}_t^P \neq 0$, and a shock, $\mathbf{e}_t^T$, as transitory if $\lim_{h\to\infty} \partial E\left(\mathbf{x}_{t+h}\right)/\partial\mathbf{e}_t^T = 0$.

Applying the methodology of King *et al.* [1991], as extended by Warne [1993] and discussed in Johansen [1995], the permanent and transitory innovations may be identified using the estimated parameters of the VEqCM representation of a cointegrated system. In particular, as explained in Johansen [1995], the matrix $\mathbf{C}\left(1\right)$ of the Wold representation (14), admits a closed-form solution in terms of the parameters of the cointegrated VAR:

$$\mathbf{C}\left(1\right) = \boldsymbol{\beta}_\perp \left(\boldsymbol{\alpha}_\perp \Gamma\left(1\right)\boldsymbol{\beta}_\perp\right)^{-1}\boldsymbol{\alpha}_\perp'. \tag{15}$$

Notice that the structure of this matrix is such that it maps reduced-form disturbances $\mathbf{u}_t$ into the space spanned by the columns of $\boldsymbol{\alpha}_\perp$ i.e. $sp\left(\boldsymbol{\alpha}_\perp\right)$. The disturbances $\boldsymbol{\alpha}_\perp'\mathbf{u}_t$ accumulate to the permanent component of $\mathbf{x}_t$, whereas transitory disturbances will be in the null-space of $\mathbf{C}\left(1\right)$. We can therefore define the permanent disturbances (permanent shocks) as:

$$\mathbf{e}_t^P = \boldsymbol{\alpha}_\perp'\mathbf{u}_t. \tag{16}$$

Then by requiring that the permanent and transitory shocks be orthogonal to each other, we can define the transitory shocks as:

$$\mathbf{e}_t^T = \boldsymbol{\alpha}'\Omega^{-1}\mathbf{u}_t. \tag{17}$$

Denoting

$$\mathbf{P}^{-1} = \begin{bmatrix} \boldsymbol{\alpha}_\perp' \\ \boldsymbol{\alpha}'\Omega^{-1} \end{bmatrix} \tag{18}$$

$$\mathbf{e}_t' = \begin{bmatrix} \mathbf{e}_t^{P\prime} & \mathbf{e}_t^{T\prime} \end{bmatrix}, \tag{19}$$

we have that

$$Var\left(\mathbf{e}_t\right) = diag\left\{Var\left(\mathbf{e}_t^P\right), Var\left(\mathbf{e}_t^T\right)\right\} = \begin{bmatrix} \boldsymbol{\alpha}'_{\perp}\Omega\boldsymbol{\alpha}_{\perp\ (n-r)\times(n-r)} & \mathbf{0}_{(n-r)\times r} \\ \mathbf{0}_{r\times(n-r)} & \boldsymbol{\alpha}'\Omega^{-1}\boldsymbol{\alpha}_{\ r\times r} \end{bmatrix} = \mathbf{P}^{-1}\Omega\mathbf{P}^{-1\prime}. \tag{20}$$

Notice that in this way we have achieved both the rotation from reduced-form shocks to permanent and transitory shocks and the orthogonalization.[11] Let $\mathbf{D}(L) = \mathbf{C}(L)\mathbf{P}$, and $\mathbf{e}_t = \mathbf{P}^{-1}\mathbf{u}_t$, the transformed Wold representation is

$$\Delta\mathbf{x}_t = \boldsymbol{\kappa} + \mathbf{D}(L)\,\mathbf{e}_t. \tag{21}$$

Thus each element of $\Delta\mathbf{x}_t$ has been decomposed into a function of $n-r$ permanent and r transitory shocks.

4 Empirical Evidence on Common Trends: Cointegration Analysis

The first step to implement the procedure described in the previous section is the determination of the cointegrating rank. Based on univariate and multivariate misspecification statistics (reported in an appendix), we choose an empirical model with two lags. In panels A and B of table 2, we report the trace test statistics for cointegration (Johansen, [1995]).[12] Note that in Panel A, we also report a variant of the test statistics that uses a small-sample correction for the degrees of freedom, proposed by Reinsel and Ahn [1992] and Reimers [1992]. In panel B, we report the trace statistics making use of the small sample correction proposed in Johansen [2002]. In both panels we also reported simulated and bootstrap critical values as well as p-values. From both panels, we see that the trace test statistics support a choice of $r = 1$, implying the existence of two common stochastic trends (Stock and Watson, [1988]). More specifically, we reject the hypothesis that there exist $n - r = 3$ common stochastic trends (no cointegration), while we do not reject the hypothesis that there is at most one cointegrating vector (see also the associated p-values reported in table 2).

`[Insert Table 2 about here]`

Having established the cointegrating rank, we obtain estimates of the cointegrating parameters $\boldsymbol{\beta} = (1, \beta_z, \beta_d)'$ using maximum likelihood (Johansen, [1995]). We estimate $\hat{\boldsymbol{\beta}} = (1, -1.759, 0.097)'$ — with associated t-statistics equal to -9.88 and 2.85 respectively. We discuss the stability these estimates in the appendix.

[11] An alternative scheme for identifying permanent and transitory shocks is due to Gonzalo and Ng [2001], which we do not follow here.

[12] Needles to say that similar results were obtained using the maximum-eigenvalue test statistics.

Observe that the coefficients $\hat{\boldsymbol{\beta}}$ have the right sign and they satisfy an important inequality. The estimated coefficient on z_t is negative and larger than one, and from our theoretical discussion in section 2 it should be equal to $(1/\rho_{D\Psi})$. Similarly, the coefficient of d_t is positive and larger than zero, and in principle should be equal to $(1/\rho_{D\Psi} - 1)$. Both estimated coefficients imply that $\rho_{D\Psi} = 1 - \overline{(D_t/\sum_{i=0}^{\infty} R_{t,t+i} Z_{t+i})} < 1$. Hence the average value term is bounded from above by unity: on average (at least within our sample), the value of net foreign debt is always smaller than the present value of net output. However, the log-linearized budget constraint in section 2 implies that these coefficients should sum to minus one. This (over-identifying) restriction was rejected at conventional significance levels, since we obtain $Q(1) = 9.20$ distributed as a $\chi^2(1)$ with asymptotic p-value of [0.002] and bootstrap p-value of [0.028] (see also Panel B and C of table 3). This is not surprising: these coefficients are unlikely to sum to minus one in empirical implementations of the model, because nondurable consumption flows are not directly observable and need to be proxied (a point stressed by Lettau and Ludvigson [2001, 2004]), but also because our measure of net foreign liabilities is a rough proxy of the true theoretical variable.

The VEqCM representation of $\mathbf{x}_t$ takes the form

$$\Delta\mathbf{x}_t = \boldsymbol{\delta} + \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{x}_{t-1} + \Gamma_1 \Delta\mathbf{x}_{t-1} + \mathbf{u}_t, \tag{22}$$

where $\Delta\mathbf{x}_t$ is the vector of log first differences, $(\Delta c_t, \Delta z_t, \Delta d_t)'$, $\boldsymbol{\delta}$ and $\boldsymbol{\alpha} \equiv (\alpha_c, \alpha_z, \alpha_d)'$ are (3×1) vectors, and $\hat{\boldsymbol{\beta}} \equiv (1, \beta_z, \beta_d)'$ is the (3×1) vector of the cointegrating coefficients discussed above. The results of estimating the first-order specification (22) are presented in table 3. Panel A of table 3 shows the estimated VEqCM, with the associated t-statistics, adjusted R^2 and a set of misspecification statistics for each equation. Panel B shows the estimated unrestricted cointegrating vector and the associated standard errors, and Panel C shows the estimated (restricted) cointegrating vector, the relevant standard errors for the coefficients as well as the likelihood ratio test statistic for the restriction on $\boldsymbol{\beta}$. Notice that all variables show evidence of strong equilibrium correction, with log net foreign liabilities displaying the largest (in absolute value) adjustment to the disequilibrium error. Specifically all variables do much of the adjustment following a shock that caused them to deviate from their long-run stochastic trends. Second, notice that consumption growth is predictable by lagged consumption growth, and the equilibrium error; net output growth is predictable by lagged net foreign liabilities growth and the equilibrium error; finally, net liabilities growth is predictable by its own growth rate and the cointegrating error term. These results imply that there is short-run predictability of all the variables in the system and much of it is attributable to the fact that all variables adjust to restore the equilibrium error, $\boldsymbol{\beta}'\mathbf{x}_t$, back to its mean.

`[Insert Table 3 about here]`

5 Dynamic Analysis of Temporary and Permanent Shocks

5.1 Permanent and Transitory Shocks on Consumption, Net Output and Net Foreign Debt

Using the permanent-transitory decomposition discussed in section 3, it is straightforward to investigate how each of the variables in our system is related to permanent and transitory shocks. Intuitively, this decomposition can be understood by looking at the properties of the matrix $\mathbf{P}^{-1}$ in (18) that achieves the rotation from the reduced form to permanent and transitory shocks. Intuitively, the variable x_j participates little in the equilibrium correction — α_j is small in absolute value — when the element of $\boldsymbol{\alpha}'_{\perp}$ that multiplies u_{jt} is large in absolute value. Thus x_j has a large weight in the permanent and a small weight in the transitory innovations. By contrast, when α_j is large the element of $\boldsymbol{\alpha}'_{\perp}$ that multiplies u_{jt} is small in absolute value, giving x_j a small weight in the permanent innovations and a large weight in the transitory innovations. Thus, the variables have a large transitory component when they do much of the adjustment needed to restore equilibrium back to its mean. This is an intuitively appealing property because — by definition — any variable that does at least some of the adjustment required to bring the equilibrium relation back to its mean must have deviated from its trend, and hence contain a transitory component. In our application, the elements of the adjustment vector $\boldsymbol{\alpha}$ are all relatively large and statistically significant (see Table 3), implying that all the variables have a non-negligible weight in the transitory innovations.

Cointegration and the assumption of orthogonality of the permanent and transitory components,[13] impose the following structure on the long-run impact matrix

$$\begin{bmatrix} \Delta z_t \\ \Delta c_t \\ \Delta d_t \end{bmatrix} : \begin{bmatrix} D_{11}(1) & D_{12}(1) & 0 \\ D_{21}(1) & D_{22}(1) & 0 \\ D_{31}(1) & D_{32}(1) & 0 \end{bmatrix} \begin{bmatrix} e^P_{1t} \\ e^P_{2t} \\ e^T_{3t} \end{bmatrix},$$

where we assume, without loss of generality, e^T_{3t} to be the transitory shock. For the interpretation of our results, we find it helpful to proceed by adopting a specific long-run structure. Namely, we obtain the identified permanent shocks η^P_{1t} and η^P_{2t} by imposing a single restriction on $D(1)$. We can assume different 'recursive' long-run structures, setting $D_{j2}(1) = 0$ for one $j \in \{1, 2, 3\}$. For instance, setting $D_{12}(1) = 0$, the new long-run impact matrix is

$$\begin{bmatrix} \Delta z_t \\ \Delta c_t \\ \Delta d_t \end{bmatrix} : \left[\begin{bmatrix} \tilde{D}_{11}(1) & 0 & 0 \\ \tilde{D}_{21}(1) & \tilde{D}_{22}(1) & 0 \\ \tilde{D}_{31}(1) & \tilde{D}_{32}(1) & 0 \end{bmatrix} \right] \begin{bmatrix} \eta^P_{1t} \\ \eta^P_{2t} \\ \eta^T_{3t} \end{bmatrix},$$

with $\eta^T_{3t} \equiv e^T_{3t}$. In what follows, we will refer our baseline model to the above matrix.

Observe that the first permanent shock is the only shock that has a long-run impact on net output per capita. Hence, it has a natural interpretation as a *permanent technology shock.*

[13] This is a rather innocuous assumption. See Quah [1992] for a discussion.

More generally, this shock can be read as a linear combination of structural shocks that would have a permanent effect on net output. Conversely, the second permanent shock in our baseline model structure has a long-run impact on consumption and foreign wealth, but no persistent effect on output. In principle, this could be consistent with *temporary output shocks* producing permanent effects on consumption and foreign wealth — as implied for instance by the textbook model of the intertemporal account for the case of infinite-horizon agents.

