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A CLASS OF DIAGNOSTICS IN THE ARIMA-MODEL-BASED DECOMPOSITION OF A TIME SERIES.

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Abstract

It is shown how, in the ARIMA-model-based approach to seasonal adjustment (as enforced in program SEATS), it is straightforward to derive tests based on the joint distribution of the optimal estimators. These tests can address a wide range of issues and, in particular, translate standard X12 ARIMA tests into the model-based framework. The diagnostics obtained with the two approaches provide, however, different information: while X12 ARIMA yields a quality assessment check, SEATS provides a model specification test.

Keywords: Time series; ARIMA models; Seasonal Adjustment;
Model-based signal extraction; Specification tests.

JEL:

1. INTRODUCTION

At present, the most widely used methods for seasonal adjustment are X12(or X11)-ARIMA (thereafter, X12A; see Findley et al, 1998), and TRAMO/SEATS (see Gómez and Maravall, 1996). The former emerged from the X11 filters, which have provided for nearly four decades an evolving paradigm. TRAMO/SEATS originated from the ARIMA-model-based (AMB) approach to seasonal adjustment of Burman (1980), Hillmer and Tiao (1982), and others.

Over the years, X12A has incorporated an important set of diagnostics and quality assessment checks, often based on prior beliefs on how “reasonable” components (such as the seasonal, trend-cycle or irregular ones) should behave. But, despite the fact that one of the large potentials of the model-based approach is the possibility of using parametric statistical inference to derive statistical tests in a straightforward manner and, despite a considerable amount of methodological work, this potential has not yet been developed. (Some references of this work are Bell, 1995; Bell and Hillmer, 1984; Box, Pierce and Newbold, 1987; Burridge and Wallis, 1985, 1984; Cleveland and Pierce, 1981; Cleveland and Tiao, 1976; Hillmer, 1985; Maravall, 1987, 1995; Maravall and Planas, 1999; Pierce, 1980, 1979; Tiao and Hillmer, 1978.)

In this paper I focus on tests derived from the joint distribution of the optimal (Minimum Mean Square Error, or MMSE) estimators of the unobserved components. Knowledge of this joint distribution (derived simply from the ARIMA model for the observed series) provides a powerful tool for inference, and it is seen how a large variety of tests can be obtained, some of which are closely related to X12A diagnostics. Yet the information provided by the two approaches is notably different. While X12A provides a quality assessment of the estimated component (is seasonality reasonably stable? is the trend contaminating the irregular? …), the SEATS diagnostics provide specification-type test (is the sample estimate of the variance of the seasonal component in agreement with the theoretical variance of the MMSE estimators? Are the sample autocorrelation estimates in agreement with these implied by the estimator model? Are empirical revisions too large for what they should theoretically be?) Viewed in this way, the information given by the two approaches can be seen as complementary. In the AMB approach, the failure of some test is likely to point out some weak part of the model, and hence the test can be of help in finding an improved specification.

2. THE MOTIVATION; AN INTRODUCTORY EXAMPLE

Some years back, when seasonally adjusting a component of a monetary aggregate (Maravall, 1987), an X11ARIMA (Dagum, 1980) **quality assessment** check said: “TOO MUCH AUTOCORRELATION IN THE IRREGULAR COMPONENT”. I asked myself: Why is it too much? How much should there be? After all, different series yield X11A-irregulars with different autocorrelation. When is the autocorrelation too much? What the diagnostic meant was that, although we do not know how much autocorrelation should be present in the irregular estimator, we do not like an irregular with that amount of correlation. It is more a Quality Assessment check (which can certainly be useful), but not a statistical test. This remark also characterizes other X12A diagnostic checks, such as, for example, the “sliding spans” diagnostics (Findley et al, 1990).

These quality assessment checks can also be applied in an identical manner to the estimates obtained with the ARIMA-model-based procedure of SEATS (and, in fact, work is being presently done at the US Bureau of the Census in this direction.) But there is also another important application that emerges from translating these checks into statistical tests, by means of the model-based structure. The (asymptotic) joint distribution of the estimators can be derived, and for example, it is possible to find the theoretical autocorrelation for the irregular estimator, from which the question: “how much is *too much*?” can be answered.

3. ONE TYPE OF MODEL-BASED TESTS (OR DIAGNOSTICS)

Assume that the variable x_t follows the ARIMA model

$$\phi(B)\ \delta(B)\ x_t = \theta(B)\ a_t \ , \tag{1}$$

where a_t is a white-noise (w.n.) variable (i.e., a $\text{niid}(0, V_a)$ variable), $\phi(B)$ denotes the stationary AutoRegressive (AR) polynomial in the backward operator B, $\delta(B)$ denotes the nonstationary polynomial containing the unit roots (or differences, such as for example, $\delta(B) = \nabla\nabla_{12}$), and $\theta(B)$ is the invertible Moving Average (MA) polynomial in B (see Box and Jenkins, 1970). To standardize units, I set $V_a = 1$.

