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## Abstract

Using an estimated life-cycle model, we quantify the role of heterogeneity in wealth returns for the response of income to marginal tax changes. In our economy, agents who are sufficiently productive can obtain higher returns by choosing to be entrepreneurs. Return heterogeneity amplifies the responsiveness of total income to marginal tax changes along the entire income distribution with the top 1 percent displaying the highest elasticities. Return heterogeneity increases the incentives to invest for the richest, high-return entrepreneurs, thus amplifying their income responses to marginal tax changes. This reallocation of capital increases aggregate productivity, generating a larger boost in equilibrium wages. This in turn strengthens the income response of the bottom 90 percent, but nevertheless, their response is smaller than at the top.

**Keywords:** risky investment, elasticity of taxable income, life-cycle, entrepreneurs, structural estimation.

**JEL classification:** E62, H21, H24.

## Resumen

Este trabajo utiliza un modelo de ciclo de vida para cuantificar el rol de la heterogeneidad en los rendimientos de las inversiones en la respuesta económica de unos cambios impositivos marginales. En este modelo, los agentes que son suficientemente productivos pueden obtener rendimientos más altos si eligen ser empresarios. La heterogeneidad en los rendimientos amplifica la respuesta de los ingresos totales a los cambios impositivos marginales sobre toda la distribución del ingreso. El 1 % más rico muestra las elasticidades más altas. La heterogeneidad de retorno aumenta los incentivos a invertir para los empresarios más ricos y de alto rendimiento, amplificando así sus respuestas de ingresos a los cambios del impuesto marginal. Esta reasignación de capital aumenta la productividad agregada, generando un mayor impulso en los salarios de equilibrio. Esto, a su vez, fortalece la respuesta del ingreso del 90 % más pobre, pero, sin embargo, su respuesta es menor que en la parte superior de la distribución.

**Palabras clave:** inversión de alto riesgo, elasticidad de la renta imponible, modelos de ciclo de vida, empresarios, estimación estructural.

**Códigos JEL:** E62, H21, H24.

# 1 Introduction

This paper studies the role of investment risk, in the form of capital return heterogeneity, for the transmission of marginal tax changes in the US. There are at least two reasons that make this analysis important for a sound evaluation of fiscal policies. First, capital income heterogeneity is a robust feature of the empirical evidence on investment return risk (e.g., Smith et al., 2019; Fagereng et al., 2020; Bach et al., 2020). Second, modeling investment return heterogeneity has been shown to be pivotal in explaining income and wealth inequality and their dynamics (e.g., Benhabib et al., 2011; Gabaix et al., 2016; Benhabib et al., 2019).

We quantify the effect of marginal tax policies in a model where households face persistent heterogeneity in returns on their wealth. In our economy, agents can obtain higher-than-average returns by choosing to be entrepreneurs, but only if they are sufficiently productive. We integrate our model of entrepreneurs into an otherwise standard life-cycle framework with incomplete markets, uninsurable labor income risk and progressive income taxes. We estimate our model to capture the right tail of the income and wealth distribution, in conjunction with a broader set of distributional and macroeconomic moments. As such, we present a suitable laboratory to study the long-run effects of marginal income tax changes.

We find that return heterogeneity amplifies the responsiveness of total income (capital and labor) to marginal tax changes, particularly for incomes in the top 1 percent. While the effect is also substantial for the bottom 90 percent, return heterogeneity alters the distributional impact of tax policy, so that it is the top 1 percent who are most responsive. With return heterogeneity, a cut in marginal tax rates increases the incentive to save, but mainly for high-return entrepreneurs in the top of the income distribution. This reallocation of capital to high-productivity agents increases aggregate productivity and generates a larger equilibrium boost in wages, in turn benefitting also the bottom 90 percent.

In our main quantitative experiment, we evaluate a reform which changes the marginal income tax rate for all taxpayers. We focus on the long-run elasticity of total taxable income with respect to net-of-tax rates (1 minus the marginal tax rate) – i.e., the elasticity of taxable income (ETI). The ETI (or policy elasticity in the sense of Hendren, 2016) measures the distortionary effects of taxation by incorporating the behavioral responses of labor supply, investment, as well as equilibrium changes in prices. The ETI is a popular measure in public finance because, under some regularity conditions, it is a sufficient statistic to evaluate the efficiency costs of tax policy reforms (e.g., Saez et al., 2012).

We show that capital return heterogeneity amplifies the ETI of the overall economy by around 34 percent (i.e., from 0.49 to 0.66).<sup>1</sup> For the top 1 percent, the ETI is more

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<sup>1</sup>In our framework, this implies an increase from 23.8 percent to 28.5 percent for the fraction of additional tax revenue lost through behavioral responses.

than doubled (i.e., from 0.37 to 0.77), while for the bottom 90 percent, the ETI increases by 19 percent (i.e., from 0.59 to 0.70). In contrast, in standard models without return heterogeneity, or in models which match the fat tail of the wealth distribution through a superstar earnings state (e.g., Kindermann and Krueger, 2020; Brüggemann, 2020), incomes at the top are less elastic to marginal tax changes. Furthermore, our results for the long-run are similar to the short-run results presented in Mertens and Montiel Olea (2018). In our case, however, higher responses at the top do not reflect any tax avoidance or intertemporal substitution from transitory changes in tax policy, relying instead on the incentives of high-return individuals to adjust their investment.

By evaluating the importance of return heterogeneity, we provide novel insights about the transmission mechanism of marginal income tax policies. In this regard, our structural approach allows us to fully control for how return heterogeneity affects the transmission of fiscal policies through individual behavior (labor supply/investment) and general equilibrium price changes. Our paper shows that return heterogeneity is both quantitatively and qualitatively important for the transmission of marginal tax changes, especially for incomes in the top 1 percent. Therefore, capturing these fiscal spillovers is crucial for policy analysis in practice.

Moreover, we estimate the long-run effects of marginal tax changes, while the empirical literature concentrates on short- to medium-run effects (e.g., Barro and Redlick, 2011; Romer and Romer, 2014; Mertens and Montiel Olea, 2018). This is mainly because estimates of the effects of marginal tax changes in the long-run (i.e., exceeding a few years) are plagued by extremely difficult issues of identification (e.g. Saez et al., 2012). For instance, the effects of a tax reform on capital income and entrepreneurial productivity may not be reached for several years and hence they are very difficult to trace back empirically to the original policy change. However, exactly for the prolonged adjustments and spillovers that follow a marginal tax change, these long-run effects are of great interest to policy-makers. We get around the identification problem by simulating the long-run effects of fiscal policy change within an estimated life-cycle model. From a quantitative point of view, while there are no truly convincing estimates of long-run ETI, the best available estimates cited in Saez et al. (2012) are in the range of 0.12 to 0.40. Interestingly, our estimates indicate a larger ETI.

One possible drawback of our approach is that our estimates are conditional on the assumed data generation process. While our model contains several important ingredients and heterogeneity over various dimensions, it almost surely misses on some features that could have a sizeable impact on the transmission of marginal tax changes. For example, we do not include human capital accumulation nor tax avoidance. These factors are naturally expected to increase the interplay between capital return heterogeneity and the distortionary effects marginal tax rates. Thus, we expect that these extra ingredients could further magnify the quantitative importance of our results. However, we leave such an analysis for future research.



Our paper is related to the literature that models uninsurable capital income risk within life-cycle heterogeneous-agent frameworks (e.g., Benhabib et al., 2011; Guvenen et al., 2019). This modeling feature has recently gained popularity for three reasons. First, the empirical evidence shows a substantial heterogeneity in capital income (Bach et al., 2020; Fagereng et al., 2020). Second, heterogeneity in capital income is a crucial ingredient in order to match the fat Pareto tail of the wealth distribution. Third, capital return heterogeneity has been shown to have important consequences for the setting of economic policies (e.g., Guvenen et al., 2019). Our contribution is to show that heterogeneity in capital income greatly modifies our understanding of the ETI along the income distribution, causing the top 1 percent to display the largest elasticities in the long-run.

Another strand of literature links aggregate productivity, entrepreneurship and taxes. In a growth model with entrepreneurs, Jaimovich and Rebelo (2017) find higher capital income tax rates reduce incentives to be entrepreneurs and long-run growth. Guvenen et al. (2019) find that a wealth tax reduces misallocation and increases aggregate TFP by re-allocating capital toward more productive entrepreneurs. Differently, we focus on the effect of progressive income taxes on entrepreneurship and productivity when capital and labor are jointly taxed. Moreover, our paper offers support within a structural approach, to a number of well established empirical results, such as the relation between tax rates and entrepreneurial activity (e.g., Djankov et al., 2010), the effects of tax changes on aggregate productivity (e.g., Cloyne, 2013) and the relationship between tax progressivity and misallocation (e.g., Fajgelbaum et al., 2019).

Our paper is also related to the literature that uses quantitative incomplete-markets life-cycle models as a laboratory to evaluate fiscal reforms (e.g., Nishiyama and Smetters, 2005; Guner et al., 2012; Chang and Park, 2020). Our focus, however, is on the importance of return heterogeneity for the long-run consequences of marginal tax changes. Moreover, several recent papers focus on the optimal tax rate for the top 1 percent (e.g., Badel et al., 2020; Kindermann and Krueger, 2020; Brüggemann, 2020). Since tax reforms frequently affect most (if not all) taxpayers rather than just the top 1 percent, we instead focus on tax changes which affect marginal tax rates along the whole distribution of income.<sup>2</sup> Furthermore, in contrast to these papers, we focus on a different mechanism (return heterogeneity) rather than human capital accumulation or a superstar state, and show that it has major implications for elasticities at the top.

Lastly, we relate to the literature on capital misallocation featuring financially constrained entrepreneurs with heterogeneous productivity (e.g., Cagetti and De Nardi, 2006; Moll, 2014; Itskhoki and Moll, 2019). On this aspect, the closest contribution is Guvenen et al. (2019), who analyze the effects of fiscal reforms, in particular a shift from capital income to wealth taxes, on capital misallocation. Instead, we focus on how marginal income tax changes affect aggregate productivity, via changes in the rate of entrepreneurship.

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<sup>2</sup>For instance, see the Jobs and Growth Tax Relief Reconciliation Act of 2003, which changed the marginal tax rates for most income brackets.

The remainder of the paper is the following. Section 2 presents our benchmark structural model. Section 3 describes the main estimation exercise. Section 4 presents the main policy experiment. Finally, Section 5 concludes.

## 2 Model

We present an incomplete-markets life-cycle model with idiosyncratic risk in both labor and capital income. The economy is populated by households who decide on consumption, saving, labor supply and whether or not to engage in entrepreneurial activity. The government taxes income, chooses public spending and provides social security benefits for retirement.

### 2.1 Environment

The economy is populated by a continuum of households, who differ by type  $i \in \{1, \dots, I\}$  and age  $j \in \{1, \dots, J\}$ . Each period, a mass of new households is born, where the rate of population growth is assumed to be  $n$ . At birth, each household learns its innate talent, indexed by  $i$ , which determines its overall level of labor productivity. We denote by  $\pi_i$  the probability that a household will be type  $i$ .

During their life, households choose consumption, savings, and labor supply and whether or not to engage in entrepreneurial activity, given idiosyncratic risk in capital and labor income. Households also pay progressive income taxes and flat social security taxes on labor earnings (up to a cap). After retirement at age  $R$ , households receive social security benefits from the government.

Households also face a risk of early death. We denote by  $s_j$  the probability of surviving to age  $j$ , conditional on surviving to age  $j - 1$ .<sup>3</sup> The demographic patterns are stable, so that age- $j$  agents make up a constant fraction  $\mu_j$  of the total population.<sup>4</sup> Accidental bequests are redistributed to all living consumers as a lump-sum transfer,  $T_b$ .

**Preferences.** All agents have identical preferences for consumption  $c_j$  and hours worked  $h_j$  over their lifetime:

$$E \left\{ \sum_{j=1}^J \beta^{j-1} \left( \prod_{k=1}^j s_k \right) u(c_j, h_j) \right\}, \quad (1)$$

where  $\prod_{k=1}^j s_k$  is the unconditional probability an age-1 agent will survive to age  $j$ . As it is standard in the literature (e.g., Conesa et al., 2009), we assume that the period utility is of the form

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<sup>3</sup>Naturally,  $s_1 = 1$  and  $s_{J+1} = 0$ .