The transitory shock η_{3t}^T only affects net output in the short and the medium run, and can therefore be read as a linear combination of structural shocks that lead to transitory changes in z_t — including *temporary technology shocks.*

5.2 Variance decompositions

Table 4 reports the fraction of the total variance in the forecast error of Δz_t, Δc_t, and Δd_t that is attributable to each of the shocks. In the table the two permanent shocks are denoted by η_{1t}^P and η_{2t}^P, the transitory shock by η_{3t}^T respectively — recall that the latter is orthogonal to the former two. We report variance decompositions and impulse responses. To quantify sampling uncertainty we have used a bootstrap Monte Carlo procedure. More specifically, Table 4 displays the fraction of the h-step ahead forecast-error variance in net output, consumption and net foreign debt that is attributable to the two permanent shocks and to the single transitory shock. For $h = 1, 2, ...$ and $h \to \infty$ we compute the portion of the total variance of each variable that is attributable to each disturbance.

For a horizon one to four quarters, the transitory shock accounts for a portion between 74% and 62% of the variance in net output, between 44% and 33% of the variance in consumption and between 29% and 15% of the variance in net foreign liabilities. At a horizon of eight to twenty quarters ahead, the transitory shock continues to contribute a considerable amount to the forecast error variance of all three variables (between 52 and 28, 24 an 10 and 10 and 6 percent respectively). However, it is the two permanent shocks that now account for the largest portion of variance. At a horizon of forty quarters, the first permanent shock, accounts for 79% of the variance of net output, 73% of the variance of consumption and 87% percent of the variance of net foreign debt. The second permanent shock accounts for 23% of the variance of consumption, whereas it has a negligible contribution to the variance of the other two variables (roughly 9%). Notably, at a horizon of forty quarters, the transitory shock still contributes 5% to the variance of all variables, but the point estimates are insignificant. Finally, the two permanent shocks account for the total of the long-run error variance in all variables,[14] with the second permanent shock having a contribution of 27% to the variance of consumption and net foreign liabilities.

It should be emphasized that the transitory shocks accounts for a respectable share of the total variation in all the variables at relatively short horizons. Although none of the shocks

[14] The property that only permanent shocks affect the variables in the long-run, wheras transitory do not, follows from cointegration and is not specific to the rotation of the shocks we have chosen. See also Gonzalo and Ng [2001] for a discussion.

should have a persistent effect on our approximate expression KA_t^* (a restriction imposed by cointegration, since KA_t^* and is a cointegrating residual), at short horizons there is significant variation in these variables, mostly attributed to the transitory shock. This means that as net output and consumption change in response to transitory shocks, the level of net foreign liabilities changes as well so that the effects of this shock on consumption are smoothed out, and the long-run equilibrium (KA_t^* or CA_t^*) is eventually restored.

[Insert Table 4 about here]

5.3 Impulse responses

Having set the boundaries of our analysis, we now turn to study impulse responses — shown in figures 3 to 5. In each figure we plot the accumulated impulse response of $\Delta z_t, \Delta c_t$ and Δd_t and the associated bootstrap confidence bands.

5.3.1 Transitory shocks

Consider first figure 2, showing the response of all the variables to a positive transitory shock raising net output. Both consumption and net output rise on impact, but consumption rises by less than net output. Correspondingly the country runs a current account surplus and accumulates foreign assets: net foreign liabilities jump down on impact and remain negative for at least ten years.

The sign of these impulse-responses is somewhat consistent with the prediction of the simplest intertemporal models of the current account, whereas national agents use foreign borrowing and lending to smooth consumption in the face of temporary shocks to their net output — see e.g. Obstfeld and Rogoff [1996], chapter 2 and especially chapter 3, where in overlapping generation models the effects on foreign assets and consumption disappear over time. While most of the analysis is developed in highly stylized models of small open economy, the same principle also applies in the case of a large open-economy such as the U.S. economy. However, it is worth stressing that the impulse responses in Figure 2 do not allow any conclusions regarding optimality of consumption and current account movements — which is instead explicitly tested by the present-value models conditional on some specification of the preferences of the national representative consumer.

Strikingly, figure 2 does not lend support to procyclical current account deficits. Temporary output expansions are not associated with a widening of the external imbalance, as implied by traditional models stressing the role of real demand shocks (e.g. government spending) in generating business cycle fluctuations.

Observe that, in light of our evidence on cointegration and our assumption of orthogonality between permanent and transitory shocks, our conclusions regarding the effects of the transitory shock are invariant to alternative ways to identify the two permanent shocks in the system (see Figure 6 and 7 below).

To check robustness, we have used alternative methodologies to calculate confidence intervals around our estimates. In all these experiments, the pattern of the impulse response to the transitory shocks is unaffected. The confidence band, if anything, becomes smaller.

`[Insert Figure 2 about here]`

5.3.2 Permanent shocks

Figure 3 shows the impulse responses to the first permanent shock — that, we argued above, can be interpreted as a technology shock. Consumption increases on impact and keeps increasing over time, while net output increases more slowly. The permanent shock increases net foreign liabilities on impact, which keep increasing for a period of four years after the shock — although they revert to a lower level in the long run, but strictly above zero.

According to the standard intertemporal model, permanent productivity shocks raise returns to domestic capital, attracting foreign investment and generating a current account deficit. This raises net foreign liabilities The capital inflow contributes to raise net output permanently.[15] The impulse responses in figure 3 is somewhat consistent with this prediction.

It is well known that if net output growth can be adequately described by an autoregressive process, a positive shock ϵ_t to net output (resulting in future net output levels that rise by more than ϵ_t) will increase permanent output more than current output. Consumption smoothing then implies that consumption increases by more than current output. As a result, a positive output innovation implies a current account deficit, rather than the surplus that is predicted by the model conditional on stationary output shocks (e.g., see Obstfeld and Rogoff, 1996, p. 84). Our impulse responses in figure 3 are strikingly consistent with this interpretation. For the first permanent shock hitting the system, c_t increases by more than net output during the transition, thereby causing a current account deficit.

Moreover, given that the U.S. is large in the world economy, we may expect that a shock raising US productivity and US demand (i.e. reducing US net saving) would put upward pressure on the world real interest rate. To the extent that the upsurge in the US demand translates into a temporary higher real rate of return, the consumption Euler equation implies that the marginal utility of US consumption should fall gradually along the transition path. Consistent with this view, as shown by the figure the US consumption increases gradually in response to the shock.

Figure 4 shows the responses of all the variables to the second permanent shock. This shock leads to a temporary small decrease in net output, which returns back to its initial level — allowing for sampling uncertainty — after two years, whereas consumption increases gradually to a new higher steady-state level (again after roughly two years). Net foreign liabilities do not respond significantly to the shock.

[15] Although not reported in the paper, we have experimented with an empirical model that allows for explicit investment dynamics, by modelling the vector of variables $\tilde{\mathbf{x}}_t = [\log(Y_t - G_t), \log(I_t), \log(C_t), \log(D_t)]'$. In response to a permanent shock, gross investment indeed increases on impact, remaining higher afterwards.

The impulse responses in Figure 4 are quite puzzling. One could observe that they are qualitatively consistent with the implications of permanent terms of trade and/or asset valuation shocks, raising long-run consumption without much effect on long-run foreign wealth and net output — the latter variable actually falls in the short run. In deriving our empirical model, however, we do not explicitly account for terms of trade and relative price effects. Most important, our constructed proxy for the time series of net liabilities d does not account for capital gains or losses. Relative goods and asset prices clearly impact the intertemporal decisions by domestic and international agents, and our result may in part capture these effects despite the empirical limitations of our variables. On the other hand, the impulse responses of Figure 4 do not seem to corroborate the hypothesis that consumption and foreign wealth permanently increase (decrease) vis-a-vis a temporary raise (drop) in net output. We conclude by noting that the second permanent shock in our baseline model explains a fairly small portion of the variance of consumption, output and net debt.

`[Insert Figure 3 and 4 about here]`

5.3.3 Alternative identification assumptions

Relative to our baseline model, there are other ways to identify permanent shocks in our system. One consists of assuming that the first permanent shock is the only shock that has a long-run effect on d_t, while the second permanent shock has only a long-run effect on z_t and c_t. Results are shown by Figure 6. For convenience we could dub the first permanent shock a *portfolio shift* shock. It turns out the effects of the first permanent shock — the *portfolio shift* shock —, are similar if not identical to the effects of *permanent technology shock* discussed in the previous section. Net output and consumption rise on impact and grow gradually until they reach the new steady state, whereas net foreign debt grows until it reaches its new steady state in five years; it seems to decline gently afterwards (see figure 6, top panel).

As discussed above, these impulse responses are qualitatively consistent with permanent productivity shocks that increase output and consumption while generating capital inflows, i.e., a current account deficit. Indeed, in this interpretation a permanent technology shock should lead to a permanent portfolio shift. The system's response to the temporary shock is also similar to our previous result: net output and consumption increase temporarily, with some accumulation of net foreign assets.

Looking at the impulse responses for the second permanent shock, note that the stock of net foreign liabilities increases on impact, reaching a peak after two years. Then it gradually decreases converging slowly to its initial level. Net output decreases somewhat on impact, but monotonically increases afterwards, becoming positive roughly two years after the shock. Recall that z_t is net of investment, so that a negative z_t could simply reflect a short-run upsurge of investment (corresponding to strong capital inflow recorded by the jump in net debt). Consumption increases on impact (consistent with expectations of higher net output) and keeps on growing over a period of ten years. One problem with these impulse responses is however the large sampling uncertainty surrounding them.

A different identification scheme assumes that the first permanent shock is the only shock that has a long-term effect on c_t, while the second permanent shock has only a long-run effect on z_t and d_t. Once again the first permanent shock can be interpreted as a permanent productivity shock, affecting positively the three variables in the system. Indeed, the impulse responses for this shock shown in Figure 7 are broadly in line with the corresponding impulse responses discussed above. The second permanent shock produces small and insignificant upward movements of debt and net output for an unchanged consumption level.

As we mentioned above, the effects of the transitory shock and its interpretation remain largely unchanged across all our models, exhibiting a similar pattern in all identification schemes (see figure 6 and 7).

[Figures 6 and 7 (in the Appendix) about here]

Our results regarding the effects of the transitory shock are robust to all the experiments that we have conducted and the alternative inferential procedures adopted.[16] Conversely, the results that are least robust to our experiments are the effects of the second permanent shock. Specifically we find that the confidence bands for the impulse responses to the second shock tend to become very wide using alternative ways of constructing the confidence intervals.

6 Concluding Remarks

Exploring the theoretical and empirical links between consumption, net output and net foreign liabilities/assets is a classic goal of international macroeconomics. The literature has shown that such link cannot be understood without distinguishing between trend and cycles of the relevant variables. In this paper, we have analyzed this issue adopting an empirical approach, focused on consumption, net output and net foreign liabilities. The intertemporal budget constraint (using appropriate transversality conditions) is sufficient to infer that (the logs of) consumption, net output and the net foreign liabilities must be cointegrated — a hypothesis for which we find empirical support. Using the restrictions implied by cointegration, we identify trend and cyclical components of these variables.

[16] These include:

- computing the standard errors of the impulse responses using bootsrap Monte Carlo
- employing the standard percentile interval for the impulse responses as $\left[s^*_{\gamma/2}, s^*_{(1-\gamma/2)}\right]$, where $s^*_{\gamma/2}$ and $s^*_{(1-\gamma/2)}$ are the $\gamma/2$ and $(1-\gamma/2)$-quantiles of the bootstrap distribution of the impulse responses
- and, using Hall's percentile interval which is determined as $\left[\hat{\phi} - \tau^*_{(1-\gamma/2)}, \hat{\phi} - \tau^*_{\gamma/2}\right]$ where $\hat{\phi}$ denotes the estimated impulse response, and $\tau^*_{\gamma/2}$ and $\tau^*_{(1-\gamma/2)}$ are the $\gamma/2$ and $(1-\gamma/2)$-quantiles of the distribution of $\hat{\phi}^{boot} - \hat{\phi}$.

See Benkwitz *et al.* [2001] and Breitüng *et al.* [2004] for a discussion.

We find that in response to a permanent shock that raises per capital net output permanently, per-capita consumption and net foreign indebtness also increase permanently. This dynamic behavior is qualitatively consistent with a permanent shock to productivity that rises the US returns above world level, thus attracting capital from abroad.

