The natural rate of interest (hereafter denoted r\*) is defined as the real rate of interest which would be observed in an equilibrium scenario in which prices and nominal wages are perfectly flexible. Thus defined, it is not a directly observable variable and the economic literature reports various ways to estimate r\* from observed data. The most commonly used method is to estimate an econometric model inspired in the neo-Keynesian theoretical framework, consisting of an aggregate demand equation (IS curve), according to which the gap between observed output and its natural level (output gap) depends on the gap between the observed real rate of interest and the natural rate of interest, and of a Phillips curve relating inflation to the output gap.<sup>1</sup>

Although this model has been widely used in the literature, it is recognised that it tends to produce inaccurate estimates of the natural rate of interest.<sup>2</sup> Chart 1 illustrates this problem for the United States: the confidence bands (at a confidence level of 90%) around the estimated r<sup>\*</sup> are very wide (around eight percentage points in total). In addition, recent work demonstrates that the accuracy of the standard model estimates declines significantly when the IS and Phillips curves are relatively flat.<sup>3</sup> The reason is that, when the slopes of both curves are equal to zero, the path of the natural interest rate cannot be identified from the available data, and the model is defined as *unobservable*.<sup>4</sup> In practice, the literature estimates of the slopes of the IS and Phillips curves often tend to be near zero, which gives rise to inaccurate estimates of r<sup>\*</sup>.

A recent study by Fiorentini, Galesi, Pérez-Quirós and Sentana proposes an alternative methodology for measuring r\* more accurately, based on a model which decomposes the observed real rate of interest as the sum of a transitory component and a permanent one, where the second component is identified as the natural rate of interest (such that the transitory component is the gap between the observed and natural rates of interest).<sup>5</sup> The underlying idea in this approach is that the natural rate of interest is that which would prevail in a hypothetical situation of the economy in which all temporary shocks are dissipated. This methodology allows more reliable estimates of r<sup>\*</sup>, even when the IS and Phillips curves are flat.

It can be seen in Chart 2 that the estimates of this model, obtained from annual data for the period 1891-2018 in 17 advanced economies show a decrease in the average natural rate of interest from the beginning of the 20th century to the decade starting in 1960. Subsequently, from the mid-1970s there is an increase which lasts until the end of the 1980s. This was followed by a gradual fall from the early 1990s which has continued almost uninterruptedly to the present, so that an average natural rate of interest for the economies analysed now seems to be negative.

What factors explain the rise and subsequent fall in the natural rate of interest? Fiorentini *et al.* estimate an error correction model with panel data which postulates a long-term relationship between the observed real rate of interest and a set of indicators of the historical behaviour of the main theoretical determinants of  $r^*$ : i) changes in productivity growth, which affect the propensity to invest; ii) demographic changes, which affect the aggregate propensity to save or the labour force participation rate of the economy, and iii) risk factors, which may increase

- 4 See R. E. Kalman (1960), "On the general theory of control systems", Proc. First International Congress on Automatic Control, Moscow.
- 5 See Fiorentini et al. (2018), op. cit.

<sup>1</sup> See T. Laubach and J. C. Williams (2003), "Measuring the natural rate of interest", *Review of Economics and Statistics*, No 85, pp. 1063-1070, and K. Holston, T. Laubach and J. C. Williams (2017), "Measuring the natural rate of interest: international trends and determinants", *Journal of International Economics*, No 108, pp. 59-S75. Also, the model assumes that r\* is the sum of two non-stationary unobserved components: the trend growth of the economy and a second component unrelated to trend growth.

<sup>2</sup> See, for example, T. E. Clark and S. Kozicki (2005), "Estimating equilibrium real interest rates in real time", The North American Journal of Economics and Finance, No 16, pp. 395-413, and R. C. M. Beyer and V. Wieland (2017), Instability, imprecision, and inconsistent use of equilibrium real interest rate estimates, Institute for Monetary and Financial Stability, Working Paper Series 110, Goethe University Frankfurt.

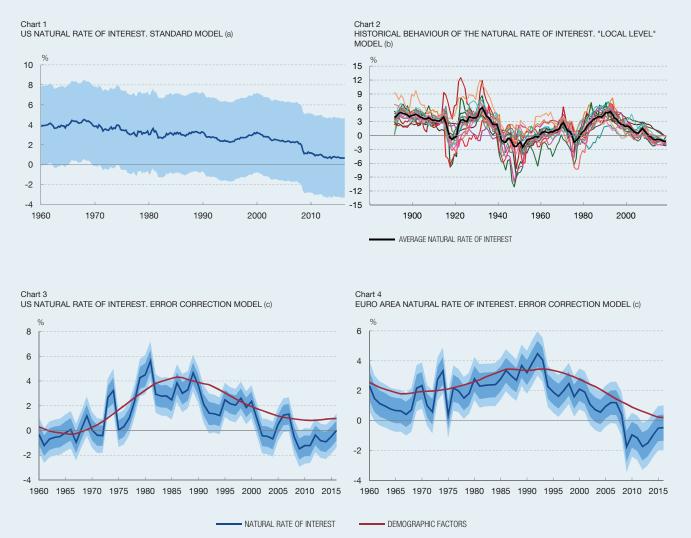
<sup>3</sup> See G. Fiorentini, A. Galesi, G. Pérez-Quirós and E. Sentana (2018), *The rise and fall of the natural interest rate*, Working Paper 1822, Banco de España.

## Box 3.1 THE ESTIMATED NATURAL RATE OF INTEREST (cont'd)

uncertainty and thus affect  $r^{\star}$  by changing the propensity to save.  $^{\rm 6}$ 

This panel model predicts an increase and subsequent fall in the natural rate of interest from the 1960s, as can

be seen in Charts 3 and 4 for the United States and the euro area, respectively. Upon decomposing the individual contribution of each factor, the demographic change proves to be most significant in explaining the rise and fall in the natural rate of interest. However, the



The charts show the natural rate of interest for the advanced economies using various econometric models.

SOURCES: Banco de España, based on the model of Holston, Laubach and Williams (2017), and Fiorentini, Galesi, Pérez-Quirós and Sentana (2018).

a The bands indicate a confidence level of 90%.

b The black line is the average of the countries.

c The bands indicate confidence levels of 68% and 90%.

<sup>6</sup> For studies which relate the natural rate of interest to productivity growth, demographics and risk factors, see, respectively, R. J. Gordon (2015), "Secular stagnation: a supply-side view", *American Economic Review*, No 105, pp. 54-59, and G. B. Eggertsson and N. R. Mehrotra (2014), *A model of secular stagnation*, National Bureau of Economic Research Working Paper 20574, and E. Fahri and F. Gourio (2018), "Accounting for macro-finance trends: market power, intangibles, and risk premia", mimeo

## Box 3.1 THE ESTIMATED NATURAL RATE OF INTEREST (cont'd)

other factors have also contributed persistently to the decrease observed in r\* since the 1990s. These results suggest that the initial increase is due to the population growth prompted by the post-war baby boom, which brought a significant, albeit temporary, increase in the participation of young people in the labour market.

When the baby boom ended, the ensuing gradual population ageing seems to have pushed the natural rate of interest down in both economies. This result is consistent with recent studies which have emphasised the role of demographics in explaining the behaviour of the natural interest rate.<sup>7</sup>

<sup>7</sup> See, for example, E. Gagnon, B. K. Johannsen and D. López-Salido (2016), *Understanding the new normal: the role of demographics,* Finance and Economics Discussion Series 2016-080, Board of Governors of the Federal Reserve System.