Let x_t be the sum of two uncorrelated unobserved components (UC)

$$x_t = s_t + n_t \ , \tag{2}$$

(as in, for example, x_t = Seasonal Component + Seasonally Adjusted (SA) series.) Finding the roots of the AR polynomials and assigning them to s_t or n_t according to their associated frequency, $\delta(B)$ and $\phi(B)$ can be factorized as

$$\delta(B) = \delta_s(B)\ \delta_n(B)$$
$$\phi(B) = \phi_s(B)\ \phi_n(B)$$

(For example, if $\delta(B) = \nabla\nabla_{12} = \nabla^2 S$, where $S = 1 + B + B^2 + ... + B^{11}$ contains the seasonal unit roots, then, $\delta_s(B) = S$, $\delta_n(B) = \nabla^2$.) Models for the UC are derived in SEATS, that are of the type:

$$\phi_s(B)\ \delta_s(B)\ s_t = \theta_s(B)\ a_{st} \ , \tag{3}$$

$$\phi_n(B)\ \delta_n(B)\ n_t = \theta_n(B)\ a_{nt} \ , \tag{4}$$

where a_{st} and a_{nt} are mutually uncorrelated UC innovations, with zero mean and variances equal to V_s and V_n, respectively. The MA polynomials $\theta_s(B)$, $\theta_n(B)$, as well as the variances V_s and V_n, are determined from the identity

$$\theta(B)\ a_t = \theta_s(B)\ \phi_n(B)\ \delta_n(B)\ a_{st} + \theta_n(B)\ \phi_s(B)\ \delta_s(B)\ a_{nt}$$

together with some additional assumptions (see Maravall, 1995). The previous identity assures consistency between the model for the observed series and the ones for the components; the rest of the discussion is valid for any admissible decomposition, independently of the identification assumptions.

Expressions (3) and (4) provide models for the theoretical components. These components are never observed and will be estimated by their MMSE estimators. For the case of s_t, and assuming a doubly infinite realization, the estimator is given by

$$\hat{s}_t = \nu_s(B,F)\ x_t \quad , \qquad (F = B^{-1}) \ ,$$

where $\nu_s(B,F)$ is the Wiener-Kolmogorov (WK) filter. In a symbolic manner, the filter can be obtained from

$$\nu_s(B,F) = V_s\ \frac{\psi_s(B)\ \psi_s(F)}{\psi(B)\ \psi(F)}\ x_t \ ,$$

where $\psi_{\cdot}(B) = \dfrac{\theta_{\cdot}(B)}{\phi_{\cdot}(B)}$. Replacing x_t by $\psi(B)\ a_t$, after cancellation of terms, it is obtained:

$$\delta_s(B)\ \hat{s}_t = V_s \left[\frac{\theta_s(B)}{\phi_s(B)}\ \frac{\theta_s(F)\ \delta_n(F)\ \phi_n(F)}{\theta(F)} \right] a_t \ , \qquad (5)$$

where the left-hand-side (l.h.s.) is the stationary transformation of the component, and the r.h.s. is an MA (convergent in B and in F) which can be easily obtained. Writing, in compact form,

$$\boxed{\delta_s (B)\, \hat{s}_t = \eta_s (B)\, a_t} \tag{6}$$

where a_t is, as before, the innovation in the observed series, $\eta_s (B)\, a_t$ is a stationary, zero mean process, and its variance and autocorrelations can be computed. In this way, the variance and autocorrelations of the (theoretical) MMSE estimator are obtained. They will be different from the variance and autocorrelations of the theoretical component, given by (3).

In summary, in the AMB approach (SEATS), we know the asymptotic distribution of (the stationary transformation) of the optimal estimator, and we can use this for testing. (A similar conclusion holds for $\delta_n (B)\, \hat{n}_t$.)

Back to the example that gave the message: “Too much autocorrelation in the irregular component”, assume the irregular component u_t is w.n. $(0,V_u)$. Letting s_t become u_t, expression (5) implies that the estimator $\hat{u}_t$ has the AutoCorrelation Function (ACF) of the model

$$\theta(B)\, y_t = \phi(B)\, \delta(B)\, a_t^* \quad , \tag{7}$$

where a_t^* is a $w.n.(0,V_u)$ variable. (Model (7) is the “Inverse Model” of model (1).) Hence, having the model for x_t, we know the ACF of the irregular component estimator.

Assume we have to treat many, many series and that no automatic model identification procedure such as TRAMO is available. We decide thus to use an “ad-hoc” fixed filter, namely the WK filter associated with the model

$$\nabla\nabla_{12}\, x_t = (1-.4B)\,(1-.6B^{12})\, a_t \;, \qquad (8)$$

for all series. (This model is the original Airline model of Box-Jenkins, 1970.) Cleveland-Tiao (1976) showed that this model yields filters not far from those of the default application of X11. Besides, it seems safe to assert that it is not far from what could be the "mode model" of a hypothetical overall empirical distribution of models.

Consider one of the treated series, with, say, 144 monthly observations we look at the **ACTUAL ESTIMATE** OF THE IRREGULAR, and compute the first-order sample autocorrelation

$$\hat{\rho}_1\,(\hat{u}_t) = .32$$

(having one year removed at both ends to attenuate end-point effects.) Is .32 "too much AC"?