Most important, the empirical characterization of the response to temporary shocks appears to provide some support to the main predictions of the consumption smoothing hypothesis — temporary output gains are associated with foreign asset accumulation. It is however at odds with traditional views associating temporary output expansions with a pro-cyclical deterioration of the external balance.

In summary, permanent shocks seem to matter a lot for net output, consumption and net foreign liabilities over long horizons. But short-term changes in net foreign liabilities that are of transitory nature seem to play a significant role in consumption smoothing. A contribution of this paper is to document the total quantity of variation in net foreign liabilities that is transitory, specifying the extent to which this follows from transitory variations of macroeconomic aggregates such as consumption and net output.

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Tables

Table 1: **Summary Statistics**

	Δc_t	Δz_t	Δd_t
Univariate Summary Statistics			
Mean ($\times$100)	0.550	0.536	1.688
Standard Deviation ($\times$100)	0.452	1.007	4.618
Correlation Matrix			
Δc_t	1.000	0.433	0.050
Δz_t		1.000	-0.081
Δd_t			1.000

NOTES for Table 1: This table reports summary statistics for quarterly growth of consumption Δc_t, net output Δz_t, and the net foreign liabilities growth rate Δd_t, where all variables are expressed in real, per-capita terms. The sample spans the first quarter of 1963 to the fourth quarter of 2002.

Table 2: **Trace (Cointegration) Statistics**

Panel A: Trace Statistics				**Panel B: Bartlett-Corrected Trace Statistics**			
$H_0 : r$	$r = 0$	$r \leq 1$	$r \leq 2$	$H_0 : r$	$r = 0$	$r \leq 1$	$r \leq 2$
$n - r$	3	2	1	$n - r$	3	2	1
$Q\left(r\|\, n\right)$	**37.998**	9.130	0.096	$Q\left(r\|\, n\right)$	**32.848**	6.189	0.057
Asymptotic p-val.	[0.004]	[0.353]	[0.757]	Asymptotic p-val.	[0.002]	[0.673]	[0.811]
Simulated p-val.	[0.002]	[0.336]	[0.745]	Simulated p-val.	[0.015]	[0.658]	[0.808]
Bootstraped p-val.	[0.011]	[0.613]	[0.848]	Bootstraped p-val.	[0.015]	[0.658]	[0.861]
$Q_{corr}\left(r\|\, n\right)$	**36.56**	8.78	0.09				
Asymptotic p-val.	[0.006]	[0.393]	[0.762]				
$Q_{95}\left(r\|\, n\right)$	29.68	15.41	3.76	$Q_{95}\left(r\|\, n\right)$	29.68	15.41	3.76
$Q_{95}^{sim}\left(r\|\, n\right)$	28.591	14.961	3.801	$Q_{95}^{sim}\left(r\|\, n\right)$	28.471	14.998	3.864
$Q_{95}^{boot}\left(r\|\, n\right)$	32.481	19.522	9.051	$Q_{95}^{boot}\left(r\|\, n\right)$	28.906	14.428	5.526

NOTES for Table 2: $Q\left(r|\, n\right)$ denotes the trace statistic as defined in Johansen [1995], i.e. $Q(r|\, n) = -T\sum_{i=r+1}^{n} \ln\left(1 - \hat{\lambda}_i\right)$ and the $Q_{corr}\left(r|\, n\right)$ is the trace statistic with the small sample correction proposed by Reinsel and Ahn [1992] and Reimers [1992], i.e. $Q_{corr}(r|\, n) = -\left(T - nk\right)\sum_{i=r+1}^{n} \ln\left(1 - \hat{\lambda}_i\right)$. The asymptotic p-values reported are calculated using the methods in Doornik [1998], while the asymptotic critical values are taken from Osterwald-Lenum [1992]. The simulated critical values and p-values are based on a Monte Carlo Simulation for T=200 with 10.000 replications. The bootstrap critical values and bootstrap p-values are based on a Bootstrap Monte Carlo with 10.000 replications. The Bartlett-corrected trace statistics are those reported in Johansen [2002], and the asymptotic and bootstrap critical values have been obtained by 10.000 replication of the empirical model used. The sample spans the first quarter of 1963 to the fourth quarter of 2002.

Table 3: **Estimates from a Cointegrated VAR(2)**

Panel A: Cointegrated VAR			
	Equation		
	Δc_t	Δz_t	Δd_t
Dependent Variable			
Δc_{t-1}	**0.326**	-0.256	1.540
$(t-stat.)$	[3.904]	[−1.297]	[3.469]
Δz_{t-1}	-0.011	-0.114	-0.241
$(t-stat.)$	[−0.323]	[−1.371]	[−1.290]
Δd_{t-1}	0.008	**0.040**	**0.832**
$(t-stat.)$	[1.076]	[2.351]	[21.579]
$\hat{\boldsymbol{\beta}}'\mathbf{x}_{t-1}$	**0.013**	**0.039**	**-0.055**
$(t-stat.)$	[3.765]	[4.862]	[−3.050]
δ	**0.086**	**0.257**	**-0.359**
$(t-stat.)$	[3.891]	[4.938]	[−3.064]
$\bar{R}^2$	0.225	0.124	0.789
$\hat{\sigma}_\varepsilon \times 100$	0.401	0.944	2.127
$LM_{HET-SQ.}$	1.791	1.430	1.802
$LM_{ARCH(2)}$	0.016	0.831	5.157**
$LM_{AR(1-12)}$	1.962*	1.517	1.528

Panel B: Cointegrating Coefficients

$$\hat{\boldsymbol{\beta}}'\mathbf{x}_t = c_t + \hat{\beta}_z z_t + \hat{\beta}_d d_t$$

$$\hat{\boldsymbol{\beta}}' = \begin{bmatrix} 1 & \mathbf{-1.759} & \mathbf{0.097} \end{bmatrix}$$

$$S.E.(\hat{\boldsymbol{\beta}}') = \begin{bmatrix} - & 0.178 & 0.034 \end{bmatrix}$$

Panel C: Restricted Cointegrating Coefficients

$$\hat{\boldsymbol{\beta}}_R'\mathbf{x}_t = c_t + \hat{\beta}_z z_t + \left(-\hat{\beta}_z - 1\right) d_t$$

$$\hat{\boldsymbol{\beta}}_R' = \begin{bmatrix} 1 & \mathbf{-0.9848} & \mathbf{-0.0152} \end{bmatrix}$$

$$S.E.(\hat{\boldsymbol{\beta}}_R') = \begin{bmatrix} - & 0.0056 & 0.0056 \end{bmatrix}$$

$\mathcal{LR}$-test: $Q(1) = \mathbf{9.200}$

Asymptotic p-value	Bootstrap p-value
[0.002]	[0.028]

NOTES for Table 3: Panel A of the table reports the estimated coefficients from a cointegrated vector autoregressive (VAR) model of the column variable on the row variable; t-statistics are given in square brackets. Estimated coefficients that are significant at the 10% level are highlighted in bold face. For each equation the adjusted $\bar{R}^2$, the estimated standard error, a $LM_{AR(1-12)}$ test for first to twelfth order autocorrelation (F(12,141)-distributed), a $LM_{ARCH(2)}$ test for second order autoregressive conditional heteroscedasticity (F(2,149)-distributed) and a $LM_{HET-SQ.}$ test for heteroscedasticity (F(8,144)-distributed) are reported. The term $\hat{\boldsymbol{\beta}}'\mathbf{x}_t \equiv c_t + \hat{\beta}_z z_t + \hat{\beta}_d d_t$ is the estimated equilibrium error (cointegrating residual) without the "symmetry" restriction imposed on the parameters. Panel B reports the unrestricted cointegrating coefficients and their associated standard errors, while Panel C reports the restricted cointegrating coefficients, their standard errors, as well as the likelihood ratio test ($\chi^2(1)$-distributed) and the associated p-value. The sample spans the first quarter of 1963 to the fourth quarter of 2002.

Table 4: **Forecast Error variance Decomposition (Orthogonalized Shocks)**

	$\Delta z_{t+h} - E_t \Delta z_{t+h}$			$\Delta c_{t+h} - E_t \Delta c_{t+h}$			$\Delta d_{t+h} - E_t \Delta d_{t+h}$		
Horizon h	η_{1t}^P	η_{2t}^P	η_{3t}^T	η_{1t}^P	η_{2t}^P	η_{3t}^T	η_{1t}^P	η_{2t}^P	η_{3t}^T
1	0.004	0.258	0.739	0.430	0.125	0.445	0.621	0.086	0.293
SE	(0.283)	(0.194)	(0.130)	(0.221)	(0.248)	(0.064)	(0.206)	(0.179)	(0.128)
2	0.005	0.308	0.687	0.464	0.138	0.399	0.718	0.055	0.228
SE	(0.284)	(0.195)	(0.121)	(0.225)	(0.255)	(0.053)	(0.214)	(0.183)	(0.113)
3	0.013	0.334	0.653	0.492	0.146	0.362	0.775	0.042	0.183
SE	(0.290)	(0.196)	(0.125)	(0.233)	(0.260)	(0.047)	(0.219)	(0.189)	(0.108)
4	0.027	0.348	0.625	0.516	0.153	0.331	0.810	0.035	0.154
SE	(0.294)	(0.197)	(0.127)	(0.239)	(0.263)	(0.042)	(0.222)	(0.193)	(0.105)
8	0.140	0.339	0.521	0.591	0.173	0.236	0.870	0.029	0.102
SE	(0.301)	(0.196)	(0.135)	(0.257)	(0.267)	(0.030)	(0.233)	(0.210)	(0.096)
12	0.283	0.294	0.423	0.638	0.188	0.174	0.888	0.032	0.080
SE	(0.296)	(0.189)	(0.134)	(0.267)	(0.268)	(0.023)	(0.242)	(0.225)	(0.088)
16	0.414	0.245	0.341	0.669	0.198	0.133	0.894	0.038	0.067
SE	(0.283)	(0.179)	(0.127)	(0.272)	(0.269)	(0.020)	(0.249)	(0.237)	(0.082)
20	0.520	0.204	0.276	0.689	0.207	0.105	0.895	0.046	0.059
SE	(0.265)	(0.167)	(0.116)	(0.275)	(0.269)	(0.017)	(0.256)	(0.248)	(0.076)
40	0.786	0.093	0.121	0.727	0.229	0.044	0.872	0.093	0.035
SE	(0.172)	(0.111)	(0.067)	(0.277)	(0.270)	(0.010)	(0.281)	(0.282)	(0.054)
∞	1.000	0.000	0.000	0.744	0.256	0.000	0.705	0.295	0.000
SE				(0.272)	(0.273)		(0.352)	(0.352)	

NOTES for Table 4: The table reports the fraction of the variance in the h step-ahead forecast error of the variable listed at the head of each column that is attributable to innovations in the permanent shocks, η_{1t}^P and η_{2t}^T, the transitory shock, η_{3t}^T. Horizons are in quarters, and the underlying VEqCM is of order 1. Each row below the reported FEVD shows the median estimate of the fraction due to each shock, the 95% confidence interval from the parameter distribution and the associated bootstrap standard errors. The sample spans the first quarter of 1963 to the fourth quarter of 2002.

The reported standard errors were computed using a bootstrap Monte Carlo procedure. Specifically, we have constructed 10000 time series of the vector $\mathbf{x}_t$ as follows. Let $\{\hat{\mathbf{u}}_t\}_{t=1}^T$ denote the vector of residuals from the estimated VEqCM. We constructed 10000 sets of new time series residuals, $\{\hat{\mathbf{u}}_t(j)\}_{t=1}^T$, $j = 1, ..., 10000$. The t^{th} element of $\{\hat{\mathbf{u}}_t(j)\}_{t=1}^T$ was selected by drawing randomly, with replacement, from the set of fitted residual vectors $\{\hat{\mathbf{u}}_t\}_{t=1}^T$. For each $\{\hat{\mathbf{u}}_t(j)\}_{t=1}^T$, we have constructed a synthetic time series $\tilde{\mathbf{x}}_t$, denoted $\{\tilde{\mathbf{x}}_t(j)\}_{t=1}^T$, using the estimated VEqCM and the historical initial conditions on $\mathbf{x}_t$. We the re-estimated the VEqCM using $\{\tilde{\mathbf{x}}_t(j)\}_{t=1}^T$ and calculated the implied forecast error variance decompositions for $j = 1, ..., 10000$.