<sup>4</sup>The measure  $\mu_j$  can be defined recursively, where  $\mu_{j+1} = s_{j+1}\mu_j / (1 + n)$  for  $j = 1, \dots, J - 1$  and  $\mu_1$  is set to normalize  $\sum_{j=1}^J \mu_j = 1$ .

$$u(c, h) = \frac{(c^\gamma(1-h)^{1-\gamma})^{1-\sigma}}{1-\sigma},$$

where  $\gamma$  is the consumption utility share and  $\sigma$  controls the household's risk aversion.

## 2.2 Labor Earnings Risk

In each period before retirement, agents receive labor earnings equal to  $w eh$ , where  $w$  is the real wage rate,  $e$  is the household's labor ability and  $h$  is hours worked. When households reach age  $R$ , they retire so that hours worked and total labor earnings become zero for ages  $j \geq R$ .

We assume ex-ante and ex-post heterogeneity in labor abilities as in, inter alia, Kaplan and Violante (2014) and Guvenen et al. (2019). A household's labor ability  $e_{i,j}(z_h)$  is given by

$$\log e_{i,j}(z_h) = \bar{e}_i + \alpha_0 + \alpha_1 j + \alpha_2 j^2 + \alpha_3 j^3 + \alpha_4 j^4 + \log z_h \quad (2)$$

A household's labor productivity depends on three factors. First, labor ability depends on a household-specific innate ability,  $\bar{e}_i$ . At birth, the household learns her type  $i \in \{1, \dots, I\}$  which indexes its overall level of labor ability. We denote by  $\pi_i$  the probability a household will become type  $i$ . Second, labor ability explicitly depends on a fourth-order polynomial in age  $j$ . Third, labor ability is also affected by an idiosyncratic shock,  $z_h$ , which follows an AR(1) process:

$$\log z'_h = \rho_h \log z_h + \varepsilon_h, \quad \varepsilon_h \sim N(0, \sigma_{\varepsilon_h}^2), \quad (3)$$

where the initial  $\log z_h$  is set to zero.

In our quantitative analysis, we parameterize the innate abilities  $\{\bar{e}_i\}_{i=1}^I$  and the type probabilities  $\{\pi_i\}_{i=1}^I$  using one parameter: the standard deviation of innate labor ability,  $\sigma_e$ . First, given  $\sigma_e$ , we set  $\{\bar{e}_i\}_{i=1}^I$  as  $I$  individual points, linearly spaced between  $-3\sigma_e$  and  $+3\sigma_e$ . Second, assuming innate labor ability is normally distributed with mean zero and variance  $\sigma_e^2$ , we construct the individual probabilities  $\{\pi_i\}_{i=1}^I$  using the approximation method of Tauchen (1986).

## 2.3 Asset Return Risk

Following Cagetti and De Nardi (2006) and Guvenen et al. (2019), we introduce a role for entrepreneurship. All households can choose to be an entrepreneur, whereby they access a "backyard technology" that uses  $k$  units of capital to produce  $q$  units of an intermediate capital service. We assume a linear technology

$$q = z_r k \quad (4)$$

where  $z_r$  can be interpreted as the household's entrepreneurial productivity.

We assume that entrepreneurial productivity follows an AR(1) process:

$$\log z'_r = \rho_r \log z_r + \varepsilon_r, \quad \varepsilon_r \sim N(0, \sigma_{\varepsilon_r}^2) \quad (5)$$

where the initial shock is drawn from the distribution  $N(0, \sigma_{\varepsilon_r}^2 / (1 - \rho_r^2))$ .<sup>5</sup> The intermediate capital service is then sold at the price  $p$  in a perfectly competitive market to the final goods producer, where it is used (along with labor) to produce the uniform final good  $Y$  (see Section 2.4 below).

All households lend on the bond market their whole wealth at the riskless rate  $r$ . Those who also choose to be entrepreneurs borrow at rate  $r$  on the same market and use their own backyard technology to produce the intermediate capital service  $q$ . Entrepreneurs must also decide how much capital  $k$  to invest in their backyard technology. They are subject to a collateral constraint ( $k \leq \lambda a$ ), where  $\lambda \geq 1$  is exogenous and controls the leverage level, while  $a$  is the individual entrepreneur's wealth (e.g., see Moll, 2014, Boar and Midrigan, 2019 and Guvenen et al., 2019). Entrepreneurs then maximize the following profit function,

$$\pi(a, z_r) = \max_{0 \leq k \leq \lambda a} \{p z_r k - (r + \delta)k\}, \quad (6)$$

where  $r + \delta$  is the rental rate of capital, with  $\delta$  representing the depreciation rate. The associated optimal capital demand is

$$k(a, z_r) = \begin{cases} \lambda a & \text{if } z_r \geq (r + \delta)/p \\ 0 & \text{if } z_r < (r + \delta)/p \end{cases} \quad (7)$$

Therefore, there exists an endogenous productivity threshold,

$$\bar{z}_r = (r + \delta)/p, \quad (8)$$

such that only households that are sufficiently productive will choose to be entrepreneurs, while the others will simply engage in lending activities.

In our framework with constant returns to scale, only a fraction of households will decide to become entrepreneurs, via an endogenous productivity cutoff, see equation (8) above. This allows the model to match the entrepreneurship rate observed in the data and also avoids the negative relationship between wealth and returns, which is counterfactual (e.g., see Bach et al., 2020).<sup>6</sup>

To summarize, all households earn  $r$  by lending their wealth on the bond market. Those households with sufficiently high entrepreneurial ability also choose to run a busi-

<sup>5</sup>In our quantitative analysis, we will technically assume that  $z_r$  is bounded by  $z_{r,min}$  and  $z_{r,max}$ .

<sup>6</sup>Specifically, with decreasing returns to scale, a household would earn a lower rate of return when its wealth is higher. Nevertheless, in equilibrium, wealthier households would earn higher returns.

ness, whereby they borrow at rate  $r$ , produce the intermediate good  $q$  and earn  $\pi(a, z_r)$ . Substituting the solution for  $\pi(a, z_r)$ , the household's total return on its wealth is given by

$$r_a(z_r) = r + \lambda \max(pz_r - (r + \delta), 0). \quad (9)$$

Therefore, there will be persistent idiosyncratic variation in returns across households, which is a crucial ingredient for the model's ability to match the fat tail of wealth (e.g., see Benhabib et al., 2011, Benhabib et al., 2019 and Guvenen et al., 2019). Furthermore, despite no explicit link between wealth and returns, high-wealth households will, on average, earn higher returns, consistent with the empirical evidence (e.g., see Bach et al., 2020 and Fagereng et al., 2020).

## 2.4 Production Technology

The final good is produced according to a Cobb-Douglas production function:

$$Y = F(Q, L) = Q^\alpha L^{1-\alpha}$$

where  $L$  is aggregate labor and  $Q$  is the aggregate of the intermediate capital good produced by entrepreneurs.

It is straightforward to derive the following aggregate relationship:

$$Y = AK^\alpha L^{1-\alpha}$$

where  $K$  is aggregate capital and  $A$  is aggregate TFP. Aggregate TFP is  $A = (Q/K)^\alpha$ , where  $Q/K$  is the average productivity of entrepreneurs. Therefore, aggregate productivity depends crucially on the allocation of capital across entrepreneurs.

The market for the intermediate good and the market for labor are both perfectly competitive. Therefore, the representative firm takes as given the prices  $(w, p)$  and chooses  $Q$  and  $L$  to maximize profits,  $\Pi = Q^\alpha L^{1-\alpha} - pQ - wL$ .

## 2.5 Government

The government taxes income in order to finance a fixed and exogenous level of government spending,  $G$ , which provides agents no utility. The government operates a balanced budget and does not use debt, implying that  $G$  is just equal to aggregate income tax revenue,  $T_y$ . The government also runs a social security system with a dedicated budget.

### 2.5.1 Income Tax

Labor and capital income are jointly taxable. Households can deduct part of the social security contribution, up to an upper limit  $\bar{y}$ . The household's taxable income is

$$y = \max \left( we_{i,j}(z_h)h + r_a(z_r)a - \frac{1}{2} \min (we_{i,j}(z_h)h, \bar{y}), 0 \right) \quad (10)$$

We impose that taxable income must be non-negative because, in principle, the asset return could be negative if the equilibrium lending rate  $r$  is negative.<sup>7</sup> We adopt a tax specification function belonging to a flexible three-parameter family, originally proposed by Gouveia and Strauss (1994) and popular in applied works (e.g., Conesa et al., 2009 and Guner et al., 2014),

$$\mathcal{T}_y(y) = \tau_0 y \left( 1 - (\tau_2 y^{\tau_1} + 1)^{-1/\tau_1} \right). \quad (11)$$

Roughly speaking,  $\tau_0$  governs the maximum tax rate, while  $\tau_1$  and  $\tau_2$  determine the progressivity of the tax schedule. For  $\tau_1 \rightarrow 0$ , the tax system reduces to a pure flat tax, while other values encompass a wide range of progressive and regressive tax functions. According to this specification, the marginal income tax rate converges to zero as taxable income converges to zero, while the marginal tax rate converges to the upper bound of  $\tau_0$  as taxable income grows large.

### 2.5.2 Social Security Scheme

The government runs a pay-as-you-go social security scheme. Taxpayers pay a social security tax only out of their labor income (at the flat tax rate  $\tau_{ss}$ ), up to an upper bound  $\bar{y}$ . The government pays a type-specific social security benefit,  $b_{i,j}$ :

$$b_{i,j} = \begin{cases} 0 & \text{if } j < R \\ \bar{b}_i & \text{if } j \geq R. \end{cases}$$

We assume that  $\bar{b}_i = \chi w L_i$ , where  $L_i$  is the average labor input of type- $i$  agents and  $\chi$  is the replacement rate.

Social security benefits are financed by a flat tax  $\tau_{ss}$  on all labor earnings  $weh$  below  $\bar{y}$ . That is, a household with labor earnings  $weh$  will pay a social security tax of  $\tau_{ss} \min(weh, \bar{y})$ . Given the tax rate  $\tau_{ss}$  and the cap  $\bar{y}$ , we internally set the replacement rate  $\chi$  so that aggregate social security tax revenue equals aggregate social security benefits.

## 2.6 Value Function

Having presented the main features of our model economy, we can now describe the household's problem in recursive form. In each period, the household chooses consumption  $c$ , savings  $a'$ , and labor supply  $h$  given idiosyncratic risk, the sequence of prices and the tax function. In retirement, households supply zero hours (i.e.,  $h = 0$ ), but they still choose consumption and savings. Let  $V_{i,j}(a, z_h, z_r)$  denote the value of a type- $i$  and age- $j$

<sup>7</sup>Nevertheless, the equilibrium rate  $r$  is positive in our analysis.

consumer with assets  $a$  and idiosyncratic shocks  $(z_h, z_r)$ . We can write the consumer's maximization problem as follows:

$$V_{i,j}(a, z_h, z_r) = \max_{c,h,a'} \{u(c, h) + \beta s_{j+1} E [V_{i,j+1}(a', z'_h, z'_r) | z_h, z_r]\} \quad (12)$$

subject to

$$\begin{aligned} y &= \max \left( we_{i,j}(z_h)h + r_a(z_r)a - \frac{1}{2} \min (we_{i,j}(z_h)h, \bar{y}), 0 \right) \\ c + a' &= a(1 + r_a(z_r)) + we_{i,j}(z_h)h - \tau_{ss} \min (we_{i,j}(z_h)h, \bar{y}) - \mathcal{T}_y(y) + T_b + b_{i,j} \\ a' &\geq 0 \\ 0 &\leq h \leq \mathbb{1}\{j < R\}. \end{aligned}$$

## 2.7 Equilibrium

We focus on a stationary equilibrium, in which capital, labor, transfers and government consumption are all constant in per-capita terms. See Appendix A for a full definition.