The AMB answer is straightforward. For model (8), the ACF of $\hat{u}_t$ is that of the model:

$$(1-.4B)\,(1-.6B^{12})\; y_t = (1-B)\,(1-B^{12})\; a_t^{*} \;,$$

so that the theoretical ρ_1 of $\hat{u}_t$ is

$$\rho_1\,(\hat{u}_t) = -.30 \;.$$

The empirical and theoretical values ($\hat{\rho}_1$ = .32 and ρ_1 = -.30, respectively) seem far, but we need a measure of distance. From Bartlett's approximation (see Box and Jenkins, 1971; Priestley, 1981),

$$V(\hat{\rho}_k) \doteq \frac{1}{T} \sum_{j=-m}^{m} \left[\rho_j^2 + \rho_{j+k}\;\rho_{j-k} + 2\,\rho_k^2\;\rho_j^2 - 4\,\rho_k\;\rho_j\;\rho_{j-k}\right] \;,$$

where all ρ 's refer to $\hat{u}_t$, and the ones in the r.h.s. are those in the theoretical ACF of $\hat{u}_t$; m denotes the truncation point, such that $\rho_j \approx 0$ for $|j| > m$.

For our example, it is obtained that SD $(\hat{\rho}_1)$ = .074 , thus a 95% Confidence Interval around $\hat{\rho}_1$, equal to $[\ \hat{\rho}_1 \pm 2\ \text{SD}\ (\hat{\rho}_1)\ ]$, yields the interval [.17, .47]. Clearly, ρ_1 = -.30 is not inside the interval, and hence we can conclude "THERE IS TOO MUCH (POSITIVE) AC IN THE IRREGULAR COMPONENT".

This (unacceptable) positive autocorrelation in the irregular component could be due to misspecification of the trend-cycle component. In fact, for this series, a (0, 2, 2) (0, 1, 1) $_{12}$ model performs considerably better. The irregular autocorrelations become $\rho_1(\hat{u}_t) = -.83$ and $\hat{\rho}_1(\hat{u}_t) = -.82$, with $\text{SD}(\hat{\rho}_1) = .04$. The value $\rho_1 = -.84$ now lies comfortably in the 95% confidence interval [-.74 , -.90] .

3. TESTING VERSUS QUALITY ASSESSMENT

In the previous example, the theoretical ρ_1 was ρ_1= -.83, which implies that $\hat{u}_t$ has important autocorrelation for the 6-times-a-year seasonal frequency, a somewhat "ugly" feature for an irregular component.

What the AMB test says is, in essence, "your empirical results are in agreement with your model". Thus it is basically a SPECIFICATION TEST.

Of course, the data and the model can be in agreement, but perhaps the model has an ugly decomposition (a clearly stationary seasonal component would provide a good example of an ugly seasonal component). Therefore, quality assessments of the X12 ARIMA type can be of help in this respect: they can, for example, discriminate among models that offer nice or nasty decomposition.

4. TESTS BASED ON THE DISTRIBUTION OF THE ESTIMATORS

The previous example provided a test for the first-order autocorrelation of the irregular component estimate. This test was based on the distribution of the stationary transformation of the estimator.

Extensions are straightforward. For example, we can compare $\rho_{12}(\hat{u}_t)$ with $\hat{\rho}_{12}(\hat{u}_t)$, test for $H_0: \ \hat{\rho}_{12} = \rho_{12}$, and, for example, when $\hat{\rho}_{12}$ is significantly larger than ρ_{12}, conclude that: "THERE IS TOO MUCH SEASONALITY IN THE IRREGULAR". The test can also be extended to other components such as, for example, the SA series (say $\hat{n}_t$). From the model for $\hat{n}_t^{ST} = \delta_n(B)\,\hat{n}_t$, we derive $\rho_{12}(\hat{n}_t^{ST})$. From the actual estimate, we compute the empirical autocorrelation $\hat{\rho}_{12}(\hat{n}_t^{ST})$. Using Bartlett's approximations, we obtain SD $(\hat{\rho}_{12})$. Again, we can test for $H_0: \ \hat{\rho}_{12} = \rho_{12}$ and, if $\hat{\rho}_{12}$ is significantly larger than ρ_{12}, conclude that: "THERE IS TOO MUCH SEASONAL AUTOCORRELATION IN THE SA SERIES" .

We can also think of more general tests. For example, a test for **UNDER/OVERESTIMATION OF SEASONALITY** (or under/over-adjustment) .

A given filter may, on occasion, capture too much variation as seasonal, or not capture all seasonal variation. This will happen when the widths of the filter squared gain peaks for the seasonal component (or dips for the SA series) are too wide or too narrow, when compared to the seasonality actually present in the series. To see what goes on, consider for example the (default) model

$$\nabla \nabla_{12} \ x_t = (1 + \theta_1 B)(1 + \theta_{12} B^{12})\, a_t \ , \qquad (9)$$