Appendix

A Empirical predictions of the intertemporal approach to the current account

Building on the consumption smoothing hypothesis, the intertemporal approach to the current account emphasizes the gains from international trade in bonds (see Obstfeld and Rogoff [1996] for a textbook comprehensive analysis). When hit by country-specific temporary shocks, national agents can choose to maintain their consumption level above current output by borrowing in the international markets. We summarize the main prediction of the intertemporal approach as follows.

For simplicity, let's abstract at first from capital accumulation. Consistent with the consumption smoothing hypothesis, the current account should respond to country-specific, temporary shocks, not to global (symmetric) or permanent shocks.

Positive shocks to current output, however, can generate either current account surpluses or deficits. In the textbook model, positive output shocks leads to accumulation of foreign wealth. But in the presence of a nontraded goods sector, the current account response to positive supply shocks depends on their sectoral incidence, as well as on the elasticity of substitution between tradables and nontradables. If the shock increases the supply of nontradables, and these goods are complement to tradables, the country will optimal run a current account deficit.

A positive output shock can also generate current account deficits in models with differentiated Home and Foreign, if Home bias in consumption and a low price elasticity of exports induce strong wealth effects from price movements — so that an increase in Home supply improves the Home terms of trade in equilibrium (see Corsetti et al. [2004]).

Accounting for investment and capital accumulation, instead, country-specific permanent shocks should have the strongest effect on external borrowing and lending — although in the presence of capital adjustment costs, the response of the current account to productivity shocks may change sign over time (see recent work by Kraay and Ventura [2000, 2002]). As discussed by Obstfeld and Rogoff [1996], consumer durables may in principle increase the volatility of the current account, since expenditure on them is 'lumpy.'

Infinite horizon models are studied under the maintained assumption that the rate of time preferences of the representative agents is equal to the market interest rate in steady state. Under this assumption, the current account only responds to stochastic disturbances. Temporary shocks have relevant long-run implications, since national consumers will want to smooth shocks over the infinite horizon, adding to their holding of foreign bonds permanently. Relative national wealth is therefore nonstationary. This is not the case in OLG models — where agents smooth consumption over their life cycle. The rate of time preferences need not be equal to the interest rate. Foreign asset holding will not be permanently affected by shocks, and the long-run distribution of national wealth is stationary. DSGE models are derived under the maintained assumption of a stationary distribution of foreign wealth — either assuming some costs associated to holding foreign bond, or letting time-preferences vary with consumption levels. Recent literature has addressed the gap between intertemporal models of the current account and portfolio models, stressing conditions under which foreign wealth is a constant share of a country portfolio (e.g. Ventura [2003]).

Although most models are written accounting for only one internationally traded asset, the main insight of this literature go through in an incomplete-market setting, whereas international bonds can be traded along with a (limited number of) other assets.

For convenience, we report below that under some conditions intertemporal models of the current account give rise to present value relations of the form

$$CA_t = -\sum_{i=1}^{\infty} \delta^i E_t \left(\Delta Y_{t+i}^p - \Delta I_{t+i} \right) \tag{23a}$$

$$= -\sum_{i=1}^{\infty} \delta^i E_t \Delta Z_{t+i}, \tag{23b}$$

where Y_t^p denotes private output ($Y_t^p = Y_t - G_t$, i.e. GDP minus government spending), I_t denotes investment and $Z_t \equiv Y_t^p - I_t$ denotes net output. This result follows from assuming quadratic utility, and combining the first order condition of the national representative consumer optimization problem with the intertemporal budget constraint. Most empirical research on consumption smoothing has gone about by testing variants of the above relation.

B Data Description

- CONSUMPTION C_t

Consumption is measured as expenditure on non-durables (PCNDGC96) and services (PCESVC96). The quarterly series are seasonally adjusted at annual rates, in billions of chain-weighted 1996 dollars. Our source is the FRED II Database of the Federal Reserve Bank of the Saint Louis.

- NET OUTPUT Z_t

Net Output is defined by the identity $Z_t \equiv Y_t - I_t - G_t$. Y_t is the real gross domestic product (GDPC1). I_t is real gross private domestic investment (GPDIC1) + real change in private inventories (CBIC1) + real personal consumption expenditure on durable goods (PCDGCC96). Finally, G_t is real government consumption expenditures & gross investment (GCEC1). All series are seasonally adjusted at annual rates, in billions of chain-weighted 1996 dollars. Our source is the FRED II Database of the Federal Reserve Bank of the Saint Louis.

- NET FOREIGN DEBT D_t

Our series for net foreign debt were obtained by cumulating the negative of the U.S. current account (BOPBCA). The original series have been scaled by 1.000, so that the series become positive throughout our sample. In the (negative) cumulated current account series, the minimum observation (largest negative in absolute value) is -699.77. So we have experimented using different additive constants (750, 800, 900, 1000, 1100), verifying the absence of any qualitative difference in our results.

- POPULATION

Our measure of population was obtained by sampling at the end of each quarter the monthly population series. Our source is the FRED II Database of the Federal Reserve Bank of the Saint Louis.

- PRICE DEFLATOR

The net foreign assets (debt) measure we employ have been deflated by the personal consumption expenditure chain-type deflator (1996=100), seasonally adjusted (PCECTPI). In principle, one would like to use the unobserved price deflator for the total flow consumption used here. Since such a deflator is unobservable, we use the total deflator expenditure as a proxy. Our source is the FRED II Database of the Federal Reserve Bank of the Saint Louis.

C The Log-Linearized Intertemporal Budget Constraint

The intertemporal budget constraint is given by

$$\sum_{i=0}^{\infty} R_{t,t+i}C_{t+i} = B_t + \sum_{i=0}^{\infty} R_{t,t+i}Z_{t+i}, \tag{24}$$

$$\sum_{i=0}^{\infty} R_{t,t+i}Z_{t+i} = D_t + \sum_{i=0}^{\infty} R_{t,t+i}C_{t+i} \tag{25}$$

where B_t is the initial (period t) level of net foreign assets and D_t net foreign debt. It can easily be seen that $D_t = -B_t$, and that $CA_t = B_{t+1} - B_t = -(D_{t+1} - D_t) = -KA_t$. We can write (25) as

$$\Psi_t = D_t + \Phi_t \tag{26}$$

where $\Phi_t = \sum_{i=0}^{\infty} R_{t,t+i}C_{t+i}$ and $\Psi_t = \sum_{i=0}^{\infty} R_{t,t+i}Z_{t+i}$. Similarly

$$1 - \frac{D_t}{\Psi_t} = \frac{\Phi_t}{\Psi_t}.$$

Taking logs

$$\log\left(1 - \frac{D_t}{\Psi_t}\right) = \phi_t - \psi_t. \tag{27}$$

The LHS of (27) can be approximated by taking a first-order Taylor approximation around the mean of the D_t/Ψ_t ratio (see Campbell *et al.* [1997]):

$$\begin{aligned} \log\left(1 - \exp\left(d_t - \psi_t\right)\right) &\approx \log\left(1 - \exp\left(\overline{d_t - \psi_t}\right)\right) \\ &\quad - \frac{\exp\left(\overline{d_t - \psi_t}\right)}{1 - \exp\left(\overline{d_t - \psi_t}\right)}\left(\left(d_t - \psi_t\right) - \left[\overline{d_t - \psi_t}\right]\right) \\ &= \log\left(1 - \exp\left(\overline{d_t - \psi_t}\right)\right) + \frac{\exp\left(\overline{d_t - \psi_t}\right)}{1 - \exp\left(\overline{d_t - \psi_t}\right)}\left[\overline{d_t - \psi_t}\right] \\ &\quad - \frac{\exp\left(\overline{d_t - \psi_t}\right)}{1 - \exp\left(\overline{d_t - \psi_t}\right)}\left(d_t - \psi_t\right). \end{aligned} \tag{28}$$

Defining

$$\rho_{D\Psi} \equiv 1 - \exp\left(\overline{d_t - \psi_t}\right),$$

we can rewrite (28) as

$$\log\left(1 - \exp\left(d_t - \psi_t\right)\right) \approx \kappa_{D\Psi} + \left(1 - \frac{1}{\rho_{D\Psi}}\right)\left(d_t - \psi_t\right), \tag{29}$$

where

$$\begin{aligned} \kappa_{D\Psi} &= \log\left(1 - \exp\left(\overline{d_t - \psi_t}\right)\right) + \frac{\exp\left(\overline{d_t - \psi_t}\right)}{1 - \exp\left(\overline{d_t - \psi_t}\right)}\overline{\left[d_t - \psi_t\right]} \\ &= \log\left(\rho_{D\Psi}\right) + \left(\frac{1 - \rho_{D\Psi}}{\rho_{D\Psi}}\right)\log\left(1 - \rho_{D\Psi}\right). \end{aligned}$$

Notice that this approximation is exact when the optimal net foreign debt to PDV of the net cash-output is constant, so that $d_t - \psi_t = \overline{\left[d_t - \psi_t\right]}$.

Hence the log-linearized intertemporal budget constraint can be written as

$$\kappa_{D\Psi} + \left(1 - \frac{1}{\rho_{D\Psi}}\right)(d_t - \psi_t) \approx \phi_t - \psi_t. \tag{30}$$

Notice that

$$\begin{aligned} \Phi_t &= C_t + \sum_{i=1}^{\infty} R_{t,t+i} C_{t+i} \\ &= C_t + (1 + r_{t+1})^{-1} \Phi_{t+1}, \end{aligned}$$

so

$$\begin{aligned} \Phi_{t+1} &= (1 + r_{t+1})(\Phi_t - C_t), \\ \text{or} &: \frac{\Phi_{t+1}}{\Phi_t} = (1 + r_{t+1})\left(1 - \frac{C_t}{\Phi_t}\right). \end{aligned}$$

Log-linearizing as above we have:

$$\Delta\phi_{t+1} \approx \kappa_{C\Phi} + r_{t+1} + \left(1 - \frac{1}{\rho_{C\Phi}}\right)(c_t - \phi_t), \tag{31}$$

where $\rho_{C\Phi}$ and $\kappa_{C\Phi}$ are defined similarly to $\rho_{D\Psi}$ and $\kappa_{D\Psi}$,[17] and $\rho_{C\Phi} < 1$. Using the trivial identity $\Delta\phi_{t+1} = \Delta c_{t+1} + (c_t - \phi_t) - (\phi_{t+1} - c_{t+1})$ and (31), equating the LHS, we obtain a difference equation in the log $C_t - \Phi_t$ ratio. Then solving forward

$$(c_t - \phi_t) = \frac{\kappa_{C\Phi}}{1 - \rho_{C\Phi}} + \sum_{i=1}^{\infty} \rho_{C\Phi}^{i} (r_{t+i} - \Delta c_{t+i}) \tag{32}$$

where the condition $\lim_{T\to\infty} \rho_{C\Phi}^{T} (c_{t+T} - \phi_{t+T}) \to 0$ has been imposed. Observe what the last condition implies. We have

$$\rho_{C\Phi} = 1 - \exp\left(\overline{c_t - \phi_t}\right) = 1 - \overline{\left(\frac{C_t}{\sum_{i=0}^{\infty} R_{t,t+i} C_{t+i}}\right)} < 0,$$

so the extent that $(c_{t+T} - \phi_{t+T})$ is a stationary process, the limit term will go to zero, at least in expectation (see Campbell et al. [1997]). To see this more clearly, using the definition of the log-differential

$$\begin{aligned} c_{t+T} - \phi_{t+T} &= \log\left(\frac{C_{t+T}}{\sum_{i=0}^{\infty} R_{t,t+T+i} C_{t+T+i}}\right) \\ &= \log\left(\frac{1}{\sum_{i=0}^{\infty} R_{t,t+T+i} (C_{t+T+i}/C_{t+T})}\right) \\ &= \log\left(\frac{1}{1 + \sum_{i=1}^{\infty} \exp\left[\sum_{j=t+T}^{T+t+i} (\Delta \log C_j - \log(1 + r_j))\right]}\right), \end{aligned}$$

[17] That is:

$$\rho_{C\Phi} \equiv 1 - \exp\left(\overline{c_t - \phi_t}\right)$$

and

$$\begin{aligned} \kappa_{C\Phi} &= \log\left(1 - \exp\left(\overline{c_t - \phi_t}\right)\right) + \frac{\exp\left(\overline{c_t - \phi_t}\right)}{1 - \exp\left(\overline{c_t - \phi_t}\right)} \overline{[b_t - \phi_t]} \\ &= \log(\rho_{C\Phi}) + \left(\frac{1 - \rho_{C\Phi}}{\rho_{C\Phi}}\right) \log(1 - \rho_{C\Phi}). \end{aligned}$$

which is a stationary process under the maintained assumption of stationary consumption growth ($\Delta \log C_t$) and stationary rates of return (r_t).