## 3 Quantitative Analysis

In this section we outline our estimation strategy, and then evaluate the model's ability to account for a number of features in the data for the US. We solve and estimate the model assuming the economy is in a steady state. One period corresponds to one year and we convert all nominal values into 2010 dollars. We refer the reader to Appendix B for a detailed description of the numerical strategy used for solving the model.

We split our parameters into two main groups: (i) a group of parameters that is externally set, either according to previous literature, via direct observation or through estimation; and (ii) a group of parameters that is internally set, either through calibration on aggregate macroeconomic statistics, or estimated by using a Simulated Method of Moments (SMM) estimator, in order to match relevant distributional moments in the Survey of Consumer Finances (SCF) for 2016.

### 3.1 Externally Set Parameters

**Externally Fixed Parameters** We fix two parameters consistently with the literature (see panel A, Table 1). The first of these is  $\sigma$ , which controls households' risk aversion. We fix this parameter to 2, consistent with a large bulk of applied works in the life-cycle literature (e.g., Nishiyama and Smetters, 2005 and Benhabib et al., 2019).<sup>8</sup> Second, we fix the capital income share  $\alpha$ , to 0.36, which is standard in the macroeconomics literature.

<sup>8</sup>Given the assumption of a Cobb-Douglas utility function, the coefficient of relative risk aversion in consumption is  $-cu_{cc}/u_c = 1 - \gamma(1 - \sigma)$ .

Then, we fix  $J$ , the maximum age in the model, to 85 and  $R$ , the retirement age, to 45. Assuming that age 1 in the model corresponds to age 21 in the real life, these choices for  $(J, R)$  correspond to ages 105 and 65 in real life/years. We set the population growth rate  $n$  to 0.7%, to be consistent with the U.S. population growth rate in the World Bank's World Development Indicators. We obtained estimates of the survival probabilities  $s_j$  from the United States Mortality Database (see Appendix C for details). Finally, we use data from the Internal Revenue Service (IRS) to set the linear social security tax,  $\tau_{ss} = 12.4\%$ , and the upper limit on the social security contribution,  $\bar{y} = 107.7k$ .

**Externally Estimated Parameters** Next, we focus on a set of parameters that we estimate outside the model (see panel B, Table 1). We estimate the parameters of the tax functions  $(\tau_0, \tau_1, \tau_2)$  via a non-linear weighted least squares method (e.g., Guner et al., 2014). Using our SCF data, we construct a measure of income that includes all income flowing to households. We then calculate federal income tax liabilities using NBER's TAXSIM program. See Appendix D for details. Figure 1 illustrates the resulting tax function.

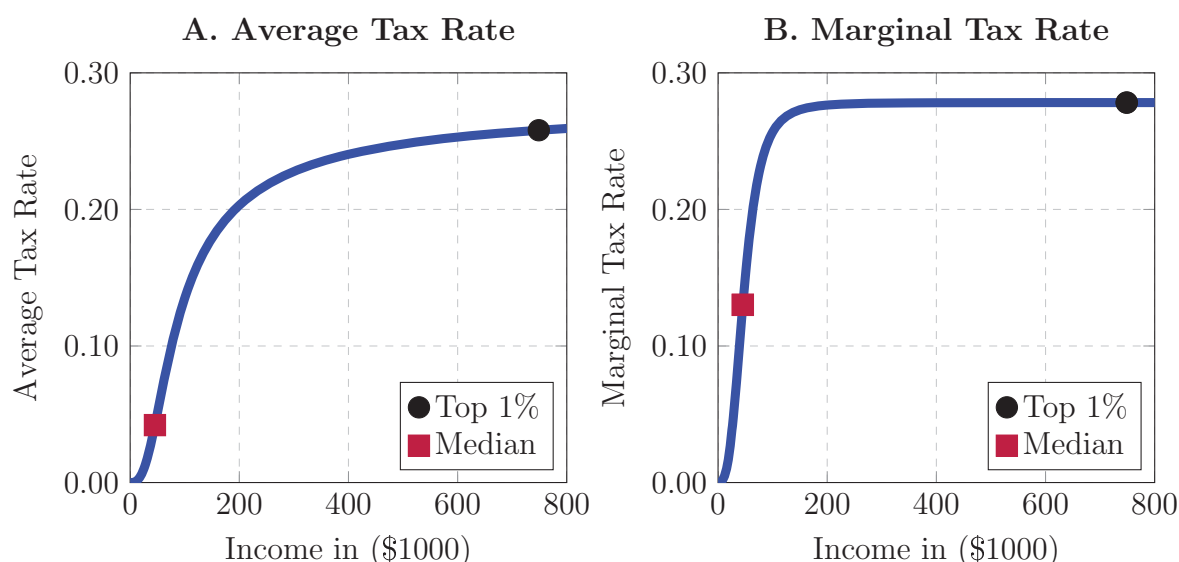
One important drawback of our analysis is that we consider a tax function for *total* income, where the tax authority does not discriminate between labor and capital income. This is mainly due to data limitation, as in microeconomic surveys like the SCF there is not a clear way to estimate distinct tax functions for labor and capital income. For this reason, our approach is standard in the empirical public finance literature, where the elasticity of taxable income generally utilizes a broad definition of the tax base (e.g., Saez et al., 2012 and Mertens and Montiel Olea, 2018). However, in reality, the US tax authority taxes differently at least part of income based on its origin (i.e., labor or capital). Our main results follow through in the alternative scenario where labor and capital incomes are taxed differently, with a progressive tax on earnings and a linear tax on capital income (see Appendix H for details).

Next, we need to face one of the biggest challenges of our quantitative exercise: the estimation of the labor ability process in (2). On the one hand, this type of fixed effect models would conform very well with the panel dimension of PSID. The problem with this approach is that the labor earnings inequality recorded in PSID is much lower than that observed on SCF and other inequality measures—e.g., the Gini coefficient is more than 10 percentage points lower in PSID than in SCF. As such, by using this method, one would lose on a fundamental aspect of inequality, particularly for the top 1 percent of the distribution. Alternatively, one could only use SCF, which is more reliable for measuring earnings at the top of the distribution. The fundamental issue with this dataset is that it lacks of a panel dimension, and so it would be very difficult to credibly estimate, for instance, dynamic features of the transitory idiosyncratic risk in (3).

We tackle these issues by adopting a hybrid approach between the two datasets. In particular, we start by estimating the parameters of the fourth order age-profile  $(\alpha_1, \alpha_2,$



Figure 1 – Tax Function



Note: We estimate an income tax function using a measure of total income in the SCF and a measure of federal tax liabilities from NBER's TAXSIM. Taxable income is expressed in 2010 dollars.

$\alpha_3, \alpha_4$ ) directly on SCF. Then, following the approach of Kaplan (2012), we recollect the process of transitory idiosyncratic risk by estimating a fixed effect model on PSID (see Appendix E for details). This method allows to exploit the panel dimension of PSID and

Table 1 – Externally Set Parameters

Parameters	Notation	Value	Std. Err.	Source
<u>A: Fixed Parameters</u>				
Risk Aversion	$\sigma$	2		Typical in lit.
Capital Share	$\alpha$	0.36		Typical in lit.
Maximum Age	$J$	85		Corresp. to age 105
Retirement Age	$R$	45		Corresp. to age 65
Survival Prob.	$s_j$	Appendix C		USMD
Pop. Growth	$n$	0.007		World Bank
Soc. Sec. Tax	$\tau_{ss}$	0.124		IRS
Soc. Sec. Cap	$\bar{y}$	107.7		IRS
<u>B: Estimated Parameters</u>				
Maximal Tax Rate	$\tau_0$	0.278	(0.003)	SCF/TAXSIM
Tax Progressivity 1	$\tau_1$	2.84	(0.10)	SCF/TAXSIM
Tax Progressivity 2	$\tau_2$	$1.14e^{-5}$	$(4.03e^{-6})$	SCF/TAXSIM
Ability Coef. 1	$\alpha_1$	0.100	(0.014)	SCF
Ability Coef. 2	$\alpha_2$	$-3.72e^{-3}$	$(1.19e^{-3})$	SCF
Ability Coef. 3	$\alpha_3$	$6.37e^{-5}$	$(3.87e^{-5})$	SCF
Ability Coef. 4	$\alpha_4$	$-4.20e^{-7}$	$(4.24e^{-7})$	SCF
Labor Ability Persist.	$\rho_h$	0.976	(0.005)	PSID
Labor Ability Std. Dev.	$\sigma_{\varepsilon_h}$	0.135	(0.006)	PSID

Note: This table reports the externally set parameters. USMD stands for the United States Mortality Database. Standard errors are reported in parentheses.

estimate a persistence component,  $\rho_h = 0.972$ , and a standard deviation  $\sigma_{\varepsilon h} = 0.135$ . Interestingly, these numbers are similar to the large body of literature estimating the process of the transitory component of labor abilities (e.g., Guvenen et al., 2019). The remaining two parameters of the ability process,  $\sigma_e$  and  $\alpha_0$  will be internally set using data from the SCF (see the discussion below).

### 3.2 Internally Set Parameters

**Internally Calibrated Parameters** We now report the calibrated parameters in Table 2. We set the discount factor,  $\beta$ , and the parameter governing the exogenous collateral constraint on the leverage level,  $\lambda$ , to target (i) an average wealth-to-output ratio ( $K/Y$ ) of 2.95, which is the average post-1980 capital-to-output ratio as reported by the Bureau of Economic Analysis (BEA); and (ii) return on safe assets ( $r$  in the model) of 1.9 percent, a value consistent with the post-1980 average US safe real return on Bills (see Jordà et al., 2019). Furthermore, we set the capital depreciation rate,  $\delta$ , to 0.05, so that the ratio of investment and durables investment to GDP is 22 percent, consistent with the post-1980

**Table 2** – Calibrated Parameters and Targeted Moments

Parameters	Notation	Value	Target
Discount Factor	$\beta$	0.989	Wealth-to-output ratio
Coll. Constraint	$\lambda$	3.04	Risk-free borrowing rate
Depreciation Rate	$\delta$	0.05	Investment-to-output ratio
Soc. Sec. Benefit	$\chi$	0.305	Balanced budget

Moments	Model	Data	Source
Capital-to-output Ratio	2.95	2.95	BEA
Investment-to-output Ratio	0.22	0.22	BEA
Borrowing Rate	0.019	0.019	Jordà et al. (2019)

*Note:* This table reports the calibrated parameters and associated targets. The social security benefit,  $\chi$ , is internally set so that the government runs a balanced budget rule, where aggregate social security tax revenue equals aggregate social security benefits.

US average in the BEA NIPA tables. Finally, we internally set the social security benefit,  $\chi$ , so that the public pension system runs a balanced budget rule.

**Internally Estimated Parameters** The five parameters that remain to be set are  $(\gamma, \sigma_e, \alpha_0, \rho_r, \sigma_{\varepsilon r})$ . The preference parameter  $\gamma$  governs the utility weight of consumption. The parameter  $\sigma_e$  is the standard deviation of permanent labor ability.<sup>9</sup> The parameter  $\alpha_0$  is the constant term in the ability profile. Finally,  $(\rho_r, \sigma_{\varepsilon r})$  govern the capital income risk faced by individuals.

<sup>9</sup>As discussed in Section 2.2, we use  $\sigma_e$  to parameterize the fixed effects  $\{\bar{e}_i\}_{i=1}^I$  and the type-probabilities  $\{\pi_i\}_{i=1}^I$ . We use  $I = 6$  types.