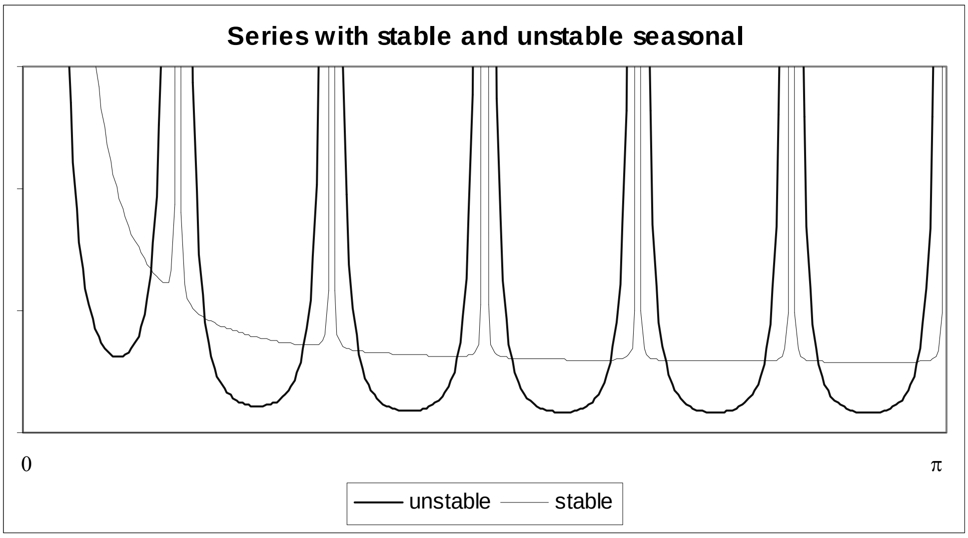
As $\theta_{12} \rightarrow -1$, in the limit, the model becomes

$$\nabla x_t = \sum_{i=1}^{11} \beta_i \, d_{it} + (1 + \theta_1 B) a_t + \mu \qquad ,$$

where d_{it} represents the seasonal dummies, and hence the series displays perfectly stable seasonality. In general, values $\theta_{12} \cong -1$ will produce stable seasonality, while values of θ_{12} far from -1 will produce unstable (highly moving) one. We set $V_a = 1$, $\theta_1 = -.6$, and consider two values for θ_{12}:

- $\theta_{12} = 0 \quad \Rightarrow$ unstable seasonality.
- $\theta_{12} = -.9 \quad \Rightarrow$ stable seasonality.

Comparison of the two spectra evidences the difference in width of the seasonal peaks.



Denote by $\nu(B,F)$ the fixed (ad-hoc) filter to estimate the seasonal component (or SA series). Set, for example, $\nu(B,F)$ = (linear) X11 default filter, which we represent as

$$\nu(B,F) = \nu_0 + \sum_{j=1}^{k} \nu_j (B^j + F^j) \; .$$

The Fourier Transform (F.T.) of the filter is called the filter gain, equal to

$$G(\omega) = \nu_0 + 2 \sum_{j=1}^{k} \nu_j \cos(j\omega) \ ,$$

where ω is the frequency measured in radians and $0 \leq \omega \leq \pi$; the squared gain of the filter is given by

$$SG(\omega) = [\, G(\omega) \,]^2 \ .$$

Let $g_y(\omega)$ denote the spectrum (or pseudo-spectrum) of the variable y_t. From $\hat{s}_t = \nu(B,F)\, x_t$, one obtains

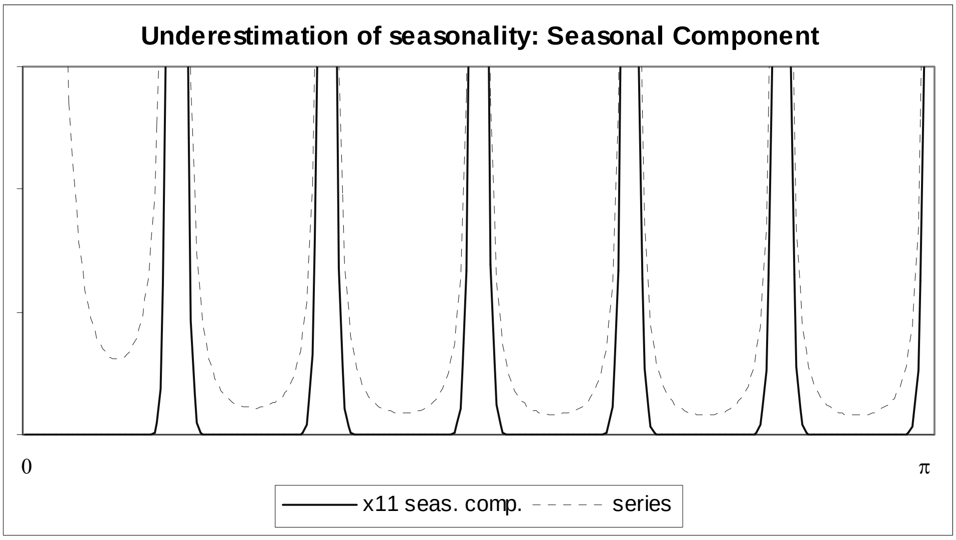
$$g_{\hat{s}}(\omega) = SG_s(\omega)\, g_x(\omega) \quad , \tag{10}$$