Notice also, that

$$\Psi_t = Z_t + (1 + r_{t+1})^{-1} \Psi_{t+1}$$

and using similar steps, we obtain

$$z_t - \psi_t = \frac{\kappa_{Z\Psi}}{1 - \rho_{Z\Psi}} + \textstyle\sum_{i=1}^{\infty} \rho_{Z\Psi}^i \left(r_{t+i} - \Delta z_{t+i} \right), \tag{33}$$

where $\kappa_{Z\Psi}$ and $\rho_{Z\Psi}$ are defined similarly to the above log-linearization parameters. Substituting (32) and (33) in (30) we find that

$$c_t - \frac{1}{\rho_{D\Psi}} z_t + \left(\frac{1}{\rho_{D\Psi}} - 1 \right) d_t - \kappa_{D\Psi} + \frac{\kappa_{C\Phi}}{1 - \rho_{C\Phi}} - \frac{\kappa_{Z\Psi}}{1 - \rho_{Z\Psi}} \approx$$

$$\textstyle\sum_{i=1}^{\infty} \rho_{C\Phi}^i \left(r_{t+i} - \Delta c_{t+i} \right) - \frac{1}{\rho_{D\Psi}} \sum_{i=1}^{\infty} \rho_{Z\Psi}^i \left(r_{t+i} - \Delta z_{t+i} \right). \tag{34}$$

$$\approx - \textstyle\sum_{i=1}^{\infty} \rho_{C\Phi}^i \Delta c_{t+i} + \frac{1}{\rho_{D\Psi}} \sum_{i=1}^{\infty} \rho_{Z\Psi}^i \Delta z_{t+i} + \sum_{i=1}^{\infty} \rho_{C\Phi}^i r_{t+i} - \frac{1}{\rho_{D\Psi}} \sum_{i=1}^{\infty} \rho_{Z\Psi}^i r_{t+i}. \tag{35}$$

Notice that if we define as an approximation of the capital account

$$\begin{aligned} KA_t^* &\equiv c_t - \frac{1}{\rho_{D\Psi}} z_t + \left(\frac{1}{\rho_{D\Psi}} - 1 \right) d_t \\ &\equiv c_t - \varphi_z z_t + (\varphi_z - 1) d_t, \end{aligned} \tag{36}$$

where

$$\varphi_z \equiv \frac{1}{\rho_{D\Psi}},$$

we have that a present value relation should hold for KA_t^*, namely[18]

$$KA_t^* = - \textstyle\sum_{i=1}^{\infty} \rho_{C\Phi}^i \Delta c_{t+i} + \frac{1}{\rho_{D\Psi}} \sum_{i=1}^{\infty} \rho_{Z\Psi}^i \Delta z_{t+i} + \left(\sum_{i=1}^{\infty} \rho_{C\Phi}^i r_{t+i} - \frac{1}{\rho_{D\Psi}} \sum_{i=1}^{\infty} \rho_{Z\Psi}^i r_{t+i} \right). \tag{37}$$

Taking conditional expectations we have that

$$KA_t^* = E_t \left\{ - \textstyle\sum_{i=1}^{\infty} \rho_{C\Phi}^i \Delta c_{t+i} + \frac{1}{\rho_{D\Psi}} \sum_{i=1}^{\infty} \rho_{Z\Psi}^i \Delta z_{t+i} + \left(\sum_{i=1}^{\infty} \rho_{C\Phi}^i r_{t+i} - \frac{1}{\rho_{D\Psi}} \sum_{i=1}^{\infty} \rho_{Z\Psi}^i r_{t+i} \right) \right\}. \tag{38}$$

Similarly, using the accounting identity

$$CA_t + KA_t \equiv 0,$$

it follows that $CA_t \equiv -KA_t$, so that for our approximate capital account expression it holds

$$CA_t^* = -KA_t^*$$

$$\text{or}: CA_t^* = E_t \left\{ - \frac{1}{\rho_{D\Psi}} \textstyle\sum_{i=1}^{\infty} \rho_{Z\Psi}^i \Delta z_{t+i} + \sum_{i=1}^{\infty} \rho_{C\Phi}^i \Delta c_{t+i} + \left(\frac{1}{\rho_{D\Psi}} \sum_{i=1}^{\infty} \rho_{Z\Psi}^i r_{t+i} - \sum_{i=1}^{\infty} \rho_{C\Phi}^i r_{t+i} \right) \right\}$$

which is an expression similar to that derived by Bergin and Sheffrin [2000] and has been used in their empirical implementation of testing for the present value relation of the current account.

[18] Disregarding linearization constants.

D Lag-Length Selection and Misspecification Statistics

Table 5: **Misspecification Statistics of VAR with k=2 lags**

Panel A: Univariate Statistics						
Equation	$\hat{\sigma}_{\varepsilon} \times 100$	AR(12)	$ARCH(2)$	$NORM(2)$	R^2	
Δc_t	0.3934	1.903*	0.040	10.347**	0.248	
Δz_t	0.9258	1.402	1.705	1.469	0.153	
Δd_t	0.2057	1.290	9.752**	63.959**	0.801	
Panel B: Multivariate Tests						
LM_1	LM_4	LM_8	LM_{12}	$L-B(39)$	$NORM\,(6)$	
11.360	19.198	20.792	12.885	468.094	73.296	
[0.252]	[0.023]	[0.014]	[0.182]	[0.000]	[0.000]	
Panel C: Lag-Length Selection						
k	$\log L$	$LR\,(k-1/k)$	$p-value$	AIC	SIC	HQ
0	288.357	N/A	N/A	-3.658	-3.600	-3.635
1	1444.607	2253.204**	[0.000]	-18.367	-18.132	-18.271
2	1562.385	**224.987****	[0.000]	**-19.761**	**-19.351**	**-19.595***
3	1568.088	10.675	[0.299]	-19.719	-19.133	-19.481
4	1574.358	11.496	[0.243]	-19.684	-18.922	-19.374
Panel D: Roots of the Companion Matrix						
$r=3$	0.9981	0.9568	0.8954	0.8954	0.2812	0.1049
$r=2$	1.000	0.9604	0.8943	0.8943	0.2790	0.1053
$r=1$	1.000	1.000	0.9527	0.8534	0.2977	0.1200

NOTES for Table 5: The R^2 can be interpreted as the fit of the model for each variable relative to a random walk with drift. AR(12) is an LM test statistic for autocorrelation (F(12,139) distributed), ARCH (2) is the test for ARCH effects ($\chi^2(2)$ distributed), and NORM is the Jarque-Bera test for normality ($\chi^2(2)$ distributed), while * (**) denotes significance at the 5% (1%) level. The LM_i are tests of i-th order autocorrelation distributed as a $\chi^2(9)$. The NORM(6) is a multivariate Normality test (Doornik-Hansen, 1994) which is distributed as a $\chi^2(6)$. The L-B(39) is the multivariate version of the Ljung-Box test for autocorrelation based on the estimated auto- & cross-correlations of the first [T/4=39] lags and is distributed as a $\chi^2(333)$. $\log L$ denotes the value of the log-likelihood, LR is sequential (i.e. k vs $k-1$ lags) LR test statistic corrected by a degrees of freedom adjustment, AIC is the Akaike information criterion, SIC denotes the Schwarz information criterion and HQ is the Hannan-Quinn information criterion. The lag order selected by each criterion, is given in boldface.

D.1 Discussion

D.1.1 Lag-Length Selection

In order for the statistical procedure to be valid we have to check that the assumptions of the underlying statistical model are satisfied. In particular we examined whether or not the estimated residuals deviate from being Gaussian. For the empirical model, two lags were chosen on the basis of univariate and multivariate tests reported in table 5, as well as conventional likelihood ratio tests and information criteria. As can be seen form table 5, the model with $k=2$ seems to exhibit residual non-normality and ARCH effects. However, the asymptotic properties of Johansen's method depends only on the i.i.d. assumption of the errors and thus normality of the errors is not crucial for inference. The same holds for the presence of ARCH.[19] The system seems to be reasonably well behaved in terms of misspecification statistics. Multivariate LM tests for first and fourth order residual auto-correlation are not significant while there seems to be some higher order autocorrelation present in the data. The presence of autocorrelation is probably due to autocorrelation in the consumption equation. Increasing the lag-length beyond two, did not seem to improve the autocorrelation properties of the residuals, while degrees of freedom were falling sharply. For this reason, we decided to use a VAR with lag-length two as an adequate description of the dynamic properties of the data.

D.1.2 Additional Evidence for the Choice of the Cointegrating Rank

In addition to using the formal trace tests, we may gain some insight by looking at the estimated roots of the characteristic polynomial of the process.[20] Juselius [1995] argues that the estimated trace statistics should be interpreted with caution, since the asymptotic critical values may not provide very good approximations in finite samples. In panel D of table 5, we have listed the $n \times k$ roots of the companion matrix (inverse of the roots of the characteristic polynomial). From table 5 it becomes quite clear that there are at least two eigenvalues close to unity and the next one is also quite high, being roughly 0.90. Choosing a cointegrating rank $r = 2$, leaves a relatively large eigenvalue (0.96) unrestricted, while similarly, choosing $r = 1$ leaves an unrestricted an eigenvalue equal to 0.92. So, in general the number of unit roots does not seem to be in accordance with the trace and maximum eigenvalue test statistics in table 2. It could well imply that at least one of the processes has a double unit root (i.e it is $I(2)$). Testing for second order of integration in the system,[21] indicated that none of the processes is $I(2)$, hence we proceeded by choosing $r = 1$ in our empirical exercise.

[19] See Gonzalo [1994] for a general dicussion of the superiority of the ML estimation method of the cointegrated VAR in the presence of non-normality and ARCH effects. See also Hansen and Rahbek [1999] for the robustness of cointegration inference in the presence of ARCH effects in the errors.

[20] See Johansen [1995], *Corollary 4.7 pp. 61-62.* The reader should also note that inspecting the number of unit roots of the process will provide insights rather than definit answers.

[21] Results available upon request.

E Temporal Stability of Long-Run Relationships

E.1 Methodological Issues and Tests

The possibility of structural change is sometimes given as a reason to question the presence of cointegration. This is due to the fact that most of the tests for cointegration, including the ones we employ here due to Johansen [1995], are based on an implicit assumption of parameter constancy in the underlying data generating process (DGP), since the presence of parameter instabilities would show up as nuisance parameters in the asymptotic distributions that have been derived. Since the existence of cointegration places restrictions on the shocks to the system, that we exploit below in order to identify the permanent and transitory shocks, a natural question that arises is whether the possibility of structural change would invalidate these assumptions. We perform formal tests of parameter stability, and our results are presented below. We argue however, that parameter instability in the cointegrating relation is unlikely to occur, for several reasons. First, the type of structural change often referred to takes the form of a shift in the time-series processes of a large number of macroeconomic variables, probably associated with significant episodes such as the oil price shock of the 1970s. It is unclear, however, why such a structural changes -if they in fact occurred- should not influence consumption, output and net foreign debt in similar ways, thereby making the conjecture of breaks in the cointegrating relation far less obvious. We would like to underline that the presence of common trends in these three variables indicates how they move together in the long-run, and follows simply from a budget constraint identity. Second, we know of no reason to conclude that structural change is likely to produce spurious evidence of cointegration. It is precisely the opposite that is likely to be the case, since it is well-known that such break-points make time series appear less stationary (Perron [1989]). Structural change in the cointegrating parameters should make it more difficult to find a stable cointegrating relation.