We use SMM to estimate these parameters by minimizing the distance between model statistics and their empirical counterparts. In particular, denoting the vector of parameters to be estimated by  $\Theta = (\gamma, \sigma_e, \alpha_0, \rho_r, \sigma_{\varepsilon r})$ , the SMM estimator solves the following minimization problem:

$$\hat{\Theta} = \arg \min_{\Theta} \left( \hat{M} - \hat{m}(\Theta) \right)' W \left( \hat{M} - \hat{m}(\Theta) \right),$$

where  $\hat{M}$  identifies the targeted moments from the SCF,  $\hat{m}(\Theta)$  represents the moments implied by the model for a given set of parameters  $\Theta$ , and  $W$  is a weighting matrix.<sup>10</sup>

**Table 3** – Estimated Parameters and Targeted Moments

Parameters	Notation	Value	Std. Err.
Utility Cons. Weight	$\gamma$	0.362	(0.004)
Labor Ability PC	$\sigma_e$	0.985	(0.023)
Labor Ability Constant	$\alpha_0$	2.741	(0.043)
Return Persistence	$\rho_r$	0.968	(0.004)
Return Shock	$\sigma_{\varepsilon r}$	0.172	(0.004)
Moments		Model	Data
Average Hours (working age)		0.299	0.304
Entrepreneurship Rate		0.087	0.085
Wealth Gini		0.862	0.860
Wealth Share, Top 1%		0.406	0.386
Wealth Share, Top 5%		0.638	0.651
Wealth Share, Top 20%		0.888	0.883
Earnings Gini		0.735	0.680
Earnings Share, Top 1%		0.145	0.172
Earnings Share, Top 5%		0.363	0.327
Earnings Share, Top 20%		0.687	0.605
Average Earnings		54.83	55.30
Revenues Tax Share, Top 1%		0.419	0.424
Revenues Tax Share, Top 5%		0.702	0.659
Revenues Tax Share, Top 20%		0.959	0.881
Wealth-Income Slope, Top 20%		1.574	1.638
Wealth-Income Slope, Top 40%		0.915	0.959
Wealth-Income Slope, Top 60%		0.693	0.717

*Note:* The top panel reports the estimated parameters with standard errors, while the bottom panel reports the moments in the model and the data. The model parameters are estimated via Simulated Method of Moments (SMM) using moments from the 2016 Survey of Consumer Finances (SCF).

Standard errors are computed via

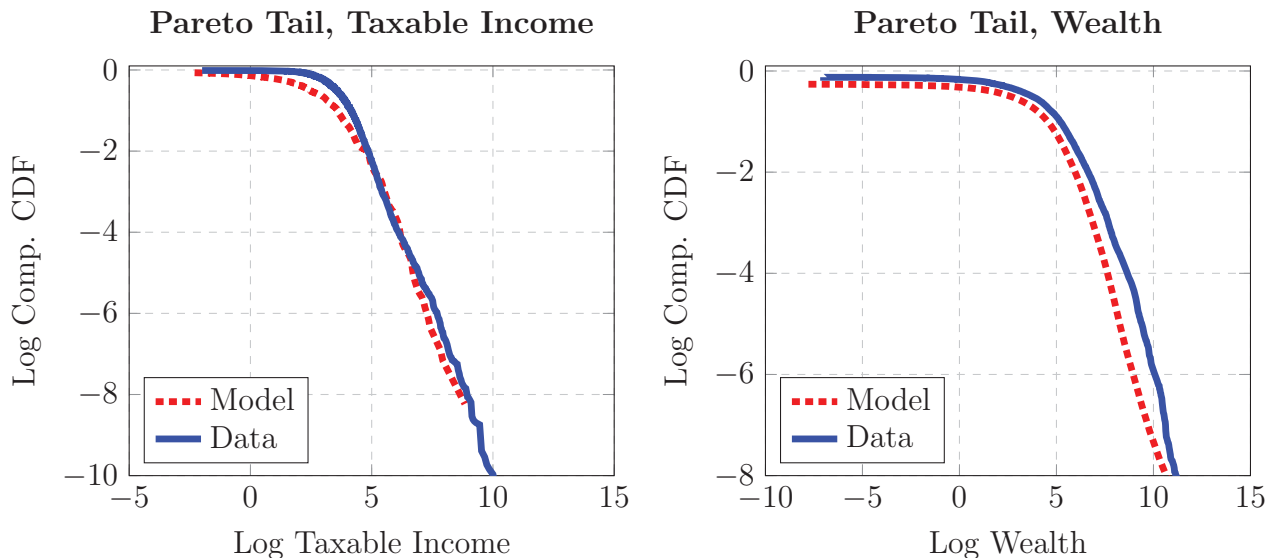
$$\sqrt{N}(\Theta - \Theta_0) \rightarrow \mathcal{N}(0, V) \quad \text{where } V = \left( 1 + \frac{1}{\omega} \right) (G'WG)^{-1}G'WSWG(G'WG)^{-1},$$

<sup>10</sup>We freely picked the weighting matrix  $W$ . In particular, we assumed the off-diagonal elements are all zero. For the diagonal elements, we assume  $W_{ii} = 1/\hat{M}_i^2$ , where  $\hat{M}_i$  is data moment  $i$ .

where  $N$  represents the number of observations, the matrix  $G$  is the gradient matrix of the moments,  $S$  is the asymptotic variance-covariance matrix of the moments and  $\omega = N_s/N$  is the ratio of the number of observations in the simulation relative to the data.

The estimated parameters are reported in the top panel of Table 3, while the moments are reported in bottom panel. We should highlight two main results from the

**Figure 2 – Fat Tail: Model vs. Data**

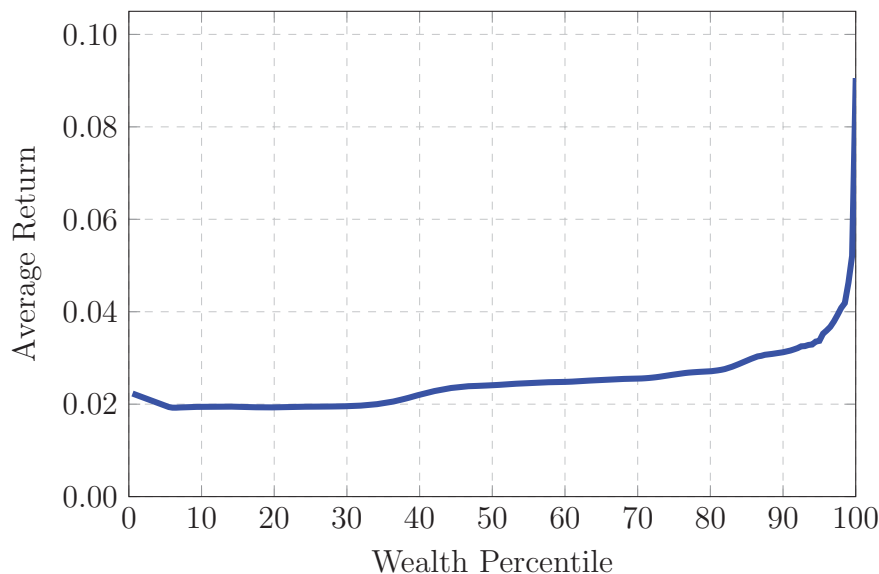


*Note:* This figure plots the complimentary cumulative distribution of taxable income and wealth, in the model and the data (SCF).

estimation exercise. First, all parameters are statistically different than zero and precisely estimated. This finding is not obvious and shows a tight link between targeted moments and structural parameters. As such, this result brings favorable evidence to our identification strategy. Second, the model does very well matching the moments from the SCF. Our model matches the wealth gini and the wealth shares of the top 1, 5 and 20 percent, respectively. Similarly, it matches the right tail in the distribution of taxable income and earnings. Interestingly, our model also captures the wealth-income slope, average hours worked, the entrepreneurial rate as well as average earnings. Finally, while they are not explicit targeted moments, our model is also able to replicate the capital income shares reported by Piketty et al. (2018) and Saez and Zucman (2020) for the richest income groups (top 1 and 10 percent) as well as for the economy as a whole (see panel D, Table 4).

Our model is also successful in generating the fat Pareto tails of the taxable income and wealth distributions observed in the data (see Figure 2). As shown in Benhabib et al. (2011), Benhabib et al. (2019) and Guvenen et al. (2019), the key ingredient to generate realistic unequal economies within life-cycle models is the presence of return heterogeneity. For this reason, the estimation of the two parameters governing entrepreneurial idiosyn-

**Figure 3** – Average Returns in Benchmark Model



*Note:* This figure plots the average asset returns by wealth percentile, as generated by the model in 2016. Average returns by wealth percentiles: top 1 percent, 7.14 percent; top 5 percent, 4.57 percent; top 10 percent, 3.93 percent; bottom 99 percent, 2.44 percent; bottom 90 percent, 2.32 percent.

cratic productivity ( $\rho_r$  and  $\sigma_{\varepsilon r}$ ) is crucial for matching the distributions of wealth and taxable income, the wealth-income slope and the share of entrepreneurs. Furthermore, the estimation of the utility parameter  $\gamma$  is useful for matching average hours of labor supply, while the two parameters of the labor ability process are important to match the average earnings ( $\alpha_0$ ) and earnings inequality ( $\sigma_e$ ).

One obvious concern is to what extent the return heterogeneity necessary to capture distributional moments is consistent with the empirical evidence on wealth returns. Figure 3 plots the wealth returns implied by the model by wealth percentile. First, qualitatively, our model captures the positive correlation between wealth and returns. The mechanism behind this result is clear: agents (entrepreneurs) enjoying high productivity in capital income have an incentive to accumulate large wealth. Interestingly, the wealth-return becomes significantly steeper for the very top percentiles of the wealth distribution (i.e., above the 95th percentile). This is a key feature in order to obtain large wealth shares for the wealthiest top 1 and 5 percent of the distribution. This qualitative aspect about the correlation between returns and wealth size is a robust feature of recent empirical studies (e.g., Benhabib et al., 2019; Fagereng et al., 2020; Bach et al., 2020).

Second, quantitatively, our model produces returns around or slightly above the risk free rate (between 2 and 4 percent) for most of the population and returns around 7.1 percent for wealthiest one percent. Comparing our model-based return profiles with the data is problematic. On the one hand, US-based data in the SCF lacks of a long-span panel dimension, and therefore one cannot fully measure for the entirety of wealth returns such as unrealized capital gains. On the other hand, countries that provide panel data

dimension on wealth returns (e.g., Norway), show that the portfolio composition of wealth (particularly at the top of the distribution) is quite different than in the US.<sup>11</sup>

To partly overcome these issues, in Appendix F, we compute returns by wealth percentile in the SCF, by adopting the technique proposed in Xavier (2020). Briefly, this consists of estimating the return for each household's portfolio in the SCF using outside estimates of the return on individual asset types. First, the data display the same qualitative patterns as that of our model: households with higher wealth obtain higher than average returns. Second, the consistency of our model-implied returns at the bottom 25 percent of the wealth distribution and, critically, in the top 1 percent is striking, while the imputed returns from the SCF seem to be higher than those implied by the model in the middle of the wealth distribution. Overall these results (particularly for the consistency at the top of the wealth distribution) confirm the ability of our model to provide a realistic explanation of the fat tail in wealth and income distributions.

## 4 Tax Policy and Return Heterogeneity

We use our structural model to analyze the long-run distortionary effects of a change in marginal income tax rates. The main policy experiment that we consider is a change in  $\tau_0$ , assuming that government spending adjusts accordingly in the long-run. This type of tax changes mimics various policy reforms implemented by the US federal government and affects the marginal tax rates of all income groups. In this sense, our policy experiment differs from Guner et al. (2016), Kindermann and Krueger (2020) and Badel et al. (2020), who instead focus on the effects of changing taxes for the top 1 percent of the income distribution.<sup>12</sup>

**ETI and Policy Elasticity** We analyze the effects of our tax change by computing the elasticity of taxable income ( $y$ ) with respect to net-of-tax rates (1 minus the average marginal tax rate) in the long-run:

$$ETI = \frac{d \ln y}{d \ln(1 - AMTR)}.$$

This measure can also be interpreted as a *policy elasticity* in the sense of Hendren (2016). As described in Saez et al. (2012), in settings without fiscal externalities and income shifting, like the one under consideration here, the ETI represents a *sufficient statistic* to

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<sup>11</sup>For example, according to Fagereng et al. (2020), in Norway, the share of housing in gross wealth held by the 95-99 percentiles (99-99.9 percentiles) is 0.73 (0.44). Meanwhile, it is only 0.33 (0.25) in the US (as reported in the 2016 SCF).