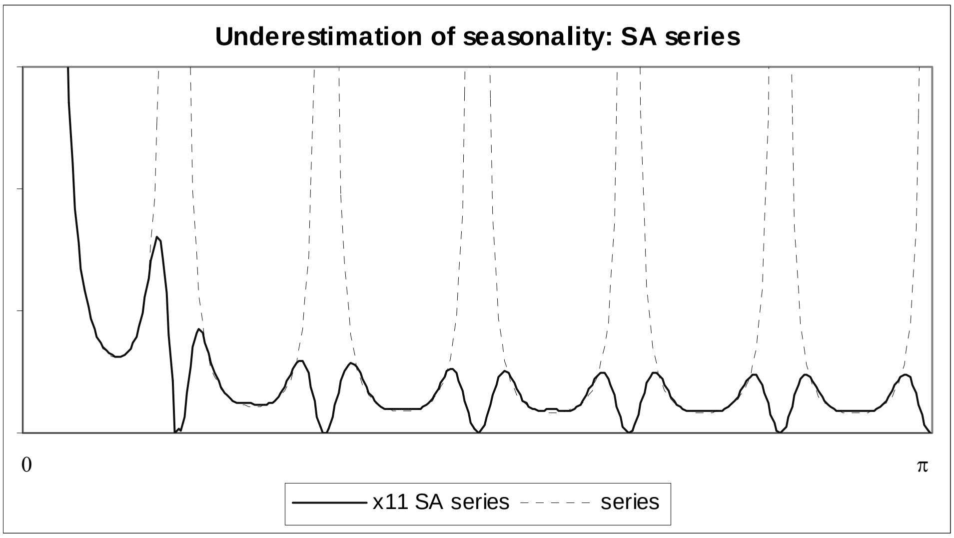
Likewise, for the SA series (n_t)

$$g_{\hat{n}}(\omega) = SG_n(\omega)\, g_x(\omega) \ . \tag{11}$$

The functions $g_{\hat{s}}(\omega)$ and $g_{\hat{n}}(\omega)$ are the spectra of the estimators of the seasonal component and SA series. We can compare them to the spectrum of the series.

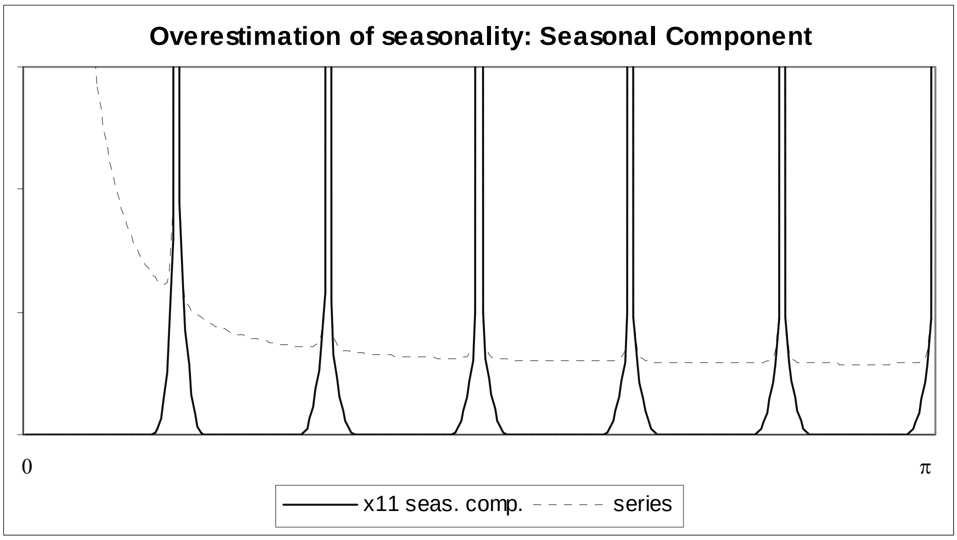
a) Unstable seasonality and underestimation

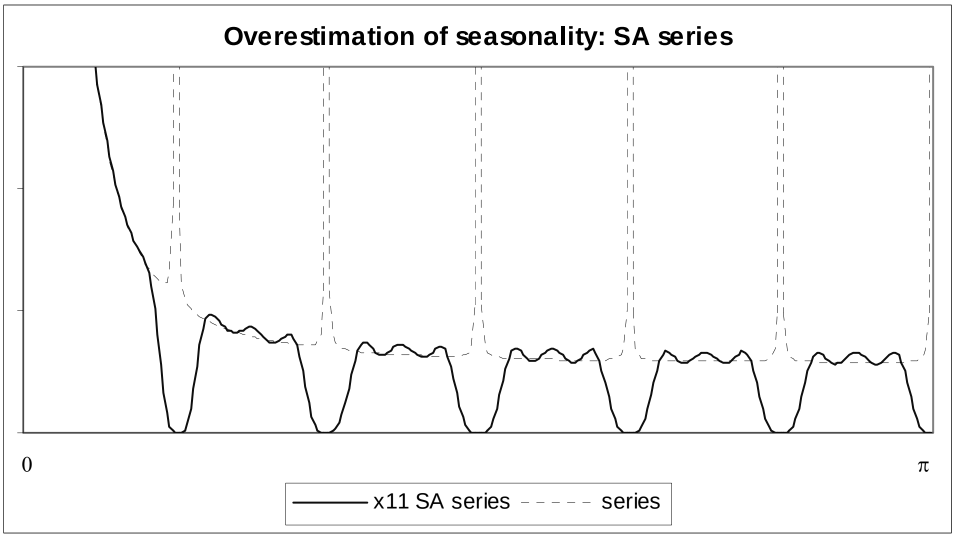




Residual seasonality (evidenced by peaks in the neighborhood of the seasonal frequencies) shows up - in an ackward manner - in the SA series.