Recent developments in the analysis of cointegrated VAR model have made it possible to test formally for the presence of parameter instability in the cointegrating relations. But before presenting findings from such tests, we should underline a few of their limitations. First, there is the issue of identifying breaks in common trends in finite samples. Any procedure that attempts to do so must necessarily divide a finite sample into subsamples. Since long samples are often required to obtain consistent estimates of cointegrating coefficients, it may be impossible to formally assess the stability of a single cointegration regime, especially when the deviations from the common stochastic trends are very persistent. Second, such methods do not propose an economic model of changes in regime that are caused by factors other than the data at hand. Therefore, they provide no guidance as to the sources of structural change or when it might occur in the future. Last but not least, one has to come to grips with the criticism, that structural break tests are inherently data-driven specification searches, that might bias inferences towards finding breaks when none actually exists (Leamer [1978]).

In order to assess the stability of the cointegrating relation we have estimated we employ tests due to Hansen and Johansen [1999]. Hansen and Johansen [1999] have suggested methods for the evaluation of parameter constancy in cointegrated VAR models, utilizing estimates obtained from the FIML estimation procedure. These tests are based on the parameterization (9). Three tests have been constructed under the two VAR representations. In the *Z-representation* all the parameters of the model (9) are re-estimated during the recursions while under the *R-representation* the short-run parameters Γ_i, $i = 1, ..., k-1$ are fixed to their full sample values and only the long run parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are re-estimated.[22]

[22] Hansen and Johansen [1999] seem to suggest that the results obtained using the "R-representation" should

The first test is the *Trace* test and it examines the null hypothesis of sample independency of the cointegration rank of the system. This is accomplished by estimating the model over each sub-sample, and the residuals corresponding to each recursive subsample are used to form the standard sample moments associated with the solution of the usual reduced rank problem (see Johansen [1995]). The eigenvalue problem is then solved directly from these subsample moment matrices. The obtained sequence of trace statistics is then scaled by the corresponding critical values, and we do not reject the null hypothesis that the chosen rank is maintained regardless of the sub-period for which it has been estimated, if the test statistic takes values greater than one.

The second test deals with the hypothesis of constancy of the cointegration space for a given cointegration rank. Hansen and Johansen [1999] have proposed a likelihood ratio test that is constructed by comparing the likelihood from each recursive subsample to the likelihood function calculated under the restriction that the cointegration vectors estimated from the full sample fall within the space spanned by the estimated vectors of each individual sample. The test statistic is χ^2 distributed with $(p-r)\,r$ degrees of freedom.

The final test, examines the constancy of each individual elements of the cointegrating vectors $\boldsymbol{\beta}$. However, when the cointegration rank is greater than one, the elements of those vectors cannot be identified, except under restrictions. But even in the case where restrictions have been imposed and the cointegration vectors are identified, if one looks at the coefficients one by one there might be problems due e.g. to the normalization chosen.[23] Fortunately, one can exploit the fact that there is a unique relationship between the eigenvalues and the cointegrating vectors.[24] Therefore, when the cointegrating vectors have undergone a structural shift this will be reflected in the estimated eigenvalues. Hansen and Johansen [1999] have derived the asymptotic distribution as well as the asymptotic variance of the estimated eigenvalues.

E.2 Stability of the Estimated Cointegrating Relation

With the caveats outlined in the previous sub-section in mind, we present the results of three tests discussed in above. We evaluate the temporal stability of our model utilizing the recursive analysis proposed by Hansen and Johansen [1999]; the aim is to establish that the results we have obtained up to here are not sample dependent. In figure 7 were our results are reported, we have chosen as a starting point the first quarter of 1983 in order for the estimated parameters to be based on a sufficient number of data points; in this way we are utilizing information for more than 19 years (79 observations). The figure shows recursively calculated trace tests scaled by the 95% critical value (top panel) and tests of the stability of $\boldsymbol{\beta}$ (lower left panel) that are based on both representations discussed above. The overall conclusion drawn from the three test is in favor of the sample independence of the cointegration results. More specifically, the top panel of figure 7 shows that the rank of the cointegration space is independent of the sample size from which it has been estimated, since the null hypothesis of a constant rank (one) could not be rejected. The plots also indicate that our choice of selecting just one cointegrating relation is the appropriate one. The plot in the lower left panel of figure 7, clearly indicates that the cointegrating space is stable over the recursive estimation sample, providing evidence of the non-rejection of the null

be prefered when the two representations give different results.

[23] In this context, multicollinearity between the variables in the system might prove to be an important problem, since there might be cases where the parameters are not estimated accurately when the variables are highly collinear.

[24] Notice that $\hat{\boldsymbol{\alpha}} = \mathbf{S}_{01}\hat{\boldsymbol{\beta}}$, hence $\hat{\boldsymbol{\alpha}}\mathbf{S}_{00}^{-1}\hat{\boldsymbol{\alpha}} = \hat{\boldsymbol{\beta}}'\mathbf{S}_{10}\mathbf{S}_{00}^{-1}\mathbf{S}_{01}\hat{\boldsymbol{\beta}} = diag\left(\hat{\lambda}_1, ..., \hat{\lambda}_r\right)$ (see Johansen [1995], Chapter 6, for a detailed analysis with all the derivations of the formula).

hypothesis for sample independency of the cointegration space for a given cointegrating rank; since for each $\tau = 1983:1,...,2003:3$ we test whether $\boldsymbol{\beta}_{2001:08} \in span(\boldsymbol{\beta}_{\tau})$, and for both representations we do not reject the null of parameter constancy. Finally, the lower left panel of figure 7 also provides substantial evidence in favor of the constancy of the estimated cointegrating vector (i.e. $\boldsymbol{\beta}_{\tau}$ above), since no substantial drift was detected on the time path of the associated eigenvalue. The last plot also seems to indicate that the maximum likelihood estimates do not display considerable instabilities in the recursive estimates. The results obtained further indicate that the empirical specification we adopt, does provide a valid framework to analyze the relative importance of permanent and transitory components in consumption, net output and net foreign debt.

`[Insert Figure 5 (in the Appendix) about here]`

F Further Robustness Results

Table 6: **Robustness Analysis: Cointegration Statistics**

Lags (k)	Scale	**Panel A: Trace Statistics**			**Panel B: Maximum Eigenvalue Statistics**		
		$r=0$	$r\leq 1$	$r\leq 2$	$r=0$	$r\leq 1$	$r\leq 2$
		$n-r=3$	$n-r=2$	$n-r=1$	$n-r=3$	$n-r=2$	$n-r=1$
4	$\kappa=700$	29.84 [0.050]	10.05 [0.282]	0.60 [0.439]	19.79 [0.076]	9.45 [0.256]	0.60 [0.439]
4	$\kappa=750$	30.95 [0.036]	8.81 [0.390]	0.40 [0.528]	22.14 [0.034]	8.41 [0.346]	0.40 [0.528]
2	$\kappa=800$	39.55 [0.002]	11.15 [0.205]	0.13 [0.716]	28.39 [0.003]	11.02 [0.155]	0.13 [0.716]
2	$\kappa=850$	38.88 [0.003]	10.31 [0.262]	0.12 [0.729]	28.56 [0.003]	10.20 [0.203]	0.12 [0.729]
2	$\kappa=900$	38.45 [0.003]	9.76 [0.305]	0.11 [0.740]	28.69 [0.003]	9.65 [0.241]	0.11 [0.740]
2	$\kappa=950$	38.18 [0.004]	9.39 [0.337]	0.10 [0.749]	28.79 [0.002]	9.29 [0.269]	0.10 [0.749]
$**2$	$\kappa=1000$	38.00 [0.004]	9.13 [0.353]	0.10 [0.757]	28.87 [0.002]	9.03 [0.290]	0.10 [0.757]
2	$\kappa=1050$	37.88 [0.004]	8.95 [0.377]	0.09 [0.765]	28.93 [0.002]	8.86 [0.305]	0.09 [0.765]
2	$\kappa=1100$	37.81 [0.004]	8.83 [0.389]	0.08 [0.772]	28.98 [0.002]	8.74 [0.315]	0.08 [0.772]
2	$\kappa=1150$	37.77 [0.004]	8.75 [0.396]	0.08 [0.778]	29.02 [0.002]	8.67 [0.322]	0.08 [0.778]
2	$\kappa=1200$	37.75 [0.004]	8.70 [0.401]	0.07 [0.785]	29.05 [0.002]	8.62 [0.326]	0.07 [0.785]
	Q_{95}	29.68	15.41	3.76	20.97	14.07	3.76

NOTES for Table 6: $Q(r|\,n)$ denotes the trace statistic as defined in Johansen [1995], i.e. $Q(r|\,n) = -T\sum_{i=r+1}^{n}\ln\left(1-\hat{\lambda}_i\right)$ and $Q(r|\,r+1)$ denotes the maximum eigenvalue, i.e. $Q(r|\,r+1) = -T \ln\left(1-\hat{\lambda}_{r+1}\right)$. The asymptotic p-values reported are calculated using the methods in Doornik [1998], while the asymptotic critical values (Q_{95}) for each test are taken from Osterwald-Lenum [1992]. The sample spans the first quarter of 1963 to the fourth quarter of 2002.

Table 7: **Robustness Analysis: Cointegrating Vector Coefficients**

Lags (k)	Scale Parameter	c_t	z_t	d_t	$Q(1)$
4	$\kappa = 700$	1	-2.874 (0.442)	0.189 (0.056)	9.220 [0.002]
4	$\kappa = 750$	1	-5.091 (0.925)	0.447 (0.130)	11.288 [0.001]
2	$\kappa = 800$	1	-1.556 (0.133)	0.060 (0.021)	9.014 [0.003]
2	$\kappa = 850$	1	-1.605 (0.145)	0.069 (0.024)	9.131 [0.003]
2	$\kappa = 900$	1	-1.655 (0.156)	0.078 (0.027)	9.181 [0.002]
2	$\kappa = 950$	1	-1.706 (0.167)	0.087 (0.031)	9.199 [0.002]
2	$\kappa = 1000$	1	-1.759 (0.178)	0.097 (0.034)	9.200 [0.002]
2	$\kappa = 1050$	1	-1.814 (0.189)	0.107 (0.037)	9.187 [0.002]
2	$\kappa = 1100$	1	-1.873 (0.202)	0.118 (0.041)	9.166 [0.003]
2	$\kappa = 1150$	1	-1.934 (0.214)	0.130 (0.045)	9.138 [0.003]
2	$\kappa = 1200$	1	-1.999 (0.228)	0.143 (0.049)	9.105 [0.003]

NOTES for Table 7: The table reports the estimated long coefficients from a cointegrated vector autoregressive (VAR) model with k lags chosen using standard information criteria, so that no residual autocorrelation is present. The last column reports the LR test statistic for the restriction . The numbers in parentheses below the coefficients are the (conditional) standard errors and the numbers in square brackets are the p-values for the LR test of the hypothesis $H_0 : \beta_z + \beta_d = -1$ ($\chi^2(1)$-distributed). The sample spans the first quarter of 1963 to the fourth quarter of 2002.

Figures

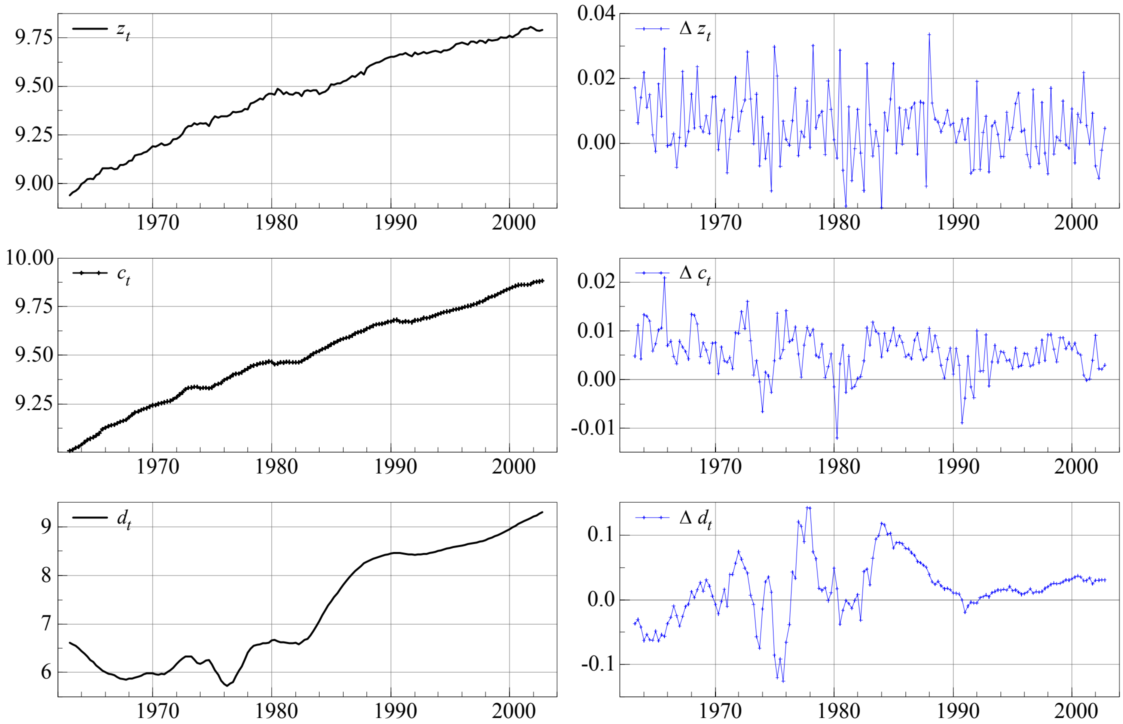

Figure 1. The Time Series Used in the Analysis.