<sup>12</sup>In Appendix I, we analyze the effect of a reform which increases marginal tax rates only for the top 1 percent. In this scenario, we obtain similar results with respect to the effect of heterogeneous returns, but marginal tax changes are more distortionary. In this sense, the exercise presented here is more conservative from a quantitative point of view.

evaluate the efficiency effects of a tax change. As such, it is a fundamental measure for policy analysis.

First, we study how the ETI varies for different income sources (i.e., capital and labor), and how it varies along the age and income distribution. Second, we study the aggregate implications of tax policies, particularly for prices, aggregate productivity and misallocation. In order to isolate the role of return heterogeneity, we compare our benchmark economy with one where capital return is constant.<sup>13</sup>

## 4.1 Effects of a Tax Change

We consider our main policy experiment, where we change  $\tau_0$  by a small amount. This change will affect the marginal income tax rate for all taxpayers.

**Elasticities by Income** In Table 4, we report the results for our main policy experiment, both in the benchmark economy and the alternative model without return heterogeneity. The main result from this exercise is that return heterogeneity substantially affects the transmission mechanism of marginal tax policies. From a quantitative point of view, return heterogeneity increases the overall ETI from 0.49 to 0.66 (i.e., by 34 percent). This effect is strongest for the top 1 percent of the income distribution, but it is also high for the bottom 90 percent, with the smallest increase accruing to the bottom 99 percent. As a result, our benchmark economy exhibits a U-shaped relationship between the ETI and income, with the top 1 percent of the income distribution displaying the highest ETI. In contrast, in the model without return heterogeneity, the ETI decreases monotonically with income.

**Elasticities by Income Source** To investigate this result, we start by considering the effects of a marginal tax change on different sources of income (e.g., capital vs. labor income, see panels B and C of Table 4). First, with return heterogeneity, the aggregate elasticity of capital income is positive (0.36), while it is close to zero (0.01) in the model without return heterogeneity. In both models, the elasticity of capital income is higher at the top of the income distribution, while it is substantially negative at the bottom of the distribution. On the one hand, a marginal tax cut generates an accumulation of wealth, most concentrated among high-income individuals. On the other hand, there is a general equilibrium effect in that the higher supply of savings will reduce the borrowing rate (see panels A and B, Table 5). This generates a negative elasticity for the bottom of the distribution.

Comparing the two models, we see that return heterogeneity amplifies the response of capital income at the top of the distribution. This, in turn, generates a larger general equilibrium decrease in the borrowing rate. As a result, the elasticity of capital income

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<sup>13</sup>Formally, we assume entrepreneurial productivity is constant and equal to average productivity in the benchmark economy, with  $z_r = Q/K$ .

**Table 4** – Effects of an Income Tax Change

Variable	No Return		
	Benchmark	Heterogeneity	Difference
<u>A. Elasticity of Taxable Income</u>			
Income Top 1%	0.77	0.37	0.41
Income Top 5%	0.62	0.40	0.22
Income Top 10%	0.60	0.43	0.17
Income Bottom 99%	0.55	0.53	0.03
Income Bottom 90%	0.70	0.59	0.11
All	0.66	0.49	0.17
<u>B. Elasticity of Capital Income</u>			
Income Top 1%	0.94	0.78	0.16
Income Top 5%	0.74	0.76	-0.01
Income Top 10%	0.64	0.70	-0.06
Income Bottom 99%	-1.76	-0.13	-1.62
Income Bottom 90%	-4.02	-0.96	-3.07
All	0.36	0.01	0.35
<u>C. Elasticity of Earnings</u>			
Income Top 1%	0.51	0.29	0.22
Income Top 5%	0.53	0.33	0.20
Income Top 10%	0.57	0.37	0.20
Income Bottom 99%	0.82	0.71	0.11
Income Bottom 90%	1.26	1.18	0.08
All	0.72	0.62	0.11
<u>D. Capital Income Share</u>			
Income Top 1%	0.60	0.16	0.44
Income Top 5%	0.39	0.16	0.23
Income Top 10%	0.32	0.16	0.16
Income Bottom 99%	0.11	0.22	-0.11
Income Bottom 90%	0.10	0.27	-0.16
All	0.24	0.21	0.03

*Note:* Panels A, B and C report the income elasticities and panel D reports the capital income share. We report the elasticities with respect to the average net of marginal tax rate. The elasticity for variable  $X$  is defined as  $dX/d(1 - AMTR)$ . Earnings is total labor income, *weh*. Income is total taxable income.

becomes more negative at the bottom of the distribution. Furthermore, in the benchmark model, the capital income share is around 60 percent for the top one percent, while it is only 16 percent in the model with no return risk (see panel D, Table 4). This, combined with more concentrated wealth and income distribution (see Appendix G), implies that in the benchmark model, the capital income elasticity of the top one percent has a bigger impact on the aggregate capital income elasticity as well on the overall ETI.

Second, the presence of return heterogeneity increases the elasticity of labor income both at the aggregate level (17.1 percent) and along the whole distribution, because the



**Table 5** – Macroeconomic Effects of an Income Tax Change

Variable	No Return		
	Benchmark	Heterogeneity	Difference
<u>A. Prices</u>			
Real Wage, $w$	0.48	0.31	0.17
Price of Capital, $p$	-0.86	-0.55	-0.31
Borrowing Rate, $r$	-3.08	-1.16	-1.92
<u>B. Aggregate Quantities</u>			
Output, $Y$	0.72	0.62	0.10
Quality-Adjusted Capital, $Q$	1.58	1.16	0.41
Capital, $K$	1.13	1.17	-0.04
Labor, $L$	0.24	0.31	-0.07
<u>C. Productivity</u>			
Aggregate TFP	0.16	0.00	0.16
Entrepreneurial Productivity	0.45	0.00	0.45
Entrepreneurial Rate	-1.87	0.00	-1.87

*Note:* We report the effects of a tax cut via  $\tau_0$  as elasticities with respect to the average net of marginal tax rate. The elasticity for variable  $X$  is defined as  $dX/d(1 - AMTR)$ .

tax cut brings about a larger increase in real wages (see panel A, Table 5). While the strongest effect is for the top 1 percent of the income distribution (78 percent), return heterogeneity does not change the shape of the elasticity of earnings, which due to income effects, remains largest for the bottom of the distribution. Nevertheless, for agents in the bottom part of the income distribution, labor earnings make up a larger part of their income (see panel D, Table 4). Therefore, since the composition of income shifts towards labor income (where the elasticity is higher), the elasticity for total income increases at the bottom of the distribution.

To summarize, the combined effects of return heterogeneity on the composition of income and its concentration, along with general equilibrium effects, explain the U-shaped relationship between the ETI and income. Return heterogeneity increases the elasticity of taxable income for both the top 1 percent (which display the highest response) and the bottom 90 percent. For the top 1 percent, it amplifies the response of savings to marginal tax changes. Moreover, the composition of income at the top shifts towards capital, where the elasticity is relatively higher. Meanwhile, return heterogeneity also amplifies the general equilibrium response of wages, which increases the labor income elasticity. Furthermore, for the bottom 90 percent, the composition of income shifts more toward labor income (where the elasticity is relatively higher), which further increases the elasticity of total income at the bottom.

Our estimates of long-run ETIs along the distribution of taxable income complement similar empirical findings for the short-run (e.g., Mertens and Montiel Olea, 2018), whereas the top 1 percent of the income distribution display the highest ETI. Nevertheless,

as our estimates are for the long-run, they do not reflect any intertemporal substitution and income shifting from transitory changes in tax policy. Furthermore, our estimates indicate a strong real response that does not stem from tax evasion or avoidance (which would potentially imply a even higher elasticity at the top). Our results, in particular, show the importance of return heterogeneity for the transmission of marginal tax policies within an estimated model consistent with the distributional characteristics of the US economy.

**Elasticities by Age and Income** Next, we turn to how the elasticity of taxable income varies by age, in addition to income. Table 6 reports the elasticity of taxable income by age and income, in both the benchmark economy and in the alternative model without return heterogeneity. Overall, there is a U-shaped relationship between the ETI and age, a pattern which is further amplified by return heterogeneity. Specifically, return

**Table 6** – Elasticities by Age and Income

Age Group	Income Group					All
	Top 1%	Top 5%	Top 10%	Bot. 99%	Bot. 90%	
<u>A. Benchmark Economy</u>						
Ages 21-30	0.35	0.44	0.56	0.95	1.53	0.93
Ages 31-40	0.35	0.40	0.43	0.58	0.88	0.56
Ages 41-50	0.47	0.44	0.43	0.55	0.69	0.53
Ages 51-64	0.67	0.55	0.53	0.64	0.82	0.66
Ages 65+	1.36	1.70	2.00	-2.96	-6.56	0.73
All Ages	0.77	0.62	0.60	0.55	0.70	0.66
<u>B. No Return Heterogeneity</u>						
Ages 21-30	0.98	0.37	0.42	0.70	0.72	0.59
Ages 31-40	0.34	0.36	0.40	0.48	0.53	0.45
Ages 41-50	0.31	0.44	0.48	0.49	0.69	0.47
Ages 51-64	0.34	0.41	0.43	0.57	0.76	0.53
Ages 65+	0.42	0.46	0.53	0.10	-0.62	0.18
All Ages	0.37	0.40	0.43	0.53	0.59	0.49

*Note:* This table reports the elasticity of taxable income by age and total income. Panel A reports the results for the benchmark economy, while panel B reports the results for the economy without return heterogeneity.

heterogeneity increases the ETI from 0.59 to 0.93 for young individuals (age 21-30). For those near retirement (age 51-64), return heterogeneity increases the ETI from 0.53 to 0.66, and generates an even larger response for retired individuals (age 65+). With return heterogeneity, marginal tax changes generate larger responses in labor earnings, particularly for the young and those nearing retirement. For retired individuals (age 65+), return heterogeneity generates a larger response of capital income to tax changes.

Return heterogeneity also affects the ETI patterns across both age and income. In our benchmark economy, the young who are in the top 1 percent tend to be less elastic relative to older agents in the top 1 percent. However, in the model without return heterogeneity, this result is flipped, so that it is instead the youngest households in the top 1 percent who are more elastic.<sup>14</sup> Moreover, it can be seen that the overall increase in elasticities for the young is driven by the higher elasticities for those in the bottom of the income

**Table 7** – Revenue Losses from an Income Tax Change

Group	Benchmark	No Return Heterogeneity	Difference
A. Total Revenue Losses			
Income Top 1%	30.9	14.7	16.2
Income Top 5%	26.1	17.2	8.9
Income Top 10%	26.3	19.2	7.1
Income Bottom 99%	26.9	26.8	0.1
Income Bottom 90%	41.0	38.0	3.1
All	28.5	23.8	4.7
B. Share of Revenue Losses			
Income Top 1%	45.0	15.2	29.8
Income Top 5%	64.1	41.2	22.9
Income Top 10%	78.0	60.5	17.5
Income Bottom 99%	55.0	84.8	-29.9
Income Bottom 90%	22.0	39.5	-17.5
All	100.0	100.0	0.0

*Note:* Panel A reports the percent of tax revenue lost to behavioral responses, given uniform change in the income tax via  $\tau_0$ , by income group. Panel B reports the share of the total revenue loss which is attributable to each income group.

distribution. Meanwhile, the overall increase in elasticities for retired individuals is driven by the stronger response of those in the top of the income distribution.

**Behavioral Revenue Losses** In our framework, there is a link between the elasticity of taxable income and the amount of additional revenue the government loses to behavioral responses (e.g., from a tax increase). Specifically, to fix concepts, suppose the government were to raise marginal taxes by a small amount (i.e.,  $d\tau_0 > 0$ ) and that the increase in total tax revenue is given by  $dT_y$ . Thus, there will be two effects on tax revenues. First, there is a “mechanical” increase in tax revenues due to the fact that all taxpayers will face a higher marginal tax rate. We denote the increase in revenues from the mechanical effect by  $dM$ . Second, the higher tax rate will trigger a behavioral response, whereby households reduce their average taxable income (by altering their labor supply or savings/investment

<sup>14</sup>It should be noted that return heterogeneity does not have much effect on the age composition of individuals within each income group or the income composition within each age group.

decisions). Intuitively, when the ETI is higher, the government will lose more revenue to the behavioral response.