b) Stable seasonality and overestimation





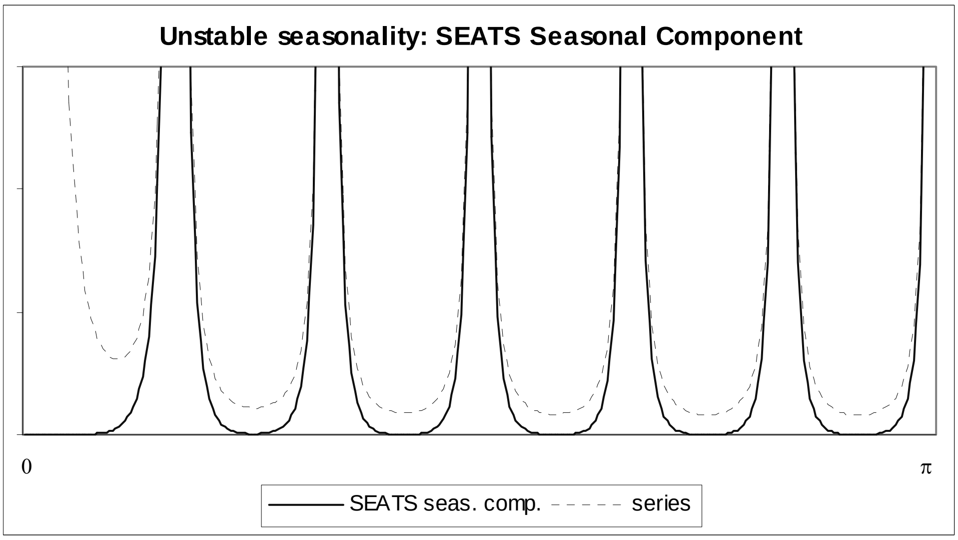
Holes induced in the SA series are clearly too wide.

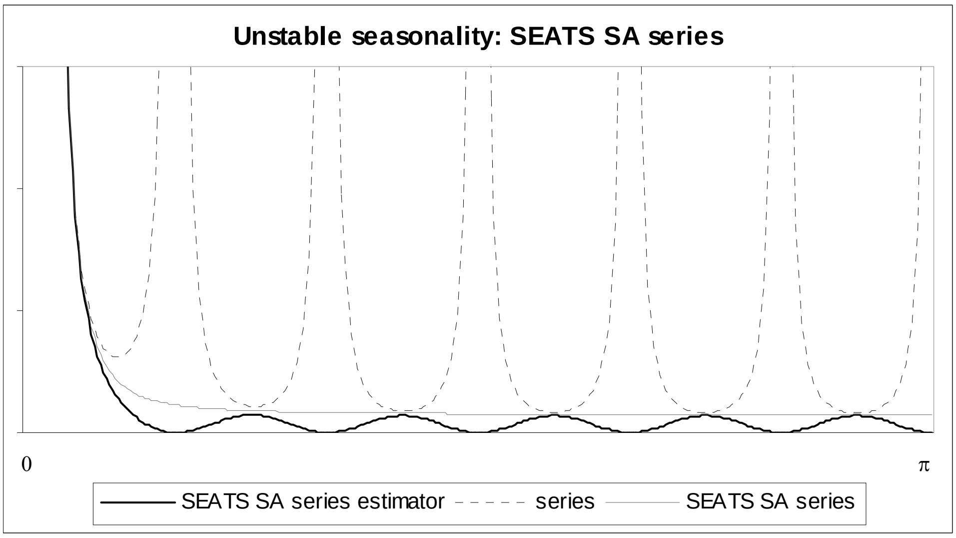
Using, on the two series, SEATS by default (in which case, the orders of the model are those of the Airline model), estimation of the parameters allows the WK-filter $\nu_s(B,F)$ to adapt to the particular stochastic structure of the series. The F.T. of $\nu_s(B,F)$ is the Gain:

$$G_s(\omega) = g_s(\omega) / g_x(\omega)$$

and, as before, the spectra of the estimators of the seasonal component and SA series are given by (10) and (11). Comparing these two spectra to the one of the series:

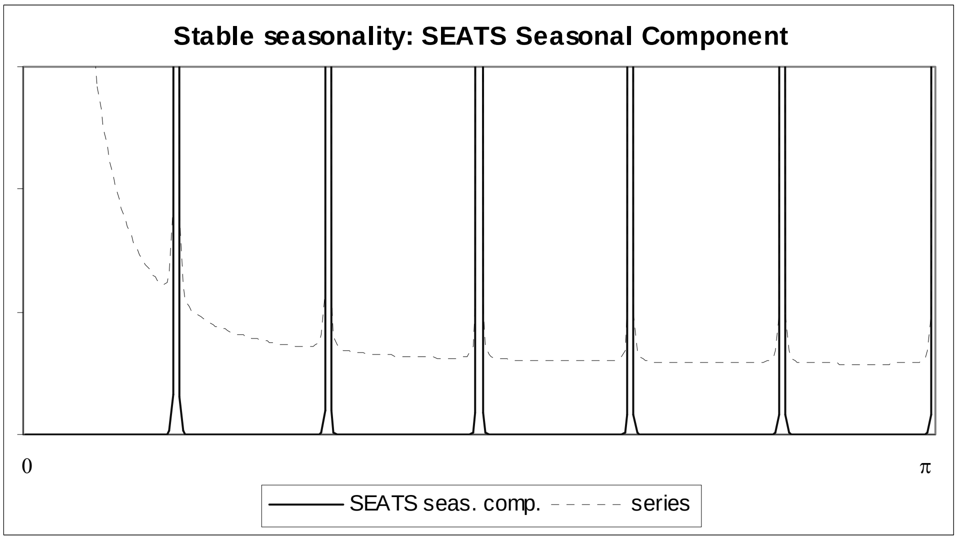
a) Unstable seasonality case (SEATS)