Notes for Figure 1: The left panel of the figure plots the logs of real per-capita values of the net output z_t, consumption c_t and the net foreign debt d_t, while the right panel plots the growth rates of the variables. The sample spans the first quarter of 1963 to the fourth quarter of 2002.

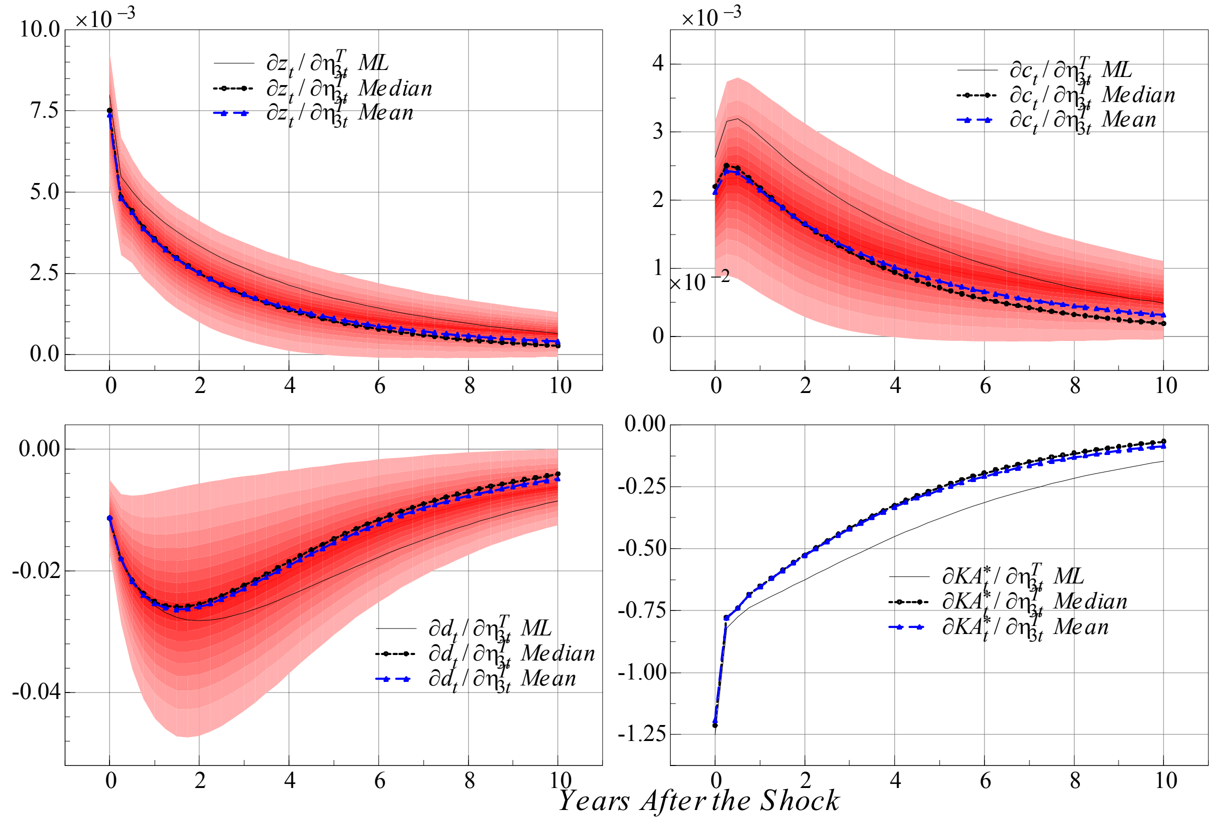

Figure 2. Impulse Responses to the Transitory Shock.

Notes for Figure 2: The first permanent shock is assumed to have a long-run impact on all three variables in the system. The second permanent shock is assumed to have a no long-run effect on z_t, but it has a long-run impact on c_t and d_t. The transitory shock is assumed to have no long-run effect on any of the variables (since transitory shocks do not persist). The horizon in the figures is measured in years after the shock. The figure shows the response of each variable to a one-unit shock. The top left panel shows the response of z_t, the top right the response of c_t, the lower-left panel the response of d_t; in all three figures the mean and median along with the associated bootsrap confidence intervals are reported. The lower-right panel shows the responce of $\beta' x_t \equiv KA^*_t$.

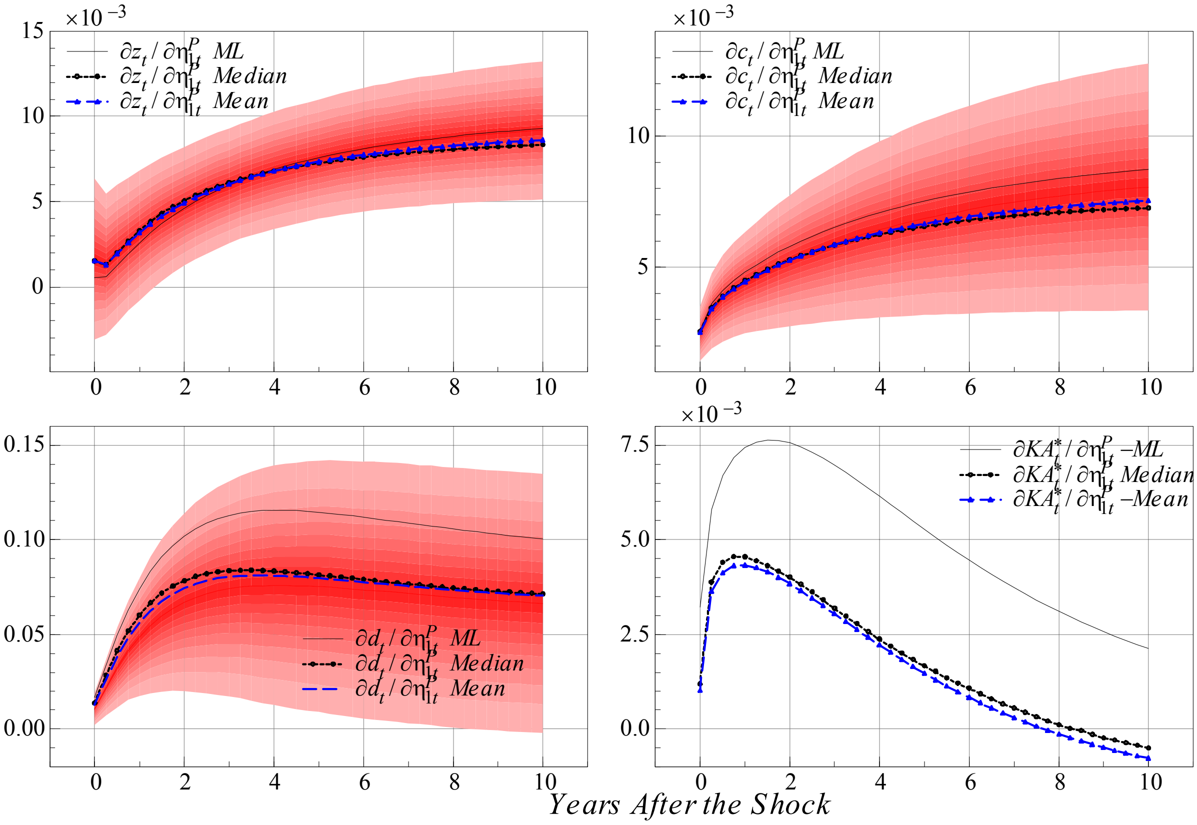

Figure 3. Impulse Responses to the First Permanent Shock.

Notes for Figure 3: See notes for Figure 3.

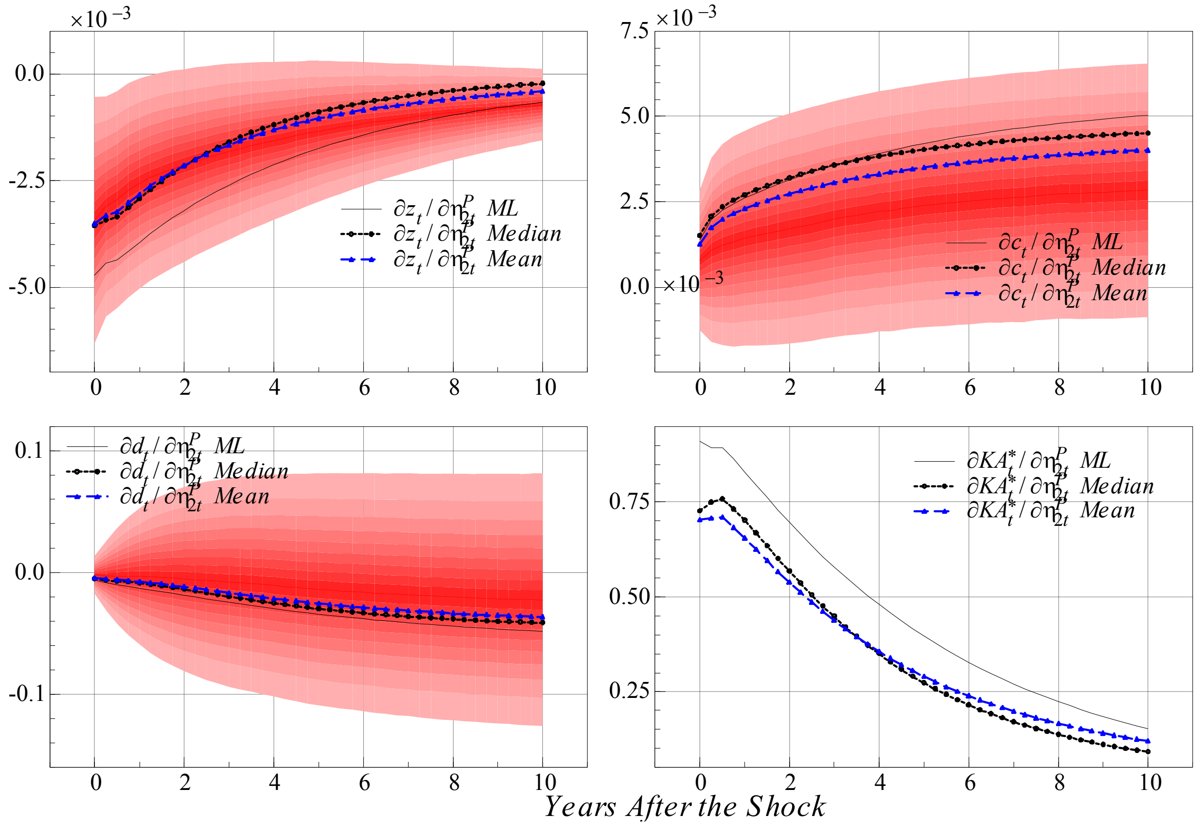

Figure 4. Impulse Responses to the Second Permanent Shock.

Notes for Figure 4: See notes for figure 2.