In Table 7, we compute the revenue loss due to the behavioral response as the percentage difference between the mechanical response and the actual revenue change (i.e.,  $(dM - dT_y)/dM$ ). Overall, in our benchmark economy, the government loses 28.5 percent of additional tax revenue because of behavioral responses. Higher losses are observed both for the top 1 percent (30.9 percent) and the bottom 90 percent (41.0 percent) of the income distribution. These patterns are further amplified by the presence of return heterogeneity (as with the elasticity of taxable income). This is especially true for individuals in the top 1 percent, where return heterogeneity more than doubles the behavioral losses from tax changes. Furthermore, almost 80 percent of the total revenue losses come from the behavioral responses of individuals in the top 10 percent. Nevertheless, a sizeable fraction of losses (20 percent) come from the bottom 90 percent.

Given our policy exercise, we can derive an approximate link between the ETI and the amount of tax revenue the government loses to behavioral responses. The marginal change in fiscal revenues,  $dT_y$ , for a marginal change in  $\tau_0$ , is approximately given by

$$dT_y \approx dM \left( 1 - \underbrace{\frac{AMTR}{1 - AMTR} \frac{AMTR}{ATR}}_{\text{behavioral response}} \times ETI \right). \quad (13)$$

See Appendix J for a derivation. The expression in (13) is a sufficient statistic to estimate the overall revenue change from the type of tax policy reforms studied here. This indicates that the behavioral revenue loss depends on the elasticity of taxable income (ETI), as well as the average marginal tax rate (AMTR) and the average tax rate (ATR).

Naturally, higher elasticities will be associated with higher revenue losses. Furthermore, due to the progressivity of the tax schedule,  $AMTR > ATR$ , and this will tend to increase the behavioral losses for a given elasticity. This suggests why revenue losses are relatively high for the bottom 90 percent. While households in the bottom 90 percent tend to have a relatively high ETI in our benchmark economy (0.70), they do also have a lower AMTR, which would lower the revenue losses. At the same time, the progressivity of the tax code generates a larger gap between the AMTR and the ATR for the bottom 90 percent, pushing up their behavioral losses.

**Entrepreneurial Productivity and TFP** Next, we analyze the effects of marginal income tax changes on entrepreneurial productivity and aggregate TFP (see panel C, Table 5). In our benchmark economy with return heterogeneity, a decrease in the marginal income tax rate reduces the entrepreneurial rate. This increases average entrepreneurial productivity,  $Q/K$ , which further increases aggregate TFP,  $A = (Q/K)^\alpha$ . The incentive to accumulate savings triggered by a marginal tax change is stronger for those agents enjoying a high capital return (i.e., the most productive entrepreneurs). By increasing

their savings, they are able to expand the amount of capital invested in their business. As a result, the most productive entrepreneurs get larger and attract more capital relative to the median business. This increases average entrepreneurial productivity, and thus aggregate TFP.<sup>15</sup> Moreover, this also implies that some low-productivity entrepreneurs would find it more convenient to quit their business simply lend their entire wealth to the most productive entrepreneurs. As a result, the share of entrepreneurs decreases. Overall, this mechanism helps generate the larger general equilibrium responses in wages, which further increases the elasticity of taxable income.

Our results for the effects of return heterogeneity on the transmission of marginal tax changes are interesting for at least two reasons. First, they complement a large literature on taxation and entrepreneurial activity (e.g., Jaimovich and Rebelo, 2017 and Itskhoki and Moll, 2019), as well as a literature on capital misallocation and wealth taxation (e.g., Guvenen et al., 2019). Second, they show the importance of wealth return heterogeneity in explaining some well-established empirical results, such as the relation between tax rates and entrepreneurial activity (e.g., Djankov et al., 2010), the effects of tax changes on aggregate productivity (e.g., Cloyne, 2013) and the relationship between tax progressivity and misallocation (e.g., Fajgelbaum et al., 2019).

## 5 Conclusion

This paper studies the role of uninsurable capital income risk for the long-run transmission mechanism of marginal tax policies. We do so by structurally estimating a life-cycle model with uninsurable labor and capital income risk. We show that our model is successful in capturing the right tail of the income and wealth distributions. Within this model, we evaluate the effects of changing marginal income tax rates.

We find that return heterogeneity has strong qualitative and quantitative effects on the transmission of fiscal policies. In particular we find that return heterogeneity strongly increases the elasticity of income for the top 1 percent (who also display the largest elasticity), but also increases the elasticity for the bottom 90 percent. We also find that with return heterogeneity, a marginal tax cut re-allocates capital to high-productivity entrepreneurs, increasing aggregate productivity.

Our framework could be extended in several ways. First, one could analyze how the long-run effects of tax changes are affected by the presence of an informal sector. When agents have the possibility of avoiding paying taxes, the effects of a tax cut should sensibly increase. Second, it would be interesting to understand the relationship between marginal tax changes and capital misallocation with an informal production sector. Third, it would be interesting to have a model with human capital accumulation. In the long run, lower taxes might push more people into acquiring education, which in turn might have a beneficial effects on entrepreneurial productivity, and income and wealth inequality.

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<sup>15</sup>The TFP losses from misallocation in our framework is 18.5 percent, which is slightly smaller than the losses (20 percent) reported by Guvenen et al. (2019).

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# Appendix

## A Definition of Equilibrium

We focus on a stationary equilibrium, in which capital, labor, transfers, and government consumption are all constant in per-capita terms. Let  $\psi_{i,j}(a, z_h, z_r)$  denote the distribution of agents with type  $i$  and age  $j$ , over assets  $a$  and idiosyncratic shocks  $(z_h, z_r)$ .

**Definition 1.** *The stationary recursive equilibrium consists of*

- (i) *the value function  $V_{i,j}(a, z_h, z_r)$ ,*
- (ii) *the policy functions  $c_{i,j}(a, z_h, z_r)$ ,  $a'_{i,j}(a, z_h, z_r)$ ,  $h_{i,j}(a, z_h, z_r)$ ,*
- (iii) *the entrepreneurial profit function  $\pi(a, z_r)$  and associated capital demand  $k(a, z_r)$ ,*
- (iv) *the prices  $(w, p, r)$ ,*
- (v) *the per-capita stocks of capital  $K$ , intermediate good  $Q$ , labor  $L$ , lump-sum transfers  $T_b$ , government spending  $G$ , and*
- (vi) *the per-capita benefit levels  $\bar{b}_i$  and labor  $L_i$  for types  $i = 1, \dots, I$ ,*
- (vii) *distributions  $(\mu_1, \dots, \mu_J)$ ,  $(\psi_{i,1}, \dots, \psi_{i,J})$  for  $i = 1, \dots, I$*

such that

1. *The value function  $V_{i,j}(a, z_h, z_r)$  solves the Bellman equation in (12) and  $c_{i,j}(a, z_h, z_r)$ ,  $a'_{i,j}(a, z_h, z_r)$ ,  $h_{i,j}(a, z_h, z_r)$  are the associated policy functions.*
2. *Household profits  $\pi(a, z_r)$  solve (6) and capital demand  $k(a, z_r)$  is given by (7).*
3. *The final goods producer maximizes its profits:*

$$F_1(Q, L) = p \quad \text{and} \quad F_2(Q, L) = w$$

4. *Markets clear:*

$$\sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \int [c_{i,j}(a, z_h, z_r) + a'_{i,j}(a, z_h, z_r)] d\psi_{i,j} + G = F(Q, L) + (1 - \delta)K$$

$$\sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \int z_r k(a, z_r) d\psi_{i,j} = Q$$

$$\sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \int a d\psi_{i,j} = K$$

$$\sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \int [k(a, z_r) - a] d\psi_{i,j} = 0$$

$$\sum_{j=1}^J \mu_j \int e_{i,j}(z_h) h_{i,j}(a, z_h, z_r) d\psi_{i,j} = L_i$$

$$\sum_{i=1}^I \pi_i L_i = L$$

5. The distribution of agents across age groups,  $\mu_1, \dots, \mu_J$ , satisfies

$$\mu_{j+1} = \frac{s_{j+1} \mu_j}{1+n} \text{ for } j = 1, \dots, J-1$$

where  $\mu_1$  is normalized so that  $\sum_{j=1}^J \mu_j = 1$ .

6. The distributions of agents within each age group  $j$  and type  $i$ ,  $\psi_{i,1}, \dots, \psi_{i,J}$ , for  $i = 1, \dots, I$ , are consistent with individual behavior. That is, the law of motion for  $\psi_{i,j}$  is

$$\psi_{i,j+1}(a', z'_h, z'_r) = \int f(z'_h | z_h) f(z'_r | z_r) \mathbb{1} \{a'_{i,j}(a, z_h, z_r) = a'\} d\psi_{i,j}(a, z_h, z_r)$$

where  $f(z'_h | z_h)$  and  $f(z'_r | z_r)$  are the conditional probabilities that a household will transition to  $z'_h$  and  $z'_r$  given that its current shocks are  $z_h$  and  $z_r$ , respectively.

Furthermore, in the initial distribution  $\psi_{i,1}(a, z_h, z_r)$  for each type  $i \in \{1, \dots, I\}$ , all age-1 agents are born with no assets (i.e.,  $a = 0$ ), the initial shock  $\log z_h$  is zero and the initial  $\log z_r$  is drawn from  $N(0, \sigma_{er}^2 / (1 - \rho_r^2))$ .

7. The government budget constraint is satisfied

$$G = T_y \equiv \sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \mathcal{T}_y(y_{i,j}(a, z_h, z_r)) d\psi_{i,j}$$

where taxable income is

$$y_{i,j}(a, z_h, z_r) = \max \left\{ we_{i,j}(z_h) h_{i,j}(a, z_h, z_r) + r_a(z_r) a - \frac{1}{2} \min (we_{i,j}(z_h) h_{i,j}(a, z_h, z_r), \bar{y}), 0 \right\}$$

8. Social security benefits equal social security taxes:

$$\tau_{ss} \sum_{i=1}^I \pi_i \sum_{j=1}^J \int \min (we_{i,j}(z_h) h_{i,j}(a, z_h, z_r), \bar{y}) d\psi_{i,j} = \sum_{i=1}^I \pi_i \bar{b}_i \left( \sum_{j=R}^J \mu_j \right)$$

9. The the type-specific benefit levels are  $\bar{b}_i = \chi w L_i$ ,

10. Lump-sum transfers  $T_b$  are consistent with individual behavior.

$$T_b = \frac{1}{1+n} \sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j (1 - s_{j+1}) \int E [a'_{i,j}(a, z_h, z_r)(1 + r_a(z'_r)) | z_r] d\psi_{i,j}$$

## B Numerical Solution Technique

The numerical solution method can be summarized as follows. First, we discretize the AR(1) processes for the idiosyncratic shocks  $(z_h, z_r)$  using the Rouwenhorst method (see Kopecky and Suen, 2010). Second, to solve for the stationary equilibrium, we define the vector function  $f_k(r, \chi, Q, T_b, \{L_i\}_{i=1}^I)$  for  $k = 1, \dots, 4 + I$  (see below for a definition). We then utilize a multi-dimensional root-finding algorithm to solve for the vector  $(r, \chi, Q, T_b, \{L_i\}_{i=1}^I)$  such that  $f_k = 0$  for  $k = 1, \dots, 4 + I$ . For a given guess for the for the vector  $(r, \chi, Q, T_b, \{L_i\}_{i=1}^I)$ , we proceed to compute  $f_k$  for  $k = 1, \dots, 4 + I$  as follows:

1. Given  $\{L_i\}_{i=1}^I$ , compute aggregate labor  $L = \sum_{i=1}^I \pi_i L_i$ .
2. Given  $Q$  and  $L$ , determine prices  $p = F_1(Q, L)$  and  $w = F_2(Q, L)$ .
3. Given  $\chi, w$  and  $L_i$ , determine the social security benefit  $b_{i,j} = \chi w L_i \times \mathbf{1}\{j \geq R\}$ .
4. Given  $w, r, p, b_{i,j}, T_b$ , solve for the policy functions  $a'_{i,j}(a, z_h, z_r), h_{i,j}(a, z_h, z_r)$  for  $i = 1, \dots, I, j = 1, \dots, J$  by iterating on the Bellman equation defined in (12).
5. Calculate the distributions  $\psi_{i,j}$  for  $i = 1, \dots, I$  and  $j = 1, \dots, \psi_J$  using Monte Carlo simulation.
6. Given  $\psi_{i,j}$  and  $(r, \chi, Q, T_b, \{L_i\}_{i=1}^I)$ , compute  $f_k$  for  $k = 1, \dots, 4 + I$ :

$$f_1 = \sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \int (k(a, z_r) - a) d\psi_{i,j}$$

$$f_2 = \sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \int \tau_{ss} \min(we_{i,j}(z_h)h_{i,j}(a, z_h, z_r), \bar{y}) d\psi_{i,j} - \chi w L \left( \sum_{j=R}^J \mu_j \right)$$

$$f_3 = Q - \sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \int z_r k(a, z_r) d\psi_{i,j}$$

$$f_4 = T_b - \frac{1}{1+n} \left[ \sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j (1 - s_{j+1}) \int E [a'_{i,j}(a, z_h, z_r)(1 + r_a(z'_r)) | z_r] d\psi_{i,j} \right]$$

$$f_{4+i} = L_i - \sum_{j=1}^J \mu_j \int e_{i,j}(z_h) h_{i,j}(a, z_h, z_r) d\psi_{i,j} \quad \text{for } i = 1, \dots, I$$

## C Survival Probabilities

The survival probabilities were obtained from the 2016 Period Life Tables from United States Mortality Database (see Table C.1). We utilized survival probabilities for both genders across the entire United States. Since the maximum age is  $J = 85$  in the model (which corresponds to age 105 in real life), we impose that  $s_{J+1} = 0$ .

**Table C.1** – Survival Probabilities

Age ( $j$ )	$s_{j+1}$	Age ( $j$ )	$s_{j+1}$	Age ( $j$ )	$s_{j+1}$
1	0.9990	31	0.9956	61	0.9483
2	0.9990	32	0.9951	62	0.9437
3	0.9990	33	0.9946	63	0.9373
4	0.9989	34	0.9942	64	0.9313
5	0.9989	35	0.9937	65	0.9251
6	0.9989	36	0.9933	66	0.9161
7	0.9988	37	0.9926	67	0.9065
8	0.9987	38	0.9921	68	0.8954
9	0.9988	39	0.9914	69	0.8826
10	0.9987	40	0.9910	70	0.8694
11	0.9986	41	0.9903	71	0.8543
12	0.9986	42	0.9895	72	0.8394
13	0.9986	43	0.9889	73	0.8226
14	0.9985	44	0.9880	74	0.8051
15	0.9984	45	0.9873	75	0.7920
16	0.9984	46	0.9867	76	0.7736
17	0.9983	47	0.9855	77	0.7543
18	0.9982	48	0.9845	78	0.7344
19	0.9981	49	0.9821	79	0.7139
20	0.9981	50	0.9826	80	0.6932
21	0.9980	51	0.9799	81	0.6720
22	0.9979	52	0.9780	82	0.6506
23	0.9977	53	0.9751	83	0.6299
24	0.9975	54	0.9738	84	0.6096
25	0.9973	55	0.9710	85	0
26	0.9971	56	0.9688		
27	0.9970	57	0.9654		
28	0.9966	58	0.9620		
29	0.9963	59	0.9582		
30	0.9959	60	0.9534		

## D Tax Function Estimation

We use the 2016 wave of Survey of Consumer Finances (SCF) to estimate the parameters of tax function, given in Equation (11). Our measure of total household income follows closely Gouveia and Strauss (1994) and includes all income flows accruing to households. In particular, it includes salaries and wages, both taxable and non-taxable income, dividend and interest income, capital gains, total pensions and annuities received

(including taxable IRA distributions), unemployment compensation and social security benefits, and alimony received. To calculate federal income tax liabilities, we use NBER’s TAXSIM program. Our notion of tax liability includes capital gains rates, surtaxes, AMT as well as refundable and non-refundable credits. In our estimation, we restrict the sample to those households whose income is strictly positive and whose measured average tax rate is less than 100%.

## E Estimation of Ability Process from PSID

To estimate the parameters  $(\rho_h, \sigma_{\varepsilon_h})$  of the labor ability process, we follow the approach of Kaplan (2012). Specifically, we utilize the the Panel Study of Income Dynamics (PSID), using data from the 1968-2017 waves. We use selection criteria similar to Kaplan (2012), where we (i) retain only the core Survey Research Center (SRC) subsample, (ii) keep only males, (iii) drop observations with missing data on years of education, (iv) keep only individuals aged between 21 and 64, (v) drop households with a second earner who earned at least half the amount earned by the male head, (vi) keep only individuals who worked between 520 and 5200 hours during the calendar year, (vii) drop observations where the nominal wage is less than 1 dollar, (viii) drop observations where real income is below \$1,500 (in 2010 dollars, deflated using the CPI). The final sample contains 62,683 individual/year observations and 7,510 distinct individuals.

We measure the household’s real earnings as the head of household’s labor income, deflated by the CPI. We divide real earnings by the head of household’s yearly hours to obtain a measure of “ability” (i.e., real wages). First, we regress log ability on a full set of year and race dummies. Let  $\log e_{i,j}$  be the residual from this regression for individual

**Table E.1** – Parameter Estimates of Ability Process from PSID

Parameter	Value	Std. Err.
$\rho_h$	0.976	(0.005)
$\sigma_{\varepsilon_h}$	0.135	(0.006)
$\sigma_e$	0.279	(0.015)

*Note:* This table reports the parameter estimates for the ability process from PSID, which are obtained using GMM. Standard errors, reported in parentheses, are obtained by bootstrap with 250 repetitions.

$i$  at age  $j$ . The benchmark specification for the statistical process governing  $\log e_{i,j}$  is

$$\begin{aligned} \log e_{i,j} &= \kappa_j + \bar{e}_i + z_{h,j} + \varepsilon_{e,j} \\ z_{h,j} &= \rho_h z_{h,j-1} + \varepsilon_{h,j} \\ z_{h,1} &= 0 \end{aligned}$$

where  $E[\varepsilon_{h,j}] = E[\bar{e}_i] = 0$ ,  $\text{Var}(\varepsilon_{h,j}) = \sigma_{\varepsilon_h}^2$ ,  $\text{Var}(\bar{e}_i) = \sigma_e^2$  and  $\text{Var}(\varepsilon_{e,j}) = \sigma_{\varepsilon_e}^2$ . This specification matches our benchmark specification for labor ability given by Equation (2) and (3)

in the text, with the exception that we allow for i.i.d. measurement error,  $\varepsilon_{e,j}$ . For notational convenience, we have slightly altered the notation here, where  $i$  indexes individual  $i$  and  $\kappa_j$  corresponds to the non-stochastic age profile of log ability. It can be shown that  $\text{Var}(z_{h,j}) = \sigma_{\varepsilon_h}^2 \left[ 1 - \rho_h^{2(j-1)} \right] / [1 - \rho_h^2]$  and  $\text{Cov}(z_{h,j}, z_{h,j+s}) = \rho_h^s \text{Var}(z_{h,j})$  for  $s > 0$ . Denote an element of the autocovariance function of  $\log e_{i,j}$  as  $\sigma_{j,j+s} \equiv \text{Cov}(\log e_{i,j}, \log e_{i,j+s})$ . The autocovariance moments for this process is then given by

$$\begin{aligned}\sigma_{j,j} &= \sigma_e^2 + \frac{1 - \rho_h^{2(j-1)}}{1 - \rho_h^2} \sigma_{\varepsilon_h}^2 + \sigma_{\varepsilon_e}^2, \\ \sigma_{j,j+s} &= \sigma_e^2 + \rho_h^s \frac{1 - \rho_h^{2(j-1)}}{1 - \rho_h^2} \sigma_{\varepsilon_h}^2 \quad \text{for } s > 0.\end{aligned}$$

Note that these moments are independent of the age profile,  $\kappa_j$ . We construct estimates of these autocovariances by age and year. We use a maximum of 25 lags and retain moments that were constructed with at least 30 observations. We assume that the variance of the i.i.d. measurement error is  $\sigma_{\varepsilon_e}^2 = 0.03$ , using the estimates of measurement error of earnings and hours in Kaplan (2012).

We use generalized method of moments (GMM) to estimate the parameters of this process. Denote the parameters to be estimated by  $\Theta = (\rho_h, \sigma_{\varepsilon_h}, \sigma_e)$ . The GMM estimator

**Table F.1** – Return on Wealth by Wealth Percentiles, Benchmark Model vs. Data

Wealth Percentile	Returns (Model)	Returns (Data)
[99-100]	0.071	0.074
[95-99)	0.039	0.066
[90-95)	0.033	0.059
[75-90)	0.029	0.053
[50-75)	0.025	0.049
[25-50)	0.021	0.040
[10-25)	0.019	0.021
[1-10)	0.021	0.028

*Note:* This table reports the resulting wealth returns by wealth percentile in the model and the data. For the data, we estimate the average return for each household's portfolio in the SCF using estimates of the average returns of different asset types between 1990 and 2016, as reported by Xavier (2020).

solves the following minimization problem

$$\hat{\Theta} = \arg \min_{\Theta} \left( \hat{M} - \hat{m}(\Theta) \right)' W \left( \hat{M} - \hat{m}(\Theta) \right)$$

where  $\hat{M}$  is the vector targeted moments in PSID, and  $\hat{m}(\Theta)$  are the corresponding model moments, and  $W$  is a weighting matrix. We assumed the off-diagonal elements of the weighting matrix  $W$  were zero. For the diagonal elements, we assumed  $W_{ii} = \sqrt{n_i}$ , where  $n_i$  is the number of observations used in the construction of moment  $i$ . To minimize the

GMM criterion, we used a scatter-search algorithm which generates random start points for the interior-point minimization algorithm. Standard errors are obtained by bootstrap with 250 repetitions. The resulting parameter estimates are reported in Table E.1.

## F Calculating Return Profiles in SCF

Here we describe the technique used to estimate the return on wealth at household level in the SCF. We define  $r_j$  and  $\zeta_j$  to be the return of asset type  $j$  and its share in each household's portfolio, respectively.<sup>16</sup> Consistently, the aggregate return on wealth can be computed as

$$r_w = \sum_j \zeta_j r_j.$$

The return  $r_j$  of each asset type is composed of its yield (income generated by the asset) and its capital gain (price changes in the asset). For each asset type, we obtain the average return over the period 1990-2016 from Xavier (2020). Finally, we compute the household's portfolio shares  $\zeta_j$  directly in the 2016 SCF.

In Table F.1, we report the resulting returns by wealth percentile in the 2016 SCF. The data display the same qualitative patterns as that of our model: households with higher wealth obtain higher than average returns. Moreover, our model implied-returns at the bottom 25 percent of the wealth distribution and the top 1 percent are quite close to the imputed returns in the SCF. In the middle of the distribution, imputed returns from the SCF seem to be higher than those implied by the model. Overall these results (particularly for the consistency at the top of the wealth distribution) confirm the ability of our model to provide a realistic explanation of the fat tail in wealth and income distributions.

## G Model with No Return Heterogeneity

Table G.1 reports how the targeted moments in our model change when we eliminate return heterogeneity. All other parameters are kept as in the benchmark model. Overall, wealth is much less concentrated, as the wealth gini falls from 0.862 to 0.706, and the top 1 percent wealth share falls from 0.406 to 0.172. As a result, tax revenues also become less concentrated, with the share of tax revenues paid by the top 1 percent of income falling from 0.419 to 0.247. Hours of work fall from 0.299 to 0.238. Furthermore, absent capital income risk, aggregate wealth-income ratio increases sensibly to 3.81 and the borrowing rate increases to 4.5 percent. This is because everyone runs a business in this alternative model. As such, capital demand is higher than in the benchmark and more evenly spread across the distribution. Higher capital demand also implies higher borrowing costs.