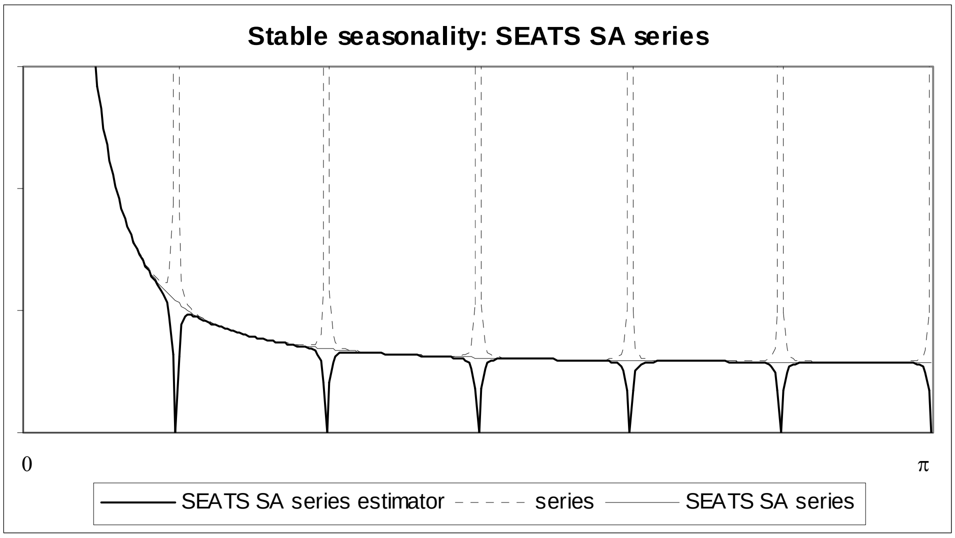




It is seen that the large peaks in the neighborhood of the seasonal frequencies have disappeared.

b) Stable seasonality case (SEATS)

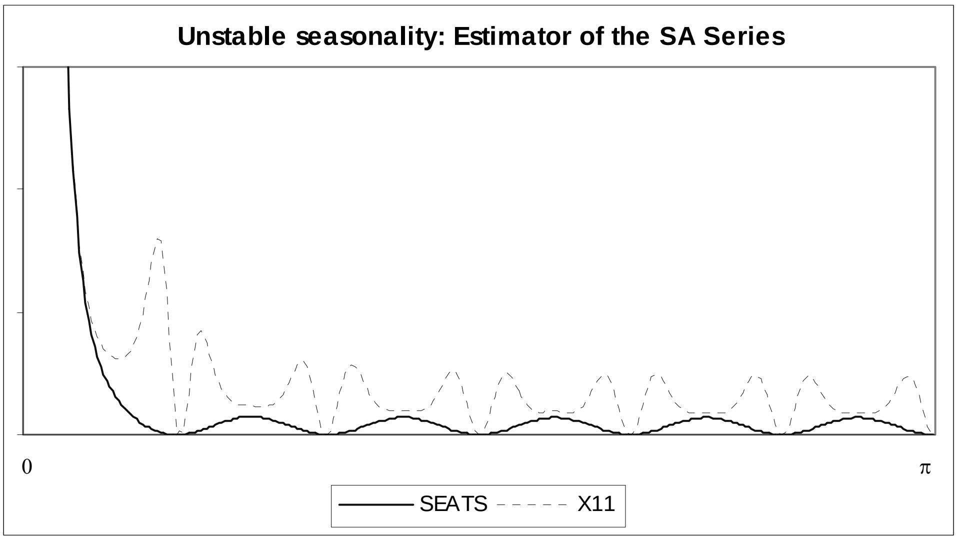


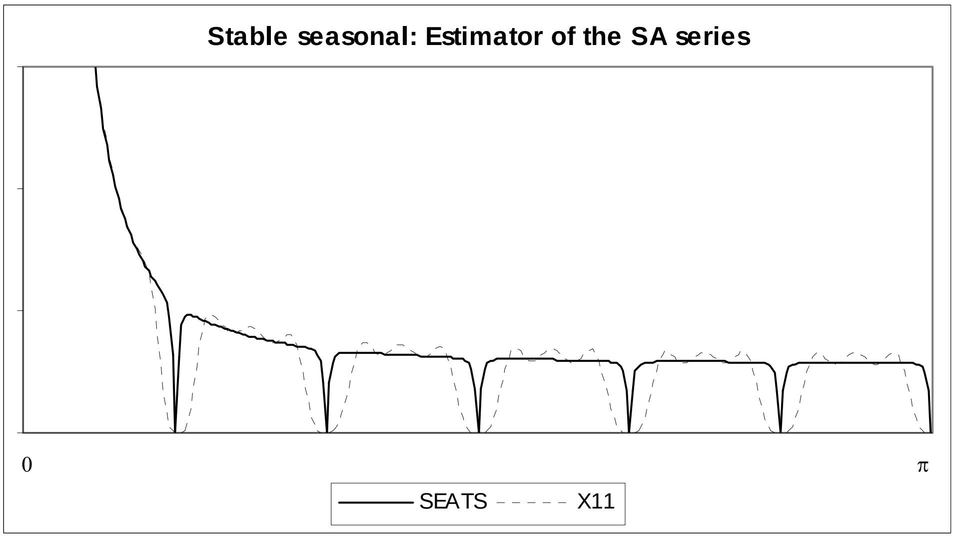


The estimator adapts to the structure of the series so that the width of the spectral holes in the SA series are determined from the width of the seasonal peaks in the series spectrum.