Appendix D: Stability Tests

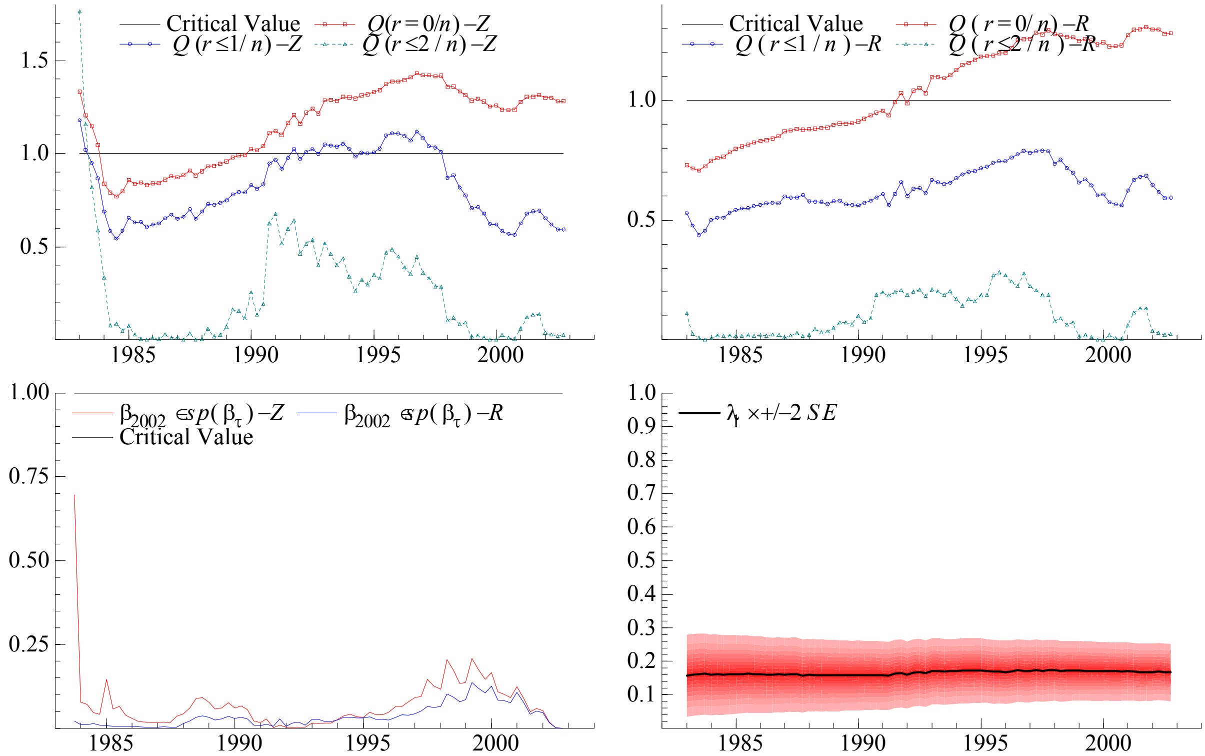

Figure 5. Recursively Calculated Trace Tests, Recursive Tests of $\boldsymbol{\beta}$ Constancy.

Notes for Figure 5: Both top panels of the figure plot the recursively calculated trace tests $Q(r|\,n) = -T\sum_{i=r+1}^{n} \ln\left(1-\hat{\lambda}_i\right)$ scaled by the asymptotic 95% critical values (Osterwald-Lenum [1992].) The top panel left shows the recursively calculated trace tests in the Z-representation where all the parameters of the VAR(2) model are re-estimated during the recursions. The top left panel shows the same test statistics under the R-representation, where the short-run parameters Γ_i, $i = 1, ..., k-1$ are fixed to their full sample values and only the long run parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are re-estimated. The lower left panel shows the two tests (based on the Z- and the R- Representation) of the stability of the cointegration space $\boldsymbol{\beta}$, i.e. we test that $\boldsymbol{\beta}_{2003:03} \in sp\left(\boldsymbol{\beta}_\tau\right)$ for $\tau = 1983:01, ..., 2002:03$, which are scaled by the asymptotic 95% citical values (the tests are $\chi^2\left(3\right)$ distributed). The lower left panel shows the recursively estimated eigenvalue of the matrix $\Pi = \boldsymbol{\alpha}\boldsymbol{\beta}'$ and the associated standard error. This allows one to evaluate the stability of the cointegrating vector and the adjustment coefficients, without imposing any normalization on the cointegration space or the adjustment coefficients, since it holds that $\hat{\Lambda} = diag\left(\hat{\lambda}_1, ..., \hat{\lambda}_r\right) = \hat{\boldsymbol{\beta}}'\mathbf{S}_{10}\mathbf{S}_{00}^{-1}\mathbf{S}_{01}\hat{\boldsymbol{\beta}} = \hat{\boldsymbol{\alpha}}'\mathbf{S}_{00}^{-1}\hat{\boldsymbol{\alpha}}$ (Johansen, [1995], Chapter 6). Recursions are running over the sample spanning the first quarter of 1983 to the fourth quarter of 2002.

Different Identifying Assumptions I

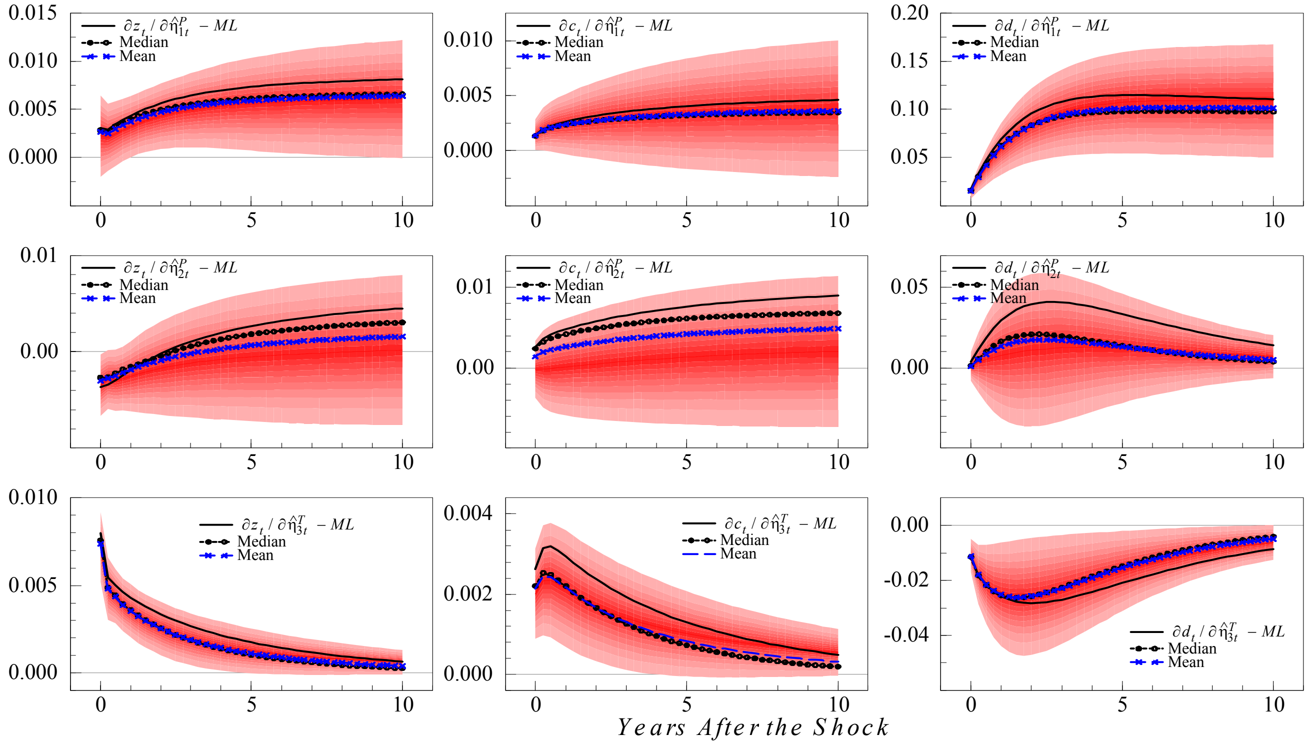

Figure 6. Alternative Identification Scheme I - Impulse Responses.

Notes for Figure 6: The first permanent shock is assumed to have a long-run impact on all three variables in the system. The second permanent shock is assumed to have a no long-run effect on d_t, but it has a long-run impact on z_t and c_t. The transitory shock is assumed to have no long-run effect on any of the variables (since transitory shocks do not persist). The top panel shows the responses to $\hat{\eta}_{1t}^P$, the mid-panel the responses to $\hat{\eta}_{2t}^P$, and the lower panel the responses to $\hat{\eta}_{3t}^T$, all with the associated bootsrap confidence bands.

Different Identifying Assumptions II

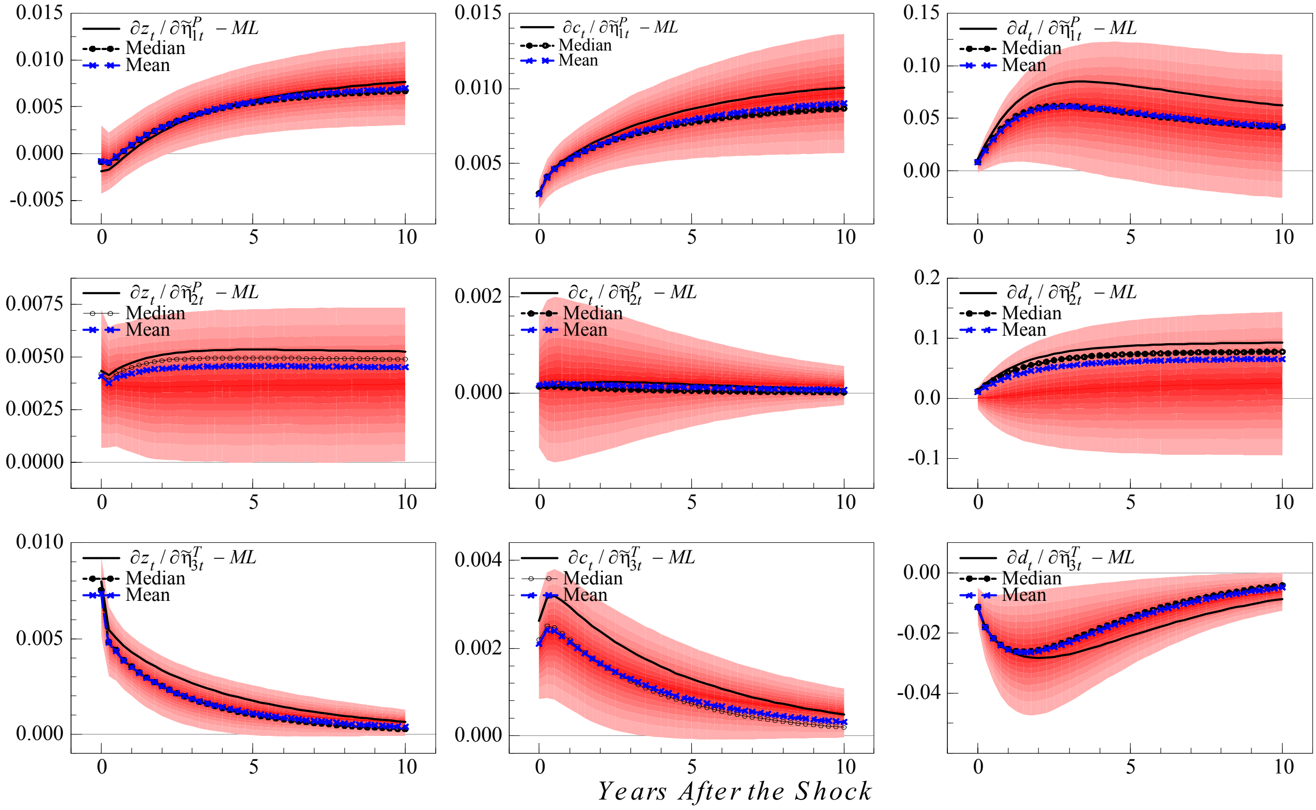

Figure 7. Alternative Identification Scheme II - Impulse Responses

Notes for Figure 7: The first permanent shock is assumed to have a long-run impact on all three variables in the system. The second permanent shock is assumed to have a no long-run effect on c_t, but it has a long-run impact on z_t and d_t. The transitory shock is assumed to have no long-run effect on any of the variables (since transitory shocks do not persist). The top panel shows the responses to $\tilde{\eta}^P_{1t}$, the mid-panel the responses to $\tilde{\eta}^P_{2t}$, and the lower panel the responses to $\tilde{\eta}^T_{3t}$, all with the associated bootsrap confidence bands.