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<sup>16</sup>The  $j$  categories are: interest earning assets, stocks, private businesses, real estate, other financial and non financial assets and debt (with a negative sign).

## H Model with Separate Capital and Labor Taxes

In our benchmark economy, both capital and labor income are taxed jointly, which is consistent with the US tax code and the empirical literature. In this section, we consider the implications for our results if labor income and capital income are taxed separately. Specifically, we now assume that taxable labor income is given by

$$y = we_{i,j}(z_h)h - \frac{1}{2} \min(we_{i,j}(z_h)h, \bar{y})$$

**Table G.1** – Targeted Moments, Benchmark Model vs. No Return Heterogeneity

<b>Moments</b>	<b>Benchmark</b>	<b>Data</b>	<b>No Return Het.</b>
Capital-to-output Ratio	2.95	2.95	3.81
Investment-to-output Ratio	0.22	0.22	0.27
Borrowing Rate	0.019	0.019	0.045
Average Hours (working age)	0.299	0.304	0.238
Entrepreneurship Rate	0.087	0.087	-
Wealth Gini	0.862	0.860	0.706
Wealth Share, Top 1%	0.406	0.386	0.172
Wealth Share, Top 5%	0.638	0.651	0.406
Wealth Share, Top 20%	0.888	0.883	0.725
Earnings Gini	0.735	0.680	0.734
Earnings Share, Top 1%	0.145	0.172	0.146
Earnings Share, Top 5%	0.363	0.327	0.362
Earnings Share, Top 20%	0.687	0.605	0.691
Average Earnings	54.83	55.30	54.81
Revenues Tax Share, Top 1%	0.419	0.424	0.247
Revenues Tax Share, Top 5%	0.702	0.659	0.571
Revenues Tax Share, Top 20%	0.959	0.881	0.914
Wealth-Income Slope, Top 20%	1.574	1.638	1.661
Wealth-Income Slope, Top 40%	0.915	0.959	1.102
Wealth-Income Slope, Top 60%	0.693	0.717	0.635

*Note:* This table reports the moments in the benchmark model and the data and compares it with a model without return heterogeneity.

The total labor income tax is then given by  $\mathcal{T}_y(y)$ , given by Equation (11) in the text. We assume this function has the same parameters as our benchmark economy. Households separately pay a flat tax on capital income, given by  $\tau_k r_a(z_r)a$ . We set  $\tau_k$  so that total tax revenue is identical to the revenue in our benchmark economy. This requires  $\tau_k = 0.226$ .

Since capital and labor are now taxed at separate rates, we cannot compute an elasticity of taxable income for total income. Therefore, we separately define an elasticity of labor income and an elasticity of capital income, where each elasticity is defined relative to the relevant net-of-marginal tax rate. We then consider the effects of a change in the labor income tax rate (i.e., a change in  $\tau_0$ ), a change in the capital income tax rate ( $\tau_k$ )



and a change in both tax rates. Table H.1 reports the resulting elasticities for labor and capital income. Table H.2 reports the resulting elasticities for the model with no return heterogeneity. Overall, we see that our main results survive. The elasticity of taxable labor income is higher for people in the bottom of the total income distribution, while

**Table H.1** – Elasticities with Separate Capital and Labor Taxation

Variable	Benchmark	Separate Taxation		
	Income Tax Change	Labor Tax Change	Capital Tax Change	Cap. & Lab. Tax Change
<u>A. Capital Income</u>				
Income Top 1%	0.94		0.72	0.65
Income Top 5%	0.74		0.50	0.50
Income Top 10%	0.64		0.41	0.41
Income Bottom 99%	-1.76		-1.49	-1.08
Income Bottom 90%	-4.02		-1.82	-1.41
All	0.36		0.12	0.18
<u>B. Labor Income</u>				
Income Top 1%	0.51	0.15		0.52
Income Top 5%	0.53	0.19		0.55
Income Top 10%	0.57	0.24		0.58
Income Bottom 99%	0.82	0.39		0.84
Income Bottom 90%	1.26	0.63		1.30
All	0.72	0.34		0.77

*Note:* This table reports the elasticities of taxable labor income and capital income in a model with separate capital and labor taxation. For both labor and capital income, we group individuals based on their position in the total income distribution.

the elasticity of taxable capital income is higher for people in the top of the total income distribution.

## I Alternative Policy Reform

Here we consider an alternative policy reform where we increase income taxes only for individuals in the top 1 percent. Specifically, we assume households face a modified income tax function,  $\hat{\mathcal{T}}_y(y)$ , given by:

$$\hat{\mathcal{T}}_y(y) = \begin{cases} \mathcal{T}_y(y) & \text{if } y < \hat{y} \\ \mathcal{T}_y(\hat{y}) + \hat{\tau}(y - \hat{y}) & \text{if } y \geq \hat{y} \end{cases}$$

For income below a threshold  $\hat{y}$ , the household's income tax is given by  $\mathcal{T}_y(y)$ , as in our benchmark economy. For income above  $\hat{y}$ , households face a flat marginal tax,  $\hat{\tau}$ . We choose the threshold  $\hat{y}$  so that the marginal tax rate  $\hat{\tau}$  applies to the top 1 percent of households and set  $\hat{\tau} = \mathcal{T}'_y(\hat{y})$ .

**Table H.2** – No Return Heterogeneity with Separate Capital and Labor Taxation

Variable	Joint Tax.	Separate Taxation		
	Income Tax Change	Labor Tax Change	Capital Tax Change	Cap. & Lab. Tax Change
<u>A. Capital Income</u>				
Income Top 1%	0.78		-0.39	0.20
Income Top 5%	0.76		-0.43	0.17
Income Top 10%	0.70		-0.39	0.12
Income Bottom 99%	-0.13		-0.19	-0.16
Income Bottom 90%	-0.96		-0.08	-0.30
All	0.01		-0.22	-0.11
<u>B. Labor Income</u>				
Income Top 1%	0.29	0.21		0.40
Income Top 5%	0.33	0.26		0.44
Income Top 10%	0.37	0.30		0.48
Income Bottom 99%	0.71	0.48		0.71
Income Bottom 90%	1.18	0.75		1.03
All	0.62	0.43		0.64

*Note:* This table reports the elasticities of taxable labor income and capital income in a model with separate capital and labor taxation and no return heterogeneity. For both labor and capital income, we group individuals based on their position in the total income distribution.

**Table I.1** – Effects of Income Tax Increase for Top 1%

Variable	Alternative Benchmark	No Return Heterogeneity	Difference
<u>A. Elasticity of Taxable Income</u>			
Income Top 1%	0.83	0.44	0.38
All	1.00	0.58	0.42
<u>B. Revenue Losses</u>			
Income Top 1%	51.0	41.8	9.2
All	53.9	36.0	17.9

*Note:* This table reports the results of the policy experiment where the top marginal tax rate is increased only for individuals in the top 1 percent of the income distribution.

We consider a policy reform in which the top marginal tax rate  $\hat{\tau}$  is increased by 1 percentage point. Compared to our main policy experiment in which income taxes were changed for all taxpayers, we see that this policy reform is more distortionary. The elasticity of taxable income, for the top 1 percent and overall, is higher in this economy relative to our main policy experiment (see panel A, Table I.1). Similarly, the behavioral revenue losses are also higher, where the government would lose a little more than half of the additional tax revenue to the behavioral response (see panel B, Table I.1). Furthermore, as in our main policy experiment, these results are amplified by the presence of return heterogeneity.

## J ETI and Revenue Loss

In this section, we derive an approximate relationship between the elasticity of taxable income and the behavioral revenue losses. Specifically, denote the tax function  $\mathcal{T}_y(y) = \tau_0 \bar{\mathcal{T}}_y(y)$ , where  $\bar{\mathcal{T}}_y(y) = y(1 - (\tau_2 y^{\tau_1} + 1)^{-1/\tau_1})$ . Let  $\Gamma(y)$  be the equilibrium distribution of agents over taxable income  $y$ .  $\Gamma$  can refer to the entire distribution, or to subsets of the entire distribution (e.g., top 1 percent, top 5 percent, etc.). Let  $T_y$  denote the aggregate amount of tax revenue:

$$T_y = \tau_0 \int \bar{\mathcal{T}}_y(y) d\Gamma(y) \quad (\text{J.1})$$

Differentiate Equation (J.1) with respect to  $\tau_0$ :

$$\frac{dT_y}{d\tau_0} = \underbrace{\int \bar{\mathcal{T}}_y(y) d\Gamma(y)}_{\text{mechanical effect}} + \tau_0 \underbrace{\frac{d \int \bar{\mathcal{T}}_y(y) d\Gamma(y)}{d\tau_0}}_{\text{behavioral effect}}. \quad (\text{J.2})$$

The first term is the mechanical effect, or the marginal increase in tax revenue absent any changes in taxable income. The second term is the behavioral response, which captures the effect of a marginal change in income on aggregate tax revenue. Let  $\bar{T}_y = \int \bar{\mathcal{T}}_y(y) d\Gamma(y)$  be normalized tax revenue and  $Y = \int y d\Gamma(y)$  denote aggregate taxable income. To simplify Equation (J.2), we assume the following approximation:

$$\frac{d\bar{T}_y}{d\tau_0} \approx \overline{AMTR} \frac{dY}{d\tau_0} \quad (\text{J.3})$$

where  $\overline{AMTR} = \int \frac{\bar{\mathcal{T}}'_y(y)y}{Y} d\Gamma(y)$  is the normalized average marginal tax rate. We also assume that  $\overline{AMTR}$  is roughly constant for a small change in  $\tau_0$ . Essentially, we are assuming that the marginal change in aggregate normalized tax revenue,  $\bar{T}_y$ , is approximately equal to the marginal change in aggregate income  $Y$  times the normalized average marginal tax rate,  $\overline{AMTR}$ . This will approximately be true when most of the higher-income individuals within the distribution  $\Gamma$  face a similar marginal tax rate. This would also be true if the marginal tax rate function,  $\mathcal{T}'_y(y)$ , could be approximated by an increasing sequence of constant marginal tax rates (e.g., consistent with the US tax code).

Substituting Equation (J.3) into Equation (J.2), we get:

$$\frac{dT_y}{d\tau_0} \approx \bar{T}_y + \tau_0 \overline{AMTR} \frac{dY}{d\tau_0} \quad (\text{J.4})$$

Let  $AMTR = \tau_0 \overline{AMTR} = \int \frac{\mathcal{T}'_y(y)y}{Y} d\Gamma(y)$  denote the average marginal tax rate. Then:

$$\frac{dY}{d\tau_0} = -\frac{dY}{d(1 - AMTR)} \overline{AMTR}. \quad (\text{J.5})$$

Substituting Equation (J.5) into (J.4) and re-arranging, we obtain:

$$\frac{dT_y}{d\tau_0} \approx \bar{T}_y \left[ 1 - \frac{\tau_0 \overline{AMTR}}{1 - AMTR} \frac{\overline{AMTR}}{\bar{T}_y/Y} \frac{dY}{d(1 - AMTR)} \frac{1 - AMTR}{Y} \right] \quad (J.6)$$

Using that the elasticity of taxable income is  $ETI = \frac{dY}{d(1-AMTR)} \frac{1-AMTR}{Y}$ ,  $AMTR = \tau_0 \overline{AMTR}$  and the average tax rate is  $ATR = T_y/Y$ , (J.6) becomes:

$$\frac{dT_y}{d\tau_0} \approx \bar{T}_y \left[ 1 - \frac{AMTR}{1 - AMTR} \frac{AMTR}{ATR} \times ETI \right]$$

For a small tax change  $d\tau_0$ , the mechanical effect is  $dM = \bar{T}_y d\tau_0$  and the marginal change in tax revenue is

$$dT_y \approx dM \left[ 1 - \frac{AMTR}{1 - AMTR} \frac{AMTR}{ATR} \times ETI \right].$$

This is Equation (13) in the text.

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