TESTING FOR UNDER/OVER ADJUSTMENT

If we compare the spectra of the estimators of the SA series in the previous two examples, it is seen how underestimation of seasonality implies excess variance in the SA series, while overestimation of seasonality implies that the variance of the SA series is too small.





The example illustrates the general fact that, when the seasonality present in the series is very stable, with little stochastic variation, the filter may OVERADJUST (remove too much variation as seasonal). On the contrary, when seasonality is unstable, and the series presents highly stochastic seasonal variation, the filter may UNDERADJUST (not remove all seasonal variation). In other words, underadjustment implies underestimation of the seasonal component variance, while overadjustment implies overestimation of the seasonal component variance; they are, thus, the result of under/over estimation of a component variance.

Proceeding as before, from model (5) for the estimator $\hat{s}_t$, we know the theoretical value of the variance of the stationary transformation of the component estimator, $V_{\hat{s}}$. We can use again Bartlett's approximation to compute the standard deviation of this variance estimator

$$\mathrm{SD}\ (\hat{V}_{\hat{s}}) = V_{\hat{s}} \left[\frac{2}{T} \left(1 + 2 \sum_{j=1}^{m} \rho_j^2 \right) \right]^{\frac{1}{2}} ,$$

where ρ_j are the autocorrelations implied by model (5). Then, we can test for whether $H_0 :\ \hat{V} = V$ is rejected or not. If, for example, s_t = seasonal component, and we have obtained

$$V_{\hat{s}} = .067 , \qquad \hat{V}_{\hat{s}} = .100 \qquad (\mathrm{SD} = .010) ,$$

we would conclude that there is "EVIDENCE OF OVERESTIMATION OF SEASONALITY".

One can think of many extensions. An important one is the following. Although the theoretical components are uncorrelated, it is well known (see Nerlove, Grether, and Carvalho, 1979) that MMSE estimation will induce some crosscorrelation between them. From model (5), and the equivalent expression for $\hat{n}_t$, we can obtain the THEORETICAL CROSSCOVARIANCE and

CROSSCORRELATION FUNCTIONS for any pair of estimators (Gómez-Maravall, 2001). Further, the Bartlett's approximations can also be extended to the estimators of these correlations. Therefore, as before, we can for example build a test that may say: "THERE IS TOO MUCH CORRELATION BETWEEN THE SEASONAL AND IRREGULAR ESTIMATORS".

As a last extension, I shall very briefly mention the SLIDING SPANS DIAGNOSTICS included in X12 ARIMA; in short, one finds first the percentage of months with unreliable adjustment. To find if the adjustment for a month is "unreliable", the program adjusts several overlapping subspans of the series, and looks at the variation of the estimated seasonality for that month with the different subspans. When that variation is larger than a preselected amount k, the adjustment for that month is judged "unreliable". If the frequency of unreliable months is more than 25%, the series should not be adjusted.

This check can also be applied, in an identical manner, to SEATS as a "quality" diagnostics in the sense of having a "nice" seasonal. But, given that the variation considered in the diagnostic is related to the revisions in preliminary estimators, in the AMB approach, for any given model, one could compute for example the measure (Maravall, 1998),

$$\mathrm{Prob}\ [\,|\hat{s}_t - \hat{s}_{t|t}| > k\,]$$

where $\hat{s}_t$ = final estimator, and $\hat{s}_{t|t}$ = concurrent estimator of s_t, obtained with the series finishing with x_t. This measure provides the probability that the variation from concurrent to final seasonal component estimator, for any given month, is larger than a threshold level k. Letting d_t denote the revision in the concurrent estimator

$$d_t = \hat{s}_t - \hat{s}_{t|t} \quad ,$$

the model for d_t is easily obtained (Pierce, 1980) and the probability $P(|d_t| > k)$ can be computed. This probability would be the theoretical value associated with the optimal estimator. Given k, $P(|d_t| > k)$ is something fully determined from the ARIMA model for the observed series. For some models the probability will be large (even above .25); for some models, it will be small. Be that as it may, optimal (MMSE) estimation of the components imply that probabilities such as the above take some particular values. Thus, as was the case with the variance and auto/cross-correlations of the estimators, comparison of this “theoretical” value with the frequency detected in the sample can provide an AMB diagnostic. Large differences between this sample frequency and the probability implied by the model, or failure of the sliding-spans diagnostic in X12 ARIMA would mean different things. So to speak, X12A would be telling us: “$\hat{s}_t$ is not nice because it moves too fast”, while failure of the test in the AMB approach would say: “The model specified is not in agreement with the empirical revisions in your concurrent estimates” (again, a specification-type test). Both the X12A and the AMB results provide relevant and different information, that complement each other. However, while a bad diagnostic in X12 ARIMA can be due to the use of an inadequate filter or to the fact that the series is wild beyond hope, in the AMB approach, failure of a test is likely to point out which parts of the model may be misspecified.

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